<u>TRACE DYNAMICS:</u> *Quantum theory as an emergent phenomenon*

"Quantum mechanics is our most successful physical theory. However it raises conceptual issues that have perplexed physicists and philosophers of science for decades. This work develops a new approach, based on the proposal that quantum theory is not a complete, final theory, but is in fact an emergent phenomenon arising from a deeper level of dynamics. The dynamics at this deeper level is taken to be an extension of classical dynamics to non-commuting matrix variables, with cyclic permutation inside a trace used as the basic calculational tool. With plausible assumptions, quantum theory is shown to emerge as the statistical thermodynamics of this underlying theory, with the canonical commutation-anticommutation relations derived from a generalised equipartition theorem. Brownian motion corrections to this thermodynamics are argued to lead to state vector reduction and to the probabilistic interpretation of quantum theory, making contact with phenomenological proposals for stochastic modifications to Schrodinger dynamics."

- Stephen Adler, 2004

TRACE DYNAMICS

Quantum Theory as an Emergent Phenomenon

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“There is no doubt that quantum mechanics has seized hold of a beautiful element of truth and that it will be a touchstone for a future theoretical basis in that it must be deducible as a limiting case from that basis, just as electrostatics is deducible from the Maxwell equations of the electromagnetic field or as thermodynamics is deducible from statistical mechanics. I do not believe that quantum mechanics will be the starting point in the search for this basis, just as one cannot arrive at the foundations of mechanics from thermodynamics or statistical mechanics.”

- Einstein (1936)

Motivations for believing that QM is incomplete

- Historical Precedent
- The quantum measurement problem
- Origin of canonical quantisation
- Infinities in quantum field theory
- Non-locality
- Unification of quantum theory with gravitation
- The cosmological constant problem

Trace Dynamics [TD] bears the same relation to Quantum Theory as Statistical Mechanics bears to Thermodynamics.

TD can be described in three well-laid out steps:

STEP 1: CLASSICAL DYNAMICS

The classical theory, which is the classical dynamics of Grassmann-valued non-commuting matrices.

As a consequence of global unitary invariance the theory possesses a unique non-trivial conserved charge of great significance.

STEP 2: STATISTICAL THERMODYNAMICS

The statistical thermodynamics of this theory, the construction of the canonical ensemble and the derivation of equilibrium.

The derivation of an all important Ward identity, as a consequence of assumed invariance under constant shifts in phase space.

From here emerge, as thermodynamic averages, the canonical commutation relations of quantum theory, the Heisenberg equations of motion, and the equivalent Schrodinger equation of quantum theory.

STEP 3: STATISTICAL FLUCTUATIONS

The consideration of Brownian motion fluctuations around the above thermodynamic approximation.

The consequent non-linear stochastic modification of the Schrodinger equation, the resolution of the quantum measurement problem and derivation of the Born probability rule.

STEP 1: CLASSICAL DYNAMICS

FUNDAMENTAL DEGREES OF FREEDOM

- The fundamental degrees of freedom are matrices living on a background space-time with complex Grassmann numbers as elements.

- Grassmann variables have the following properties:

$$\theta_i\theta_j + \theta_j\theta_i = 0, \qquad \theta_i^2 = 0;$$
$$\chi = \theta_R + i\theta_I, \qquad \{\chi_r, \chi_s\} = 0.$$

- Thus we have a matrix field with the help of which to each space-time point we can associate a matrix.

FUNDAMENTAL DEGREES OF FREEDOM

- Some further properties of Grassmann elements:

(i) Product of an even number of Grassmann elements commutes with all the elements of Grassmann algebra.

(ii) Product of an odd number of Grassmann elements anti-commutes with any other odd number product.

- Therefore we have two disjoint sectors:

Bosonic Sector- B: Consists of the identity and the Even Grade elements of the algebra.

Fermionic Sector-F: Consists of the Odd Grade elements of the algebra.

FUNDAMENTAL DEGREES OF FREEDOM

- Therefore, the fundamental degrees of freedom of the trace dynamics theory are the matrices made out of elements from these sectors.

$$B_I \in \{M; M_{ij} \in B\} \qquad\qquad F_I \in \{M; M_{ij} \in F\}$$

- A general matrix can be decomposed as

$$M = \mathcal{A}_1(\in B_I) + \mathcal{A}_2(\in F_I)$$

into bosonic and fermonic sectors.

