QCD in the cores of neutron stars

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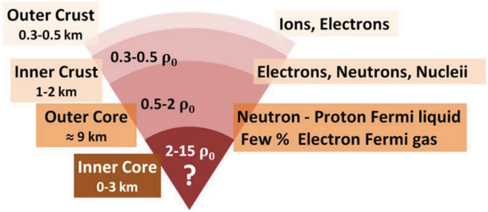
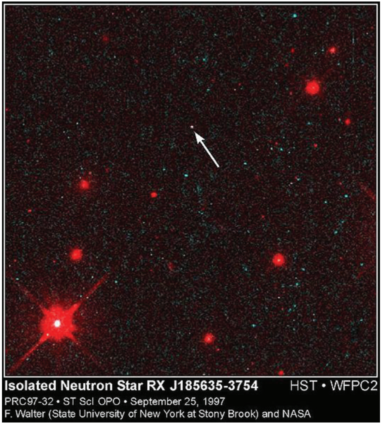
April 2019, Bangalore

Neutron stars

- Masses $\lesssim 2.0 M_\odot$
- Radii $\sim 10\text{km}$
- $T \lesssim KeV \sim 10^7 K$
- $n \lesssim 15 n_0 \quad (n_0 = 0.16\text{fm}^{-3})$





Can we understand these objects from 1st principles?

Structure

Competition:

- Gravity tries to pull the star into a black hole

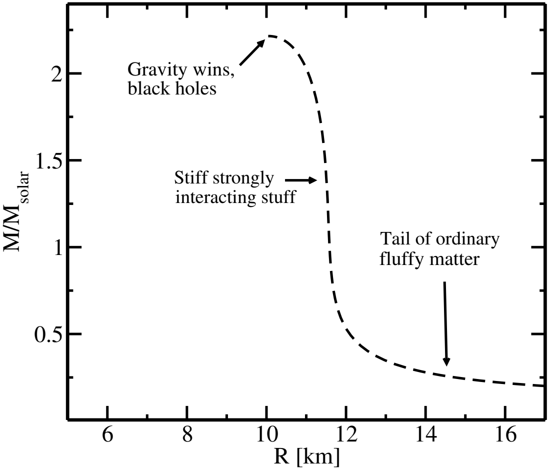
$$
\begin{aligned}
\frac{dP}{dr} &= -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1} \\
\frac{dM}{dr} &= 4\pi r^2 \epsilon(r)
\end{aligned}
$$

