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Weak force detection & Quantum Foundations

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Some remarks, before starting

- We will discuss experiments with mechanical resonators aiming at detecting weak forces.

- Most of these experiments feature essentially “classical” features, and quantum effects are actually not much important (with the exception of quantum limits on the detection process)

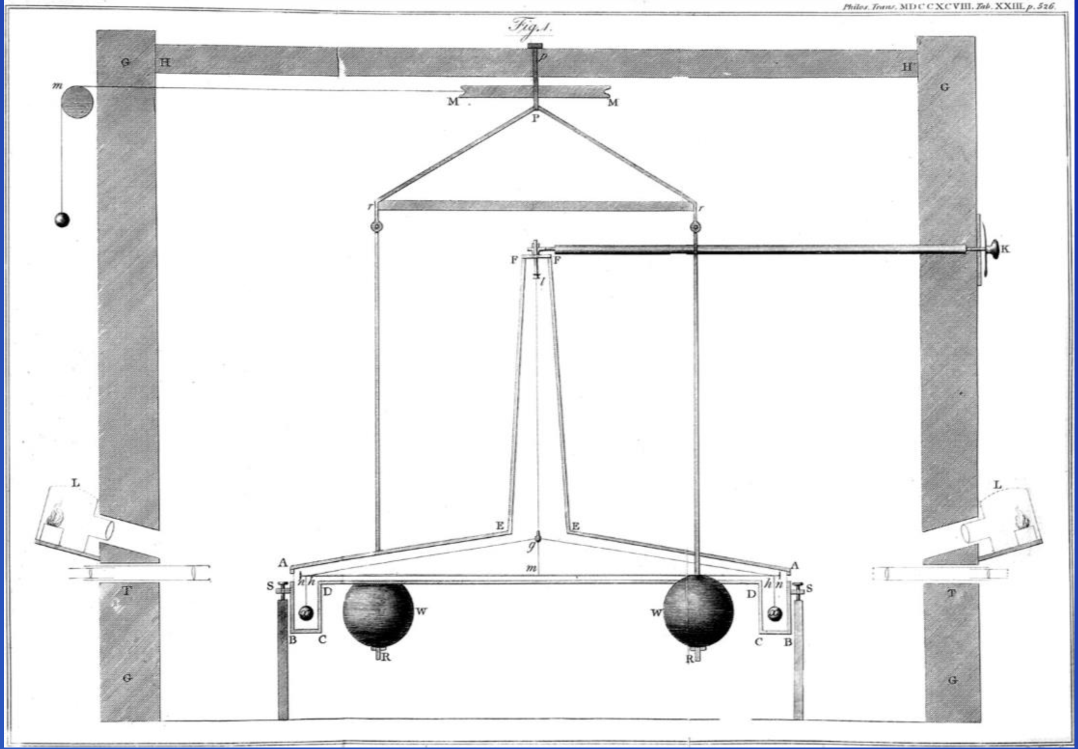
- Surprisingly, it turns out that they can be used to investigate the quantum to classical crossover in some relevant framework. In particular we will focus on noninterferometric testing of collapse models based on mechanical systems.

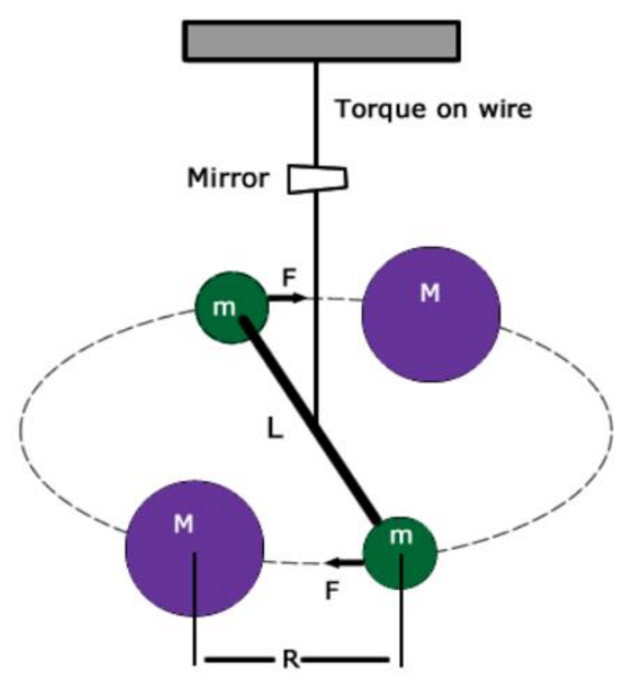
Outline

1) General features
    - fluctuation-dissipation & thermal noise
    - dissipation in mechanical systems
    - detector noise
    - quantum limits and beyond

2) Some relevant technologies and applications to quantum foundational problems
    - nanomechanical systems
    - gravitational wave detectors
    - test of collapse models

A bit of history



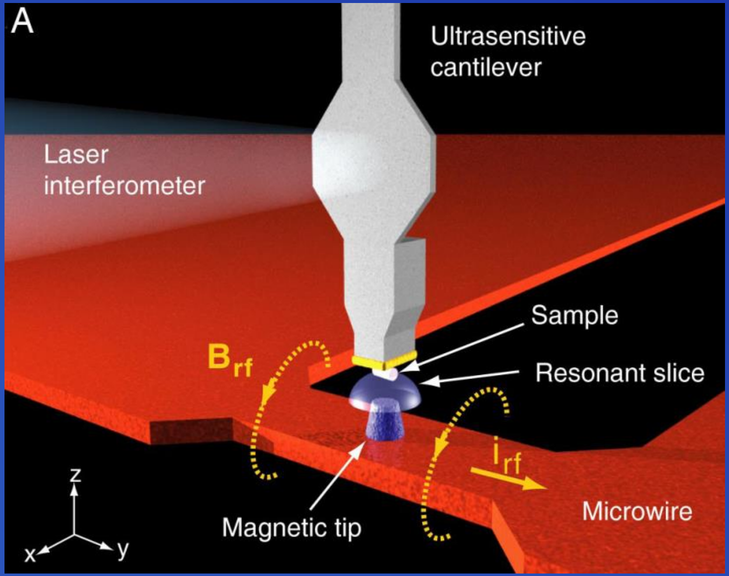


Cavendish Torsional Balance (1798)
[actually conceived by geologist J. Michell, 1783]

Remarkable: Best measurement of G today are done in the same way!

Some more modern stuff

Ultrasensitive force microscopy: detect forces from single spins



Gravitational wave detectors: detect fluctuations in spacetime metrics

What forces can one measure?

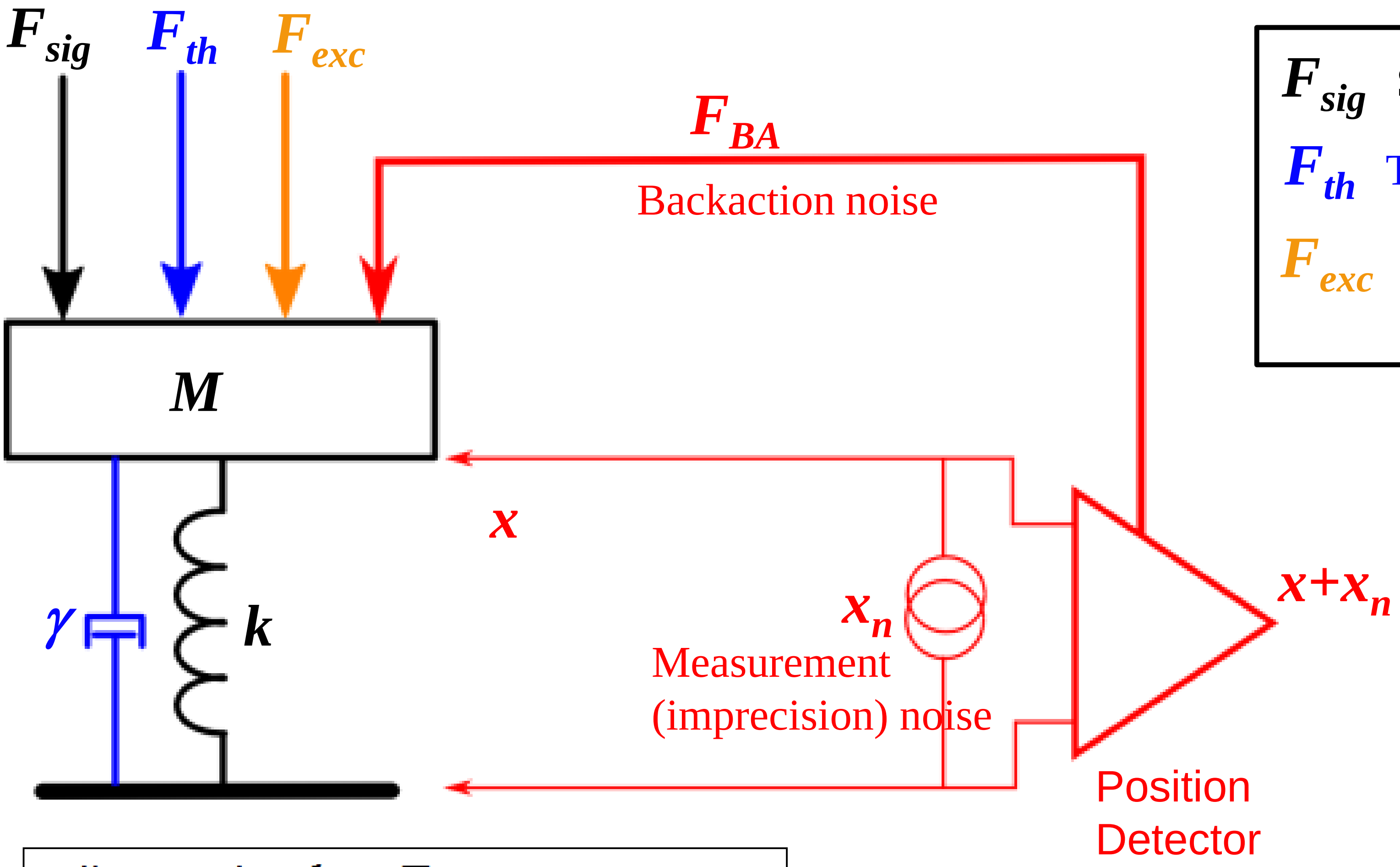
Fundamental Physics

- Gravitational forces
- Gravitational waves
- Low distance deviations of Newton law (hidden dimensions)
- Hidden or Dark-matter like particles (axions, dilatons, spin-gravity models …)
- Casimir forces
- Radiation pressure forces (optomechanics)

Applied Physics

- Atomic force microscopy (surface imaging with atomic resolution)
  & Variations of AFM (i.e. magnetic forces)
- Forces due to manipulation of single spins (MRFM)
- MEMS: anything that can be transduced into a force
  (pressure, acceleration, mass, …)

General scheme



F_{sig}	Signal force
F_{th}	Thermal noise
F_{exc}	Excess noise (seismic, acoustic …)

$$m\ddot{x} + m\gamma\dot{x} + k = F_{TOT}$$

$$x(\omega) = \frac{1}{-m\omega^2 - im\gamma\omega + k} F_{TOT}(\omega)$$

$$x_{out} = x + x_n$$

Fluctuation-Dissipation Theorem (FDT)

Valid for macroscopic variables with linear response + thermal equilibrium with a bath

$$x(\omega) = \chi(\omega) F(\omega)$$

“Coordinate” “Force”

$$\chi = \chi' + i\enclose{circle}{\chi''}$$

Susceptibility

FDT formulas

$$\boxed{S_{xx}} = \frac{2k_B T \enclose{circle}{\chi''}}{\omega}$$

$$S_{FF} = \frac{2k_B T}{\omega} \frac{\chi''}{|\chi|^2}$$

- Fluctuation & dissipation reflect the same physical mechanism: the coupling to the thermal bath
- Two-sided → one-sided (only ω>0) spectrum: makes sense only for classical noise, as S(-ω)=S(ω)

  $$2k_B T \rightarrow 4k_B T$$

- Classical → Quantum (don’t need for this lecture)

  $$k_B T \rightarrow \hbar\omega \coth\left[\frac{\hbar\omega}{2k_B T}\right]$$

Relevant examples

1) Mechanical resonator (brownian noise)

$$m\ddot{x} + m\gamma\dot{x} + k = F$$

$$\chi^{-1}(\omega) = -m\omega^2 - im\gamma\omega + k$$

$$S_{FF} = 4k_BTm\gamma$$

2) Magnetization fluctuations in a macroscopic magnet

$$M = \chi(\omega)B$$

$$S_{MM} = \frac{4k_BT\chi''}{\omega}$$

3) Noise voltage across a resistor (Nyquist-Johnson noise)

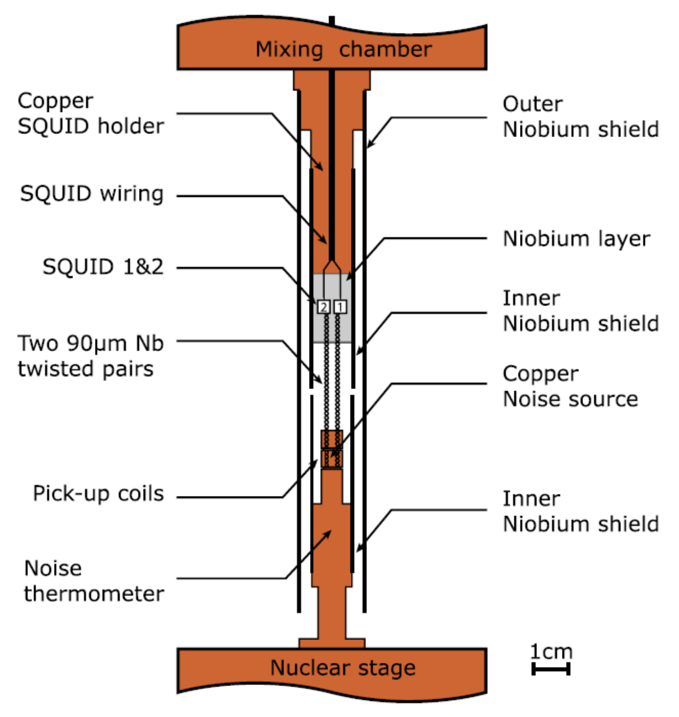
$$V = RI = R\dot{Q}$$

$$\chi = \frac{i}{\omega R}$$

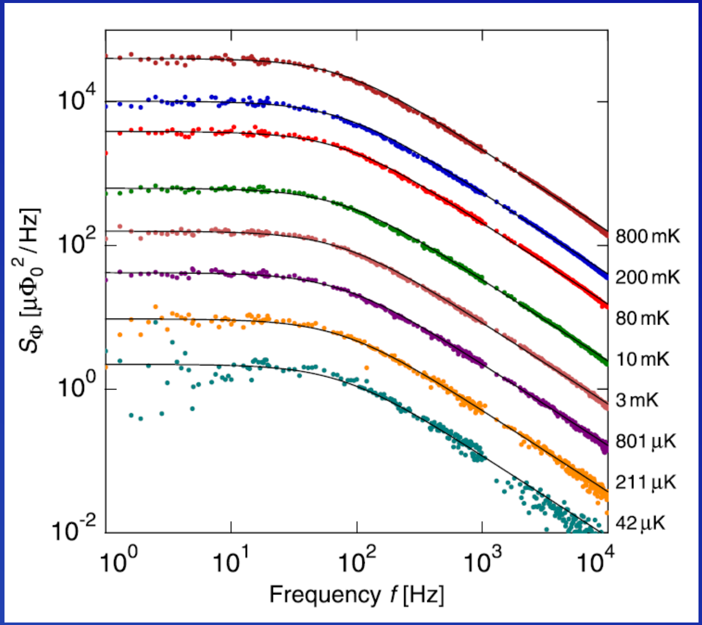
$$S_{VV} = 4k_BTR$$

Note: FDT assumes [Force * Coordinate] = [Energy]

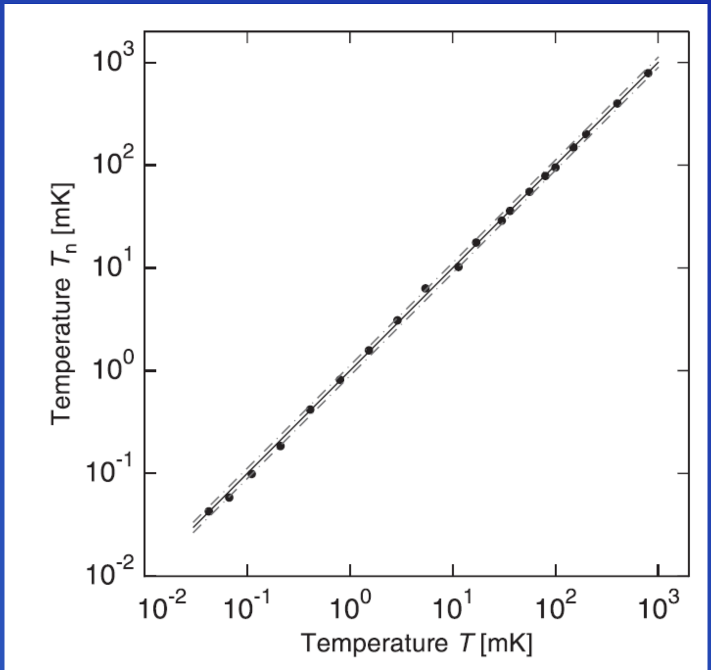
Application of FDT: noise thermometry



Enss et al, Appl. Phys. Lett. 103, 052605 (2013)



Magnetic noise from a piece of copper (thermal conducting currents)



- Primary thermometer
- Linear over 5 decades

Back to mechanical resonator

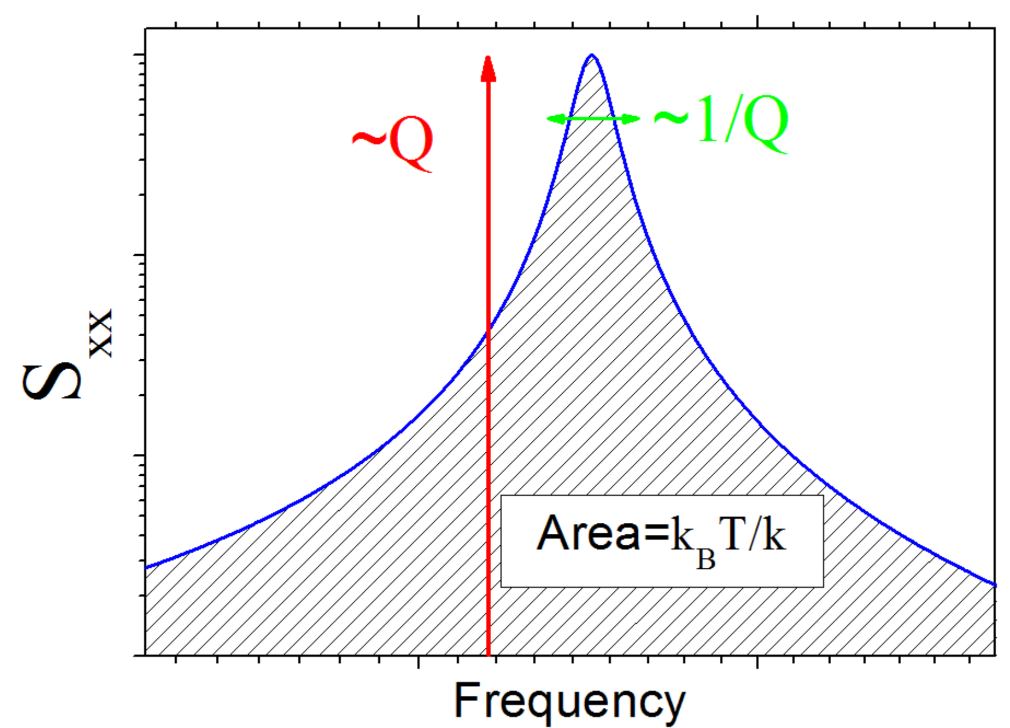
$$\omega_0^2 = \frac{k}{m} \qquad Q = \frac{\omega_0}{\gamma}$$

$$\chi^{-1}(\omega) = m\left(-\omega^2 + \omega_0^2 - i\frac{\omega\omega_0}{Q}\right)$$

$$S_{FF} = 4k_B T\frac{m\omega_0}{Q}$$

$$S_{xx}(\omega) = \frac{4k_B T\omega_0 / mQ}{\left(-\omega^2 + \omega_0^2\right)^2 + \left(\omega\omega_0 / Q\right)^2}$$

$$\left\langle x^2 \right\rangle = \frac{1}{2\pi}\int_0^\infty S_{xx}(\omega)d\omega = \frac{k_B T}{k}$$



Classical Equipartition result : $\frac{1}{2}k_B T = \frac{1}{2}k\left\langle x^2 \right\rangle$

Area is always equal to $k_B T/k$ (independently of Q) BUT

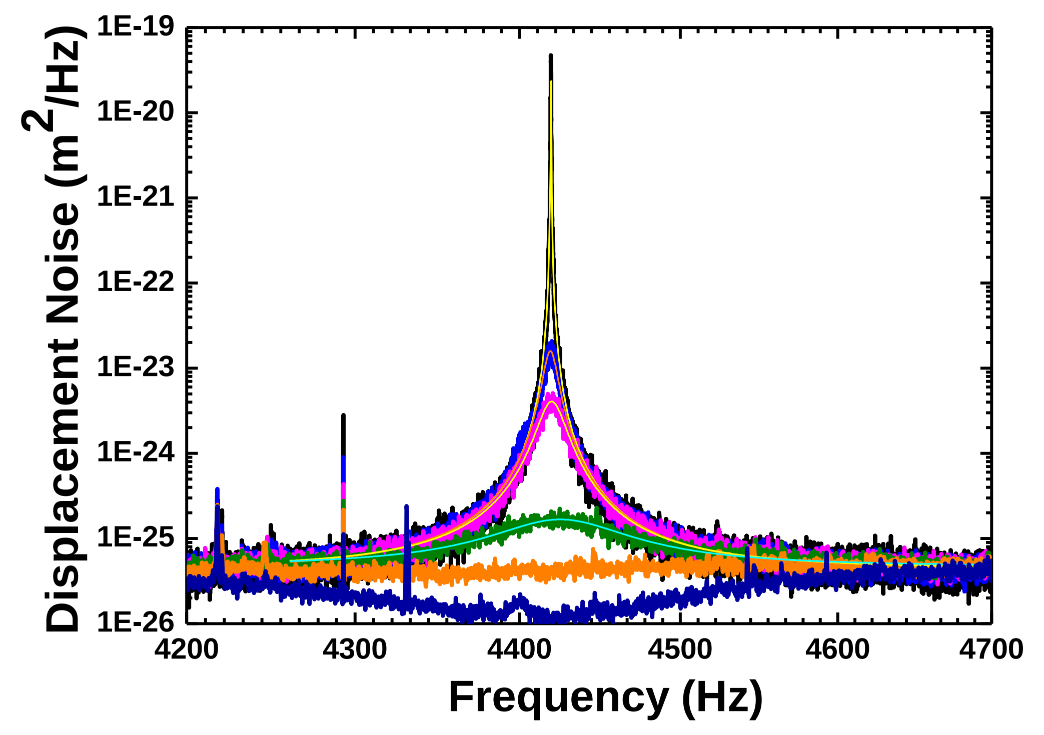
- the characteristic (averaging) time is $\tau = Q/\omega_0$
- the force noise is proportional to T/Q !

Cold Damping

The actual quality factor is modified by the measurement system.

Two possible mechanisms:

- Optomechanical (cavity) cooling
- Feedback cooling



A. Vinante et al, APL 2012

Change the variance of the noise
(By changing the dynamical Q to Qa)

$$S_{xx} = \frac{4k_BT\,\omega_0/mQ}{\left[\left(\omega^2-\omega_0{}^2\right)^2+\left(\omega\omega_0/Q_a\right)^2\right]}$$

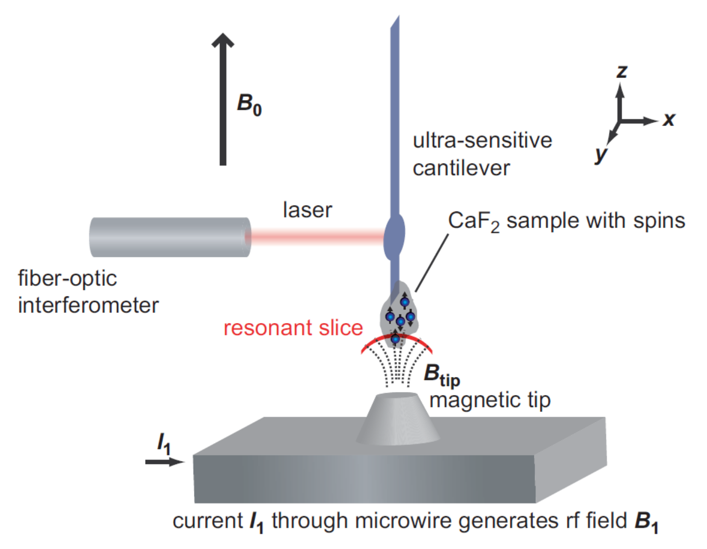
$$\left\langle x^2\right\rangle = \frac{k_BT}{k}\frac{Q_a}{Q}$$

DOES NOT CHANGE:

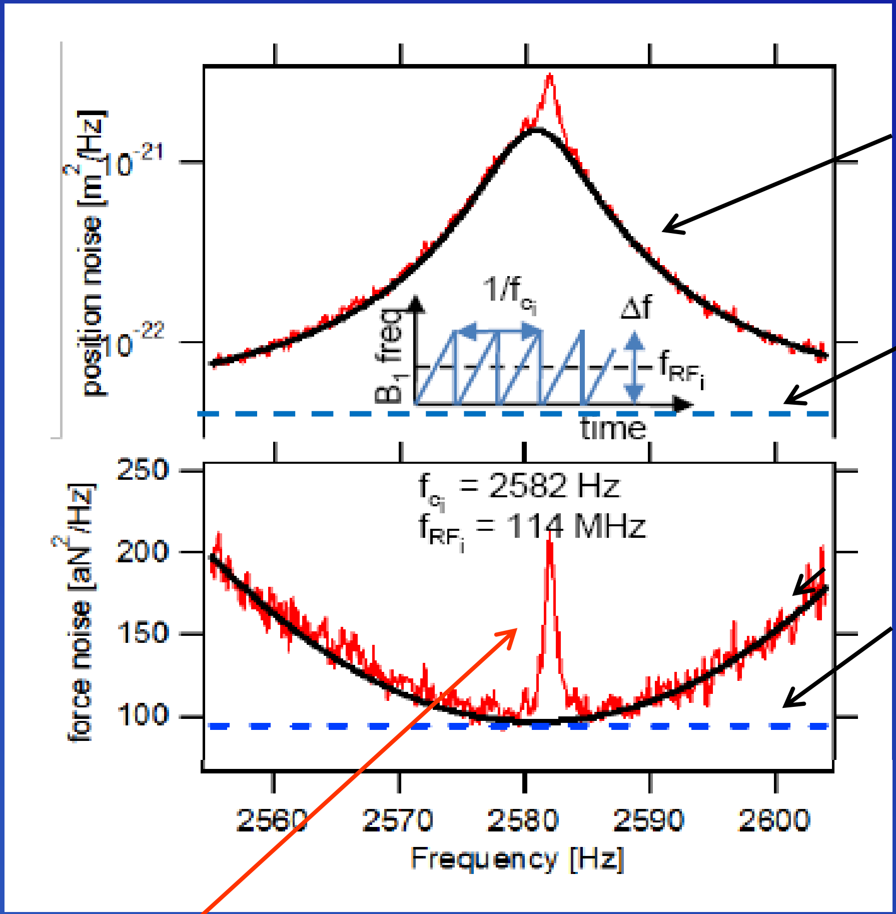
1) The force noise (numerator)
2) The Signal to Noise Ratio
   (Force signal and force noise change the same way)

How a real force measurement works

Displacement



T. Oosterkamp et al,
Appl. Phys. Lett. 96, 083107 (2010)



Thermal motion of cantilever

Measurement noise floor

Thermal force noise

Force

Force signal due to rf-manipulated spins in the sample

Important parameters

- Thermal force noise spectral density

$$S_{FF} = 4k_B T \frac{m\omega_0}{Q} = 4k_B T \frac{\sqrt{km}}{Q}$$

- If we wish to measure the lowest possible force, for a given mass, we need:

  low damping (low frequency – high Q, low k)

  low temperature

- Obvious, but important:

  What really matters is the Signal/Noise ratio, not the noise alone !

