The measurement problem

and some mild solutions

ICTS meeting „Fundamental Problems in Quantum Physics“

Tata Institute, Bangalore, Nov. 2016

Dustin Lazarovici

Université de Lausanne

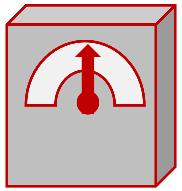
The measurement problem

Most simply put, the measurement problem is this: Quantum mechanics involves only one equation – the Schrödinger equation – and this equation does not describe the phenomena.

$$i\hbar\partial_t \Psi = H\Psi$$

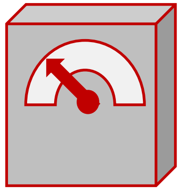
Consider a quantum system in the state φ_i , $\mathrm{i} = 1,2, \ldots$

Couple it to a measurement apparatus in the ready state ϕ_0



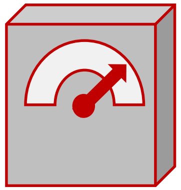
If the system is in the state φ_1, the apparatus shows the measurement result 1

$$\varphi_1 \phi_0 \rightarrow \varphi_1 \phi_1$$



If the system is in the state φ_2, the apparatus shows the measurement result 2

$$\varphi_2 \phi_0 \rightarrow \varphi_2 \phi_2$$



Due to linearity of the S.E. the superposition is also a solution

$$(\varphi_1 + \varphi_2) \phi_0 \rightarrow \varphi_1 \phi_1 + \varphi_2 \phi_2$$

?

Consider a quantum system in the state φ_i , $\mathrm{i} = 1,2,\ldots$

Couple it to a measurement apparatus in the ready state ϕ_0

If the system is in the state φ_1, the apparatus shows the measurement result 1

$$\varphi_1\phi_0 \rightarrow \varphi_1\phi_1$$

If the system is in the state φ_2, the apparatus shows the measurement result 2

$$\varphi_2\phi_0 \rightarrow \varphi_2\phi_2$$

Due to linearity of the S.E. the superposition is also a solution

$$(\varphi_1+\varphi_2)\phi_0 \rightarrow \varphi_1\phi_1 + \varphi_2\phi_2$$

Schrödinger‘s „Cat Article“

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be decided by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

- Erwin Schrödinger, „The present situation in Quantum Mechanics“

The measurement problem

The following three claims are **mutually inconsistent:**

1. The wave-function of a system is *complete,* i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
2. The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
3. Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up).

Tim Maudlin, “Three Measurement Problems” (1995)

The orthodox answer: collapse postulate

When the quantum system is “measured” or “observed”, the Schrödinger evolution is suspended and the wave-function collapses into one of the possible “eigenstates” (with probabilities given by Born’s rule)

John Bell, „Against `Measurement‘“

It would seem that the theory is exclusively concerned about “results of measurement”, and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of “measurer”? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a Ph.D.? If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less “measurement-like” processes are going on more or less all the time, more or less everywhere. Do we not have jumping then all the time?

Three „mild“ solutions

We have to give up one of the three principles

1. The wave-function of a system is *complete,* i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
2. The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
3. Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up).

Three „mild“ solutions

We have to give up one of the three principles

1. The wave-function of a system is *complete,* i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
2. The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
3. Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up).

→ Specify additional variables that provide a complete description of the physical system
→ Obvious choice: particle positions $(\boldsymbol{\Psi}, \boldsymbol{Q})$
→ Bohmian mechanics

Bohmian mechanics

The complete state of an N particle system at time t is described by

$$(\Psi_{\mathrm{t}}, Q_t)$$

where $\mathrm{Q_t} = (X_1(t), X_2(t), \ldots, X_N(t)) \in \mathbb{R}^{3N}$ is the actual configuration of particles.

The particle configuration moves according to a law of motion that is defined by the wave function

$$\dot{Q}_t = v^{\Psi_{\mathrm{t}}}(Q_t)$$

Particles are the ontology of Bohmian mechanics.

Three „mild“ solutions

We have to give up one of the three principles

1. The wave-function of a system is *complete,* i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
2. The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
3. Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up).

→ Give up the linear time-evolution of the wave function
→ Specify a precise collapse law for Ψ
→ GRW theory / collapse models

GRW theory

- Between jumps, the wave function evolves according to the Schrödinger equation
- When a collapse event occurs, the wave function changes according to

$$\psi \to \frac{\psi_x^i}{\|\psi_x^i\|}, \quad \psi_x^i := \hat{L}_x^i \psi, \quad \hat{L}_x^i := \frac{1}{(\pi r_c)^{3/4}} \exp[-\frac{(q_i - x)^2}{2r_c^2}]$$

- The jumps are random in time (Poisson process with frequency λ) and random in space (distribution of the collapse center $\sim |\psi|^2$).
- For a single „particle“, the wave function collapses about once every 10^{15} seconds
- For a macroscopic system with $N \sim 10^{24}$ „particles“, superpositions are almost immediately destroyed

Three „mild“ solutions

We have to give up one of the three principles

1. The wave-function of a system is *complete,* i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
2. The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
3. Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up).

→ Accept macroscopic superpositions as a genuine prediction of QM („Branching universe“)
→ Try to reconcile a non-collapsing Ψ with empirical evidence
→ Many-Worlds theory (Everett Interpretation)

If the system is in the state φ_1, the apparatus shows the measurement result 1

$$\varphi_1\phi_0 \rightarrow \varphi_1\phi_1$$

If the system is in the state φ_2, the apparatus shows the measurement result 2

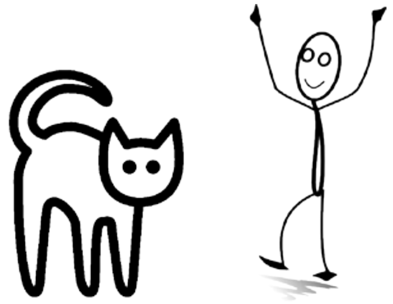
$$\varphi_2\phi_0 \rightarrow \varphi_2\phi_2$$

Due to linearity of the S.E. the superposition is also a solution

$$(\varphi_1+\varphi_2)\phi_0 \rightarrow \varphi_1\phi_1 + \varphi_2\phi_2$$

If the system is in the state φ_1, the apparatus shows the measurement result 1

$$\varphi_1\phi_0\chi_0 \rightarrow \varphi_1\phi_1\chi_1$$

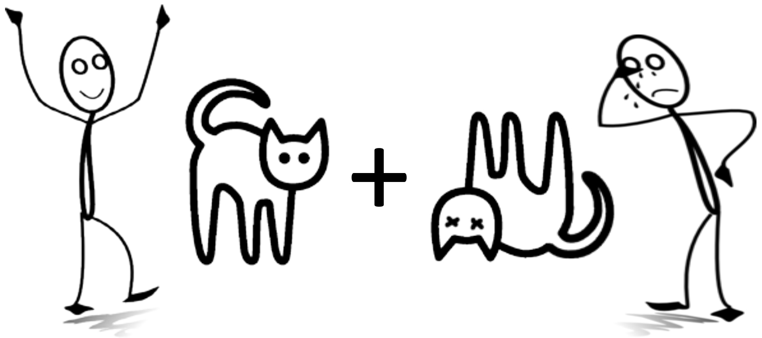


If the system is in the state φ_2, the apparatus shows the measurement result 2

$$\varphi_2\phi_0\chi_0 \rightarrow \varphi_2\phi_2\chi_2$$

Due to linearity of the S.E. the superposition is also a solution

$$(\varphi_1+\varphi_2)\phi_0\chi_0 \rightarrow \varphi_1\phi_1\chi_1 + \varphi_2\phi_2\chi_2$$



Challenges of the Many-Worlds Theory

- How do we explain quantum probabilities, if all possible measurement outcomes occur?
- How can a branch of the universal wave function represent our world – or any world at all?

Ontologies for the GRW theory

- GRW0 – wave function ontology
- GRWm – mass density

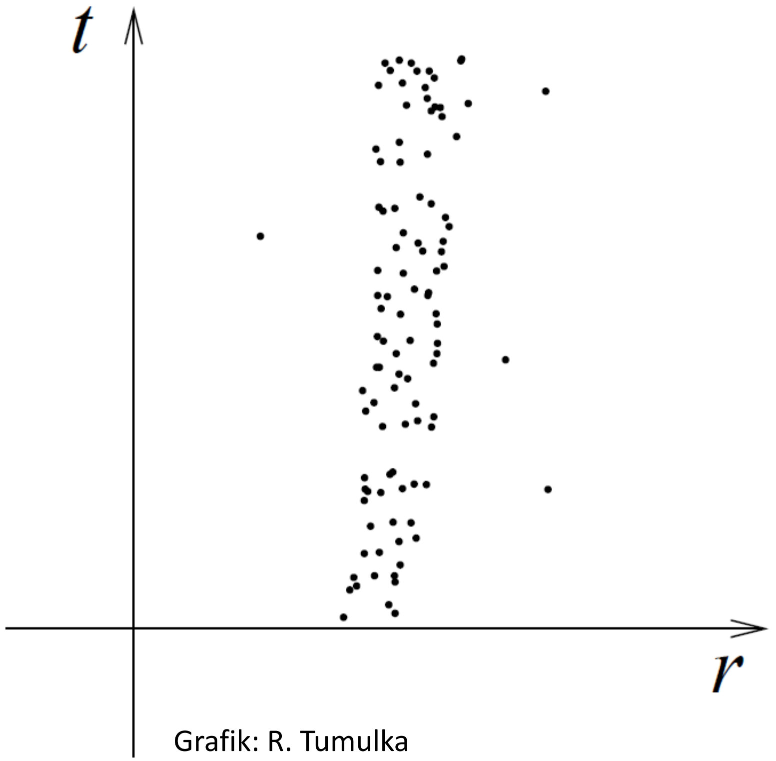
$$m(t,x)=\sum_{i=1}^{N} m_i \int dq_1 \dots dq_N \, \delta(q_i - x)|\Psi(q_1,\dots,q_N)|^2$$

- GRWf – flash ontology

GRW flash ontology

John Bell, „Are there quantum jumps?“

„[T]he GRW jumps (which are part of the wave-function, not something else) are well localised in ordinary space. Indeed each is centered on a particular spacetime point (**x**,t). So we can propose these events as the basis of the `local beables‘ of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the `observables' of other formulations of quantum mechanics, for which we have no use here.) A piece of matter then is a galaxy of such events.”



Grafik: R. Tumulka

What is quantum mechanics about?

Bohmian mechanics

The wave function is a function on **configuration space.**

$$\Psi(x_1, x_2, \dots, x_N), \qquad (x_1, x_2, \dots, x_N) \in \mathbb{R}^{3N}$$

N is the number of **particles**. $\mathrm{q} = (x_1, x_2, \dots, x_N)$ describes a possible configuration of **particles**.

When you say "particles", mean **particles**.

The complete state of an N particle system at time t is described by

$$(\Psi_{\mathrm{t}}, Q_t)$$

where $\mathrm{Q_t} = \ (X_1(t), X_2(t), \dots, X_N(t)) \in \mathbb{R}^{3N}$ is the actual configuration of particles.

Bohmian mechanics

The particle configuration evolves according to a law of motion

$$\dot{X}_k = \frac{\hbar}{m}\, Im \frac{\nabla_k \Psi}{\Psi}(X_1, X_2, \ldots, X_N)$$

The wave function defines a velocity field v^{Ψ} along which the particle configuration moves. The wave function “guides” the particles.

Bohmian mechanics is defined by two equations

Schrödinger equation

$$i\hbar\partial_t \Psi = (-\frac{\hbar^2}{m}\Delta_x + V)\Psi$$

Guiding equation

$$\dot{X}_k = \frac{\hbar}{m}\, Im \frac{\nabla_k \Psi}{\Psi}(X_1, X_2, \ldots, X_N)$$

Where does the guiding equation come from?

E.g. quantum probability flow.