- Pressure of strong interactions resists the gravity

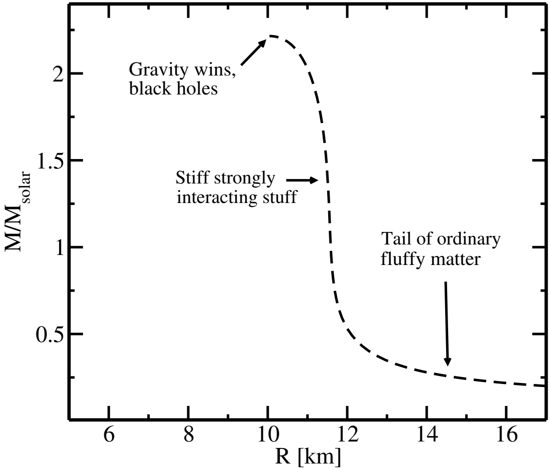
$$\epsilon(P)$$

or $P(\mu)\ldots$

A map from micro to macro



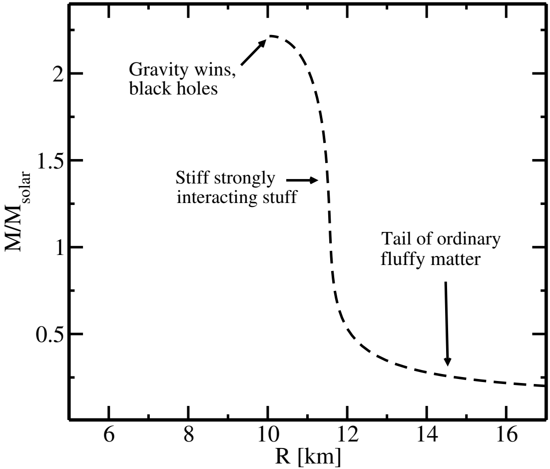
A map from micro to macro



Neutron stars are *femtoscopes*

$$10^{-15}\text{m} \rightarrow 10^{4}\text{m}$$

A map from micro to macro

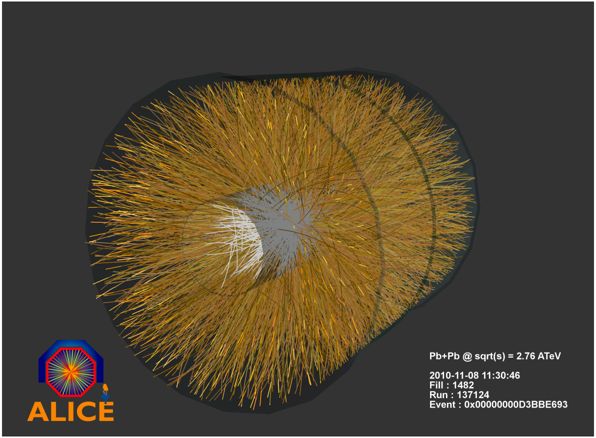


Neutron stars are *femtoscopes*

$10^{-15}\text{m} \rightarrow 10^{4}\text{m}$

...but 10^{19}m away

The other femtoscope:



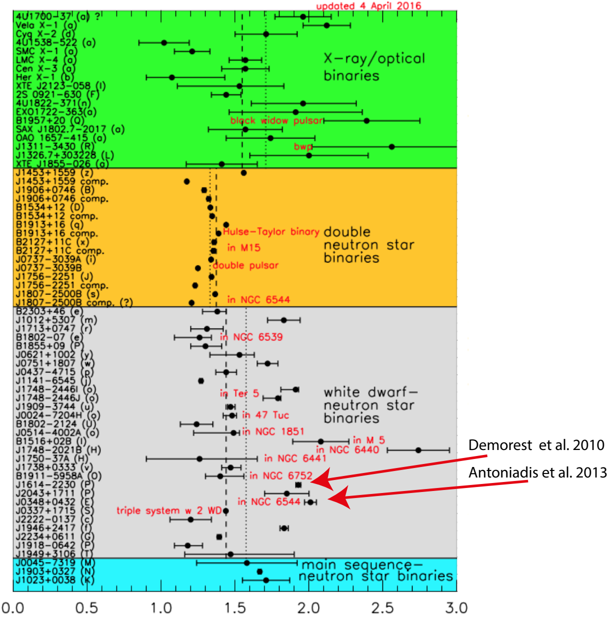
- Transition to hot quark matter around $\epsilon \sim 500\text{MeV}/\text{fm}^3$.
- The big question:

  Is there cold quark matter inside neutron stars?

Outline

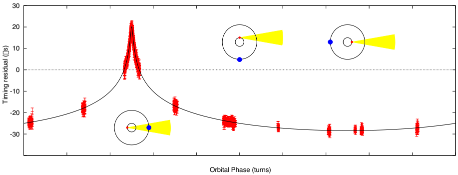
- Overview of neutron star observations
- What we know about the equation of state?
- Astrophysical constraints on the equation of state
- Is there quark matter in neutron stars?

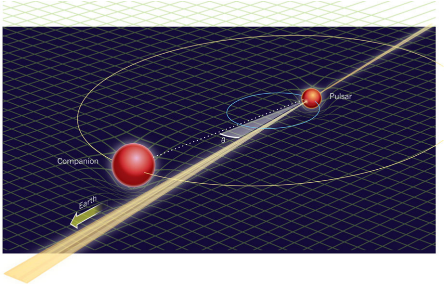
Mass measurements:



Lattimer Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

Mass measurements, PSR J1614-2230





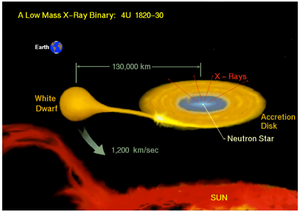
Shapiro delay:

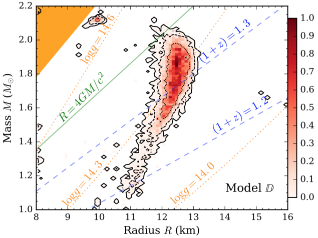
- Binary pulsar
  period: 3.1508076534271(6)ms
- Nearly edge-on orbits
  $i = 89.17(2)^\circ$
- Increase in light travel time through the curved space-time near a white dwarf.

$$M_{\max} > \begin{cases} 1.97 \pm 0.04 & \text{J1614-2230} \\ 2.01 \pm 0.04 & \text{J0348+0432} \\ 2.17 \pm 0.11 & \text{J0740+6620}? \end{cases}$$

Demorest et al. Nature 467, 1081-1083 (2010); Antoniadis et al., Science 240, 448 (2013)
1904.06759 Cromatrie et al.

