

Quantum Equilibrium

Probabilities and the role of operators in Quantum Mechanics

Dustin Lazarovici

Université de Lausanne

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How to derive probabilistic predictions from a deterministic theory?

Bohmian Mechanics

Bohmian mechanics is defined by two (deterministic) equations

$$\begin{cases} \frac{d}{dt}Q(t) = v^\Psi(Q(t)) \sim \Im \frac{\nabla \Psi}{\Psi} & \text{(guiding equation)} \\ i\hbar \frac{\partial}{\partial t} \Psi_t = H \Psi_t & \text{(Schrödinger's equation)} \end{cases}$$

Where Ψ is the universal wave function and $Q(t) = (q_1(t), q_2(t), \dots, q_N(t)) \in \mathbb{R}^{3N}$ the particle configuration.

Statistical predictions are about subsystems.

Splitting of configuration space:

$$q = (x, y) \in \mathbb{R}^{3n} \times \mathbb{R}^{3(N-n)}$$

Configuration of the universe:

$$Q(t) = (\underbrace{X(t)}_{\text{subsystem}}, \underbrace{Y(t)}_{\text{rest of universe}})$$

Conditional wave-function:

$$\psi^{Y(t)}(x) = \Psi(x, Y(t))$$

The x-system has the *effective wave function* $\varphi(x)$ if

$$\Psi(x, y) = \varphi(x)\phi(y) + \Psi^\perp(x, y)$$

and $\Psi^\perp(x, Y(t)) \approx 0, \forall x \in \mathbb{R}^{3n}$

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$\Rightarrow$ Effective description of a “nice” subsystem of a Bohmian universe is again Bohmian:

$$\left\{ \begin{array}{l} \frac{d}{dt}X(t) = v^\varphi(X(t)) = \frac{\hbar}{m} \Im \nabla \varphi \\ i\hbar \frac{\partial}{\partial t} \varphi_t = H \varphi_t \end{array} \right.$$

Quantum Equilibrium Hypothesis

In an ensemble of independent subsystems with (conditional) wave function ψ , the particle configuration Q is distributed according to the Quantum Equilibrium measure

$$\mathbb{P}^\psi(Q \in A) = \int_A |\psi(q)|^2 d^{3n}q$$

where $\mathbb{P}^\psi$ is the measure with density $|\psi|^2$ on configuration space $\mathbb{R}^{3n}$.

(Born's rule)

Equivariance

For a subsystem with effective wave function φ , let T_t be the Bohmian flow induced by v^ψ . If $\psi(t)$ solves the Schrödinger equation, then $\mathbb{P}^\psi$ is **equivariant**, i.e.

$$\mathbb{P}^{\psi(t)} = \mathbb{P}^{\psi(0)} \circ T_t^{-1},$$

or

$$\int_{T_t^{-1}(A)} |\psi_0|^2 d^{3n}q = \int_A |\psi_t|^2 d^{3n}q$$

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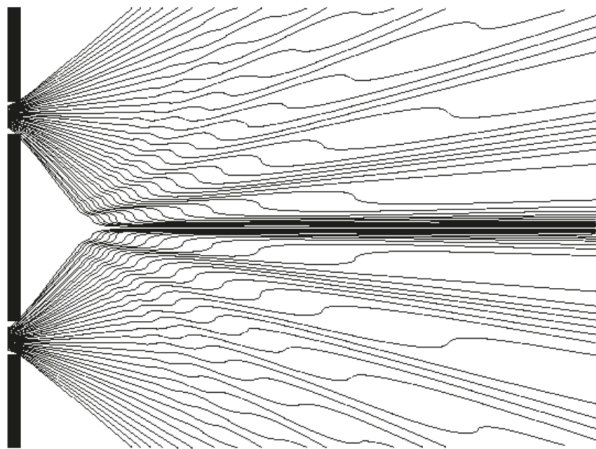
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$$\int_{T_t^{-1}(A)} |\psi_0|^2 d^{3n}q = \int_A |\psi_t|^2 d^{3n}q$$

$\Rightarrow$ If the QEH holds at one time, it holds at every time t .

