Quantum Nonlocality and Relativity

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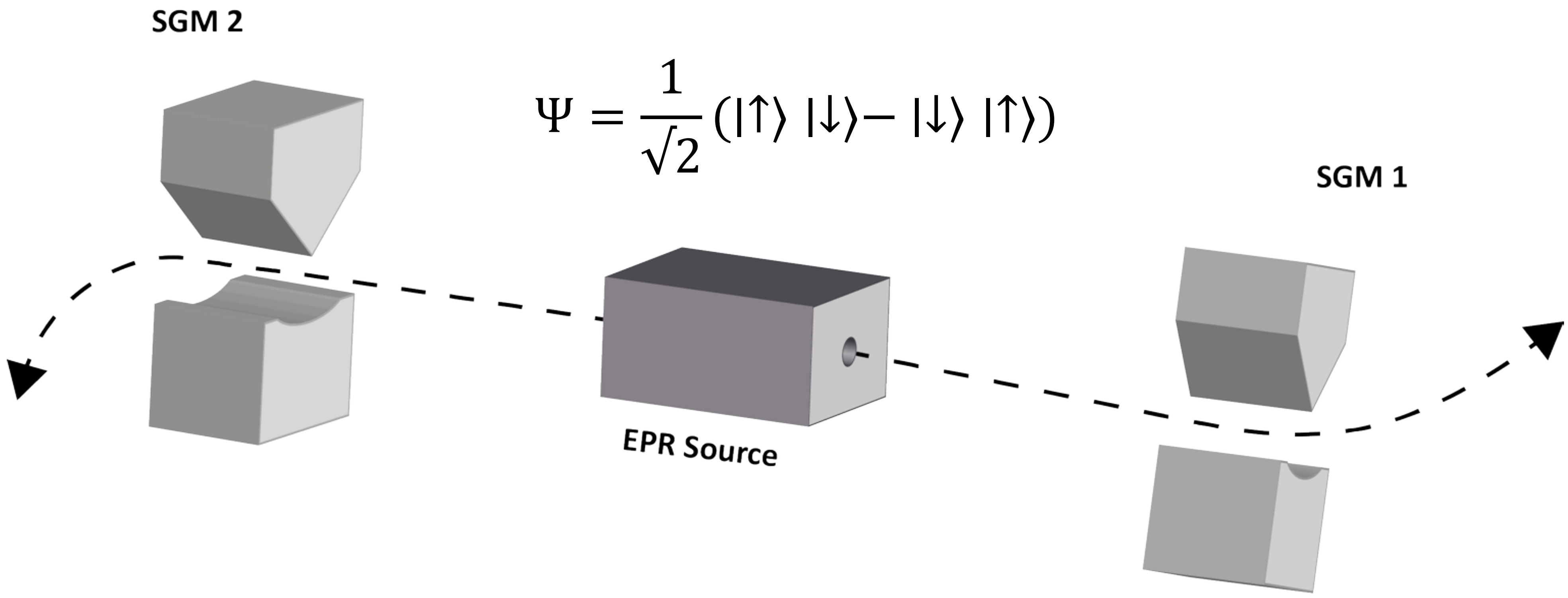
ICTS meeting „Fundamental Problems in Quantum Physics“

„The most profound discovery in science“

Nature is nonlocal.

The EPRB Experiment

$$\Psi = \frac{1}{\sqrt{2}}(|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle)$$



The EPR Dilemma

If we measure a-Spin UP for particle 1, we know immediately that the same measurement on particle 2 will yield a-Spin DOWN.

Either the a-Spin of particle 2 was determined before → **QM is incomplete**

Or the measurement on particle 1 has determined the a-Spin of particle 2 → **Nonlocality**

The Bell Theorem

The statistical predictions of quantum mechanics cannot be reproduced by assuming (locally) pre-determined spin values.

→ **Nonlocality**

Suppose there exists random variables

$$X_{\vec{a}}^{(1)}, X_{\vec{a}}^{(2)} \in \{-1, 1\}, \text{ s.t. } X_{\vec{a}}^{(1)} = -X_{\vec{a}}^{(2)}$$

For arbitrary parameters $\vec{a}, \vec{b}, \vec{c}$ we have

$$\begin{aligned}
&\mathbb{P}(X_{\vec{a}}^{(1)} = -X_{\vec{b}}^{(2)}) + \mathbb{P}(X_{\vec{b}}^{(1)} = -X_{\vec{c}}^{(2)}) + \mathbb{P}(X_{\vec{c}}^{(1)} = -X_{\vec{a}}^{(2)}) \\
&\quad = \mathbb{P}(X_{\vec{a}}^{(1)} = X_{\vec{b}}^{(1)}) + \mathbb{P}(X_{\vec{b}}^{(1)} = X_{\vec{c}}^{(1)}) + \mathbb{P}(X_{\vec{c}}^{(1)} = X_{\vec{a}}^{(1)}) \\
&\quad \geq \mathbb{P}(X_{\vec{a}}^{(1)} = X_{\vec{b}}^{(1)} \text{ or } X_{\vec{b}}^{(1)} = X_{\vec{c}}^{(1)} \text{ or } X_{\vec{c}}^{(1)} = X_{\vec{a}}^{(1)}) \\
&\quad = \mathbb{P}(\text{„sure event“}) \\
&\quad = 1
\end{aligned}$$

Bell inequality

For the Spin Singlet State $\psi = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$ with
e.g. $\vec{a} = 0°, \vec{b} = 120°, \vec{c} = 240°$, QM predicts

$$\mathbb{P}(X_{\vec{a}}^{(1)} = -X_{\vec{b}}^{(2)}) + \mathbb{P}(X_{\vec{b}}^{(1)} = -X_{\vec{c}}^{(2)}) + \mathbb{P}(X_{\vec{c}}^{(1)} = -X_{\vec{a}}^{(2)}) = \frac{3}{4} < 1$$

→ **No local model can explain the correlations predicted by QM**

→ **Experiments confirm the violation of the Bell inequality**
(first „loophole free“ EPR Experiment: Hensen et. al. 2015)

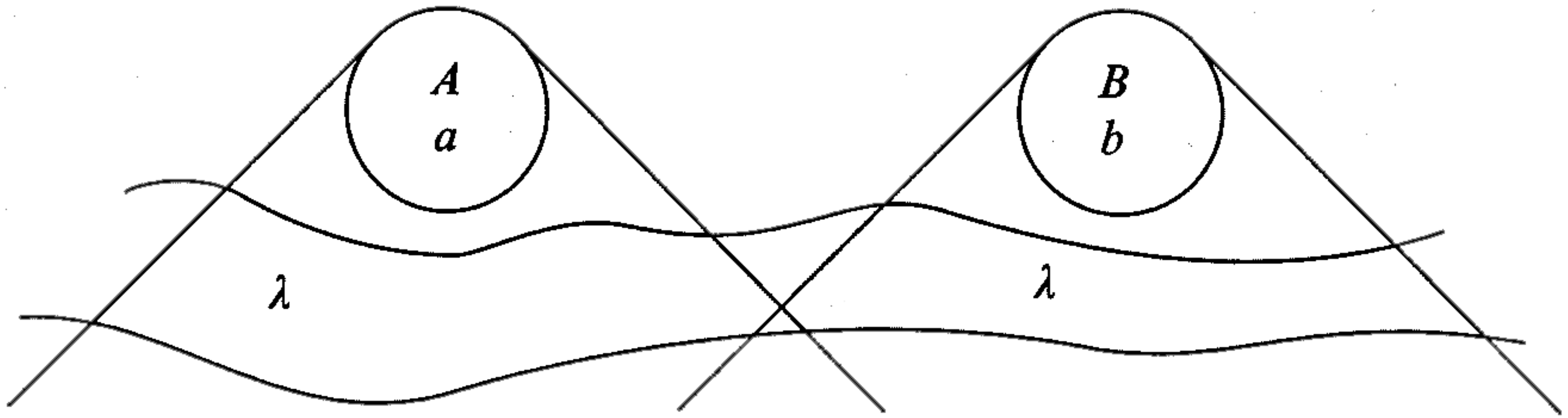
John Bell, „Bertlmann‘s socks and the nature of reality“

It is important to note that to the limited degree that *determinism* plays a role in the EPR argument, it is not assumed but *inferred*. What is held sacred is the principle of “local causality” or “no action at a distance”. […]

It is remarkably difficult to get this point across, that determinism is not a *presupposition* of the analysis. There is a widespread and erroneous conviction that for Einstein determinism was always *the* sacred principle. The quotability of his famous ‘God does not play dice’ has not helped in this respect.

Nature is nonlocal.

CHSH inequality: Bell‘s theorem without perfect correlations



In the EPRB experiment, the measurement outcomes A and B are correlated:

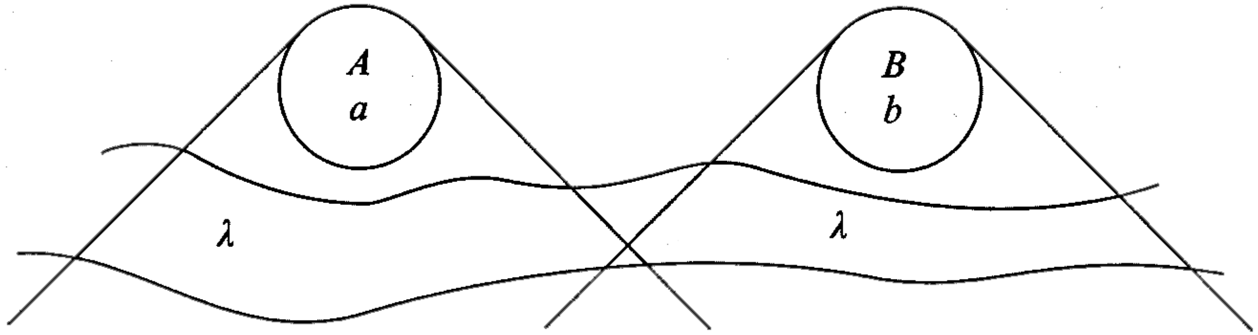
$$P(A, B \,|a, b) \neq P(A \,|a)\, P(B \,|b)$$

Correlation does not imply causation. We call the correlations **locally explicable** iff

$$P(A, B \,|a, b, \lambda) = P(A \,|a, \lambda)\, P(B \mid b, \lambda)$$

where the variable λ encodes all factors („common causes“) in the past of A and B that could explain the correlations, and

$$\int_\Lambda P(A, B \,|a, b, \lambda)\, dP(\lambda) = P(A, B \,|a, b)$$



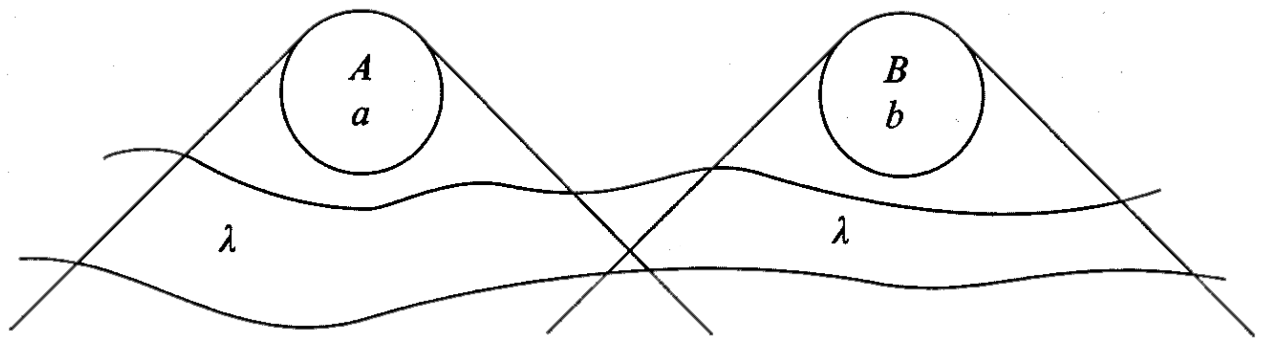
Why „local“?

From conditional probabilities:

$$P(A, B \,|a, b, \lambda) = P(A \,|B, a, b, \lambda)\, P(B \,|a, b, \lambda)$$

But if λ specifies the complete (relevant) state if the system, locality implies

$$P(A, B \,|a, b, \lambda) = P(A \,|\not{B}, a, \not{b}, \lambda)\, P(B \,|\not{a}, b, \lambda)$$



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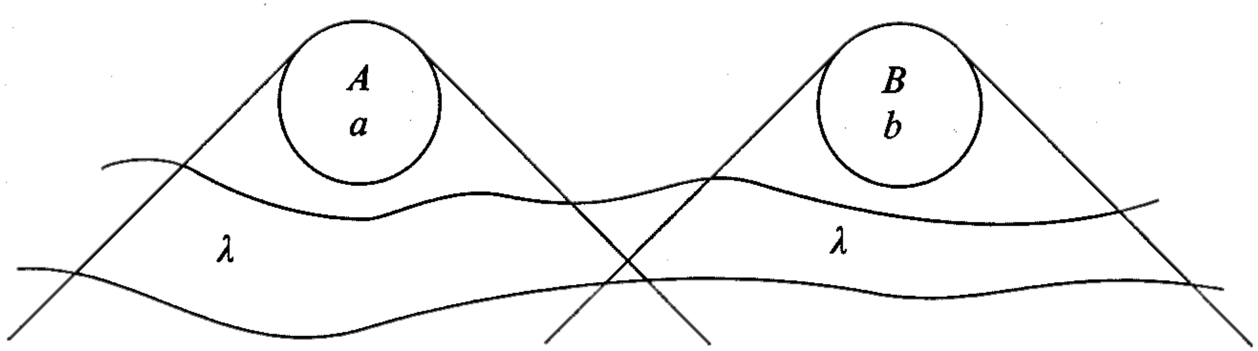
{ Parameter independence
Outcome independence



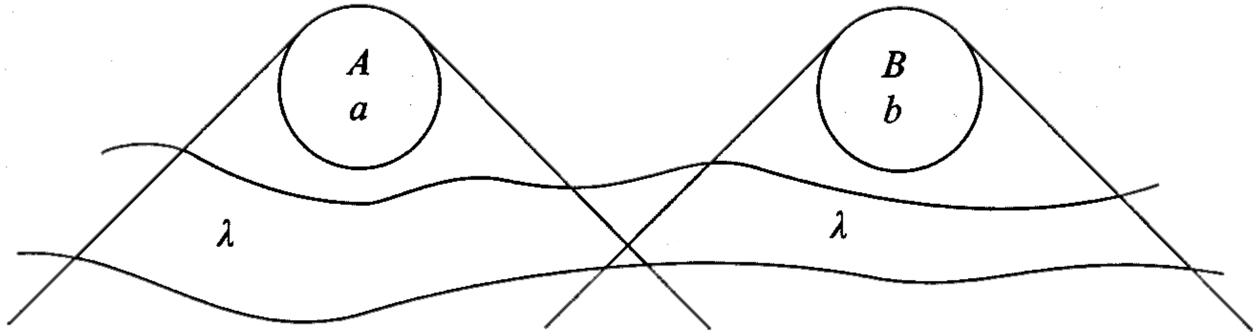
Standard Quantum Mechanics

In standard „Copenhagen" QM, the relevant variables describing the past state of the system include the wave function Ψ and possibly some „classical variables" $X_1, \dots, X_n$ describing the particle source, measurement apparatus etc.

$$\lambda_{QM} = (\Psi, X_1, \dots, X_n)$$

But this is not enough to provide a local explanation of the EPR correlations:

$$P(A, B \,|a, b, \lambda_{QM}) = P(A \,|a, \lambda_{QM})\, P(B \mid b, \lambda_{QM})$$



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→ Standard Quantum Mechanics is nonlocal in the sense of Bell (just check the Definition!)

