QUANTUM CHROMODYNAMICS

Program

A. Basics of QCD:
   color d.o.f. of quarks
   non-Abelian field theory of quarks and gluons
   asymptotic freedom

B. QCD @ short distances:
   nucleon structure functions
   e^+e^- annihilation into hadrons
   Drell–Yan processes
   jet physics in e^+e^- annihilation and hadron-hadron scattering
   quarkonium physics

C. Effective Field Theories:
   Fermi Theory
   Heavy Top Quark
   SMEFT

Literature

1. "Foundations of Quantum Chromodynamics", T. Muta, World Scientific, ISBN 981-02-2674-8

2. "QCD and Collider Physics", R.K. Ellis, W.J. Stirling, B.R. Webber, Cambridge University Press, ISBN 0521-581893

3. "Quantum Chromodynamics", G. Dissertori, I. Knowles, M. Schmeling, Oxford University Press, ISBN 0-198-50572-8

4. "Gauge Theories of the Strong and Electroweak Interactions", M.Böhm, A. Denner, H. Joos, B.G. Teubner, ISBN 3-519-23045-3

5. "Gauge Theories of the Strong, Weak and Electromagnetic Interactions", C. Quigg, Benjamin-Cummings, ISBN 0-8053-6020-4

6. "Gauge Field Theories", P.H. Frampton, Benjamin-Cummings, ISBN 0-8053-2584-0

7. "Gauge Theory of Elementary Particle Physics", T.-P. Cheng, L.-F. Li, Oxford University Press, 0-19-851961-3

§1. Introduction of Color

historical definition of strong interactions:

- binding force of nucleons inside nucleus
- force in nucleon-nucleon scattering

Spin-statistics problem of the quark model

$\Delta^{++}\left(s_z = \frac{3}{2}\right) = u(\uparrow)u(\uparrow)u(\uparrow) \leftarrow$ totally symm. spin wave funct.

$\in$ decuplet $\uparrow$ Fermi statistics: totally antisymm. wave function

(i) ground state: P-waves $\lightning$ to naive/exp. experience

(ii) magnetic moments of nucleons: $\vec{\mu} = \frac{eQ}{2m}\left[\vec{\ell} + 2\vec{S}\right]$

nucleon moments are built up additively from quark moments

ratio: $\boxed{\frac{\mu_p}{\mu_n} = -\frac{3}{2} \qquad \text{exp} = -1.46}$

Solution: quarks carry 3-valued differentiator so that symmetric quark model possible

I. Color Hypothesis (Greenberg '64)
Next to flavor charges quarks carry color charges; each quark appears in exactly 3 colors (red, blue, green = 1,2,3): $q = (q_1, q_2, q_3)$

color trafos: max. mixing group of the 3 color d.o.f. ($\neq$ comm. phase)

$$q \to q' = e^{-i\sum_{k=1}^{8}\alpha_k \frac{\lambda_k}{2}}\; q \leftarrow SU(3)_{\mathcal{C}} \text{ transformations}$$
$$= \text{unimodular, unitary } 3\times 3 \text{ matrices [non-Abelian group]}$$

<u>Gell-Mann matrices:</u> $\lambda_k \quad k = 1, 2, \ldots, 8$ (3-dim. ext. of $\vec{\sigma}$ in $SU(2)$)

$\lambda_k^\dagger = \lambda_k \quad \Rightarrow \quad e^{-i\alpha_k \frac{\lambda_k}{2}}$ unitary: $U^\dagger U = 1$

$Tr\lambda_k = 0 \quad \Rightarrow \quad$ unimodular: Det $U = +1$

<u>explicit representation:</u>

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

<u>properties:</u> $T^k = \dfrac{\lambda_k}{2}$

$$\begin{aligned} [T^a, T^b] &= i f_{abc} T^c \quad [A_2 \text{ algebra}] \\ \{T^a, T^b\} &= \frac{1}{3}\delta_{ab} 1 + d_{abc} T^c \\ Tr(T^a T^b) &= \frac{1}{2}\delta_{ab} \qquad Tr(T^a) = 0 \end{aligned}$$

I'. Color Hypothesis (Gell-Mann '72)

The $SU(3)_{\mathcal{C}}$ symmetry is exact. All physical (free) states, observables and int. are $SU(3)_{\mathcal{C}}$ singlets.

(a) quarks as color triplets do not appear as free particles.