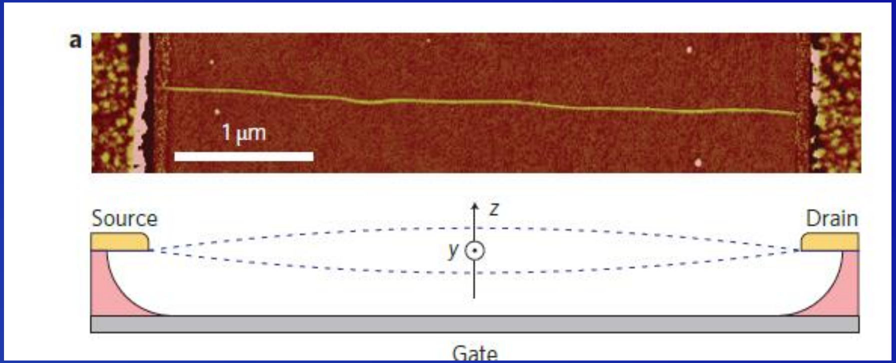
$$SNR = \frac{\left|F_{Sig}(\omega)\right|^2}{S_{FF}}$$

This depends on which kind of force you are looking for

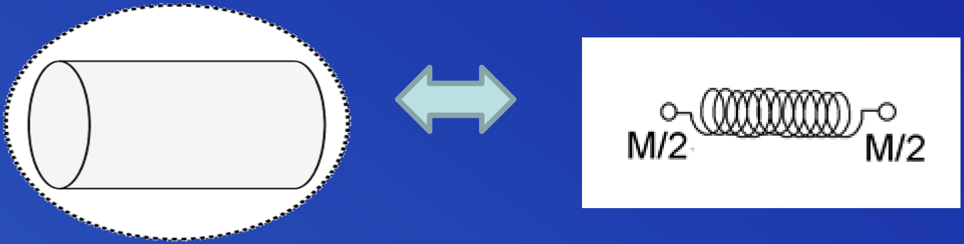
- Carbon Nanotube (A. Bachtold et al, Nature Nanotech, 2013)

$$m = 10^{-20}\ \mathrm{kg}\ ,\ Q = 5\times 10^{4}\ ,\ f_0 = 5\ \mathrm{MHz}$$

$$\sqrt{S_{FF}} = 1.2\times 10^{-20}\ \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}$$



- Resonant Bar gravitational wave detectors (AURIGA detector, Italy)

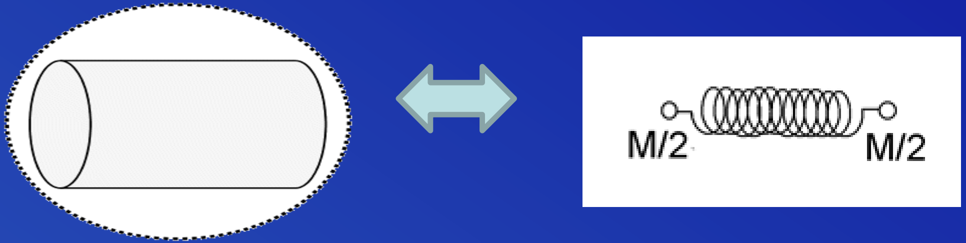


$$m = 10^{3}\ \mathrm{kg}\ ,\ Q = 5\times 10^{6}\ ,\ f_0 = 900\ \mathrm{Hz}\ ,\ T{=}100\ \mathrm{mK}$$

$$\sqrt{S_{FF}} = 3\times 10^{-12}\ \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}$$

BUT: Which one is better to detect gravitational forces?

What minimum gravitational wave ?



$$\frac{\Delta L}{L} = \frac{1}{2} h \quad \Rightarrow \quad F = k\Delta L = \frac{1}{2} m\omega_0{}^2 L h \quad (L \approx 1\,\mathrm{m})$$

$$\sqrt{S_{FF}} = 3\times10^{-12} \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}} \quad \Rightarrow \quad \sqrt{S_{hh}} = \frac{2\sqrt{S_{FF}}}{m\omega_0{}^2 L} = 3\times10^{-22} \frac{1}{\sqrt{\mathrm{Hz}}}$$

or equivalently

$$\sqrt{S_{LL}} = 3\times10^{-22} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$$

Large and rigid: Small absolute displacements

Small and soft: Small absolute forces

Temperature

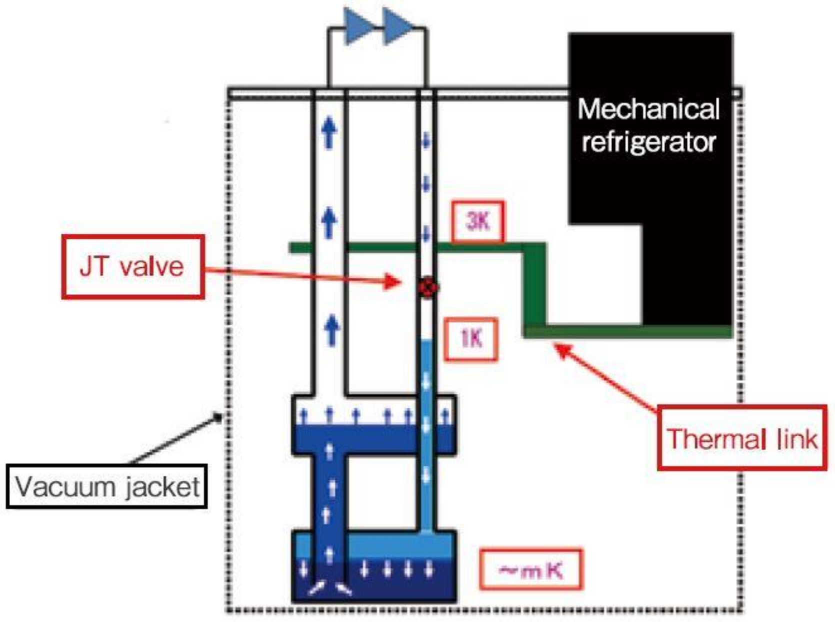
$$S_{FF}=4k_BT\frac{m\omega_0}{Q}=4k_BT\frac{\sqrt{km}}{Q}$$

Possible options:

- Room temperature: T=300 K
- Intermediate (Mechanical refrigerators): T=3-300 K
- Cryogenic (Liquid Helium): T=1-5 K
- Millikelvin (Dilution Refrigerators): T=10 mK – 1 K
- Microkelvin (Adiabatic Nuclear Demagnetization): T=10 μK-10 mK
- Nanokelvin (Cold Atoms): ~ nK

^{3}He-^{4}He Dilution Refrigerators

- Standard tool to work in the temperature 10 mK – 1 K (superconducting qubits, ultrasensitive bolometers, etc)
- Closed cycle refrigerator exploiting two phases of ^{3}He-^{4}He liquid mixtures.
- Precooling the circulating fluid to 3K by closed cycle mechanical refrigerator.
  (also a source of unwanted vibrational noise !)



- ^{3}He rich phase → ^{3}He diluted phase

⇩

- Liquid → Vapour

(Entropy Absorption ⇨ cooling)

Quality factor - dissipation/loss angle

$$S_{FF} = 4k_BT\frac{m\omega_0}{\boxed{Q}} = 4k_BT\frac{\sqrt{km}}{Q}$$

Total dissipation 1/Q is composed of many independent components

$$\frac{1}{Q} = \phi + \frac{1}{Q_{surf}} + \frac{1}{Q_{clamping}} + \frac{1}{Q_{gas}} + \ldots$$

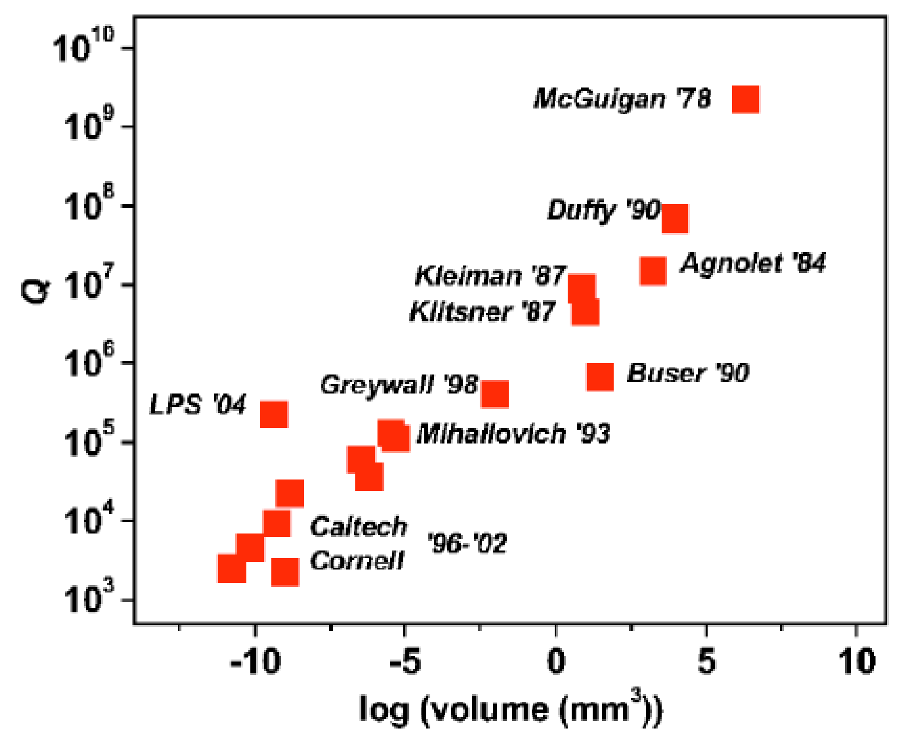
- $\phi(\omega)$: material loss angle (phonon-phonon, thermoelastic, phonon-defects …)

$$k = k_0(1+i\phi)$$

$$Y = Y_0(1+i\phi)$$

- Surface dissipation: higher losses (more defects, two-level systems, ...)
- Clamping losses: phonons irradiated into the support
- Gas losses: scales with gas density (in molecular regime)

Empirical scaling with size



Ekinci and Roukes, Rev. Sci. Instrum. **76**, 061101 (2005)

Very Roughly

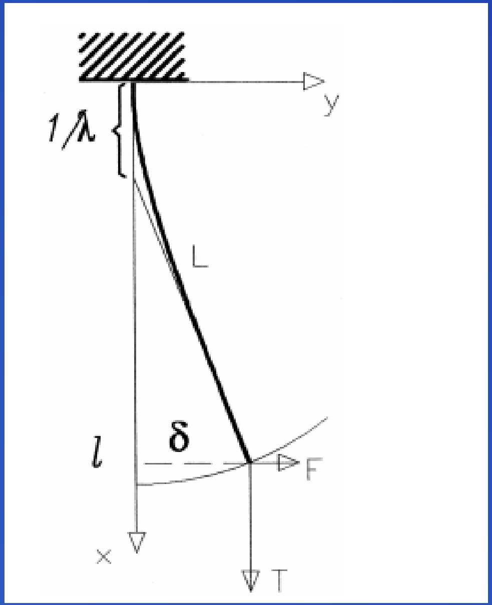
$$Q \propto L \quad \text{linear size}$$

$$\frac{1}{Q} \propto \frac{1}{L} = \frac{\text{Surface}}{\text{Volume}}$$

- Data suggest that surface dissipation is dominating in nanomechanical resonators
- There are notable exceptions, we will discuss soon

Dissipation dilution

- Virtually lossless springs exist in nature
- Simple pendulum: gravitational spring



Cagnoli et al, Physics Letters A 272, 39 (2000)

There is still elastic dissipation due to bending of the wire at the clamping point.

The elastic energy is only a small fraction of the total restoring energy ($k_{el} << k_{grav}$)

Effective loss:

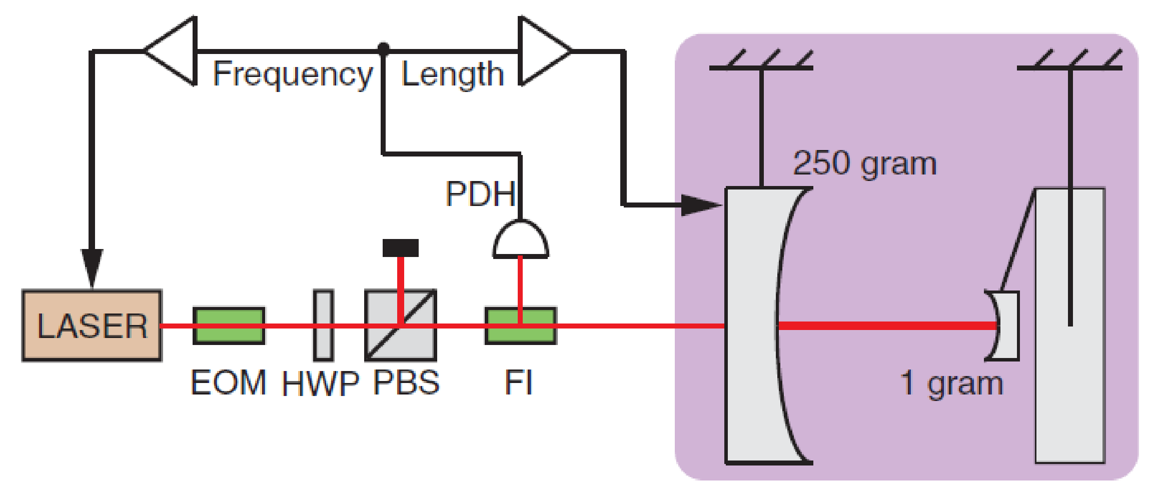
$$\frac{\phi_{eff}}{\phi} = \frac{k_{el}}{k_{el} + k_{grav}} = \sqrt{\frac{Y_0 I}{16 m g L^2}} << 1$$

Effect exploited in pendulum suspensions of LIGO gravitational wave detector

$$\phi \approx 10^{-6} \Rightarrow \phi \approx 10^{-8}$$

Optical dilution

- Optomechanical spring is also virtually lossless

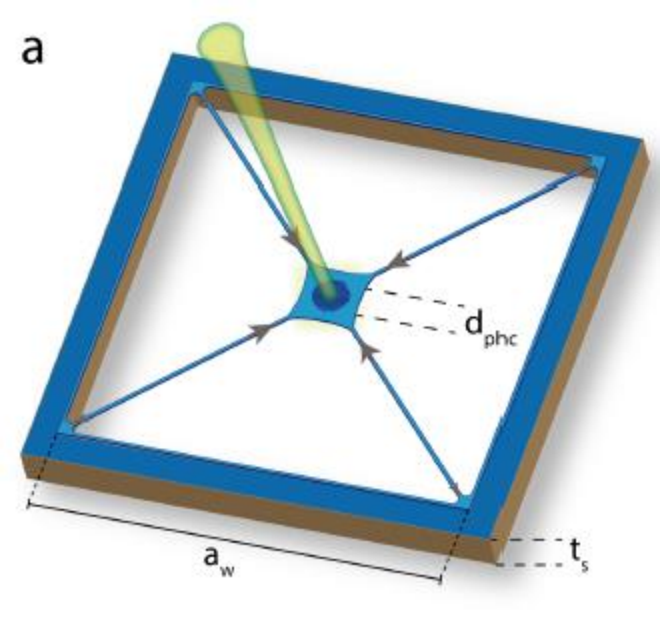


T. Corbitt et al, PRL **99,** 160801 (2007)

Low LASER power: ω_0=2π×12.7 Hz, Q=20000
High LASER power: ω_0=2π×1 kHz, Q=1.6×10^6

However note that the ratio $\gamma = \frac{\omega_0}{Q}$ (damping rate) does not change !

Dilution in stressed membrane/strings



Groblacher et al, PRL 2016

The strings are fabricated with a built-in tension, close to the elasticity limit of the material.

The spring constant is induced by the built-in tension is much larger then the intrinsic bending elasticity

$$\omega_0 = 2\pi \times 150\ \mathrm{kHz}$$

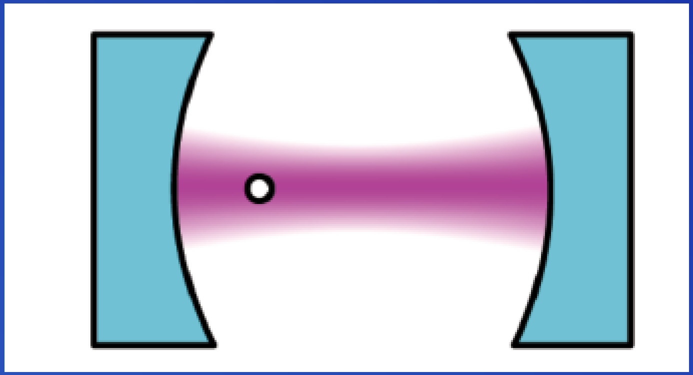
$$\mathrm{Q} \approx 10^8$$

REMARKABLE: at room temperature

Product $\omega_0 \times Q$ becomes very large → very good for quantum optomechanics

Ratio $\gamma = \frac{\omega_0}{Q}$ is unchanged by high tension → not useful for force detection

Levitation (total dilution)



Optomechanical levitation
Vienna (Aspelmeyer)
Southampton (Ulbricht)
…

Electrical levitation – Paul traps
London (Barker)

Magnetic levitation
Vienna

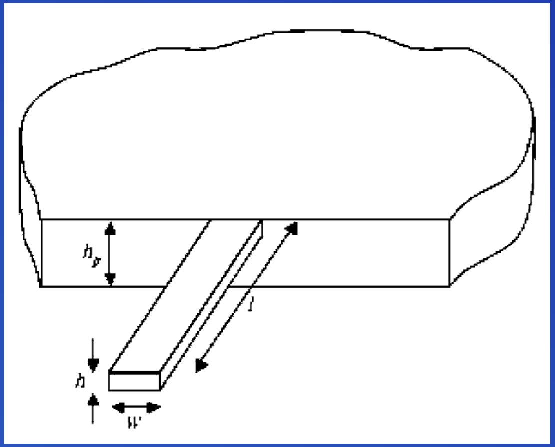
The killing solution ?

- Only center of mass motion
  extreme decoupling of C.M. from internal modes
  (material dissipation very ineffective)
- No clamping losses
- Dissipation only by residual gas
  (in principle can be made negligible too)
- Highest Q factor so far $Q \approx 10^8$
  Ballistic regime
  (single collisions with gas particles)

More on this from other speaker here. Let’s go on

Clamping losses

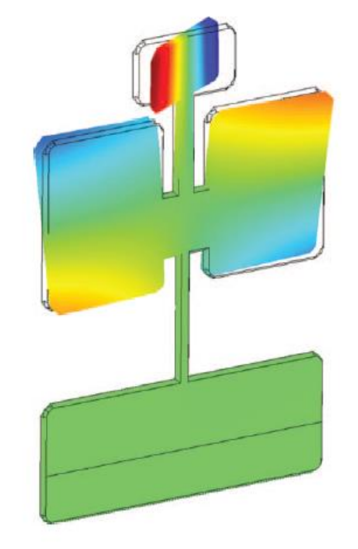
Depend on the motion-induced stress in the vicinity of the clamping point.



Simple cantilever sticking out from support .
Strong stress at the clamping point

$$Q_{clamping} < 10^6$$

Solution: minimize the stress at the clamping point



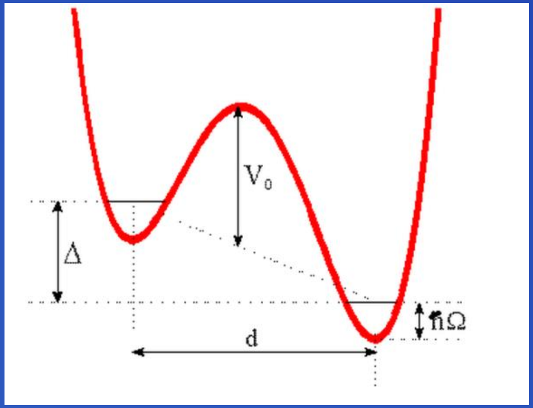
$$Q_{clamping} > 10^8$$

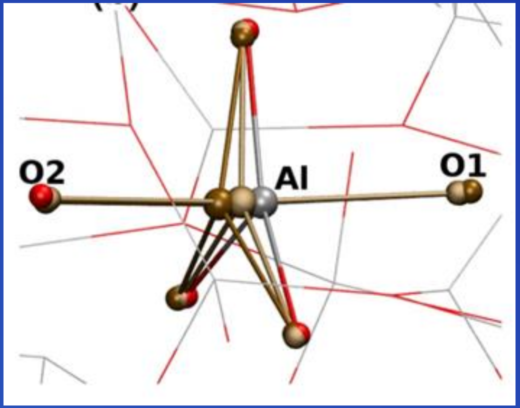
Metcalf et al, Rev. Sci. Instrum. **84**, 075001 (2013)

Dependence of Q on T

Typically, the dissipation decrease when reducing temperature.
Although non-monotonically (Debye/thermoelastic peaks)

At very low temperature (T < 1K), interaction with two-level systems become dominant

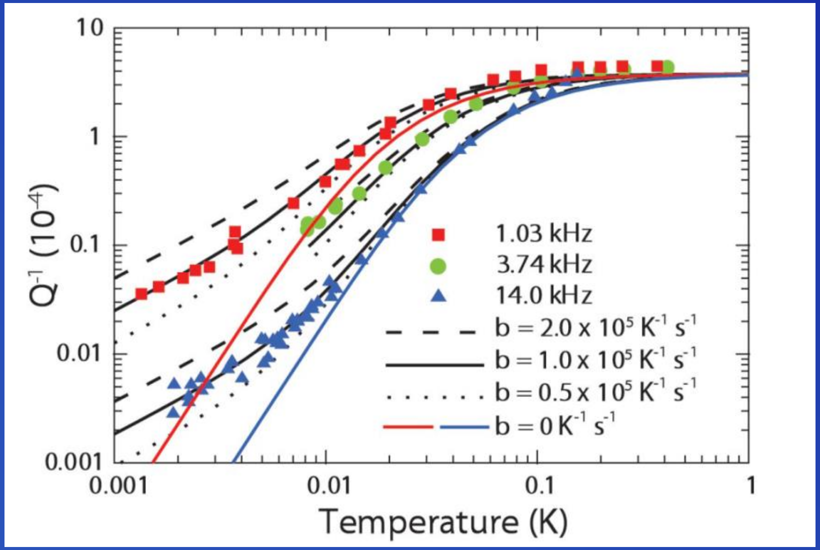




Assuming a broad range of tunneling two-level systems explains
all properties of glasses at low temperature: thermal conductivity – heat capacity
but also mechanical dissipation !

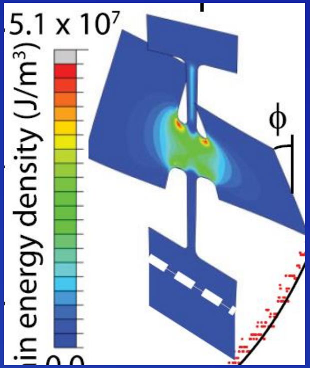
Anderson, Halperin, Varma, Philos. Mag. (1972)
Phillips, J. Low Temp. (1972)

Experimentally

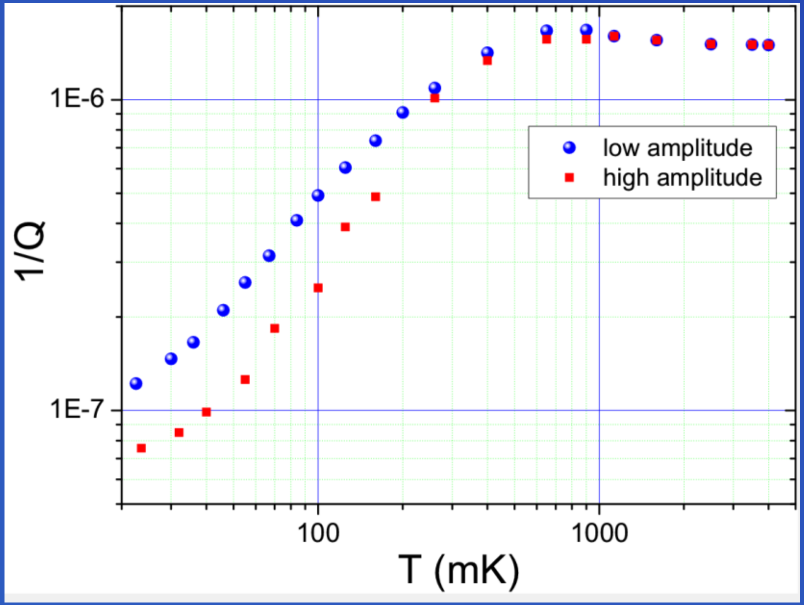


SiO_2 thick torsional resonator

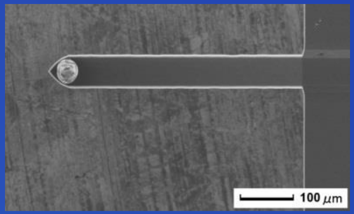
Fefferman et al, PRL 195501 (2008)



Effect is dramatic in glasses !