→ The question is, if we can save locality by introducing additional „hidden“ variables

CHSH inequality

Consider the expactation value

$$E(a,b) \coloneqq E(A \cdot B\ |a,b) \quad = P(A=+1, B=+1|a,b) + P(A=-1, B=-1|a,b) \\ - P(A=+1, B=-1\ |a,b) \ - P(A=-1, B=+1\ |\ a,b)$$

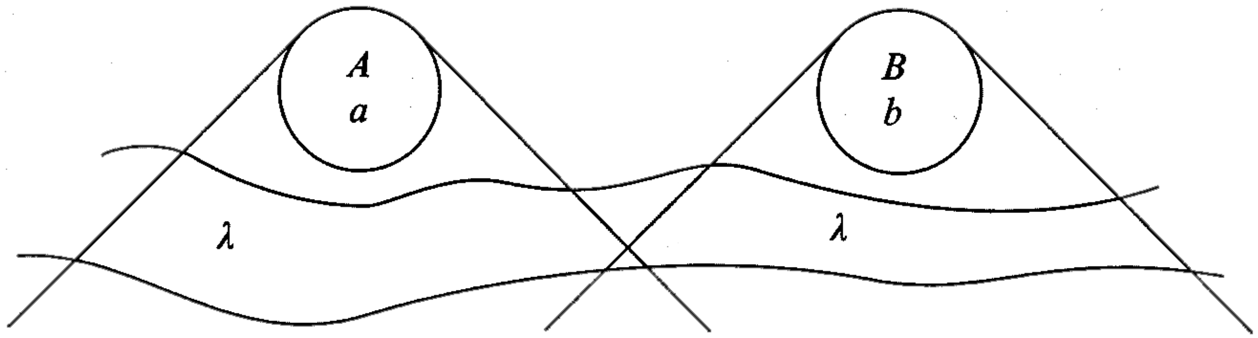
Then it holds for arbitrary parameter choices a, a', b, b'

$$\boxed{S \coloneqq |\ E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \leq 2}$$

Two Assumptions

$Locality$: $P(A, B \mid a, b, \lambda) = P(A \mid a, \lambda)\, P(B \mid b, \lambda)$

$No\ Conspiracy$: $P(\lambda \mid a, b) = P(\lambda)$



CHSH-inequality

Quantum Mechanics:

$$E(a, b) = -\boldsymbol{a} \cdot \boldsymbol{b}$$

Maximal violation of the CHSH inequality for (e.g.) $a = 0°, a' = 90°, b = 45°, b' = -45°$

$$S = 2\sqrt{2} > 2$$

Experiment:

E.g. Hansen et. al. 2015 „Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres"

$$S = 2{,}42 \pm 0{,}20$$

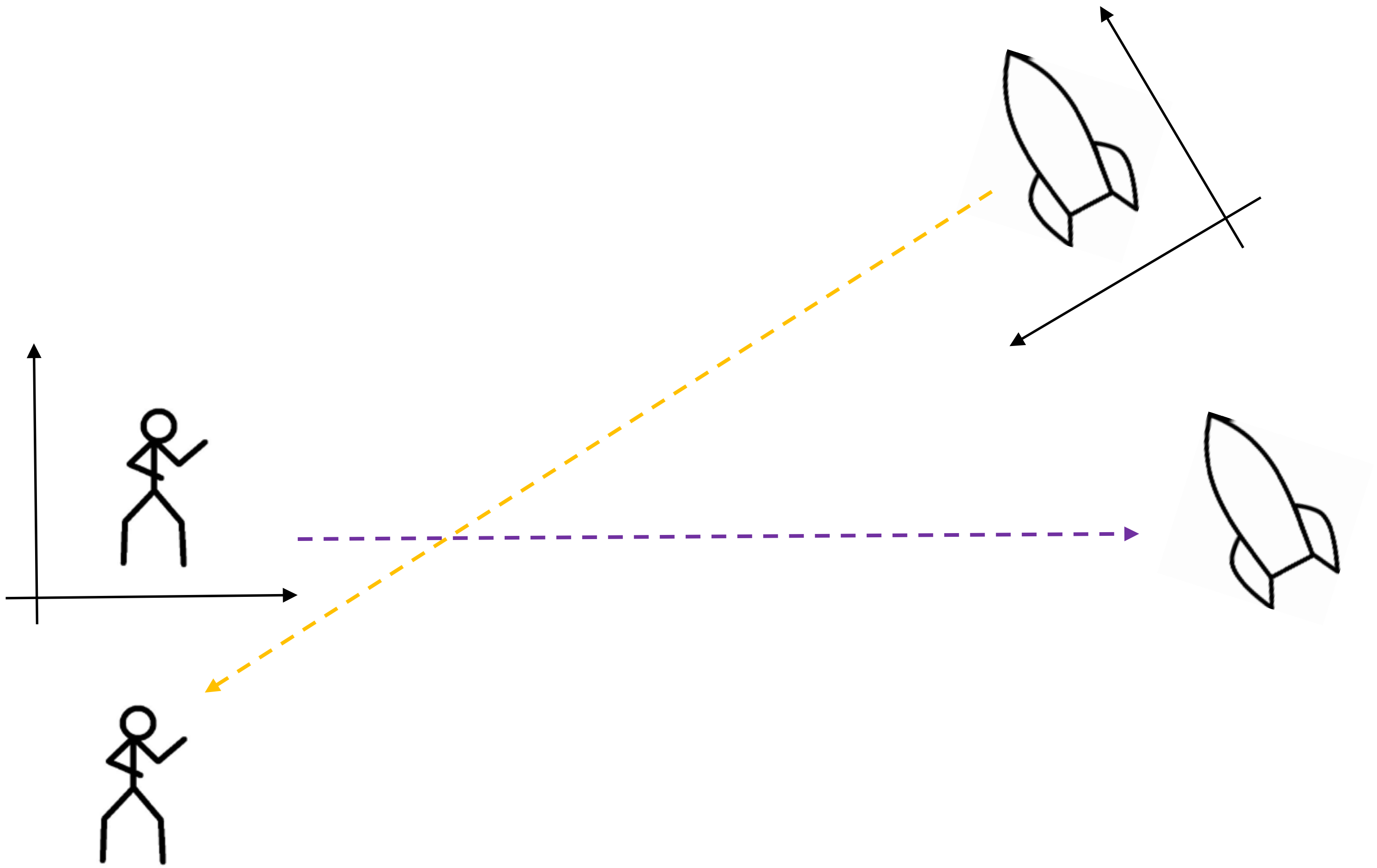
Who‘s afraid of signaling?

Can we use Quantum nonlocality for faster than light signalling?

And if we could, why would it be so bad?

Who‘s afraid of signaling?

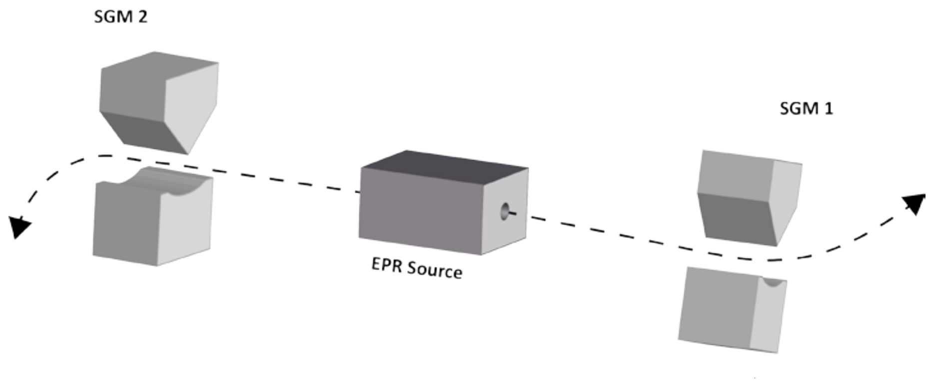
Who‘s afraid of signaling?



A faster than light communication protocoll

- A and B prepare entangled particles in the spin singlet state
- The SGM of B is oriented in z-direction
- The SGM of A is oriented in x-direction

→ A and B measure spin up/down with probability 1/2



- Suppose A knows the exact Bohmian position of the particles
- To send a signal, he turns his SGM into z-direction iff he knows that he will measure spin up

→ B measures spin down with probability 3/4

→ By a sufficiently long series of measurements, B can determine with arbirtrary certainty if A signals or not

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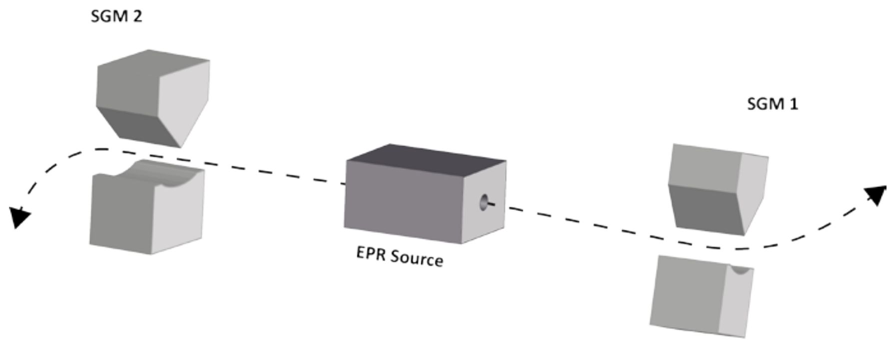

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Impossible in quantum equlibrium!

No signalling in Quantum Equilibrium

Absolute Uncertainty: A and B cannot have more precise knowledge of the particle position than given by $\rho = |\psi|^2$

No signalling condition:

$$\sum_{A=\pm 1} P(B|A,a,b)P(A|a) = P(B\,|b)$$

No signalling and local commutativity

If the measurements A and B are space-like separated, the associated operators (POVMs) must commute:

$$[P_A^a, P_B^b] = 0$$

Then and only then is the joint probability defined independent of the order in which the measurements are performed:

$$\mathbb{P}(A, B) = \langle \psi \mid P_A^a P_B^b \mid \psi \rangle = \langle \psi \mid P_B^b P_A^a \mid \psi \rangle$$

Local Commutativity implies No Signalling:

$$\sum_A \mathbb{P}(B \mid A, a, b)\, \mathbb{P}(A \mid a) = \sum_A \frac{\langle \Psi \mid P_B^b P_A^a \mid \Psi \rangle}{\langle \Psi \mid P_A^a \mid \Psi \rangle} \langle \Psi \mid P_A^a \mid \Psi \rangle$$

$$= \sum_A \langle \Psi \mid P_B^b P_A^a \mid \Psi \rangle = \sum_A \langle \Psi \mid P_A^a P_B^b \mid \Psi \rangle = \langle \Psi \mid \Big(\sum_A P_A^a \Big) P_B^a \mid \Psi \rangle$$

$$= \langle \Psi \mid P_B^b \mid \Psi \rangle = \mathbb{P}(B \mid b)$$

nonlocality ↯ relativity

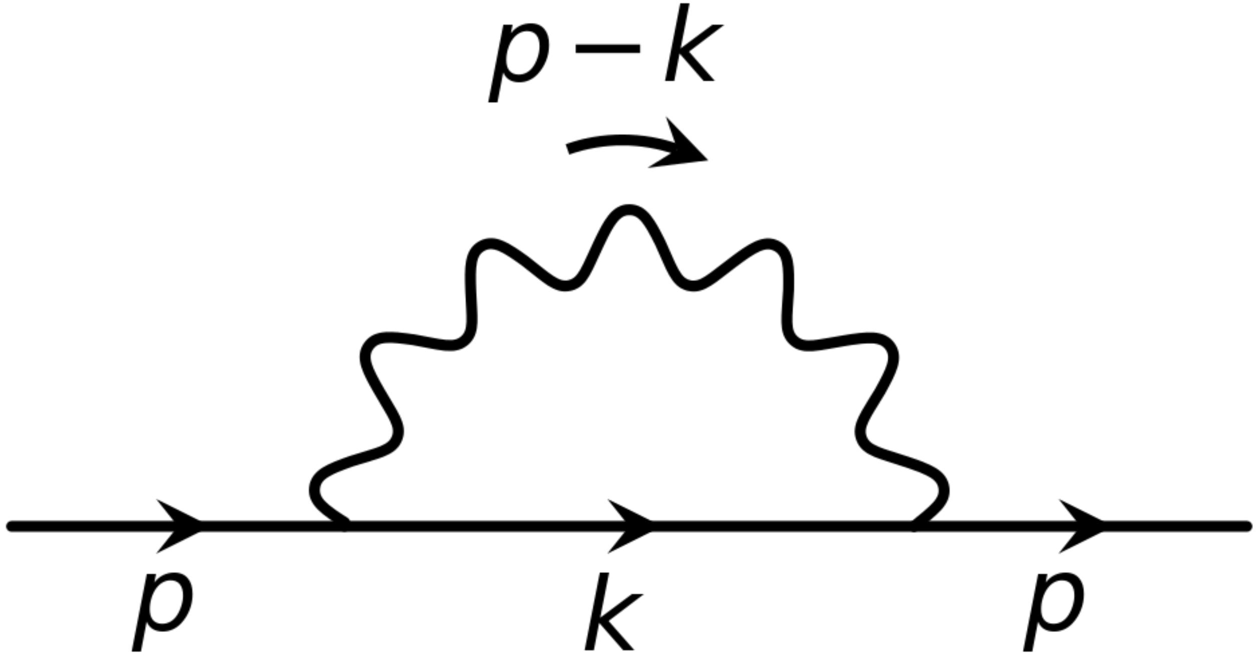
Conflicts with Relativity

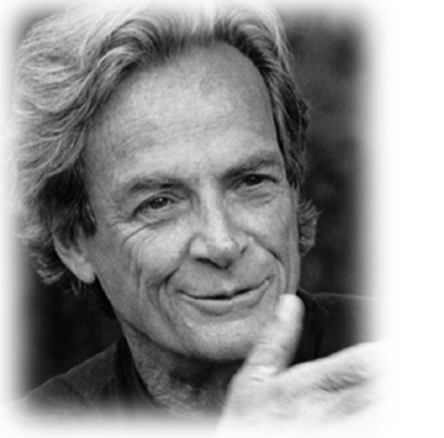
I. Problem of interactions
II. Problem of synchronisation
III. Problem of probabilitites

I. The problem of interactions

Self-interaction

$\rightarrow$ Ultraviolet divergence

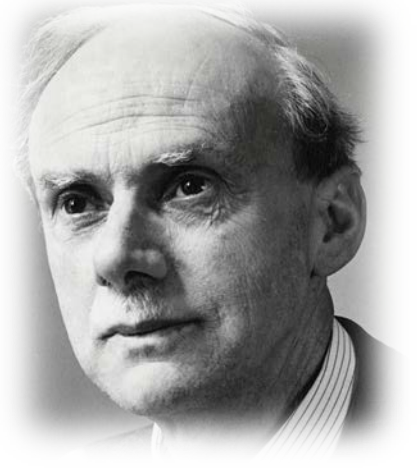




"I don't think we have a completely satisfactory relativistic quantum-mechanical model, even one that doesn't agree with nature, but, at least, agrees with the logic that the sum of probability of all alternatives has to be 100%. Therefore, I think that the renormalization theory is simply a way to sweep the difficulties of the divergences of electrodynamics under the rug. I am, of course, not sure of that."