Schrödinger equation → continuity equation

$$\partial_t \rho + \nabla \cdot j = 0$$

With ρ = | Ψ |^2 the probability density and

$$j = \frac{\mathrm{i}\hbar}{2m} \left(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*\right) = \frac{\hbar}{m}\, Im\, \Psi^* \nabla \psi$$

the probability current.

The current is the density transported with the velocity flow:

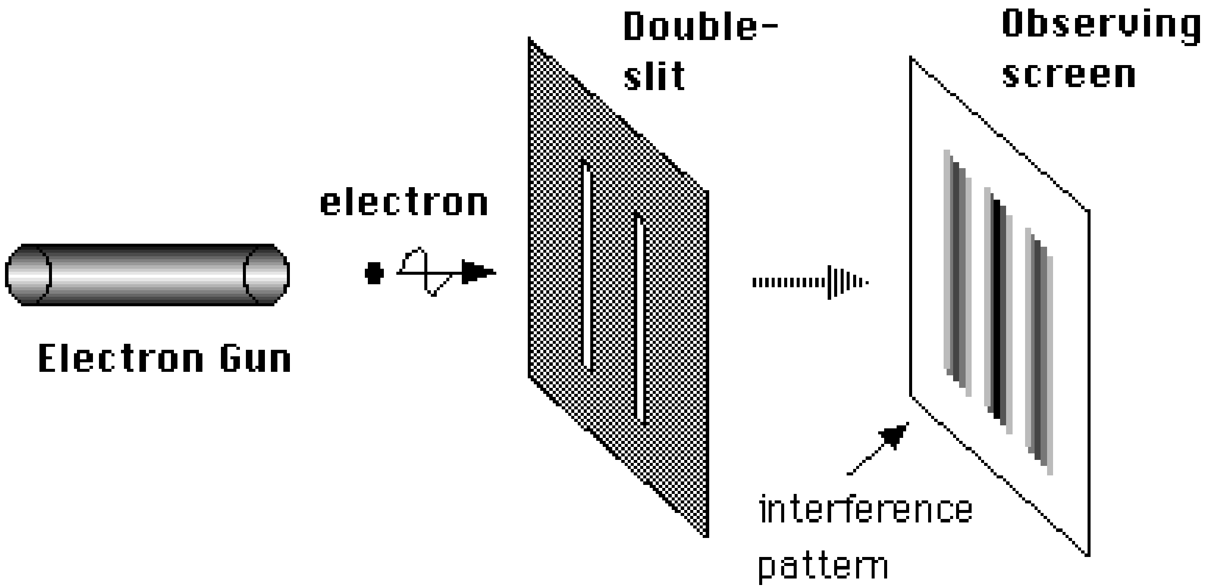
$$j = \rho v^{\Psi}$$

from which one can read off the guiding equation.

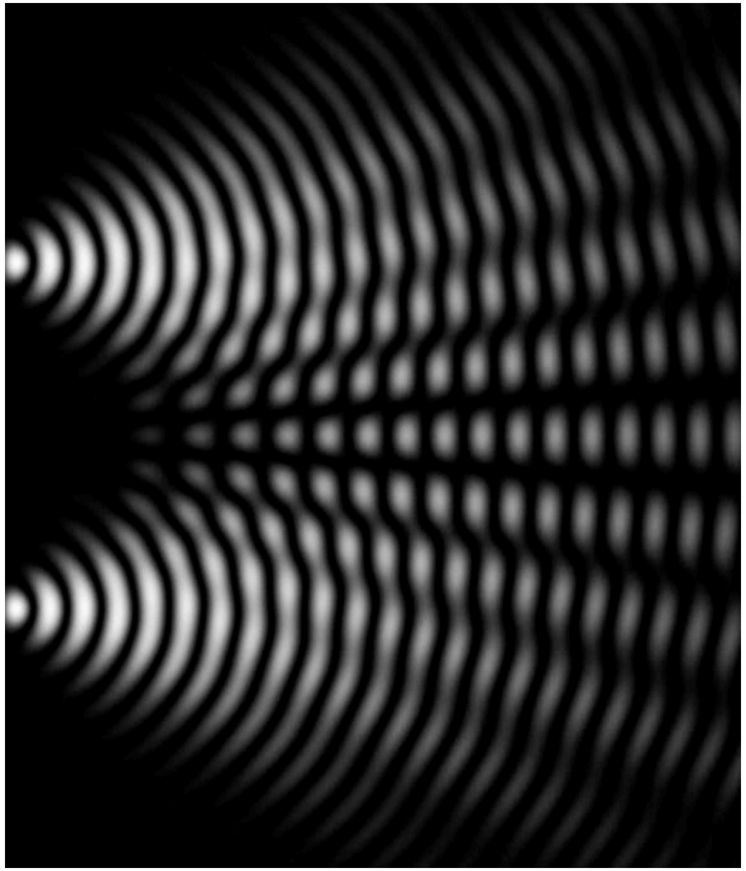
Of course, there is only one question that really matters:

Do these two equations describe the phenomena?

The Double Slit Experiment

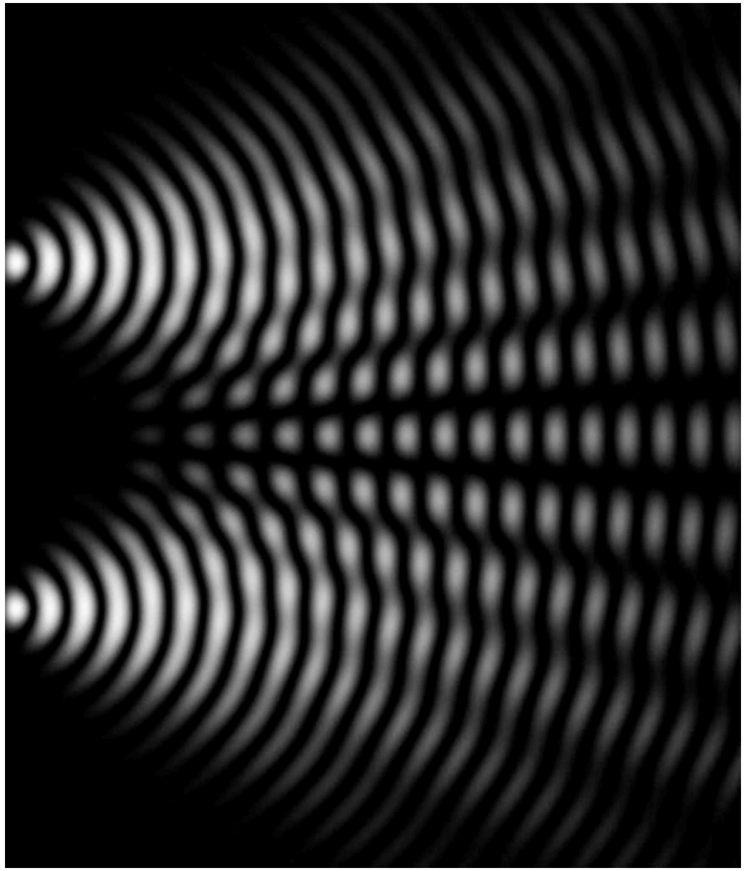


The Double Slit Experiment



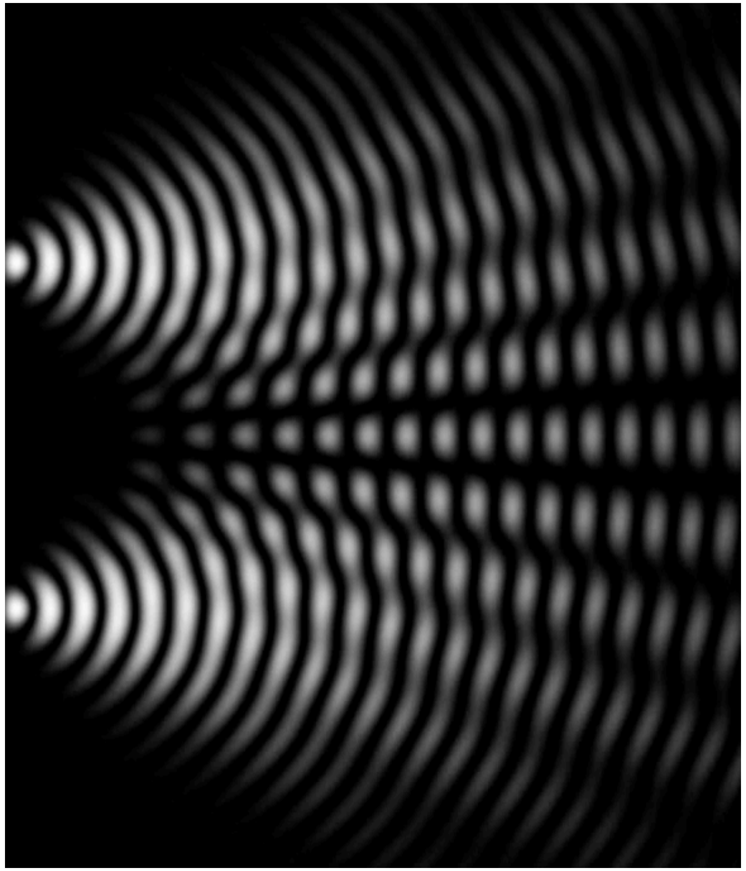
Tonomura et. al., Am. J. Physics 57, 1989