- We define an operation Trace on this matrix field as follows:

$$Tr : \mathcal{G}_M \to \mathcal{G}_C$$

TRACE DERIVATIVE

- We can construct a polynomial P from these non-commuting matrices (say O) and obtain the trace (indicated in bold) of the polynomial: $\mathbf{P} = TrP$

- Trace derivative of **P** with respect to the variable O defined as

$$\delta\mathbf{P} = Tr\frac{\delta\mathbf{P}}{\delta O}\delta O$$

- Example: $P = AOB$ $\delta P = A(\delta O)B$

$$\delta TrP = \epsilon_B TrBA\delta O$$

$$\frac{\delta\mathbf{P}}{\delta O} = \epsilon_B BA$$

LAGRANGIAN & HAMILTONIAN DYNAMICS

- We write the Lagrangian of a theory as a Grassmann even polynomial function of bosonic/fermionic operators and their time derivatives.

$$\mathbf{L}[\{q_r\},\{\dot{q}_r\}] = TrL[\{q_r\},\{\dot{q}_r\}]$$

- And the trace action $\mathbf{S} = \int dt\mathbf{L}$
- Equations of motion: $\dfrac{\delta \mathbf{L}}{\delta q_r} = \dfrac{d}{dt}\dfrac{\delta \mathbf{L}}{\delta \dot{q}_r}$
- Canonical momentum: $p_r \equiv \dfrac{\delta \mathbf{L}}{\delta \dot{q}_r}.$
- Trace Hamiltonian: $\mathbf{H} = Tr\sum_r p_r\dot{q}_r - \mathbf{L}$

$$\frac{\delta \mathbf{H}}{\delta q_r} = -\dot{p}_r, \qquad \frac{\delta \mathbf{H}}{\delta p_r} = \epsilon_r\dot{q}_r$$

LAGRANGIAN & HAMILTONIAN DYNAMICS

- We define a generalized Poisson Bracket over the phase space $\{q_r, p_r\}$

$$\{\mathbf{A}, \mathbf{B}\} = Tr \sum_r \epsilon_r \left(\frac{\delta \mathbf{A}}{\delta q_r} \frac{\delta \mathbf{B}}{\delta p_r} - \frac{\delta \mathbf{B}}{\delta q_r} \frac{\delta \mathbf{A}}{\delta p_r} \right)$$

which satisfies the Jacobi identity

$$\{\mathbf{A}, \{\mathbf{B}, \mathbf{C}\}\} + \{\mathbf{C}, \{\mathbf{A}, \mathbf{B}\}\} + \{\mathbf{B}, \{\mathbf{C}, \mathbf{A}\}\} = 0.$$

- Also

$$\dot{\mathbf{A}} = \frac{\partial \mathbf{A}}{\partial t} + \{\mathbf{A}, \mathbf{H}\}.$$

CONSERVED PARAMETERS

- The Trace Hamiltonian is conserved.

- Trace Fermion Number is conserved:

$$\mathbf{N} = \frac{1}{2} i Tr \sum_{r \in F} [q_r, p_r]$$

- As a result of global unitary invariance of the trace Hamiltonian

$$\mathbf{H}[\{U^\dagger q_r U\}, \{U^\dagger \dot{q}_r U\}] = \mathbf{H}[q_r, \dot{q}_r]$$

there exists a remarkable conserved charge:

$$\tilde{C} = \sum_{r \in B} [q_r, p_r] - \sum_{r \in F} \{q_r, p_r\}$$

PHASE SPACE

- One can define a measure on the matrix operator phase space:

$$d\mu = \prod_{r,m,n} d(x_r)^A_{mn}; \qquad A = 0, 1$$

$$(x_r)_{mn} = (x_r)^0_{mn} + i(x_r)^1_{mn}.$$

- This measure is invariant under canonical transformations. Liouville's theorem holds. Hence under a dynamical evolution

$$d\mu[\{x_r + \delta x_r\}] = d\mu[\{x_r\}].$$

- The bosonic and fermionic measures can be separated:

$$d\mu = d\mu_B d\mu_F$$

STEP 2: STATISTICAL THERMODYNAMICS

CANONICAL ENSEMBLE

- We study the evolution of phase space distribution.

We assume that a large enough system rapidly forgets its initial distribution and the time averages of physical quantities are equal to the statistical averages over an equilibrium ensemble which is determined by maximizing the combinatoric probability subject to conservation laws.

