Analogue gravity
and
Hawking radiation

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LIVING REVIEWS
in relativity

Analogue Gravity

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Laser pulse analogues for gravity and analogue Hawking radiation

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Lecture Notes in Physics 870

Daniele Faccio · Francesco Belgiorno
Sergio Cacciatori · Vittorio Gorini
Stefano Liberati · Ugo Moschella *Editors*

Analogue Gravity Phenomenology

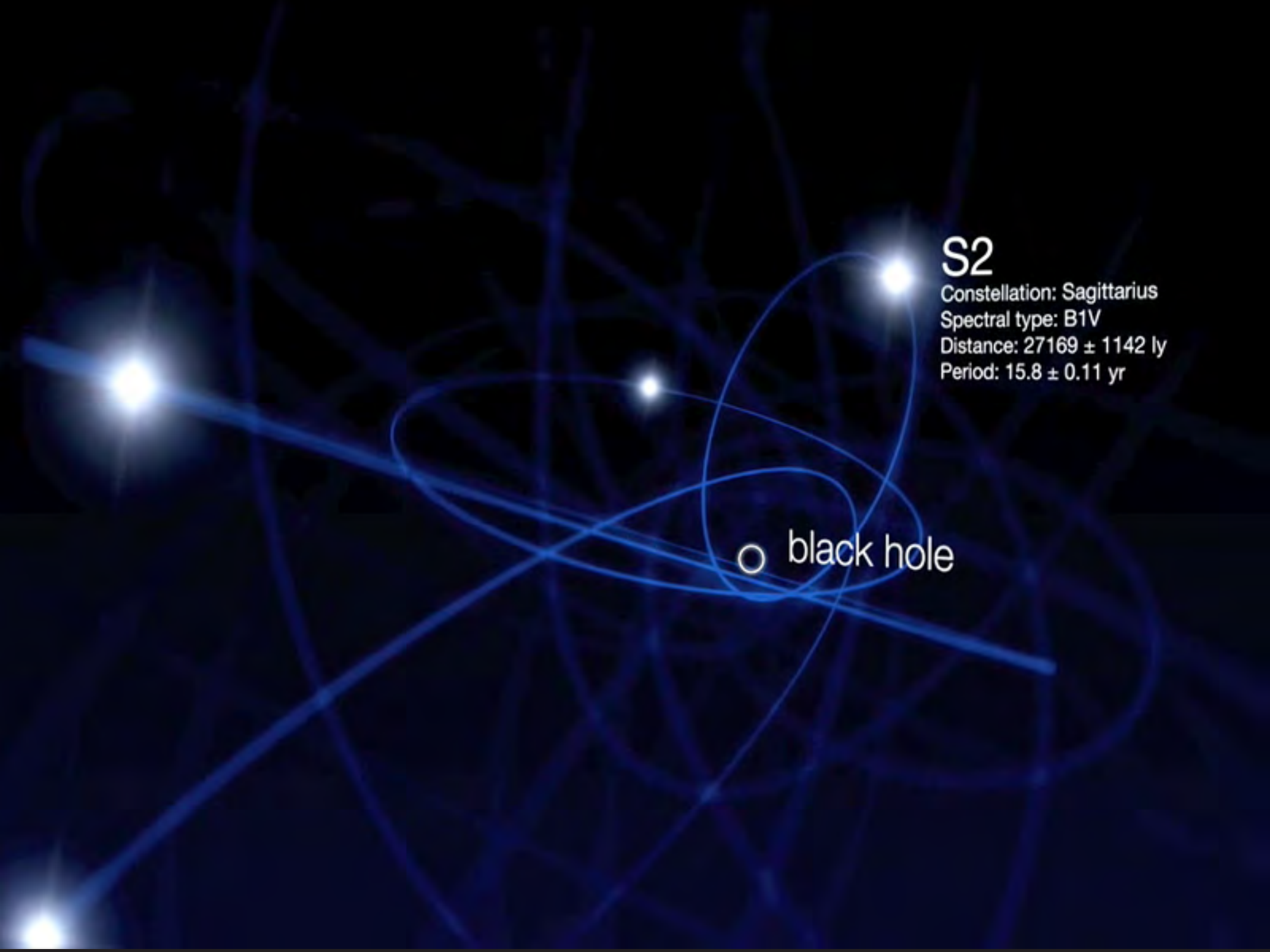
Analogue Spacetimes and Horizons, from Theory to Experiment

Evidence of a Black hole at the centre of our galaxy ?

Evidence of a Black hole at the centre of our galaxy ?

28 stellar orbits around SgrA* monitored over 20 years

ESO VLT array, Chile


S2
Constellation: Sagittarius
Spectral type: B1V
Distance: 27169 ± 1142 ly
Period: 15.8 ± 0.11 yr
black hole

Black holes & Horizons on Earth ?

gravitational black holes & event horizons

- River model for black holes

Painlevé-Gullstrand metric for a BH

$$ds^2 = c^2 dt^2 - (dr - V dt)^2$$

$$V = -\sqrt{\frac{2GM}{r}}$$

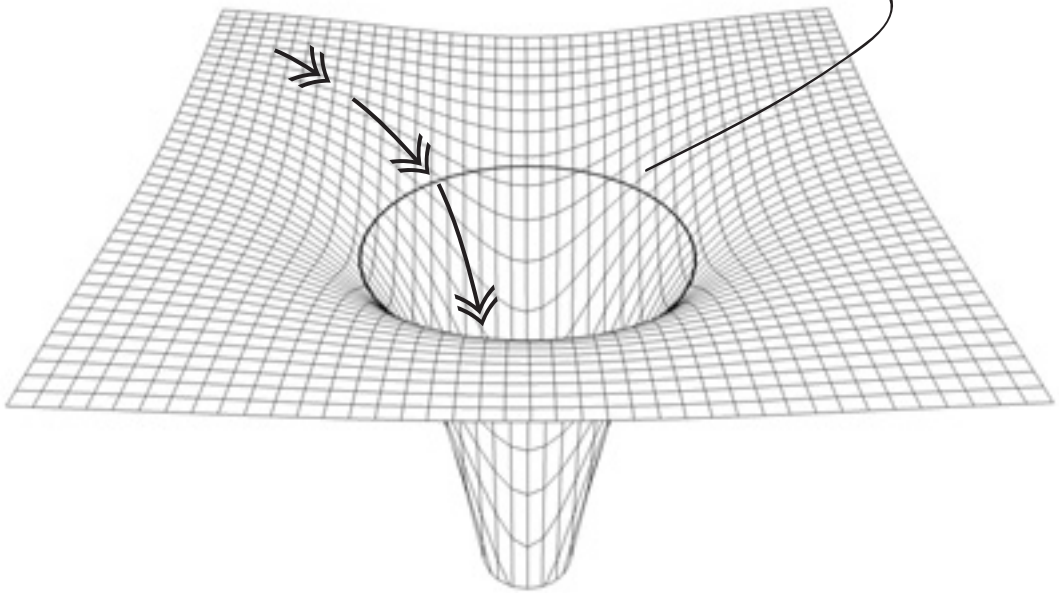
- Space behaves like a flowing river with galilean velocity *V*