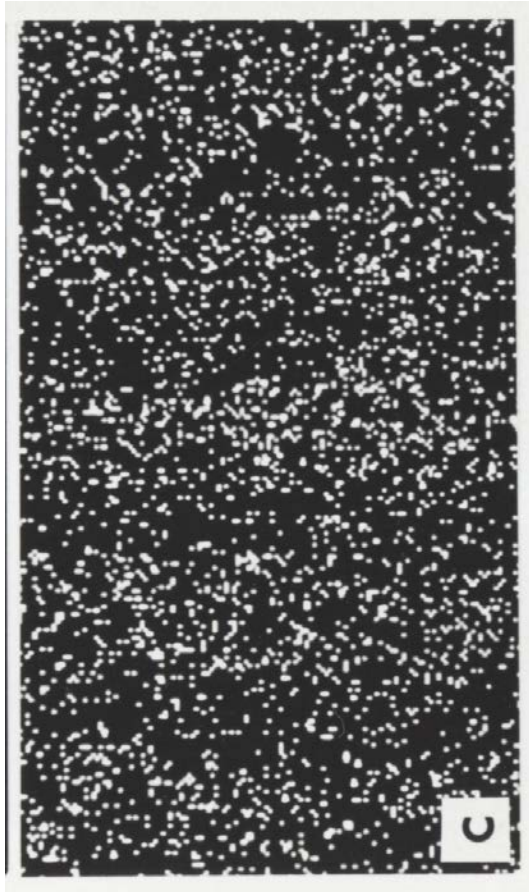
The Double Slit Experiment



Tonomura et. al., Am. J. Physics 57, 1989

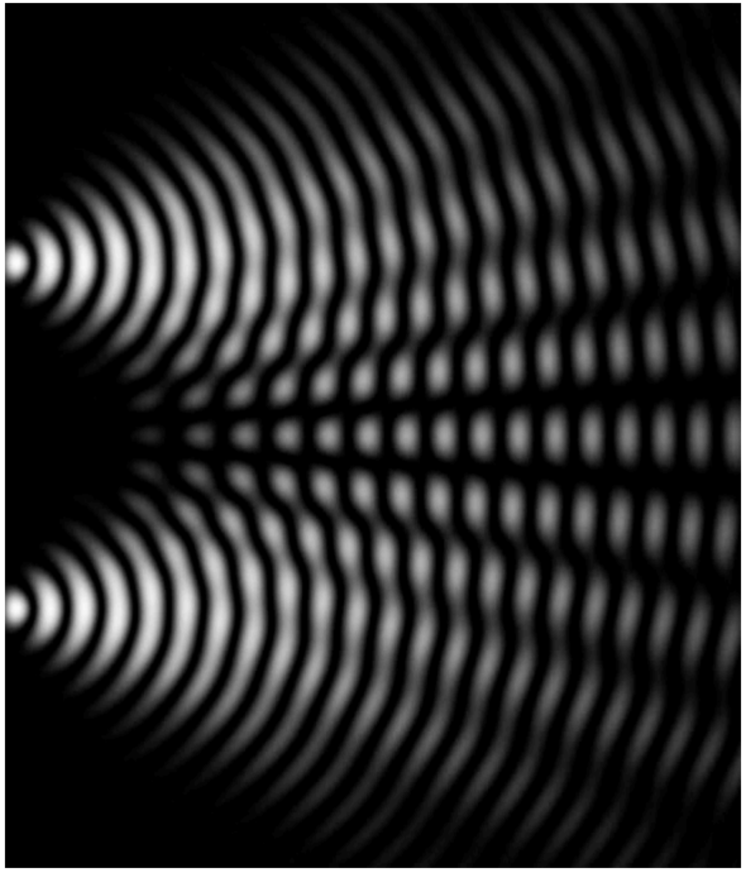
The Double Slit Experiment

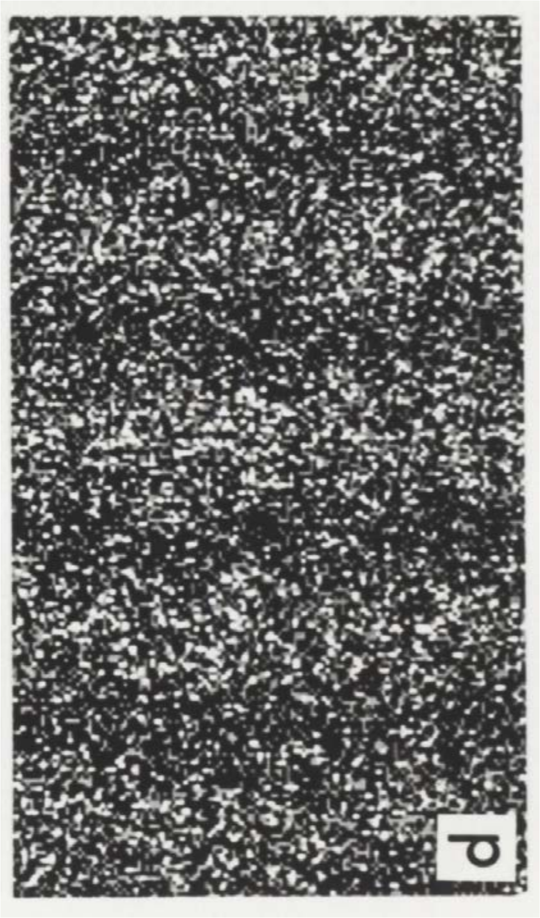




Tonomura et. al., Am. J. Physics 57, 1989

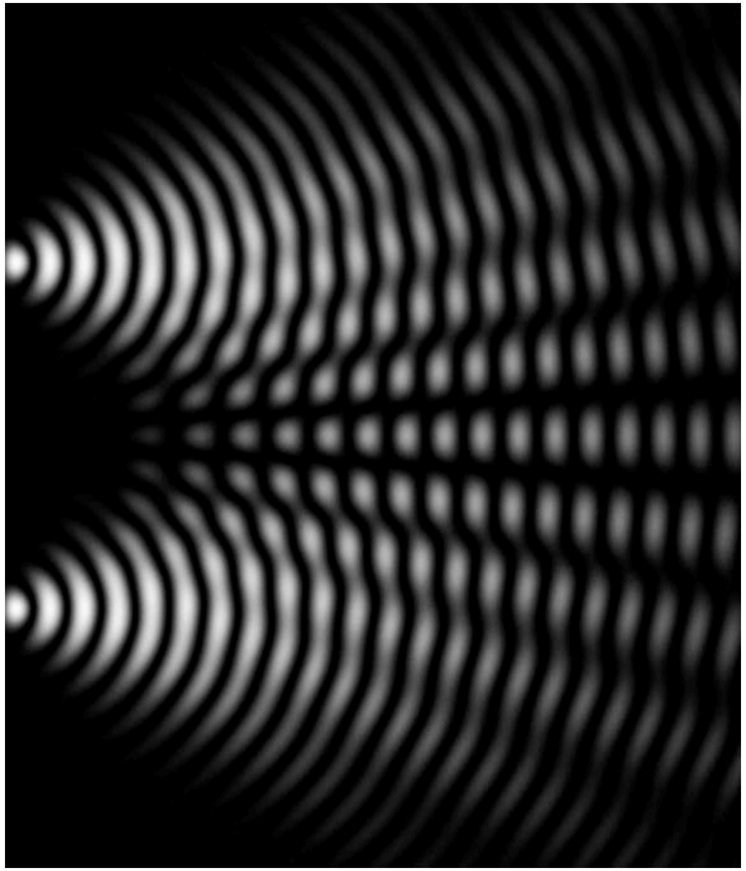
The Double Slit Experiment

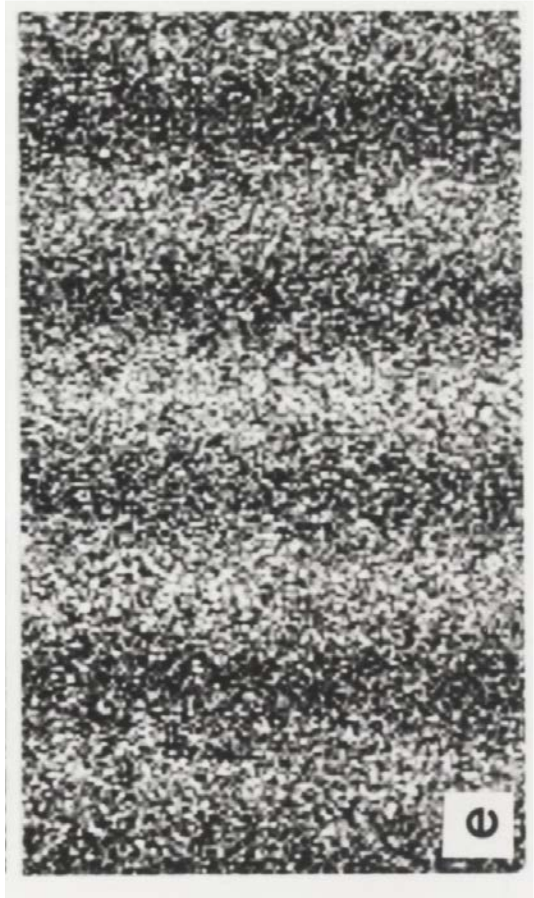




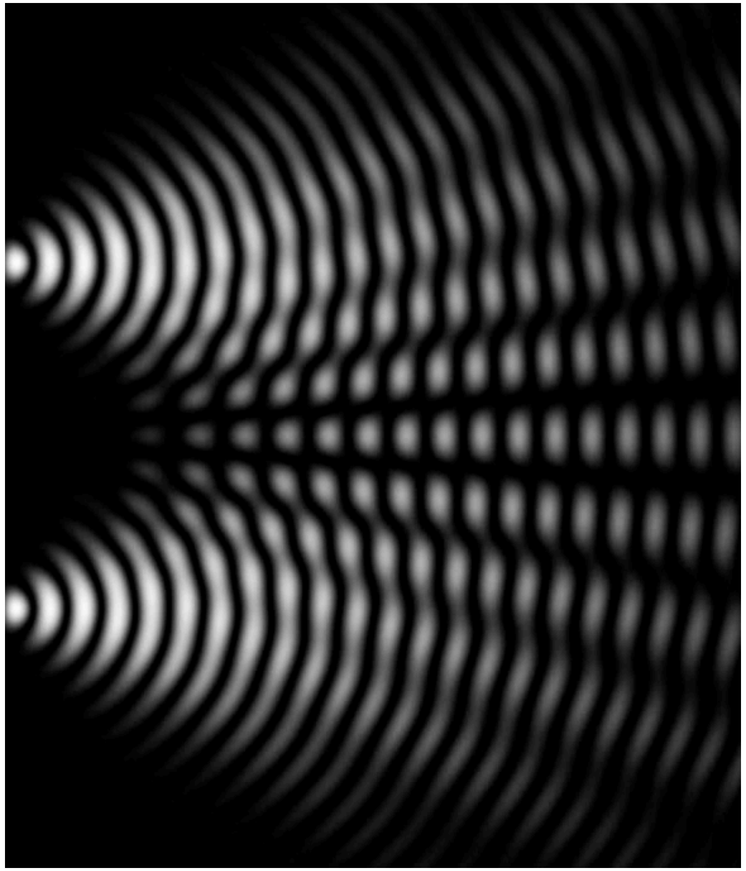
Tonomura et. al., Am. J. Physics 57, 1989