- Probability of finding the system in operator phase space volume element $d\mu[\{x_r\}]$

$$dP = d\mu[\{x_r\}]\rho[\{x_r\}$$

- For a system in statistical equilibrium, phase space density distribution is constant:

$$\dot{\rho}[\{x_r\}] = 0$$

CANONICAL ENSEMBLE

- Hence, ρ depends only upon conserved operators, conserved trace functionals and constant parameters.

- By going to a frame where the system is not translating, accelerating or rotating the charges associated with Poincare symmetry can be put to zero. In that case

$$\rho = \rho(\tilde{C}, \mathbf{H}, \mathbf{N})$$

- In addition the distribution function of dynamical variables can depend on constant parameters:

$$\rho = \rho(Tr\tilde{\lambda}\tilde{C}; \mathbf{H}, \tau; \mathbf{N}, \eta)$$

- Significant consequences follow from this general form.

CANONICAL ENSEMBLE

- We define the ensemble average of an operator O as

$$\langle O \rangle_{AV} = \int d\mu \; \rho O$$

- It can be shown that $\langle O \rangle_{AV} = F_O(\bar{\lambda})$ and so $[\bar{\lambda}, \langle O \rangle_{AV}] = 0$

- Now we choose $O = \tilde{C}$ It can be then shown that $\langle \tilde{C} \rangle_{AV}$

can be written in terms of a real diagonal and non-negative `magnitude' operator $D_{eff}$ and a unitary diagonal `phase' i_{eff}

$$\langle \tilde{C} \rangle = i_{eff} D_{eff}, \quad Tr(i_{eff} D_{eff}) = 0,$$

$$i_{eff} = -i^{\dagger}_{eff}, \quad i^2_{eff} = -I, \quad [i_{eff}, D_{eff}] = 0$$

- If the ensemble does not favour any one state over the other:

$$\langle \tilde{C} \rangle_{AV} = i_{eff} \hbar, \quad Tr\; i_{eff} = 0$$

$$i_{eff} = i[diag(1, -1, 1, -1, ..., 1, -1)$$

CANONICAL ENSEMBLE

- To obtain the equilibrium distribution, maximise the entropy

$$S = \int d\mu \; \rho \log \rho$$

subject to the constraints

$$\int d\mu \rho = 1, \qquad \int d\mu \rho \tilde{C} = \langle \tilde{C} \rangle$$

$$\int d\mu \rho \mathbf{H} = \langle \mathbf{H} \rangle, \qquad \int d\mu \rho \mathbf{N} = \langle \mathbf{N} \rangle,$$

to obtain

$$\rho = Z^{-1} \exp(-Tr \tilde{\lambda} \tilde{C} - \tau \mathbf{H} - \eta \mathbf{N})$$

$$Z = \int d\mu \exp(-Tr \tilde{\lambda} \tilde{C} - \tau \mathbf{H} - \eta \mathbf{N})$$

THE EMERGENCE OF QUANTUM FIELD DYNAMICS

- Motivation: equipartition theorem: Consider an $H(\{x_r\})$

$$\int d\mu \, \frac{\partial[x_r \exp(-\beta H)]}{\partial x_s} = \int d\mu \, \delta_{rs} \exp(-\beta H) - \int d\mu \, x_r \frac{\partial[\beta H]}{\partial x_s} \exp(-\beta H)$$

- Equipartition theorem: $\delta_{rs} = \dfrac{\int d\mu \, x_r \beta(\partial H/\partial x_s) \exp(-\beta H)}{\int d\mu \, \exp(-\beta H)}$

- The conserved charges of TD play a role analogous to that of energy in classical statistical physics.

- Motivated by this, and by the equipartition theorem, one derives a general Ward identity for the canonical ensemble.

- This leads to the emergence, at low energies, of an effective quantum field dynamics.

THE EMERGENCE OF QUANTUM FIELD DYNAMICS

- Under a constant shift of any matrix variable $x_r \to x_r + \delta x_r$,

$$\int d\hat{\mu}\delta_{x_r}(\rho_j O) = 0.$$

- Now consider the operator $O = \{\tilde{C}, i_{eff}\}W$

$$\int d\hat{\mu}\delta_{x_s}[\exp(-Tr\tilde{\lambda}\tilde{C} - \tau\mathbf{H} - \eta\mathbf{N}$$

$$-\sum_r Tr j_r x_r) Tr\{\tilde{C}, i_{eff}\}W] = 0.$$

- One makes the following assumptions:

(i) The energy scale τ^{-1} is the Planck scale and we are working at much lower energies.

(ii) The chemical potential η can be neglected.