At $r_S = \frac{2GM}{c^2}$ *V* equals speed of light -> point of no return

gravitational black holes & event horizons

Black hole

$$V = -\sqrt{\frac{2GM}{r}} = c$$

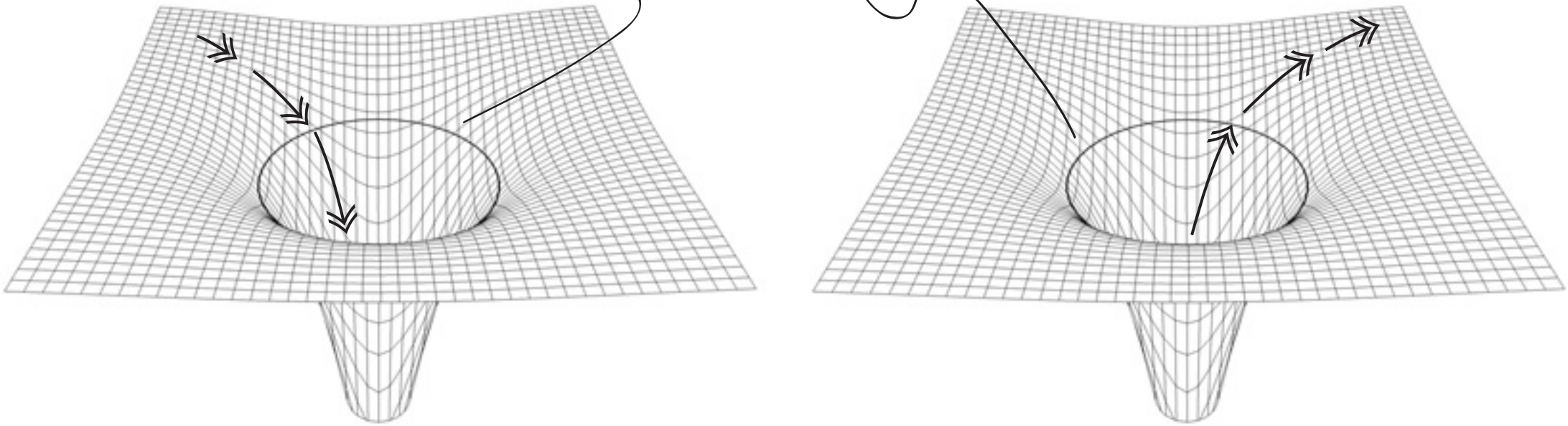


gravitational black holes & event horizons

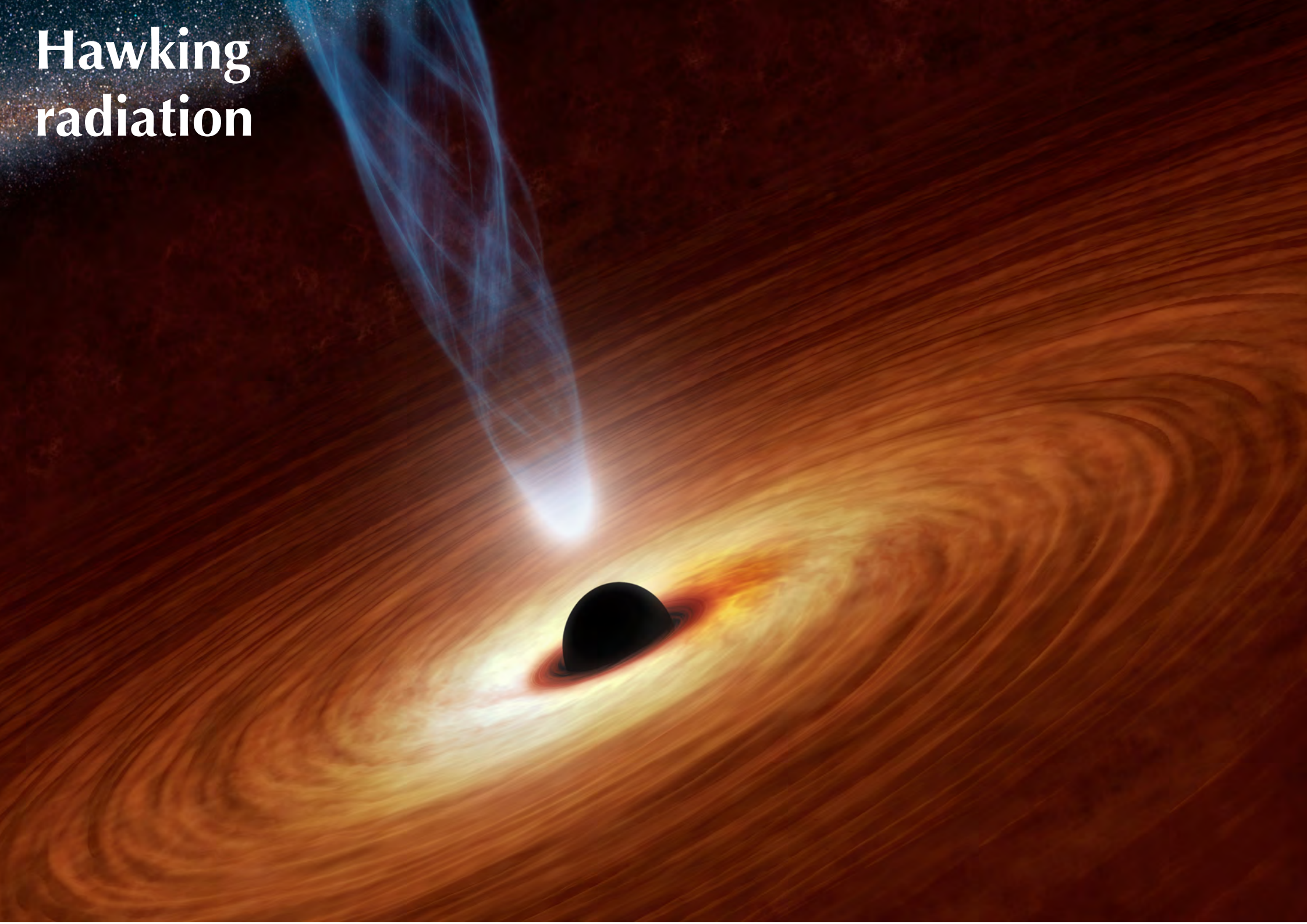
Black hole

White hole

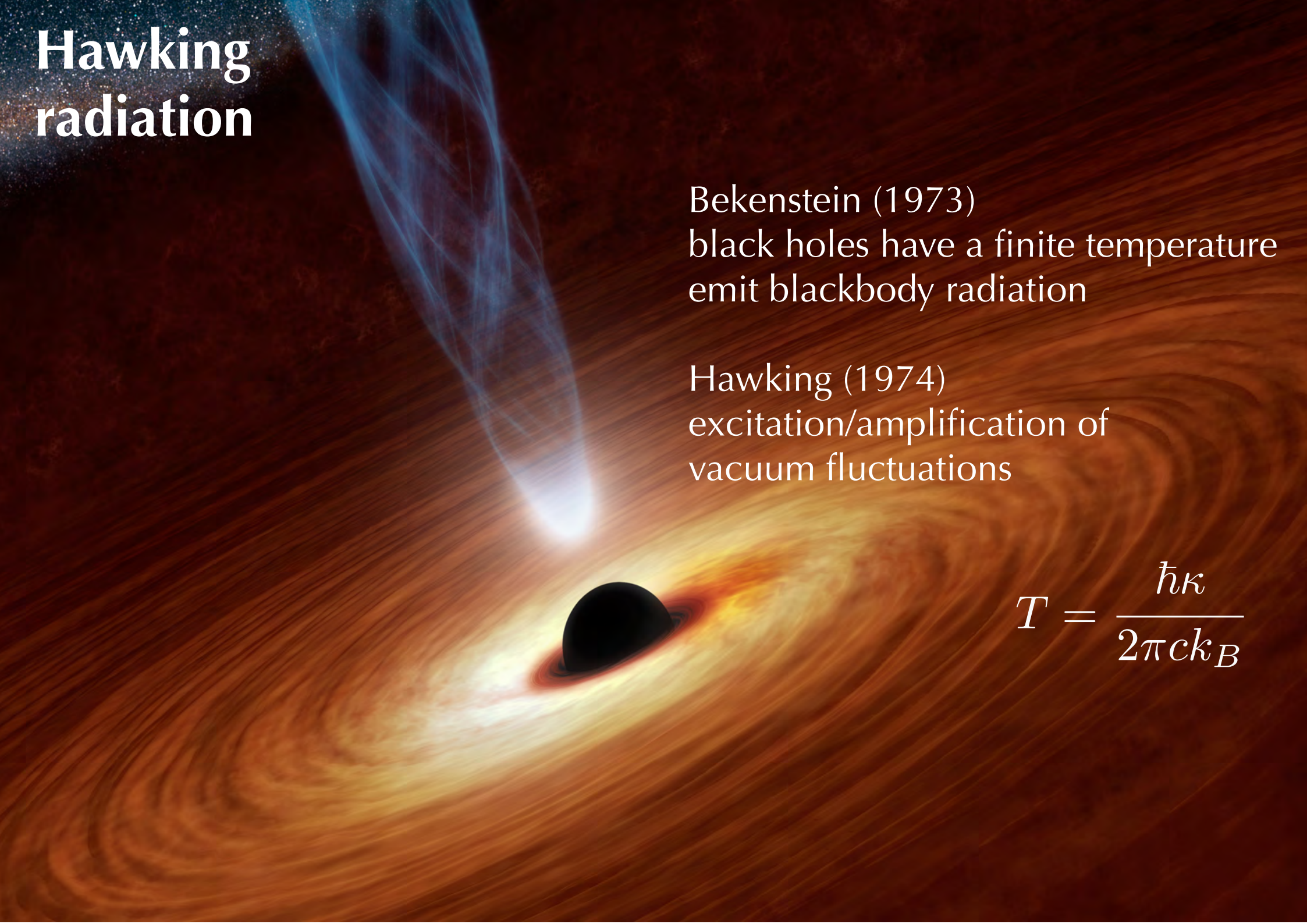
$$V = -\sqrt{\frac{2GM}{r}} = c$$



Hawking radiation



Hawking radiation

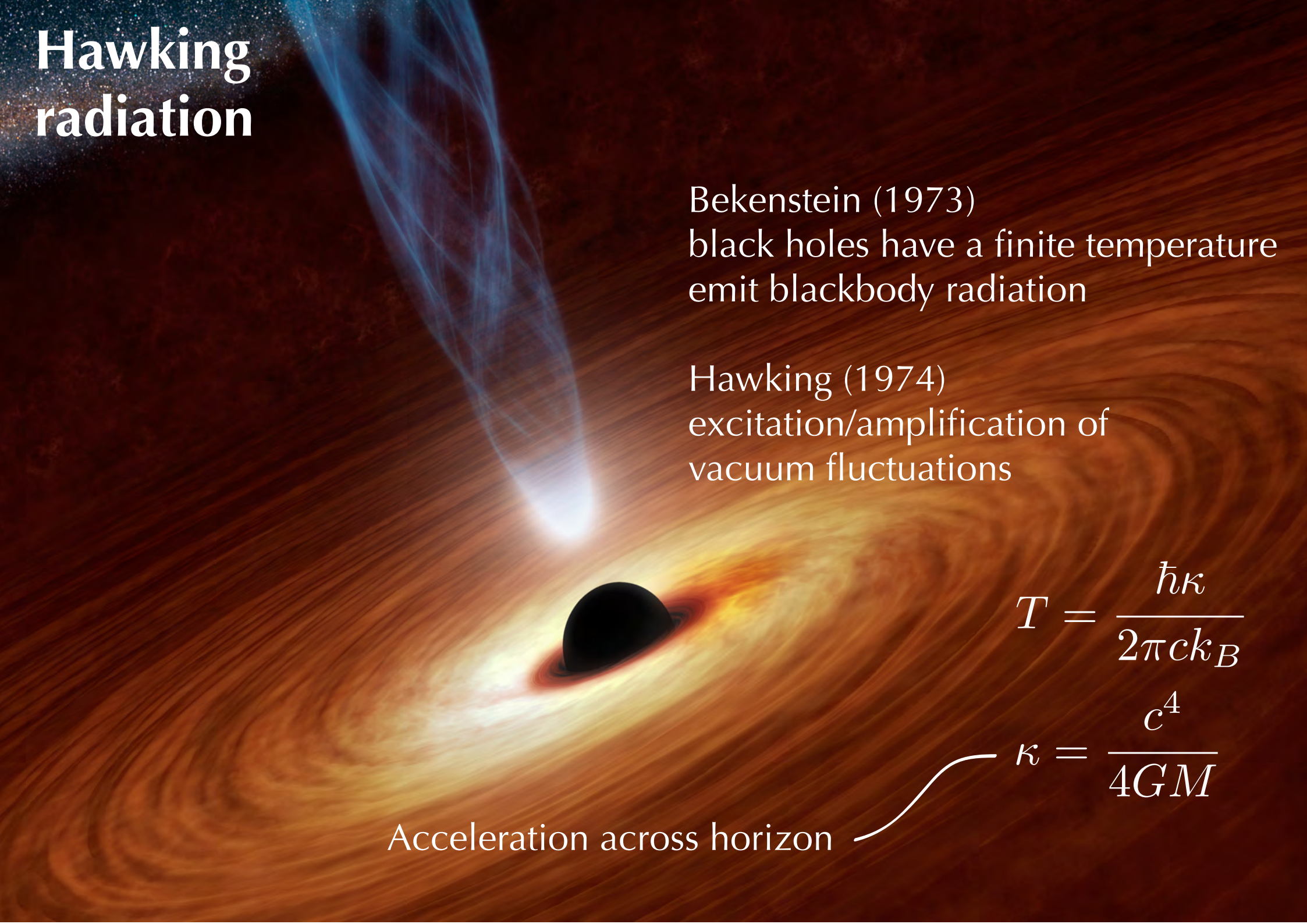


Bekenstein (1973)
black holes have a finite temperature
emit blackbody radiation

Hawking (1974)
excitation/amplification of
vacuum fluctuations

$$T = \frac{\hbar \kappa}{2\pi c k_B}$$

Hawking radiation



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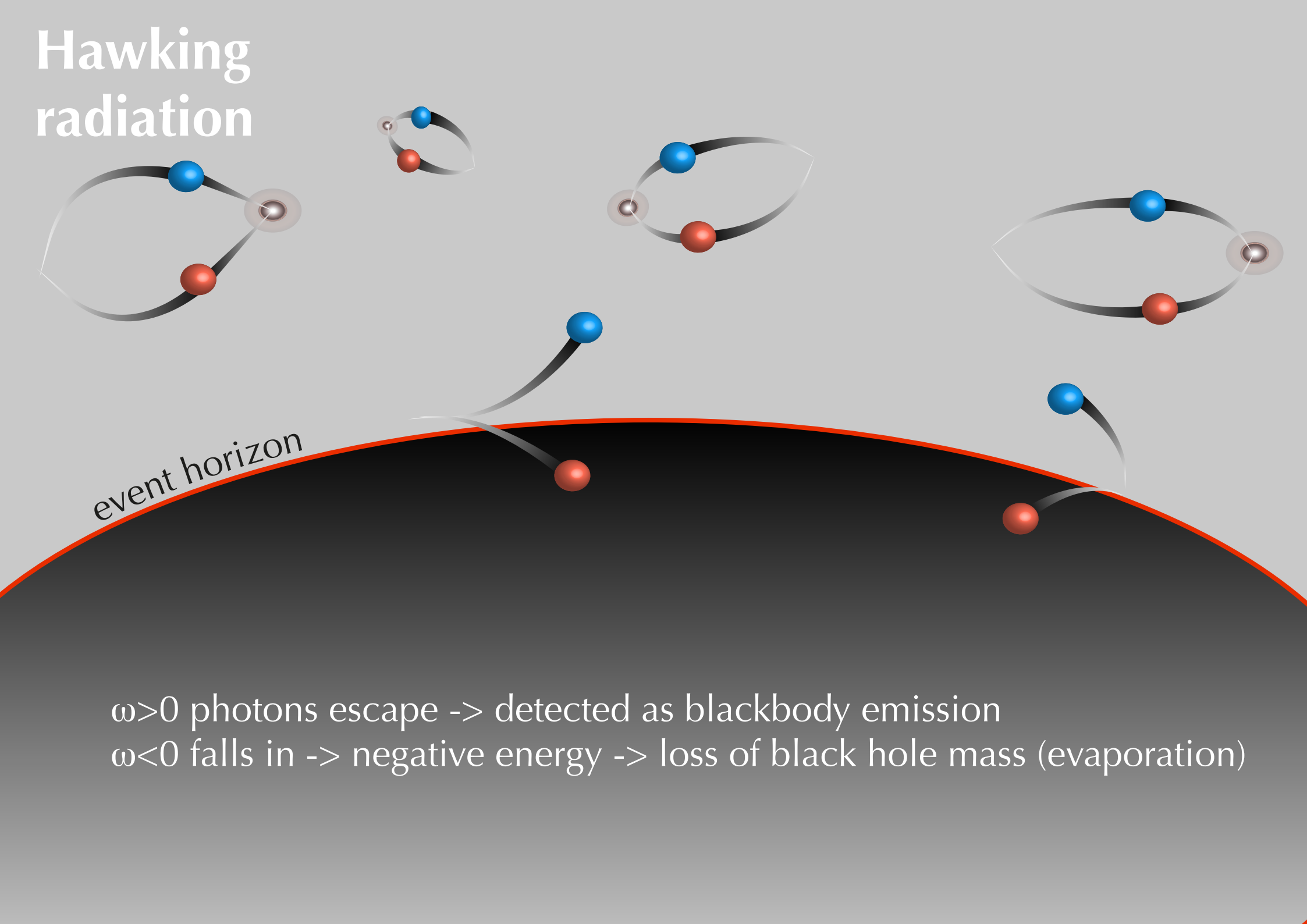
$$T = \frac{\hbar\kappa}{2\pi c k_B}$$

$$\kappa = \frac{c^4}{4GM}$$

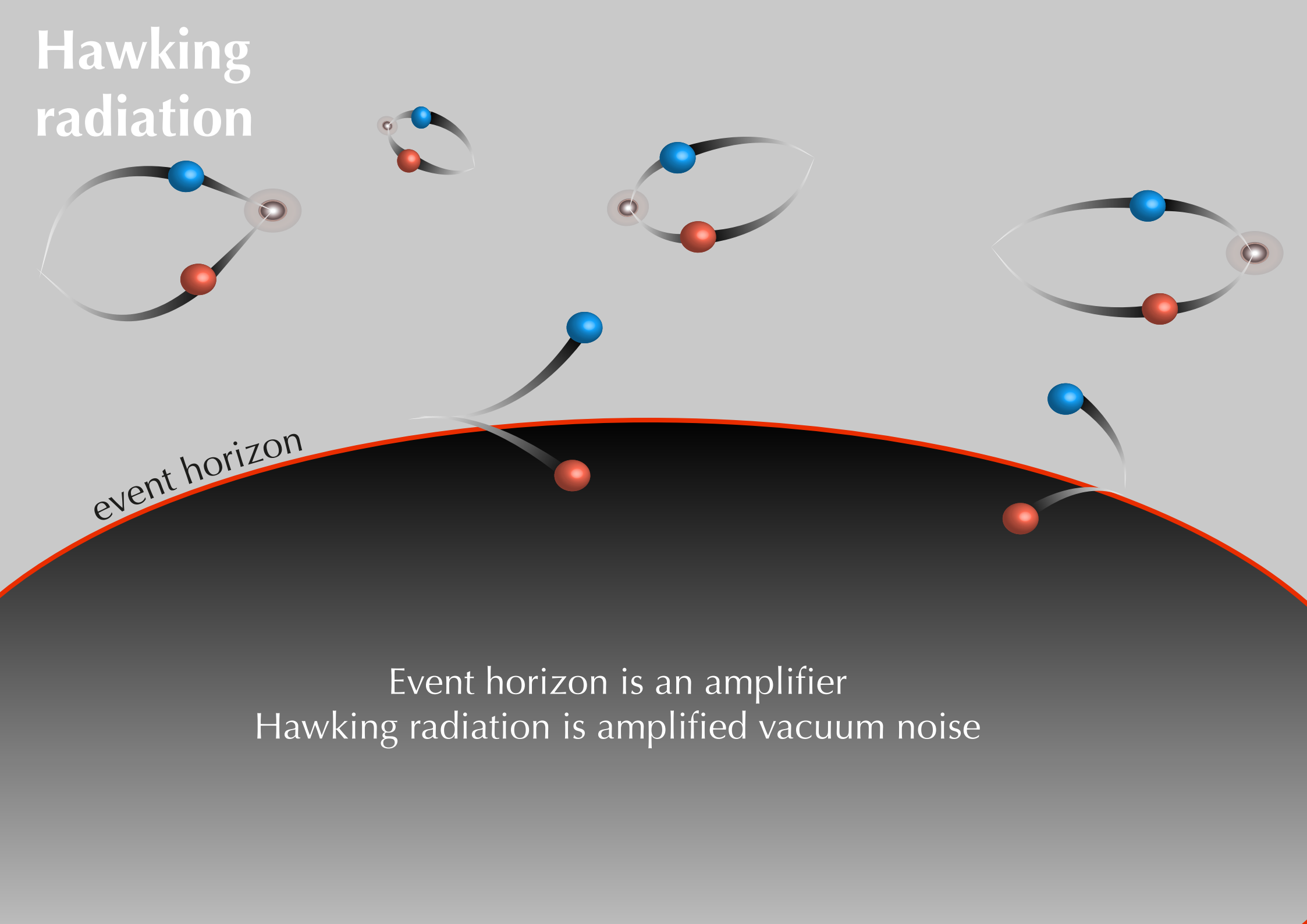
Acceleration across horizon

Hawking radiation



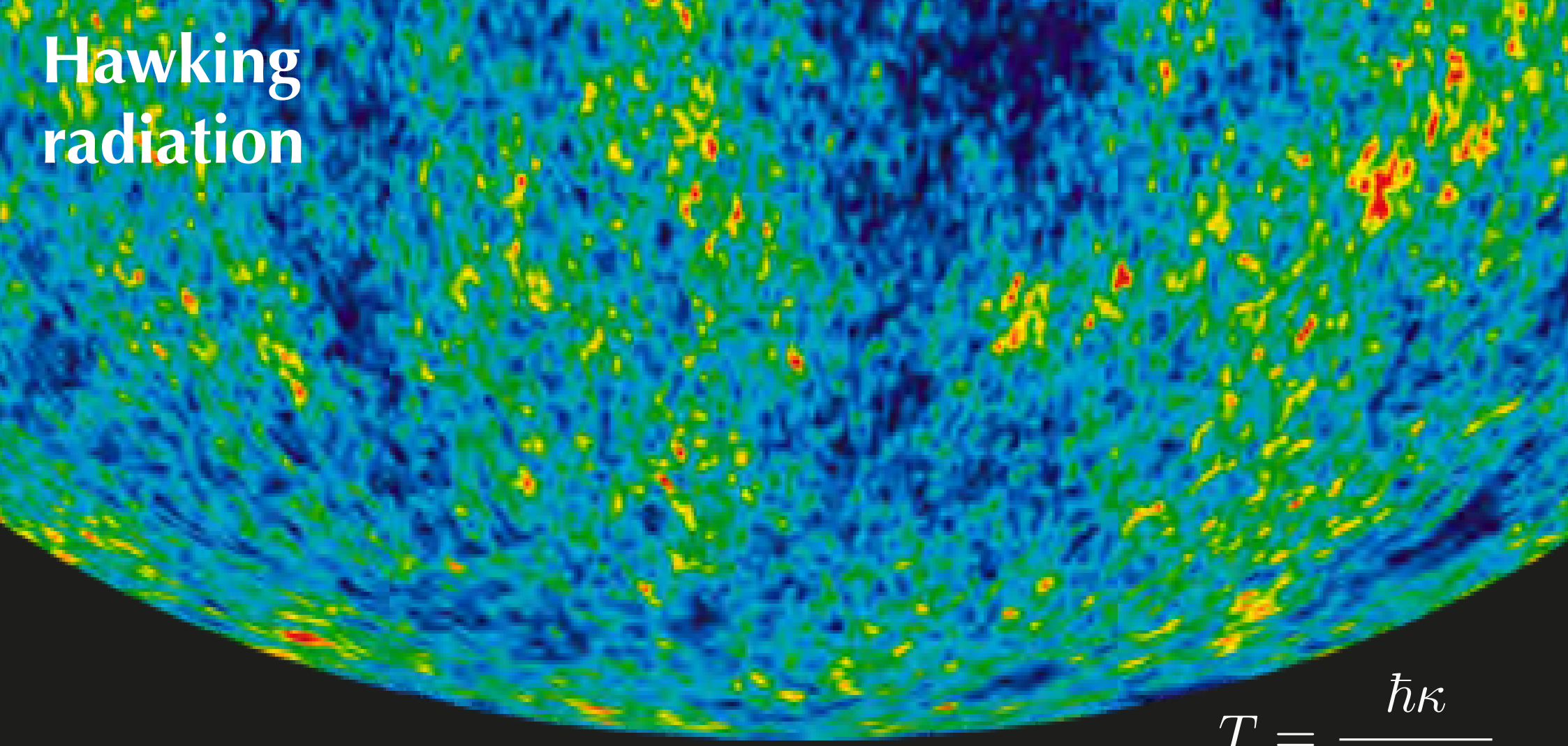
ω>0 photons escape -> detected as blackbody emission
ω<0 falls in -> negative energy -> loss of black hole mass (evaporation)

Hawking radiation



Event horizon is an amplifier
Hawking radiation is amplified vacuum noise

Hawking radiation



Cosmic microwave background is 9-10 orders of magnitude hotter

$$T = \frac{\hbar\kappa}{2\pi c k_B}$$

$$\kappa = \frac{c^4}{4GM}$$

Acceleration across horizon

Black hole Temperature

Black hole Temperature

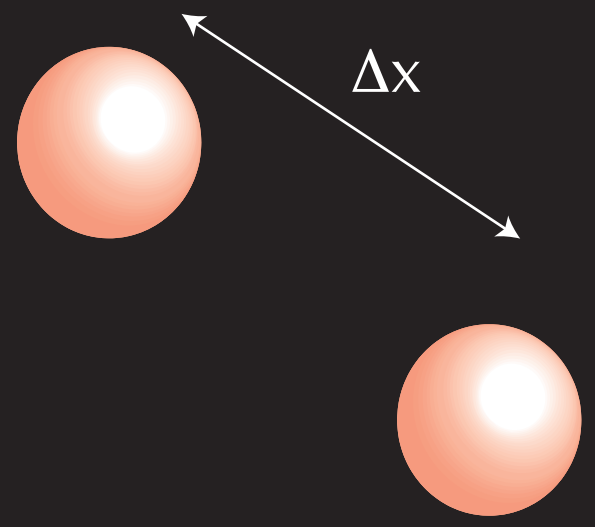
Heisenberg uncertainty relation

$$\Delta x \Delta p = h$$

Black hole Temperature

Heisenberg uncertainty relation

$$\Delta x \Delta p = h$$



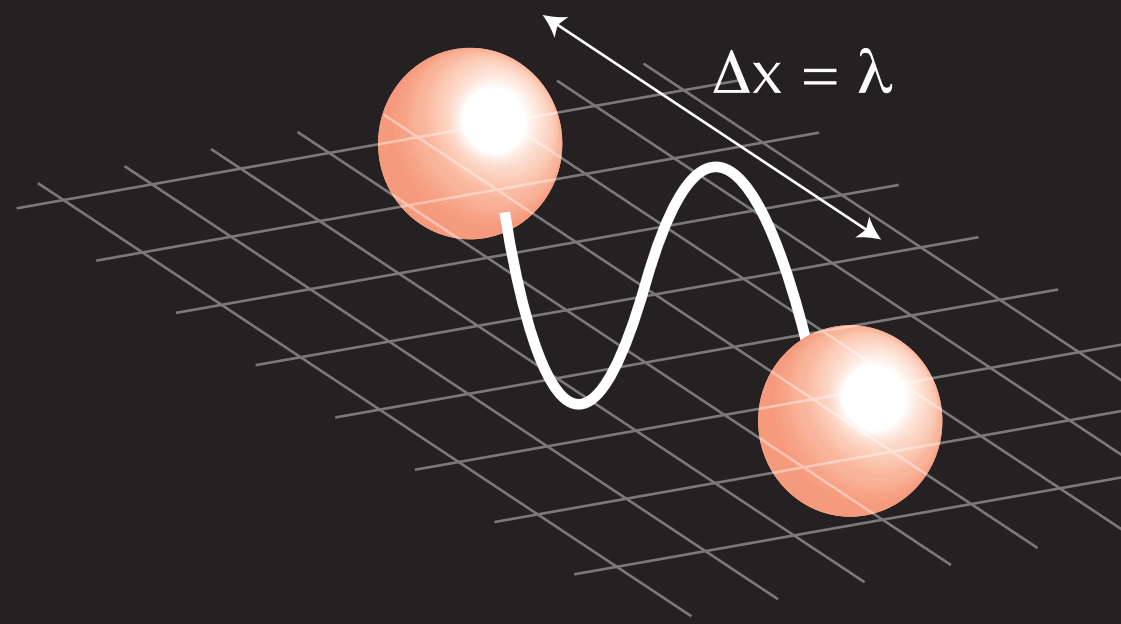
Black hole Temperature

Heisenberg uncertainty relation

$$\Delta x \Delta p = h$$

in flat spacetime

$$\Delta x \sim \lambda$$



Black hole Temperature

Heisenberg uncertainty relation

$$\Delta x \Delta p = h$$

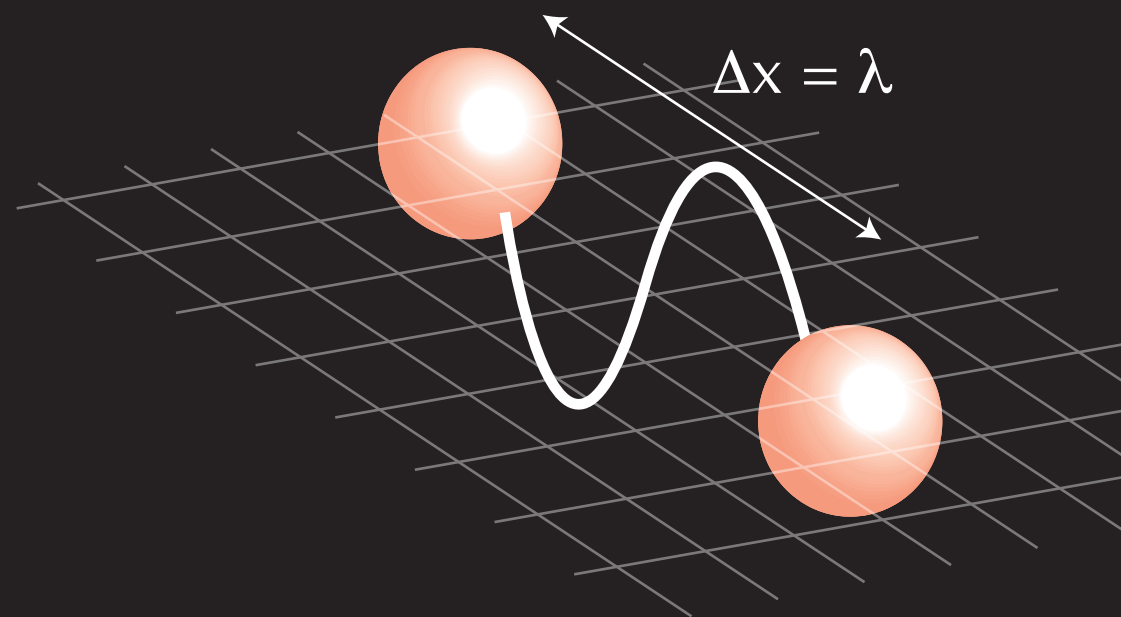
in flat spacetime

$$\Delta x \sim \lambda$$

$$\rightarrow \quad \Delta p = \frac{h}{\lambda}$$

Energy is given by

$$E = \Delta p \times c$$



Black hole Temperature

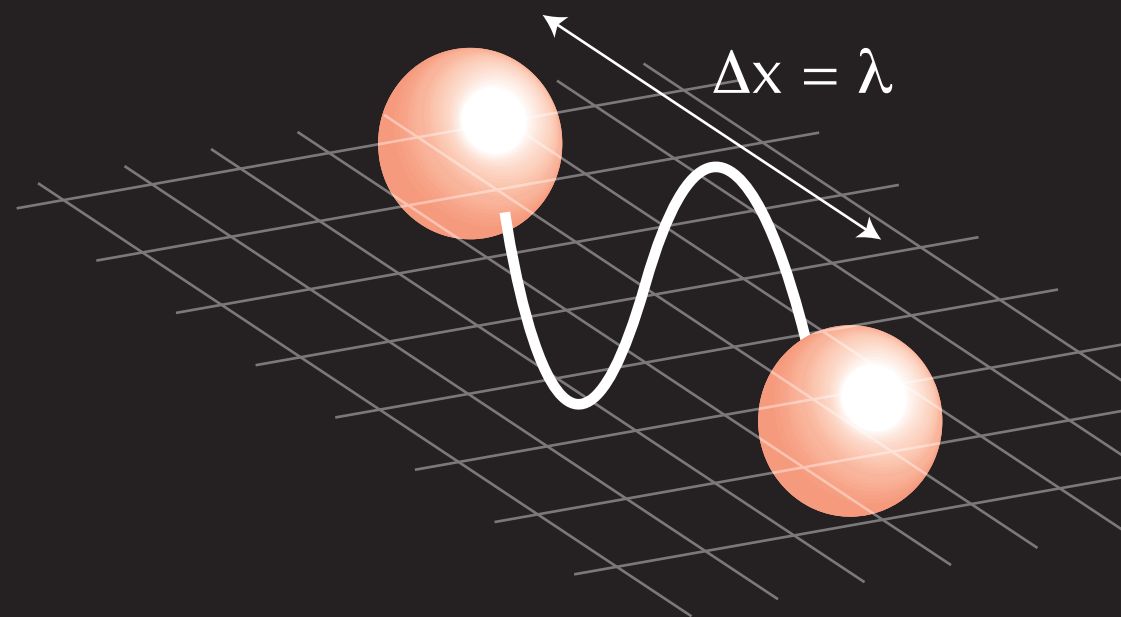
Heisenberg uncertainty relation

$$\Delta x \Delta p = h$$

in flat spacetime

$$\Delta x \sim \lambda$$

$$\longrightarrow \Delta p = \frac{h}{\lambda}$$



Energy is given by

$$E = h\frac{c}{\lambda} = h\nu$$

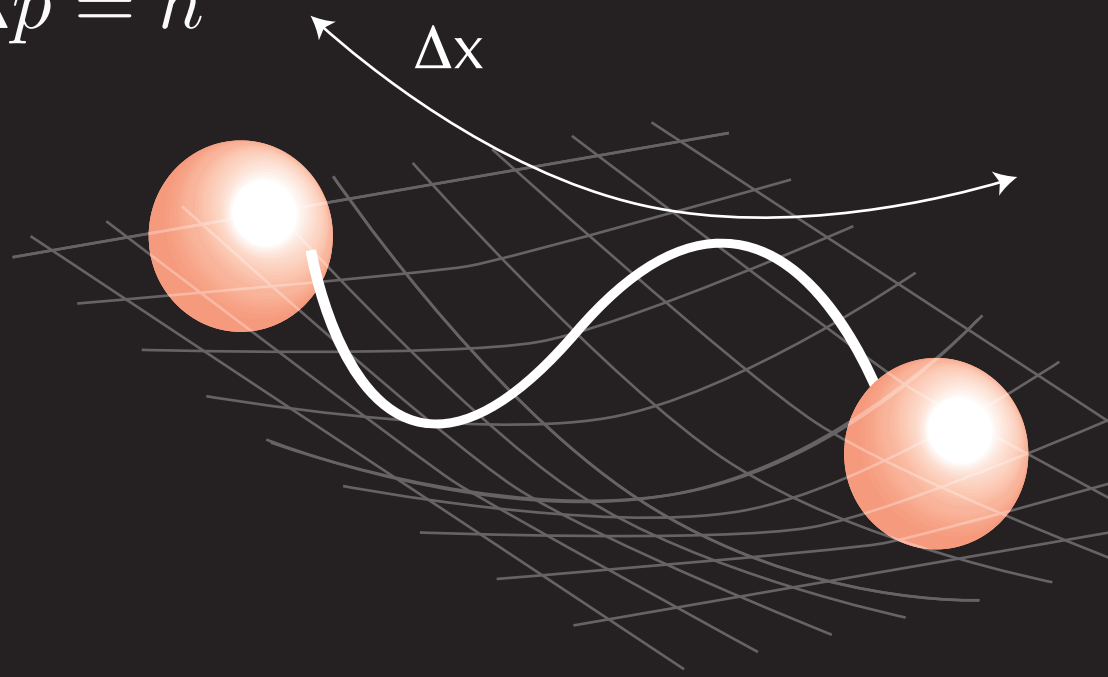
vacuum fluctuations separated by λ or more have E=hν