The Double Slit Experiment

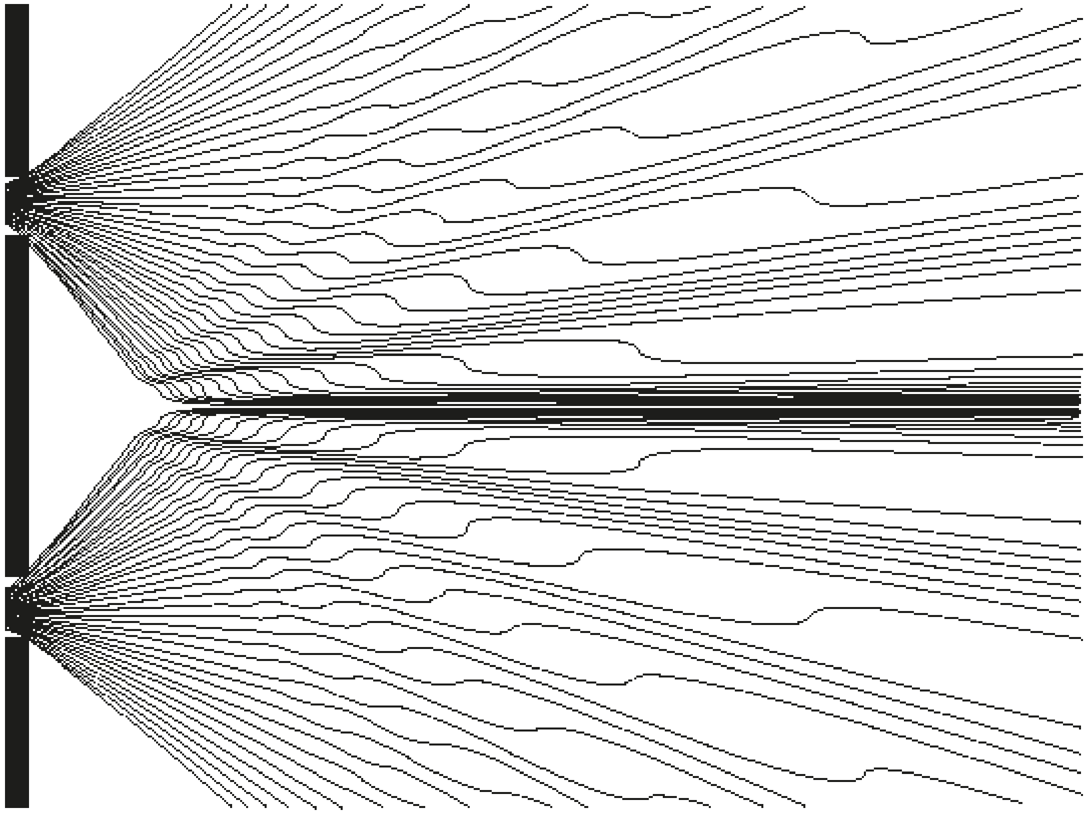




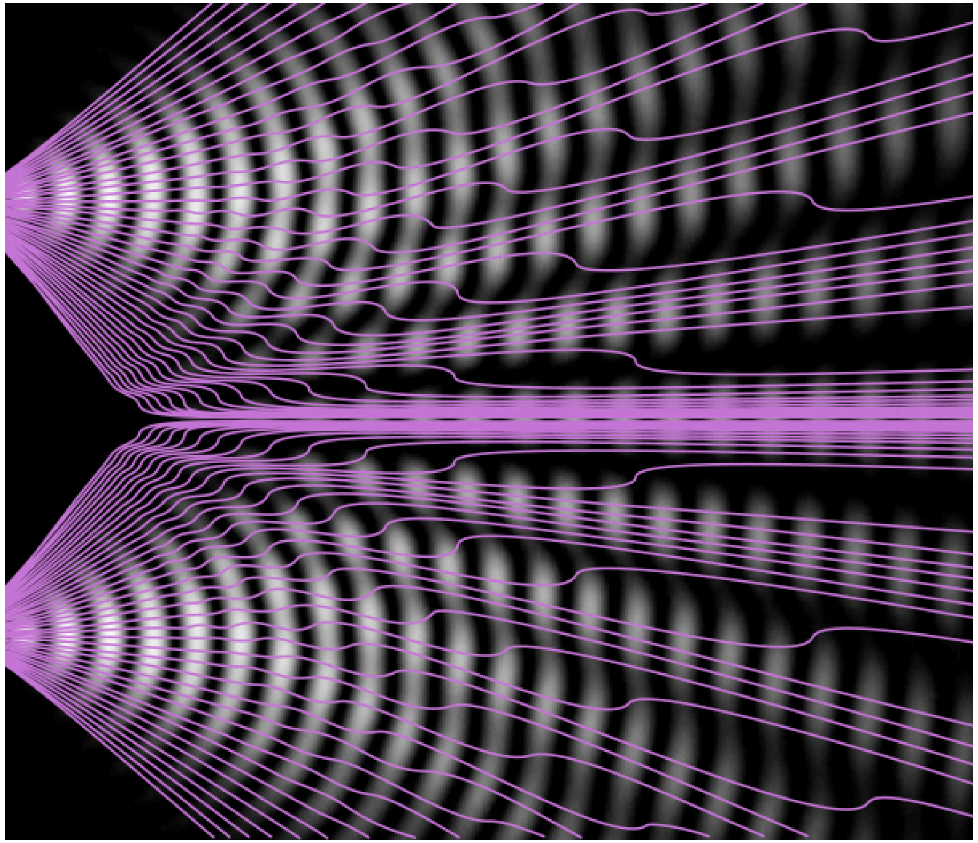
Tonomura et. al., Am. J. Physics 57, 1989

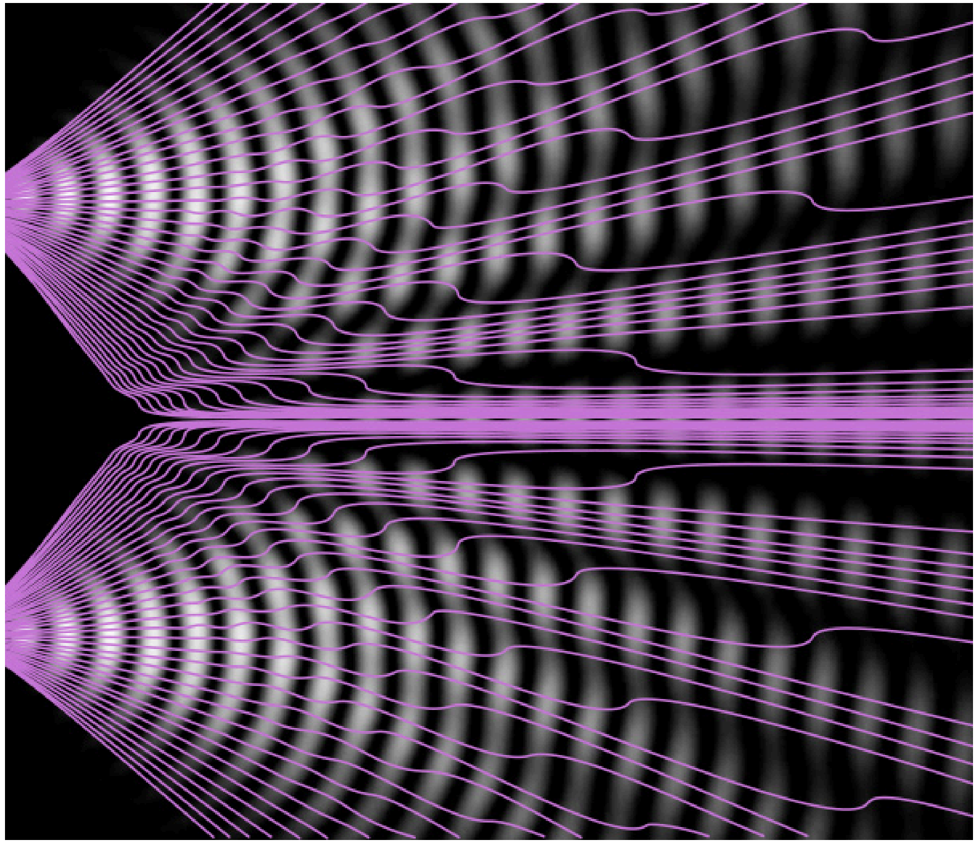


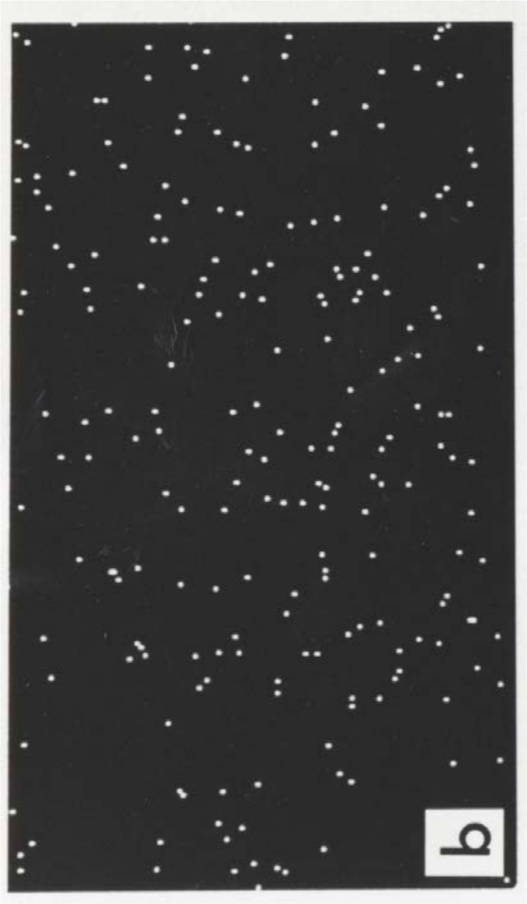
Diffraction of a wave
through a double slit

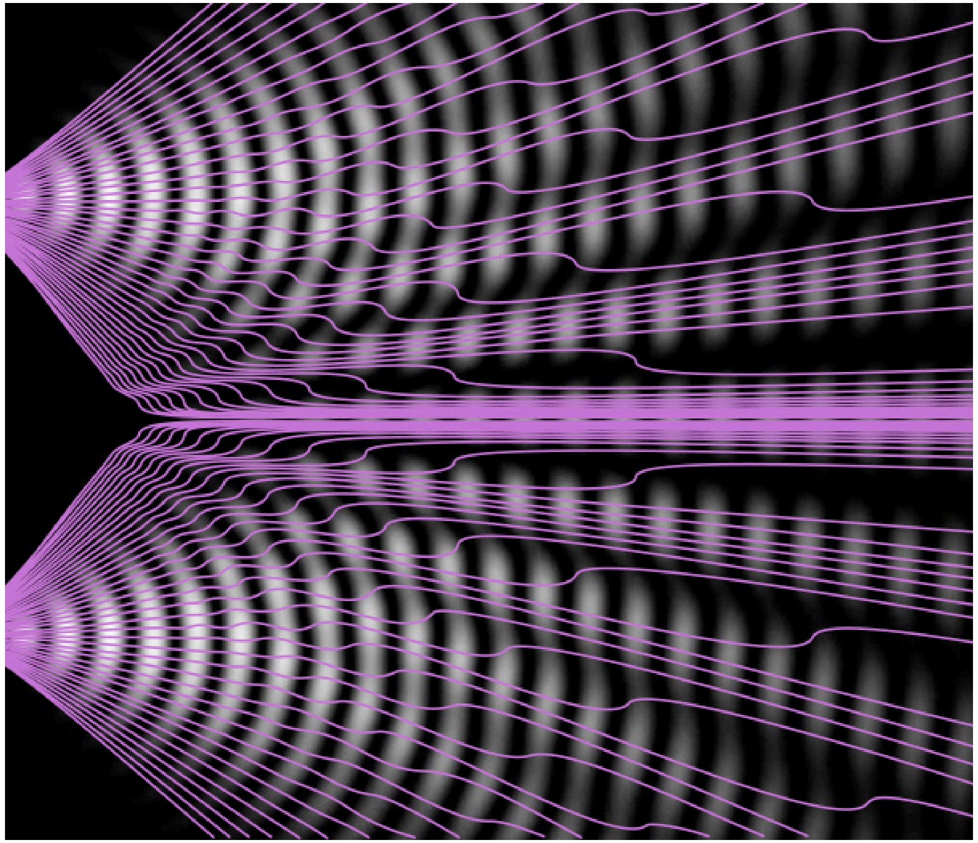


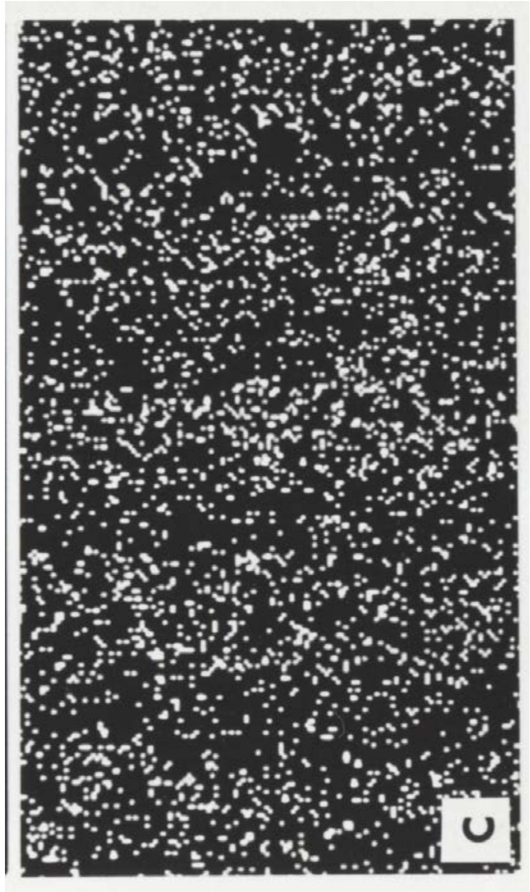
Bohmian trajectories on a double slit
for different initial positions