(iii) The charge $\tilde{C}$ is replaced by its average $\langle\tilde{C}\rangle_{AV} = i_{eff}\hbar$

THE EMERGENCE OF QUANTUM FIELD DYNAMICS

- One obtains the Ward identity:

$$\langle \mathcal{D}x_u \rangle_{AV} = 0. \quad \mathcal{D}x_u = i_{eff}[W, x_u] - \hbar \sum_s \omega_{us} \left(\frac{\delta \mathbf{W}}{\delta x_s} \right)$$

- If we take W=H we obtain

$$\mathcal{D}x_u = i_{eff}[H, x_u] - \hbar \dot{x}_u$$

which gives the effective Heisenberg equations for the dynamics when canonically averaged over the ensemble.

- If we take $W = \tilde{\sigma}_v x_v \;\rightarrow\; i_{eff}\mathcal{D}x_u = [x_u, \tilde{\sigma_v} x_v] - i_{eff}\hbar\omega_{uv}\tilde{\sigma}_v$

effective canonical commutation relations emerge on averaging

$$\langle\langle q_u, q_v \rangle\rangle = \langle\langle p_u, p_v \rangle\rangle = 0 \qquad \langle\langle q_u, p_v \rangle\rangle = i_{eff}\hbar\delta_{uv}$$

- In this sense is quantum dynamics recovered.

THE EMERGENCE OF QUANTUM FIELD DYNAMICS

- We make the following correspondences between operator polynomials in trace dynamics and operator polynomials in quantum field theory:

$$S(\{x_r\}) \Leftrightarrow S(\{X_r\})$$

$$\psi_0^\dagger \langle S(\{x_r\}) \rangle_{AV} \psi_0 = \langle vac | S(\{X_r\}) | vac \rangle$$

$$[X_u, \tilde{\sigma}_v X_v] = i_{eff} \hbar \omega_{uv} \tilde{\sigma}_v$$

$$\dot{X}_u = i_{eff} \hbar^{-1} [H, X_u]$$

- The transition to the Schrodinger picture is made as usual.

STEP 3: STATISTICAL FLUCTUATIONS

- In deriving the Schrodinger equation we have made certain approximations valid at equilibrium.

- We replaced $\tilde{C}$ by its canonical average. If we also consider the fluctuations about the average quantities we have possibility of obtaining a stochastic equation of evolution

- By treating quantum theory as a thermodynamic approximation to a statistical mechanics, the theory opens the door for the ever-present statistical fluctuations to play the desired role of the non-linear stochasticity which impacts on the measurement problem.

STATISTICAL FLUCTUATIONS

- We consider fluctuations in the Adler-Millard charge:

$$\Delta\tilde{C} = \tilde{C} - \langle\tilde{C}\rangle_{AV} = \tilde{C} - i_{eff}\hbar = -\hbar(\mathcal{K} + \mathcal{N})$$

- and obtain the modified stochastic Schrodinger equation:

$$|\dot{\Phi}\rangle = [i\hbar^{-1}\{-1 + (\mathcal{K}_0(t) + i\mathcal{K}_1(t))\}H$$
$$\frac{1}{2}i(\mathcal{M}_0(t) + i\mathcal{M}_1(t))]|\Phi\rangle,$$

$$H = \sum_r \sum_l \frac{1}{2} i m_r [\psi^\dagger_{rl}, \psi_{rl}] \qquad \mathcal{N} = \sum_r \mathcal{N}_{rl}$$

$$\mathcal{M}(t) = \sum_{r,l} m_r \mathcal{N}(t)_{r,l}$$

- Stochastic process: Brownian motion case

$$(dW_t^n)^2 = \gamma_n dt,$$
$$dW_t^n dW_t^m = 0, m \neq n,$$
$$dW_t^n dt = dt^2 = 0.$$

Stochastic Modification of Schrodinger Equation

- For the c-number fluctuations:

$$i\hbar^{-1}\mathcal{K}_0 dt = i\beta_I dW_t^I,$$
$$-\hbar^{-1}\mathcal{K}_1 dt = \beta_R dW_t^R,$$

- For the fluctuating matrix with spatial correlations:

$$\frac{1}{2}i\mathcal{M}_0 dt = i\int d^3x dW_t^I(\vec{x})\mathcal{M}_t^I(\vec{x}),$$
$$-\frac{1}{2}\mathcal{M}_1 dt = \int d^3x dW_t^R(\vec{x})\mathcal{M}_t^R(\vec{x}),$$