Black hole Temperature

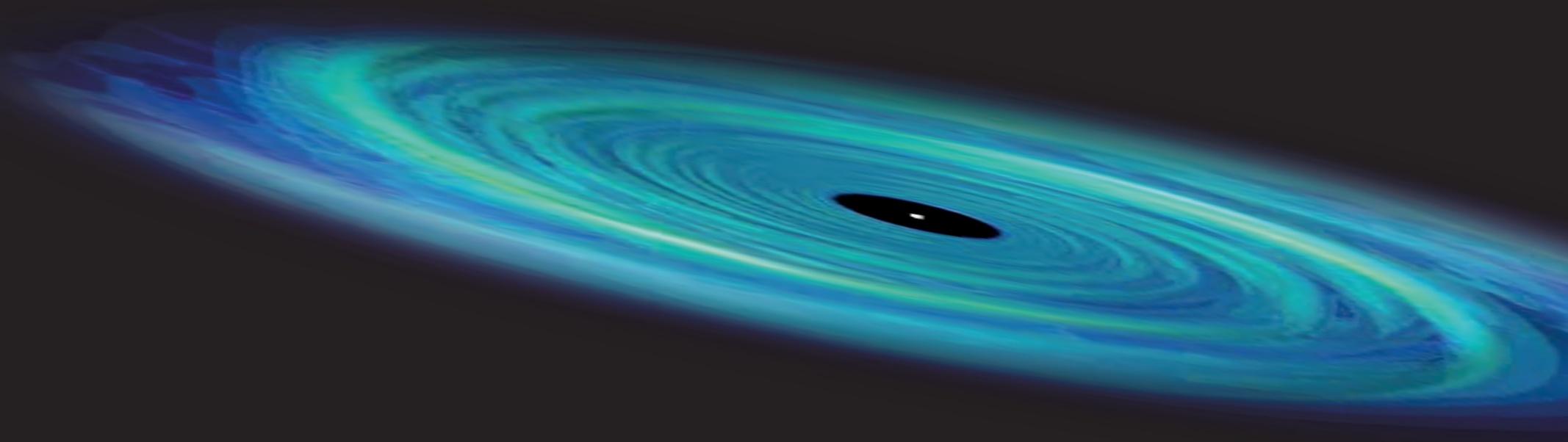
Heisenberg uncertainty relation

$$\Delta x \Delta p = h$$

in **curved** spacetime



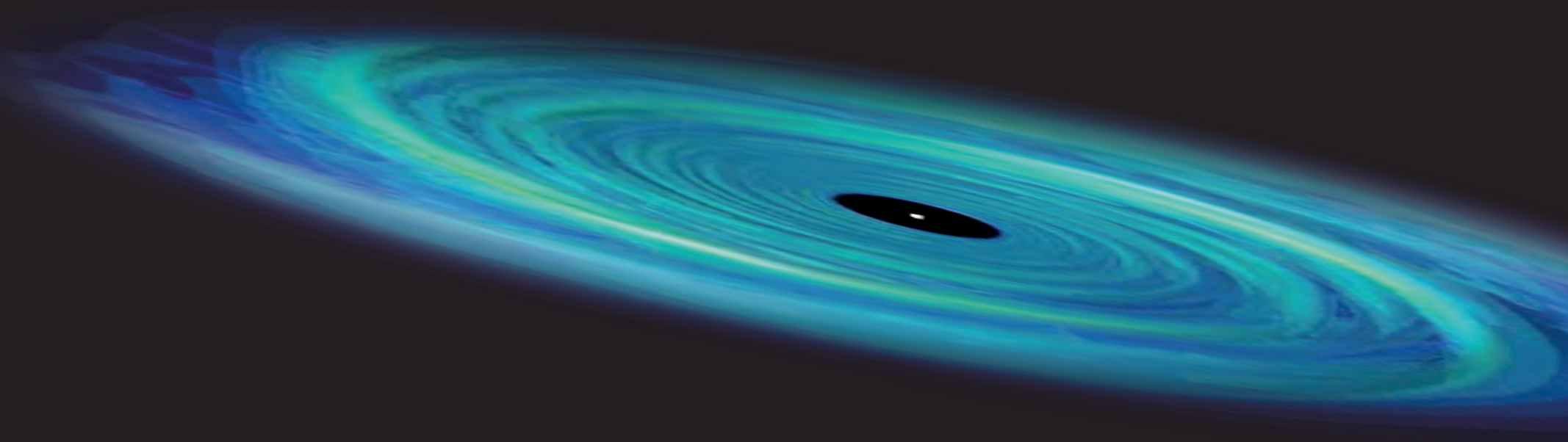
Black hole Temperature



Black hole Temperature

No-hair theorem

1) Charge
2) Angular momentum, spin
3) Mass



Black hole Temperature

No-hair theorem

1) Charge
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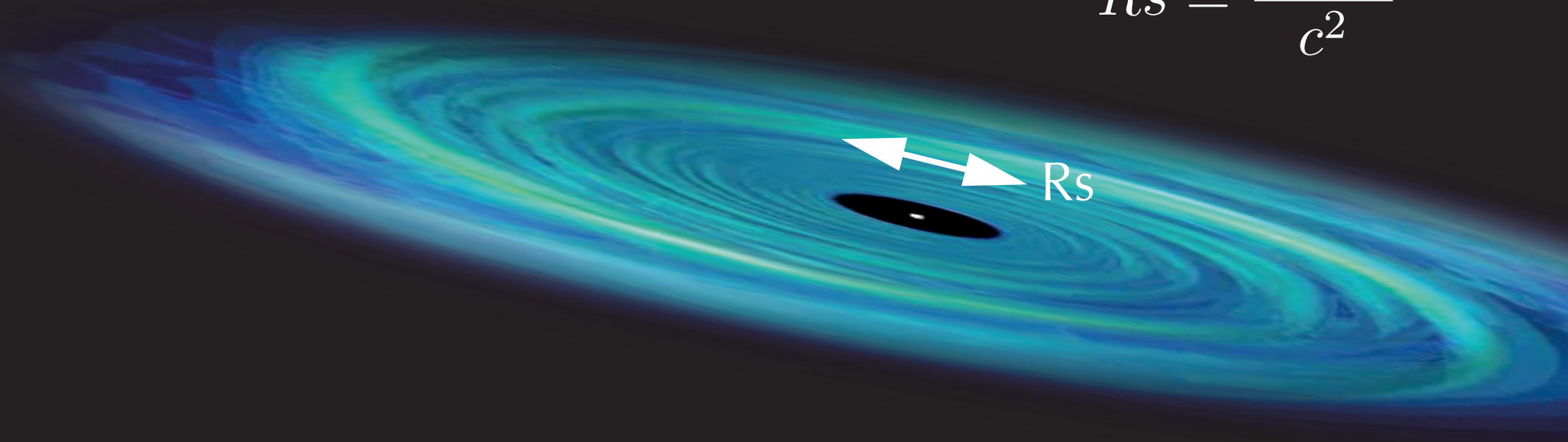
$$Rs = \frac{2GM}{c^2}$$



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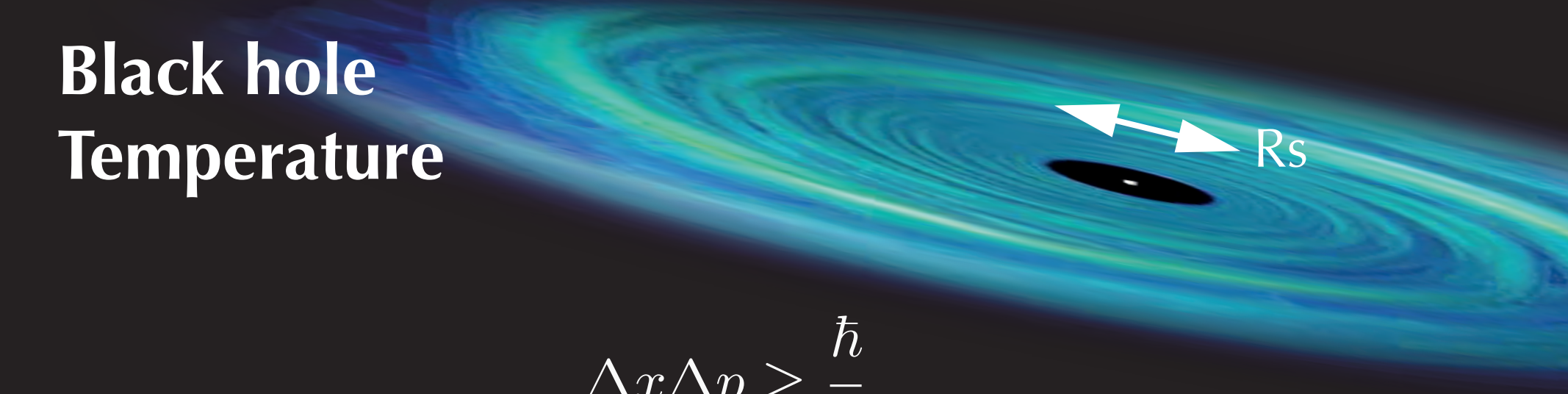
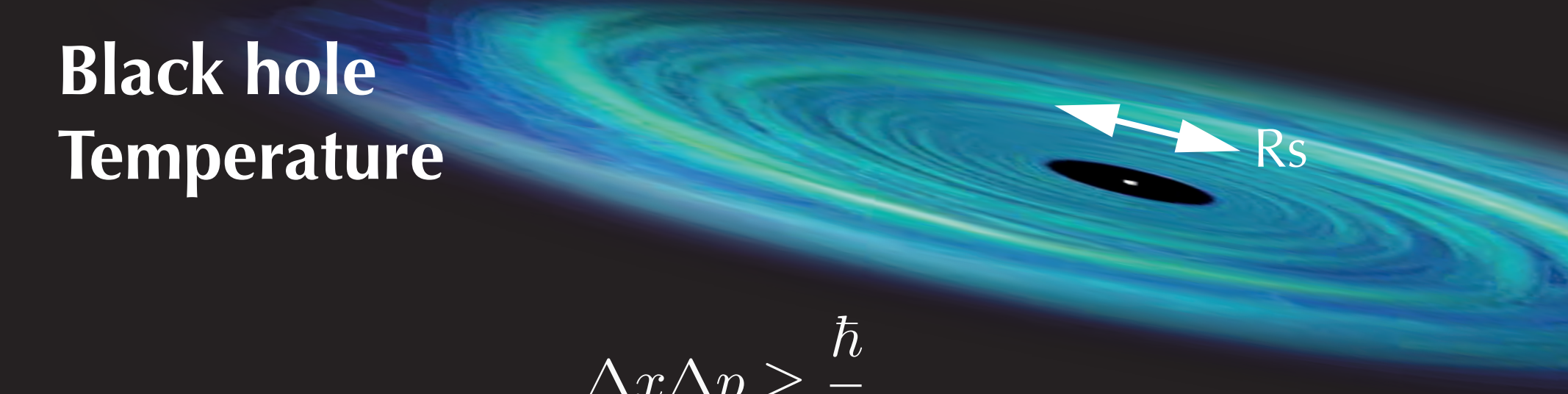


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Black hole Temperature



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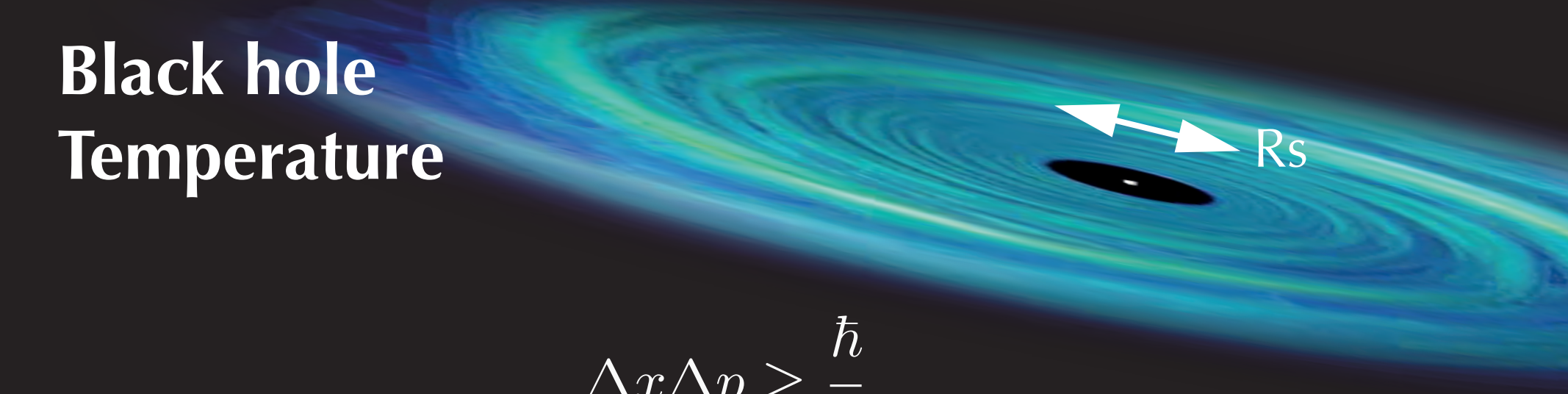
No-hair theorem $\Delta x = Rs = \frac{2GM}{c^2}$

Black hole Temperature



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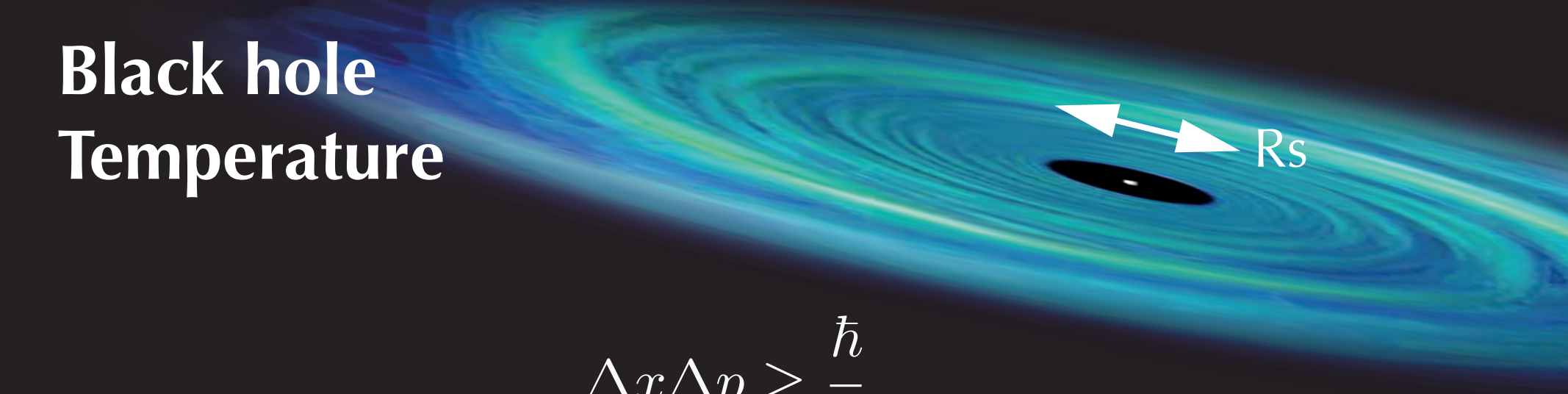
Black hole Temperature



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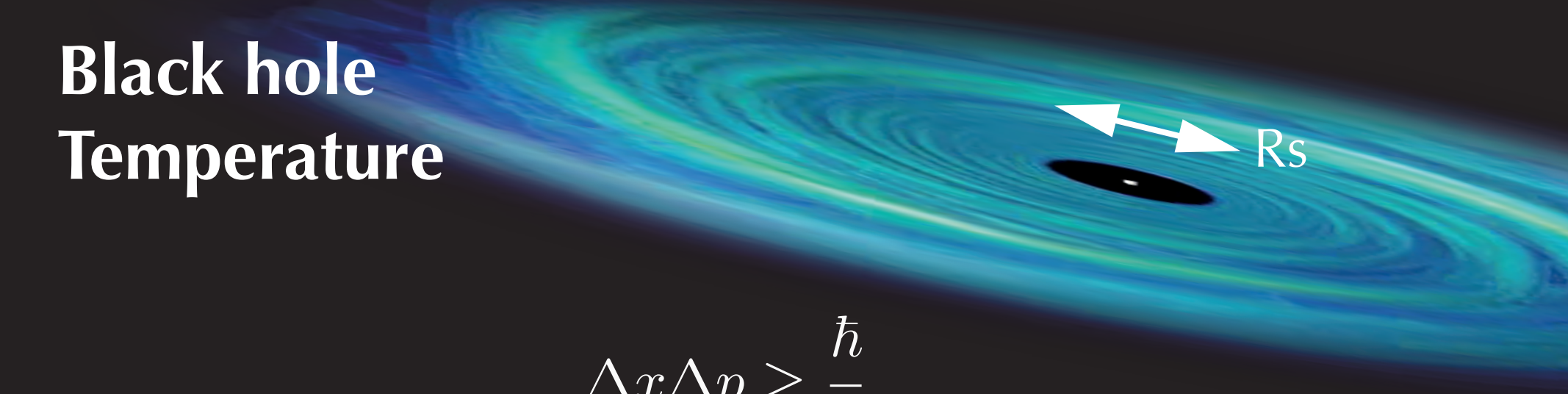