But can be seen also in crystals
(likely due to surface glassy oxide)



Single crystal silicon cantilever
(Vinante et al, recent measurement)

To summarize

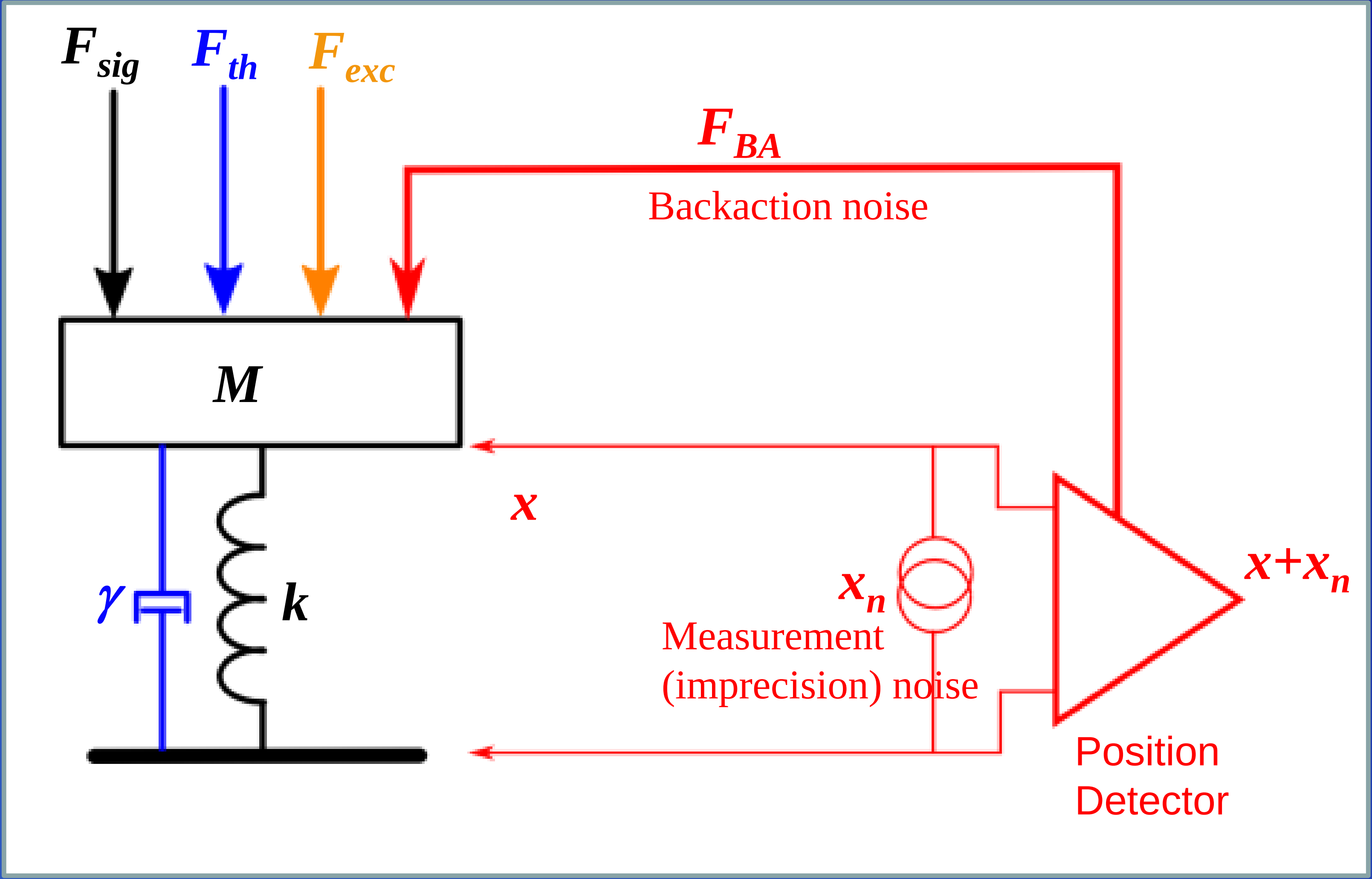
Conventional (clamped) mechanical resonators

$Q \approx 10^4 - 10^6$	typical of nanomechanical resonators
Q up to $10^7 - 10^8$	achievable by proper optimization (material, temperature, clamping, stress …)

Levitated systems

Q up to 10^8	with moderately high vacuum
$Q >> 10^8$	potentially achievable with UH vacuum

Detector noise



Any linear position detector features 2 conjugate noise sources:

- Imprecision position noise x_n (adds incoherently to the signal)
- Backaction force noise F_{BA} (real force acting on the resonator)

Heisenberg microscope

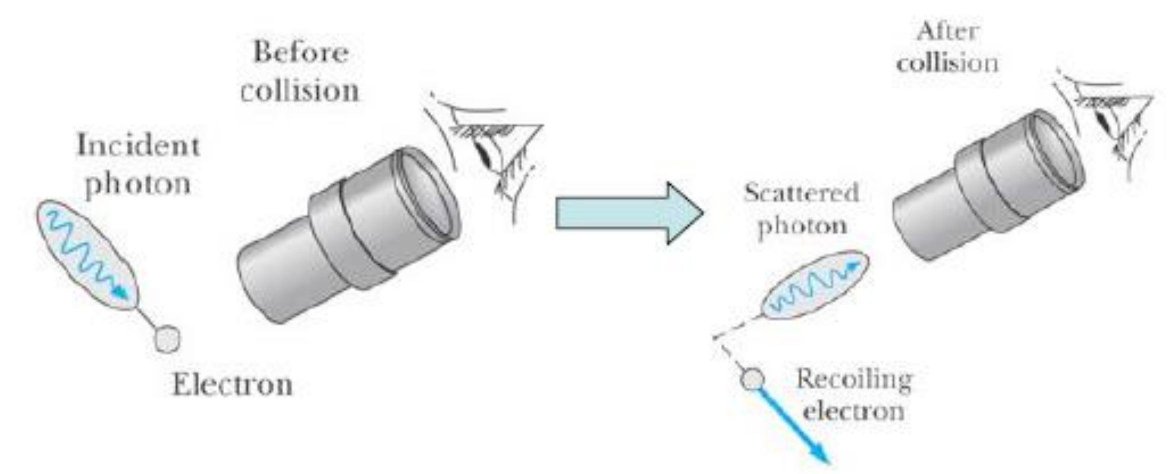
$$S_{FF}S_{xx} \geq \hbar^2$$

(one-sided)

- Lower limit achieved by ideal quantum limited detectors
- Holds strictly for linear detector at high power gain

C. Caves, PRD 26 1817 (1982)

Original Heisenberg thought experiment

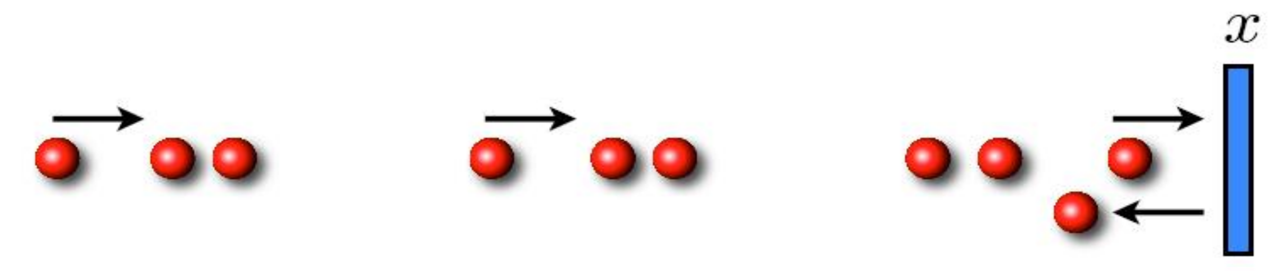


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Note: this is different from the standard uncertainty relation (Zero-point fluctuation of system)

Heisenberg limit comes from ZPFs of the measurement apparatus (the photon) !

Example: optomechanical sensing



Displacement x $\Rightarrow$ phase shift of reflected photons: $\varphi = 2kx$

Momentum transferred by a reflected photon: $p = 2\hbar k$

$\Delta N = \sqrt{N}$ (Poisson statistics)

$\Delta\varphi = \frac{1}{2\sqrt{N}}$ because, for coherent states $\Delta N \Delta\varphi = \frac{1}{2}$

$$\Big\Downarrow$$

$$\Delta p \Delta x = \frac{\hbar}{2} \Longrightarrow S_{FF} S_{xx} = \hbar^2$$

$$S_{FF} \propto \dot{N} \propto \text{Power}$$

$$S_{xx} \propto \frac{1}{\dot{N}} \propto \text{Power}$$

Standard quantum limit (SQL)

Total displacement due to the detector is sum of imprecision and backaction-induced term

$$S_{xx}^{(added)} = S_{xx} + S_{FF} \left|\chi(\omega)\right|^2$$

$$S_{FF} S_{xx} \geq \hbar^2$$

$$S_{FF} \propto \dot{N} \propto \text{Power}$$

$$S_{xx} \propto \frac{1}{\dot{N}} \propto \text{Power}$$

Minimized when
two terms are equal

$$S_{xx}^{(added)} \geq 2\hbar\chi(\omega)$$

SQL for displacement

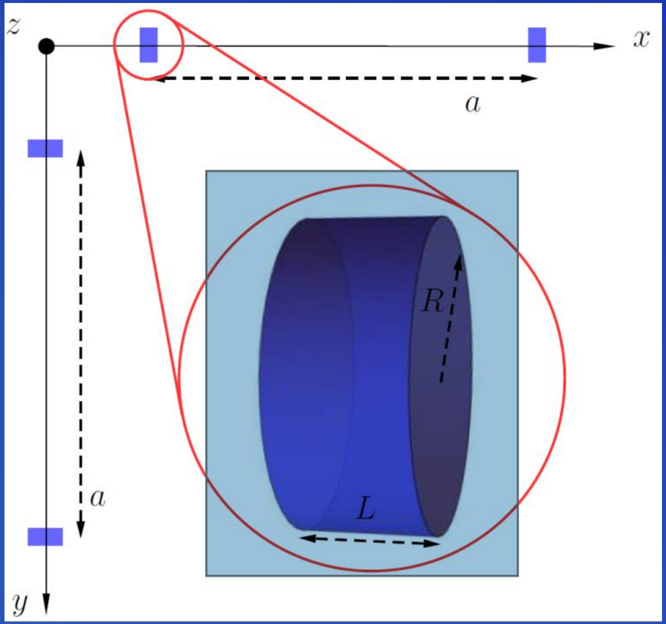
By same argument, applied to force

$$S_{FF}^{eq} = S_{FF} + \frac{S_{xx}}{\left|\chi(\omega)\right|^2}$$

SQL for force

$$S_{FF}^{eq} \geq \frac{2\hbar}{\chi(\omega)}$$

Example: LIGO



Free masses as mirrors:

$$\chi(\omega) = \frac{x(\omega)}{F(\omega)} = \frac{1}{m\left(-\omega^2 + \omega_0{}^2 - i\frac{\omega\omega_0}{Q}\right)} \approx -\frac{1}{m\omega^2}$$

$$S_{xx}{}^{(added)} \geq 2\hbar\left|\chi(\omega)\right| = \frac{2\hbar}{m\omega^2}$$

Effective experimental values: m=10 kg, $\omega/2\pi$ =100 Hz

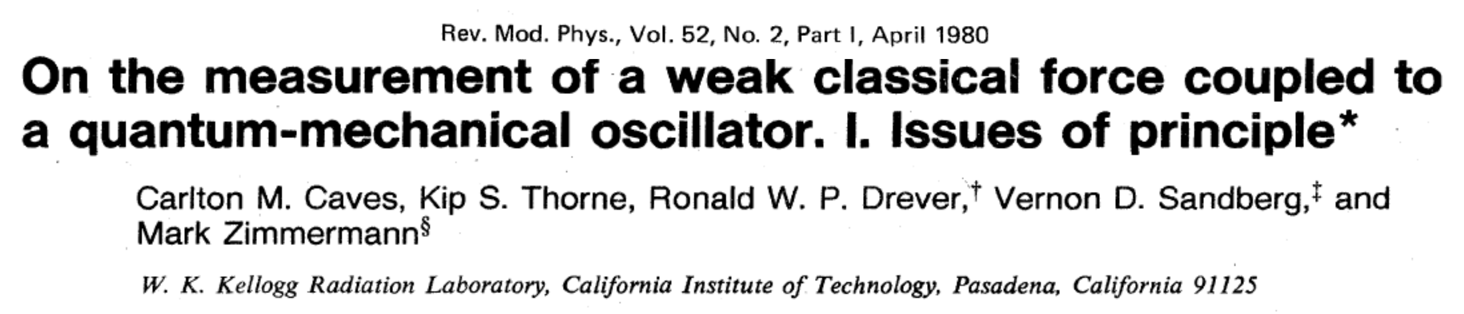
$$S_x{}^{(added)} = \sqrt{S_{xx}{}^{(added)}} \approx 10^{-20}\ \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$$

$$S_h = \frac{S_x{}^{(added)}}{L} \approx 10^{-23}\ \frac{}{\sqrt{\mathrm{Hz}}}$$

Is SQL a fundamental limit?

SQL comes from Heisenberg: $\Delta p \Delta x = \frac{\hbar}{2}$

However, Heisenberg does not forbid to get arbitrarily large accuracy on the continuous measurement of one variable, provided that you give up completely information about the conjugate.



Rev. Mod. Phys., Vol. 52, No. 2, Part I, April 1980

On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle*

Carlton M. Caves, Kip S. Thorne, Ronald W. P. Drever,[†] Vernon D. Sandberg,[‡] and Mark Zimmermann[§]

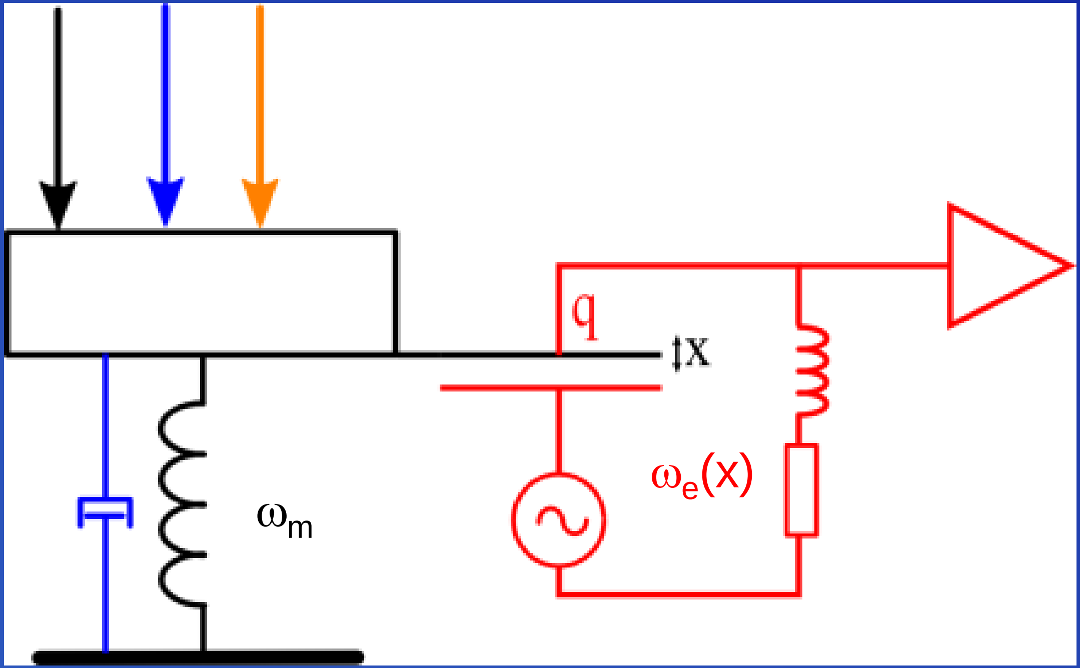
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

No quantum limit to detect a classical force with a quantum oscillator !

RECIPE: the measurement device must couple to a variable X such that $[H,X]=0$ (Quantum Nondemolition Measurement).

Unfortunately, most measurement devices couple to position x:
$[H,x]\neq 0$ (measure x $\Rightarrow$ perturbs p, which influences next measurement of x)

QND in a mechanical resonator



$$\omega_e >> \omega_m$$

$$H_{\text{int}} = Exq$$

Standard linear optomechanical setup (with LC cavity, E pump electric field)

$$E = E_0 \cos\left[\left(\omega_e - \omega_m\right)t\right]$$

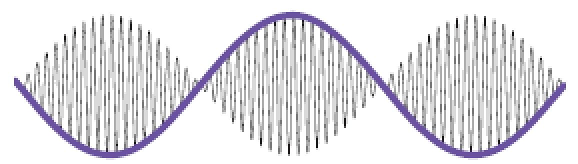
Parametric phonon-photon upconverter

$$E = E_0 \cos\left[\left(\omega_e - \omega_m\right)t\right] + E_0 \cos\left[\left(\omega_e + \omega_m\right)t\right]$$

Back-action evasion measurement (only possible linear QND measurement)

How does it work?

$$E = E_0 \cos\left[\left(\omega_e - \omega_m\right)t\right] + E_0 \cos\left[\left(\omega_e + \omega_m\right)t\right]$$
$$= 2E_0 \cos\left(\omega_e t\right)\cos\left(\omega_m t\right)$$



$$x(t) = X_1(t)\cos(\omega_m t) + X_2(t)\sin(\omega_m t)$$
$$q(t) = Q_1(t)\cos(\omega_e t) + Q_2(t)\sin(\omega_e t)$$

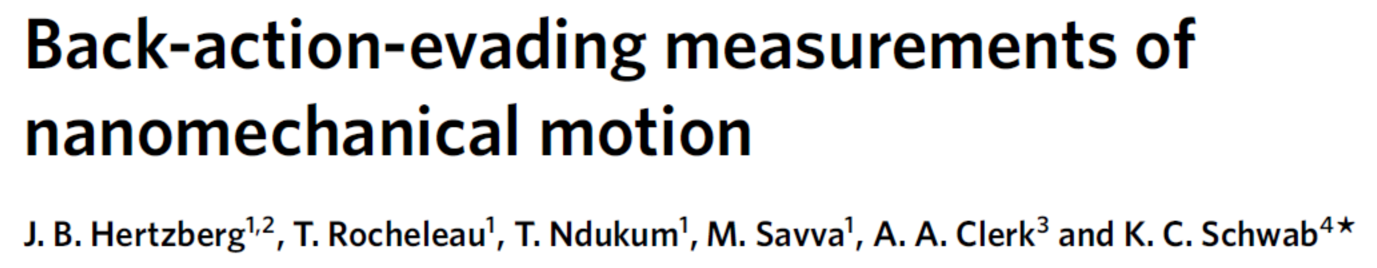
$$H_{\text{int}} = Exq$$
$$= 2E_0 \cos(\omega_e t)\cos(\omega_m t)x(t)q(t)$$
$$\approx \frac{E_0}{2} X_1(t) Q_1(t)$$

(neglecting high frequency terms)

Equations in absence of noise

$$\dot{X}_1 = 0$$
$$\dot{X}_2 = -\frac{E_0}{2m\omega_m} Q_1$$
$$\dot{Q}_1 = 0$$
$$\dot{Q}_2 = -\frac{E_0}{2L\omega_m} X_1$$

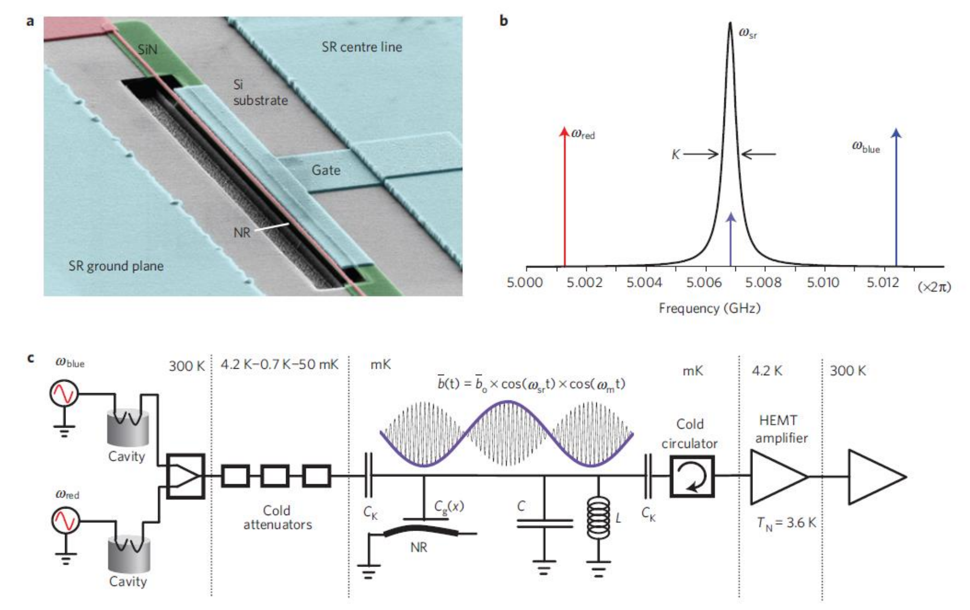
Measured quadrature X_1 is unperturbed !

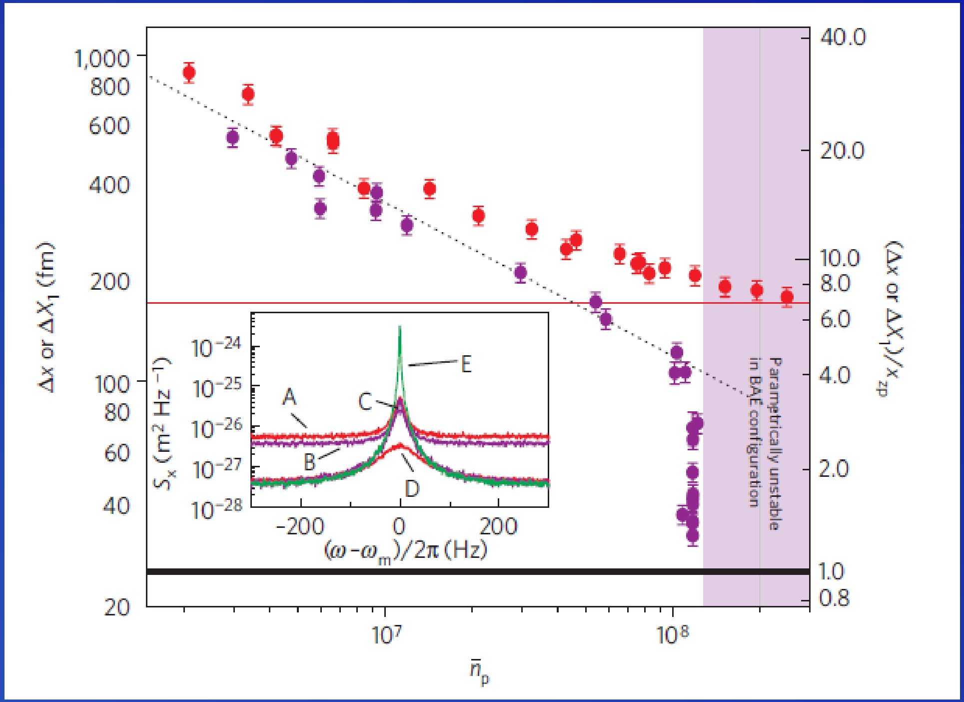


Back-action-evading measurements of nanomechanical motion

J. B. Hertzberg[1,2], T. Rocheleau[1], T. Ndukum[1], M. Savva[1], A. A. Clerk[3] and K. C. Schwab[4]★

Nature Physics 6, 217 (2010)





More recently: squeezing of measured quadrature below zero point fluctuations (several groups)

Detection of weak forces & quantum foundations:

2^{nd} part

Outline

- The measurement problem (in short)
- The CSL model
- Experiments with nanomechanical resonators (MRFM)
- Macroscopic experiments (gravitational wave detectors)
- Other models/experiments

The Measurement Problem

Two different dynamics in Standard Quantum Mechanics

1) Ordinary evolution: Linear and Deterministic

$$\psi = a\psi_1 + b\psi_2$$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

2) Measurement process: Nonlinear (Reduction postulate) & Stochastic (Born Rule)

$$\psi = a\psi_1 + \cancel{b\psi_2}$$

$$P(\psi_1) = |a|^2$$

Standard Quantum Mechanics works well with reduction postulate
as long as you don't care about conceptual problems
(*"Shut-up and calculate!"* , D. Mermin)

What is precisely a measurement ?
At which level does the reduction take place (if it ever takes place) ?
How "fundamental" is the reduction postulate ?
…

Some possible ways out

1) Interpretations (Copenhagen, Many-Worlds, and a lot more)
Physics ⇒ Metaphysics

2) Decoherence
Entanglement of any system to an environment naturally destroys genuine quantum effects (like quantum interference)

Issues
- Decoherence does not describe collapse (just more and more entanglement)
- Cannot explain definite outcomes of measurements

3) Quantum mechanics is incomplete ⇨ Hidden variables (Böhmian Mechanics)

4) Quantum mechanics is an approximated theory ⇨ Collapse models (CSL et al)

Spontaneous wavefunction collapse models

- Quantum and Classical are micro-macroscopic limits of a more general theory, which merges the two dynamics

- Random collapses are intrinsic to quantum evolution
  = <u>dynamical reduction instead of istantaneous reduction</u>

- Mass-proportional (larger size ⇒ faster collapse)
  May be related with gravity (Diosi-Penrose model)

- Natural Micro-Macro transition @ ~ 10^{-7} m

- Everything else is naturally derived
  - Quantum Mechanics at microscale
  - Reduction Postulate and Born rule ⇒ Measurement problem solved !
- - Classical Mechanics at macroscale

Continuous Spontaneous Localization (CSL)

Schrödinger equation + Stochastic term (collapse field)

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}H\,dt + \sqrt{\lambda}\int d^3x(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)dW_t(\mathbf{x}) - \frac{\lambda}{2}\int d^3x(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)^2 dt\right]|\psi_t\rangle$$

2 phenomenological constants

- Correlation Length r_C

  conventionally $r_C = 10^{-7}$ m, but in principle no bound
- Collapse rate λ

  Lower bounds (to guarantee collapse at “macroscopic” scale)

  $\lambda \sim 10^{-16}$ s^{-1} following Ghirardi, Rimini, Weber (GRW)

  $\lambda \sim 10^{-8}$ s^{-1} following Adler (latent image formation as CSL effect)

Experimental tests of collapse models

Collapse models CAN BE TESTED !
(unlike interpretations of quantum mechanics)

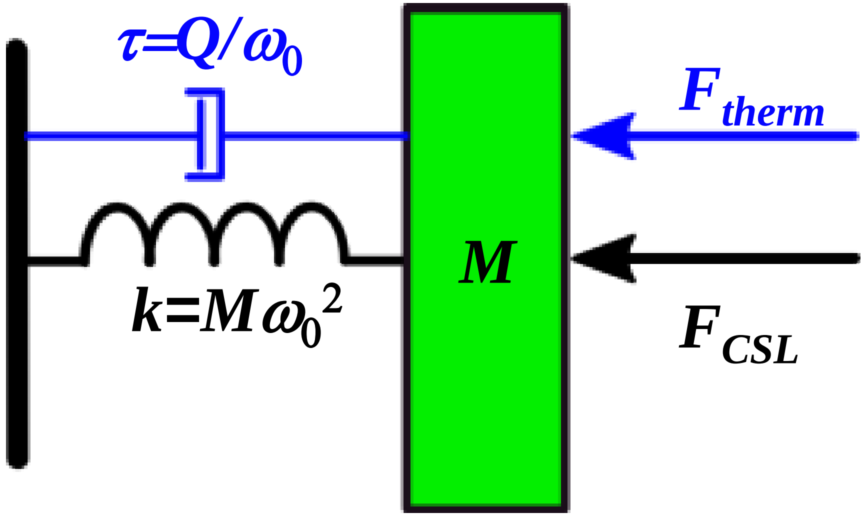
Direct (Interferometric): macroscopic quantum superposition