– Richard P. Feynman (1965)

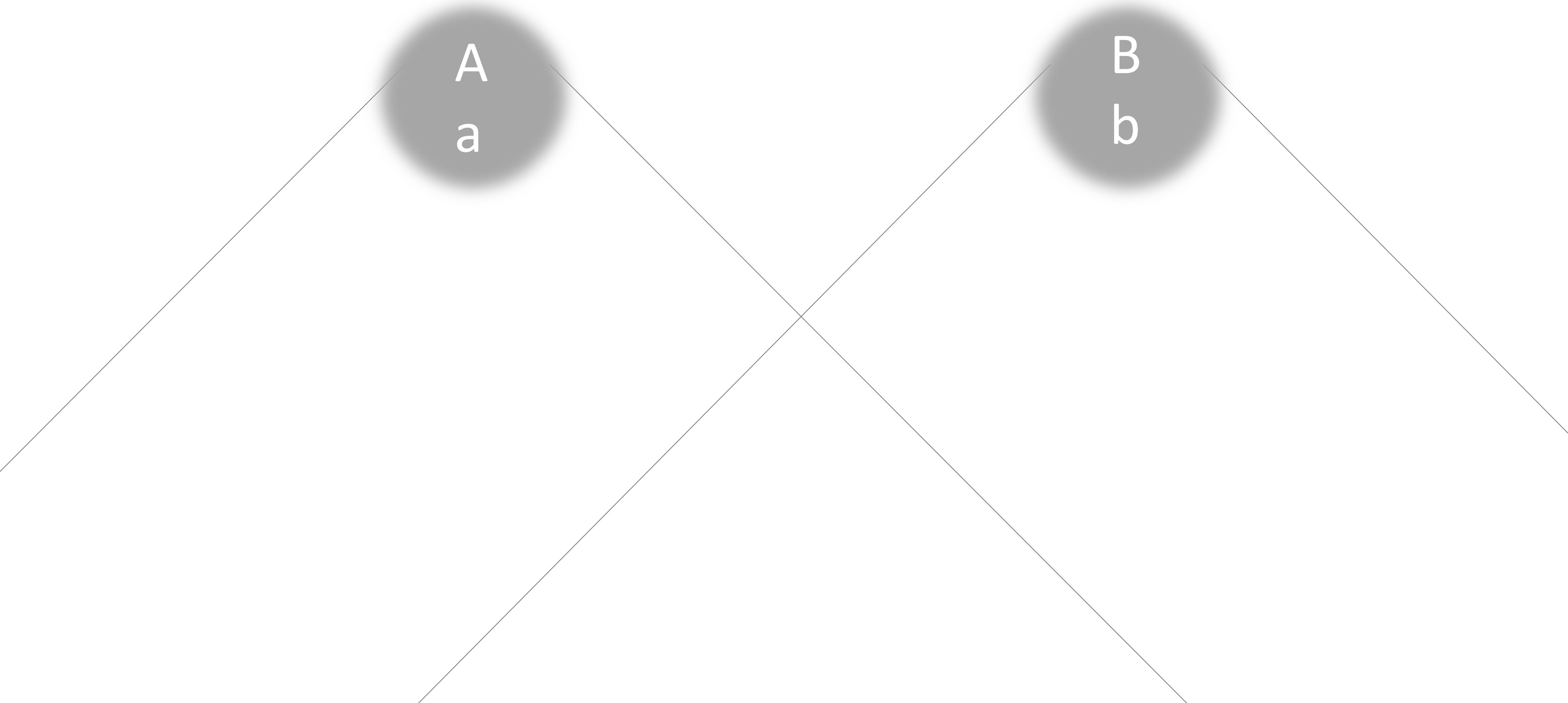
“Most physicists are very satisfied with the situation. They say, Quantum Electrodynamics is a good theory, and we do not have to worry about it any more. I must say that I am very dissatisfied with the situation, because this so-called good theory does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small - not neglecting it just because it is infinitely great and you do not want it!”

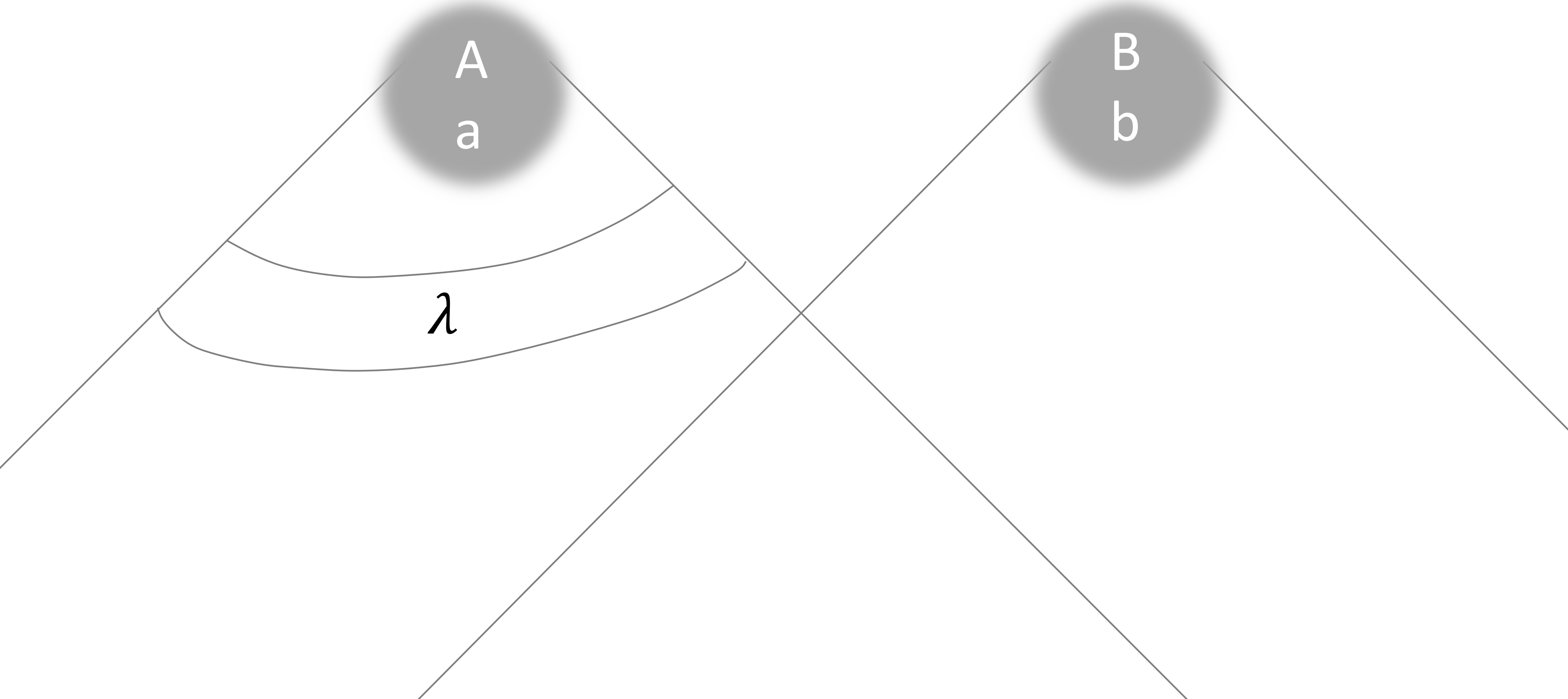


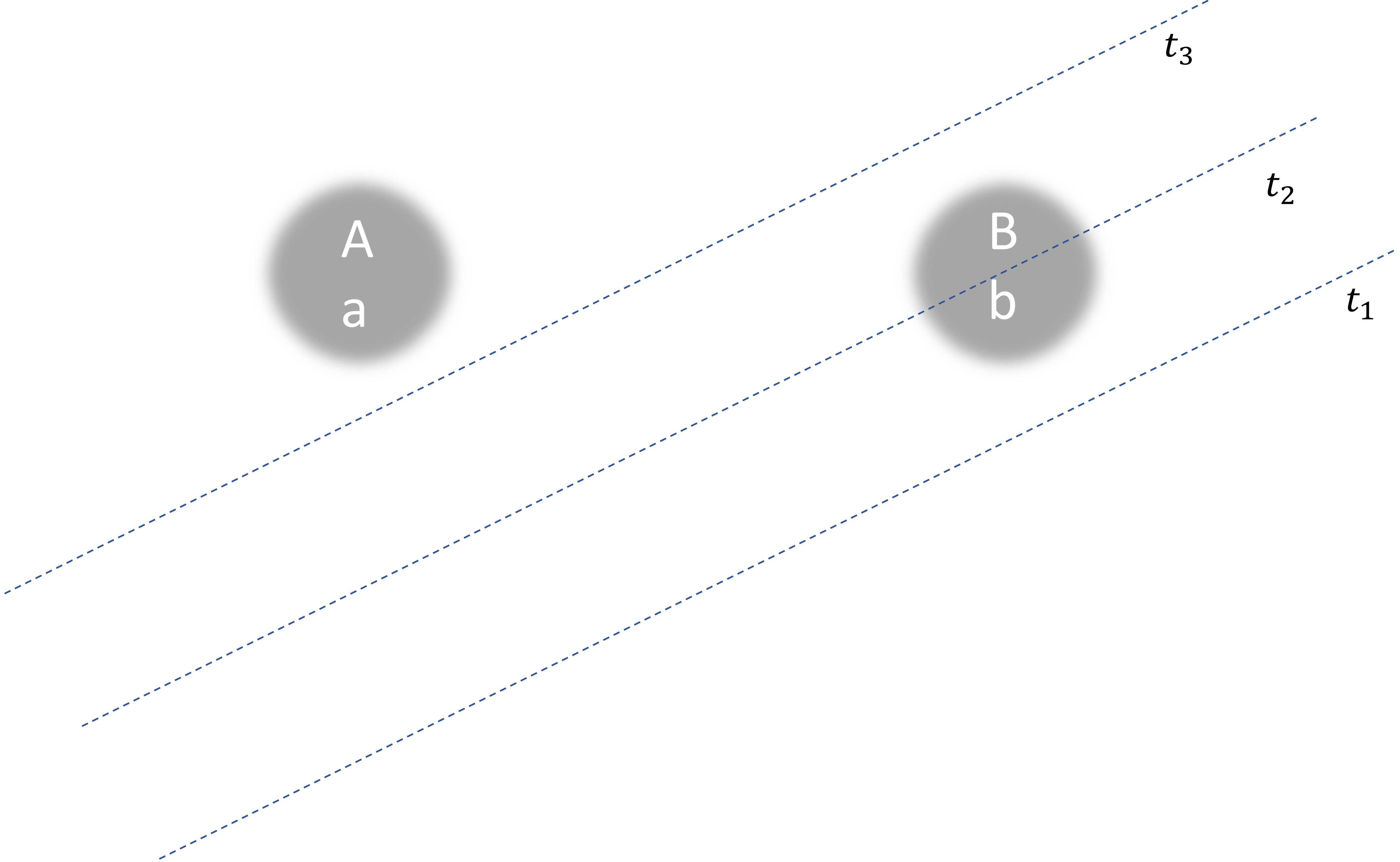
– P.A.M. Dirac (1975)