Quantum Equilibrium Hypothesis

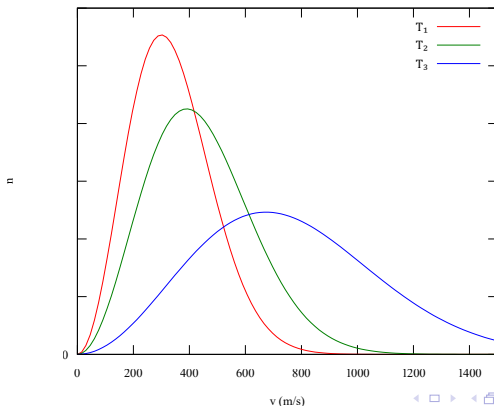


How can we justify the Quantum Equilibrium Hypothesis?

Classical statistical mechanics

Maxwell-Boltzmann distribution of velocities in an ideal gas

$$f(v)dv = \left(\frac{2\pi k_B T}{m}\right)^{-3/2} \exp\left(-\frac{1}{k_B T} \frac{mv^2}{2}\right) dv$$

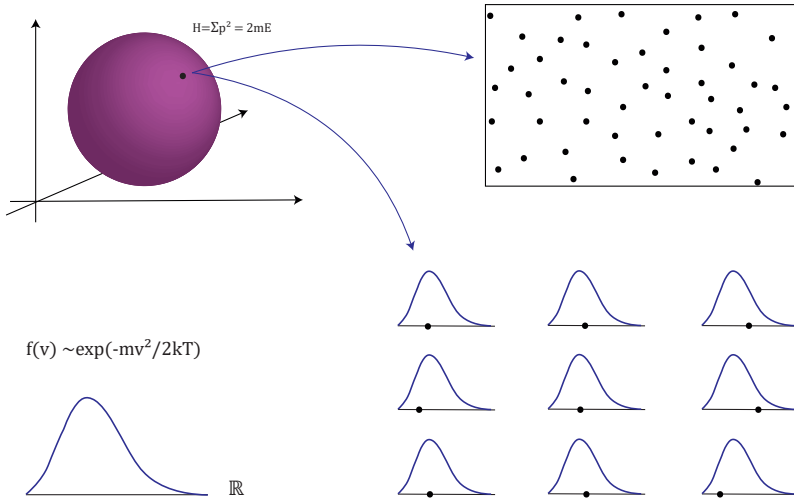


The Maxwell-Boltzmann distribution is a statistical hypothesis about the distribution of particle velocities in an ideal gas.

How can we justify (derive) the Maxwell-Boltzmann distribution?

Ludwig Boltzmann:

The ensuing, most likely state [...] which we call that of the Maxwellian velocity distribution, since it was Maxwell who first found the mathematical expression in a special case, is not an outstanding singular state, opposite to which there are infinitely many more non-Maxwellian velocity-distributions, but it is, to the contrary, distinguished by the fact that by far the largest number of possible states have the characteristic properties of the Maxwellian distribution, and that compared to this number the amount of possible velocity-distributions that deviate significantly from Maxwell's is vanishingly small.



Compute:

$$\lim_{N \rightarrow \infty, \frac{E}{N} = \frac{3}{2} k_B T} \lambda_E \left(\left\{ X \in \Gamma_E : v_{i,X} \in [a, b] \right\} \right) = \int_a^b \frac{\exp \left(-\frac{1}{k_B T} \frac{mv^2}{2} \right)}{\left(\frac{2\pi k_B T}{m} \right)^{3/2}} dv$$

λ_E is the microcanonical measure on the energy shell.