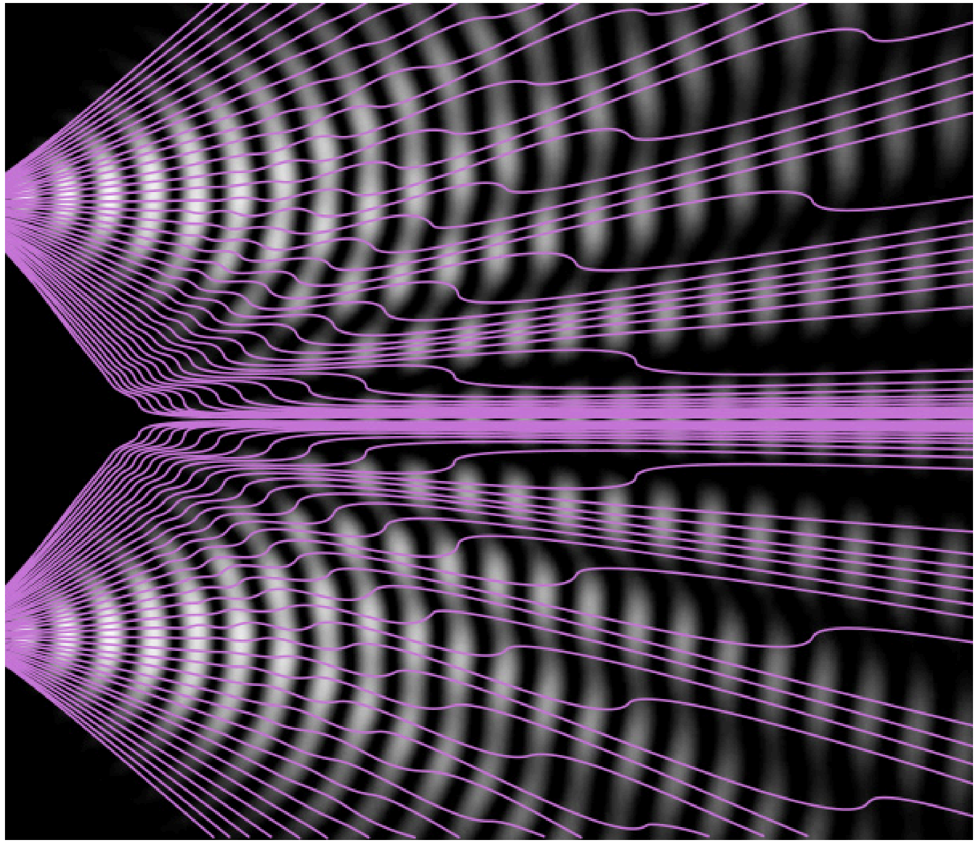


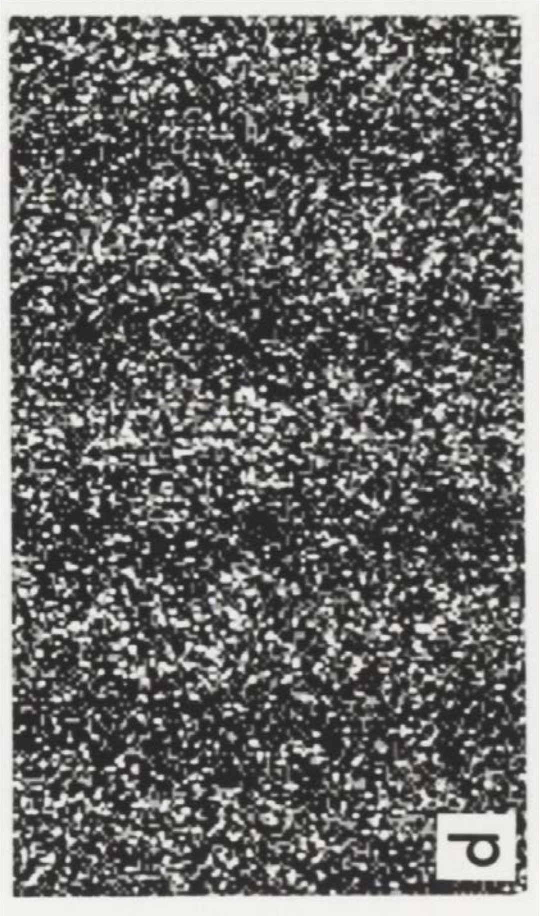


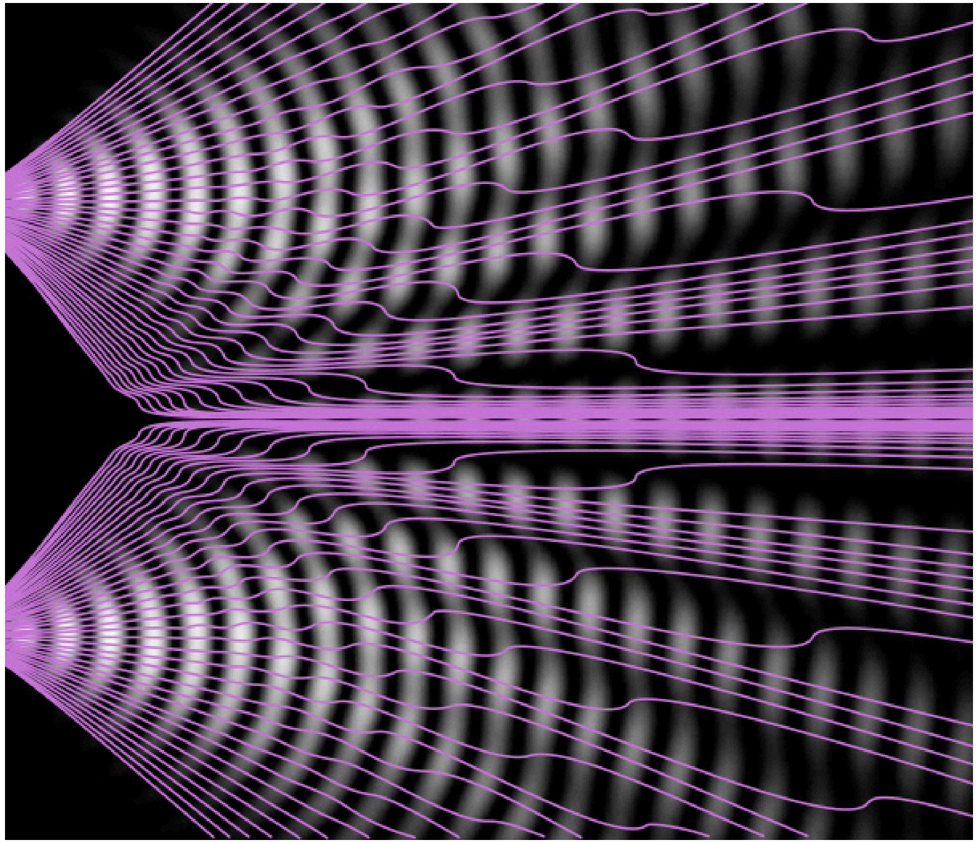


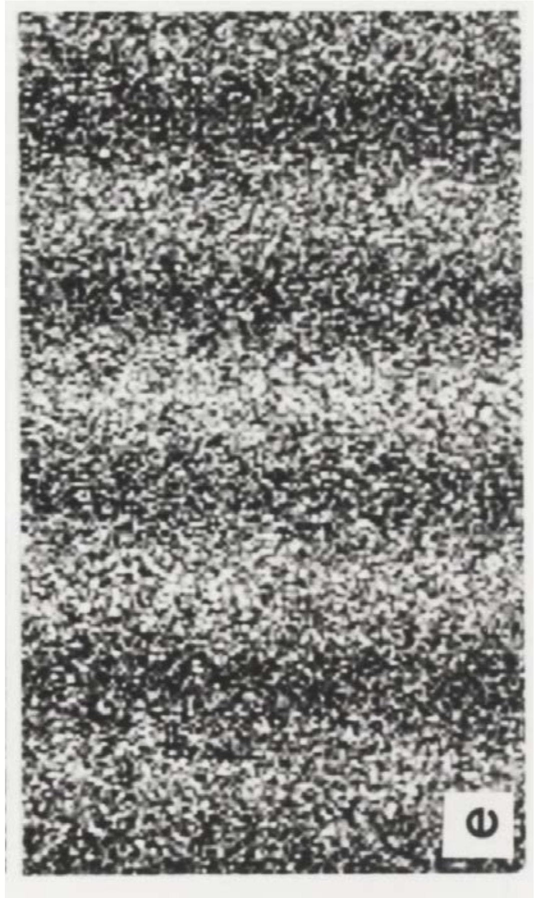




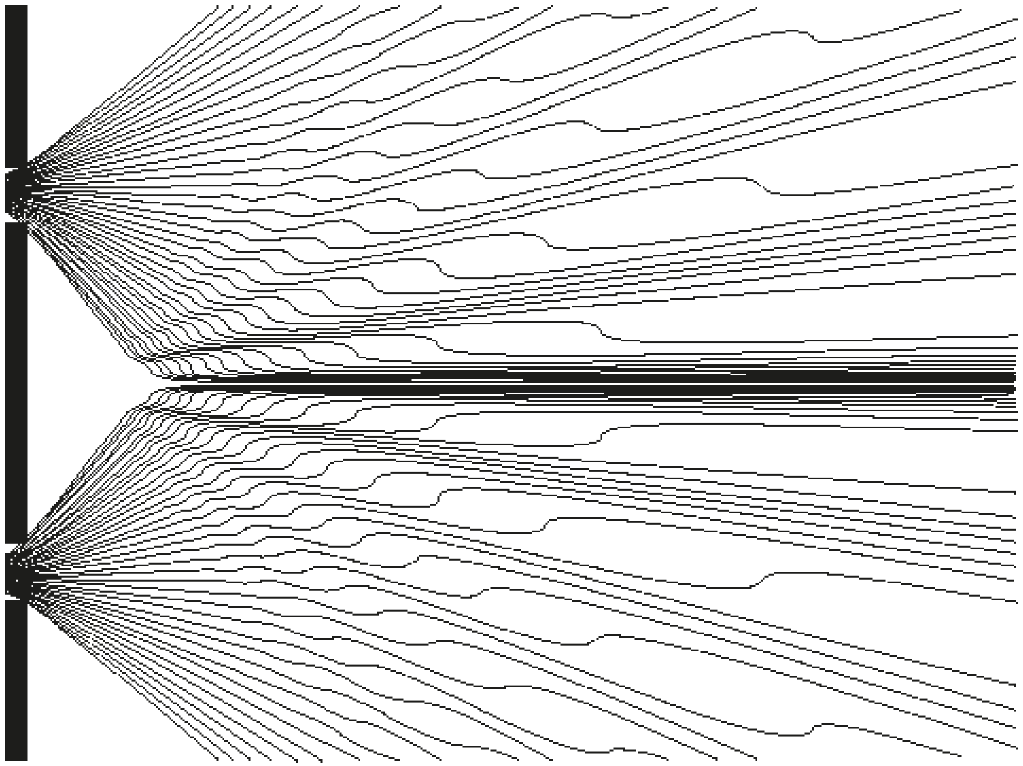


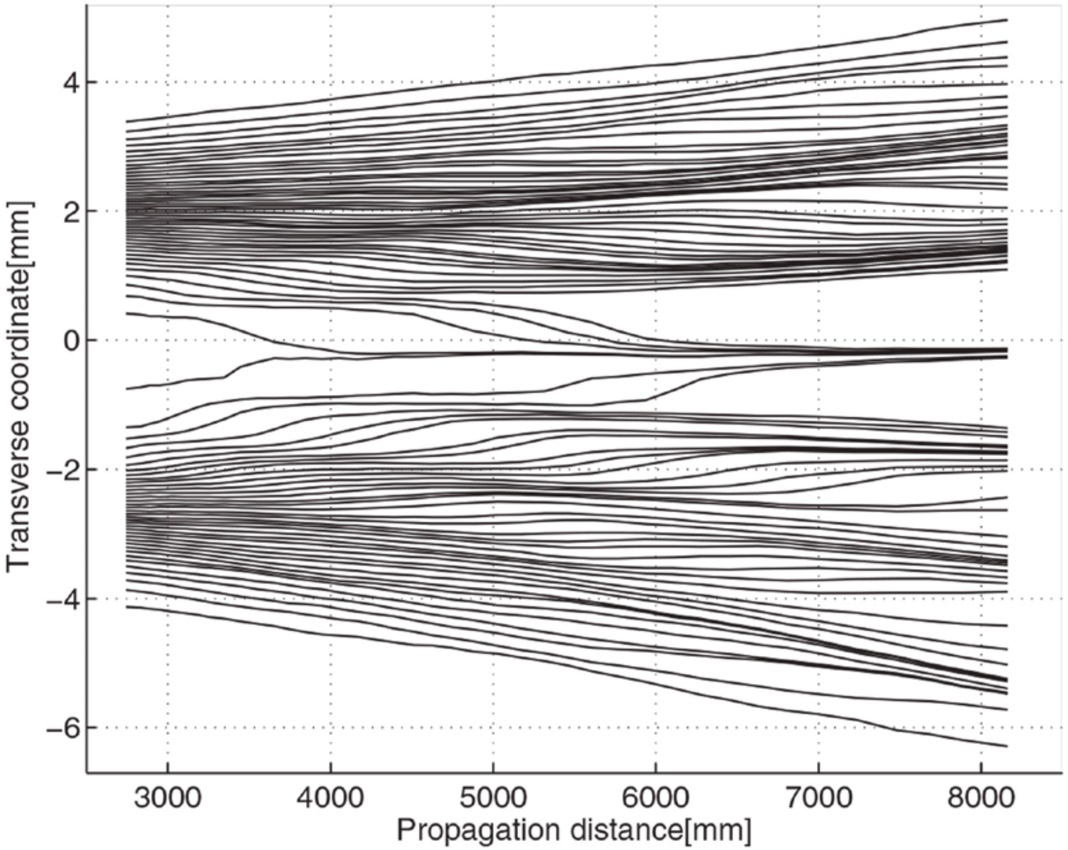





e

Weak measurements of photon trajectories at the double slit





S.Kocsis, B.Braverman, S.Ravets, M.J.Stevens, R.P.Mirin, L.K. Shalm and A.M. Steinberg. Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer. Science, 332:1170-1173, 2011.