(b) color wave functions:

$$\left.\begin{array}{l} \text{baryon: } \frac{1}{\sqrt{6}}\epsilon_{ijk} \\ \text{meson: } \frac{1}{\sqrt{3}}\delta_{ij} \end{array}\right\} \epsilon_{ijk}, \delta_{ij}\ SU(3)_{\mathcal{C}} \text{ singlets}$$

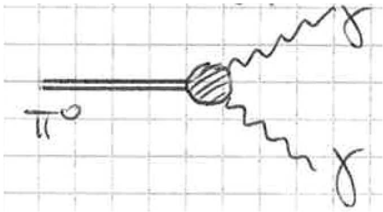
$$\begin{aligned} \text{Ex.: } \Delta^{++}\left(s_z = \frac{3}{2}\right) &= \frac{1}{\sqrt{6}}\epsilon_{ijk} u_i(\uparrow) u_j(\uparrow) u_k(\uparrow) \\ \Phi(s_z = +1) &= \frac{1}{\sqrt{3}}\delta_{ij} s_i(\uparrow)\bar{s}_j(\uparrow) \end{aligned}$$

(c) elm. int.: $\mathcal{L}_{elm} = -e j^{\mu} A_{\mu}$

$$j_{\mu} = \sum_{fl} \bar{q}\gamma_{\mu} Q_q q \equiv \sum_{fl}\sum_{c} \bar{q}_c \gamma_{\mu} Q_q q_c \quad \Rightarrow SU(3)_{\mathcal{C}} \text{ singlet}$$

TESTS OF THE COLOR HYPOTHESIS

1.) $\pi^0 \to \gamma\gamma$ decay



$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2}{32\pi^3}\frac{m_\pi^3}{f_\pi^2}(Q_u^2 - Q_d^2)^2 N_{\mathcal{C}}^2$$

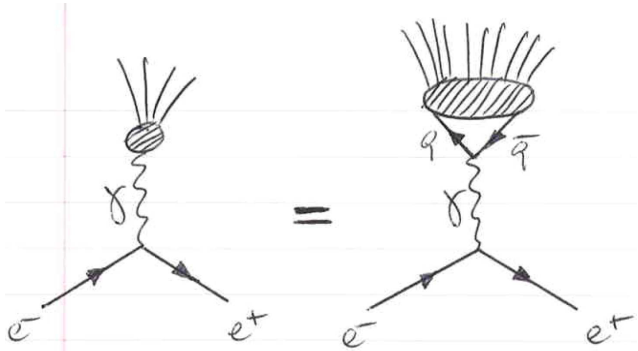
w/o color $N_{\mathcal{C}} = 1$: $\Gamma = 0.868 \pm 0.065$ eV