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Law of large numbers

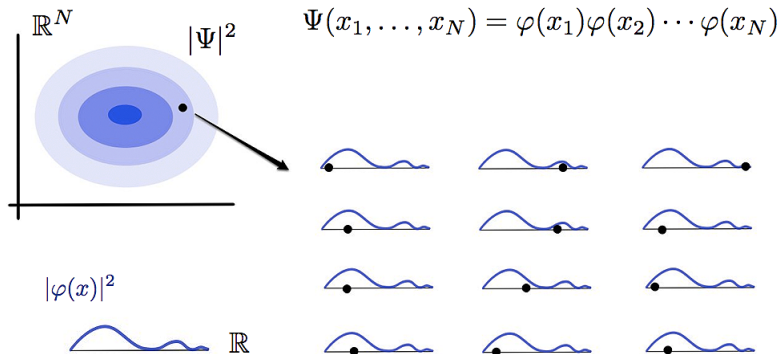
$$\lambda_E \left(\left\{ X : \left| \frac{1}{N} \sum_{i=1}^N \chi_{\{v_{i,x} \in [a,b]\}}(X) - \int_a^b \frac{\exp \left(-\frac{1}{k_B T} \frac{mv^2}{2} \right)}{\left(\frac{2\pi k_B T}{m} \right)^{3/2}} dv \right| > \epsilon \right\} \right) \rightarrow 0$$

Typicality

The stationary microcanonical measure λ_E is a **typicality measure**.

Its role is to define what we mean by “**by far the largest number of possible states** have the characteristic properties of the Maxwellian distribution”, i.e. to define **equilibrium**.

Quantum Equilibrium



Quantum Equilibrium

Conditional probability

$$\mathbb{P}^{\Psi}(\{Q = (X, Y), X \in d^n x\} | Y) = \frac{|\Psi((x, Y))|^2 d^n x}{\int |\Psi((x, Y))|^2 d^n x} = |\psi^Y(x)|^2 d^n x.$$

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We do not know Y , but we (may) know ψ^Y

$$\mathbb{P}^{\Psi}(\{Q = (X, Y), X \in d^n x\} | \psi^Y = \psi) = |\psi|^2 d^n x$$

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Law of large numbers

$$\mathbb{P}_t^{\Psi} = \left(\left\{ Q : \left| \frac{1}{N} \sum_{i=1}^N \chi_{\{X_i \in A\}}(Q) - \int_A |\varphi(x)|^2 \right| < \epsilon \right\} \right) \rightarrow 0, N \rightarrow \infty$$

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$\mathbb{P}^{\Psi}$ is the measure with density $|\Psi|^2$ defined by the [universal wf.](#)

Born's rule holds for ensembles of subsystems in typical Bohmian universes (Quantum Equilibrium).

Typicality

The “universal” $|\Psi|^2$ -measure is a **typicality measure**.

Its role is to define “**typical Bohmian universes**”, i.e. Quantum Equilibrium.

It is distinguished as the unique stationary (equivariant) measure for the Bohmian dynamics.

Absolute Uncertainty

Theorem: In Quantum Equilibrium, you cannot have more information about the configuration of a system with (conditional) wave function ψ than what is given by the $|\psi|^2$ -distribution.

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Proof: Conditional probability formula

$$\mathbb{P}^\Psi(\{Q = (X, Y), X \in d^n x\} | Y) = |\psi^Y(x)|^2 d^n x.$$

The “information” about the subsystem must be stored, somehow, in the configuration of the environment. But this is already taken into account, when we condition on Y .

Part II: Measurements and the role of operators

A weird notion: “Observables”

- Why should some laboratory procedure be associated with a linear operator on Hilbert space?
- How does the experimentalist know what she has to do to “measure an operator” $\hat{A}$?
- Do *all self-adjoint operators correspond to measurable quantities? (If not, which one do?)*
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Ideal Measurements

Definition: A *measurement apparatus* is a macroscopic system that interacts with another system and evolves (depending on the state of the measured system) into one of several macroscopically distinct states (“pointer positions”) corresponding to a measurement result $\alpha_k \in \mathcal{A}$.

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In QM, the macroscopically distinct states $\alpha_k \in \mathcal{A}$ correspond to well-localized wave functions Φ_k with mutually disjoint supports.