Radius measurements





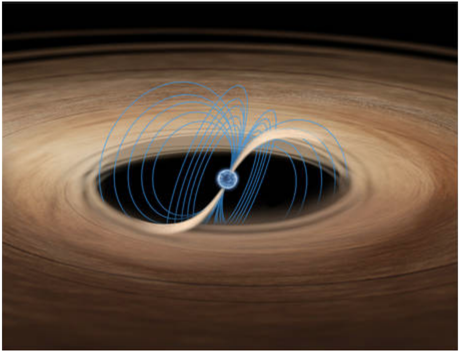
Nättilä et al.
Astron.Astrophys. 608 (2017)

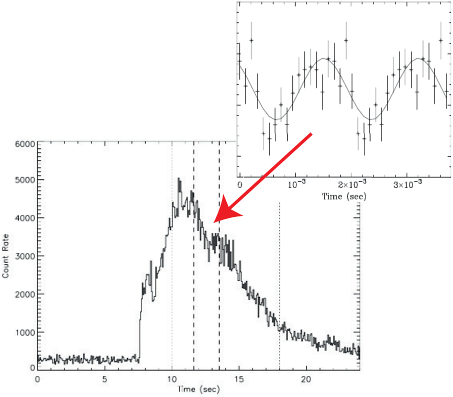
- NS accretes matter from a companion
- Ignition of the envelope generates thermonuclear explosion

$$A = \frac{F_\infty}{\sigma T_{bb}^4} = f_c^{-4} \left(\frac{R}{D}\right)^2 (1-2\beta)^{-1}$$

- Many systematic uncertainties:
  PRE, distance, screening by accretion disk
  atmospheric composition
  interstellar absorption...

X-ray pulse profiling and burst oscillations





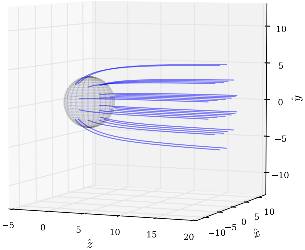
Rossi X-ray timing explorer
Strohmeyer et al. ApJ 486 (1997) 355

- Concentrated the accretion at magnetic poles
  $\rightarrow$ X-ray hot spot

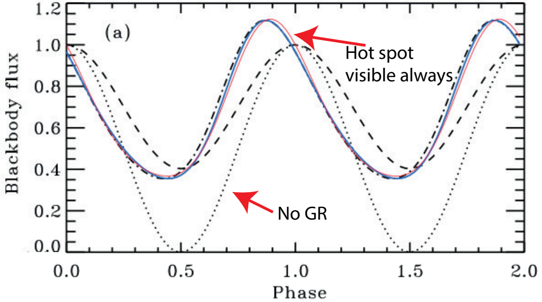
$\sim 30 - 60 keV$

- Rotation causes modulated X-ray emission

X-ray pulse profiling and burst oscillations



Nättilä, Pihajoki A&A 615, A50 (2018)

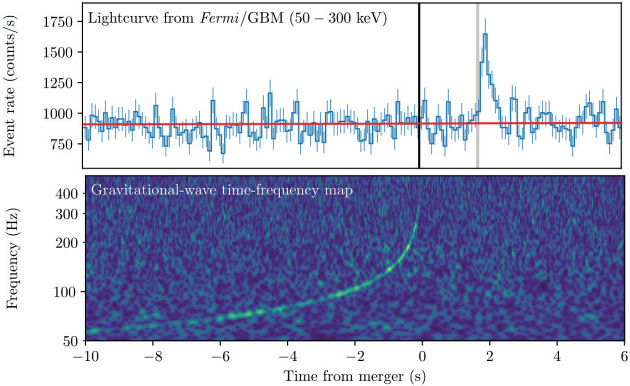


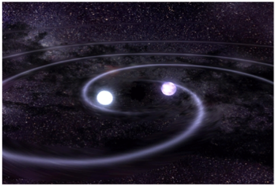
Poutanen & Beloborodov
Mon.Not.Roy.Astron.Soc. 373 (2006)