w/ color $N_{\mathcal{C}} = 3$: $\Gamma = 7.81 \pm 0.60$ eV $\leftarrow$

experimental: $\Gamma_{exp} = 7.84 \pm 0.56$ eV $\leftarrow$

2.) $e^+e^- \to$ hadrons

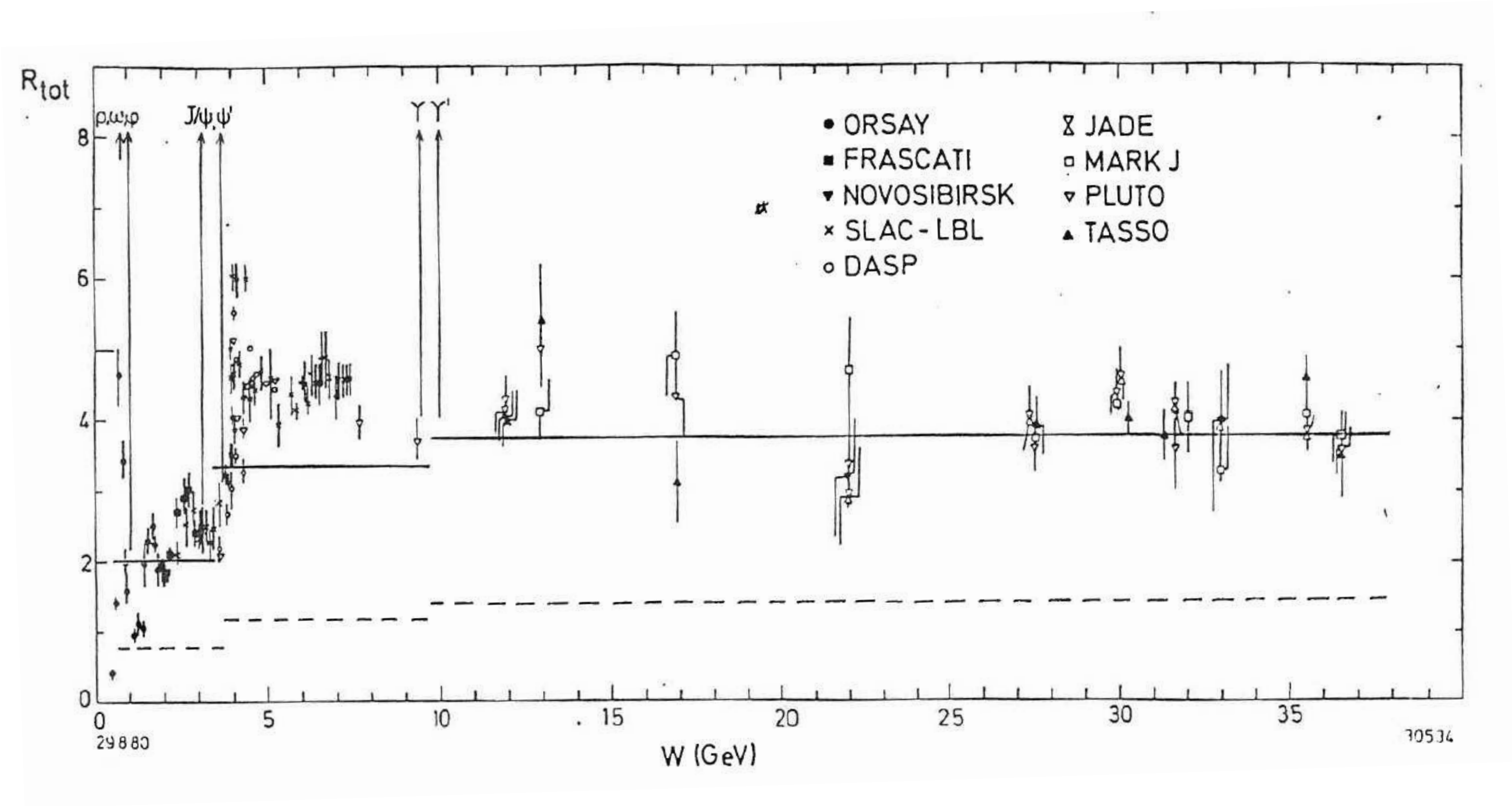
In the quark-parton model the production probability in $e^+e^- \to$ hadrons determined by the one for $q\bar{q}$ pairs; final-state int. negligible for $\frac{d_{prod\ q\bar{q}}}{d_{hadron}} \sim \frac{1\ \text{GeV}}{E} \to 0 \quad (E \to \infty)$.



$$R = \frac{\sigma(e^+e^- \to \text{had.})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{fl,c} \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_{fl} e_q^2$$

q	e_q
u, c, t	$+\frac{2}{3}$
d, s, b	$-\frac{1}{3}$

energy	prod. q's	R w/o color	R w/ color
< 3 GeV	u, d, s	$\frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$	2
> 5 GeV	$+c$	$\frac{6}{9} + \frac{4}{9} = \frac{10}{9}$	$\frac{10}{3}$
> 10 GeV	$+b$	$\frac{10}{9} + \frac{1}{9} = \frac{11}{9}$	$\frac{11}{3}$



§2. Gluon Gauge Fields

> II. Color Hypothesis (Nambu '66, Fritzsch+Gell-Mann '72, Leutwyler '73)
> Color charges are sources of gauge fields ($\Rightarrow$ gluons)
> that build up the strong interaction between quarks.

Lagrangian for color triplet:

$$\mathcal{L}_q = \bar{q}(x)(i\not{D} - m_q)q(x) \quad \text{with } q = (q_1, q_2, q_3)$$

$$m_{q_1} = m_{q_2} = m_{q_3} \; SU(3)_{\mathcal{C}} \text{ singlet}$$

— invariant w.r.t. local, non-Abelian $SU(3)_{\mathcal{C}}$ transformations

$$\left.\begin{array}{l} q(x) \to Sq(x) \\ \bar{q}(x) \to \bar{q}(x)S^{-1} \end{array}\right\} S = e^{-i\alpha_k T^k} \qquad \left(T^k = \frac{\lambda_k}{2}\right)$$

8 minimally coupled gluon fields $G_\mu^k(x)$ $(k = 1, \ldots, 8) \longrightarrow G_\mu = G_\mu^k T^k$

$$i\partial_\mu \to i\partial_\mu - g_s G_\mu = iD_\mu$$

$$G_\mu(x) \to SG_\mu S^{-1} - \frac{i}{g_s} S\partial_\mu S^{-1}$$

ROT. SHIFT

$\Rightarrow$ $D \to D' = SDS^{-1}$ ROTATION

Gluon field Lagrangian:

curl: $G_{\mu\nu} = D_\nu G_\mu - D_\mu G_\nu = \partial_\nu G_\mu - \partial_\mu G_\nu - ig_s[G_\mu, G_\nu]$

gauge trf.: $G_{\mu\nu} \to G'_{\mu\nu} = S G_{\mu\nu} S^{-1}$ pure rotation

from $G_{\mu\nu} = \frac{i}{g_s}[D_\mu, D_\nu]$ [no observable]