Ideal Measurements

System couples to apparatus

$$\varphi_k \Phi_0 \xrightarrow{\text{Schrödinger Evolution}} \varphi_k \Phi_k$$

The system may be in a superposition

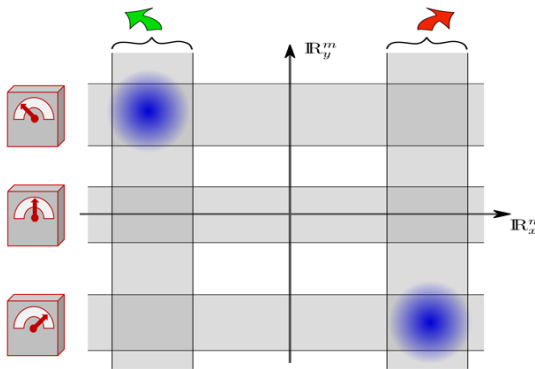
$$\varphi = \sum c_k \varphi_k, \quad \sum_k |c_k|^2 = 1$$

The Schrödinger evolution of system+apparatus yields

$$\varphi \Phi \xrightarrow{\text{Schrödinger Evolution}} \sum_k c_k \varphi_k \Phi_k$$

Ideal Measurements

The apparatus wave functions Φ_k correspond to different “pointer positions”. Therefore, they have macroscopically disjoint support:
 $\text{supp } \Phi_k \cap \text{supp } \Phi_j \approx \emptyset$. ($\rightarrow$ Decoherence)



In BM, system and apparatus have an **actual configuration** $Q = (X, Y)$.

The probability for the measurement outcome α_j is:

$$\begin{aligned} \mathbb{P}(Q = (X, Y) \mid Y \in \text{supp } \Phi_j) \\ &= \int_{\{x, y \mid y \in \text{supp } \Phi_j\}} \left| \sum_k c_k \varphi_k(x) \Phi_k(y) \right|^2 d^m x d^n y \\ &\approx |c_j|^2 \int |\varphi_j|^2(x) d^m x \int |\Phi_j|^2(y) d^n y = |c_j|^2. \end{aligned}$$

Conservation of measure:

$$\begin{aligned}\|\varphi\Phi_0\|^2 &:= \int |\varphi(x)\Phi_0(y)|^2 d^m x d^n y \\ &= \int \left| \sum_k c_k \varphi_k(x) \Phi_0(y) \right|^2 d^m x d^n y \\ \stackrel{\text{time-evolution}}{=} &\int \left| \sum_k c_k \varphi_k(x) \Phi_k(y) \right|^2 d^m x d^n y \\ &= \sum_k |c_k|^2 \int |\varphi_k|^2(x) \int |\Phi_k|^2(y) d^m x d^n y \\ &= \sum_k |c_k|^2.\end{aligned}$$

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This implies

$$\sum_{k \neq k'} c_k^* c_{k'} \int \varphi_k^*(x) \varphi_{k'}(x) d^m x = 0.$$

Since this holds for arbitrary coefficients $c_k, c_{k'}$, each summand must vanish separately:

$$\int \varphi_k^*(x) \varphi_{k'}(x) d^m x = 0, \text{ for } k \neq k'.$$

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Proposition: If the system wave functions $\varphi_1, \varphi_2, \dots$ lead to macroscopically distinguishable measurement results, they must form an *orthonormal system*:

$$\langle \varphi_i | \varphi_j \rangle = \delta_{ij}.$$

Introduce the orthogonal projectors

$$P_{\varphi_k} := |\varphi_k\rangle\langle\varphi_k|$$

Corresponding to the measurement values $\mathcal{A} = \{\alpha_k\}$.