- Matter-wave interferometry with molecules or nanoparticles

Indirect (non-interferometric): energy non-conservation effects

- X-ray spontaneous emission from free electrons
- Force noise/Spontaneous heating in mechanical resonators

Random Collapses → Momentum kicks → Stochastic driving force



$$\frac{d\langle E\rangle}{dt} = \frac{k_B \mathrm{T}}{\tau} - \frac{\langle E\rangle}{\tau}$$

$$\frac{d\langle E\rangle}{dt} \propto F_{CSL}$$

$$\langle E\rangle = k_B \mathrm{T} + \Delta E_{CSL} = k_B(T + \Delta T_{CSL})$$

S. Nimmrichter et al, PRL 113 020045 (2014)
L. Diosi, PRL 114, 050403 (2015)
A. Vinante et al, PRL 116, 090402 (2016)

CSL heating of a mechanical resonator

$$\Delta T_{\rm CSL} = \frac{\hbar^2 Q}{2m\omega_0 k_B}\eta$$

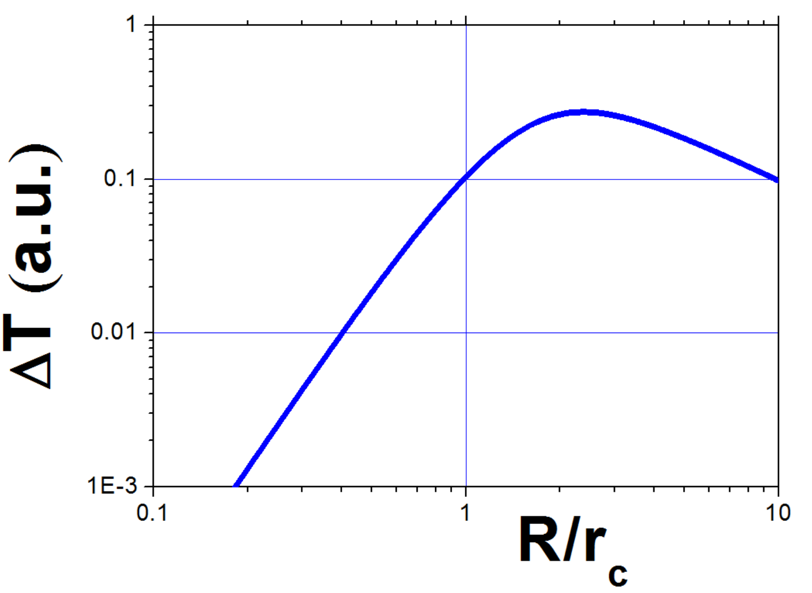
$$\begin{aligned}\eta_j &= \frac{\gamma_{\rm CSL}}{m_0^2}\iint \frac{e^{-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4r_C^2}}}{(2\sqrt{\pi}\,r_C)^3}\,\frac{\partial\varrho(\mathbf{r})}{\partial r_j}\,\frac{\partial\varrho(\mathbf{r}')}{\partial r'_j}\,\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{r}' \\ &= \frac{\gamma_{\rm CSL}}{m_0^2}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\,\mathbf{k}_j^2\,e^{-\mathbf{k}^2 r_C^2}\,|\tilde\varrho(\mathbf{k})|^2\end{aligned}$$

[$\gamma_{CSL} = (4\pi r_c^2)^{3/2}\lambda$]

Exact solution for a sphere

$$\eta_{\rm s} = \frac{2\sqrt{\pi}\,\gamma_{\rm CSL}\,\varrho_{\rm sphere}^2\,R^2}{3m_0^2 r_C}\left(1-\frac{2r_C^2}{R^2}+e^{-\frac{R^2}{r_C^2}}\left(1+\frac{2r_C^2}{R^2}\right)\right)$$

Collett, Pearle, Found. Phys. 33, 1495 (2003)



$$k_B \Delta T_{CSL} = \boxed{\frac{Q}{2m\omega_0}} \cdot \boxed{\hbar^2 \eta}$$

Spectral density
of CSL
force noise
S_{ff}

Mechanical resonator
response

MAXIMIZATION of $\Delta T_{CSL}/T$ requires:

High $\tau = Q/\omega_0$
Low T

Minimize
Thermal noise

$R \simeq r_C$
High ϱ

Maximize CSL signal

NOTE: most collapse models suggest
$r_c=10^{-7}$-10^{-6} m

NOTE: This holds as long as the thermal noise is dominant source.
Present state-of-art experiments are in this limit !

Possible experimental approaches

1) Mesoscopic mechanical systems (ex. nanomechanical resonators):

+ Can work at very low temperature (down to 10 mK)

+ Optimal size (at rc ~10^{-7} – 10^{-6} m).

- Usually not very low frequency (kHz to GHz)

2) Macroscopic mechanical systems for precision measurements (ex. Gravitational wave detectors)

+ Extremely low frequency (mHz – Hz)

- High temperature (usually 300 K)
- Size >> optimal (at rc ~10^{-7} – 10^{-6} m)

3) Ultracold atoms

+ Extremely low temperature (<nK)

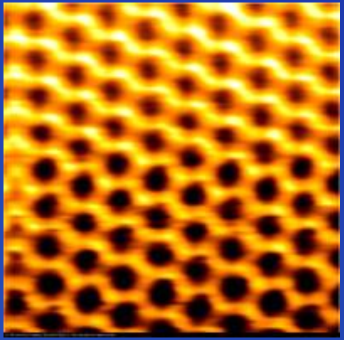
- Size << optimal (at rc ~10^{-7} – 10^{-6} m)

Micro-nanomechanical systems

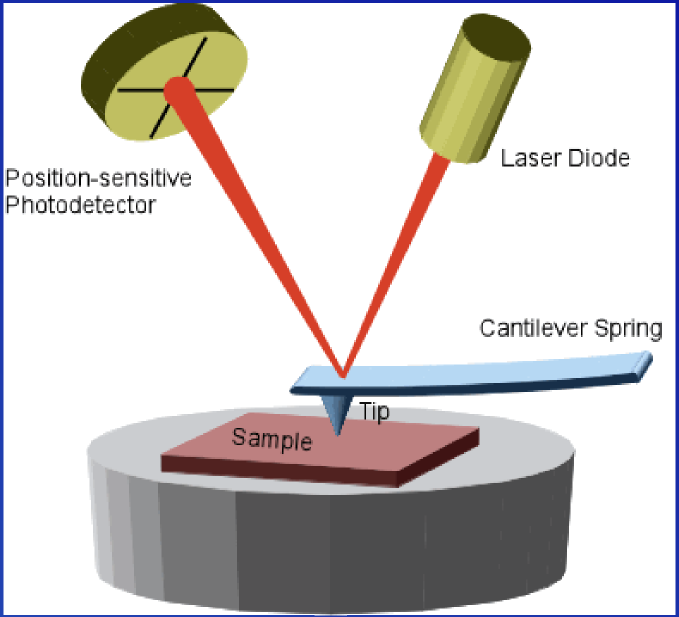
Atomic Force Microscopy (AFM)

“Feels” the force between a sharp tipped cantilever and the sample surface (Binnig, 1985)

+ Atomic resolution!
- “Feels” only surface (2D)



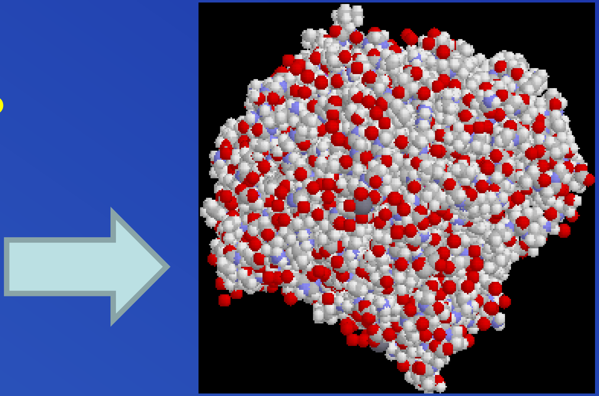
AFM on Graphene



What is the limitation? The tip is sensitive only to the atoms on the surface

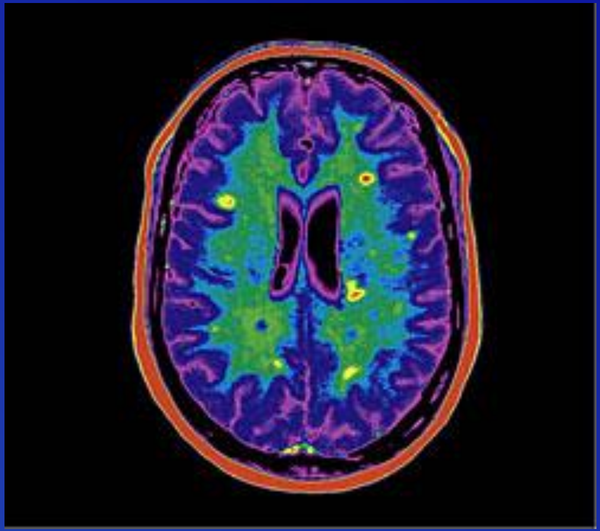
Is there a way to achieve 3D imaging with atomic resolution?

If possible, then



Solution: combine with MRI?

Magnetic Resonance Imaging (MRI)
Coil-detection and imaging of hydrogen spins in matter (for instance in human body). Needs Field gradients!



- \+ Threeedimensional!
- \- Poor resolution (10-100 microns)

What is the limitation? You need the coherent magnetic signal from a lot of spins ($> 10^{12}$) in order to be detectable by a coil receiver

Magnetic Resonance Force Microscopy (MRFM)

Idea (John Sidles, 1991): Instead of measuring the spin electromagnetic signal, measure the gradient-dipole force between the sample spin and a ferromagnetic tip attached to a sensitive cantilever.

1) High spatial resolution

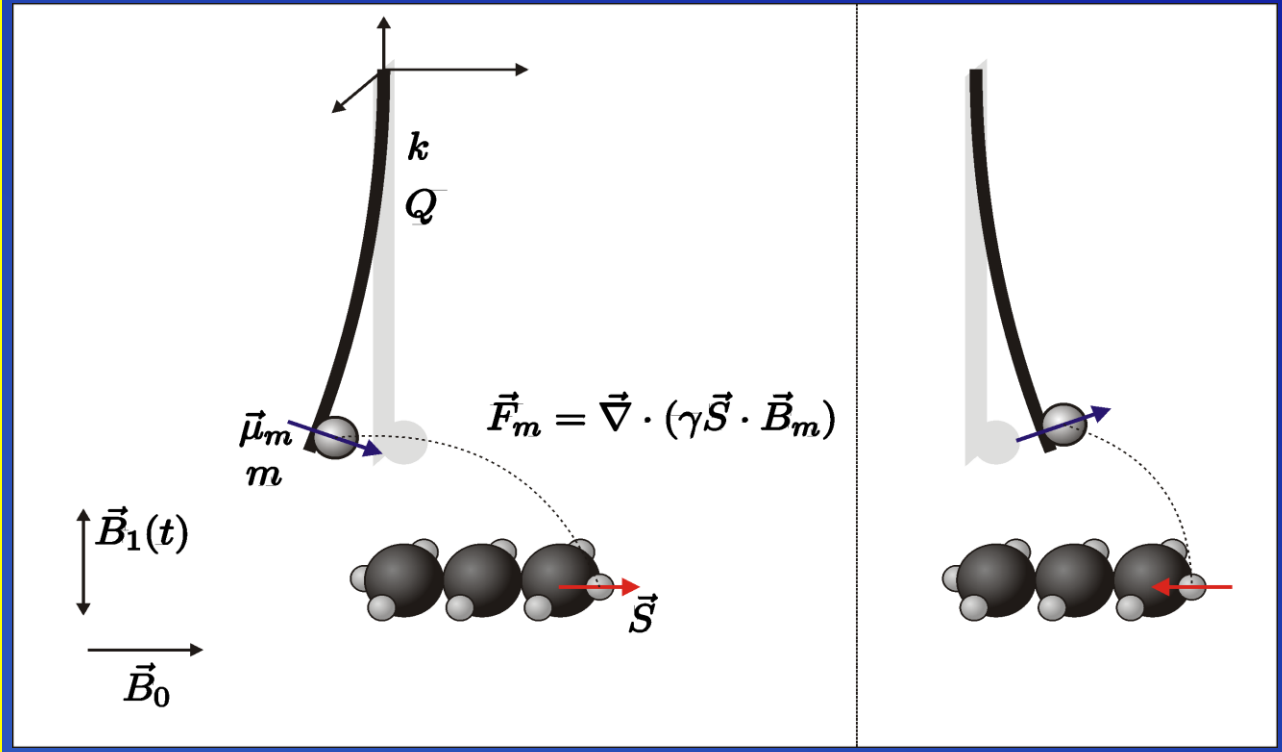
Magnetic resonance ($\omega=\gamma B$) only in a thin Resonant Slice

Thickness $\propto$ 1/G

2) High spin sensitivity

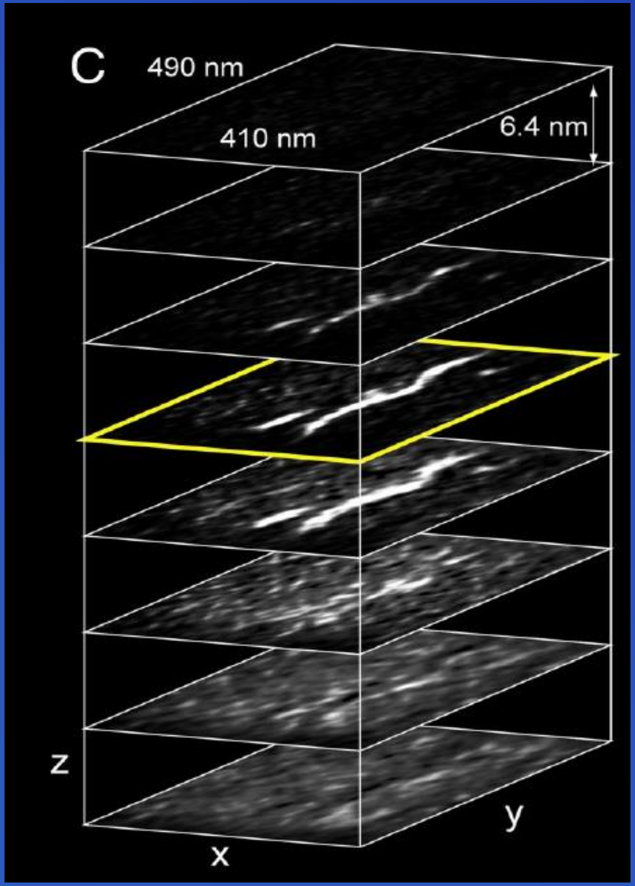
Force $\propto$ G

Key element: strong field gradient G=∇B

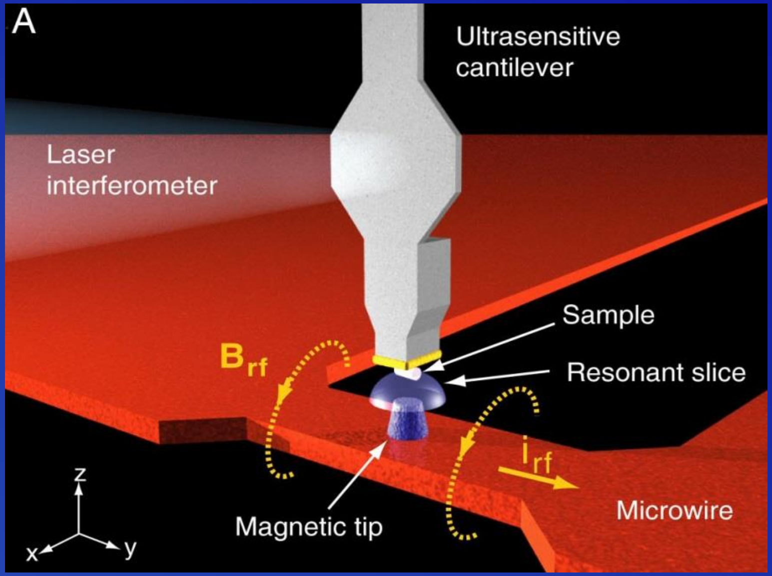


MRFM: some milestone results

Imaging of a virus with
~6 nanometers resolution
(C. Degen et al, PNAS 2009)



IBM Almaden (D. Rugar group)

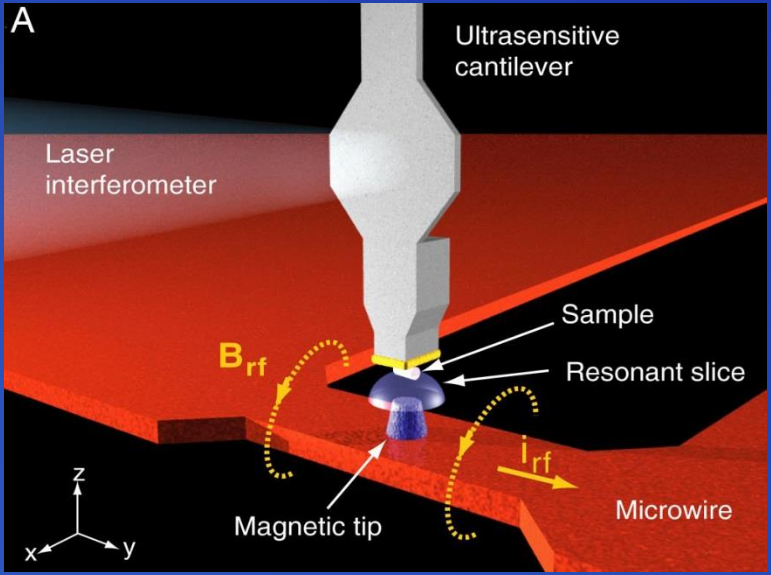


Ultrasoft cantilever (k~10^{-4} N/m)
Cryogenic temperature (T~1K)
Optical readout

Detection of a single electron spin

(**D. Rugar et al,** Nature 430, 329-332, 2004)

The cantilever



Has to be designed in order to achieve lowest possible force noise.

Force signal of order
1E-18 N (electron spins)
1E-20 N (nuclear spins)

$$S_{FF} = 4k_BT\frac{m\omega_0}{Q} = 4k_BT\frac{k}{\omega_0 Q} = 4k_BT\frac{\sqrt{mk}}{Q}$$

Solution: very high aspect ratio (very low k)

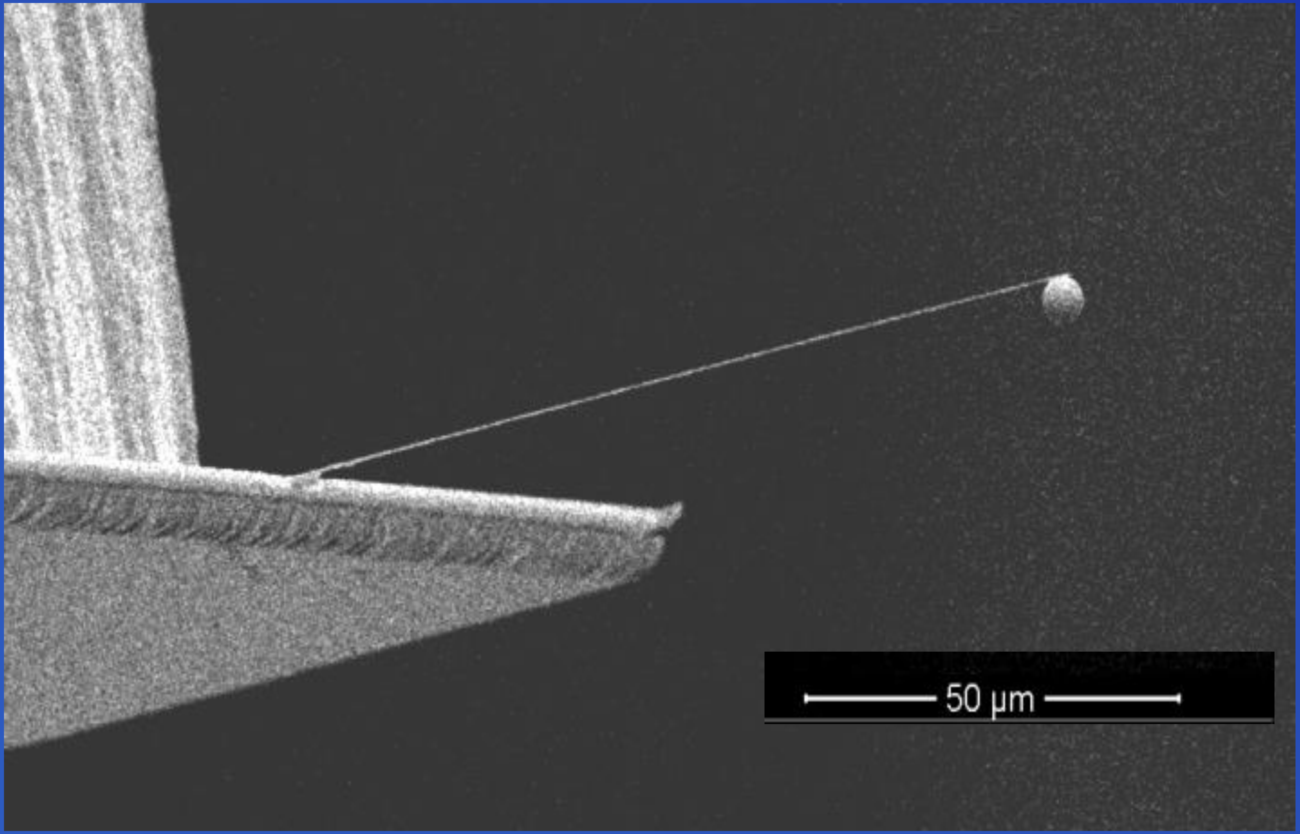
Thickness / Length = 100 nm / 100 mm

Low frequency (3-10 kHz)

Leiden MRFM Experiments

2011 @ (Kamerlingh Onnes Laboratory, T. Oosterkamp)

Silicon nanocantilever (IBM type)



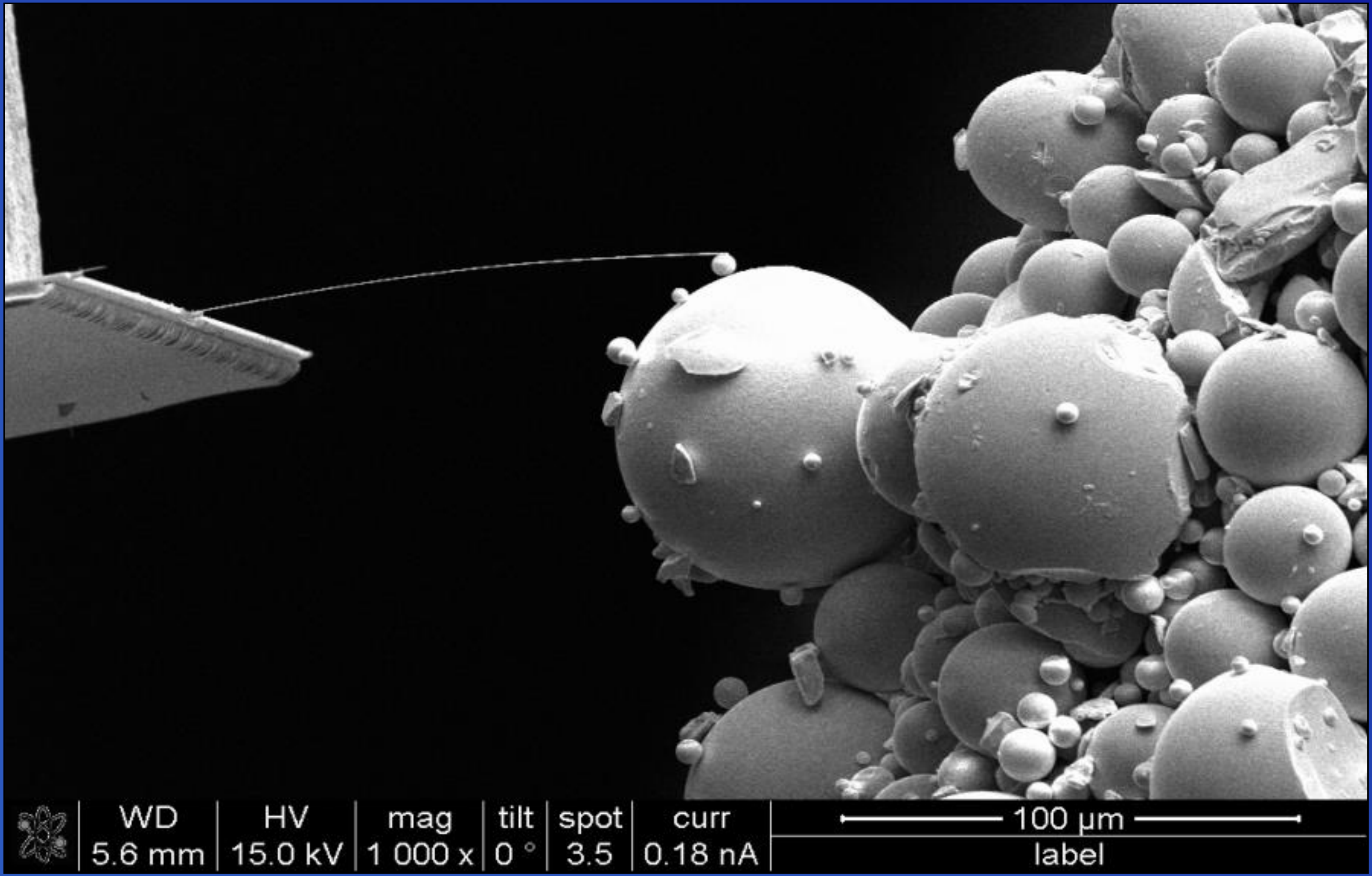
Very high aspect ratio

Thickness=100 nm
(close to standard rc)

Width=5 μm
Length=100 μm

f_0=3084 Hz
Q=4x10^4

Attaching the Magnetic Particle



The challenge

- Very weak forces ($<10^{-18}$ N).
- Force resolution limited by thermal force noise: $S_{FF} = \frac{4k_B T m \omega_0}{Q}$

⇓

Try to cool to lowest possible temperature (~ mK range)

PROBLEM:

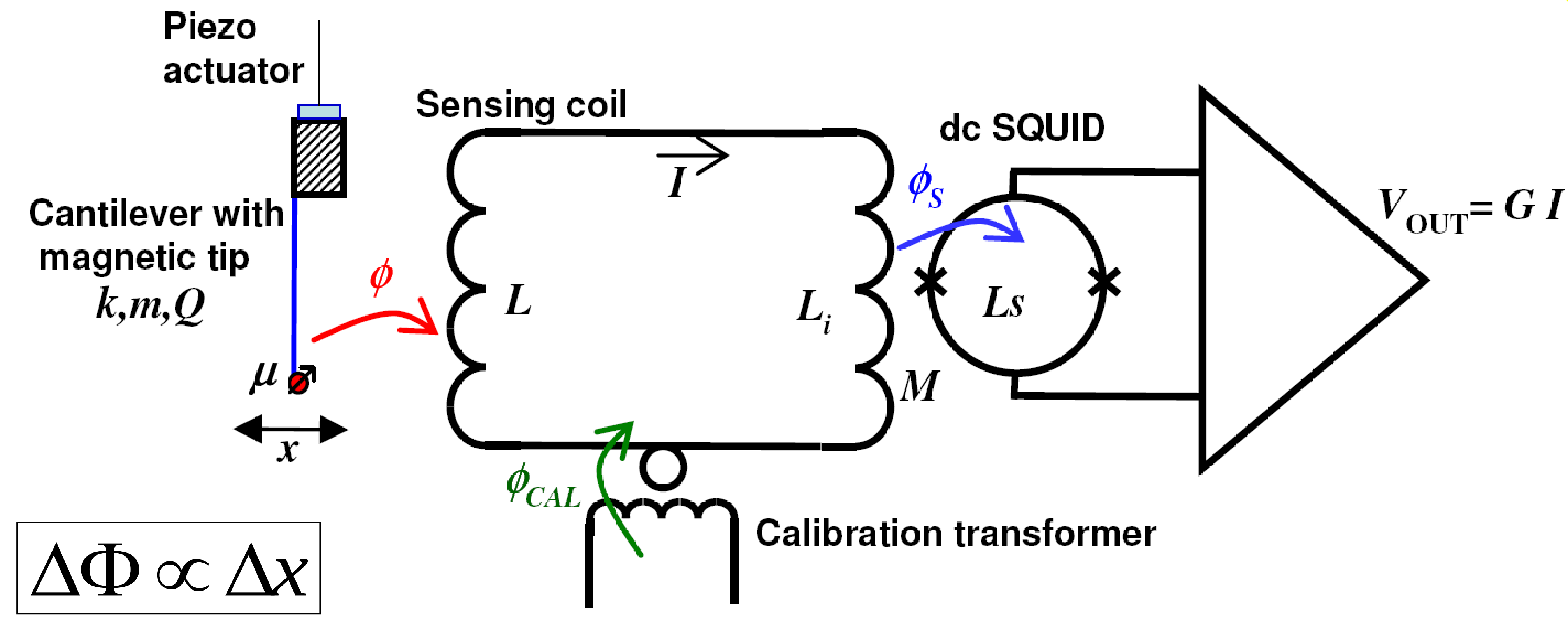
Standard optomechanical techniques not very suitable

(mechanical resonators can be hardly cooled below 1 K because of heat absorption)

⇓

Look for a detection technique compatible with mK temperature

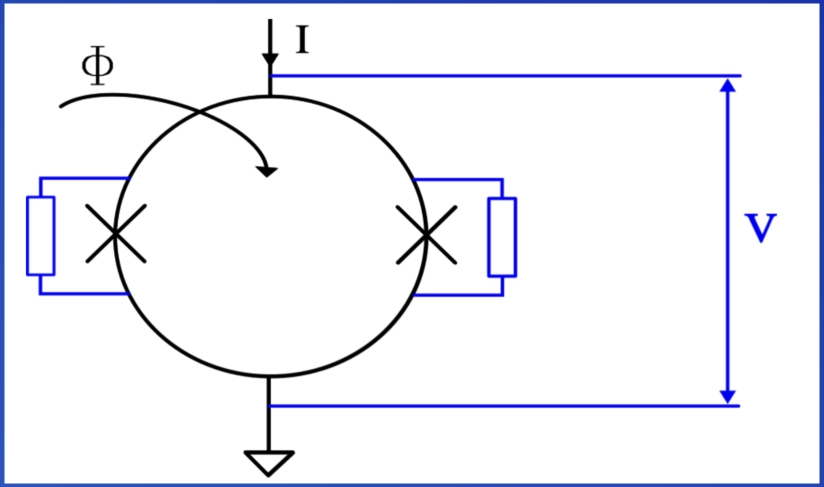
SQUID-based detection



dc SQUID (Superconducting Quantum Interference Device)
Most sensitive magnetic flux sensor

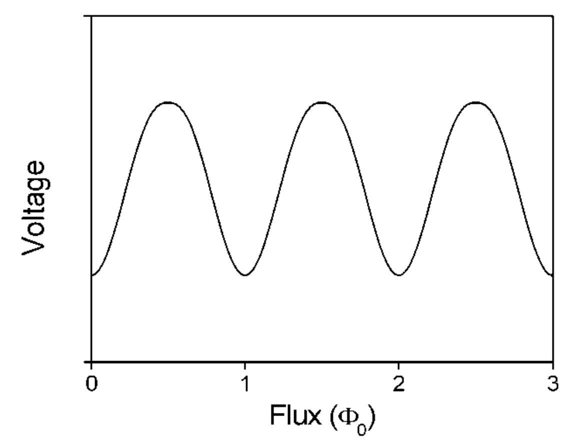
Displacement sensitivity ~ 1 pm/√Hz
O. Usenko et al., Appl. Phys. Lett. 98, 133105 (2011)

The dc SQUID: a sort of "cavity"



- overdamped nonlinear «LC resonator»
- Voltage state ($I>I_{cr}$) ⇒ oscillator driven by Josephson oscillations

- Magnetic Flux Φ modulates resonator frequency f_J
- Output voltage: $V = \frac{h}{2e} f_J = \Phi_0 f_J$
- Noise close to Heisenberg limit

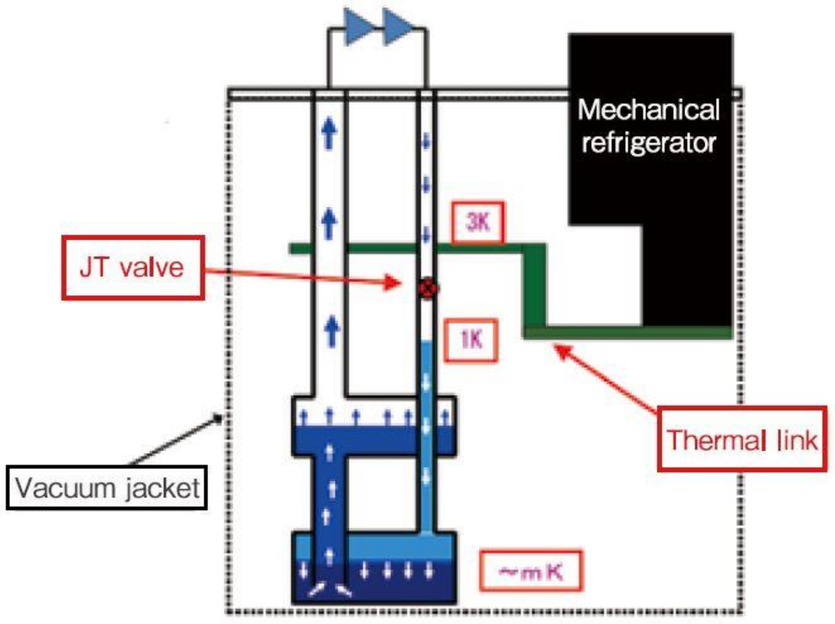


A quantum description can be done by treating the dc SQUID as a nonlinear parametric amplifier

A. Kamal, J. Clarke, M. Devoret, Phys. Rev. B 86, 144510 (2012)

He3-He4 Dilution Refrigerators

- Standard tool to work in the temperature 10 mK – 1 K (superconducting qubits, ultrasensitive bolometers, etc)
- Closed cycle refrigerator exploiting two phased of He3-He4 liquid mixtures



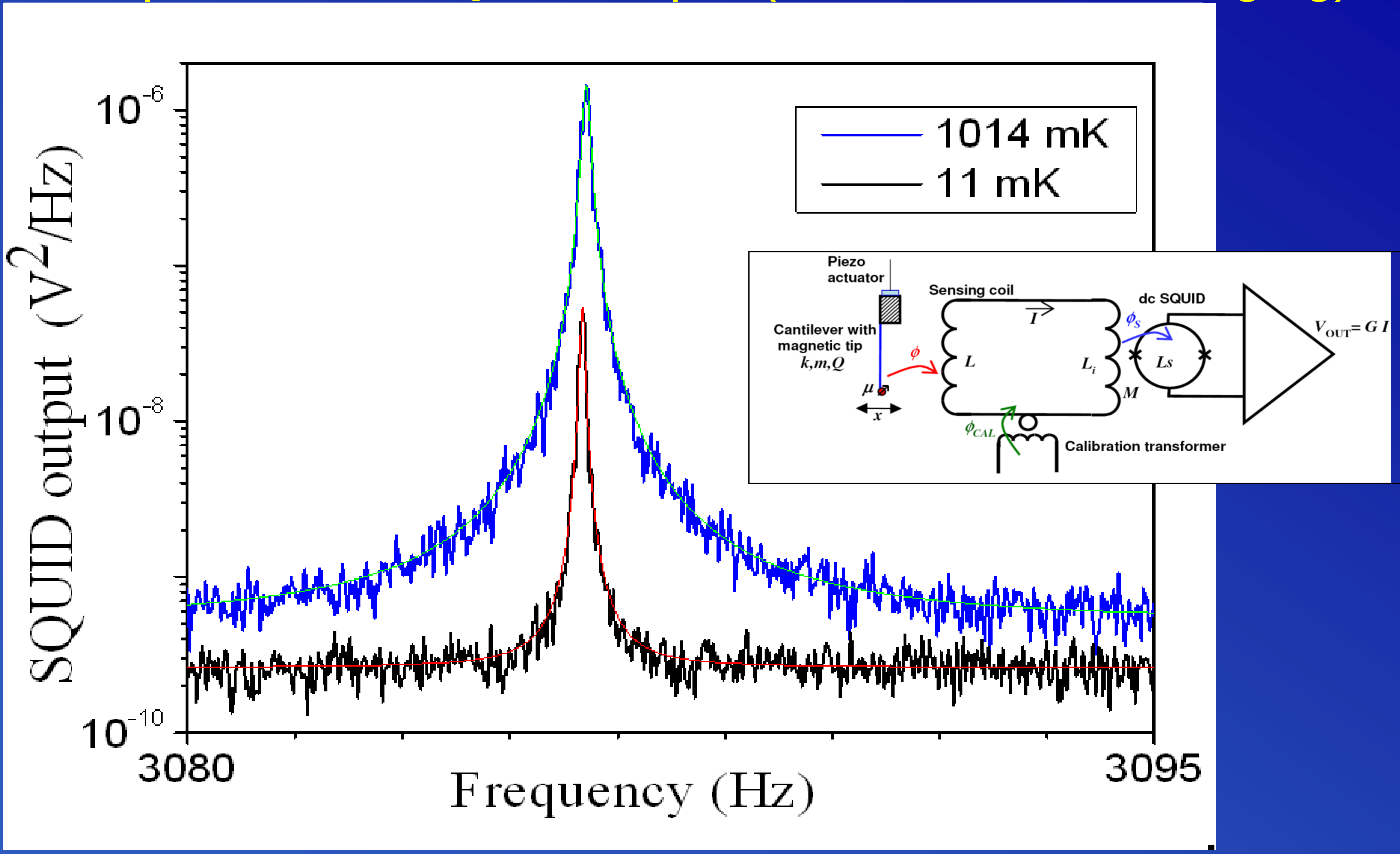
- He3 rich phase → He3 diluted phase

⇩

- Liquid → Vapour

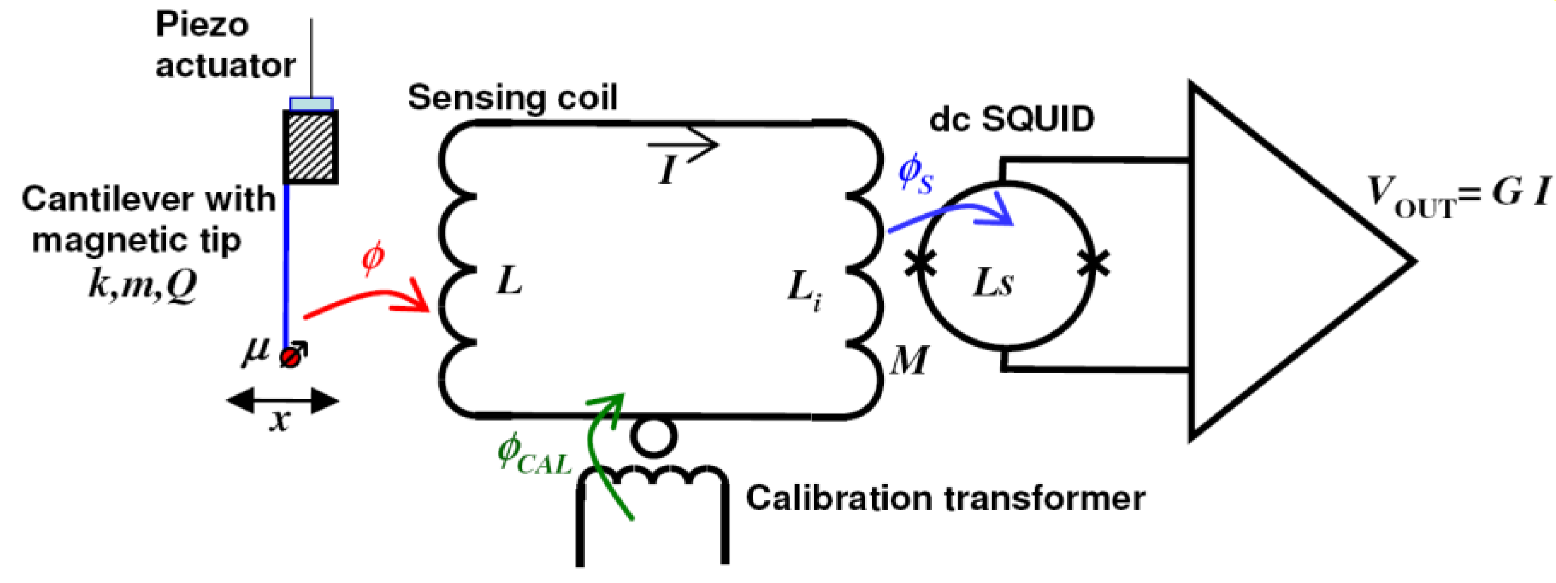
(Entropy Absorption ⇨ cooling)

Noise spectrum at SQUID output (~ 10 minutes averaging)



Area under peak $\propto$ Mean Resonator Energy

Independent calibration of mechanical energy



$$\left\langle V_{th}{}^{2} \right\rangle = \alpha\, \beta^{2} k_B T$$

L_{tot} : Total loop inductance

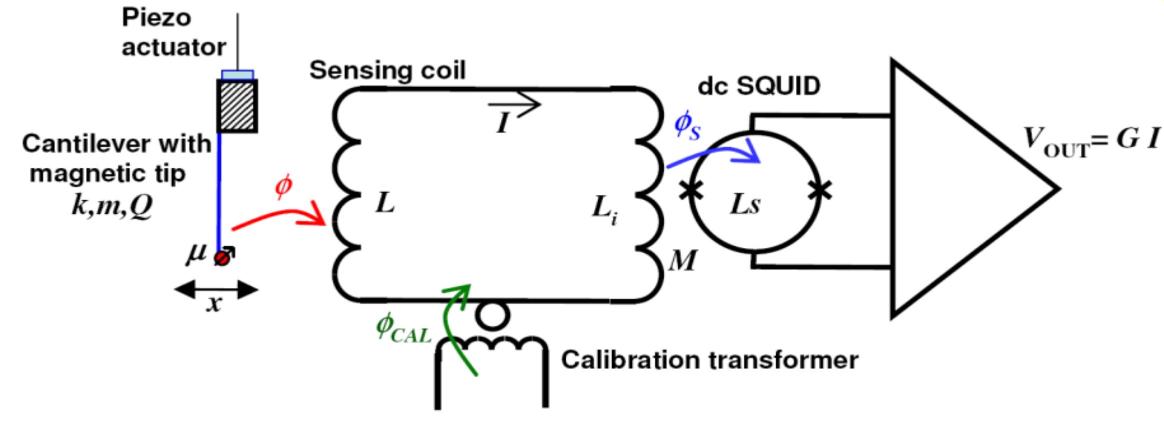
$$\beta^{2} = \frac{1}{kL_{tot}} \left(\frac{\partial \phi}{\partial x} \right)^{2}$$

adimensional coupling

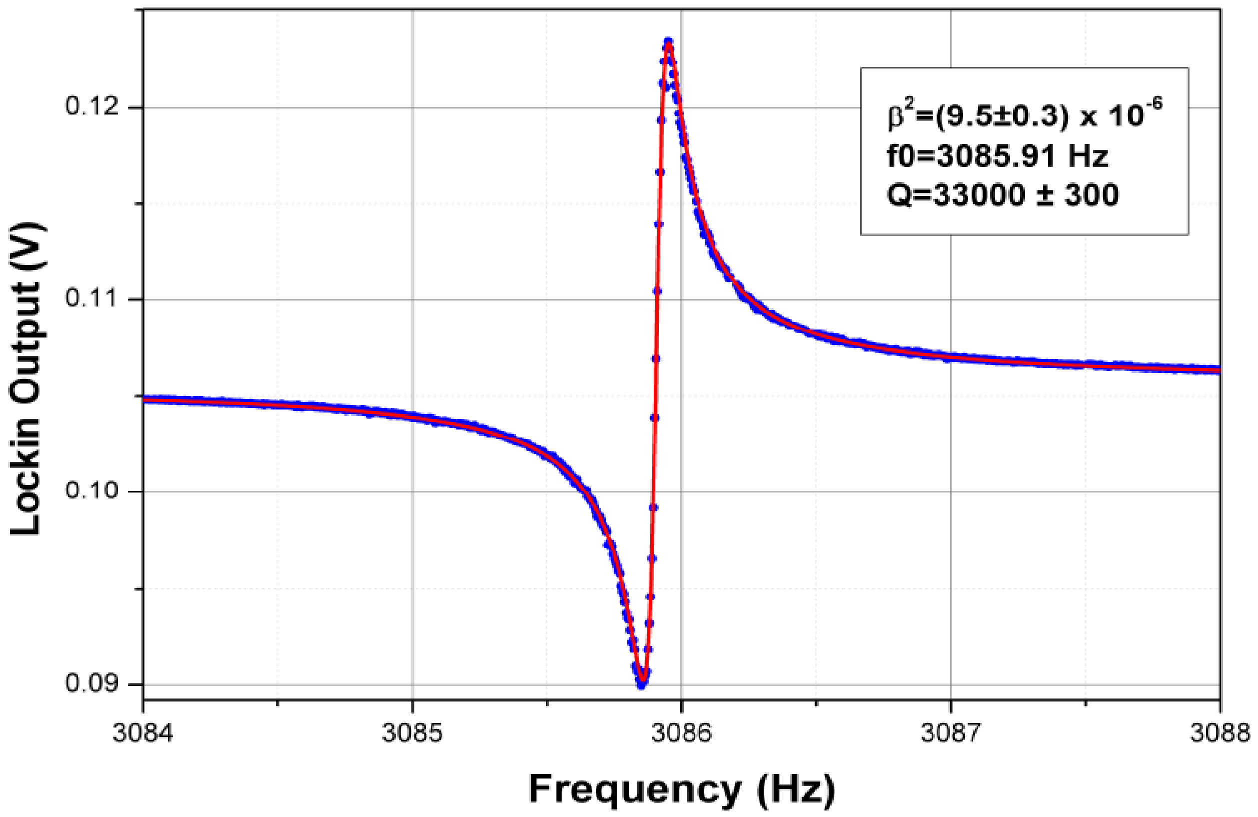
$$\alpha = G^{2} \frac{M^{2}}{L_{tot}}$$

SQUID gain + Superconducting loop

Measuring β^2



Inject calibration flux ϕ_{CAL}



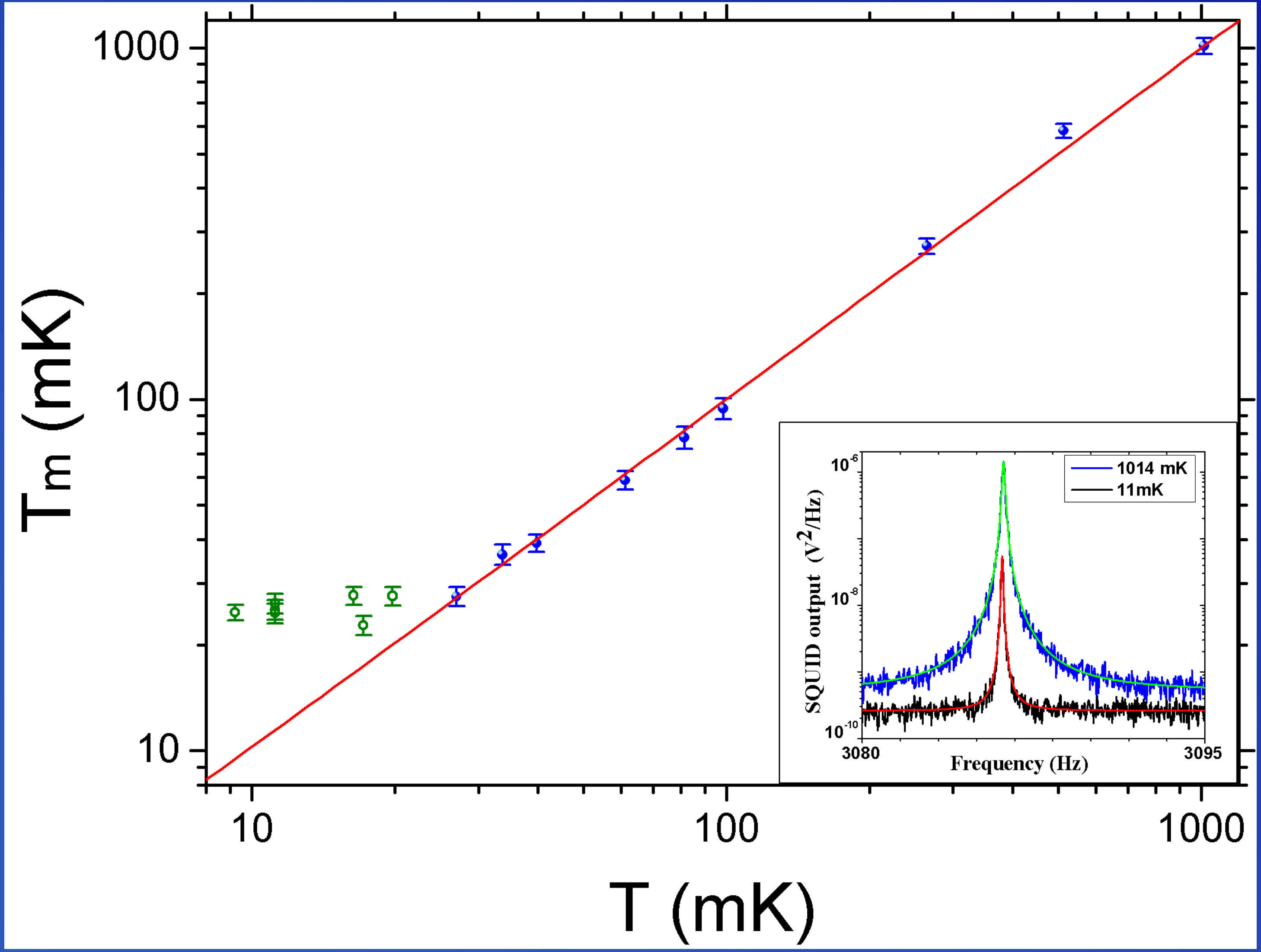
$$\frac{V}{\phi_{cal}} \propto \frac{f_1{}^2 - f^{\,2} + i\,\dfrac{ff_0}{Q}}{f_0{}^2 - f^{\,2} + i\,\dfrac{ff_0}{Q}}$$

$$f_1{}^2 = f_0{}^2 \times (1-\beta^2)$$

↓

β^2

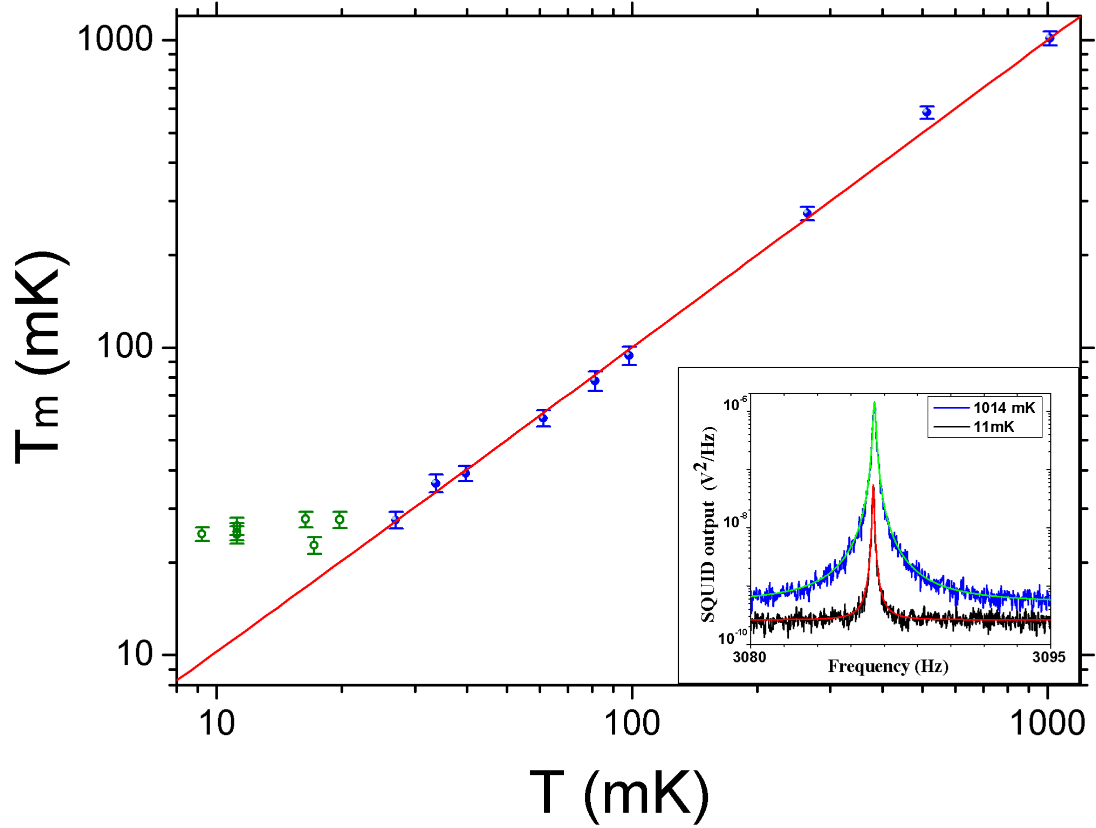
Mean Energy $\frac{\langle E \rangle}{k_B}$ vs Temperature



Force noise $S_{ff}=5\text{x}10^{-19}$ N/√Hz @ $T_m \sim 25$ mK

Non-thermal energy: how much?