$\mathcal{L}_g = -\frac{1}{2} Tr G^2_{\mu\nu} = -\frac{1}{4}(G^k_{\mu\nu})^2$ ← gauge invariant:

↑ no mass term $\left(+\frac{1}{2} m_g^2 Tr G_\mu^2\right)$

consists of: (a) kinetic part $= -\frac{1}{4}(\partial_\nu G^k_\mu - \partial_\mu G^k_\nu)^2$

(b) trilinear coupling $\sim g_s GGG$

(c) quartic coupling $\sim g_s^2 GGGG$

— self-interaction of gluon fields: color-charged gluons are sources of gluons ($\neq \gamma$)

— g_s universal, fixed in gauge sector: color charges quantized

Lagrangian of QCD: $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{GF} + \mathcal{L}_{FPG}$:

$\mathcal{L}_{QCD} = qg$ Lagrangian

$\mathcal{L}_{GF}$ = gauge fixing

$\mathcal{L}_{FPG}$ = ghost Lagrangian

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D} - m_q)q - \frac{1}{2} Tr G^2_{\mu\nu}$$

$$= \bar{q}(i\not{\partial} - m_q)q - \frac{1}{2} Tr(\partial_\nu G_\mu - \partial_\mu G_\nu)^2 \quad \text{kinet. part}$$

$$-g_s \bar{q} \not{G} q \quad q - g \text{ coupling}$$

$$+ig_s Tr(\partial_\nu G_\mu - \partial_\mu G_\nu)[G_\mu, G_\nu] \quad 3g \text{ coupling}$$

$$+\frac{g_s^2}{2} Tr[G_\mu, G_\nu]^2 \quad 4g \text{ coupling}$$

Lorenz gauge:

$$\mathcal{L}_{GF} = -\frac{1}{\xi} Tr(\partial G)^2$$

$$\mathcal{L}_{FPG} = \partial^\mu c^{a*}(\partial_\mu + g_s f_{abc} G^b_\mu) c^c$$

axial gauge:

$$\mathcal{L}_{GF} = -\frac{1}{\xi} Tr(nG)^2$$

for $\xi \to 0$

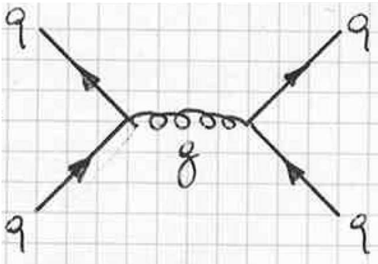
$$\mathcal{L}_{FPG} = 0$$

§3. Asymptotic Freedom

QCD: quark-quark scattering: $\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 + \cdots$

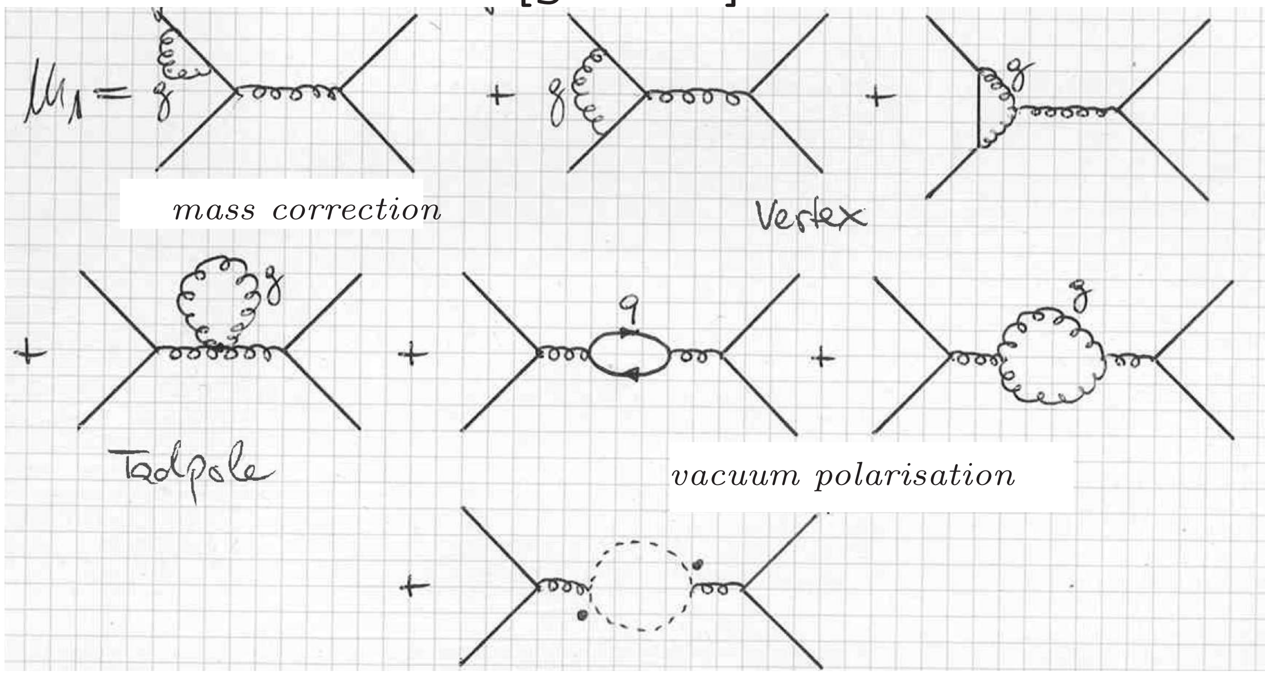
Bornterm:

$\mathcal{M}_0 =$



$= \dfrac{4\pi\alpha_s(\mu^2)}{Q^2} \cdots \quad [Q^2 = -q^2]$

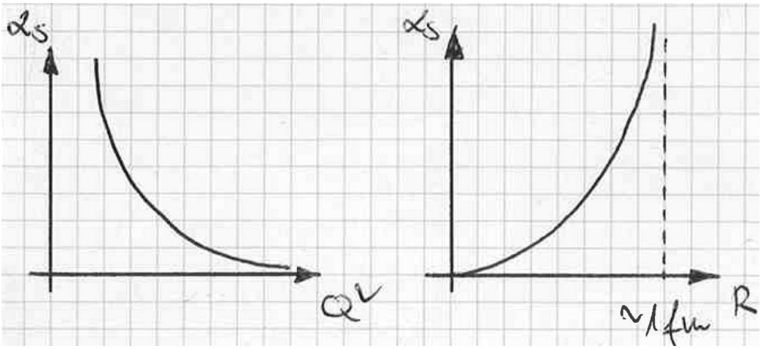
Radiative corrections [generic]:



$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{33-2N_F}{12}\frac{\alpha_s}{\pi}\log\frac{Q^2}{\mu^2}}}$$

With increasing Q^2 the effective color charge vanishes: asymptotic freedom

[non-Abelian $SU(3)$: $N_F \leq 16$] consequence of non-Abelian gauge boson loops; contrary to $U(1)$ [and all other theories] [Politzer '73, Gross & Wilczek '73; ('t Hooft '72)]



~ confinement radius

Scale parameter of QCD: quantum theory introduces a scale into unscaled classical chromodynamics [for $m_q = 0$] via renormalization: introduction of coupling constant at default distance:

$\alpha_s = \alpha_s(\mu^2)$ [← exp. determined]

INTERMEZZO

Reformulation:

$$\frac{1}{\alpha_s(Q^2)} = \underbrace{\frac{1}{\alpha_s(\mu^2)} - \frac{33-2N_F}{12\pi}\log\mu^2}_{\equiv \frac{33-2N_F}{12\pi}\log\frac{1}{\Lambda^2}} + \frac{33-2N_F}{12\pi}\log Q^2$$

$$\Rightarrow \boxed{\boxed{\alpha_s(Q^2) = \frac{12\pi}{(33-2N_F)\log\frac{Q^2}{\Lambda^2}}}}$$

$\Lambda^{-1} \sim 1$ fm $\sim$ conf. rad.

$\Rightarrow$ $\boxed{\Lambda \sim 100 - 300 \text{ MeV}}$

$$\frac{\alpha_s(Q^2)}{\pi} \lesssim 10^{-1} \text{ for } Q^2 \gtrsim 2\text{GeV}^2$$

$\Rightarrow$ region of ensured perturbation theory

Renormalization group equation:

$$\mu^2 \ \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = \beta\left(\alpha_s(\mu^2)\right) \qquad \beta(\alpha_s) = -\beta_0 \frac{\alpha_s^2}{\pi} + \mathcal{O}(\alpha_s^3)$$

Solution: $\log \frac{Q^2}{\mu^2} = \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{d\alpha_s}{\beta(\alpha_s)} = -\frac{\pi}{\beta_0}\left[\frac{1}{\alpha_s(\mu^2)} - \frac{1}{\alpha_s(Q^2)}\right]$