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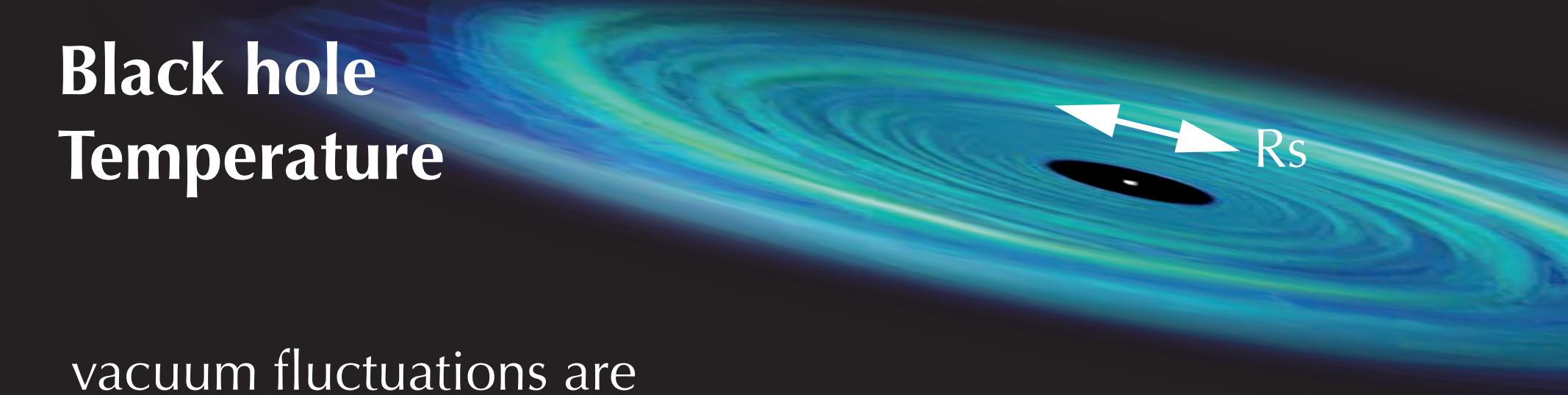
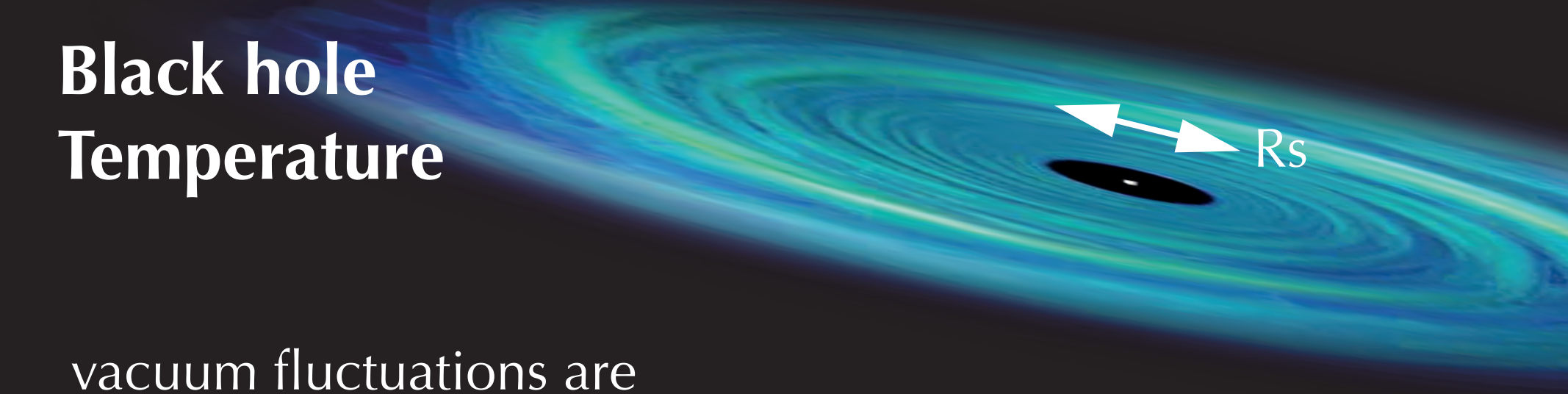
$$T = \frac{\hbar c^3}{4GMk_B}$$ Hawking temperature

Black hole Temperature



vacuum fluctuations are spread out to Rs

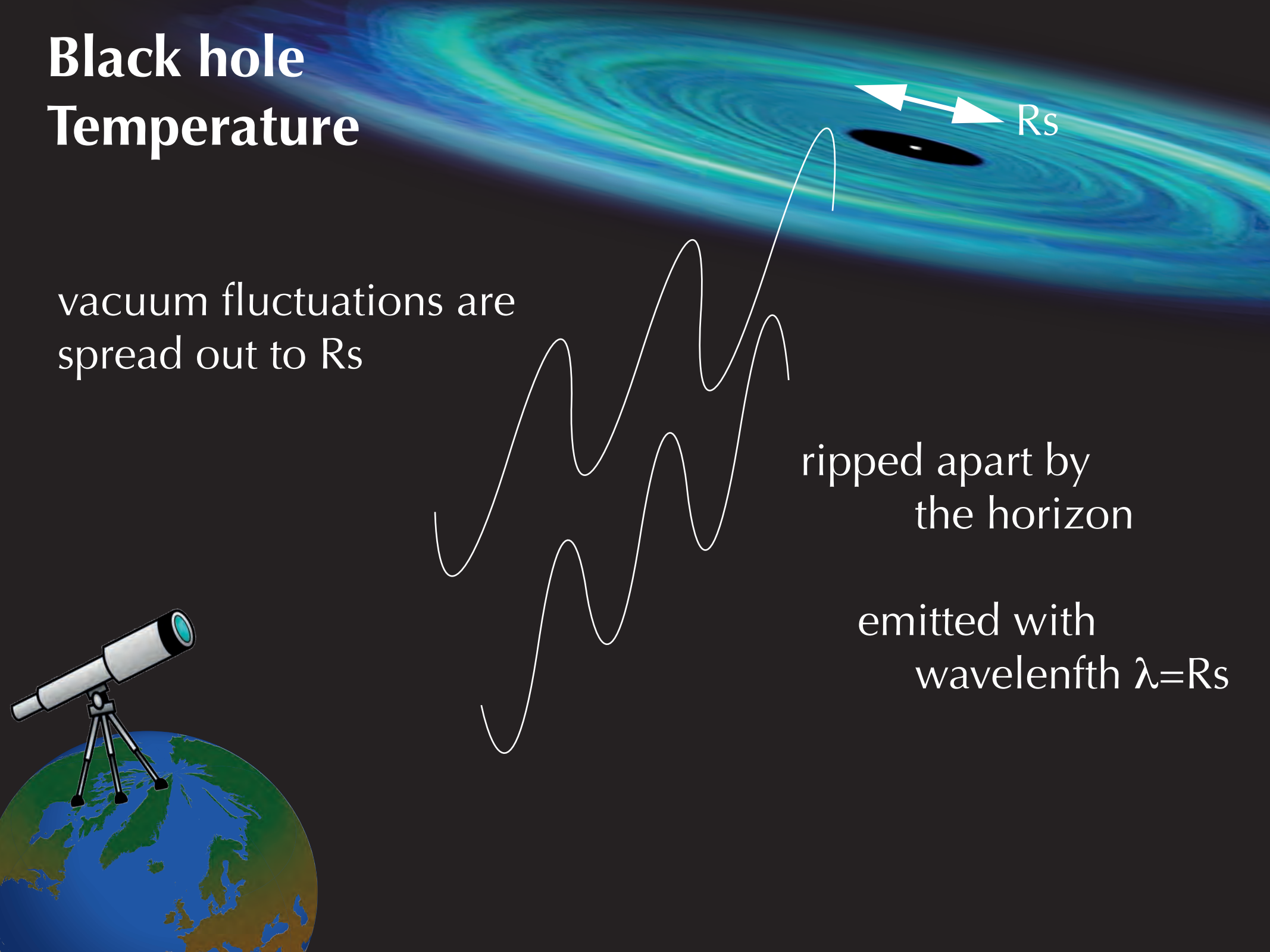
Black hole Temperature



vacuum fluctuations are spread out to Rs

ripped apart by the horizon

Black hole Temperature



Analogue black holes & event horizons

Create a flowing river or medium such that

$$ds^2 = c^2 dt^2 - (dr - V dt)^2$$

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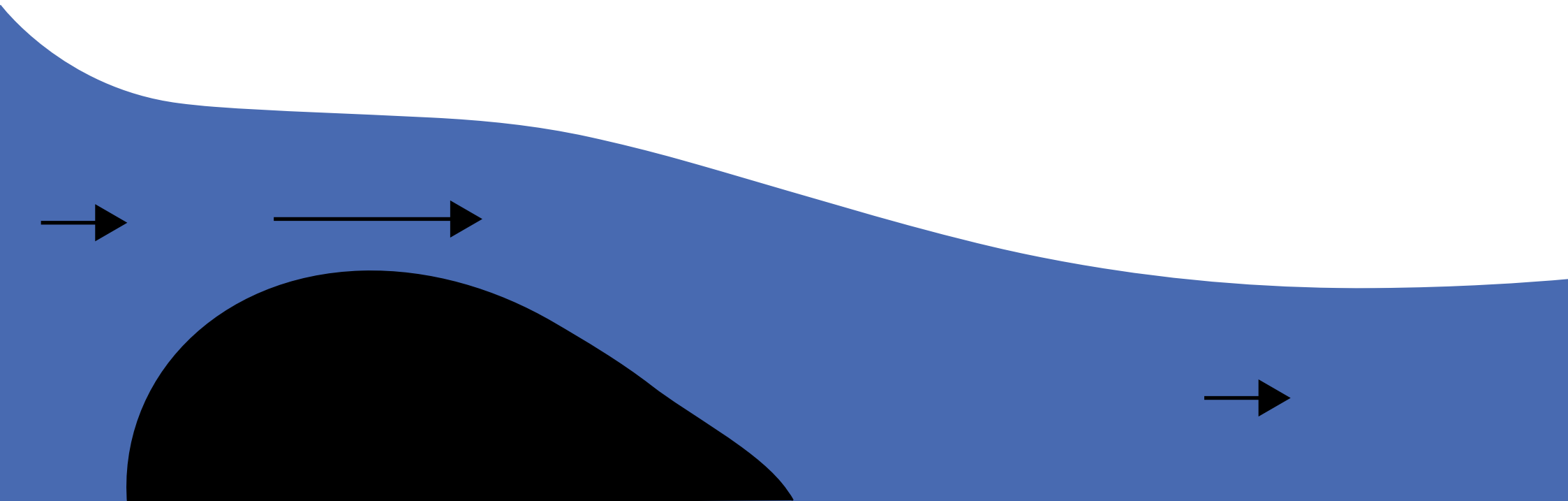

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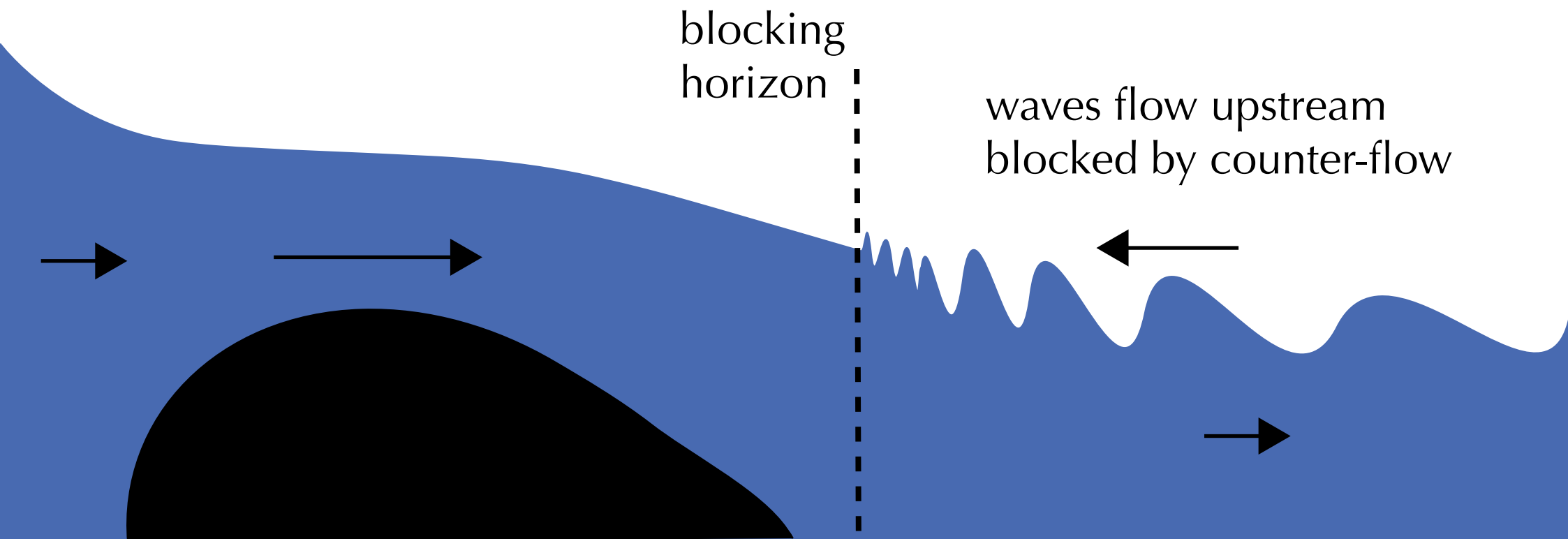
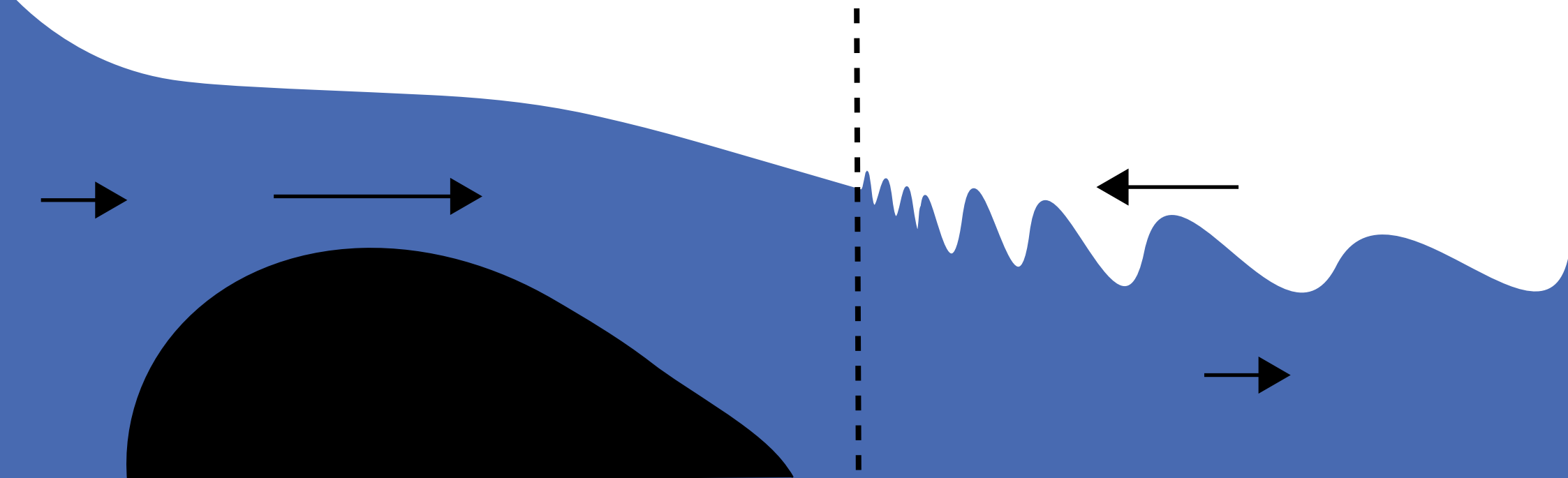


Analogue black holes & event horizons

First proposed by Unruh (1981) predicted Hawking emission from horizon

experiments with
white hole horizon -> $T=10^{-12}$ K << black hole T



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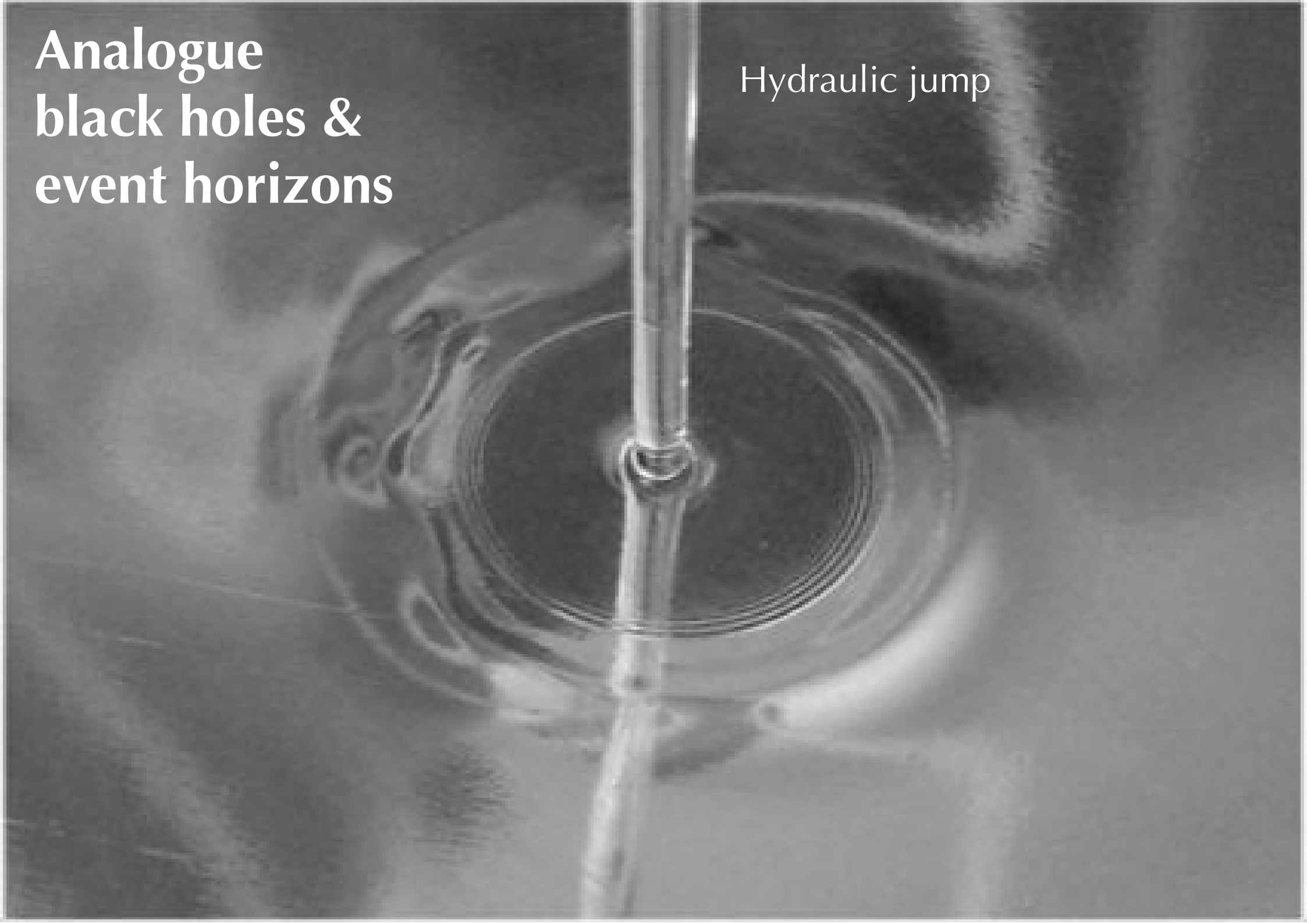

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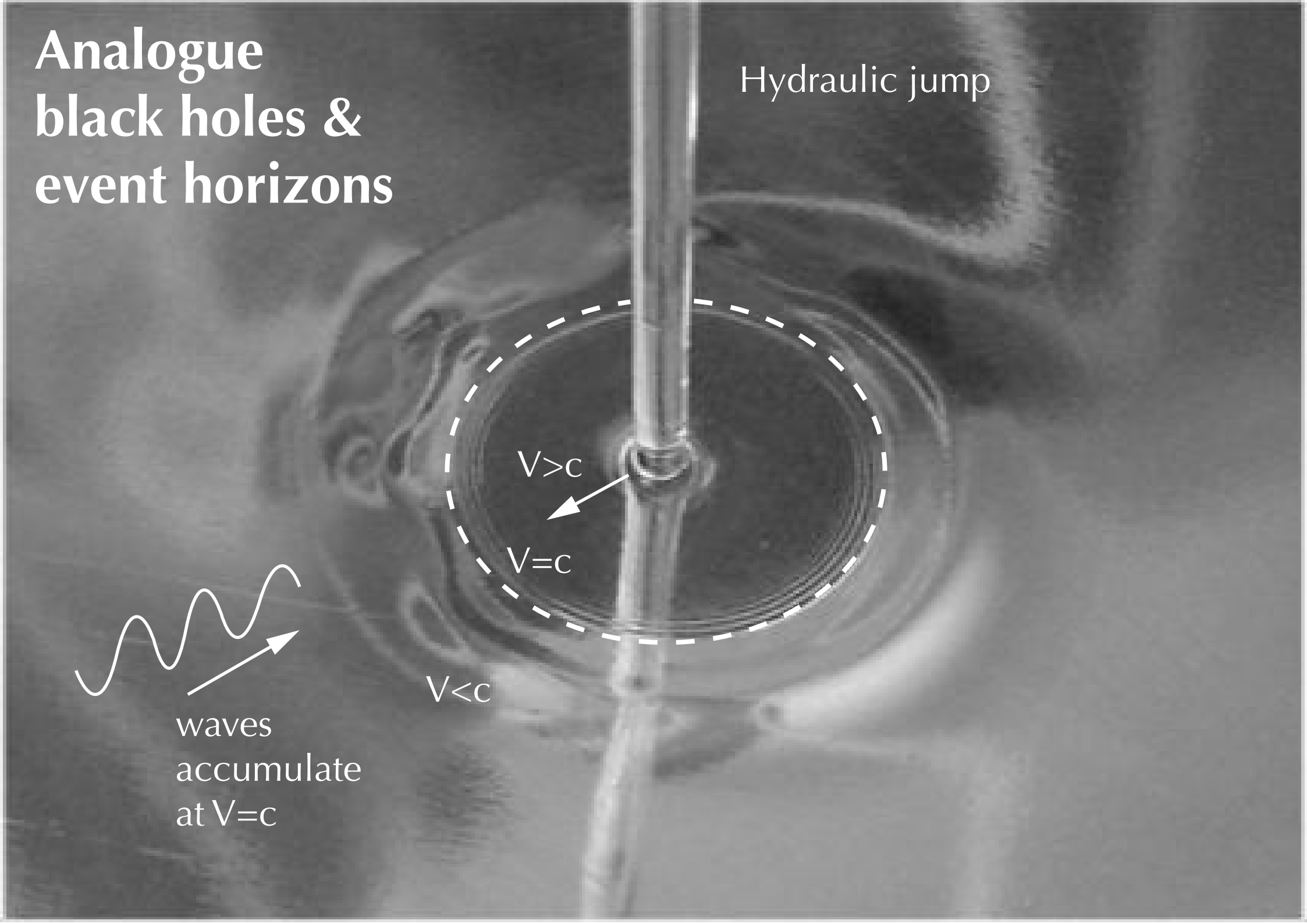