CSL (as other effects...) would cause a finite positive intercept



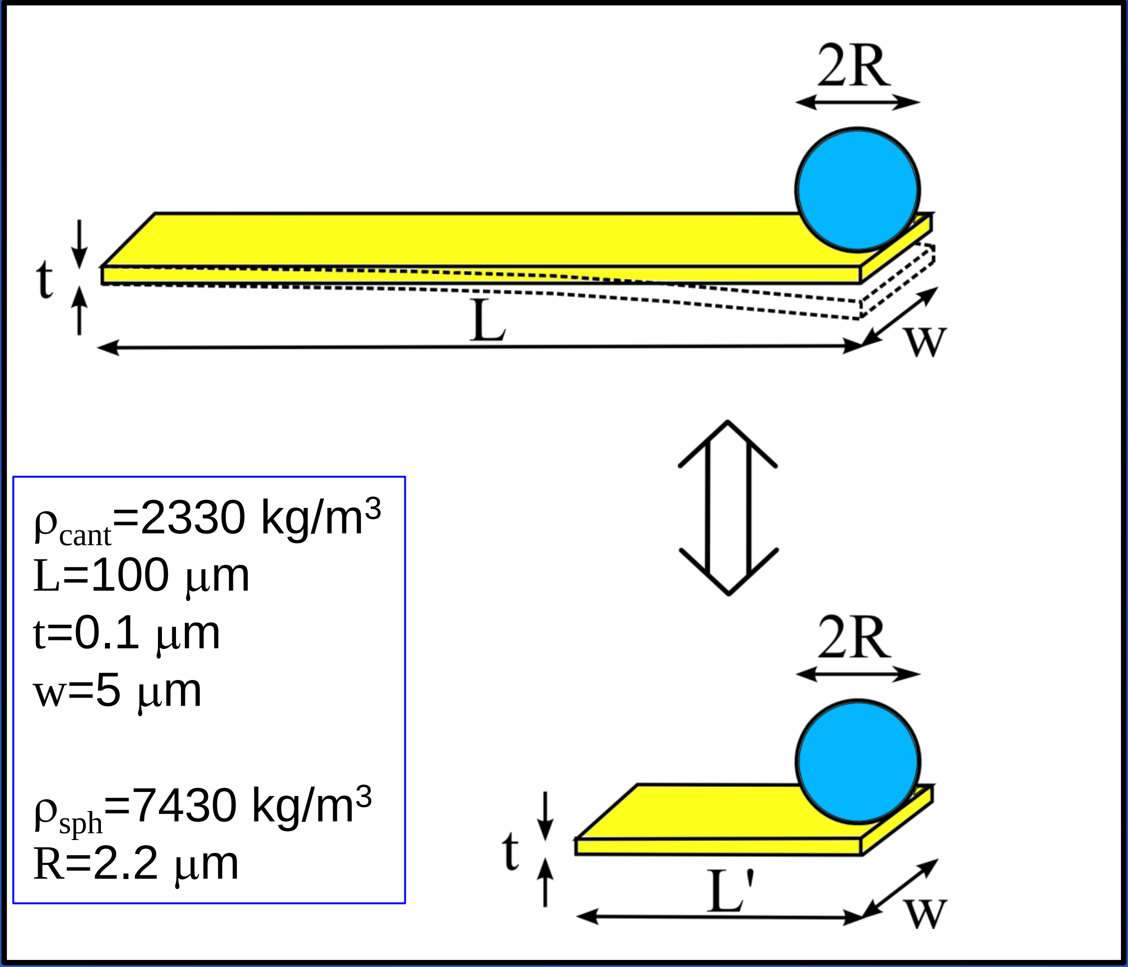
$T_m = T + \Delta T_{CSL}$

$\Delta T_{CSL} < 2.5$ mK
(95% C.L.)

Connect to CSL parameters

Technical issues:

- Composite object : CSL force noise acts sphere + cantilever (correlations)
- Bending mode (flexural). Standard CSL formulas hold for rigid motion



ρ_{cant}=2330 kg/m^3
L=100 μm
t=0.1 μm
w=5 μm

ρ_{sph}=7430 kg/m^3
R=2.2 μm

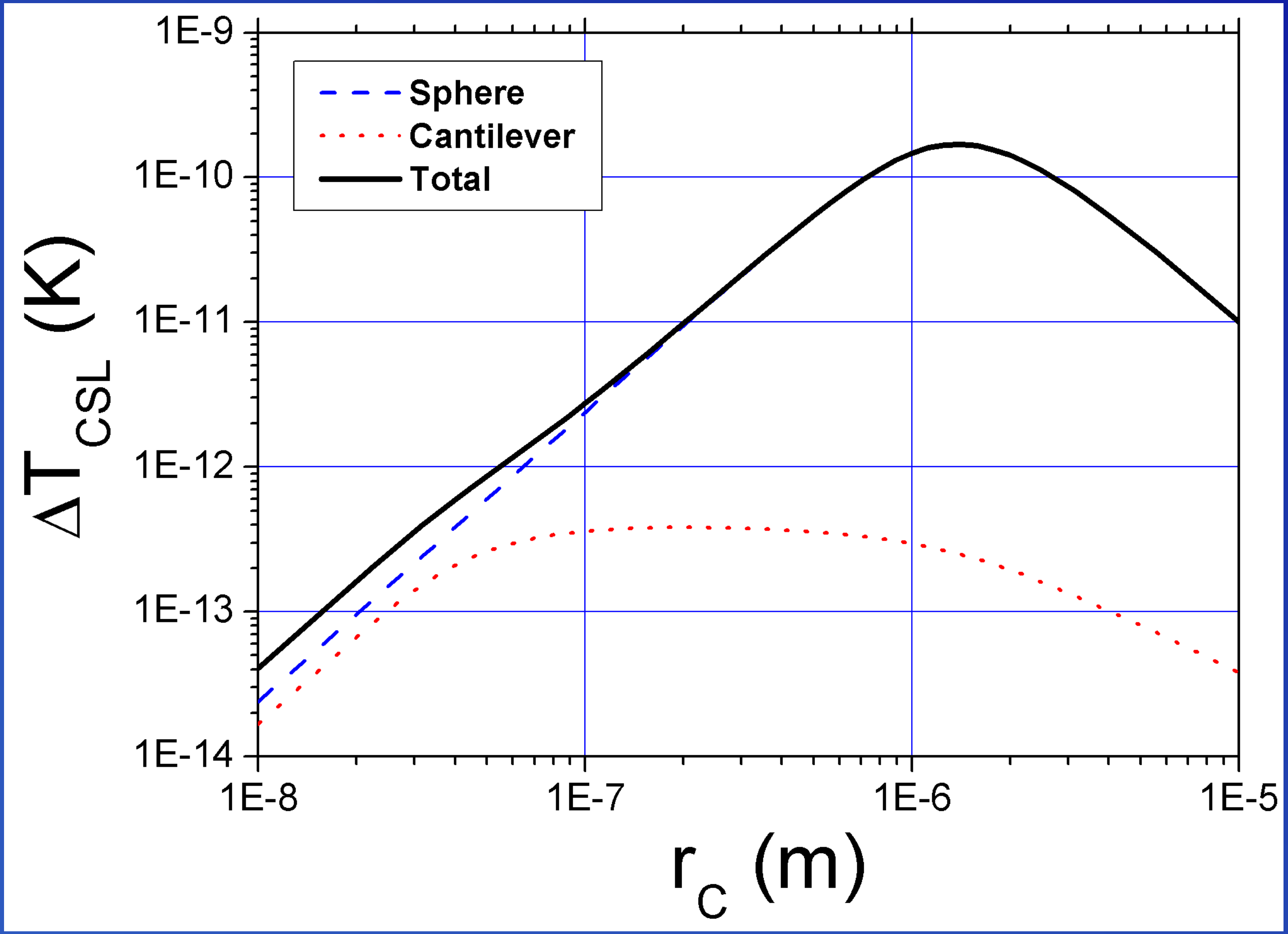
Solution:

- Approximate cantilever bending motion with a rigid translation of a slab with effective mass/length:

$$L' \approx 0.236\, L$$

Collaboration with Trieste group
(M.Bahrami , A. Bassi)

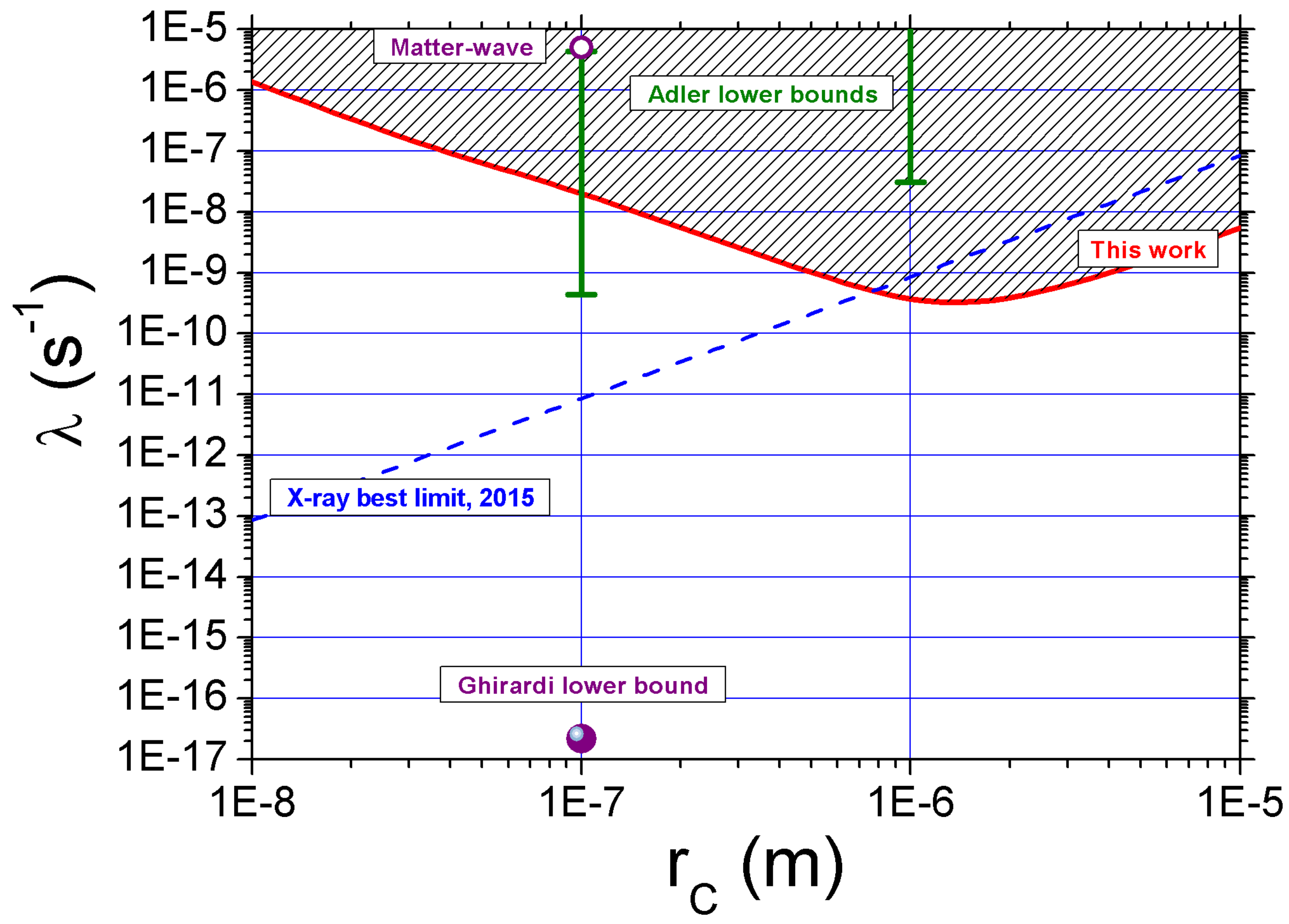
Computed CSL-induced heating vs r_C



Assuming Collapse rate from Ghirardi et al: $\lambda=2.2\text{x}10^{-17}$ Hz

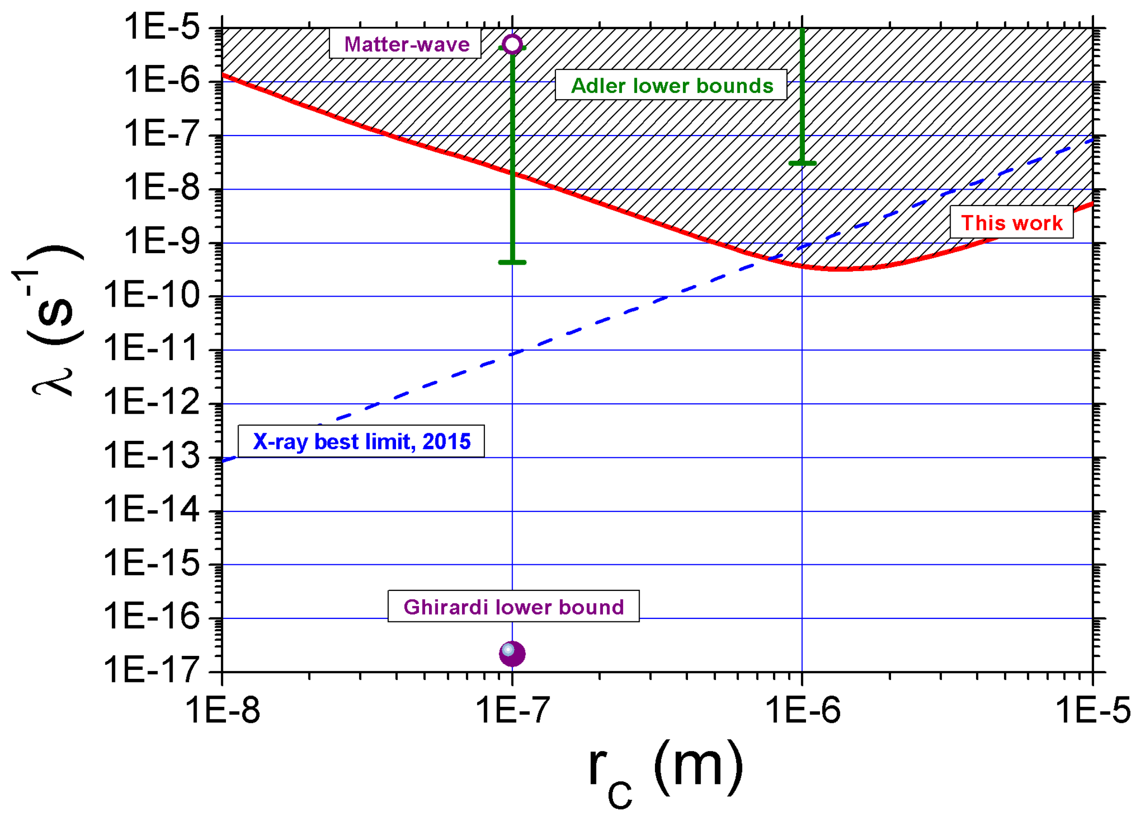
Upper Limit

[A. Vinante et al, Phys. Rev. Lett. 116, 090402 (2016)]



Mechanical vs X-ray

- Adler model totally excluded by X-ray
- Following Adler, X-ray limits could be evaded by additional hypothesis, i.e. CSL spectrum with cutoff



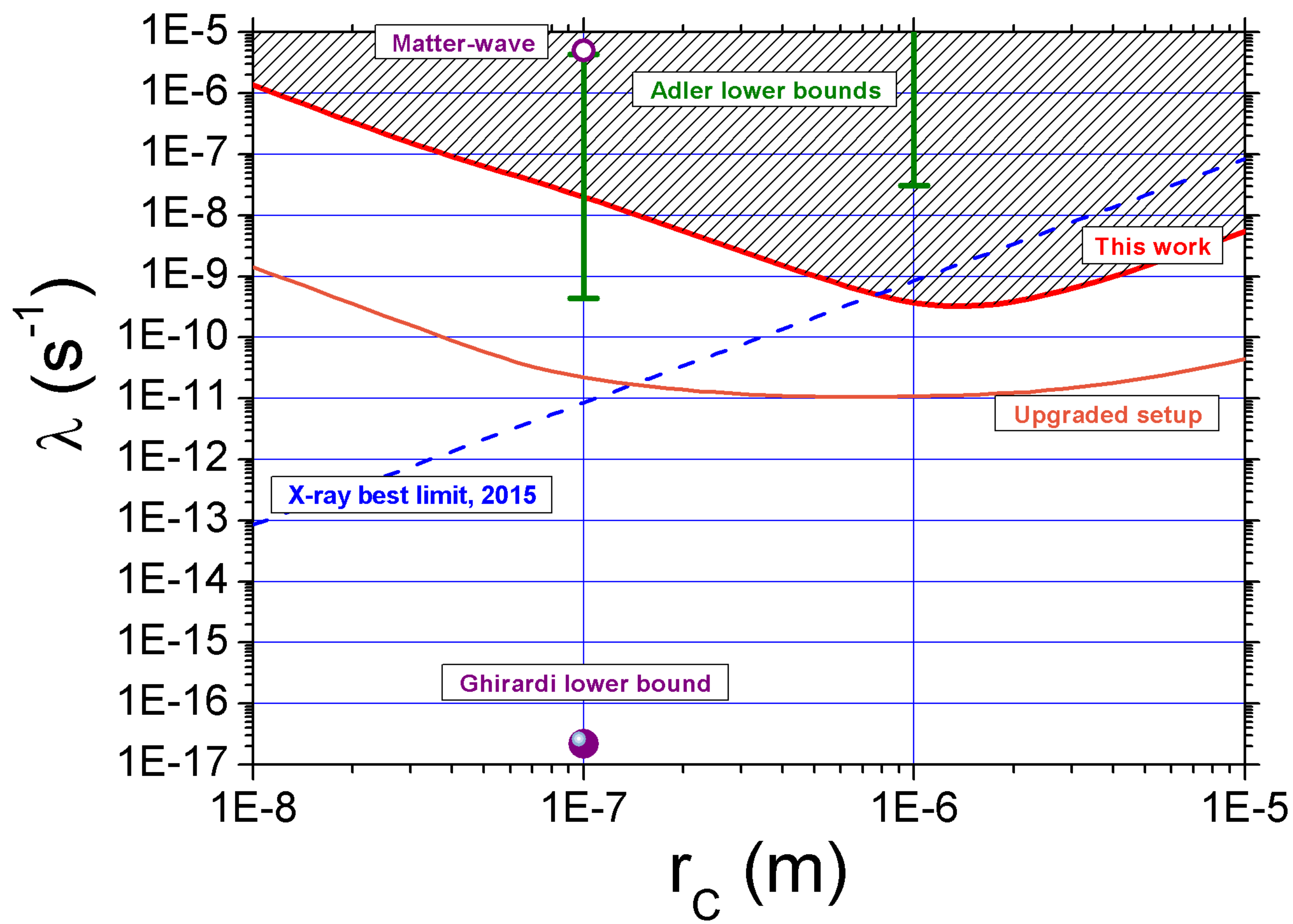
Upper limits set by mechanical resonators are stronger, in this respect.

~ same timescale , ms to s, of Adler effects (photographic process)

Outlook (1 year ago, discussing with T. Oosterkamp)

Same scheme - improved setup (but existing technology !)

- $Q \sim 10^5 \Rightarrow Q \sim 10^7$ (Diamond cantilevers)
- Heavier materials (Pb - Pt - FePt)



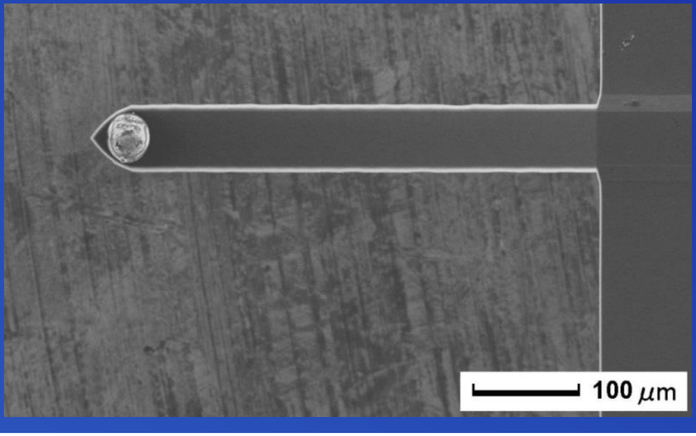
Still far from standard CSL…

BUT

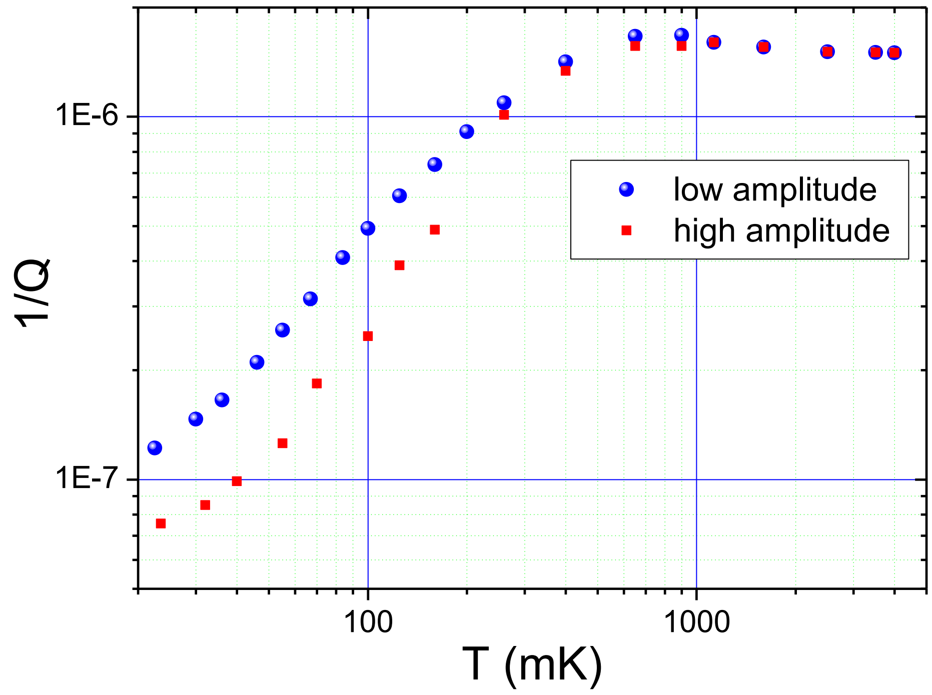
experiment will not be so easy …

- Vibrational noise
- Back-action noise

Most recent experiment in Trento



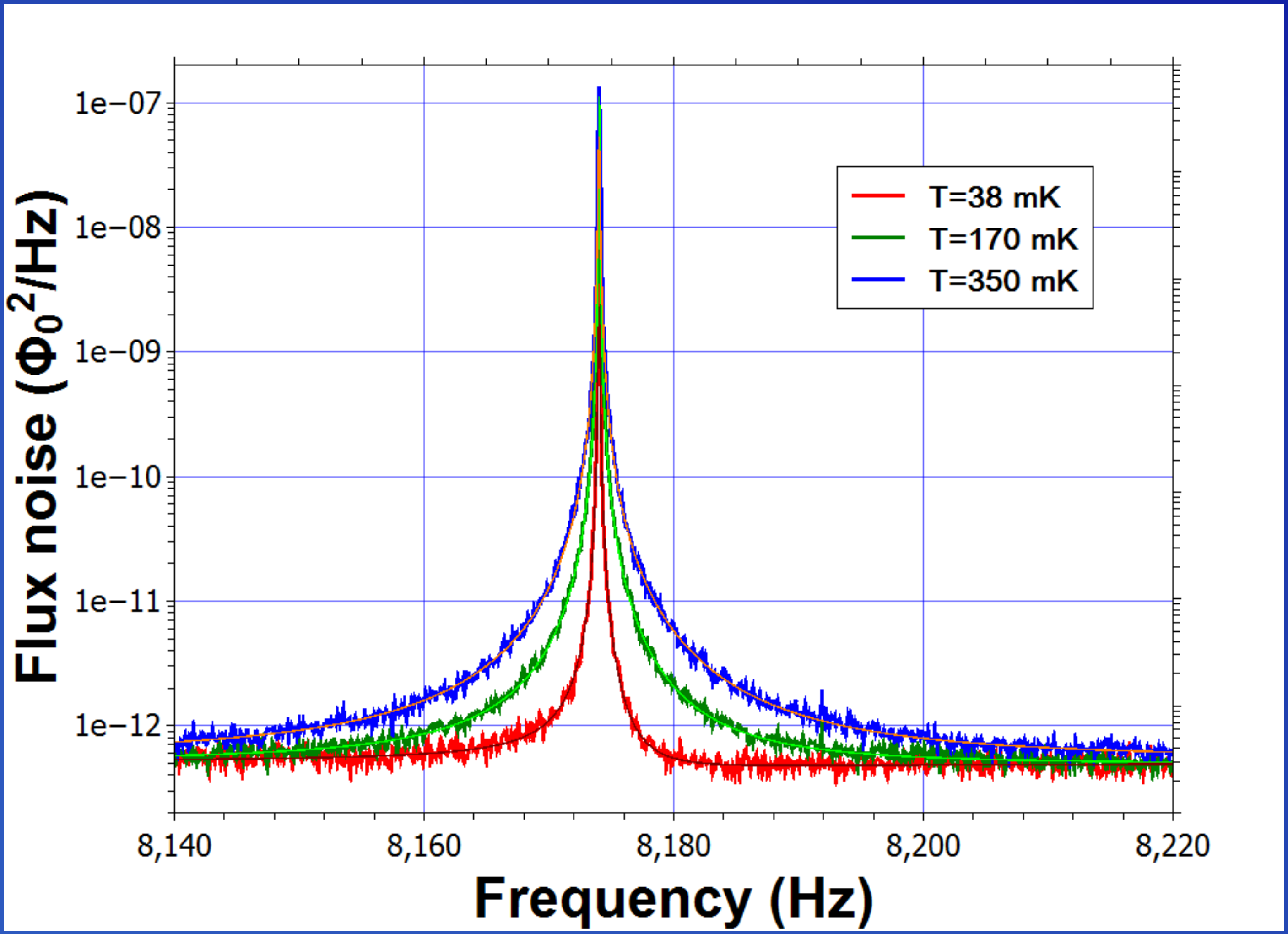
- Same idea, but thicker cantilever with higher Q
- AFM Silicon cantilever with bigger magnet (450x50x2 µm). Much stiffer (k=0.4 N/m)
- SQUID readout