- Stochastic Schrodinger Equation:

$$|d\Phi\rangle = [-i\hbar^{-1}H dt + i\beta_I dW_t^I H_{eff}$$
$$+\beta_R dW_t^R H + i\int d^3x dW_t^I(\vec{x})\mathcal{M}_t^I(\vec{x})$$
$$+\int d^3x dW_t^R(\vec{x})\mathcal{M}_t^R(\vec{x})]|\Phi\rangle.$$

Stochastic Modification of Schrodinger Equation

- The previous equation is not norm-preserving. Define a physical state with a conserved norm: $|\Psi\rangle = \frac{|\Phi\rangle}{\langle\Phi|\Phi\rangle^{1/2}}$
- Along with the criterion of no superluminal signalling, this leads to the non-linear stochastic dynamics:

$$
\begin{aligned}
d|\Psi\rangle = \Big[-i\hbar^{-1}H dt + i\beta_I H dW_t^I \\
-\frac{1}{2}[\beta_I^2 H^2 + \beta_R^2 (H - \langle H\rangle)^2] dt \\
+\beta_R (H - \langle H\rangle) dW_t^R \\
+i\int d^3x \mathcal{M}^I(\vec{x}) dW_t^I(\vec{x}) - \\
\frac{\gamma}{2} dt \int d^3x [\mathcal{M}^I(\vec{x})^2 + (\mathcal{M}^R(\vec{x}) - \langle \mathcal{M}^R(\vec{x})\rangle)^2] \\
+\int d^3x (\mathcal{M}^R(\vec{x}) - \langle \mathcal{M}^R(\vec{x})\rangle) dW_t^R(\vec{x}) \Big] |\Psi\rangle.
\end{aligned}
$$

- Compare with Continuous Spontaneous Localisation model:

$$d\psi_t = \Big[-\frac{i}{\hbar} H dt$$
$$+\frac{\sqrt{\gamma}}{m_0}\int d\mathbf{x}(M(\mathbf{x}) - \langle M(\mathbf{x})\rangle_t) dW_t(\mathbf{x})$$
$$-\ \frac{\gamma}{2m_0^2}\int d\mathbf{x}\,(M(\mathbf{x}) - \langle M(\mathbf{x})\rangle_t)^2 dt\Big]\,\psi_t$$

- This can be obtained from TD by choosing:

$$\mathcal{M}^R(\mathbf{x}) = \int d^3y\; g(\mathbf{x}-\mathbf{y}) \sum_r m_r N_r(\mathbf{y})$$

Collapse of the wave-function, and the Born rule

- Consider a simplified version of the stochastic equation:

$$d|\Psi\rangle = (-i\hbar^{-1}H - \frac{1}{2}[\beta_R^2(A-\langle A\rangle)^2 + \beta_I^2 A^2])|\Psi\rangle dt + \beta_R(A-\langle A\rangle)|\Psi\rangle dW_t^R + i\beta_I A|\Psi\rangle dW_t^I$$

- The corresponding equation for the density matrix is:

$$d\hat{\rho} = i\hbar^{-1}[\hat{\rho}, H_{eff}]dt - \frac{1}{2}|\beta|^2[A,[A,\hat{\rho}]dt + \beta_R[\hat{\rho},[\hat{\rho},A]]dW_t^R + i\beta_I[A,\hat{\rho}]dW_t^I.$$

- Define E[ ] as the expectation value w.r.t. stochastic process

Collapse of the wave-function, and the Born rule

- And define the variance of A:

$$V = \langle (A - \langle A \rangle)^2 \rangle = Tr\hat{\rho}A^2 - (Tr\hat{\rho}A)^2$$

$$dE[V] = E[dV] = -4\beta_R^2 E[V^2]dt$$

$$E[V(t)] = E[V(0)] - 4\beta_R^2 \int_0^t ds E[V(s)^2]$$

$$0 \leq E[(V - E[V])^2] = E[V^2] - E[V]^2$$

$$E[V(t)] \leq E[V(0)] - 4\beta_R^2 \int_0^t ds E[V(s)]^2.$$

- Non-negativity of variance suggests that

$$E[V(\infty)] = 0, \qquad V[\infty] \to 0$$

- The system hence results in one of the eigenstates of A.
- Also: $E[V(t)] \leq \dfrac{V[0]}{1 + 4\beta_R^2 V[0]t}$