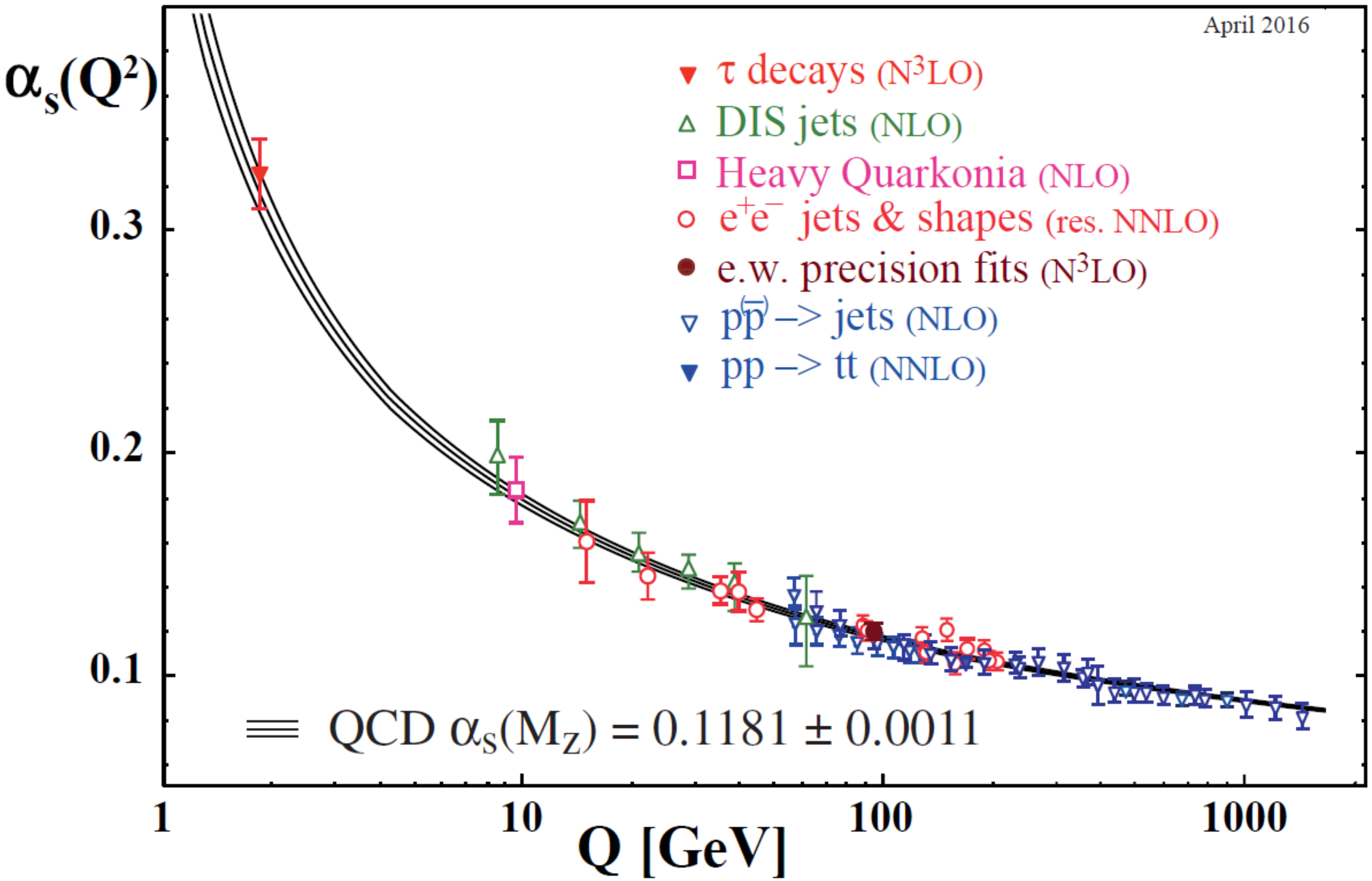
$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_0 \frac{\alpha_s}{\pi} \log \frac{Q^2}{\mu^2}} \qquad \beta_0 = \frac{33 - 2N_F}{12}}$$

RGG det. asymptotic behavior of running cplg.

higher orders: $\beta(\alpha_s) = -\frac{\alpha_s^2}{\pi}\left[\beta_0 + \beta_1 \frac{\alpha_s}{\pi} + \beta_2 \frac{\alpha_s^2}{\pi^2} + \cdots\right]$