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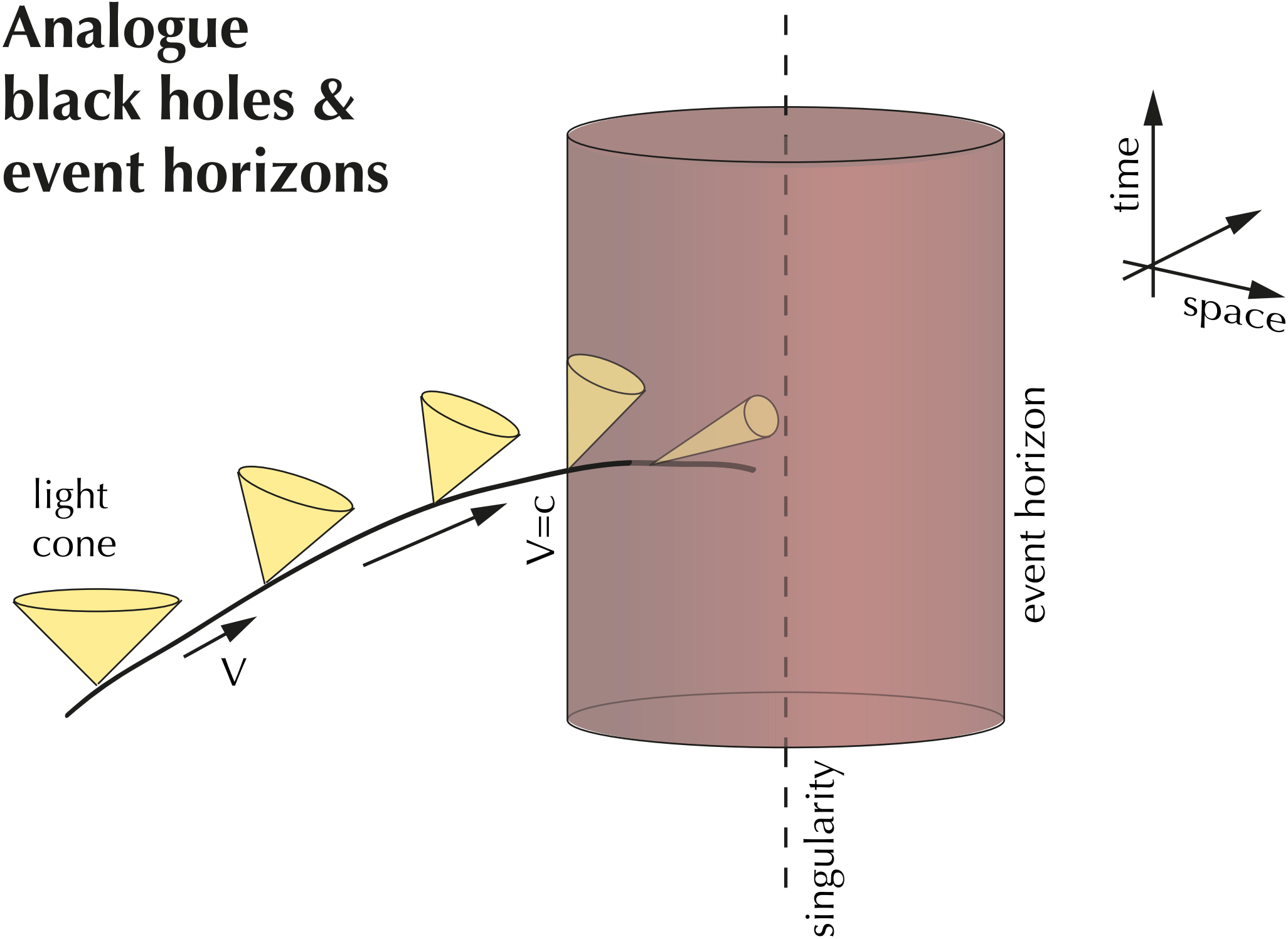

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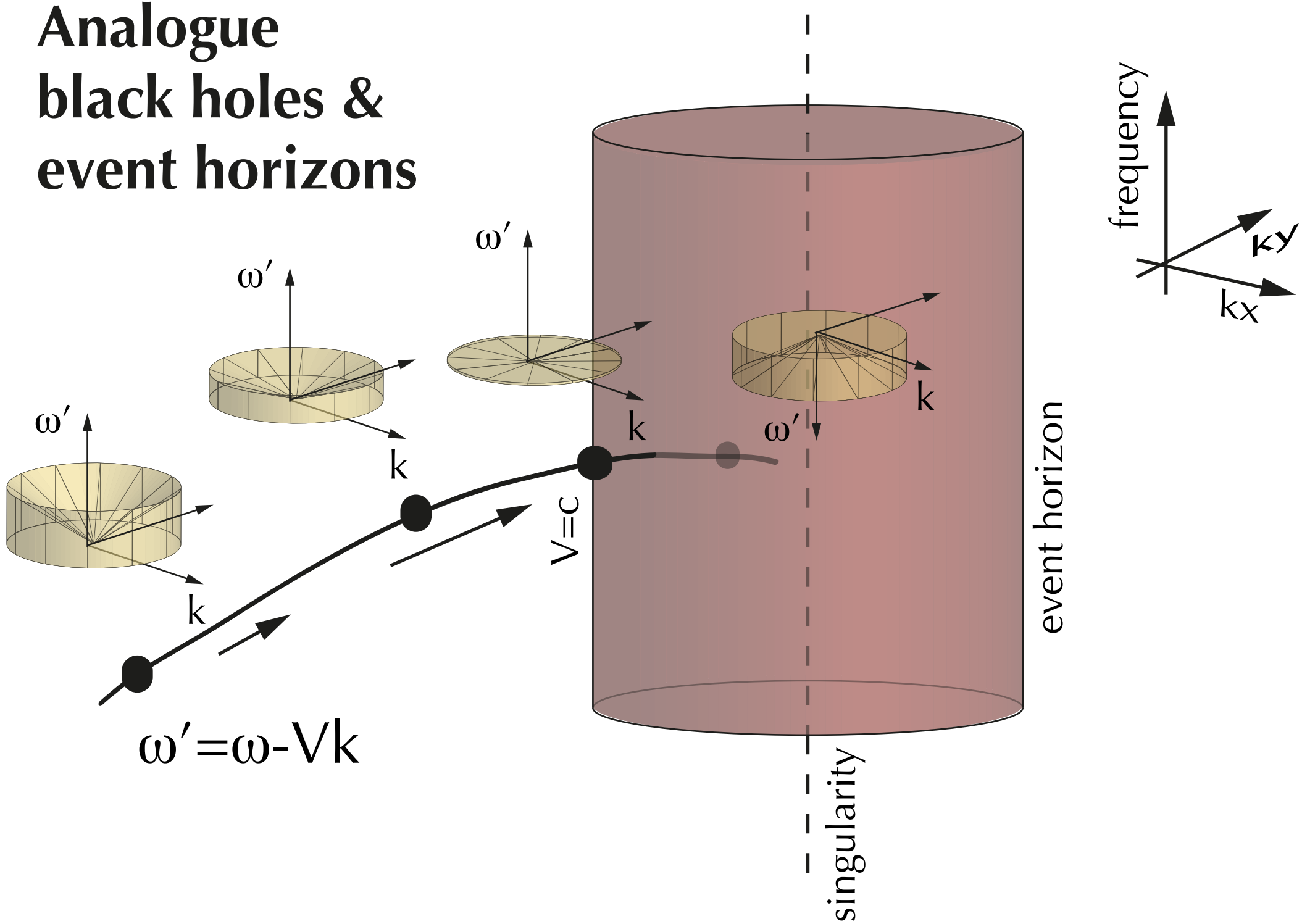

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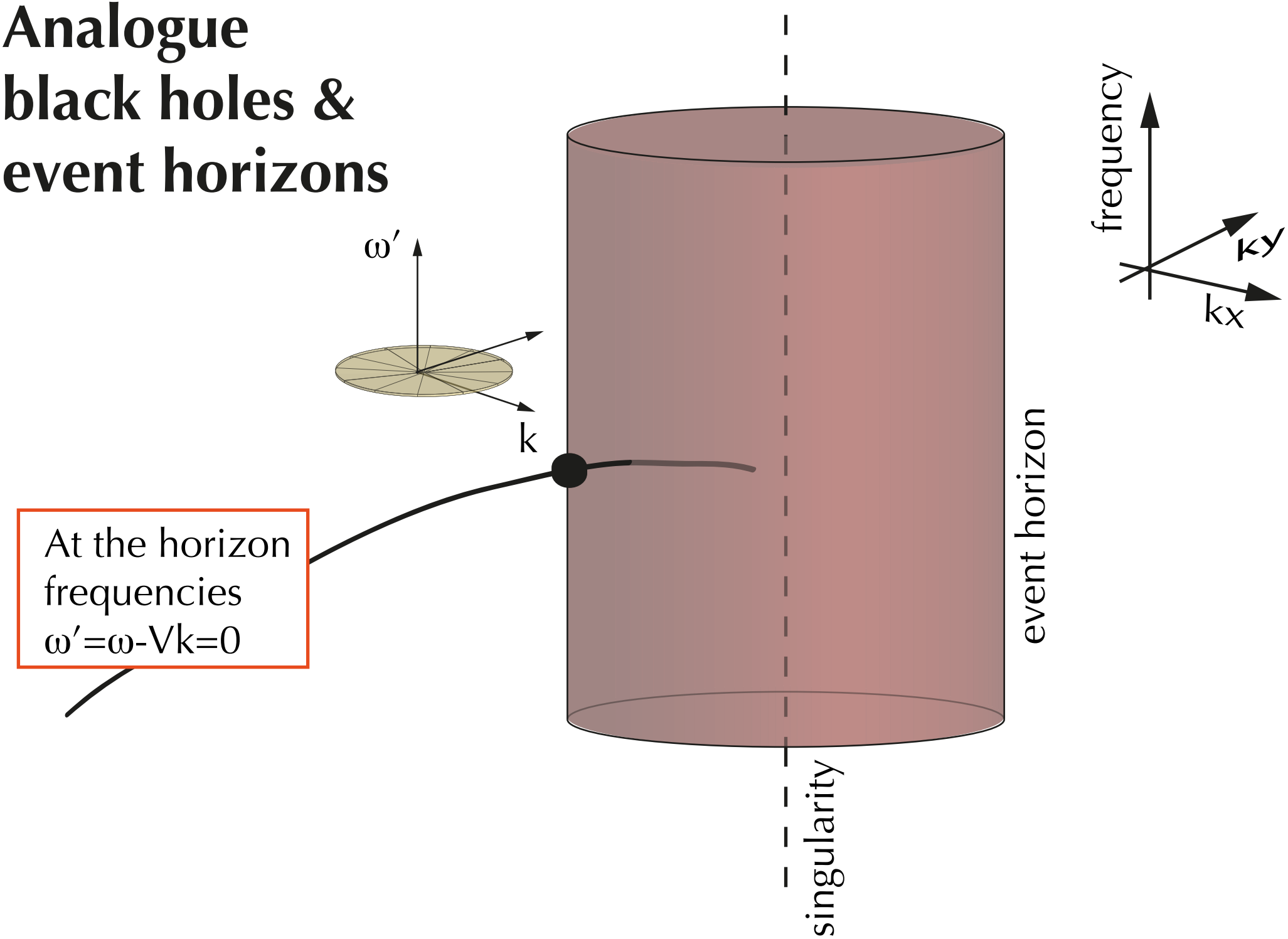

Analogue black holes & event horizons

Artificial horizons in the research lab:

Analogue black holes & event horizons

Artificial horizons in the research lab:

acoustic systems
gravity waves in water
phonon oscillations in Bose-Einstein Condensate
Waveguide systems
Polaritons
Liquid Helium
SQUID arrays
Light in refractive media

- horizon effects,
Rousseaux, Leonhardt et al.,
- classical Hawing mode conversion
Unruh et al.,

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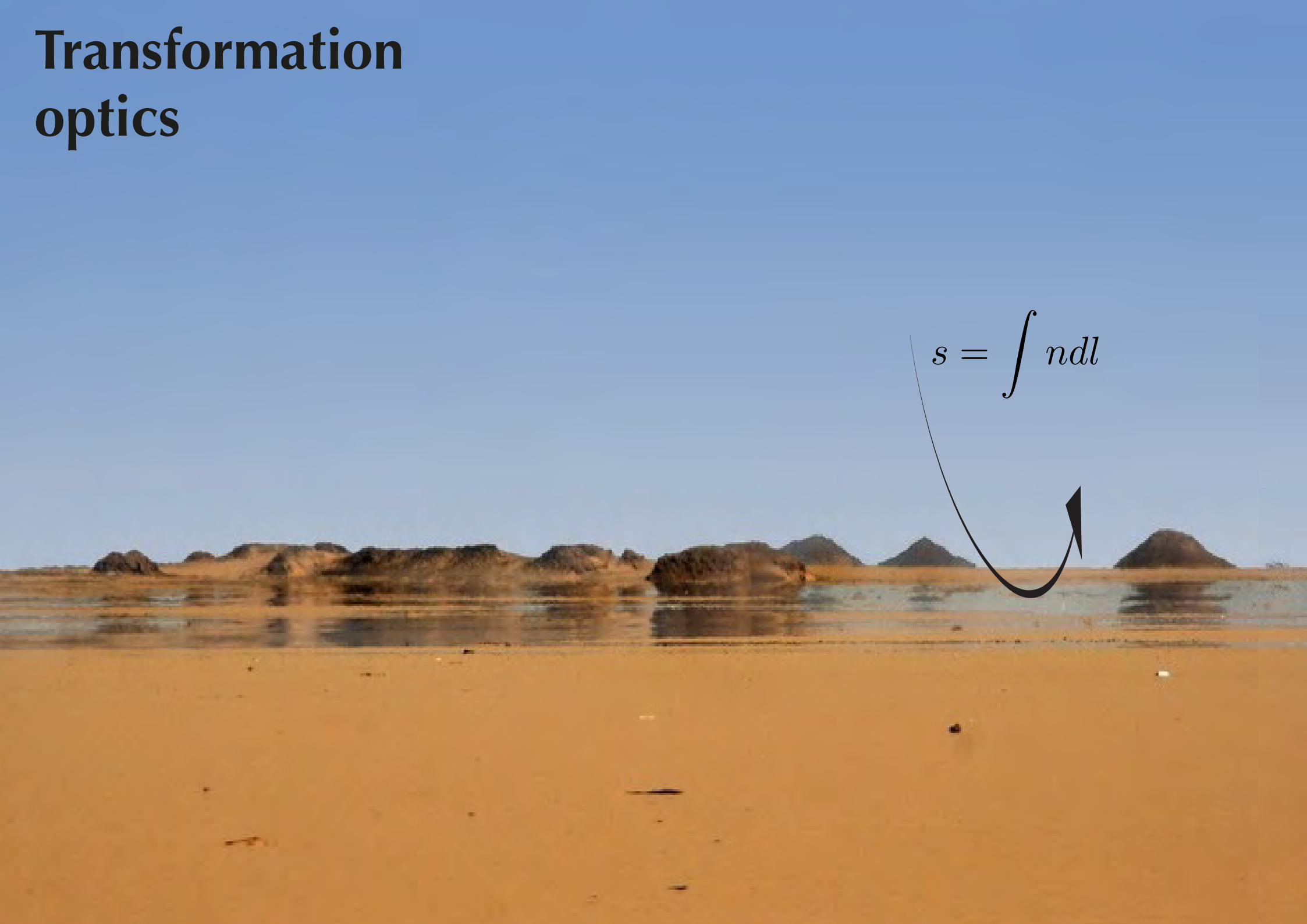
Leonhardt et al., Science, 2008
- Hawking emission
Faccio et al., PRL 2011

Transformation optics

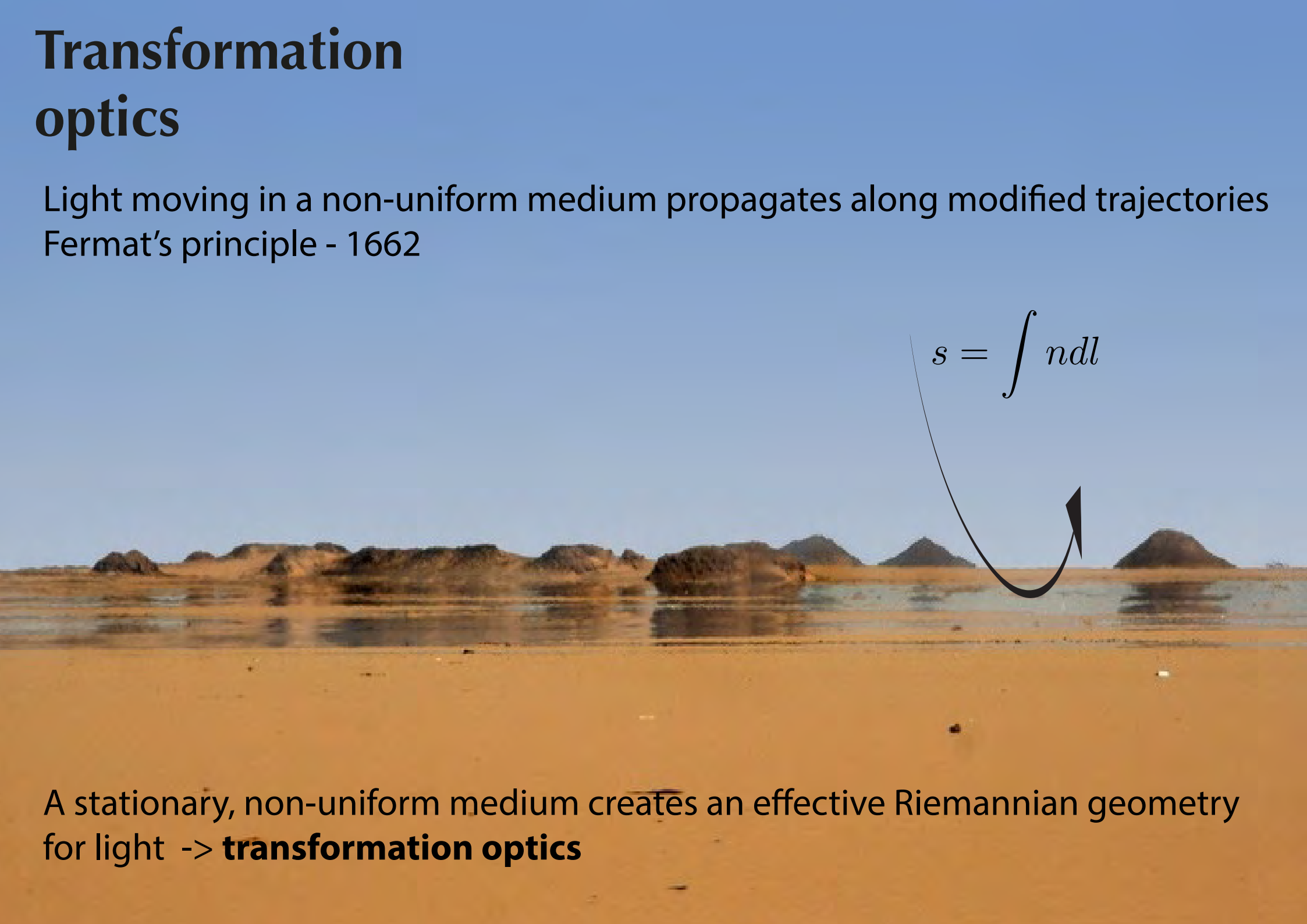
Transformation optics



Transformation optics

Light moving in a non-uniform medium propagates along modified trajectories
Fermat's principle - 1662

$$s = \int n dl$$



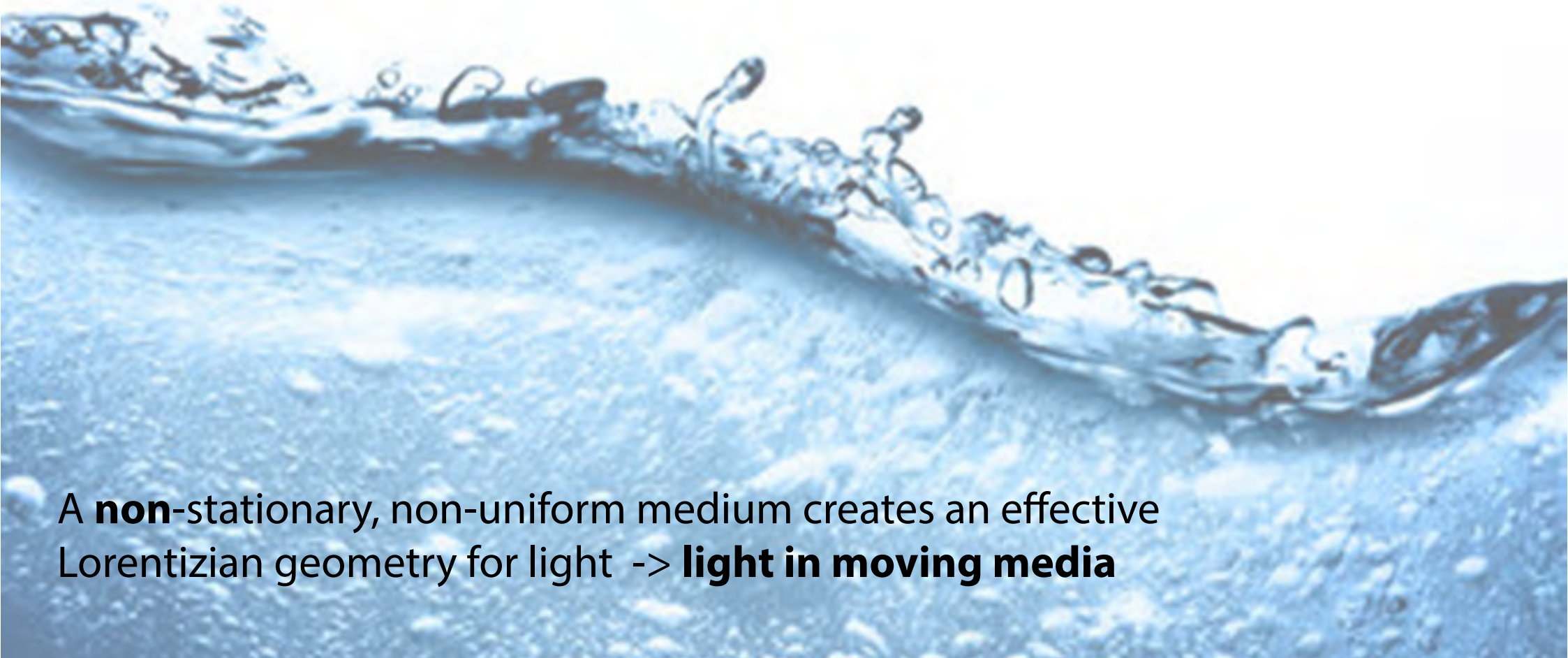
A stationary, non-uniform medium creates an effective Riemannian geometry for light -> **transformation optics**