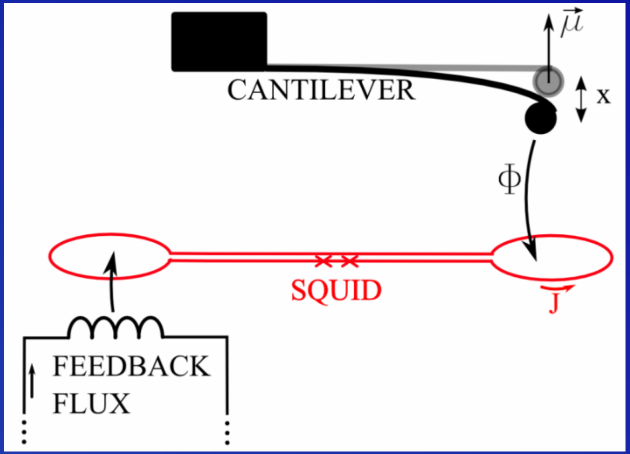
Very high Q~10^7
@ T<< 1K

(~10^5 with submicron devices)

Force noise at millikelvin temperature



Measurement scheme



$$Fit = A + B(T,Q)\frac{f_0^{\,4}}{\left(f^2 - f_0^{\,2}\right)^2 + \left(f_0 f / Q_a\right)^2}$$

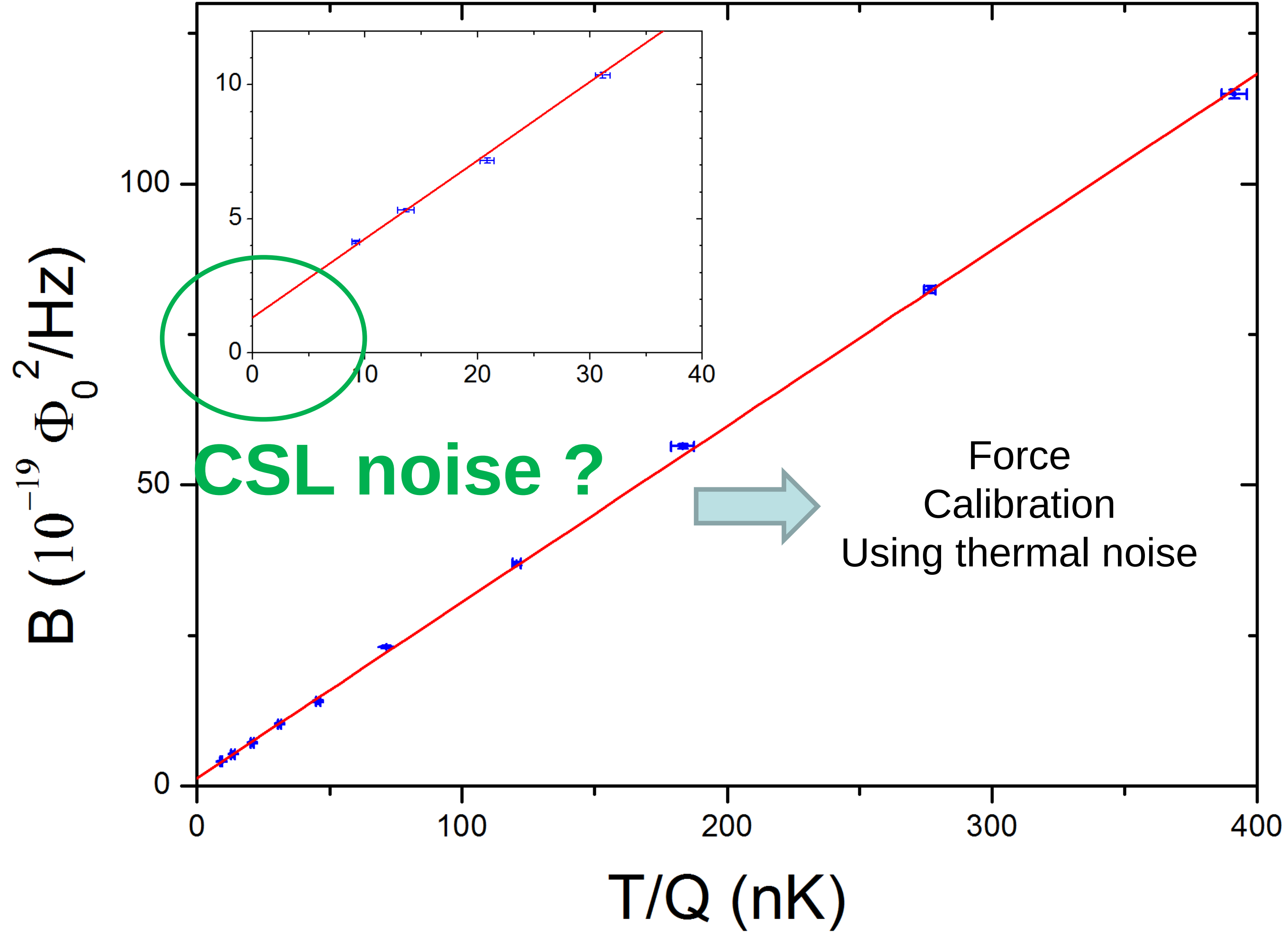
$$B \propto S_{ff}$$

Cantilever thermal noise

$$B \propto S_{ff} \propto \frac{T}{Q}$$

Nonzero Intercept !

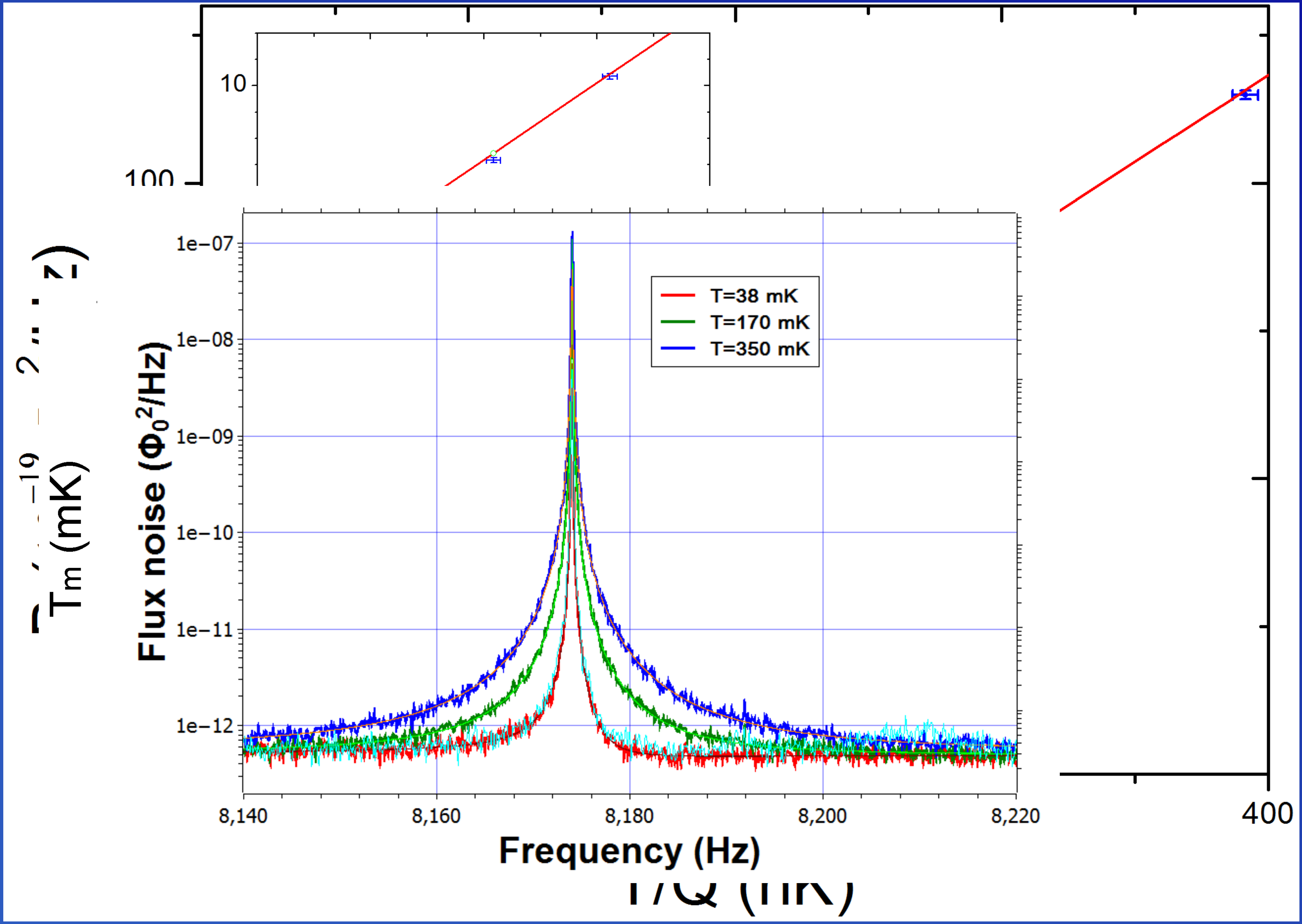
(1.24±0.14)E-19 Φ_0^2/Hz



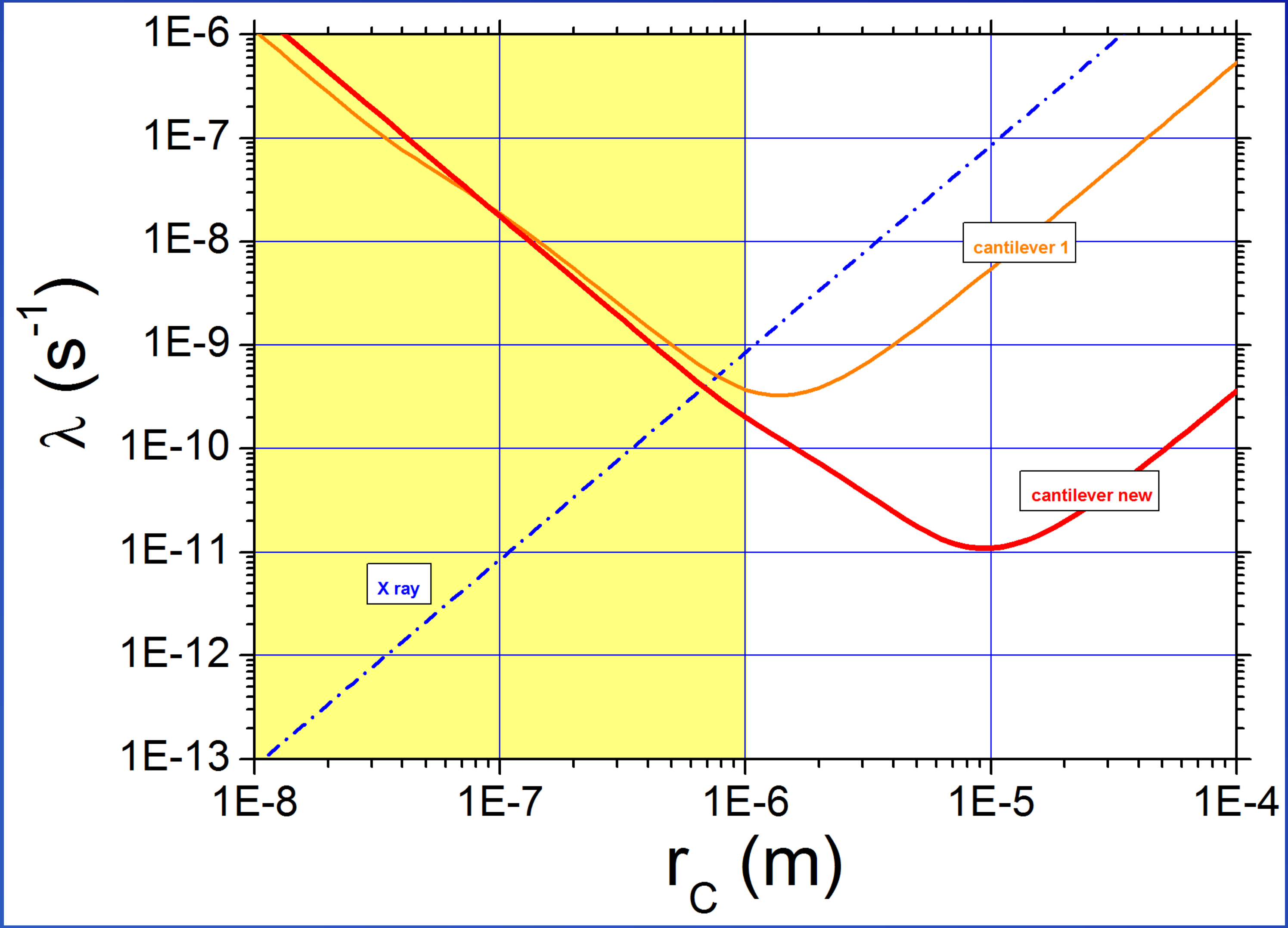
CSL noise ?

Force
Calibration
Using thermal noise

Potential sources of nonthermal noise



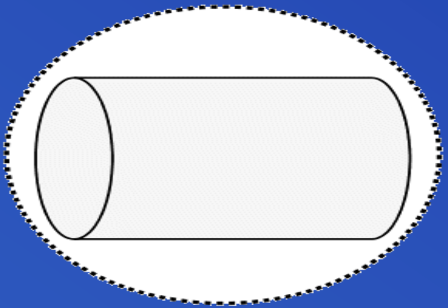
Preliminary upper limit
(or measured CSL noise, depending on interpretation…)



2) Bounds on CSL from macroscopic experiments: Gravitational wave detectors

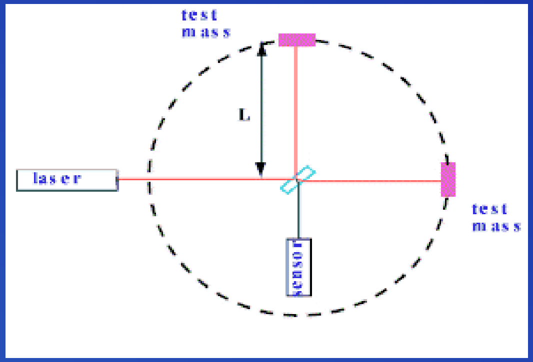
Our recent work: M. Carlesso et al, arXiv:1606.04581 (2016)

Resonant mass detectors



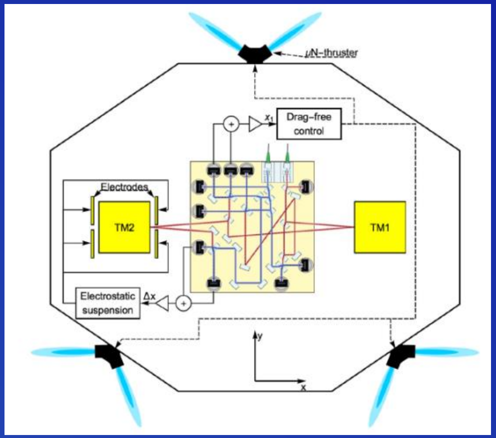
AURIGA (INFN, Italy)

Interferometric detectors



LIGO

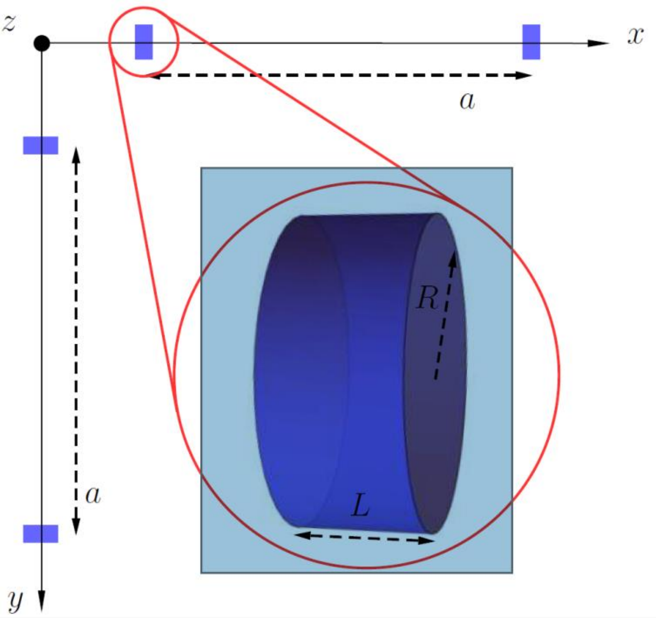
Space missions

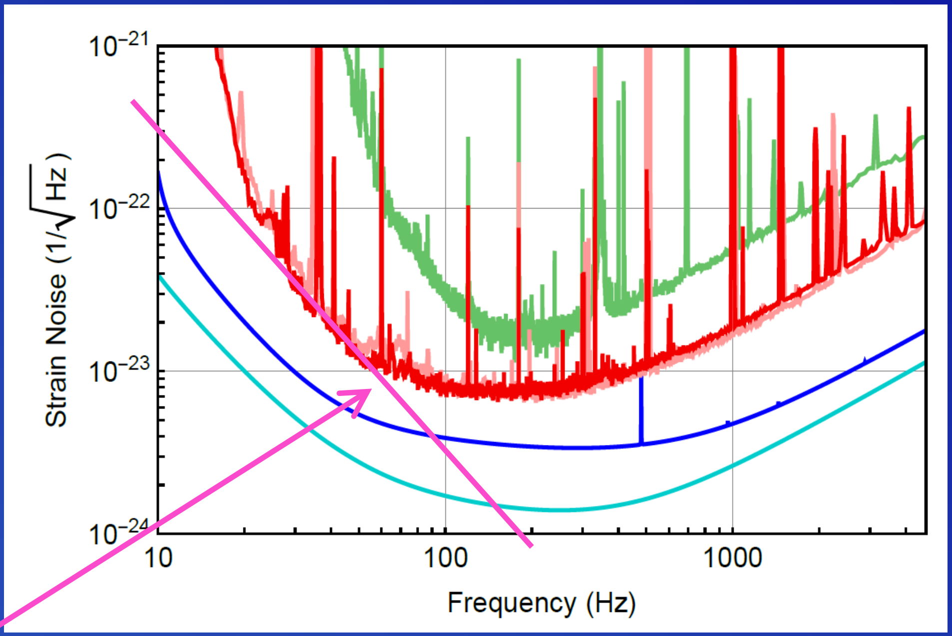


LISA Pathfinder

Advanced LIGO

- 4 Cylindrical test masses in near free-fall at f>10Hz (SiO$_2$, R=20 cm, L=17cm)





B.P. Abbott et al., Phys. Rev. Lett. 116, 061102 (2016)

- Residual force noise on test masses can be inferred by strain noise S$_h$ curve

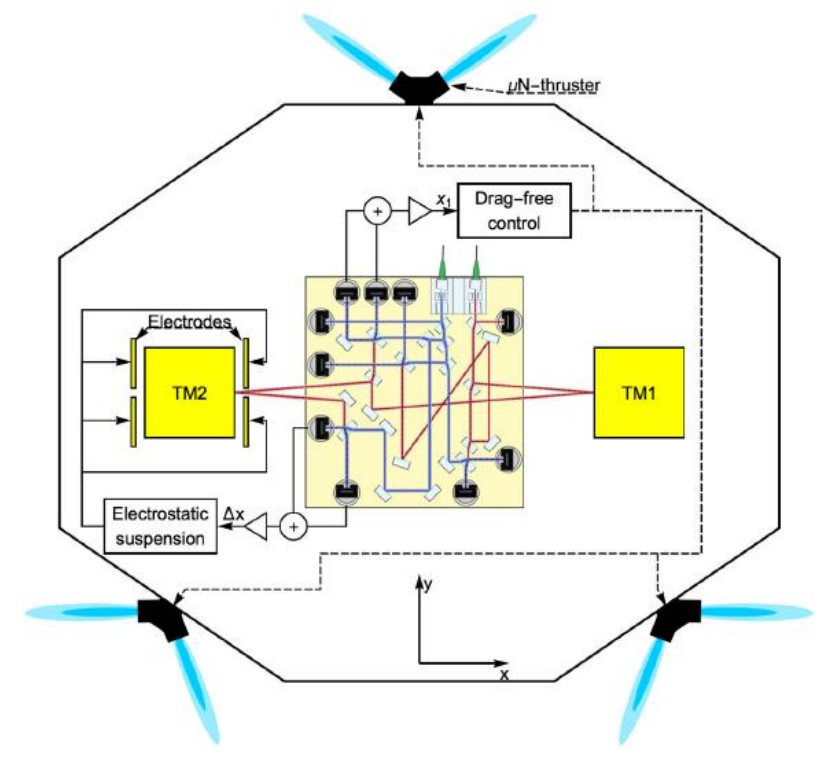
$$S_h = L S_x = \frac{L}{M\omega^2} S_f$$

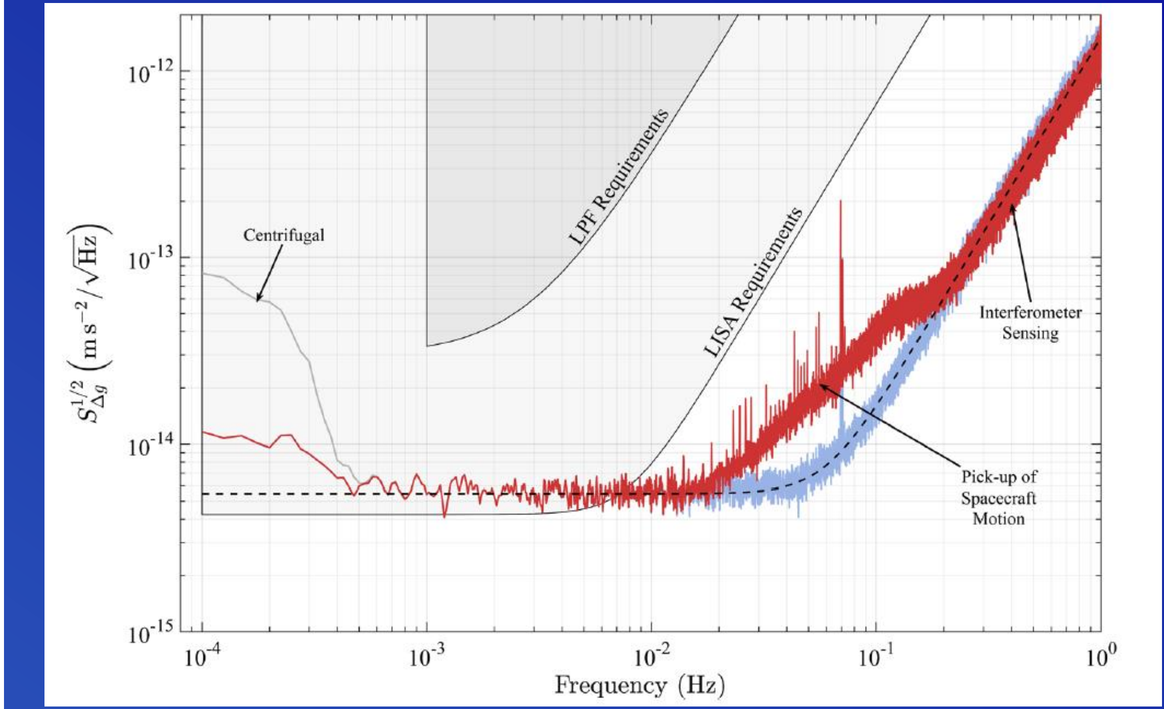
Lowest force noise S$_f$ ≈ 95 fN/√Hz @ 35 Hz

CSL force can be calculated from known geometry and material

LISA Pathfinder

- 2 cubic test masses in near free-fall @ f>1 mHz (AuPt, L=4.6 cm, M=2 kg)