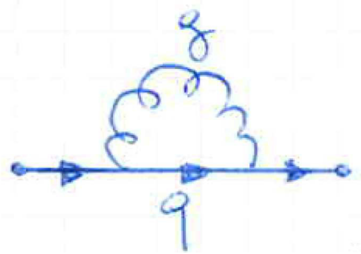
$$\beta_1 = \frac{153 - 19N_F}{24} \qquad \beta_2 = \frac{1}{128}\left[2857 - \frac{5033}{9}N_F + \frac{325}{27}N_F^2\right]$$

$$\alpha_s(Q^2) = \frac{\pi}{\beta_0 \log \frac{Q^2}{\Lambda^2}} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \ \frac{\log \log \frac{Q^2}{\Lambda^2}}{\log \frac{Q^2}{\Lambda^2}} + \cdots \right\}$$


April 2016
$\alpha_s(Q^2)$
τ decays (N³LO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
e⁺e⁻ jets & shapes (res. NNLO)
e.w. precision fits (N³LO)
pp̄ –> jets (NLO)
pp –> tt (NNLO)
0.3
0.2
0.1
QCD $\alpha_s(M_Z)$ = 0.1181 ± 0.0011
1
10
100
1000
Q [GeV]

Quark masses

quark self-energy:



$$\Sigma(\not p = m) = m\ C_F\ \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_0^2}{m^2}\right)^{\epsilon}\left(\frac{3}{4\epsilon}+1\right)$$

$m = m_0 + \Sigma(\not p = m)$ pole mass

$\overline{m}(\mu^2) = m_0 + \delta\overline{m}$ $\overline{\text{MS}}$ mass

$$\delta\overline{m} = m\,C_F\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_0^2}{\mu^2}\right)^{\epsilon}\frac{3}{4\epsilon}\quad \text{[only divergence]}$$

Relation pole mass $\leftrightarrow$ $\overline{\text{MS}}$ mass:

$$\begin{aligned}\overline{m}(\mu^2) &= m - [\Sigma(\not p = m) - \delta\overline{m}] = m\left[1 - C_F\frac{\alpha_s}{\pi}\left(\frac{3}{4}\log\frac{\mu^2}{m^2}+1\right)\right]\\ &= m\left[1 - C_F\frac{\alpha_s}{\pi}\right]\left[1 - \frac{3}{4}C_F\frac{\alpha_s}{\pi}\log\frac{\mu^2}{m^2}\right]\end{aligned}$$

$$\begin{aligned}\overline{m}(m^2) &= m\left[1 - C_F\frac{\alpha_s(m^2)}{\pi}\right]\\ \overline{m}(\mu^2) &= \overline{m}(m^2)\left[1 - \frac{\alpha_s}{\pi}\log\frac{\mu^2}{m^2}\right]\end{aligned}$$

renormalization group equation:

$$\mu^2 \frac{\partial \overline{m}(\mu^2)}{\partial \mu^2} = -\gamma_m(\alpha_s(\mu^2))\ \overline{m}(\mu^2)$$

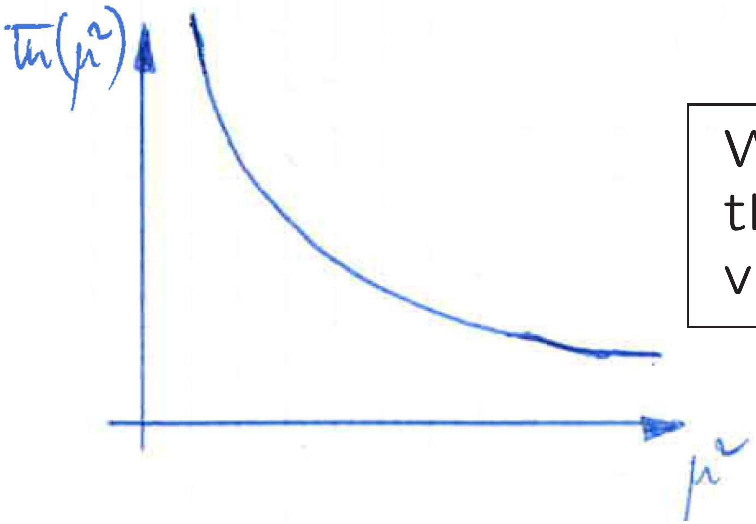
$$\gamma_m(\alpha_s) = \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \quad \text{anomalous mass dimension}$$

$$\alpha_s(\mu^2) = \frac{\pi}{\beta_0 \log \frac{\mu^2}{\Lambda^2}}$$

$$\Rightarrow \text{ solution: } \overline{m}(\mu^2) = \overline{m}(m^2) \exp\left\{ -\frac{1}{\beta_0} \int_{m^2}^{\mu^2} \frac{dQ^2}{Q^2 \log \frac{Q^2}{\Lambda^2}} \right\}$$

$$= \overline{m}(m^2) \left[\frac{\alpha_s(\mu^2)}{\alpha_s(m^2)} \right]^{\frac{1}{\beta_0}}$$

$$\boxed{\begin{aligned} \overline{m}(\mu^2) &= \hat{m}[\alpha_s(\mu^2)]^{\frac{1}{\beta_0}} \\ \hat{m} &= \overline{m}(m^2)[\alpha_s(m^2)]^{-\frac{1}{\beta_0}} \end{aligned}}$$



With growing μ^2 $(R \to 0)$ the effective quark mass vanishes.