- Light bends around neutron star, spot visible most of the time
- Pulse profile sensitive to compactness R/M.
- Missions:
  - NICER, June 2017, R within 2% for PSR J0437-4715
    Proc.SPIE Int.Soc.Opt.Eng. 9144 (2014)
  - eXTP, STROBE-X (< 2025), $\mathcal{O}(10)$ %-level measurements
    Proc.SPIE Int.Soc.Opt.Eng. 9905 (2016)

Gravitational waves from neutron star mergers

Breakthrough in gravitational wave astronomy: GW170817



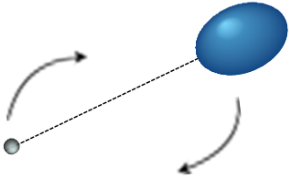


Gravitational waves: PRL. 119, 161101 (2017), γ-ray: APJ. 848 (2017)
Aslo: X-ray, UV, Optical, IR, Radio APJL, 848 (2017) L12

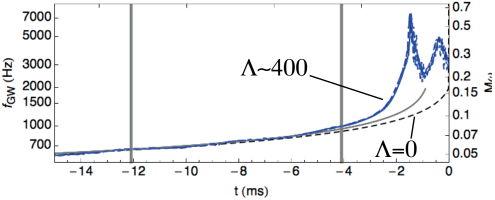
Gravitational waves from neutron star mergers

Tidal deformability during inspiral:

$$\Lambda = \frac{\text{(Quardupole moment) } Q_{ij}}{\text{(tidal field) } \mathcal{E}_{ij}}$$



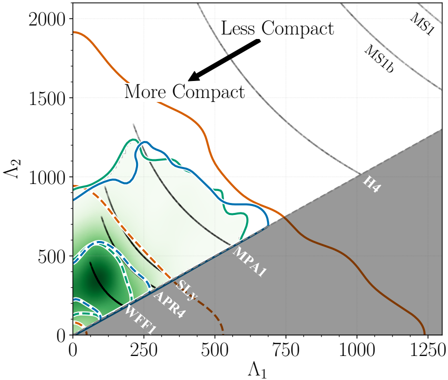
Linear response of the quadrupole moment to an external quadrupolar gravitational field



Read et al. PRD88 (2013) 044042

Gravitational waves from neutron star mergers

Tidal deformability during inspiral:



Nondetection by LIGO leads to upper limit

- Using linearized framework:

$$\text{LIGO/Virgo:} \quad \Lambda(1.4M_\odot) < 580$$

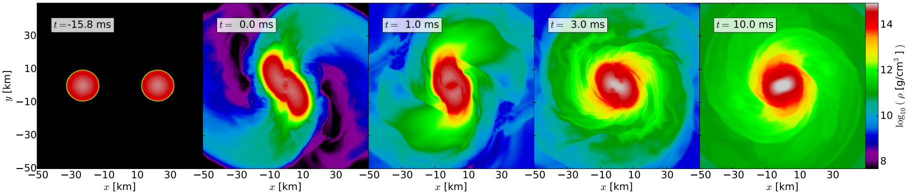
90% credibility, low spin prior, same EoS for both stars, LIGO+VIRGO PRL 121, (2018)

How can mergers be used to teach us about QCD

2) Existence of associated electromagnetic signal:

- Binary merger product either:
  - Black hole
  - Hypermassive neutron star $\rightarrow$ BH
  - HMNS $\rightarrow$ Supramassive neutron star $\rightarrow$ BH

- Observation of the EM counterpart limits the maximal masses supported by the EoS: $M_{\max} \lesssim 2.16 M_{\odot}$
  Rezzolla et al. APJ 852 (2018), Margalit, Metzger APJ 850 (2017), Bauswein et al. APJ 850 (2017)

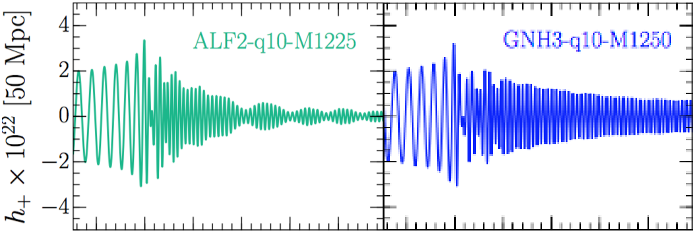


Takami et al. PRD91 (2015)

How can mergers be used to teach us about QCD

3) Post-merger hypermassive neutron star ringdown:

Not observed in GW170817



Takami et al. PRD91 (2015)

- Ringdown sensitive to EoS up to

$$T \lesssim 100\text{MeV}$$

Shen et al, Nucl. Phys. A 637 (1998)

Outline

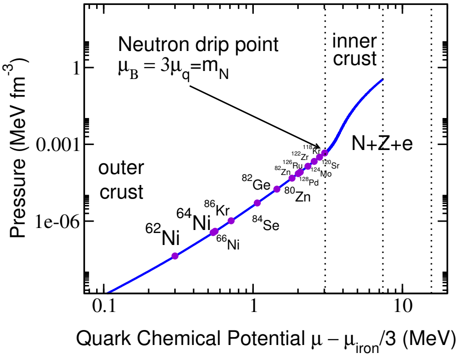
- Overview of neutron star observations
- What we know about the equation of state?
- Astrophysical constraint on the equation of state
- Is there quark matter in neutron stars?

Equation of state:

$$
\begin{aligned}
P(\mu) &= -\log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{QCD}} \\
\mathcal{L}_{QCD} &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i \gamma_0)\psi_i .
\end{aligned}
$$

- At $T \neq 0$ and $\mu \lesssim T$: Lattice field theory
- At $\mu \gtrsim T$ simulations become unfeasible due to sign problem.
  - Low energy effective theories at low densities
  - Perturbation theory at high densities $\alpha_s(\mu) \sim 1/\log(\mu^2)$

Equation of state:



Atmosphere and ocean: atoms, ions, electrons, molecules (∼ 10m)

Outer crust: (∼ 100m)

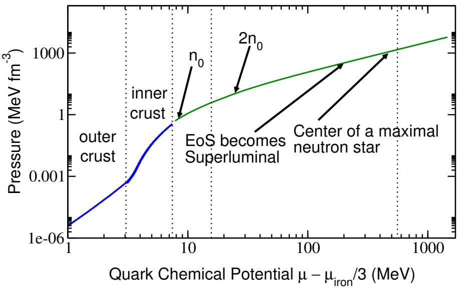
- Degenerate electron gas with nuclei

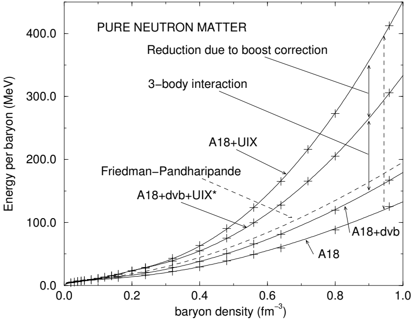
Inner crust: (∼ 1km)

- Neutron gas + Z + e

Negele & Vautherin Nucl.Phys. A207 (1973); Baym et al. Nucl.Phys. A207 (1973);

Equation of state:



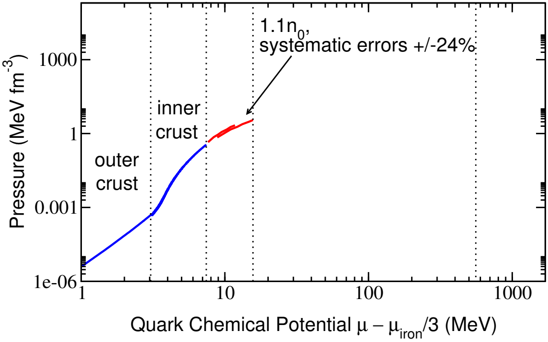


- Try to extrapolate to higher densities: many body schrödinger Eq. with 3N, boost corrections, etc.
- Large corrections:

  Not simply interacting neutrons, something more complicated...

Akmal, Pandharipande & Ravenhall PRC58 (1998)

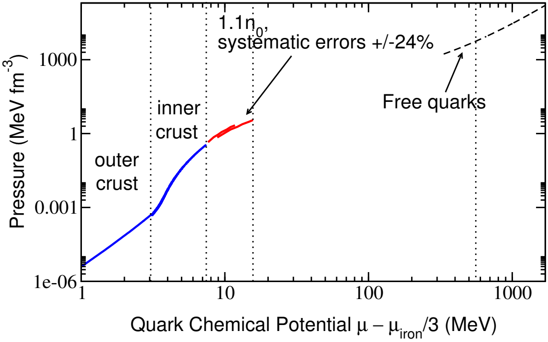
Equation of state:



- Relativistic Weinberg EFT, includes 3N, 4N
- Breakdown scale: $\Lambda_b \sim 500\text{MeV} < M_\rho$
- Systematic error estimation through resolution scale $\pm 24\%$ at $n = 1.1 n_0$.