Transformation optics in moving media

Light in moving/flowing medium - Fresnel 1818
Fizeau 1851

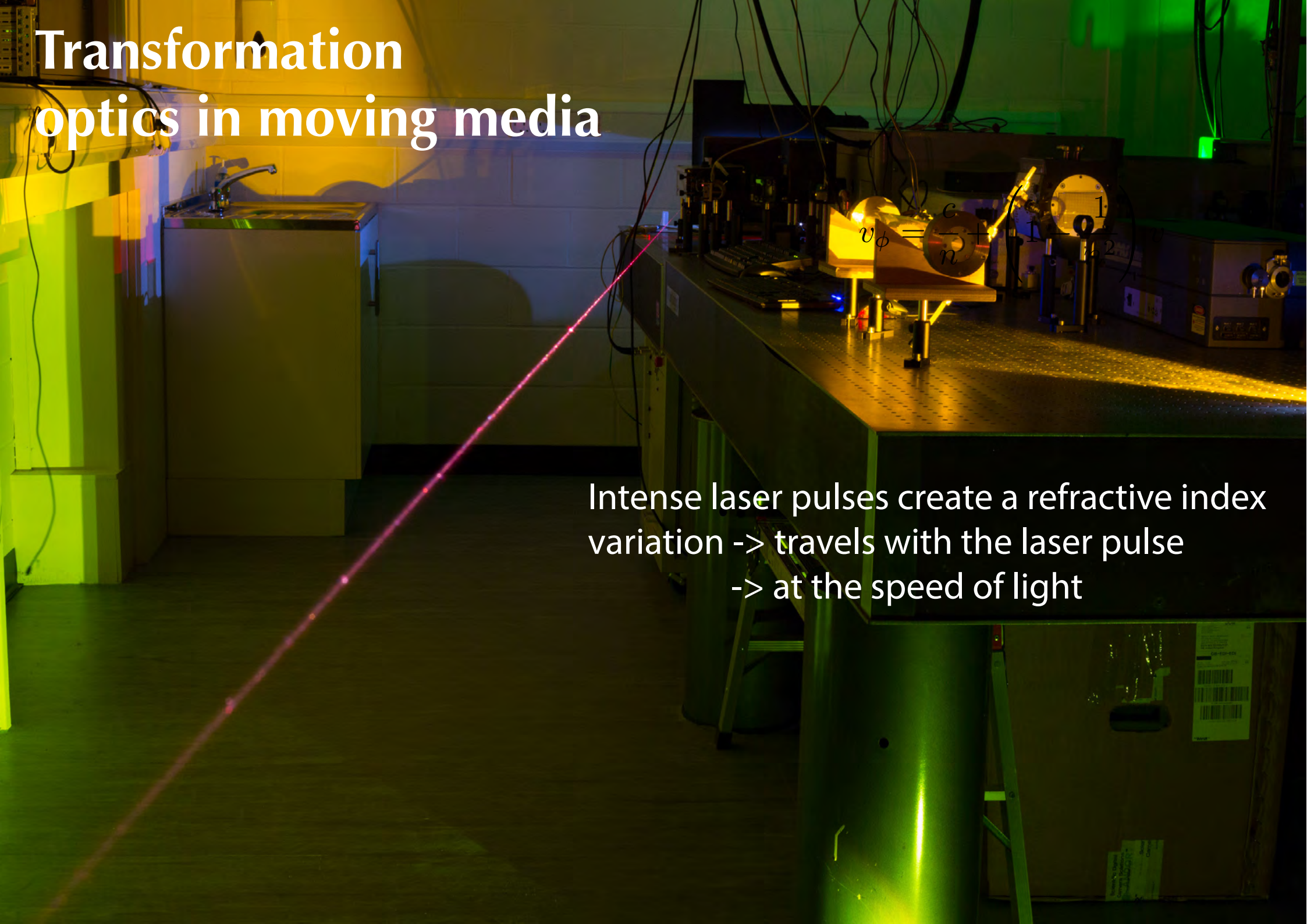
$$v_\phi = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) v$$

Connection with general relativity - Gordon metric, 1923



A **non**-stationary, non-uniform medium creates an effective Lorentizian geometry for light -> **light in moving media**

Transformation optics in moving media



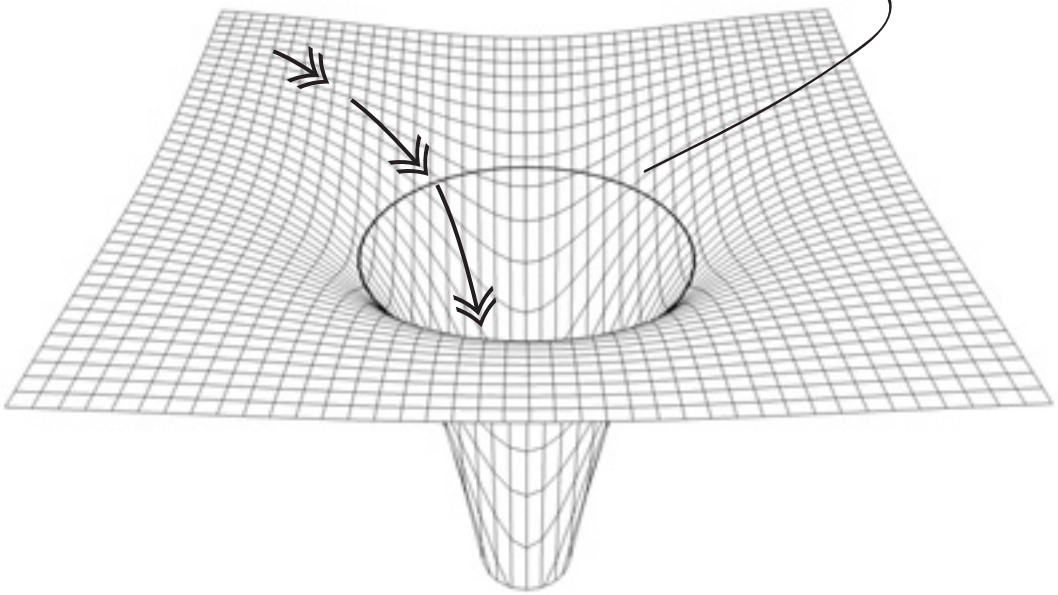
$$v_\phi = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)$$

Intense laser pulses create a refractive index variation -> travels with the laser pulse
-> at the speed of light

analogue gravity in moving media

Black hole

$$V = -\sqrt{\frac{2GM}{r}} = c$$

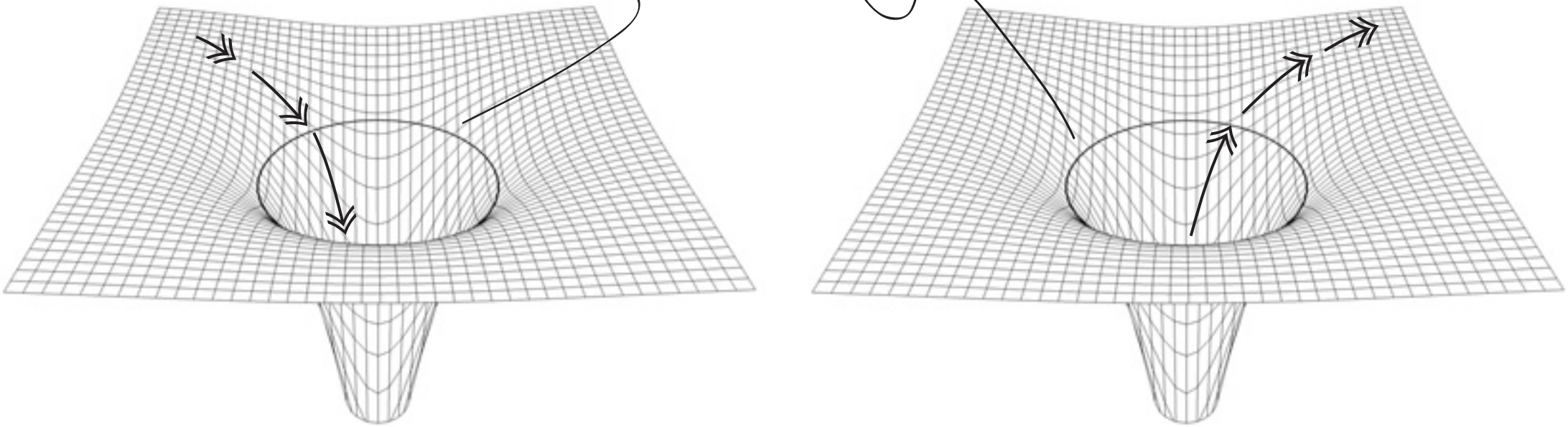


analogue gravity in moving media

Black hole

White hole

$$V = -\sqrt{\frac{2GM}{r}} = c$$

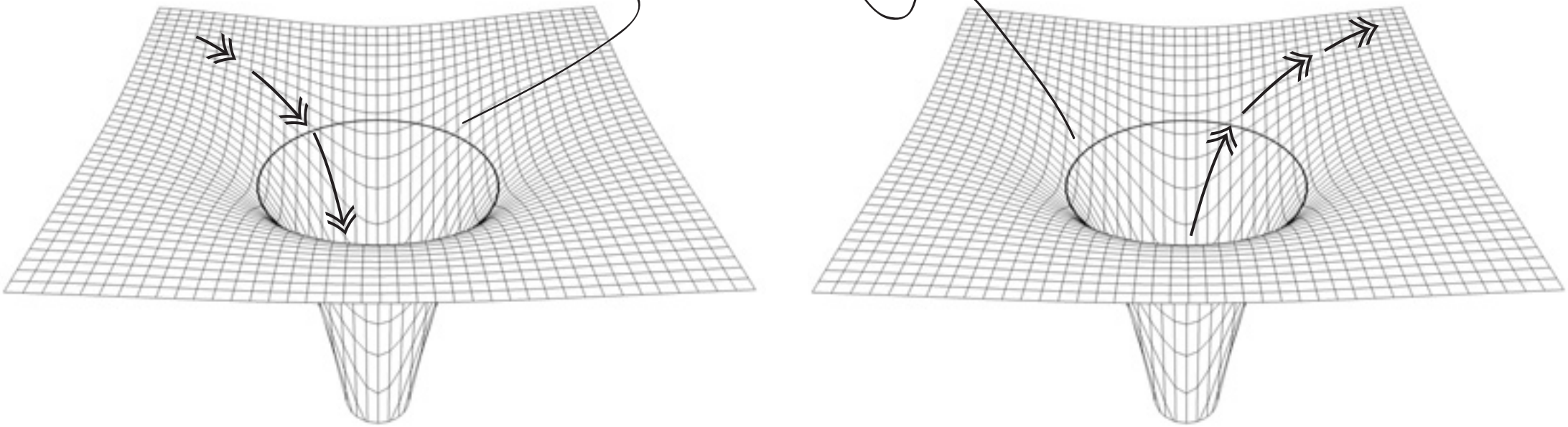


analogue gravity in moving media

Black hole

White hole

$$V = -\sqrt{\frac{2GM}{r}} = c$$



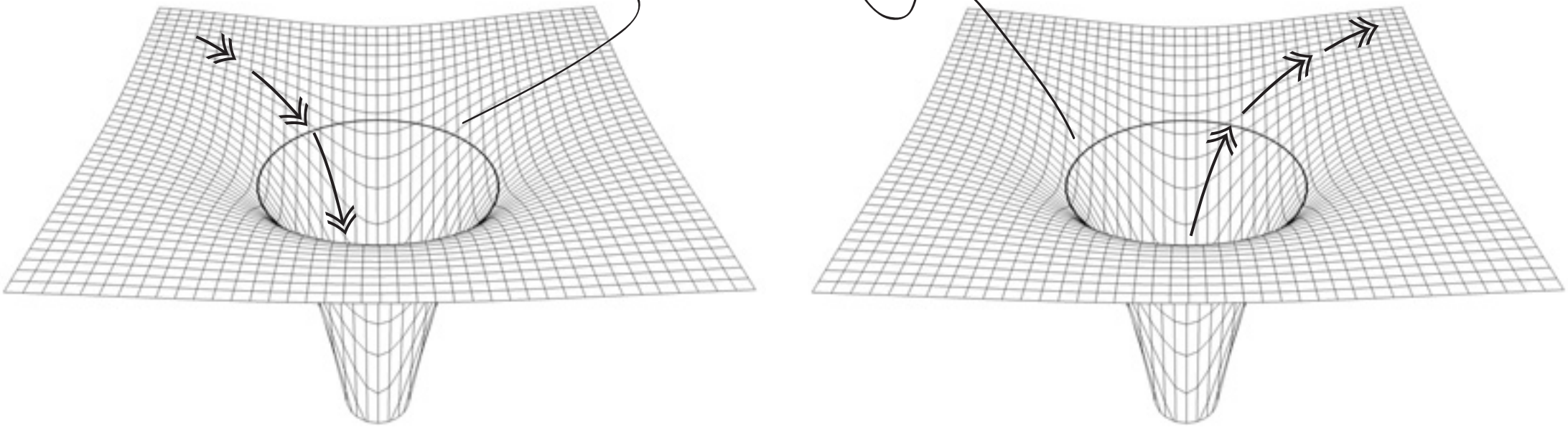
- White holes are time-reversed Black holes:
  at thermal equilibrium, WH emission = BH Hawking radiation (*Hawking, PRD - 1976*)

analogue gravity in moving media

Black hole

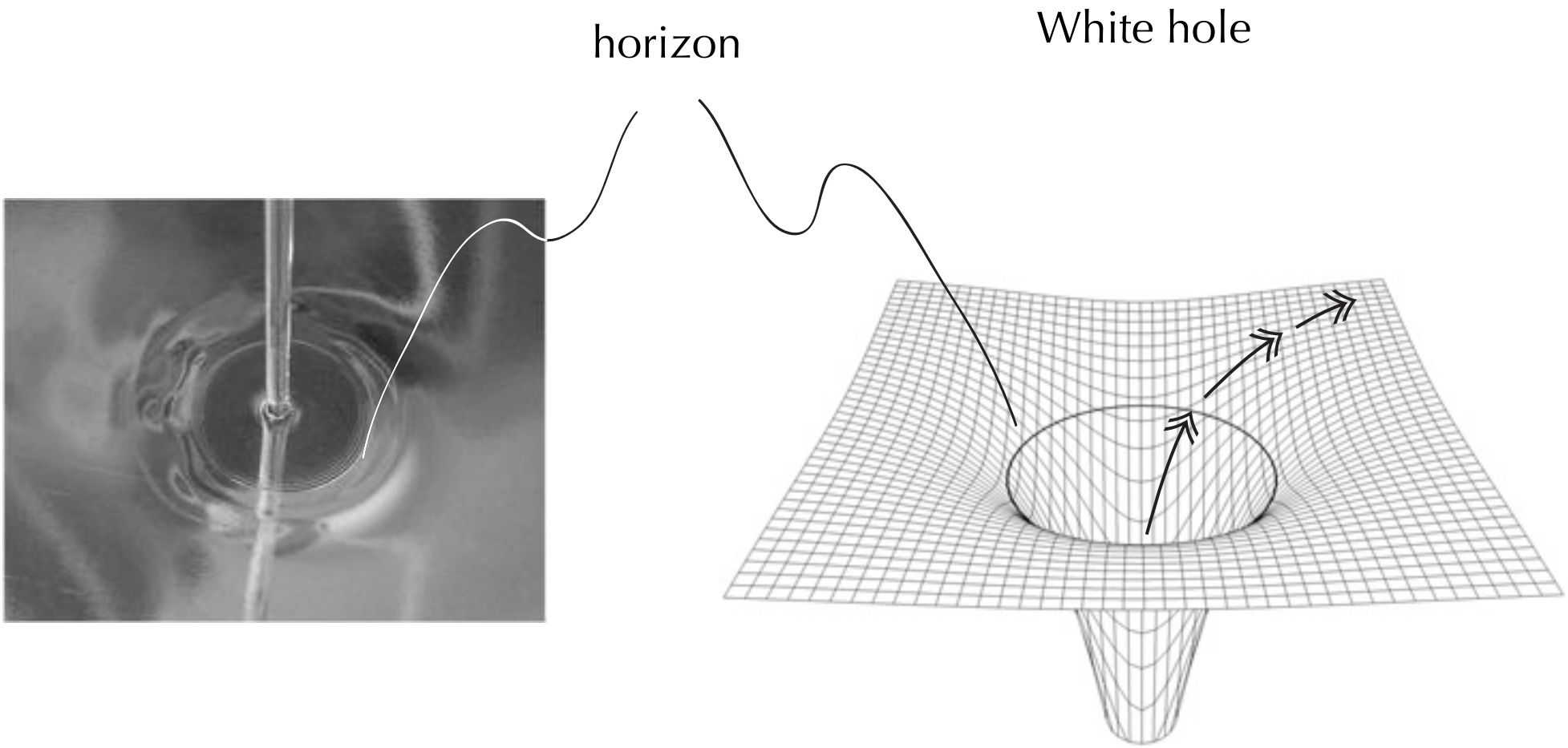
White hole

$$V = -\sqrt{\frac{2GM}{r}} = c$$



$$T_H = \frac{\hbar\kappa}{2\pi c k_B} \qquad \kappa = \frac{c^4}{4GM} \sim \frac{3 \cdot 10^{43}}{M} \; m/s^2 \qquad T_H \sim 10 \; nK$$

analogue gravity in moving media

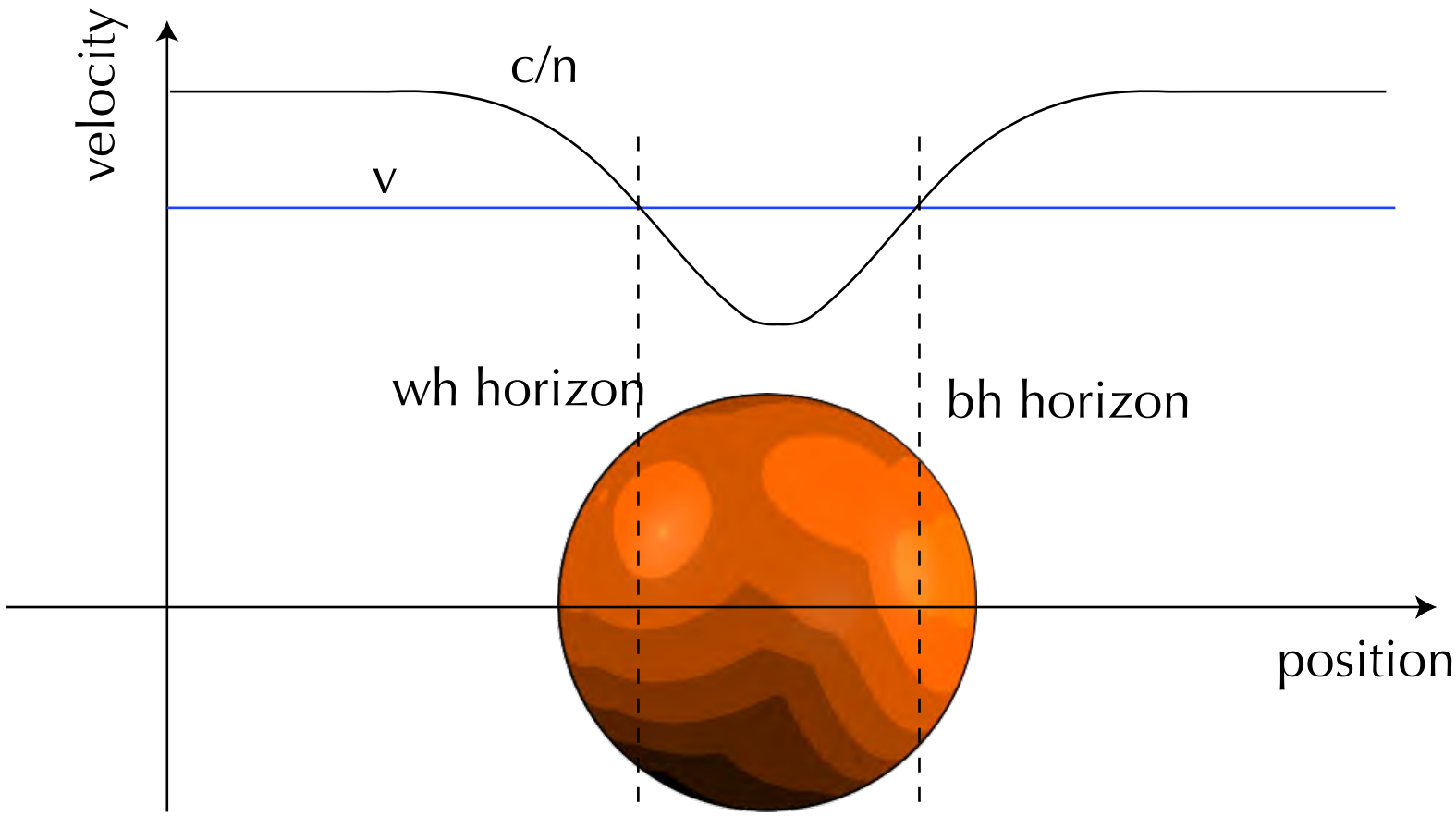


$$T_H = \frac{\hbar\kappa}{2\pi c k_B} \qquad \kappa = \frac{c^4}{4GM} \sim \frac{3 \cdot 10^{43}}{M} \; m/s^2 \qquad T_H \sim 10\ nK$$

analogue gravity in moving media

- create a flowing medium that can trap light

  waves are blocked when v=c/n -> horizon



analogue gravity in moving media

- Gordon metric -> “river” metric (*Belgiorno et al., PRD - 2011*)

$$ds^2 = c^2 dt'^2 - (d\tilde{x}'^2 - V dt')^2$$

$$V = \gamma^2 v \frac{n^2 - 1}{n}$$

analogue gravity in moving media

- Gordon metric -> “river” metric (*Belgiorno et al., PRD - 2011*)