II. The problem of synchronisation

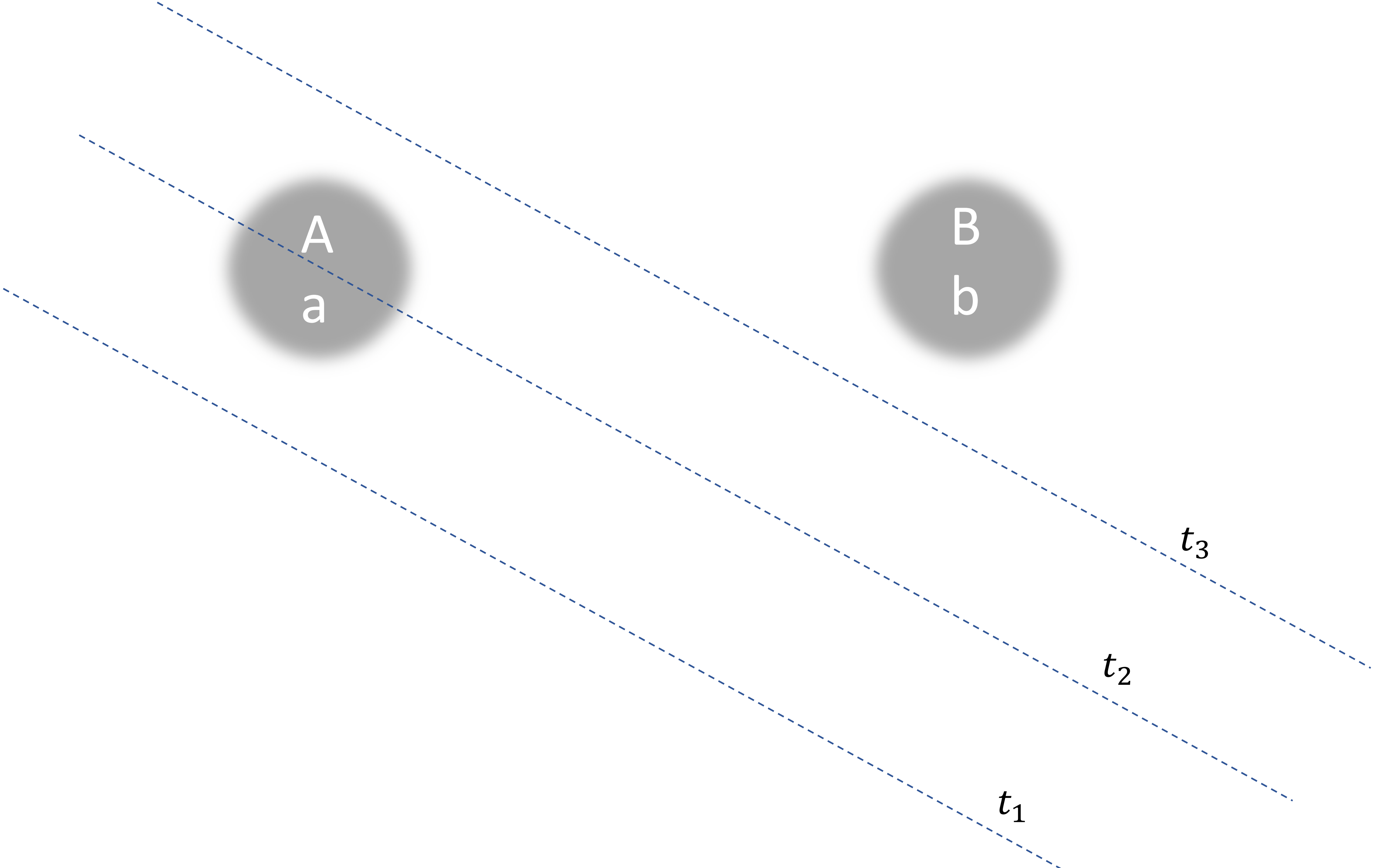

A
a
B
b


A
a
B
b


A
a
B
b
λ



t_3
t_2
t_1
A
a
B
b


A
a
B
b
t_3
t_2
t_1

Covariant (deterministic) model

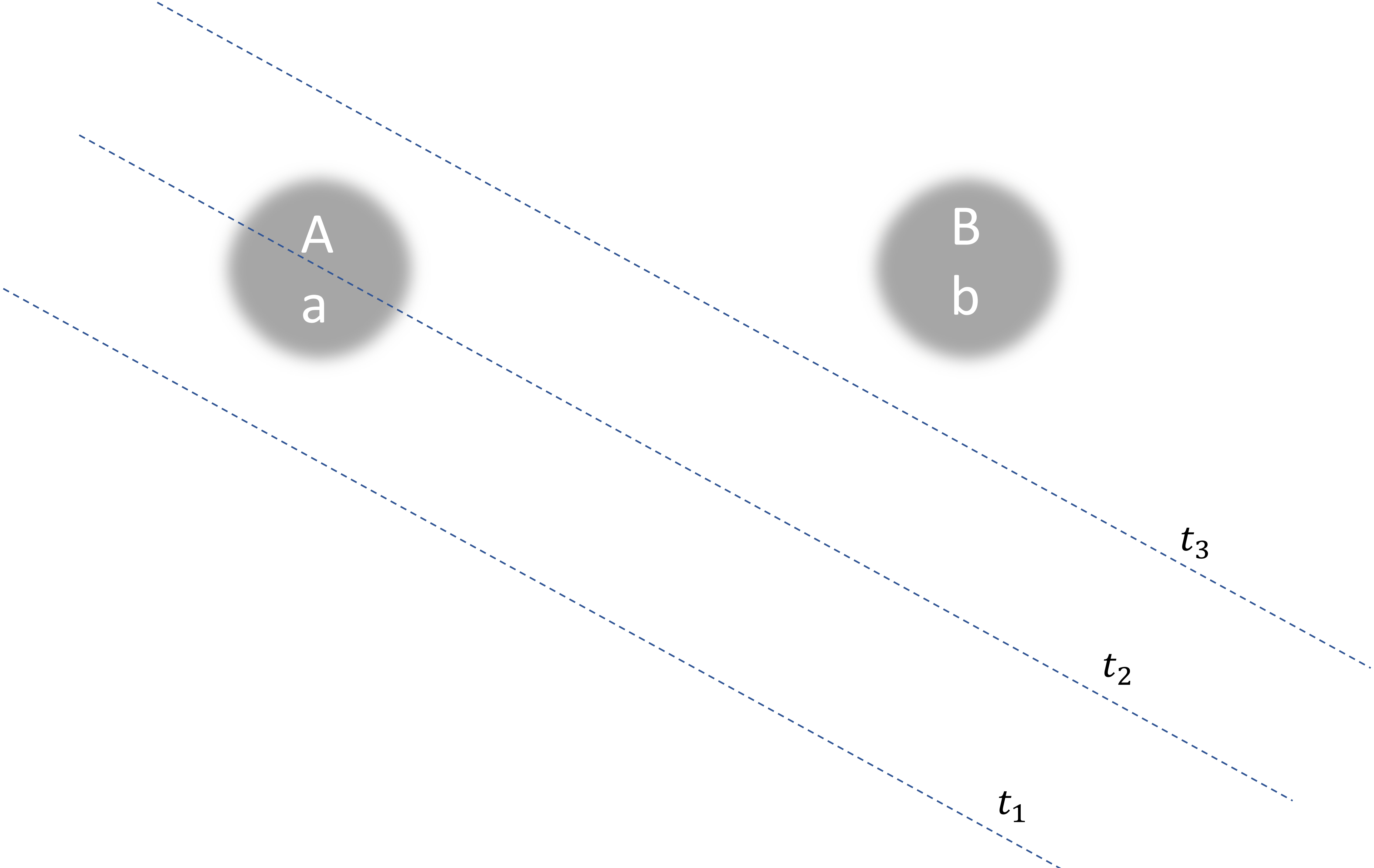
$\Rightarrow$ Bell-local model

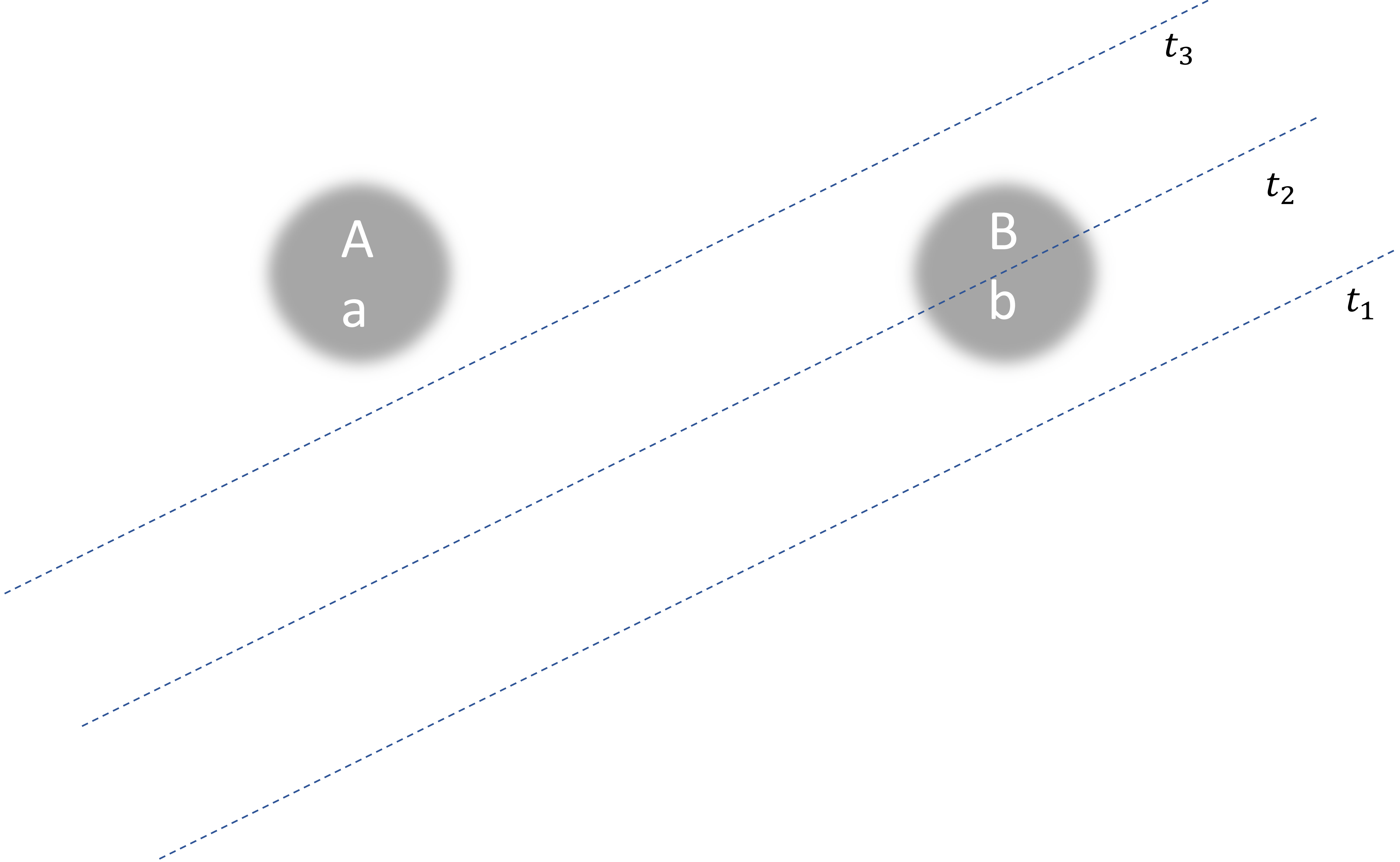
$\Rightarrow$ *Ruled out by experiment*

Possible Solutions

- All frames are equal, but some are more equal than others

  $\rightarrow$ Preferred foliation


A
a
B
b
t_3
t_2
t_1



t_3
t_2
t_1
A
a
B
b

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- All frames are equal, but some are more equal than others
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- Future events can influence the system in the past
  - → Retrocausality

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- All frames are equal, but some are more equal than others
  - → Preferred foliation
- Future events can influence the system in the past
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- Stochastic Collapse Model
  - → rGRWf (Tumulka, 2006)

III. The problem of probabilities

Quantum equilibrium (Born‘s rule) cannot hold in all Lorentz frames

Hardy‘s State

$$\Psi = \frac{1}{\sqrt{3}}\left(|\uparrow\rangle_z^1|\downarrow\rangle_z^2 - \sqrt{2}|\downarrow\rangle_x^1|\uparrow\rangle_z^2\right)$$

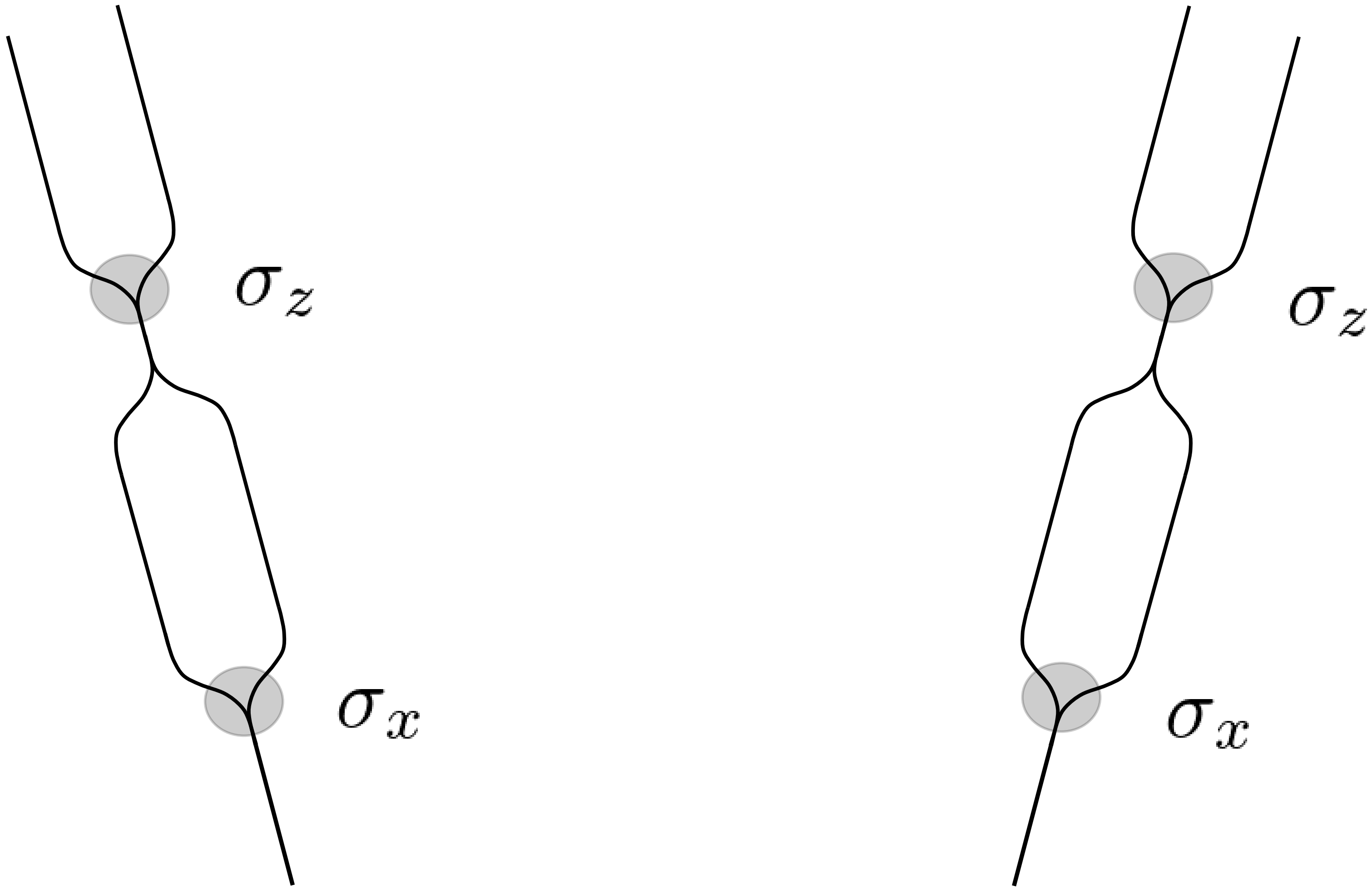
Hardy‘s State

i) $$P\left(\sigma_x^1 = +1, \sigma_x^2 = +1\right) > 0$$

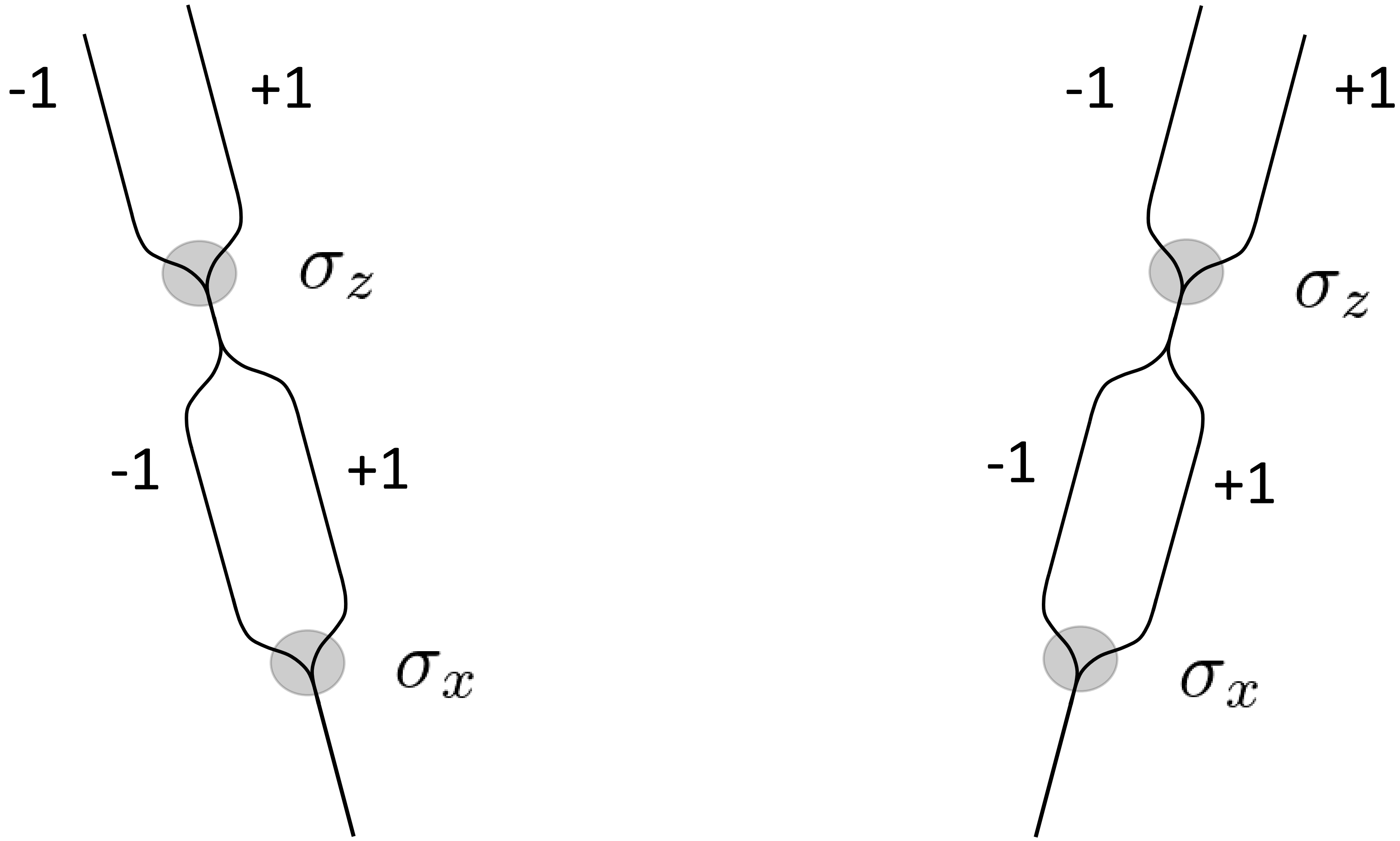
ii) $$\sigma_x^2 = +1 \Rightarrow \sigma_z^1 = -1$$