The Born Probability Rule

- Take $\Pi_a = |a\rangle\langle a|$ as the projector into the a-th eigenstate of A
- If the initial state of the system is $|\Psi_i\rangle = \sum_a p_{ia}|a\rangle$

at t=0 when the stochastic evolution has not started,

$$E[\langle\Pi_a\rangle]_{t=0} = \langle\Pi_a\rangle_{t=0} = |p_{ia}|^2.$$

- The system results in a particular eigenstate $|f\rangle$ with some probability $P_f \rightarrow E[\langle\Pi_a\rangle]_{t=\infty} = \sum_f \langle f|\Pi_a|f\rangle P_f = P_a.$
- Now for any operator G commuting with H and A

$$E[d\langle G\rangle] = Tr(-i\hbar^{-1}[G, H_{eff}]E[\hat{\rho}]$$
$$-\frac{1}{2}|\beta|^2[G, A][A, E[\hat{\rho}]])dt = 0$$

- Therefore: $E[\langle\Pi_a\rangle]_{t=0} = \langle\Pi_a\rangle_{t=0} = E[\langle\Pi_a\rangle]_{t=\infty}$

giving the Born probability rule: $P_a = |p_{ia}|^2$

REALITY CHECK

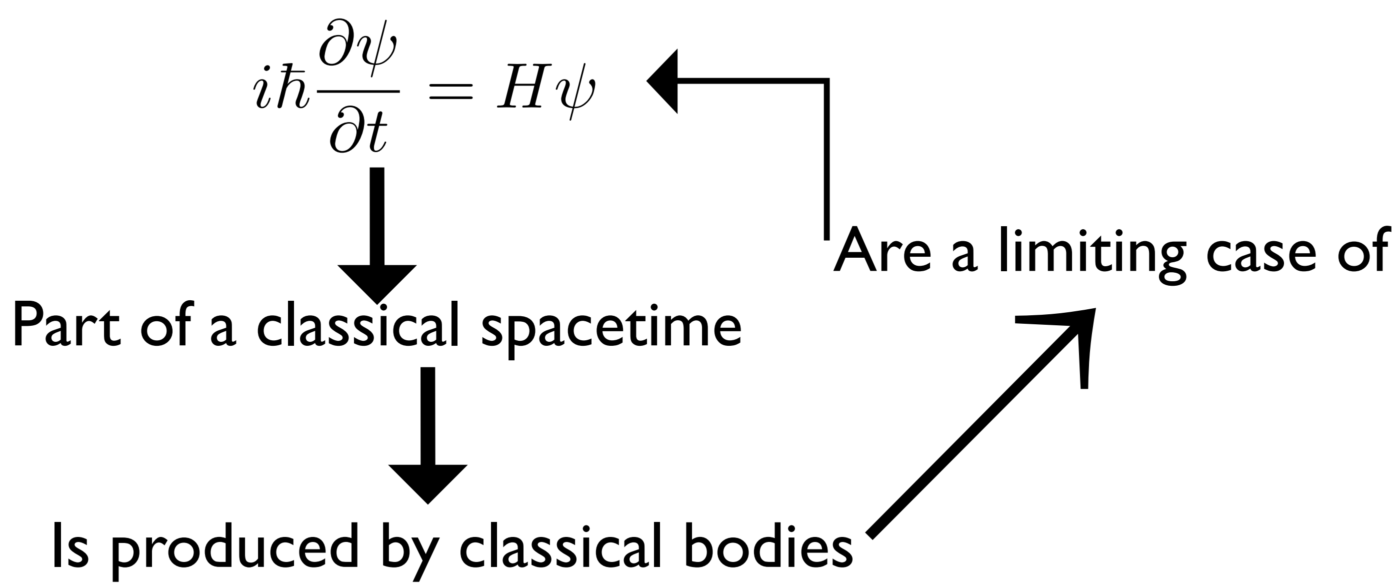
- Canonical quantisation is emergent
- Potential resolution of the measurement problem
- Underlying dynamics is nonlocal
- Cosmological constant?
- If quantum theory is emergent we should not `quantize' gravity

Outstanding Problems

- Why should norm be preserved?
- How to uniquely predict CSL?
- The issue of the preferred frame and Lorentz invariance
- How to test TD experimentally?
- Relativistic collapse models?
- No preferred quantum state in the ensemble.
- The form of the Hamiltonian.
- The significance of the `second' universe in TD?
- Where does the thermal reservoir come from?
- What about gravity?
- The problem of time in quantum theory

The Problem of Time in Quantum Theory

The problem of time in quantum theory



Quantum theory depends on classical time.
Classical time comes from quantum theory!