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Probability for the measurement result α_k :

$$\langle\varphi|P_{\varphi_k}|\varphi\rangle = \sum_j \sum_i c_i^* c_j \langle\varphi_i||\varphi_k\rangle\langle\varphi_k||\varphi_j\rangle = |c_k|^2$$

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Expectation value:

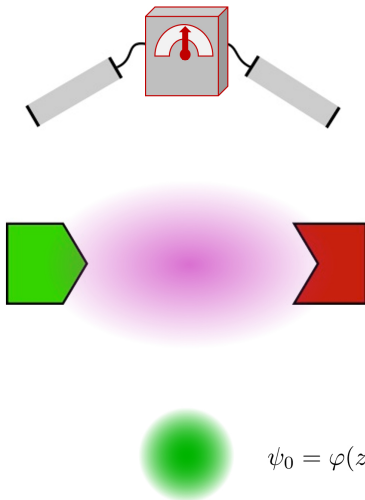
$$\langle\varphi|\sum_k \alpha_k P_{\varphi_k}|\varphi\rangle =: \langle\varphi|\hat{A}|\varphi\rangle = \sum_k \alpha_k |c_k|^2$$

Where we introduced the self-adjoint operator

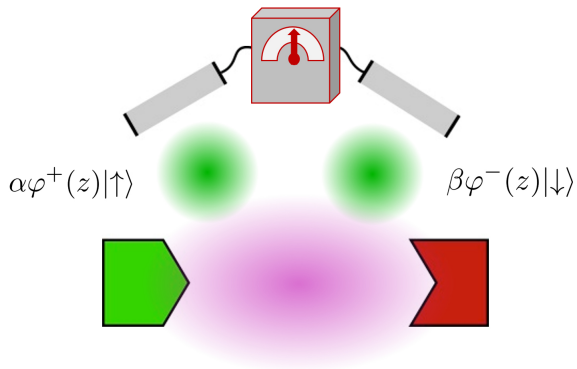
$$\hat{A} := \sum_k \alpha_k P_{\varphi_k}.$$

Operators are book-keepers of measurement statistics

Spin measurement



Spin measurement



Example: Spin measurement

$$\psi_0 = \varphi(z)(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$$

Time-evolution (Pauli equation) in an inhomogeneous magnetic field:

$$\psi_t = \alpha\varphi^+(z)|\uparrow\rangle + \beta\varphi^-(z)|\downarrow\rangle$$

Probability for “Spin up”

$$\mathbb{P}(z \in \text{supp } \varphi^+) = \int_{\text{supp } \varphi^+} |\psi_t|^2 = |\alpha|^2 = \langle \psi_t | \uparrow \rangle \langle \uparrow | \psi_t \rangle$$

Probability for “Spin down”

$$\mathbb{P}(z \in \text{supp } \varphi^-) = \int_{\text{supp } \varphi^-} |\psi_t|^2 = |\beta|^2 = \langle \psi_t | \downarrow \rangle \langle \downarrow | \psi_t \rangle$$

Expectation value:

$$\frac{\hbar}{2}|\alpha|^2 - \frac{\hbar}{2}|\beta|^2 = \langle \psi_t | \frac{\hbar}{2}\sigma_z | \psi_t \rangle$$

Spin is not a predetermined “property” of the particles.

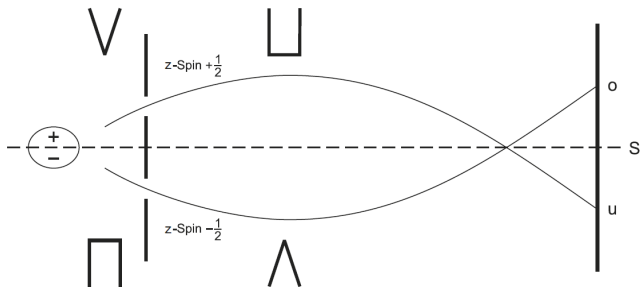


Fig. 8.6. Combination of two Stern-Gerlach-experiments. The curved lines represent the paths of the wave packets

General scheme

$$\varphi(x) \xrightarrow{\text{system couples to apparatus}} \Psi(x, y) = \varphi(x)\Phi(y)$$

$$\text{Schrödinger Evolution} \xrightarrow{\quad} \Psi_T(x, y)$$

$$\text{Born's rule} \xRightarrow{\quad} \rho^{\Psi_T} = |\Psi_T|^2(x, y)$$

$$\text{we care about} \xrightarrow{\quad} \mathbb{P}^\varphi(d\alpha) = \mathbb{P}^{\Psi_T}(F^{-1}(d\alpha)) = \int_{F^{-1}(d\alpha)} |\Psi_T(x, y)|^2 dx dy$$

where $F : \mathbb{R}^{3n} \times \mathbb{R}^{3m} \rightarrow \mathcal{A}$ is some coarse graining function (random variable) from the particle configuration to measurement outcomes.