iii) $$\sigma_x^1 = +1 \Rightarrow \sigma_z^2 = -1$$

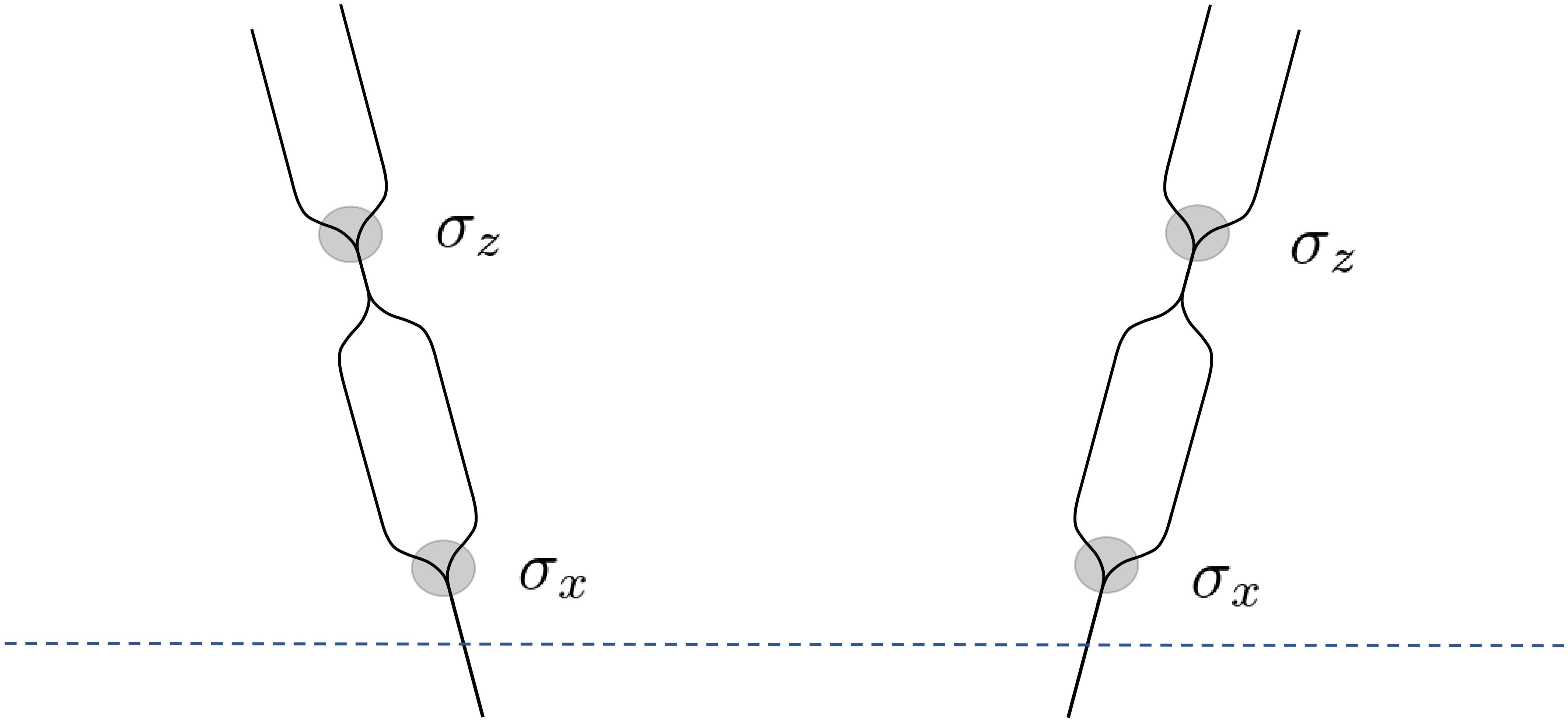
iv) $$P\left(\sigma_z^1 = -1, \sigma_z^2 = -1\right) = 0$$