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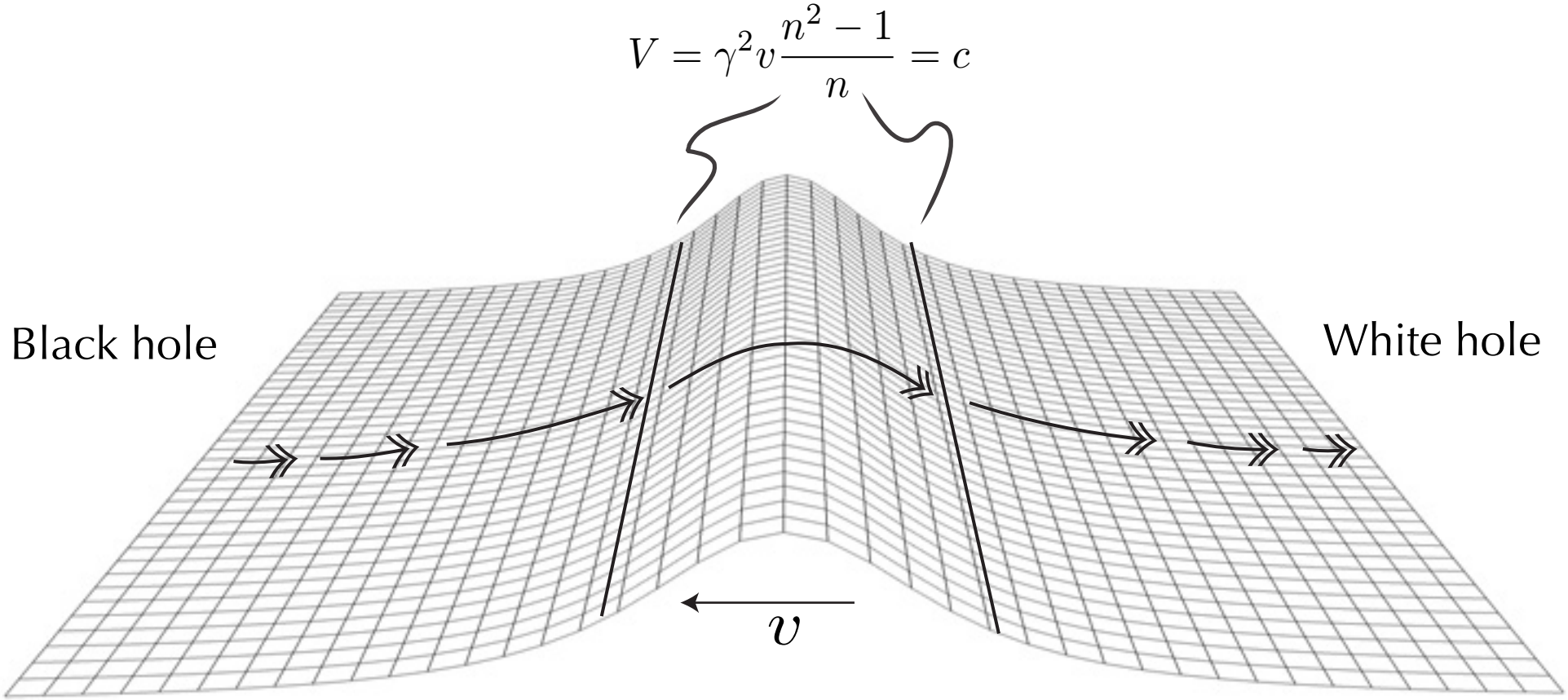
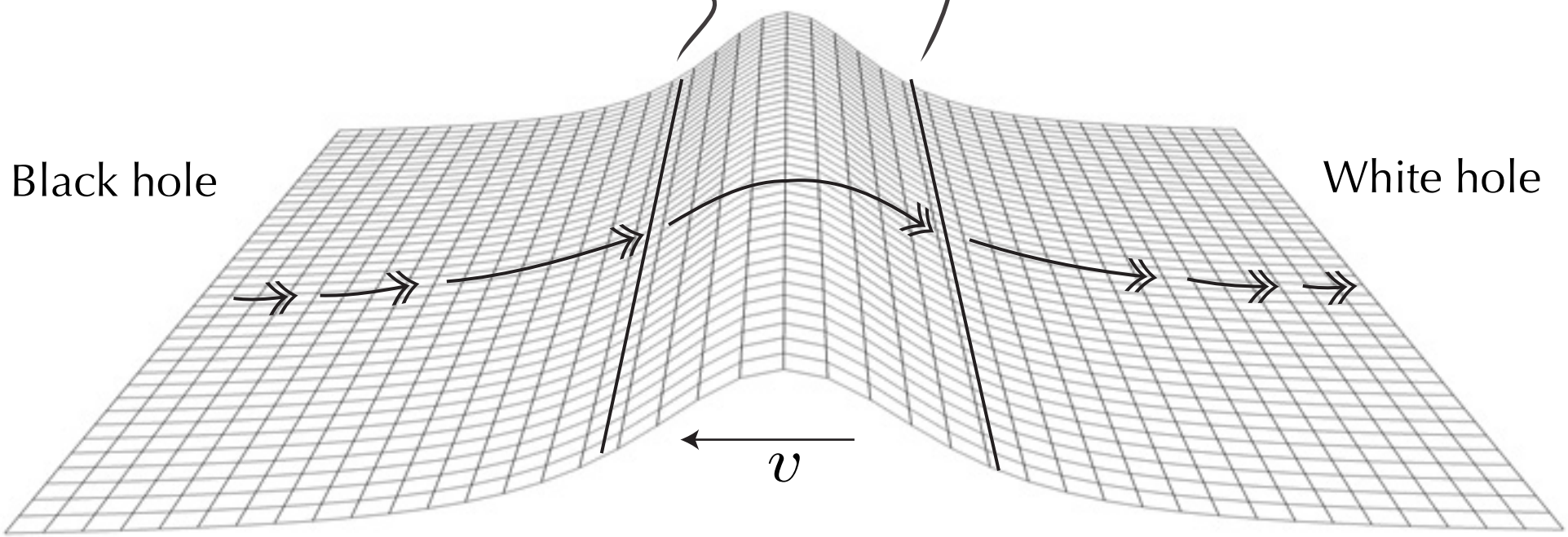


analogue gravity in moving media

- Analogue of a surface gravity (*Belgiorno et al., PRD - 2011*)

$$\kappa = \gamma^2 v^2 \left.\frac{dn}{dx}\right|_H \longrightarrow T_H = \frac{\hbar\kappa}{2\pi c k_B}$$

$$V = \gamma^2 v \frac{n^2 - 1}{n} = c$$



analogue gravity in moving media

- Quantum properties of the optical horizon (*Belgiorno et al., PRD - 2011, Philbin et al., Science - 2008*)

in collaboration with
S. Cacciatori, V. Gorini
F. Belgiorno, E. Rubino

Quantum properties of the optical horizon

MAXWELL equations...

compare input (δn=0) and output (δn≠0) vacuum states

approximation:

full 4D model but dn has no transverse dependence

$$\frac{n(x-vt)^2}{c^2}\partial_t\Phi - \partial_x^2\Phi = 0$$

$$\downarrow$$

$$\Phi = Ae^{ip}$$ WKB approximation

$$\downarrow$$

$$u = x - vt$$ fwd propagating coordinate

$$w = x + vt$$ bwd propagating coordinate

$$\downarrow$$

$$p_\pm = F_\pm\left(w - \int^u dy \frac{1 \pm n(y)n/c}{1 \mp n(y)v/c}\right)$$ fwd/bwd propagating solutions

analogue gravity in moving media

- Quantum properties of the optical horizon

$$p_+ = F_+\left(w - \int^u dy \frac{1+n(y)\frac{v}{c}}{1-n(y)\frac{v}{c}}\right)$$

$$n = n_0 + \delta n$$

$$\delta n = \delta n_0 e^{-(x-vt)^2}$$

$$\downarrow$$

$$\frac{c}{n_0+\delta n} < v < \frac{c}{n_0}$$

fwd solution diverges = horizon condition
(bwd solution does not diverge)

$$\downarrow$$

$$\int^u dy \frac{1+n(y)n/c}{1-n(y)v/c} \sim \frac{2c}{n'}\log(u)$$

log divergence -> blackhole-like

Quantum properties of the optical horizon

$$p_+ = F_+\left(w - \int^u dy \frac{1+n(y)\frac{v}{c}}{1-n(y)\frac{v}{c}}\right)$$

$$n = n_0 + \delta n$$

$$\delta n = \delta n_0 e^{-(x-vt)^2}$$

write "input" (no δn) state and "out" (with δn) state
-> Bogoliubov transformation
-> excess photons in the "out" state
-> blackbody emission

$$\langle N_\omega \rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

Bogoliubov coefficients relate output to input states
-> thermal distribution

$$T = \frac{\hbar v^2}{2\pi c(1 - vn/c)k_B}\frac{dn}{dx}$$

Blackbody temperature

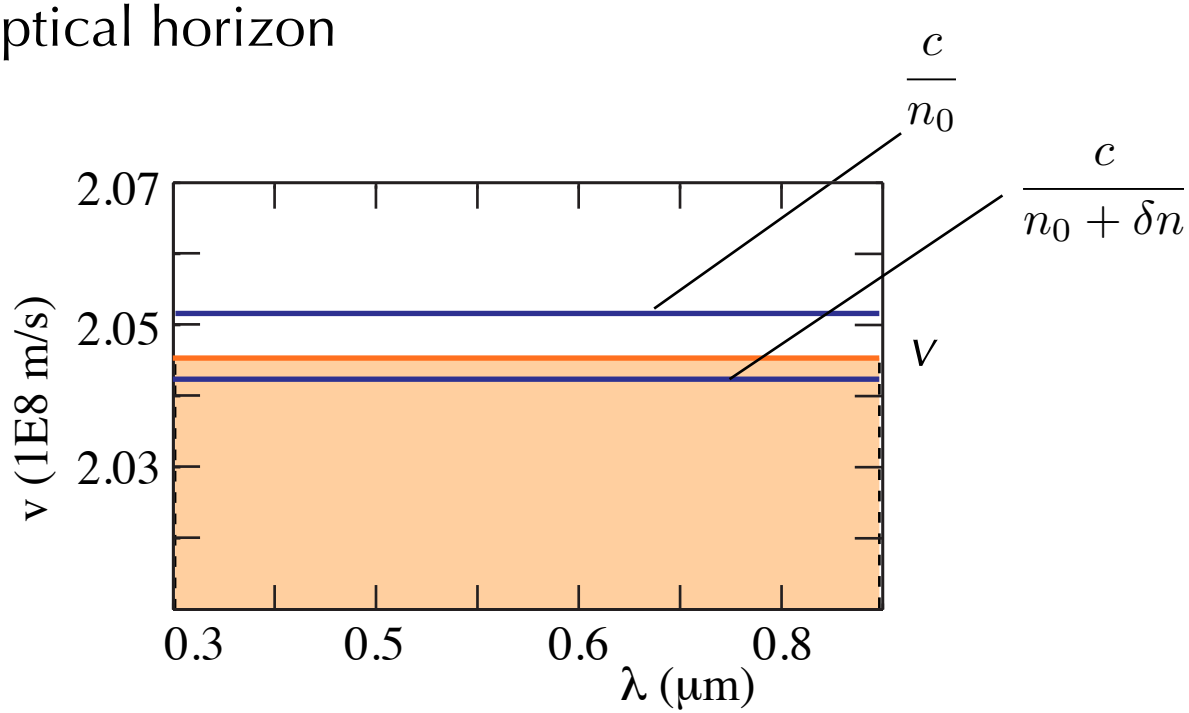
horizon emits radiation analogous to Hawking radiation!!

analogue gravity in moving media

- Dispersive properties of the optical horizon

$$\frac{c}{n_0 + \delta n} < v < \frac{c}{n_0}$$

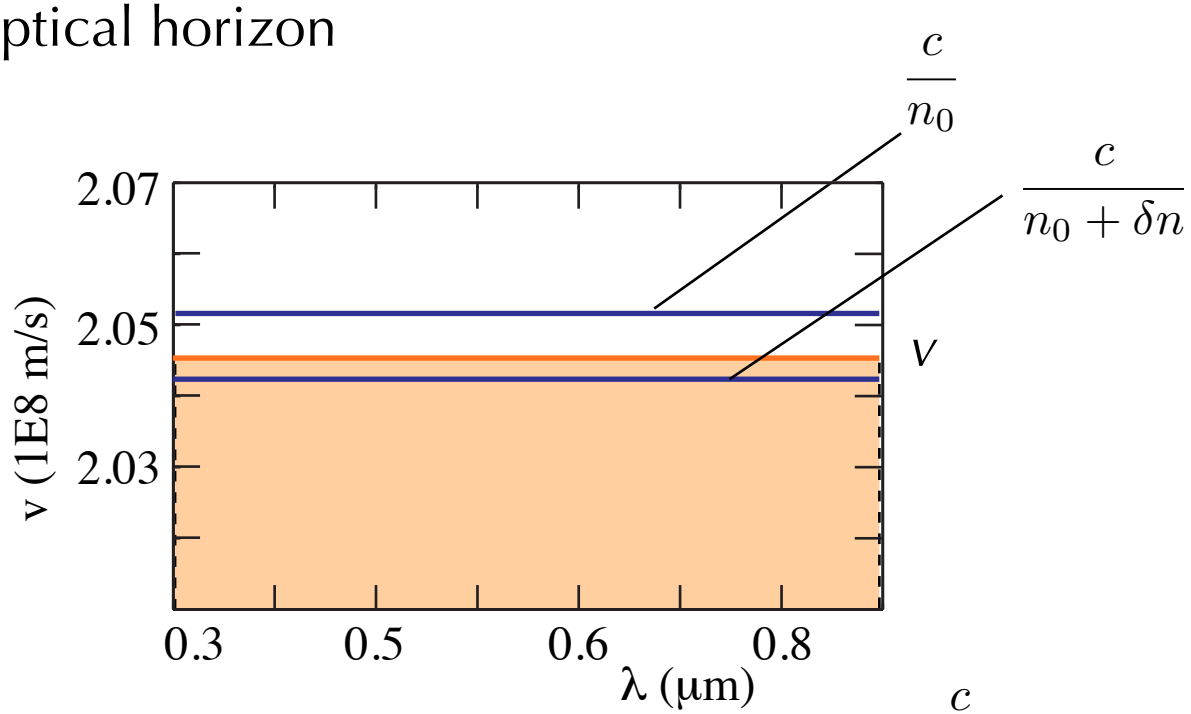
NO dispersion



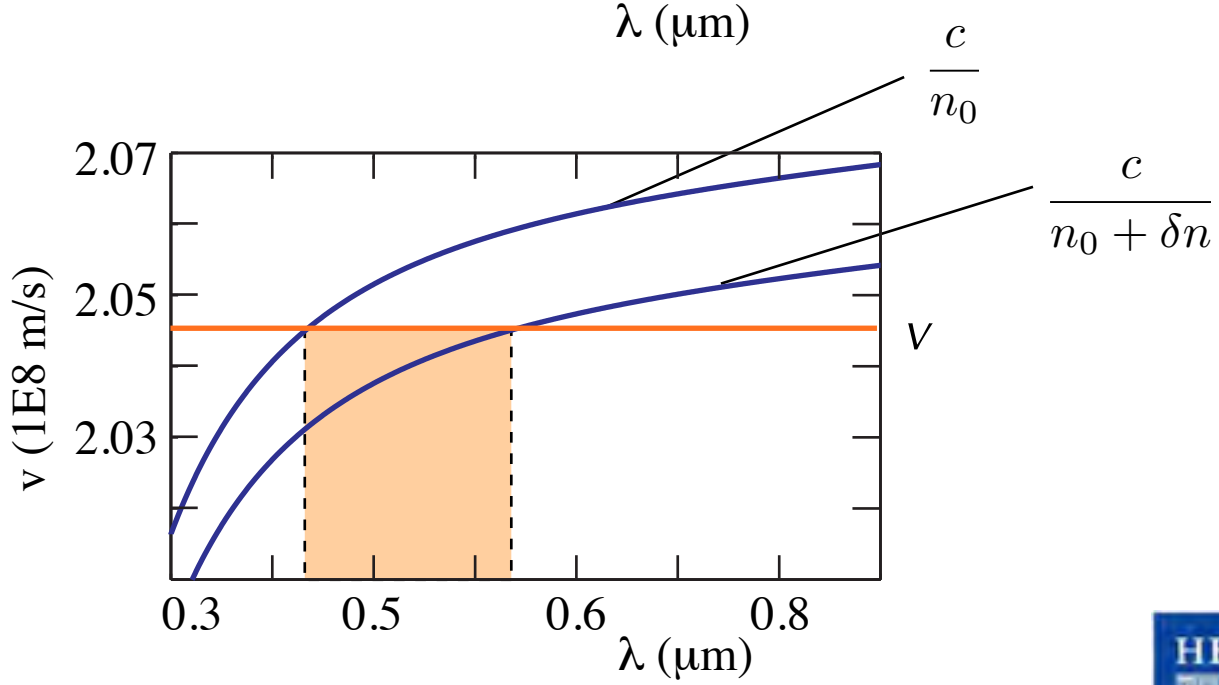
- Dispersive properties of the optical horizon