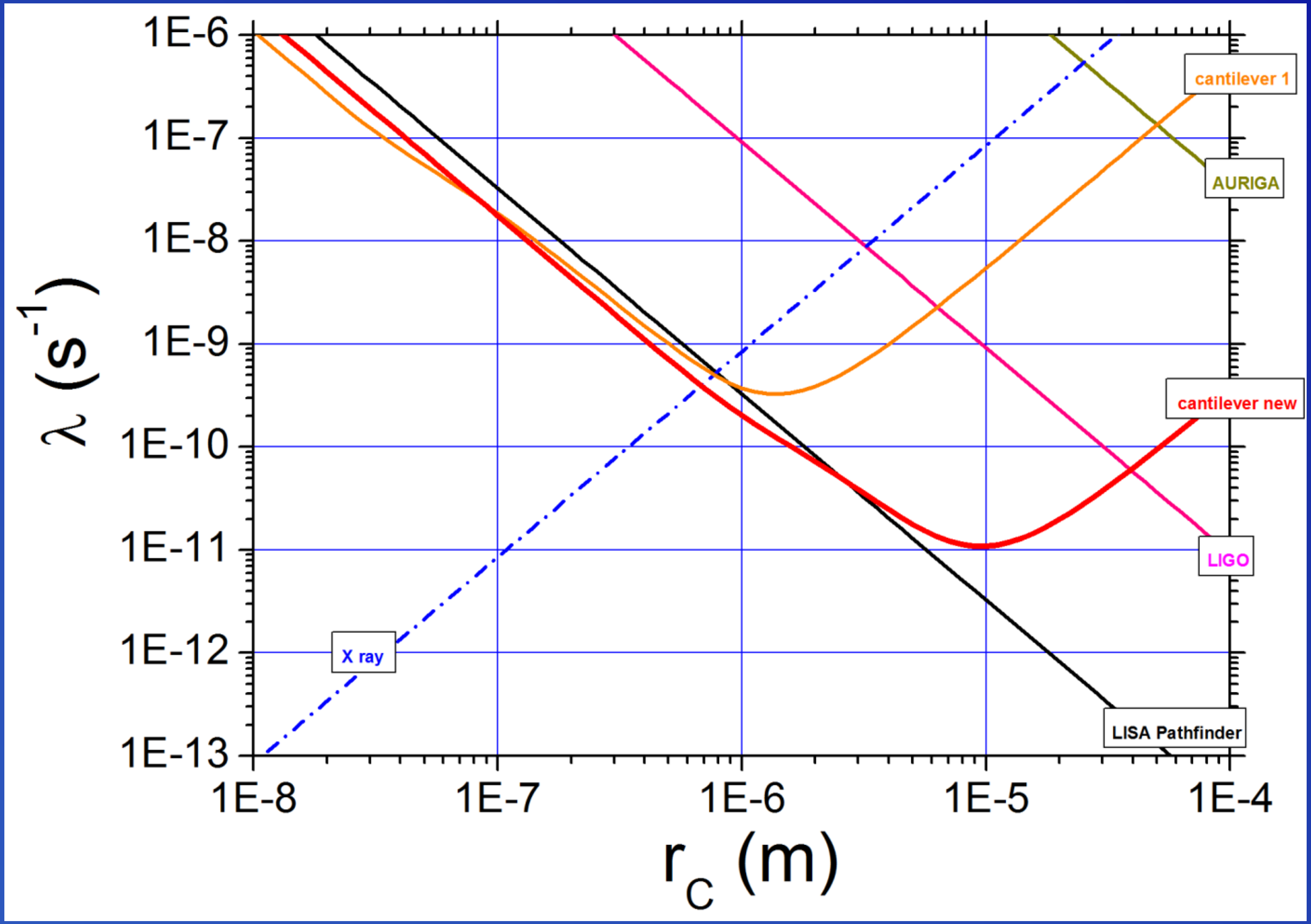
Lowest differential acceleration noise S_g=5.2 fm/s²/√Hz

force noise on single mass S_f=7.3 fN/√Hz

- material almost 10x denser than LIGO
- force noise 10x better than LIGO

M. Armano et al, Phys. Rev. Lett. 116, 231101 (2016)

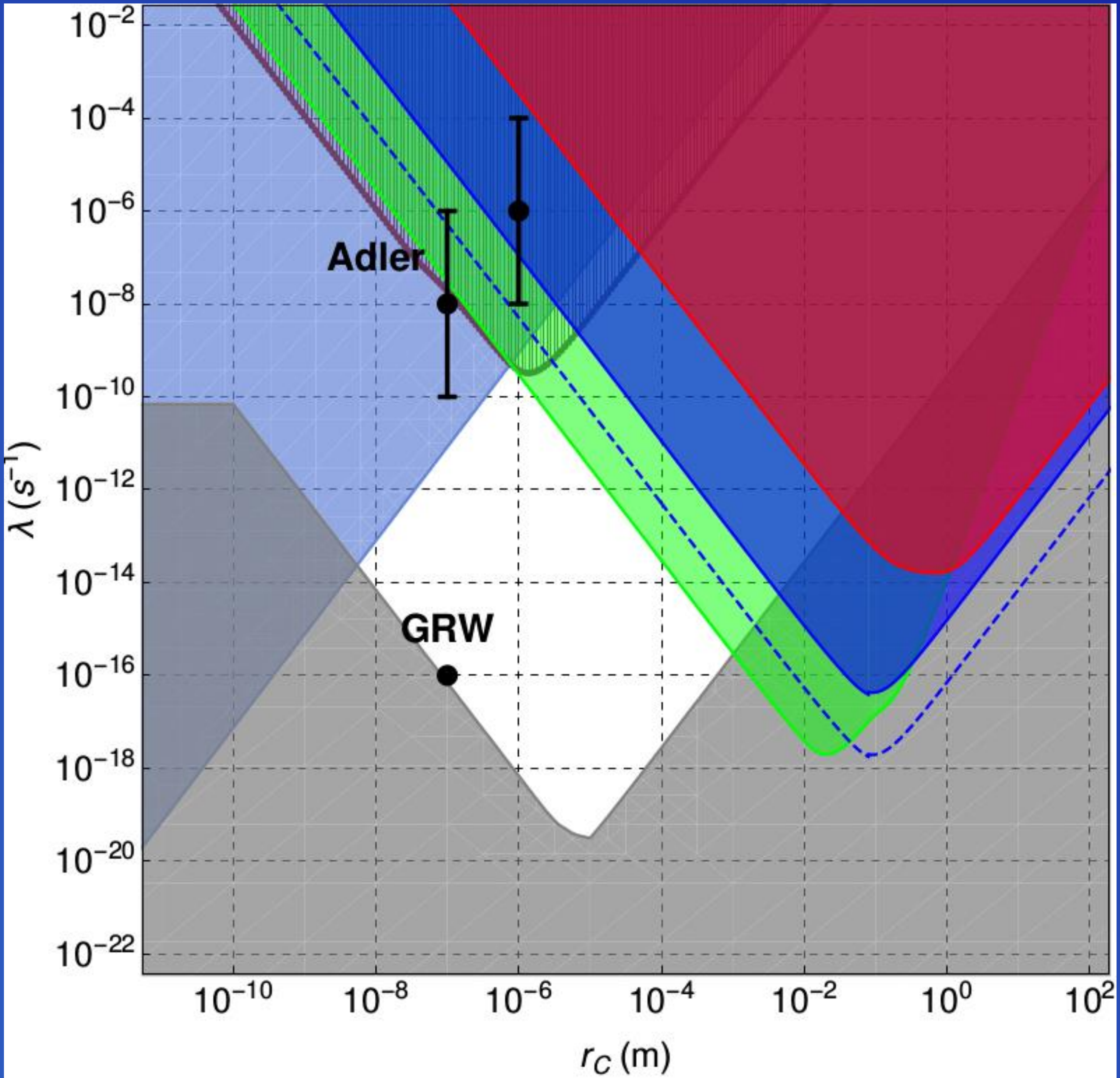
Upper limits on CSL



REMARKABLE: bound from LISA is comparable to nanomechanical systems at microscale!

Please also note that: Cantilever experiment cost $\sim 10^4$ €
LISA Pathfinder cost $\sim 10^9$ €

Zoom out (to make it more impressive)



M. Carlesso et al, arXiv:1606.04581 (2016)

Gray Region

CSL model is no more “natural” (collapse of macroscopic objects not granted anymore)

The Ellis model

- Proposed by people from high energy physics
  Inspired by ideas from Quantum Gravity
  Decoherence-like collapse of wavefunction would be caused by a bath of space-time wormholes at Planck length scale (spacetime "foam")
  J. Ellis, S. Mohanty and D.V. Nanopoulos, Phys. Lett. B 221, 113 (1989).

- Somehow resembles CSL, but no free parameters.
  Effective diffusion constant:

$$\eta_{\mathrm{Ellis}} = \frac{(cm_0)^4 m^2}{(\hbar m_{\mathrm{Pl}})^3}$$

- Present data from AURIGA-LIGO-LISA exclude Ellis model by 10 orders of magnitude !

  M. Carlesso et al, arXiv:1606.04581 (2016)

- NOTE: Ellis model also recently excluded by matter-wave interferometry !
  J. Minar et al, arXiv:1604.07810

The Diosi-Penrose (DP) model

- According to Penrose, the superposition principle is incompatible with the covariance principle of General Relativity. Massive superposition collapse is determined by gravity.

- DP model tries to incorporate this idea, but is essentially similar to the CSL model. In contrast with the original Penrose proposal, there must be a free parameter (r_C as in CSL) to suppress “spontaneous heating” effects.

- Diffusion constant as in CSL (force noise):

$$\eta = \frac{Gm\rho}{6\pi^{1/2}\hbar}\left(\frac{a}{r_C}\right)^3$$

a: lattice constant

- LISA Pathfinder data provides a lower bound on r_c

$$r_c > 40 \text{ fm}$$

B. Helou et al, arXiv:1606.03637

Other models

Several variations of collapse models have been recently introduced:

- Dissipative CSL.

The noise behaves as an effective thermal bath at unknown T_{CSL}

Force noise is the same as standard CSL

NO infinite energy gain. Equilibrium energy is eventually achieved
Energy monitoring methods may fail.

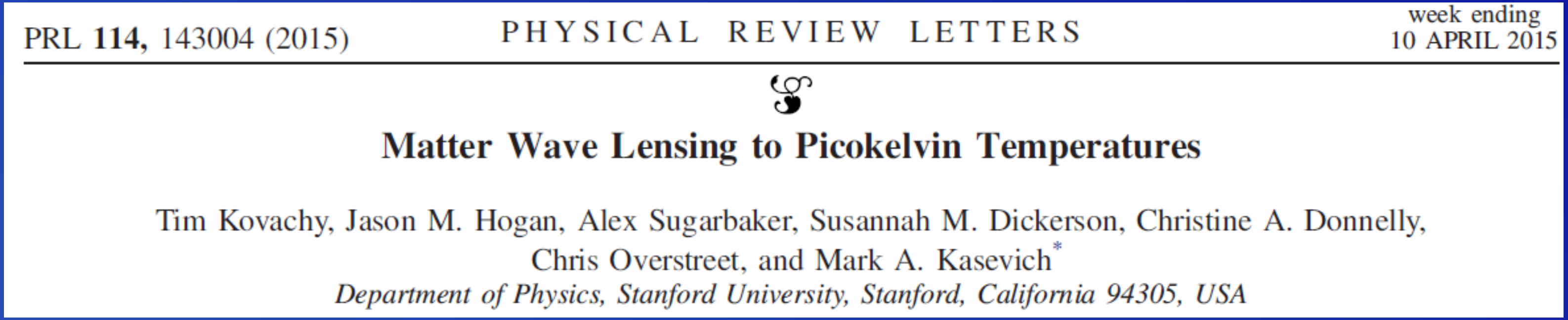
A fundamental dissipation mechanism should appear!

- Coloured noise CSL

CSL noise may depend on frequency, for instance a cutoff may exist

Need to test at different frequencies

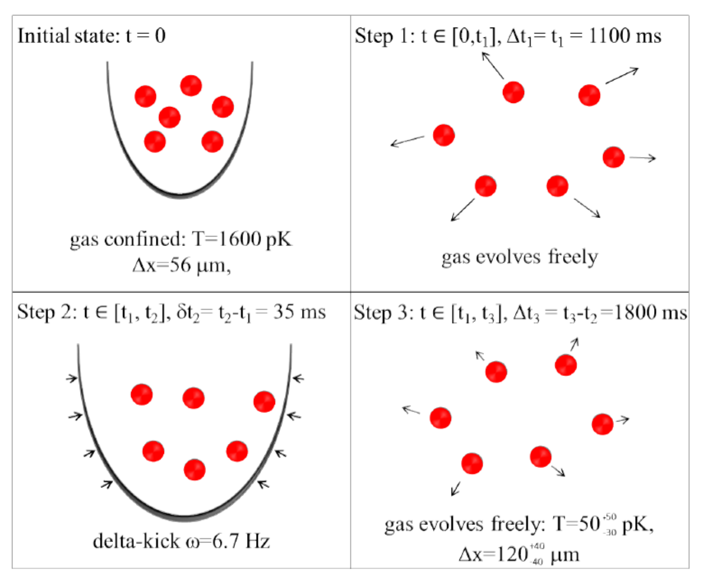
3) Upper bounds from cold atoms experiments

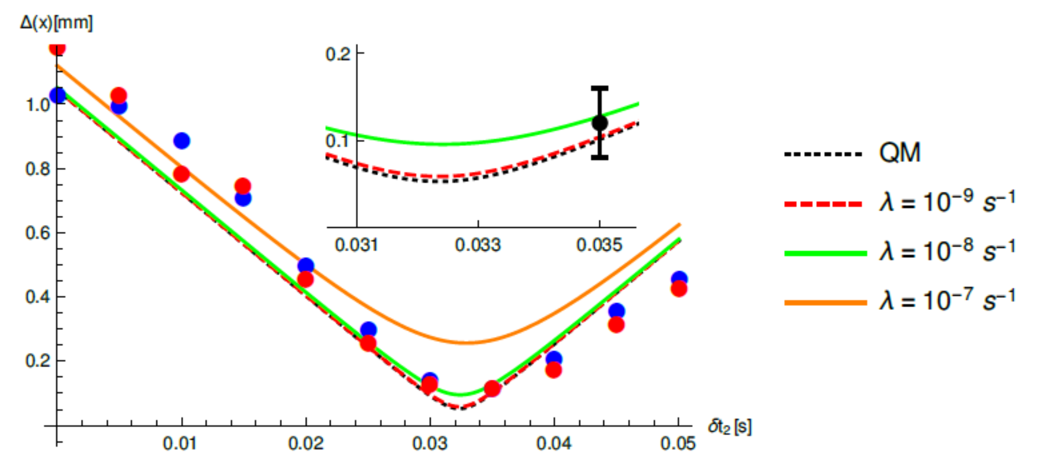


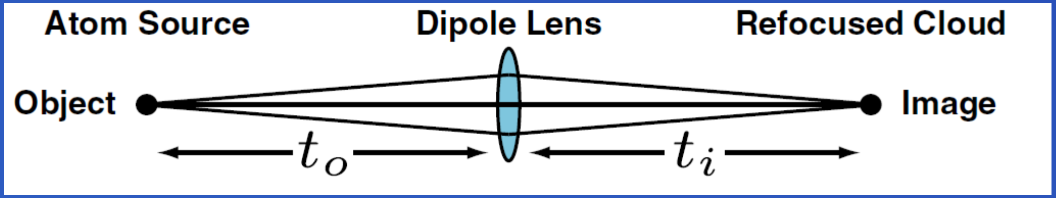
PRL **114,** 143004 (2015) PHYSICAL REVIEW LETTERS week ending 10 APRIL 2015

Matter Wave Lensing to Picokelvin Temperatures

Tim Kovachy, Jason M. Hogan, Alex Sugarbaker, Susannah M. Dickerson, Christine A. Donnelly,
Chris Overstreet, and Mark A. Kasevich*
Department of Physics, Stanford University, Stanford, California 94305, USA



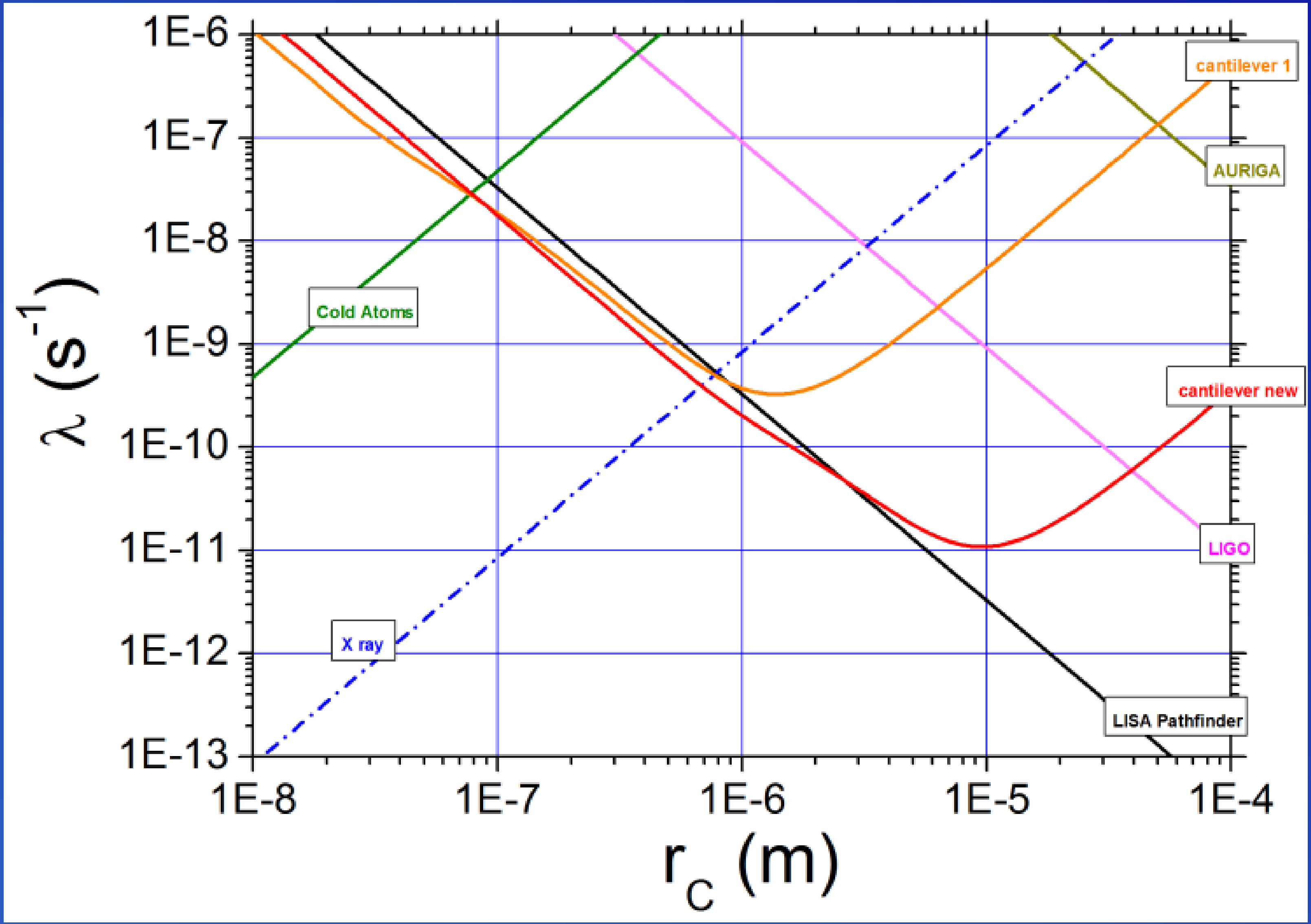




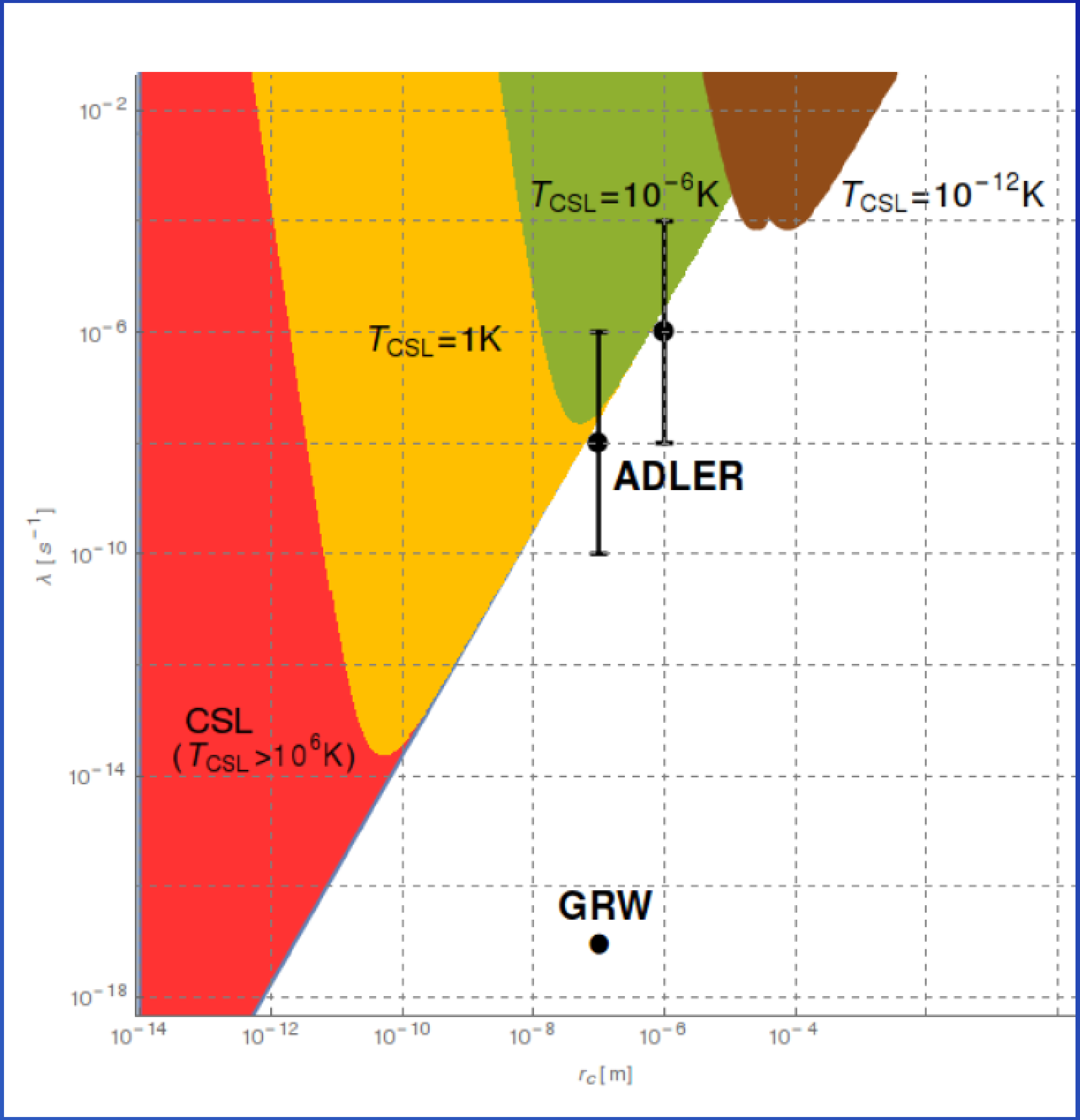
$$\frac{dT}{dt} < 50\,\mathrm{pK/s}$$

Upper bound on CSL from cold atoms

M. Bilardello et al, Physica A 462, 764 (2016)



Bound on the dCSL (dissipative CSL)



How to probe parameter space down to GRW parameters ?

- Conventional resonators can quickly improve 2 orders of mag, but hard to do much better.
  Quantum limit becomes relevant !
  Need to strongly improve the Q factor. Hard!

- Optically/magnetically levitated nano/microparticles
  In principle ultrahigh Q achievable
  Very intense research area
  (More in this school/workshop)

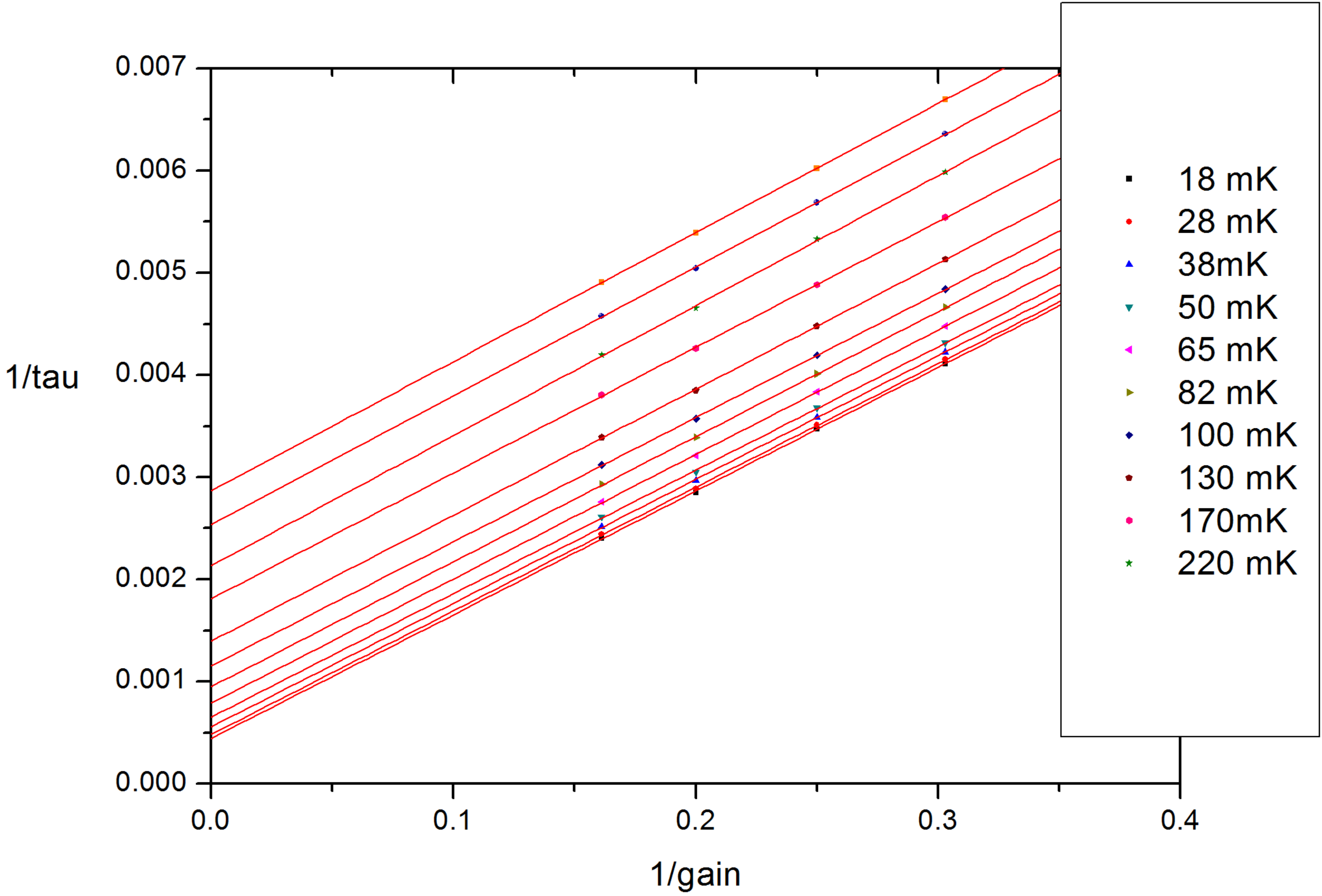
- Levitated micro/nano particles in space
  Seems very promising, after LISA results
  (MAQRO)

- Direct Interferometric tests with molecules or nanoparticles.

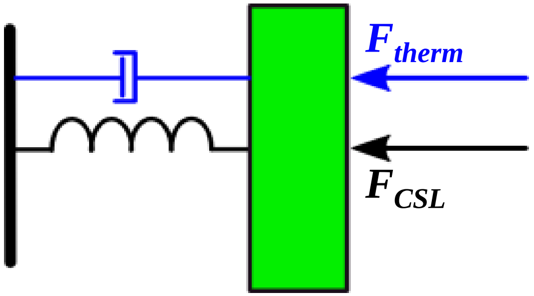
Will they overcome indirect tests, at the end of the day ?

Thank you all for attention !

SQUID magnetic spring



CSL Spontaneous heating of a solid mass



$$\Delta T_{\mathrm{CSL}} = \frac{\hbar^2 Q}{2m\omega_0 k_B}\eta$$

$$\eta_j = \frac{\gamma_{\mathrm{CSL}}}{m_0^2} \iint \frac{e^{-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4r_C^2}}}{(2\sqrt{\pi}\, r_C)^3} \frac{\partial \varrho(\mathbf{r})}{\partial r_j} \frac{\partial \varrho(\mathbf{r}')}{\partial r'_j} \, \mathrm{d}^3\mathbf{r}\, \mathrm{d}^3\mathbf{r}'$$

$$= \frac{\gamma_{\mathrm{CSL}}}{m_0^2} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \mathbf{k}_j^2 \, e^{-\mathbf{k}^2 r_C^2} \, |\tilde{\varrho}(\mathbf{k})|^2$$

FOR A CUBOID WITH SIDES L_1, L_2, L_3
(rc<< L_i) , rigid motion along L_1

$$\left.\frac{dE}{dt}\right|_{CSL} = W_{CSL} = \frac{4\pi\lambda\hbar^2 r_C{}^2 \rho^2 L_2 L_3}{m_0{}^2 m}$$

A solid body also features 3N-6 normal modes (phonons). For a cube, assuming only longitudinal modes:

$$k_n = (n_1, n_2, n_3)\frac{\pi}{L} \qquad \omega_n = |\vec{n}|\, v_s$$

1) How much power does CSL inject in a given normal mode ?????

2) Can we estimate (order of magnitude) the total power injected in the body ?

3) Can we devise an ultralow temperature experiment to detect this “spontaneous” heating?