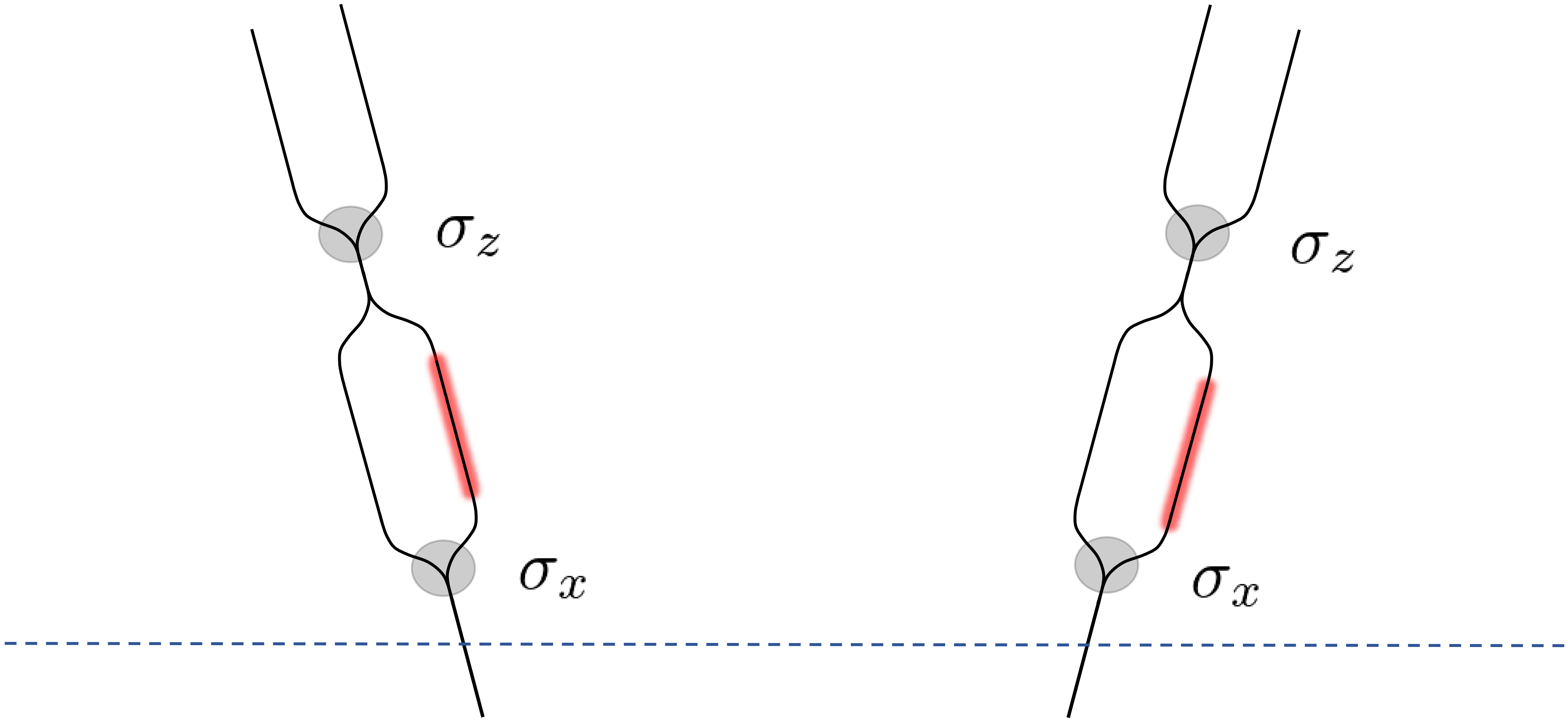
σ_z
σ_x
σ_z
σ_x



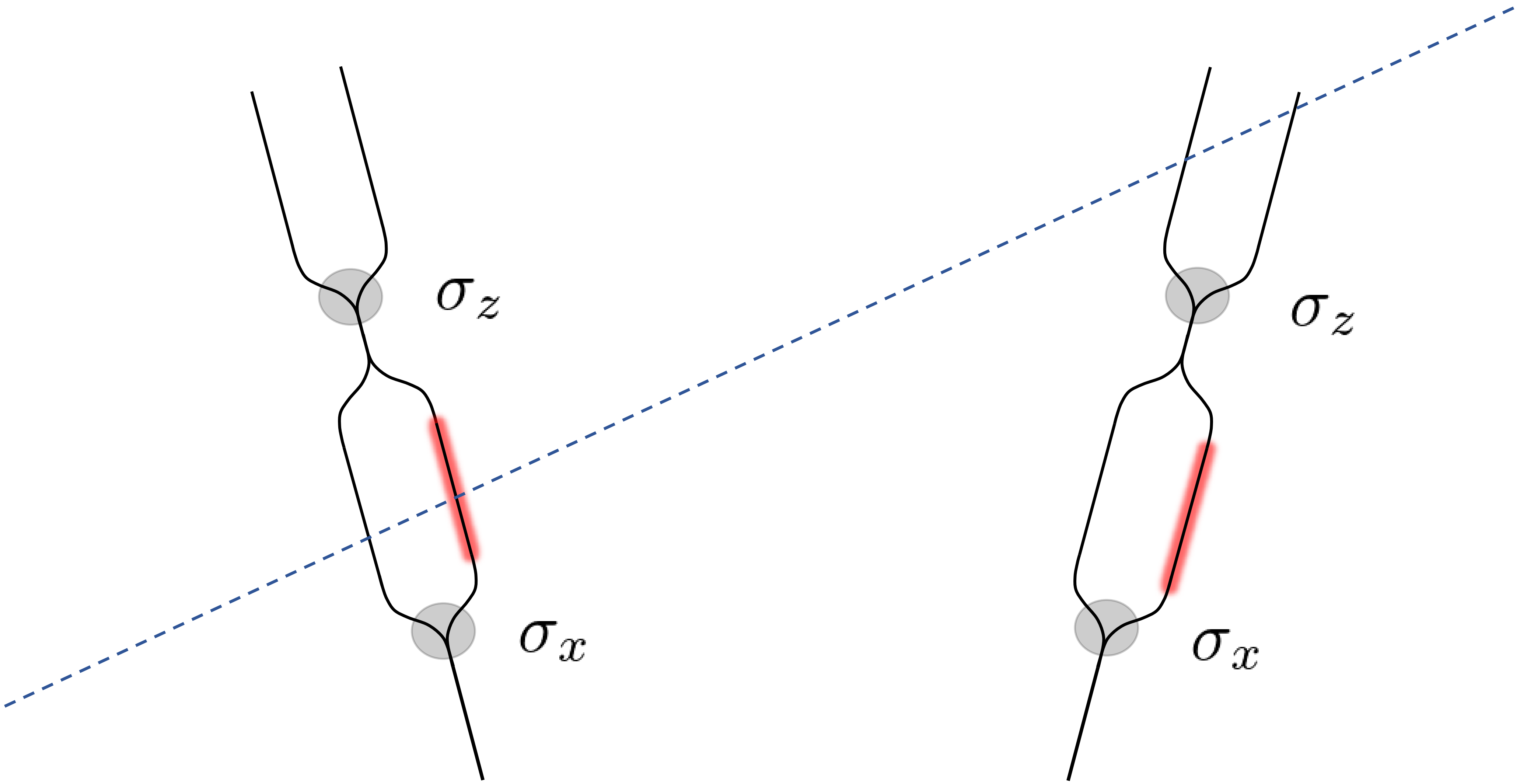
-1
+1
σ_z
-1
+1
σ_x
-1
+1
σ_z
-1
+1
σ_x



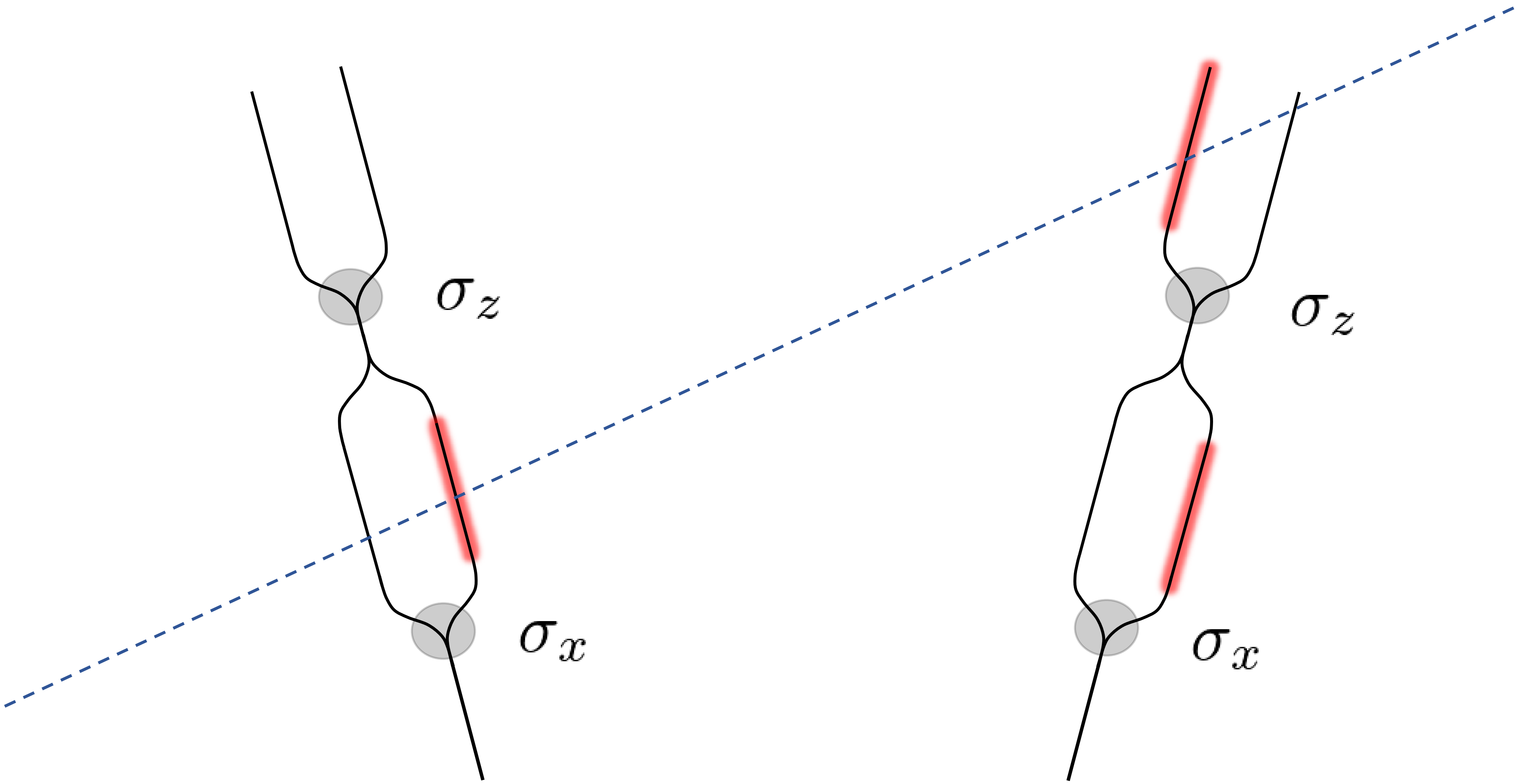
σ_z
σ_x
σ_z
σ_x



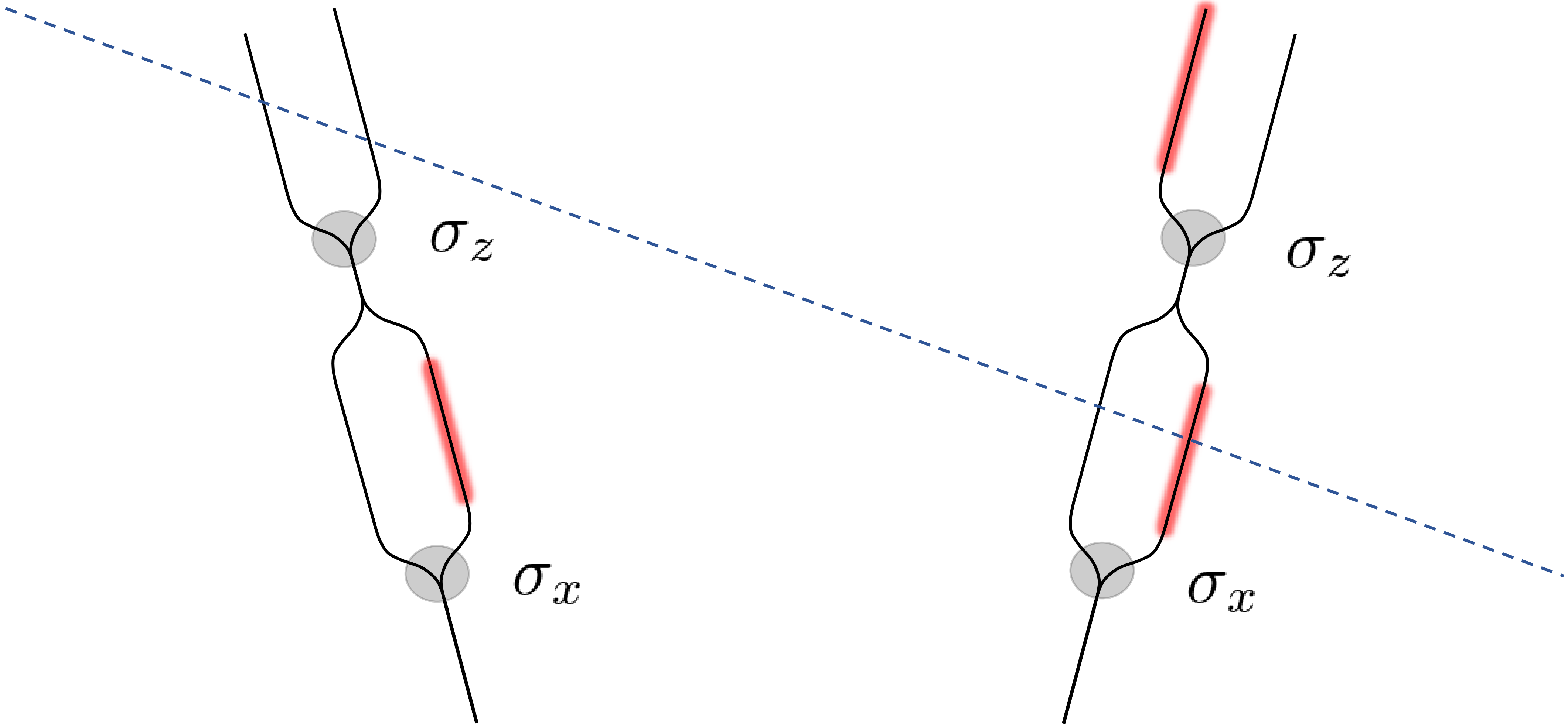
σz
σx
σz
σx

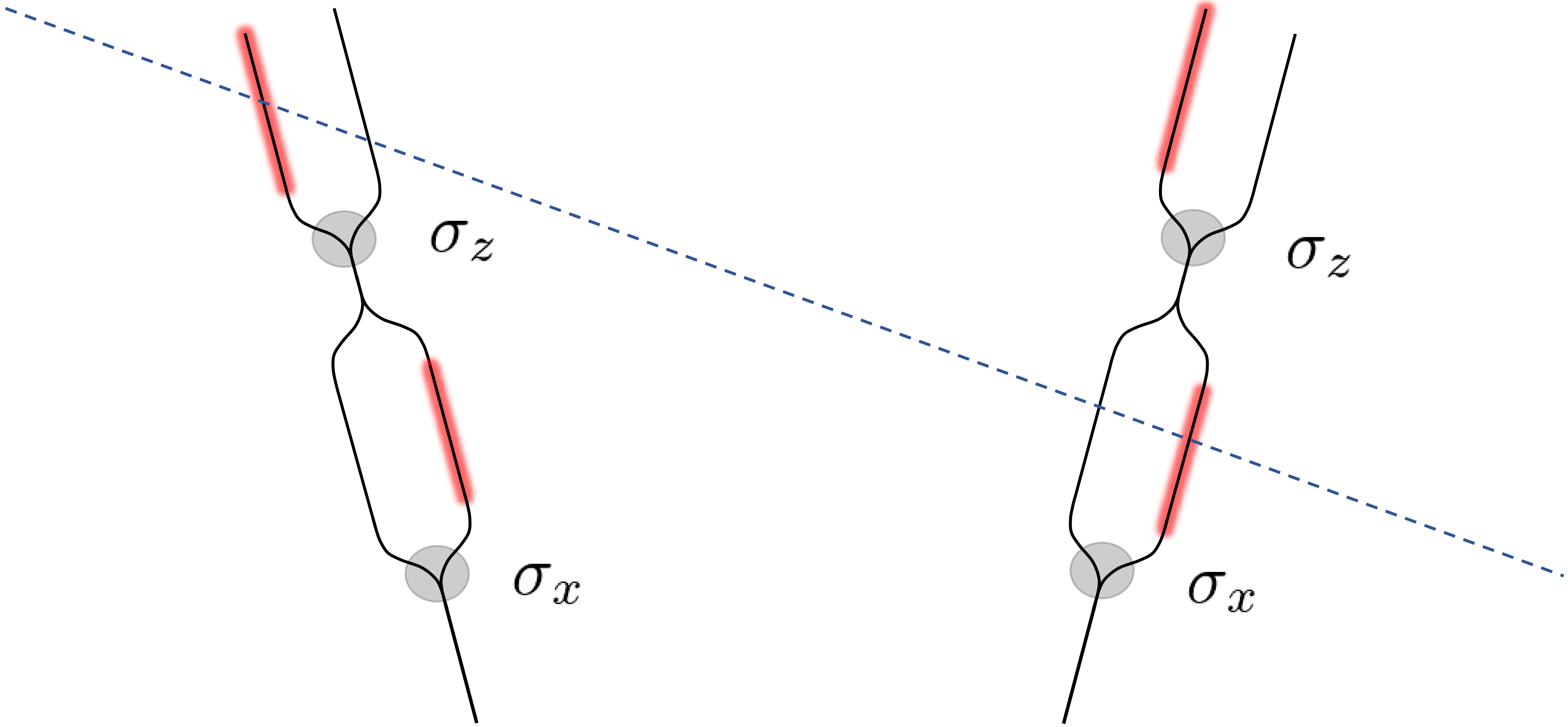


σ_z
σ_x
σ_z
σ_x

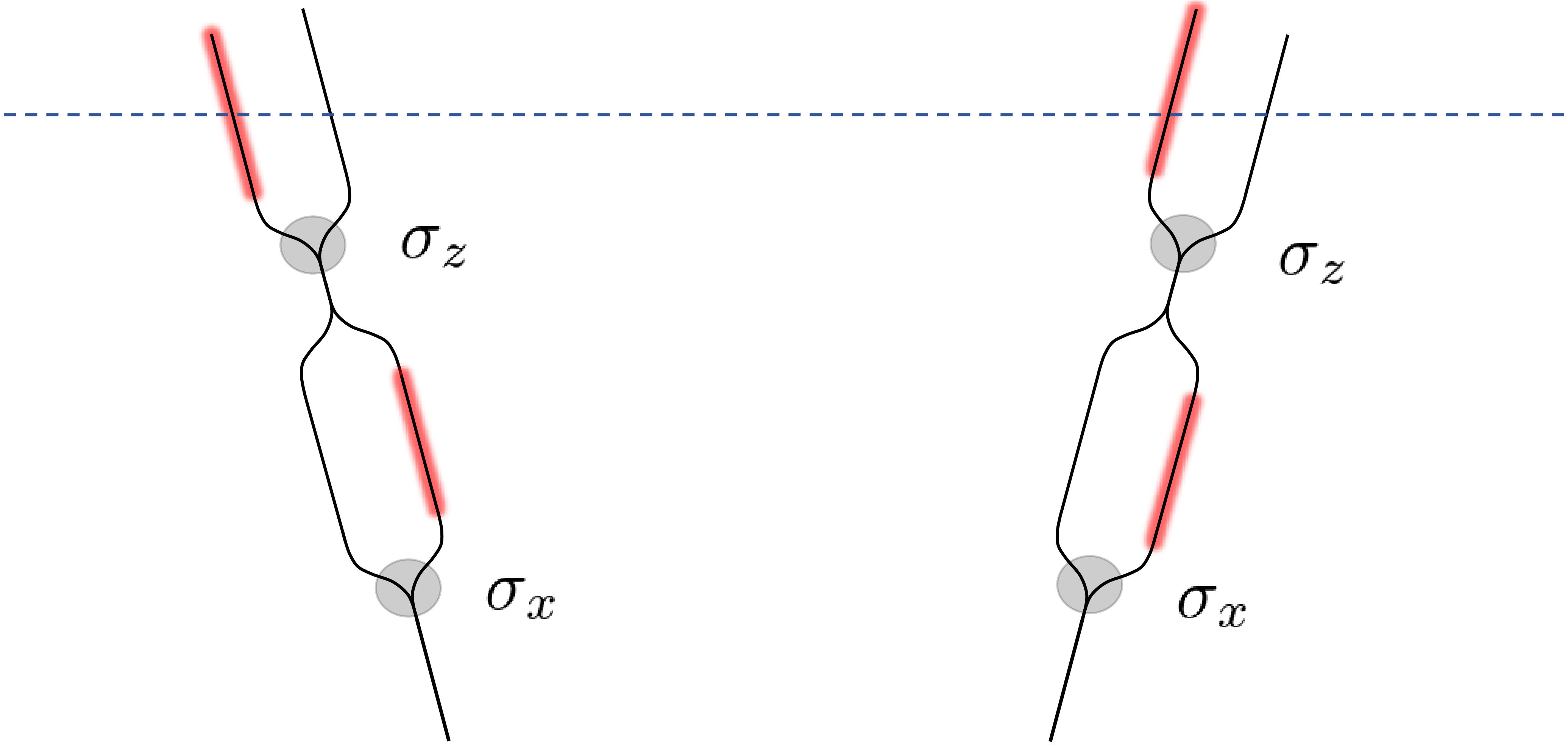


σ_z
σ_x
σ_z
σ_x

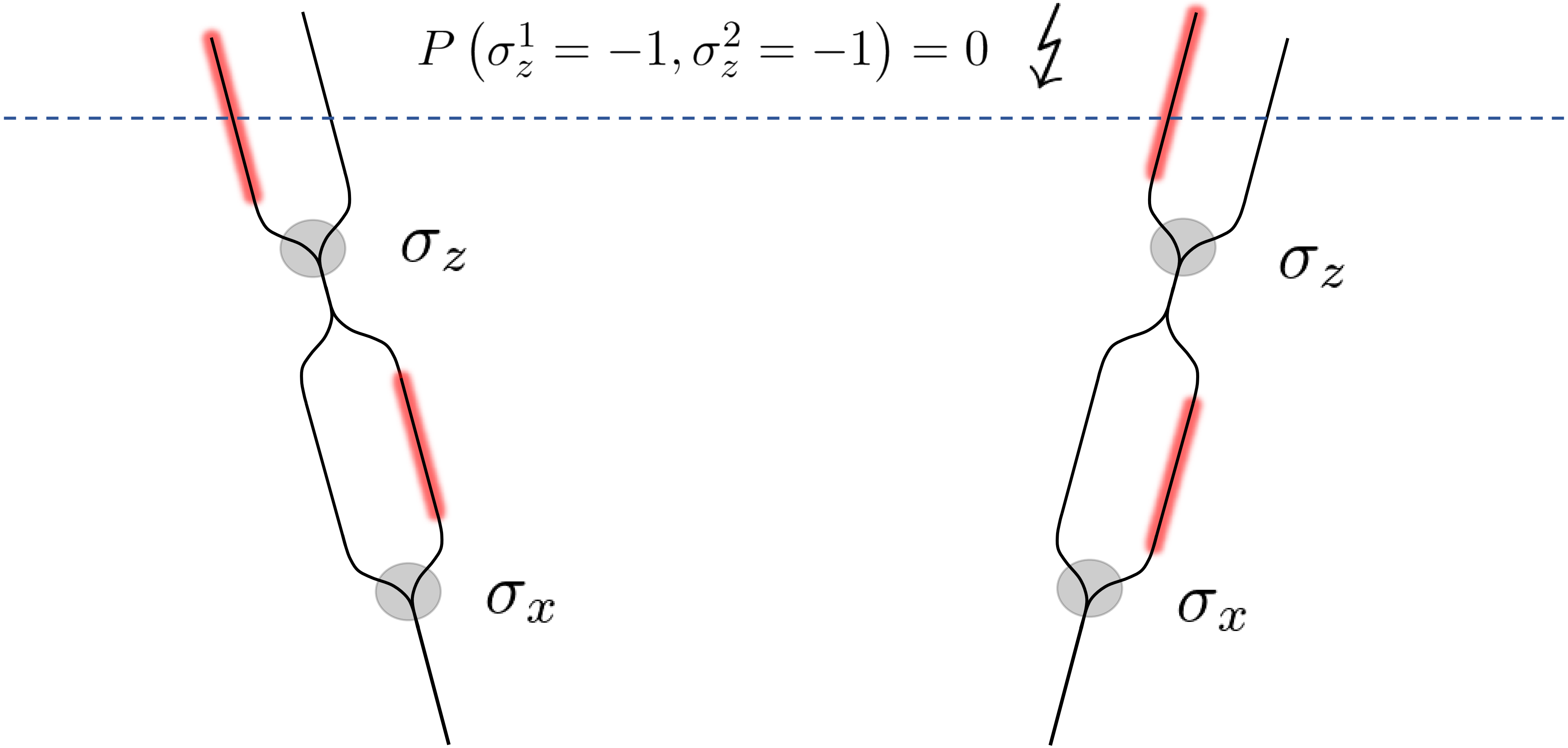

σ_z
σ_z
σ_x
σ_x



σ_z
σ_x
σ_z
σ_x



σ_z
σ_x
σ_z
σ_x


$P\left(\sigma_z^1 = -1, \sigma_z^2 = -1\right) = 0$
σ_z
σ_x
σ_z
σ_x

Problems of ``realism“ or ``hidden variables“?

Towards a relativistic quantum theory:

Multi-time wave functions

Single-time wave function (non-relativistic)

$$\Psi(\mathrm{t}, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_\mathrm{N}), \qquad \mathbf{x}_\mathrm{i} \in \mathbb{R}^3$$

Single-time wave function (non-relativistic)

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$\rightarrow$ Multi-time wave function

$$\Psi(x_1, x_2, \dots, x_N), \qquad x_i \in \mathbb{R}^4$$

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→ Multi-time wave function

$$\Psi(x_1, x_2, \ldots, x_N), \qquad x_i \in \mathbb{R}^4$$

$$= \Psi(\mathrm{t}_1, \mathbf{x}_1, \mathrm{t}_2, \mathbf{x}_2, \ldots, \mathrm{t}_\mathrm{N}, \mathbf{x}_\mathrm{N})$$

Single-time from multi-time wave function

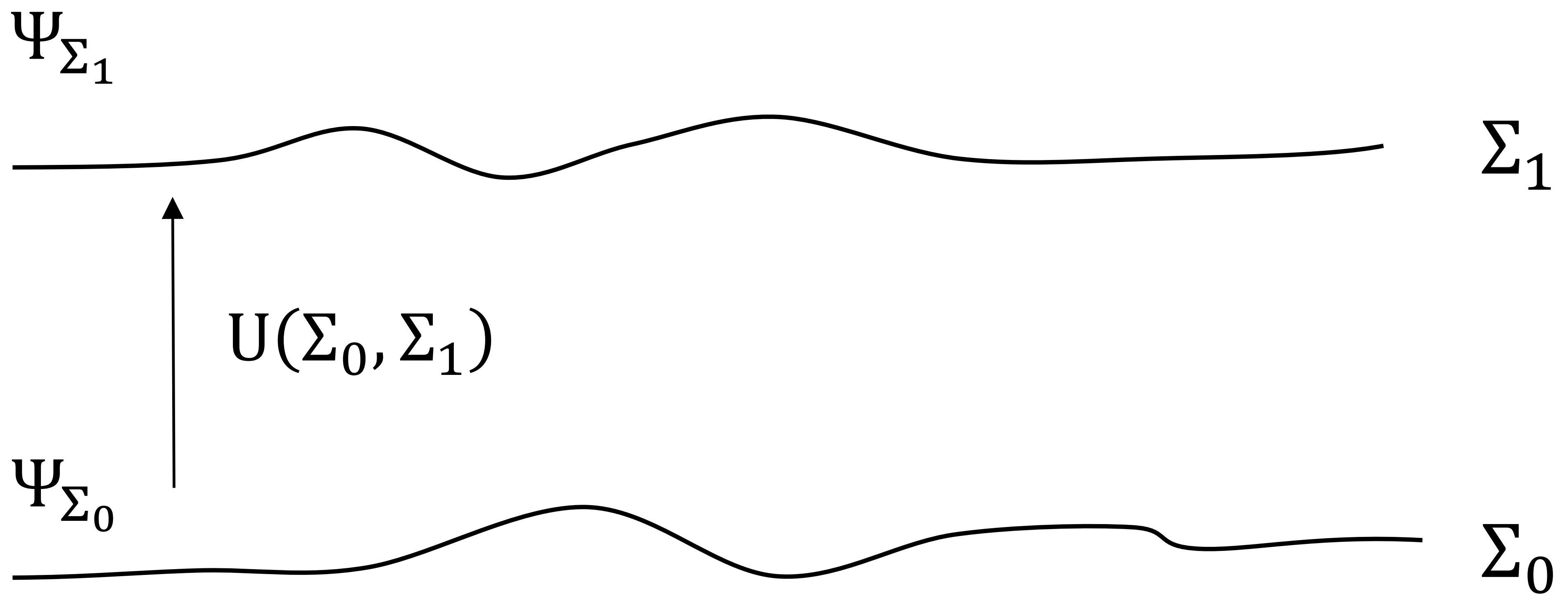
$$\Phi(\mathrm{t}, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\mathrm{N}) \coloneqq \Psi(\mathrm{t}, \mathbf{x}_1, \mathrm{t}, \mathbf{x}_2, \ldots, \mathrm{t}, \mathbf{x}_\mathrm{N})$$