The wave function of a subsystem

- Bohmian mechanics is, fundamentally, a theory about the universe as a whole
- The formulation of the theory *contains only one wave function*, the universal wave function

$$\Psi : \mathbb{R}^{3N} \to \mathbb{C}^{d}$$

- To describe a subsystem of the universe, split the particle coordinates into

$$q = (x, y) \in \mathbb{R}^{3n} \times \mathbb{R}^{3m}$$

where x are the degrees of freedom of the subsystem

and y the configurations of the rest of the universe, i.e. the environment.

- Analogously, the actual configuration of the universe is split into

$$Q(t) = \Big(\underbrace{X(t)}_{\text{subsystem}} \, , \, \underbrace{Y(t)}_{\text{rest of universe}} \Big)$$

The wave function of a subsystem

Now we can define the **conditional wave function** of the subsystem as

$$\Phi_t^Y(x) \;\coloneqq \Psi_t(\mathrm{x}, \mathrm{Y}(\mathrm{t}))$$

It generates the guiding field for the subsystem according to

$$\dot{X} = \frac{\hbar}{m}\; Im \frac{\nabla_k \Phi^Y}{\Phi^Y}(X)$$

The conditional wave function is the Bohmian analogon to the usual wave functions in QM:

- If the system is isolated, Φ^Y solves an independent Schrödinger equation
- If the system interacts with a macroscopic environment, Φ^Y collapses
- Statistical predictions for the subsystems are formulated in terms of Φ^Y

Effective collapse

Suppose that the universal wave function has the form

$$\Psi(\mathrm{x},\mathrm{y}) = \varphi(x)\psi(y) + \Psi^{\perp}(x,y)$$

where ψ and $\Psi^{\perp}$ have disjoint y-support and $Y(t) \in supp\ \psi$.

Then the conditional wave function becomes an **effective wave function**

$$\Phi_t^Y(x) \ := \Psi_t\big(\mathrm{x},\mathrm{Y}(\mathrm{t})\big) = \ \varphi_t(x)\psi_t(Y(t)) + \cancel{\Psi_t^{\perp}(x,Y(t))}$$

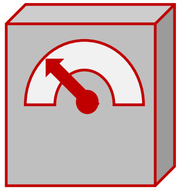
Or, after normalization

$$\Phi_t^Y(x) \ := \varphi_t(x)$$

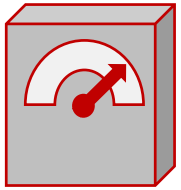
E.g. after spin measurement

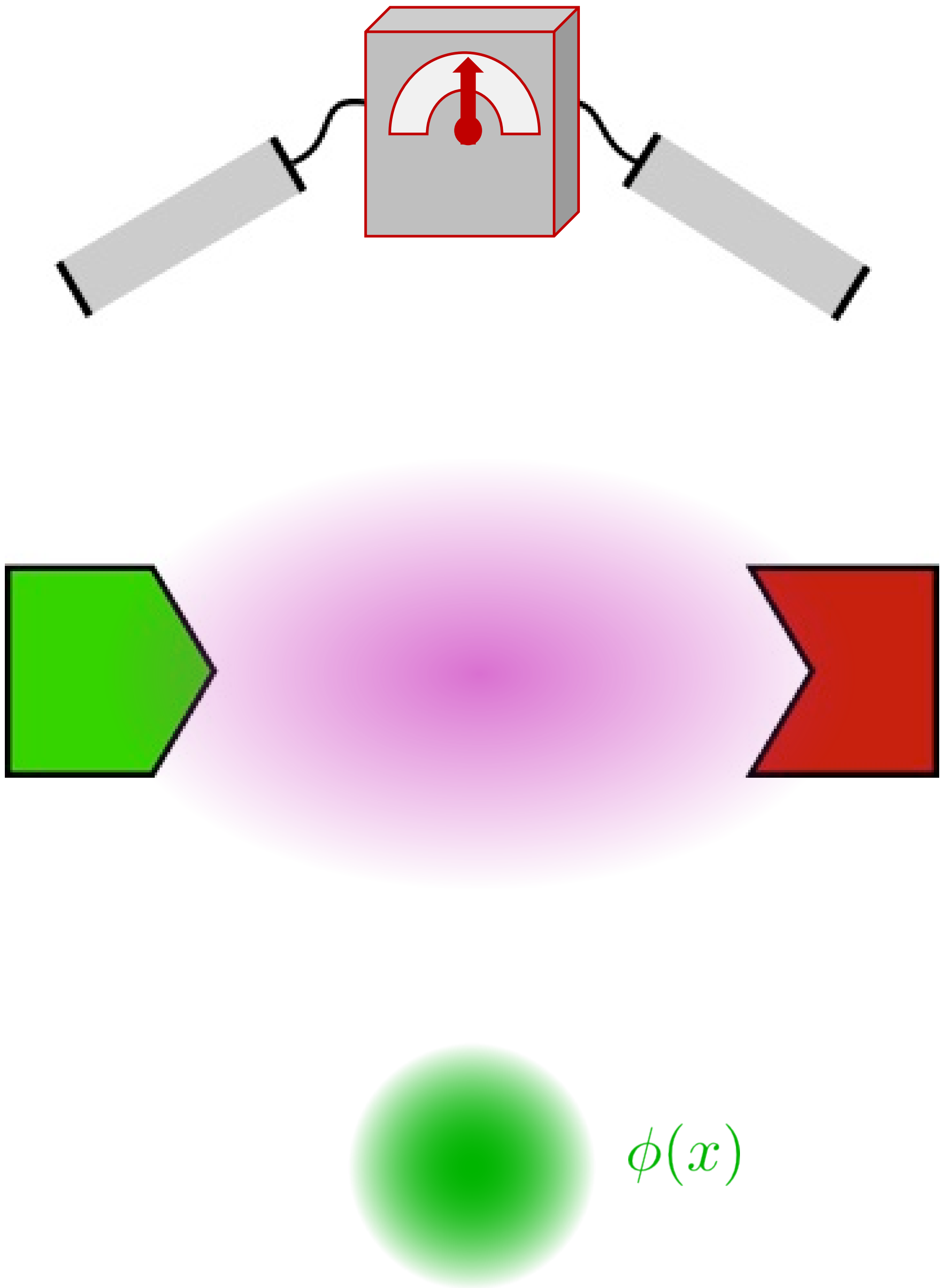
$$(\varphi_1+\varphi_2)\psi_0 \rightarrow \ \varphi_1\psi_1 + \varphi_2\psi_2$$