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For an arbitrary (measurable) result set $A \subset \mathcal{A}$:

$$O(A) = \int_A O(d\alpha) \text{ i.e. } \mathbb{P}^\varphi(A) = \langle \varphi, O(A) \varphi \rangle = \int_A \langle \varphi, O(d\alpha) \varphi \rangle.$$

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It is straight forward to check the following properties:

$$\begin{aligned} O(\mathcal{A}) &= \int_{\mathcal{A}} O(d\alpha) = \mathbb{P}^{\Psi_T}(F^{-1}(\mathcal{A})) = \mathbb{P}^{\Psi_T}(\mathcal{Q}) = 1 \\ A \cap B = \emptyset &\Rightarrow O(A \cup B) = O(A) + O(B) \end{aligned}$$

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The family of operators defines a **positive operator valued measure (POVM)**.

Example: Position measurement

Here: $F = Id$ and $\mathcal{A} = \mathbb{R}^3$. The quadratic form is $\varphi(x) \implies \rho^\varphi(x)$ with probability distribution $\mathbb{P}^\varphi(d^3x) = |\varphi(x)|^2 d^3x$. We get

$$\mathbb{P}^\varphi(A) = \int \chi_A |\varphi|^2 d^3x = \int_A \langle \varphi | \chi_{\{d^3x\}} | \varphi \rangle = \langle \varphi | \chi_A | \varphi \rangle,$$

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The position POVM is thus defined by

$$\mathcal{B}(\mathbb{R}^3) \rightarrow \text{Sym}(\mathcal{H}), \quad A \mapsto \chi_A$$

The position POVM is a **PVM (projector valued measure)**, since $O(A)^2 = O(A)$ for all A .

Example: Imprecise position measurement

The measured position is the sum $X + Y$, where X is the actual position and Y is a random measurement error with distribution r . The distribution for the measured position $\tilde{X}$ is then

$$\tilde{\rho}(x) = \int r(x - y) |\varphi|^2(y) dy .$$

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This means

$$\begin{aligned} \mathbb{P}^\varphi(\tilde{X} \in A) &= \int_A \tilde{\rho}(x) dx = \int_A \int r(x - y) |\varphi|^2(y) dy dx \\ &= \int \left(\int \chi_A(x) r(x - y) dx \right) |\varphi|^2(y) dy =: \langle \varphi | \tilde{O}(A) | \varphi \rangle \end{aligned}$$

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The corresponding POVM is thus

$$\tilde{\mathcal{O}}(A) = \int p(x - y) \chi_A(y) dy$$

Example: Imprecise position measurement

This POVM is not an PVM, since $\tilde{O}(A)^2 \neq \tilde{O}(A)$, in general.

→ The measurement is not described by a self-adjoint operator.

John Bell - The theory of local beables

The concept of 'observable' lends itself to very precise mathematics when identified with 'self-adjoint operator'. But physically, it is a rather woolly concept. It is not easy to identify precisely which physical processes are to be given status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in 'classical terms' because they are there. The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments. 'Observables' must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.

Some Takeaways I

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Born's rule is a regularity of typical Bohmian universes.

If the quantum formalism is correct, Bohmian mechanics is (most likely) the correct microscopic theory that grounds this formalism.

Some Takeaways II

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The mathematical attractiveness of the operator formalism can easily suggest that operators must play an important and fundamental role in quantum theory.

They don't.

Some Takeaways III

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If a theory describes (correctly) the distribution of matter in time and space, everything else follows.

In the end, every measurement is a position measurement.