Single-time from multi-time wave function

$$\Phi(t, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) \coloneqq \Psi(t, \mathbf{x}_1, t, \mathbf{x}_2, \dots, t, \mathbf{x}_N)$$

$\downarrow$ Lorentz transformation

$$\Psi'(t'_1, \mathbf{x}'_1, t'_2, \mathbf{x}'_2, \dots, t'_N, \mathbf{x}_N), \quad t'_i \neq t'_j$$



(Tomonaga-Schwinger)

Configuration space

$$\Omega_0 := \mathbb{R}^{4N}$$

Configuration space

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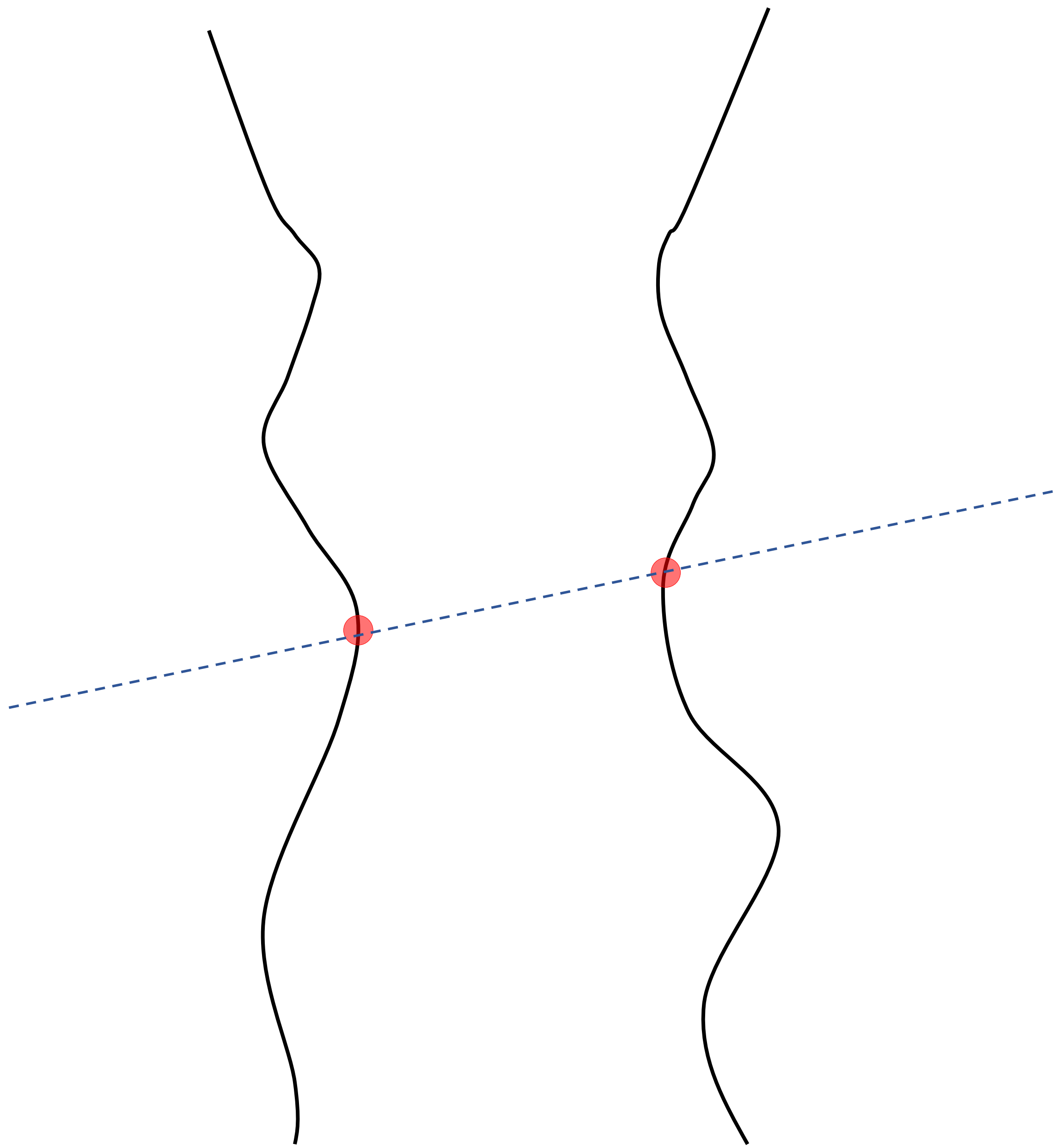

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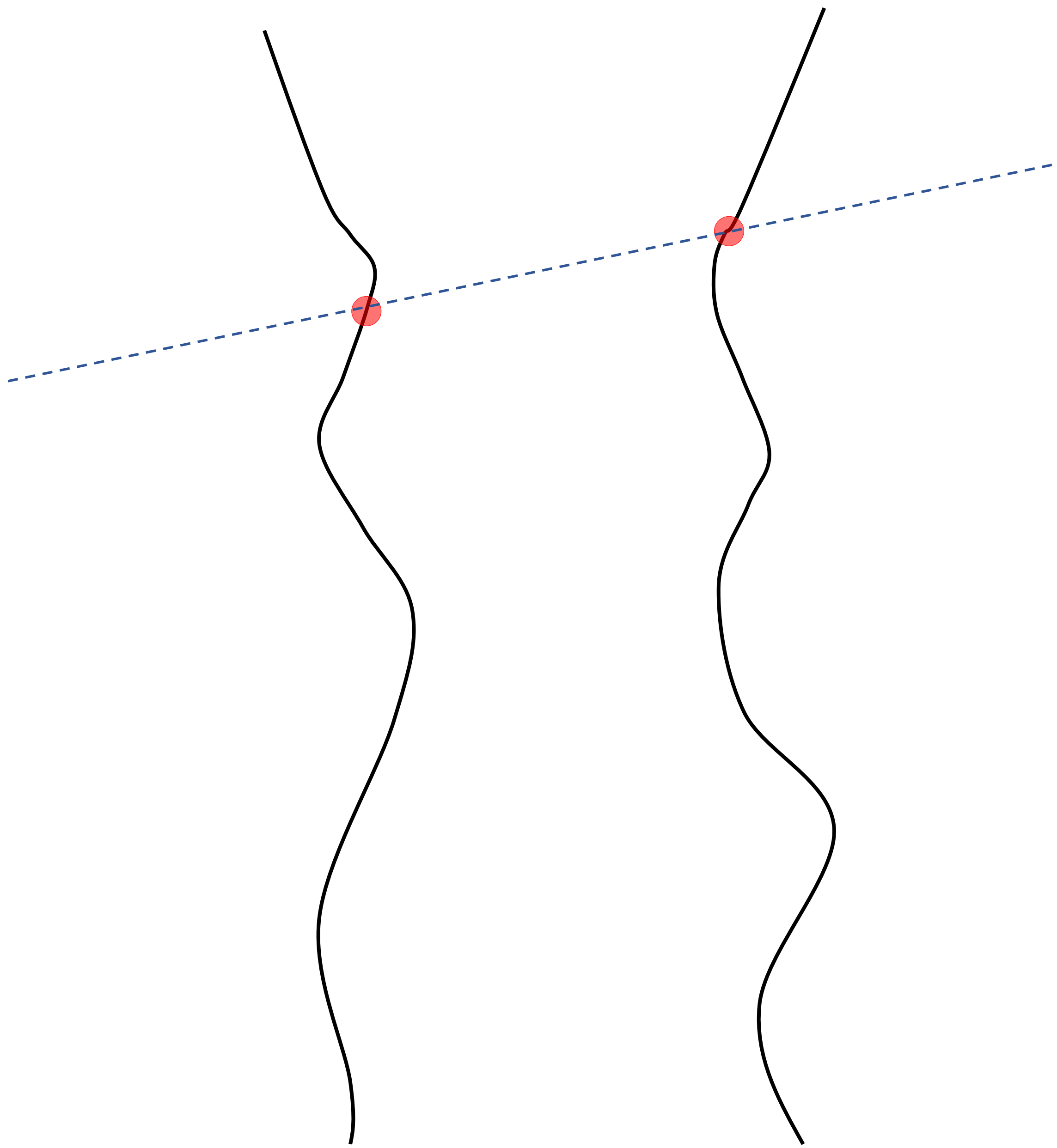

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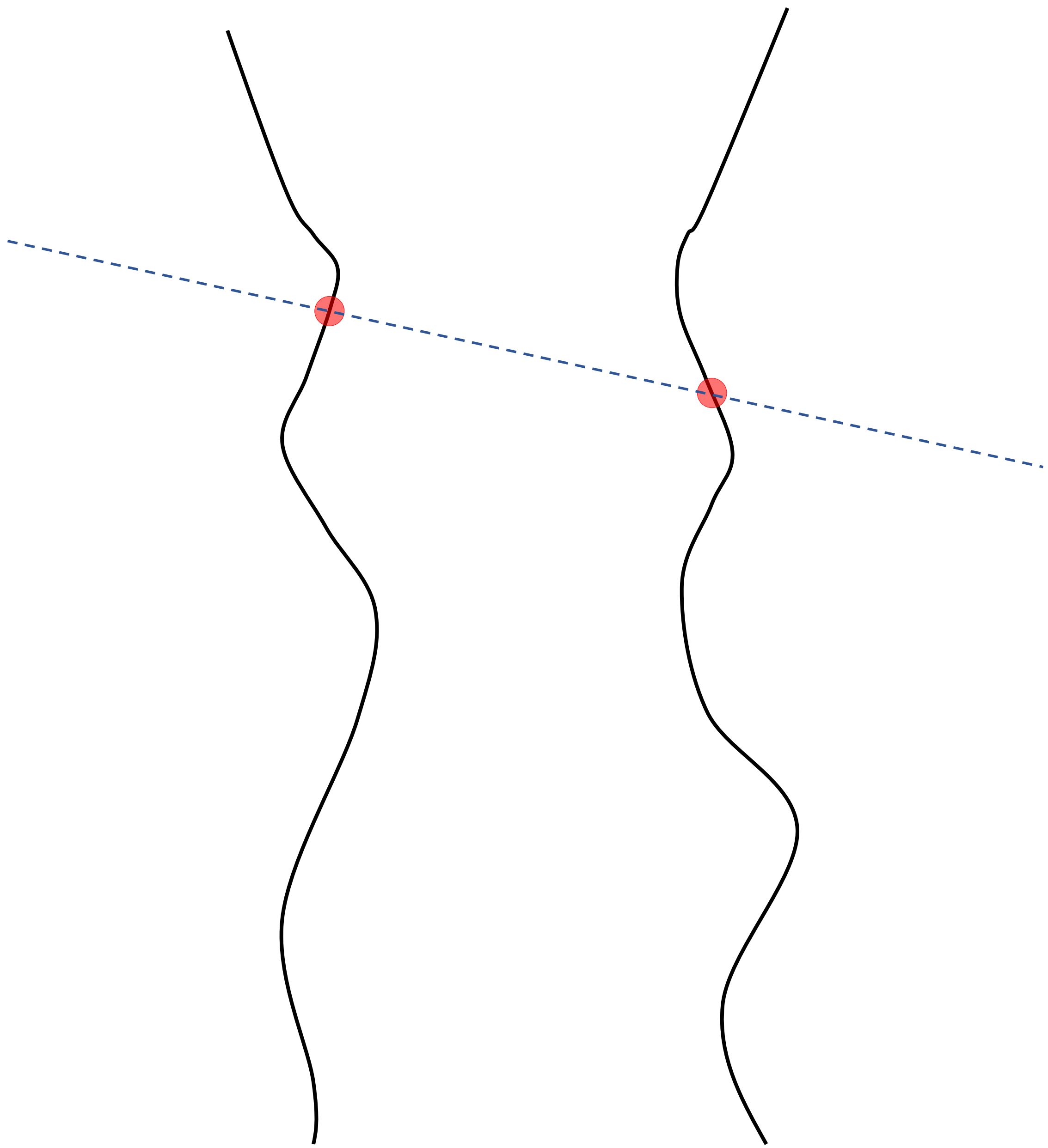