If $Y(t) \in supp\ \psi_1$, the effective wave-function „collapses“ to φ_1

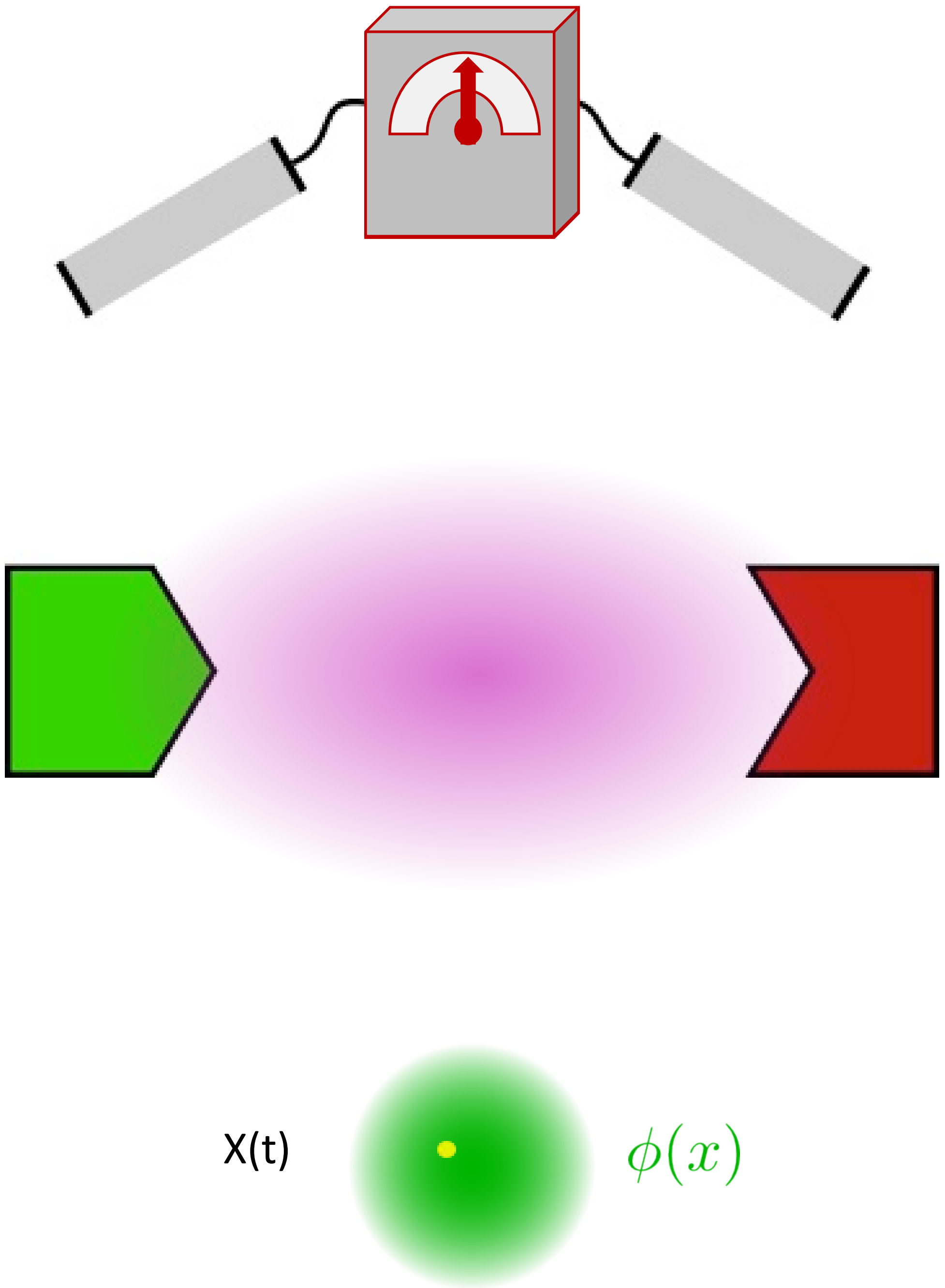


If $Y(t) \in supp\ \psi_2$, the effective wave-function „collapses“ to φ_2

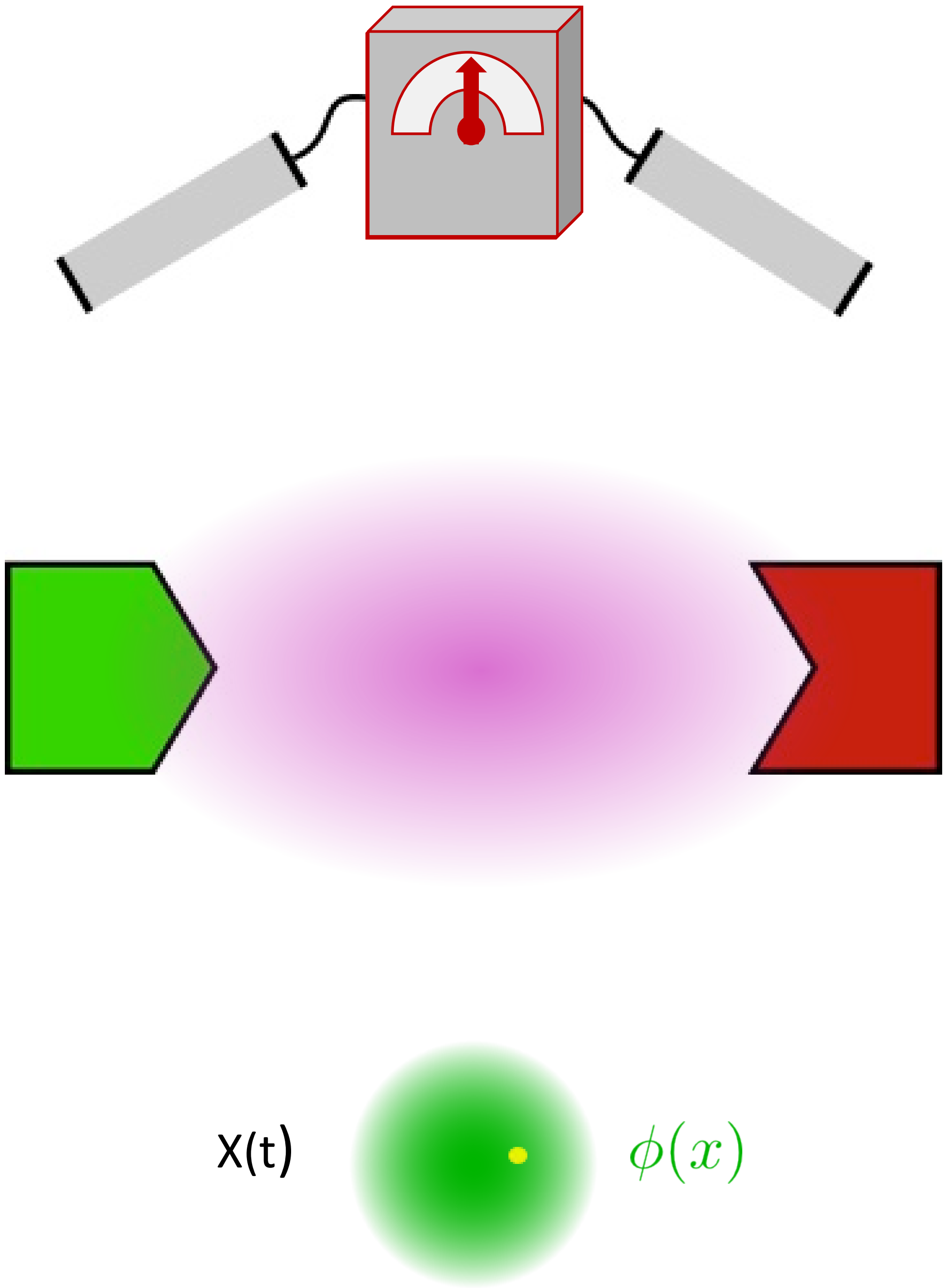




Animation: Stefan Teufel

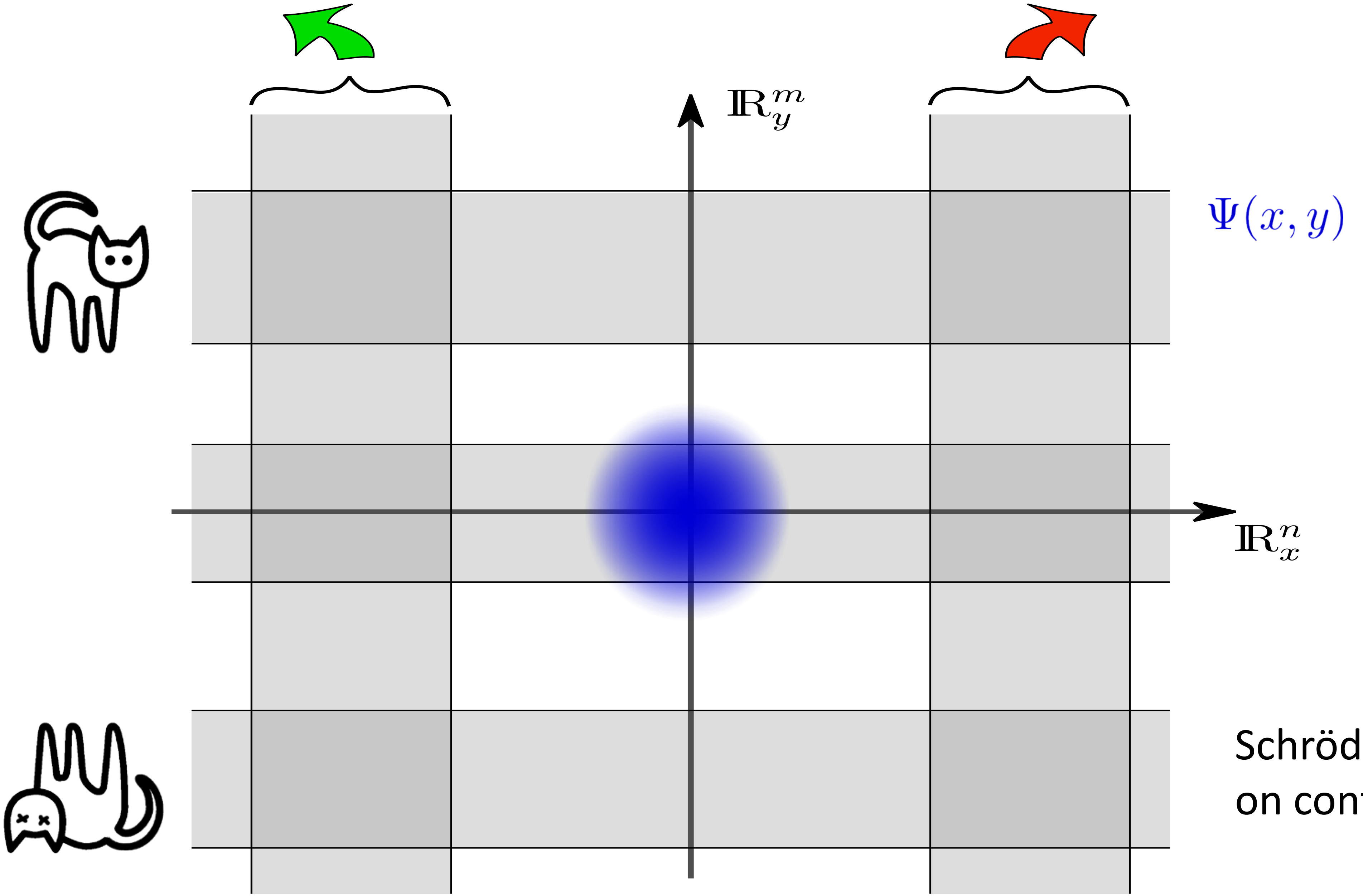


Animation: Stefan Teufel



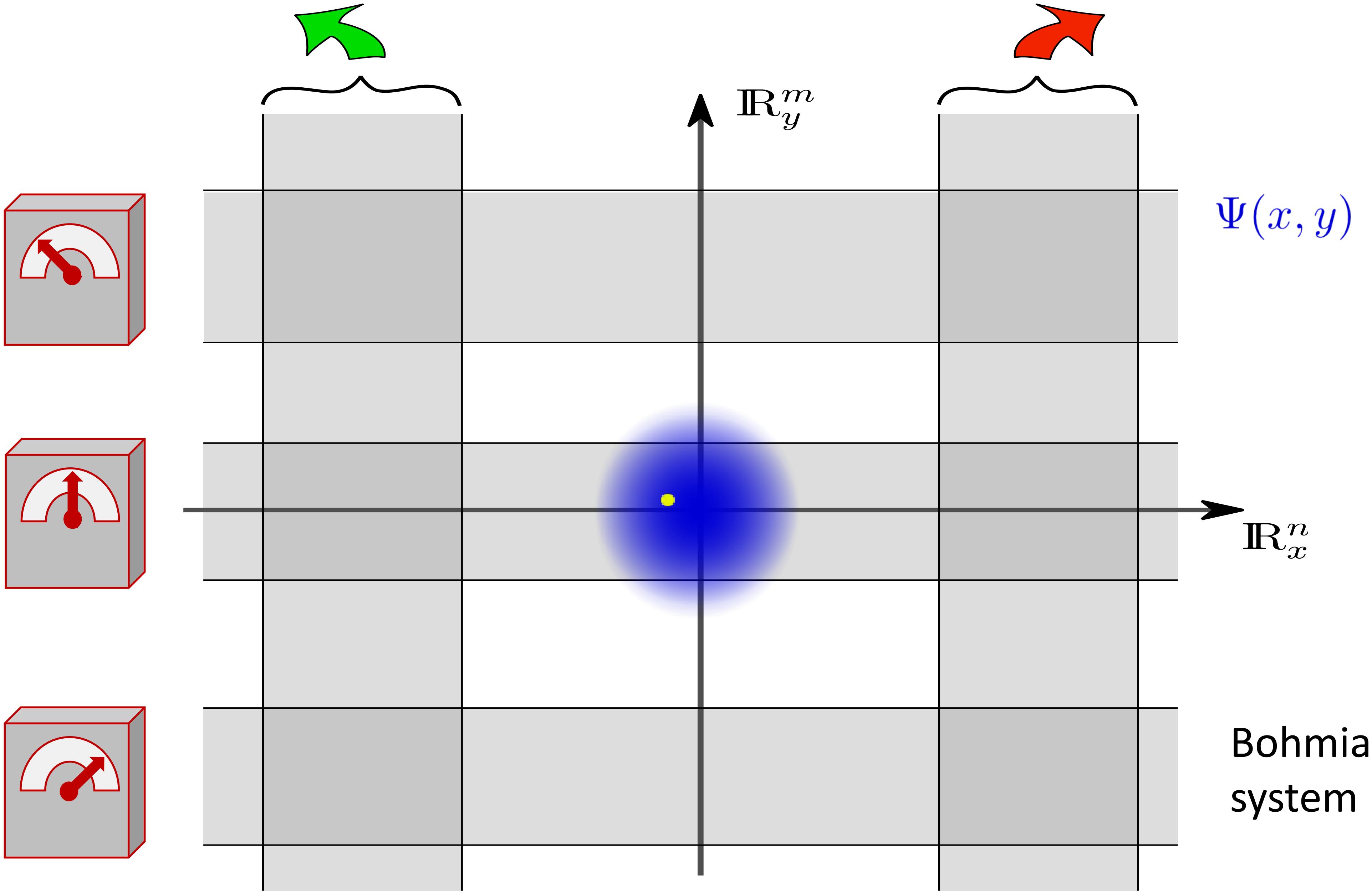
Animation: Stefan Teufel

Animation: Stefan Teufel



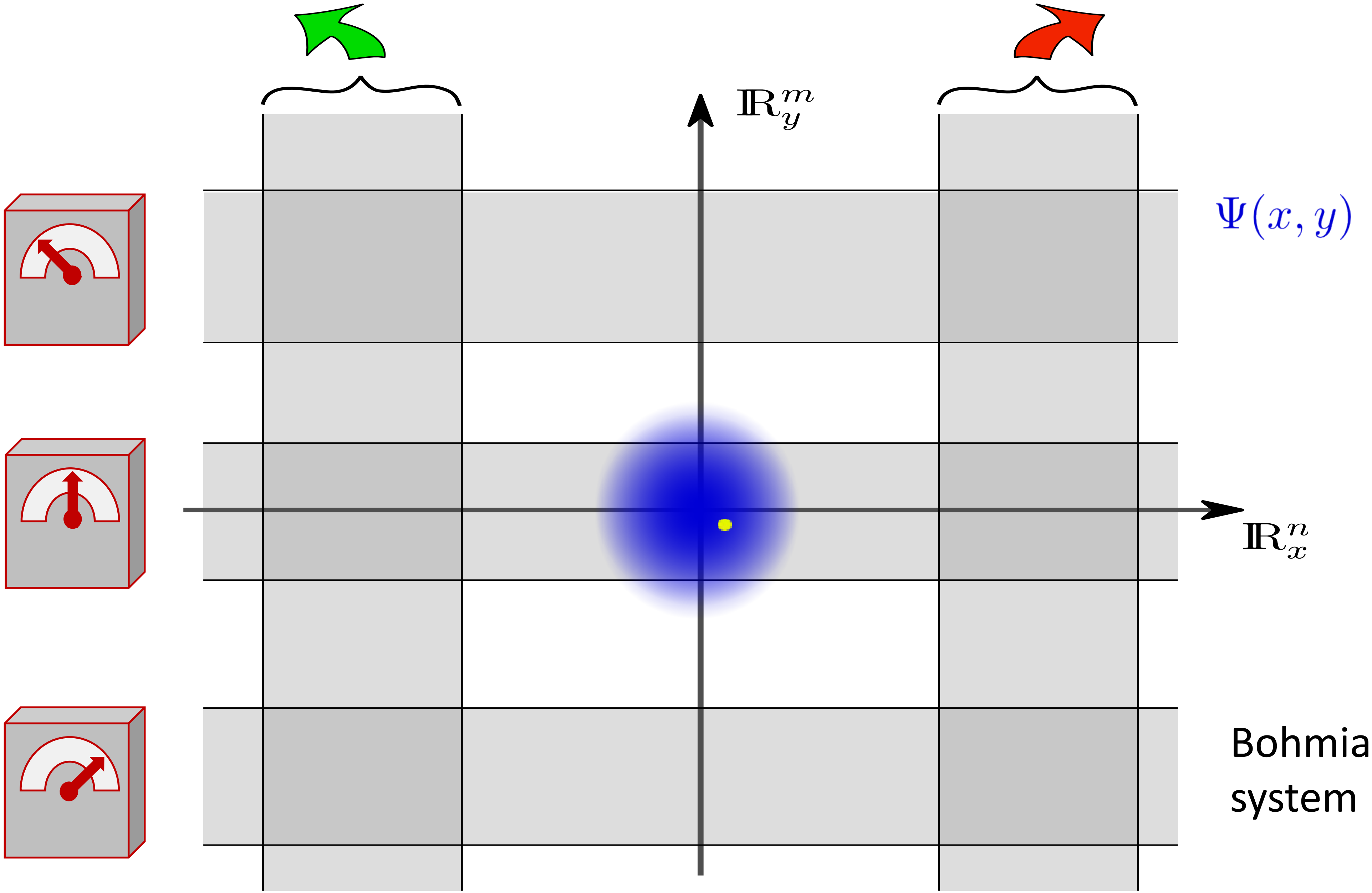
Schrödinger evolution
on configuration space

Animation: Stefan Teufel



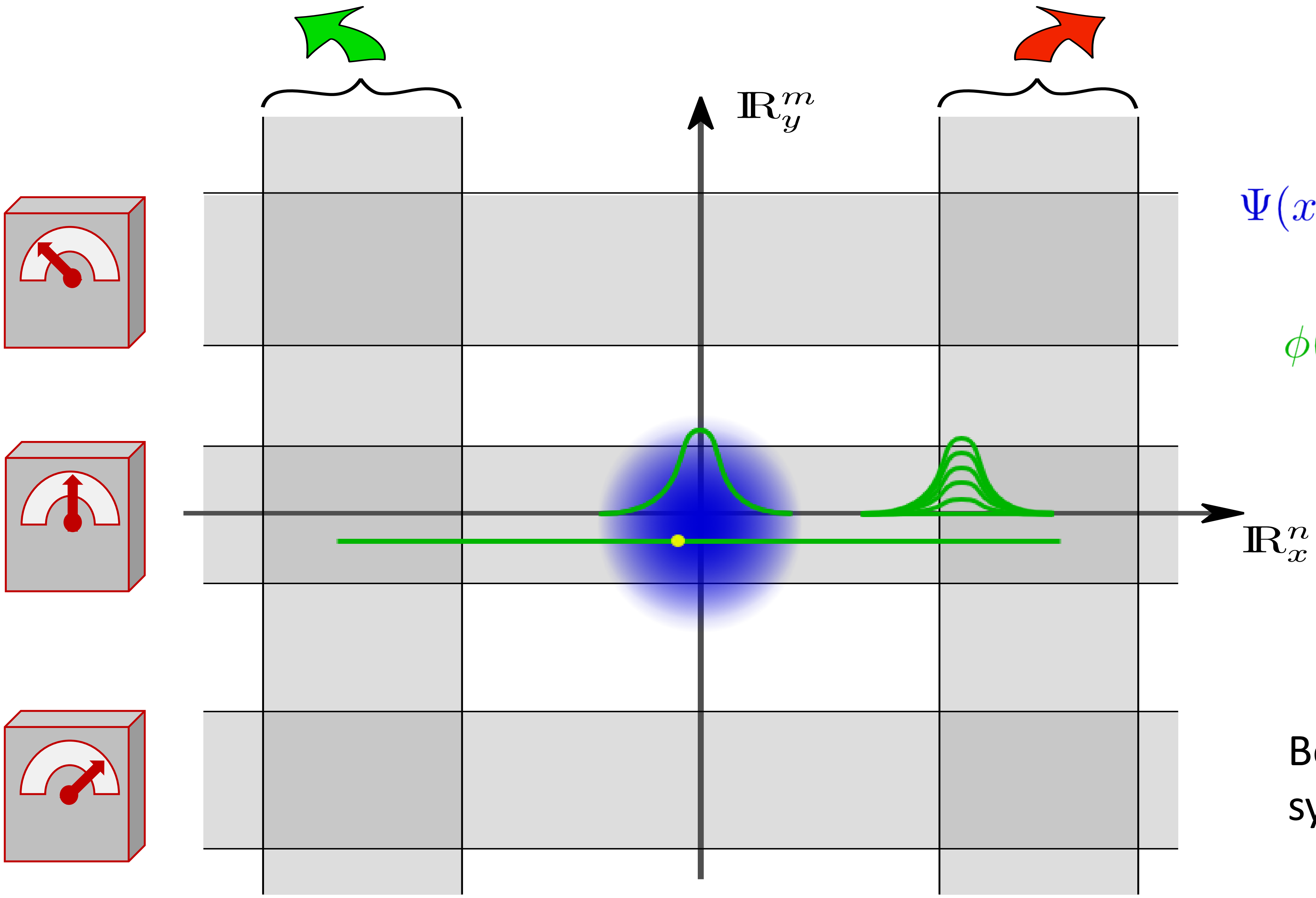
Bohmian mechanics for system plus apparatus