Examples:

bottom quark: $m_b = 4.8$ GeV $\quad \overline{m}_b(m_b^2) = 4.2$ GeV

$\overline{m}_b(M_Z^2) = 2.9$ GeV

charm quark: $m_c = 1.6$ GeV $\quad \overline{m}_c(m_c^2) = 1.2$ GeV

$\overline{m}_c(M_Z^2) = 0.6$ GeV

light quarks: $\overline{m}_u(1\,\text{GeV}^2) \sim 5$ MeV Gasser, Leutwyler

[QCD sum rules] $\overline{m}_d(1\,\text{GeV}^2) \sim 8$ MeV

$\overline{m}_s(1\,\text{GeV}^2) \sim 200$ MeV

Higher orders:

$$\overline{m}(m^2) = \frac{m}{1 + C_F \frac{\alpha_s(m^2)}{\pi} + K \left(\frac{\alpha_s(m^2)}{\pi}\right)^2}$$

Gray, Broadhurst, Grafe, Schilcher

$K_t \sim 10.9 \qquad K_b \sim 12.4 \qquad K_c \sim 13.4$

$$\overline{m}(\mu^2) = \overline{m}(m^2) \frac{c\left[\frac{\alpha_s(\mu^2)}{\pi}\right]}{c\left[\frac{\alpha_s(m^2)}{\pi}\right]}$$

$$\begin{aligned}
c(x) &= \left(\frac{9}{2}x\right)^{\frac{4}{9}} \left[1 + 0.895x + 1.371x^2 + 1.952x^3\right] \quad m_s < \mu < m_c \\
c(x) &= \left(\frac{25}{6}x\right)^{\frac{12}{25}} \left[1 + 1.014x + 1.389x^2 + 1.091x^3\right] \quad m_c < \mu < m_b \\
c(x) &= \left(\frac{23}{6}x\right)^{\frac{12}{23}} \left[1 + 1.175x + 1.501x^2 + 0.1725x^3\right] \quad m_b < \mu < m_t \\
c(x) &= \left(\frac{7}{2}x\right)^{\frac{4}{7}} \left[1 + 1.389x + 1.793x^2 - 0.6834x^3\right] \quad m_t < \mu
\end{aligned}$$

Chetyrkin

Larin, van Ritbergen, Vermaseren

INTERMEZZO: IR renormalon

$\overline{m_t}(\overline{m_t}) = 163.643$ GeV, $\alpha_s(m_t) = 0.1088$:

$m_t = (163.643 + 7.557 + 1.617 + 0.501 + 0.195 + \cdots)$ GeV

$$\alpha_s(Q) = \frac{1}{b_0 \log \frac{Q^2}{\Lambda_{QCD}^2}} = \frac{\alpha_s(m)}{1 - \alpha_s b_0 \log \frac{m^2}{Q^2}} = \sum_{n=1}^{\infty} \left(\alpha_s(m) b_0 \log \frac{m^2}{Q^2} \right)^n \qquad b_0 = \frac{33 - 2N_F}{12\pi}$$

IR-contribution to the last loop integration at $(n+1)$-loop order

$$\delta m^{(n+1)} \propto m \alpha_s^{n+1}(m) \int^m dQ b_0^n \log^n \frac{m^2}{Q^2} = m(2b_0)^n \alpha_s^{n+1}(m) n!$$

expansion up to $n_0 \approx 1/(2b_0\alpha_s)$ converges, then diverges (asympt. series)
$\Rightarrow$ minimal term (Stirling's formula):

$$\delta m^{(n_0+1)} \approx m\alpha_s(m) n_0^{-n_0} \sqrt{2\pi} n_0^{n_0+\frac{1}{2}} e^{-n_0} \approx m\alpha_s \sqrt{2\pi n_0} e^{-n_0} \approx m \sqrt{\frac{\pi\alpha_s}{b_0}} \exp\left(-\frac{1}{2b_0\alpha_s} \right)$$

$$= m \sqrt{\frac{\pi\alpha_s}{b_0}} e^{\log \frac{\Lambda_{QCD}}{m}} = \sqrt{\frac{\pi\alpha_s}{b_0}} \Lambda_{QCD}$$

$\Rightarrow$ ambiguity of $\mathcal{O}(\Lambda_{QCD})$: $\delta m_t \approx 110$ MeV