Configuration space

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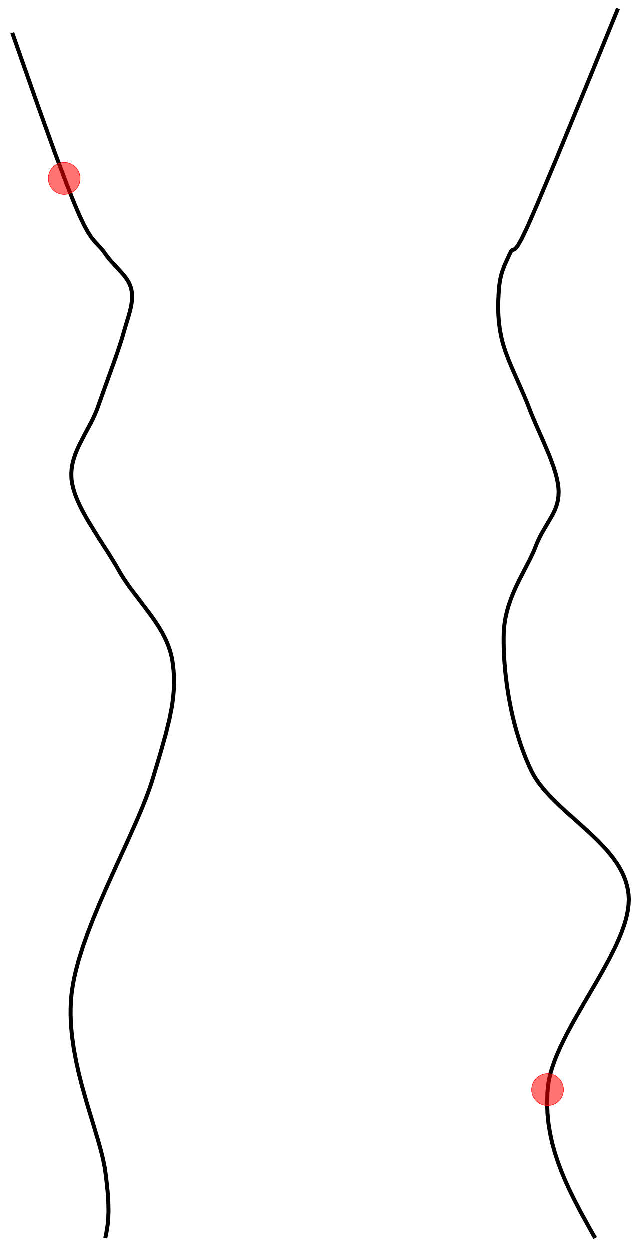
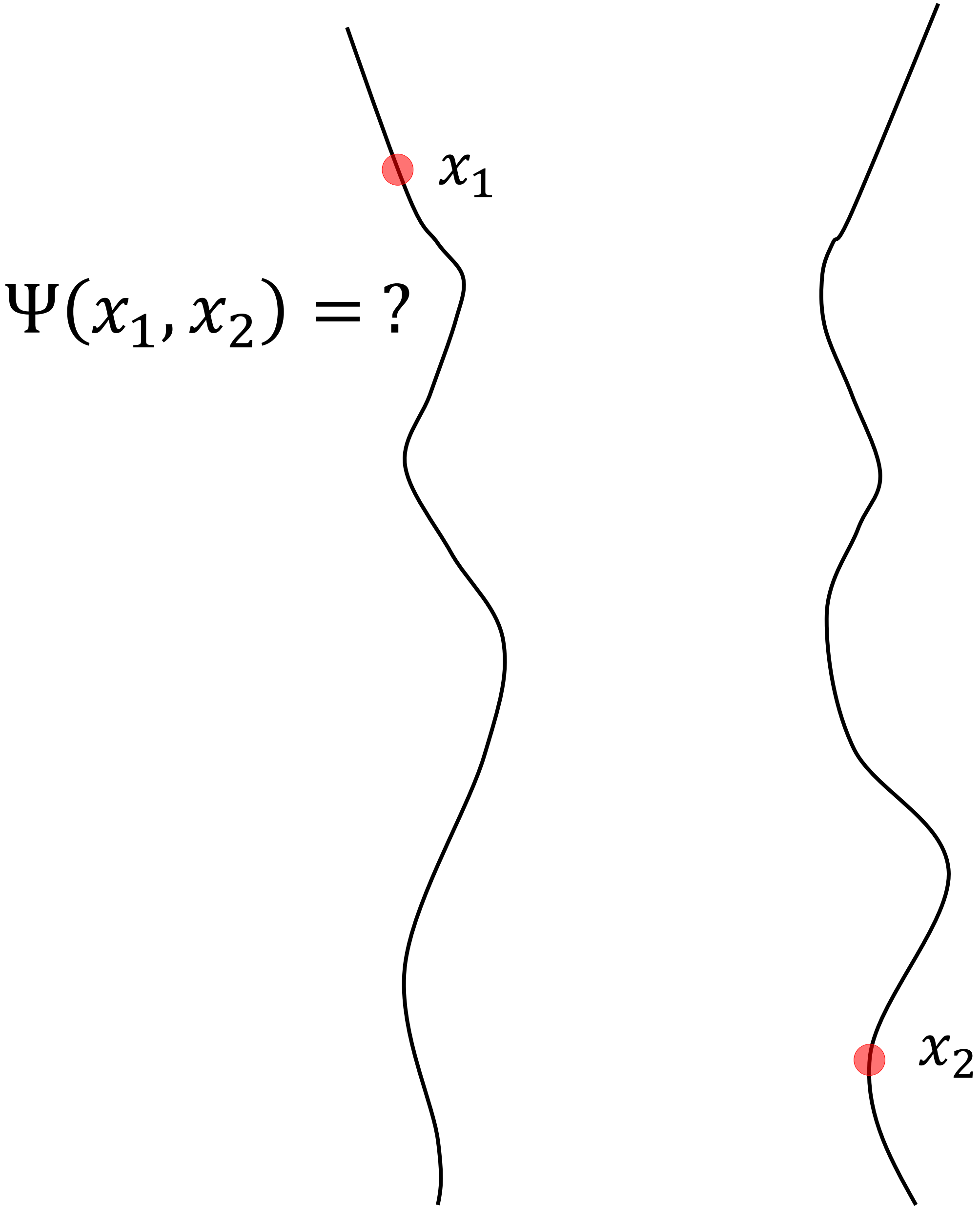


Configuration space

$$\Omega_0 := \mathbb{R}^{4N}$$



Configuration space

$$\Omega_0 := \mathbb{R}^{4N}$$

$$\Omega_1 := \{(x_1, \ldots, x_N) \in \mathbb{R}^{4N} \mid \forall i \neq j\colon \left(x_i - x_j\right)^2 < 0\}$$

Time evolution for multi-time wave functions

- Interaction potentials
  → only trivial dynamics (Petrat & Tumulka)

- Particle creation / annihilation (gauge bosons)
  → ultraviolet divergences (Dirac)

- Interactions from boundary conditions
  → zero range interaction (Lienert, Teufel)

- Integral equations / retarded action at a distance
  → not Hamiltonian (Bethe & Salpeter, Feynman, Barut)

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Hamiltonian dynamics:

$$i\partial_{t_j}\Psi(t_1,\mathbf{x}_1,\dots,t_N,\mathbf{x}_N) = H_j\Psi(t_1,\mathbf{x}_1,\dots,t_N,\mathbf{x}_N)$$

Consistency condition:

$$\left[i\partial_{t_j} - H_j, i\partial_{t_k} - H_k\right] = 0$$

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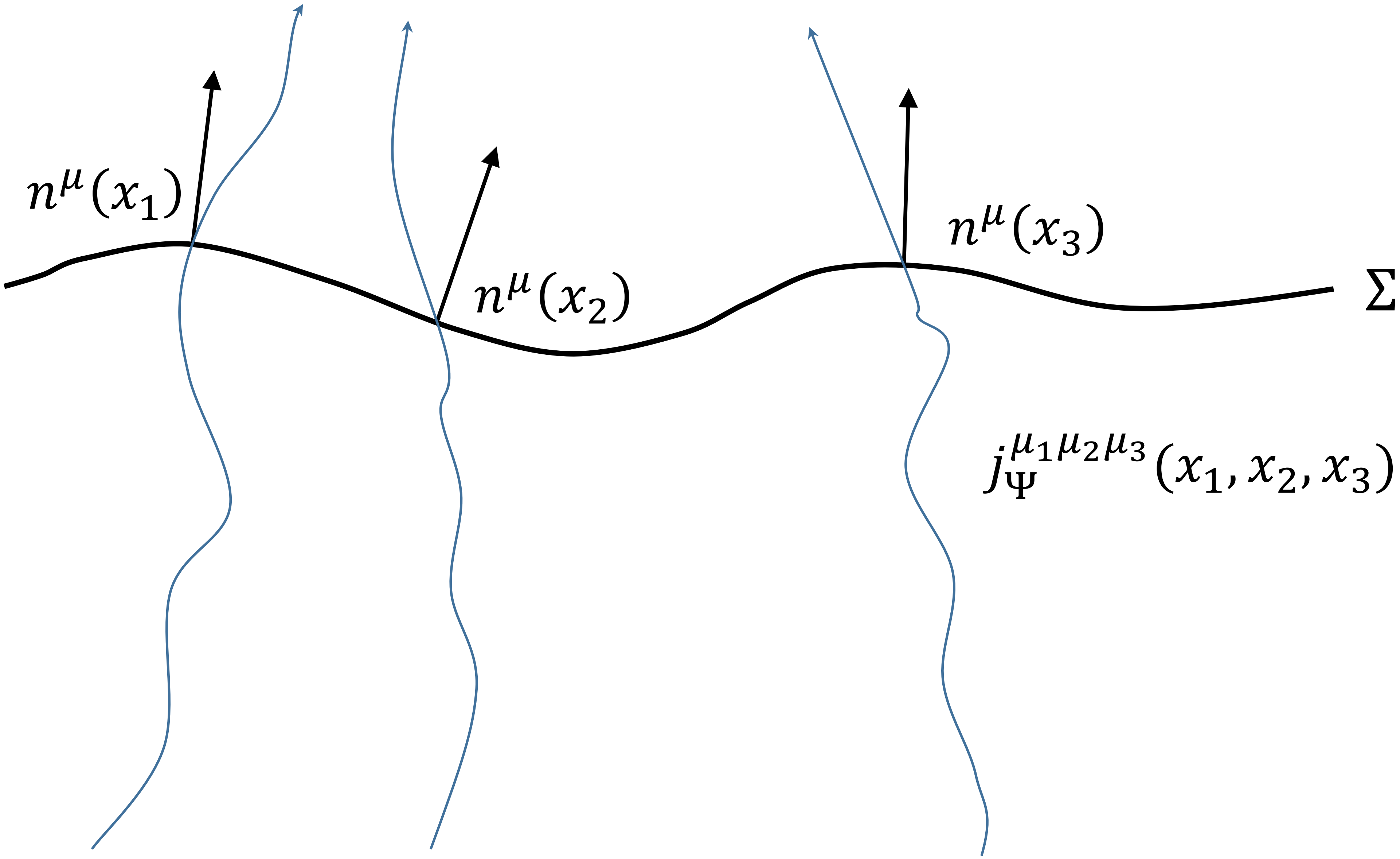
$$\Psi(x_1, x_2) = \alpha \int \mathrm{d}^4 y \int \mathrm{d}^4 z \, K(x_1 - y) K(x_2 - z) \delta((y - z)^2) \Psi(y, z)$$

Conserved, positive-definite tensor current

$$j_{\Psi}^{\mu_1 \mu_2 \ldots \mu_N}(x_1, \ldots, x_N)$$

E.g. for Dirac spinors

$$j_{\Psi}^{\mu_1 \ldots \mu_N}(x_1, \ldots, x_N) = \overline{\Psi}(x_1, \ldots, x_N)\, \gamma_1^{\mu_1} \ldots \gamma_N^{\mu_N}\, \Psi(x_1, \ldots, x_N)$$



$n^\mu(x_1)$
$n^\mu(x_2)$
$n^\mu(x_3)$
Σ
$j_\Psi^{\mu_1\mu_2\mu_3}(x_1, x_2, x_3)$

Hypersurface Bohm-Dirac Theory

$$\frac{\mathrm{d}X_k^{\mu_k}(s)}{\mathrm{d}s} \propto j_\Psi^{\mu_1 \ldots \mu_k \ldots \mu_N}(x_1, \ldots, x_N) \prod_{j \neq k} n_{\mu_j}(x_j) \Big|_{x_j = X_j(\Sigma)}$$

Hypersurface Bohm-Dirac Theory

$$\frac{\mathrm{d}X_k^{\mu_k}(s)}{\mathrm{d}s} \propto j_\Psi^{\mu_1 \dots \mu_k \dots \mu_N}(x_1, \dots, x_N) \prod_{j \neq k} n_{\mu_j}(x_j) \Big|_{x_j = X_j(\Sigma)}$$

Requires a preferred foliation! $\Sigma \in \mathfrak{F}$

$\rightarrow$ Conflict with Relativity ?

Foliation from the wave function

$$\Psi \rightarrow \mathfrak{F} = \mathfrak{F}[\Psi]$$

Foliation from the wave function

$$\Psi \to \mathfrak{F} = \mathfrak{F}[\Psi]$$

E.g. from energy-momentum tensor (Struyve):

$$n^{\mu} \propto P^{\mu} = \int_{\Sigma} T^{\mu\nu}\,(x)\,d\sigma_{\nu}(x)\,, \qquad T^{\mu\nu} = \langle\, \Psi \mid t^{\mu\nu}(x) \mid \Psi \,\rangle$$

Can Bohmian mechanics be made relativistic?

Can Bohmian mechanics be made relativistic?

What do you mean by relativistic?

- ✓ Lorentz invariance
- ✓ No faster than light signalling
- ✓ Local commutativity
- ✓ No preferred frame detectable

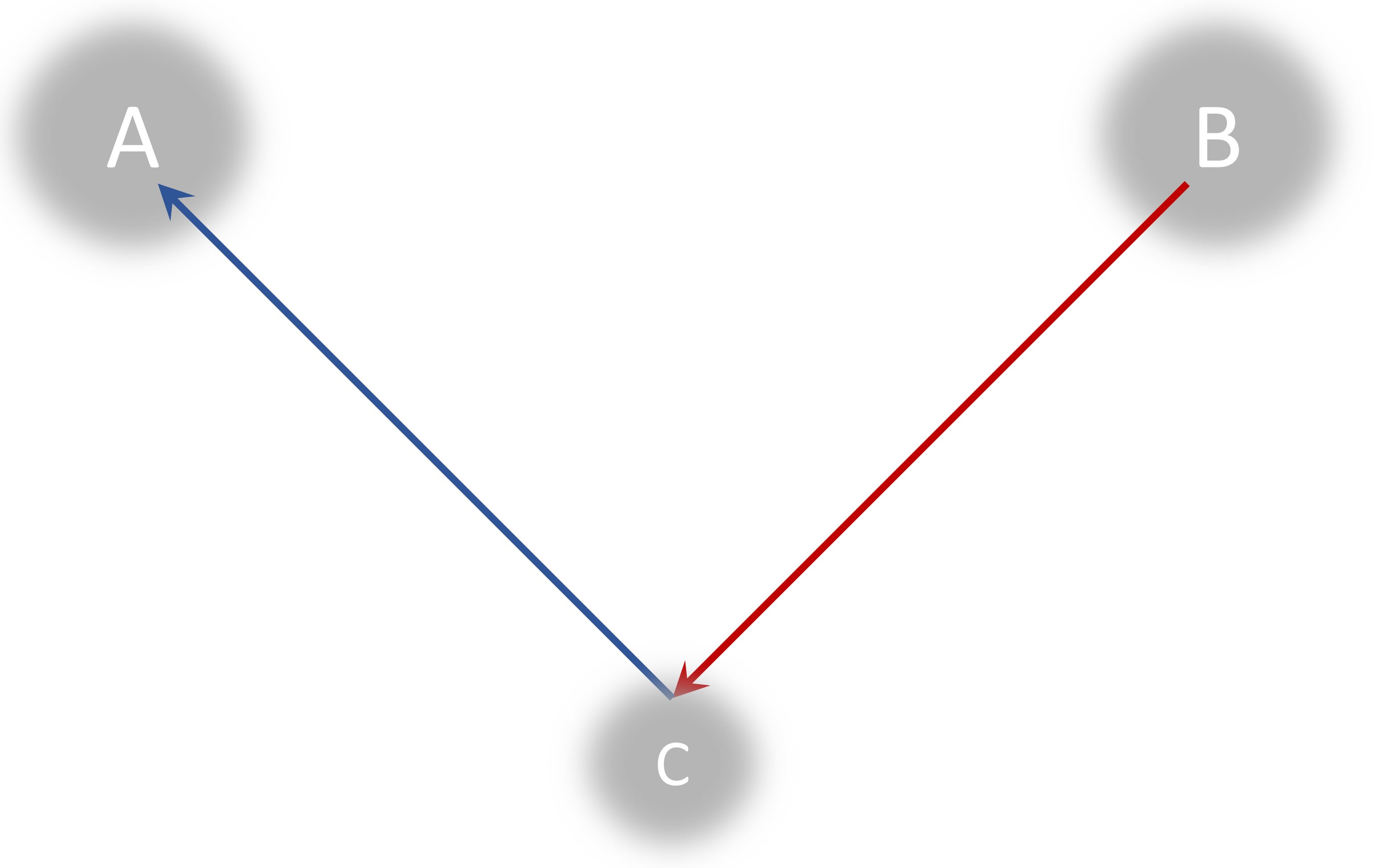
If either Relativity or Nonlocality has to go…

Can backwards causation explain quantum nonlocality?

(Costa de Beauregard, Cramer, Price, Sutherland, Wharton, Reznik & Aharanov, Goldstein & Tumulka, D.L.)


A
B


A
B


A
B
C

“[The usual quantum] paradoxes are simply disposed of by the 1952 theory of Bohm, leaving as *the* question, the question of Lorentz invariance. So one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics – the real problems of quantum mechanics – they must be talking about Lorentz invariance.” - J.S. Bell

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