$$\frac{c}{n_0+\delta n} < v < \frac{c}{n_0}$$

NO dispersion

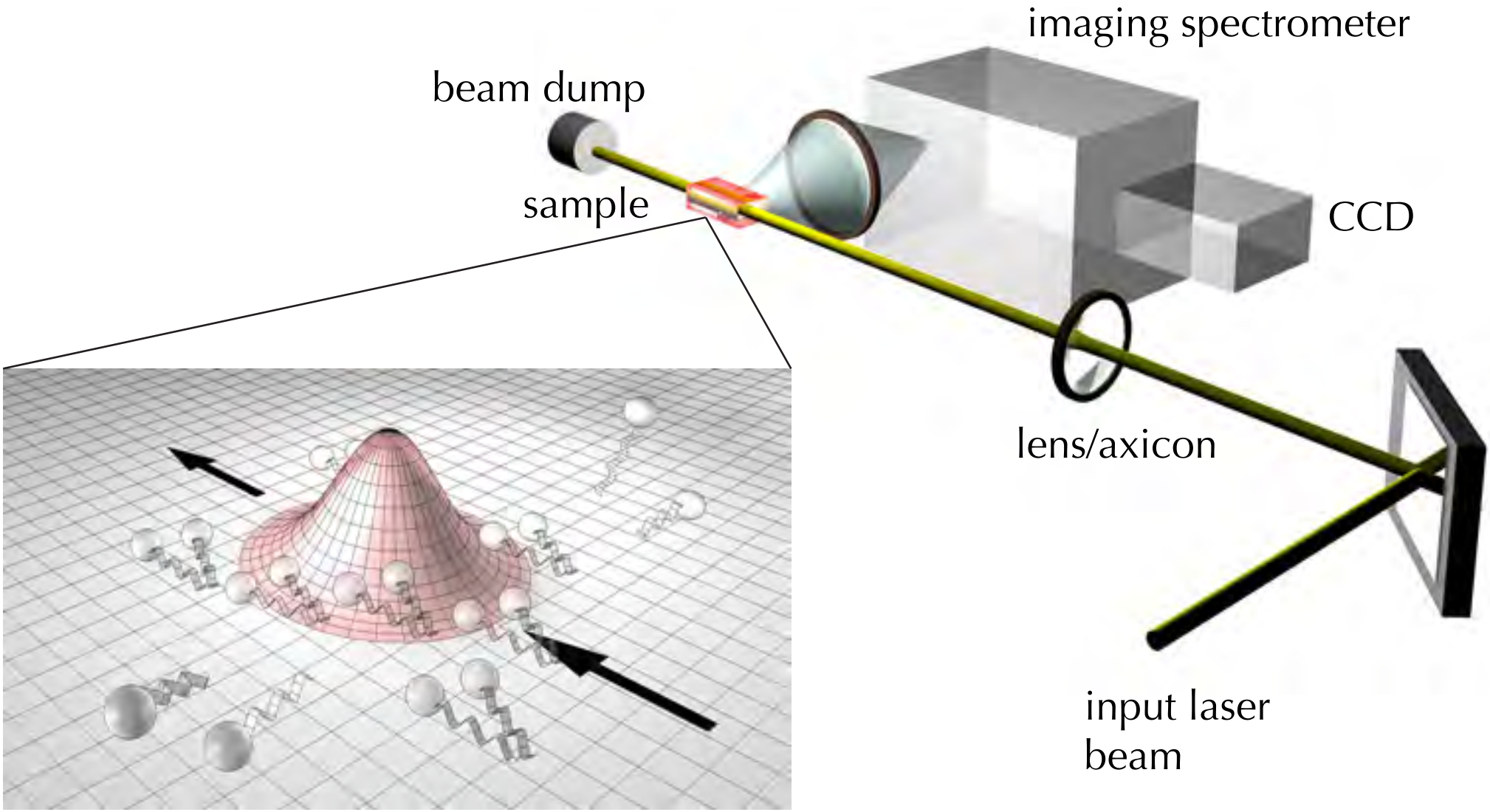


WITH dispersion



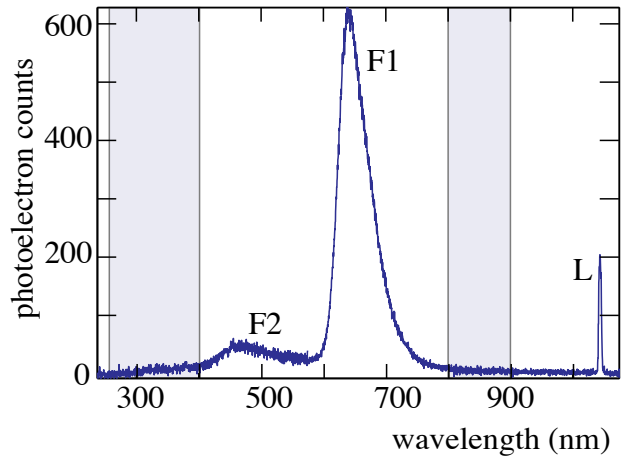
analogue gravity in moving media

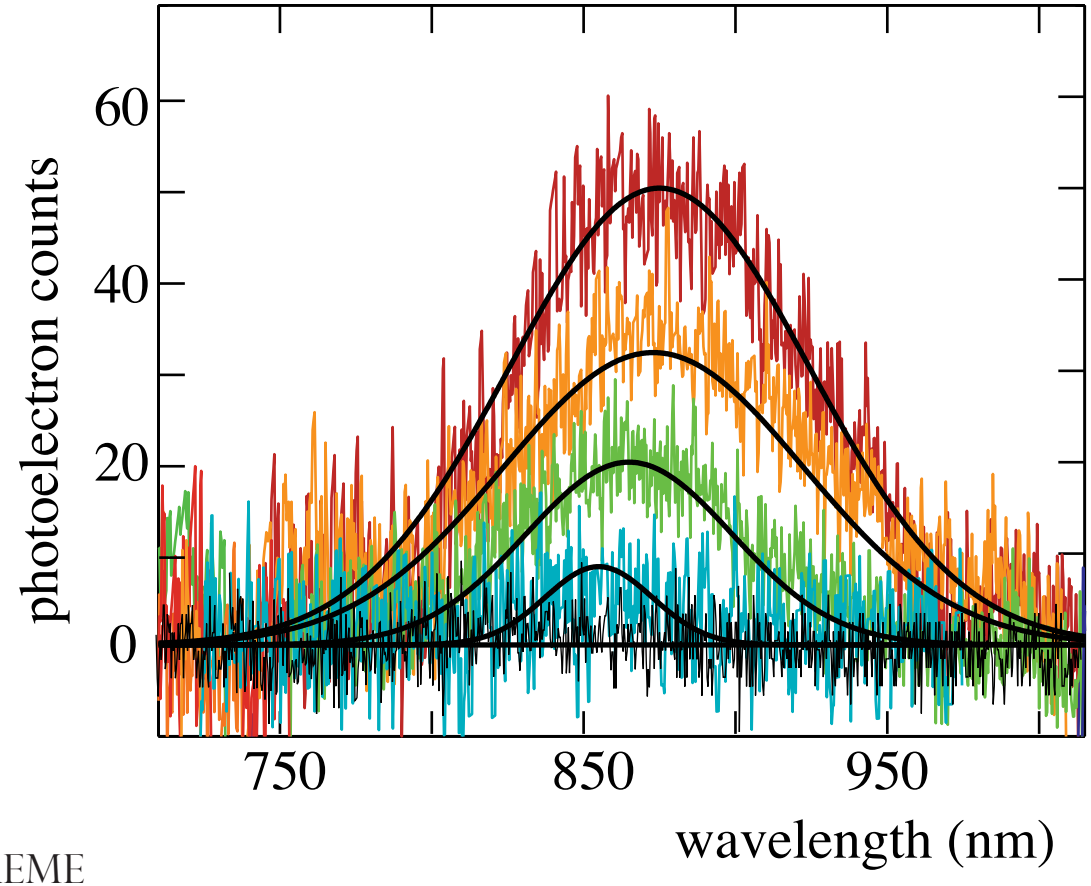
- experiments



experiments

measurements with Bessel pulses
-> control vg
-> control peak intensity

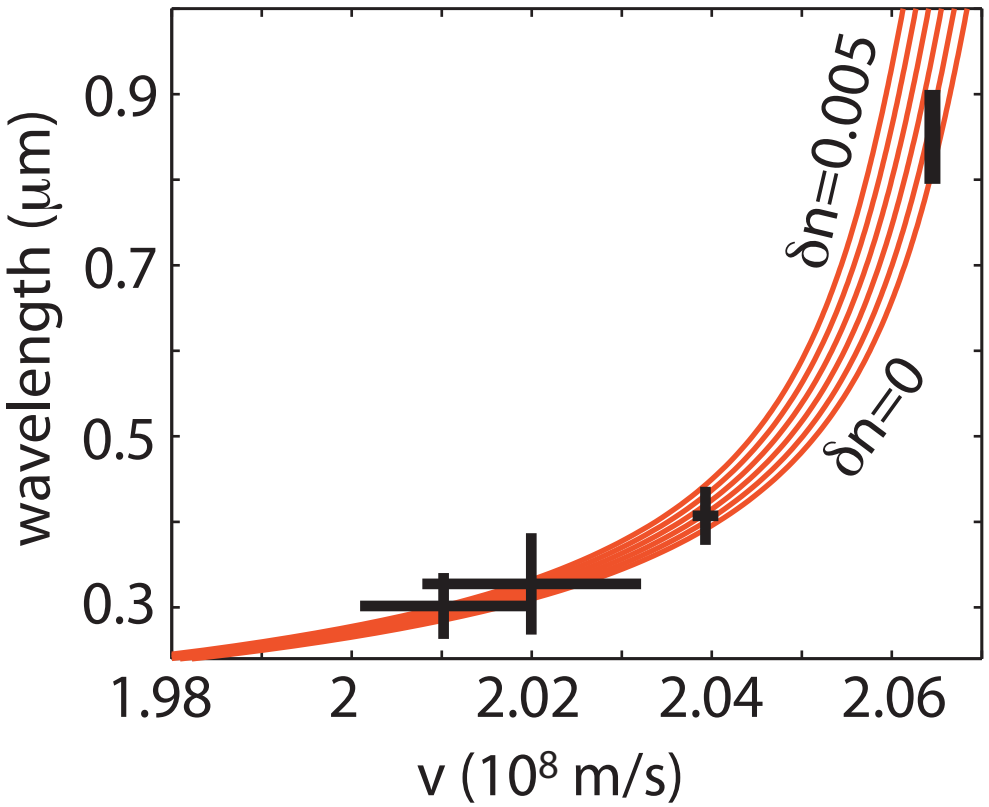


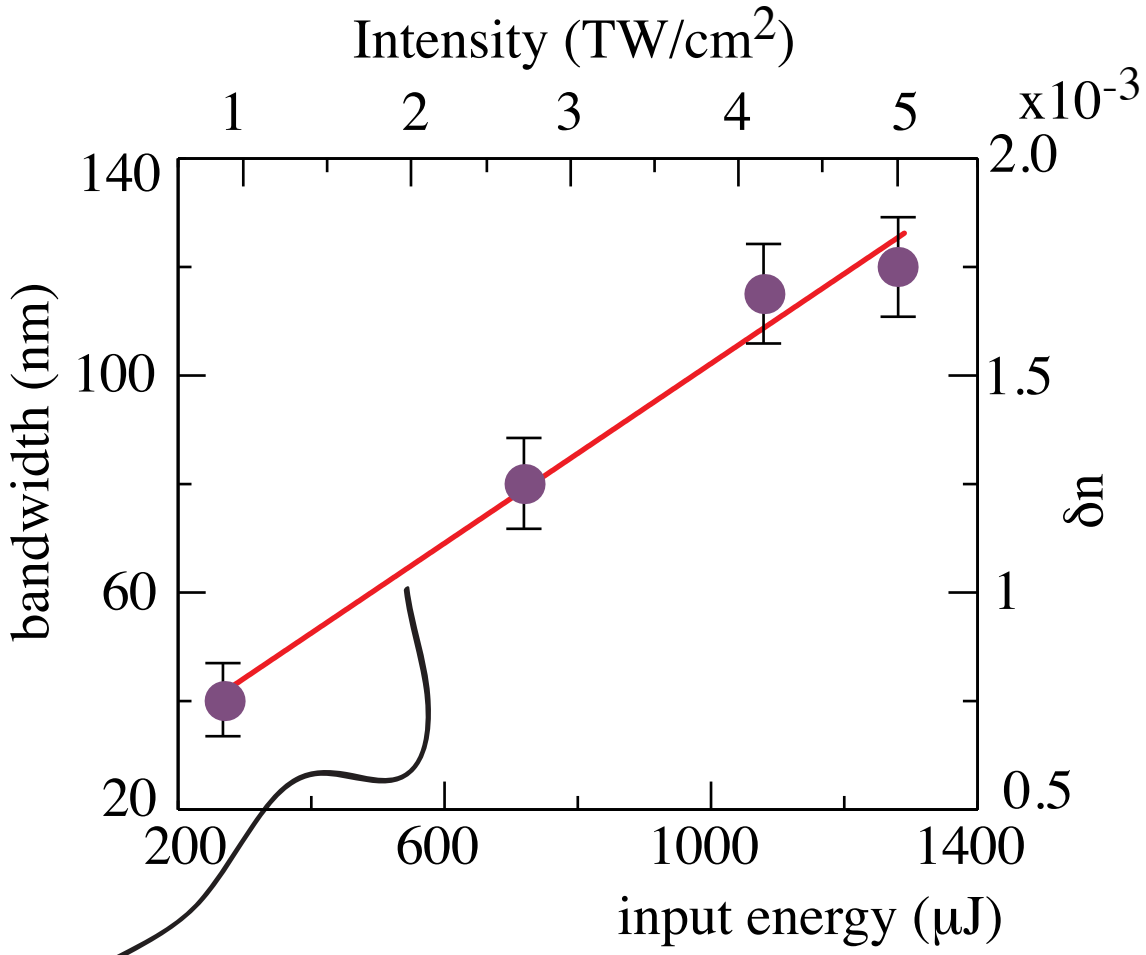


input E (microJ):
- 1280
- 1080
- 720
- 270
- Gaussian excitation

analogue gravity in moving media

experiments

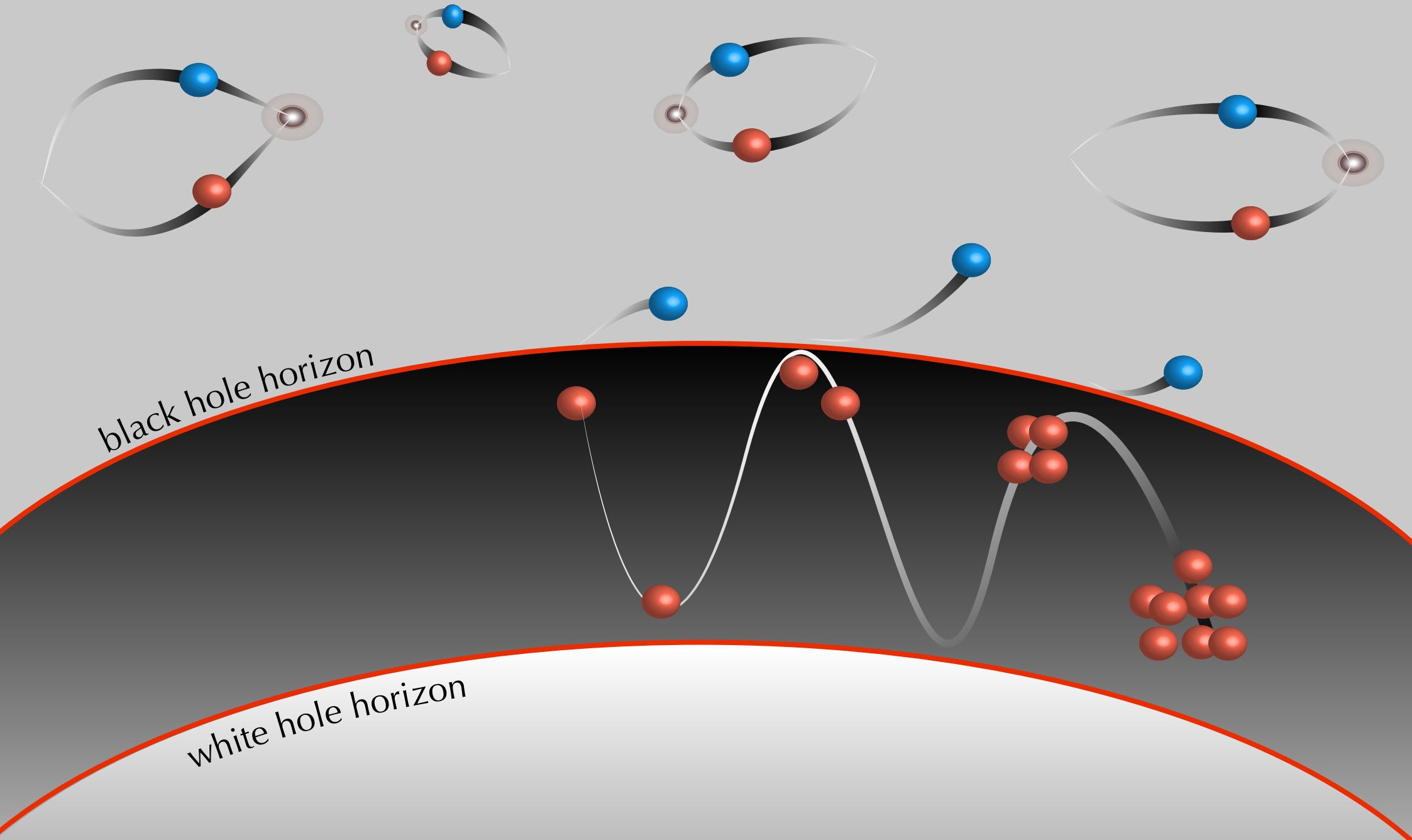




slope gives directly n_2!!

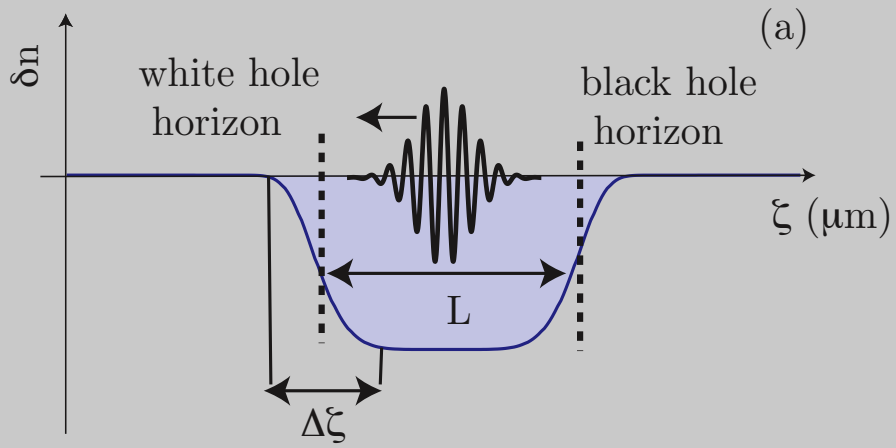
$n_2 = 2.8 +/- 0.5\ 10^{-16}$ W/cm^2

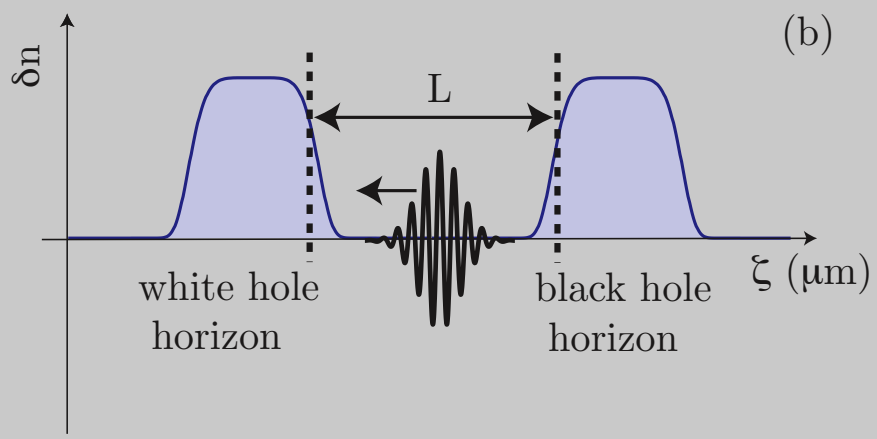
The Black Hole Laser



The Black Hole Laser

Diamond - ideal medium -> huge transparency range
can be used to build a **black hole laser**





[1] Faccio et al., “Optical black holes”, Class. Quantum Grav., submitted (2012)

Living Rev. Relativity, **14**, (2011), 3
http://www.livingreviews.org/lrr-2011-3
(Update of lrr-2005-12)

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Analogue Gravity

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Observation of self-amplifying Hawking radiation in an analogue black-hole laser

Jeff Steinhauer

nature physics
ARTICLES
PUBLISHED ONLINE: 15 AUGUST 2016 | DOI: 10.1038/NPHYS3863

Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

We observe spontaneous Hawking radiation, stimulated by quantum vacuum fluctuations, emanating from an analogue black hole in an atomic Bose–Einstein condensate. Correlations are observed between the Hawking particles outside the black hole and the partner particles inside. These correlations indicate an approximately thermal distribution of Hawking radiation. We find that the high-energy pairs are entangled, while the low-energy pairs are not, within the reasonable assumption that excitations with different frequencies are not correlated. The entanglement verifies the quantum nature of the Hawking radiation. The results are consistent with a driven oscillation experiment and a numerical simulation.

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Laser pulse analogues for gravity and analogue Hawking radiation

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Lecture Notes in Physics 870

Daniele Faccio · Francesco Belgiorno
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Analogue Gravity Phenomenology

Analogue Spacetimes and Horizons, from Theory to Experiment