Animation: Stefan Teufel



Bohmian mechanics for system plus apparatus

Animation: Stefan Teufel

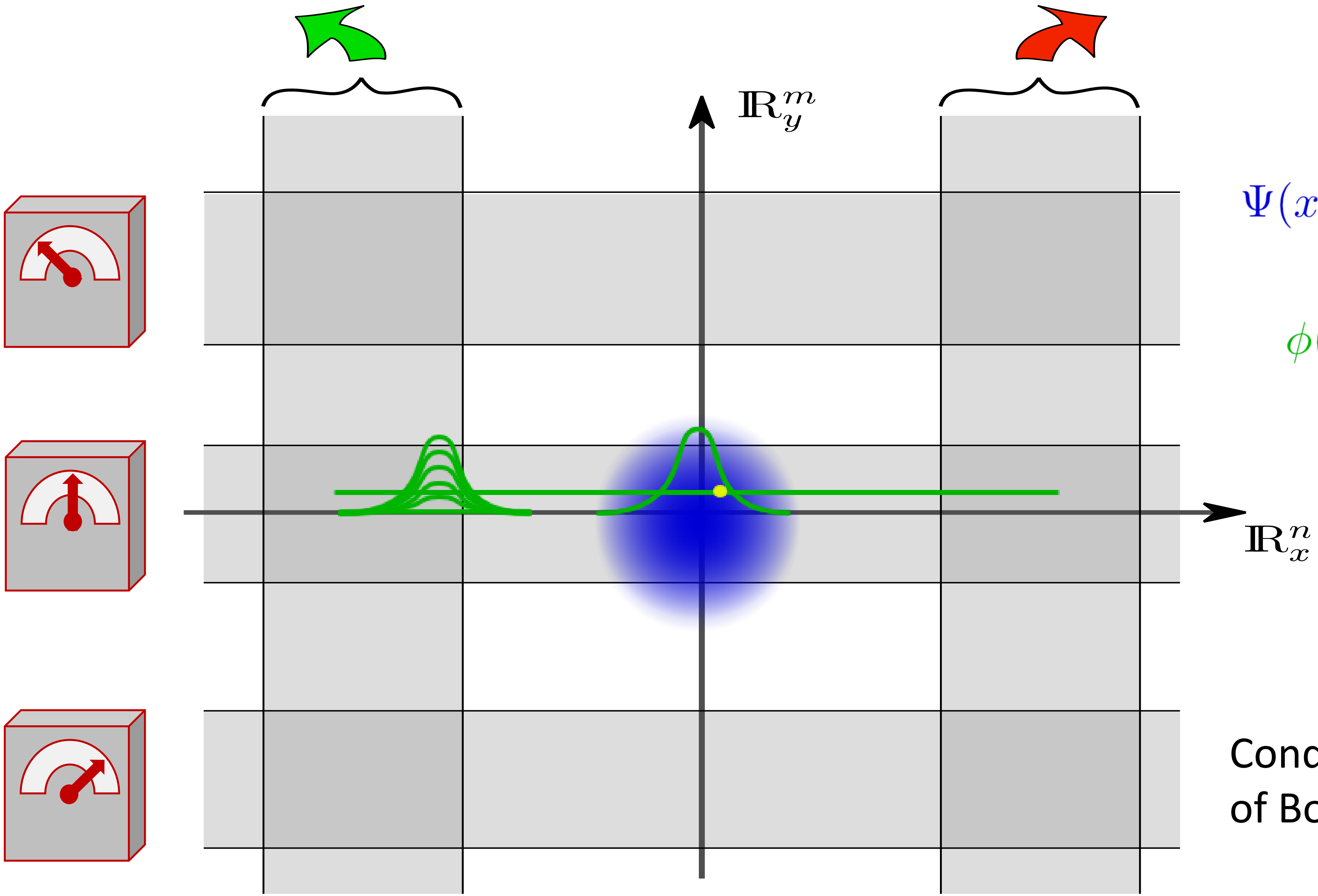


$$\Psi(x,y) = \phi_1(x)\psi_1(y) + \phi_2(x)\psi_2(y)$$

$$\phi(x) = \Psi(x, Y(t))$$

Bohmian mechanics for system plus apparatus

Animation: Stefan Teufel



$$\Psi(x,y) = \phi_1(x)\psi_1(y) + \phi_2(x)\psi_2(y)$$

$$\phi(x) = \Psi(x, Y(t))$$

Conditional wave function of Bohmian mechanics

Other results in Bohmian mechanics

- Operators and Observables
- Probabilities and Absolute Uncertainty
- Identical Particles
- Classical Limit
- Scattering theory
- Arrival time measurements
- …