Tews et al. PRL. 110 (2013)
Hebeler et al. ApJ 773 (2013)

Equation of state



- At high densities: $\alpha_s(\mu_B) \approx 0$, free fermi gas of quarks

Higher order corrections:

$$P(\mu_B) = \int \frac{d^3p}{(2\pi)^4} E(p)\theta(\mu_q - E(p))$$

NLO:

- Interactions cause corrections to disp. rel.:

$$E^2(p) \sim p^2 + g^2\mu^2$$
$$P(\mu_B) \sim P_{\text{free}}(1 + c_1 g^2)$$

NNLO:

- Corrections to $E^2(p) \sim p^2 + g^2\mu^2 + g^4\mu^2$.
- $p \sim g\mu$ contribute at $\int d^3p\, p \sim g^4\mu^4$.
  - $\mathcal{O}(1)$ modification to disp. rel. $\Rightarrow$ non.pert.
  - Integral over scales gives a log:

$$P(\mu_B) \sim P_{\text{free}}(1 + c_1 g^2 + c_2 g^4 + c_2' g^4 \log\left[\frac{g\mu}{\mu}\right])$$

Freedman & McLerran PRD16 (1977), Baluni PRD17 (1978)
Full NNLO with mass dependece: AK et al. PRD81 (2010)
Full T-dependence: AK, Vuorinen PRL 117 (2016)

Higher order corrections:

N^{3}LO:

- Multiple logarithmically enhanced phase space regions:

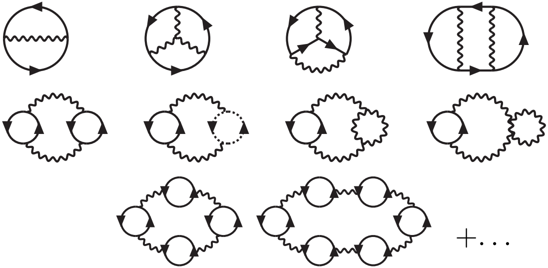
$$P(\mu_B)/P_{\text{free}} \sim 1 + c_1 g^2 + c_2 g^4 + c_2' g^4 \log g + c_3' g^6 \log^2 g + c_3'' g^6 \log g + c_3 g^6$$

Leading-log N^{3}LO: Gorda, AK, Vuorinen, Romatschke, Säppi, PRL 121 (2018)
Next-leading-log N^{3}LO: Work in progress, Gorda, AK, Paatelainen, Säppi, Vuorinen
Full N^{3}LO: In the future...

- Note: some non-perturbative effects (confinement, color super conductivity, etc.) don't come at any order in g.

Higher order corrections:

In practice, effective potential from vacuum graphs:



- Discrete Matsubara modes:

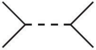
$$\int \frac{d\omega}{(2\pi)} \to T \sum_{\omega_n}, \qquad \omega_n = (2n+1)\pi T i + \mu$$

Higher order corrections:

Our strategy:

- perform p_0 integrals first to get vacuum feynman diagrams with μ-dependent phase space integrals:

 $\rightarrow$  $-2\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}$ 

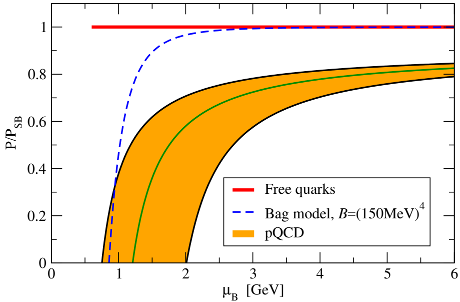
$+ \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}\int \frac{d^3\vec{q}}{(2\pi)^3}\frac{\theta(\mu - E(\vec{q}))}{2E(\vec{q})}$ 

Ghisou et al. NPB915 (2017)

- Automatized integral reduction of vacuum diagrams
- Deal with the resummation of diagrams in an EFT framework: Hard-Loop theory

Braaten & Pisarski NPB 337 (1990) 569, Blaizot et al. PRD63 (2001)
AK, Vuorinen PRL 117 (2016)
Gorda, AK, Vuorinen, Romatschke, Säppi, PRL 121 (2018)

Reliability of N^2LO pQCD:

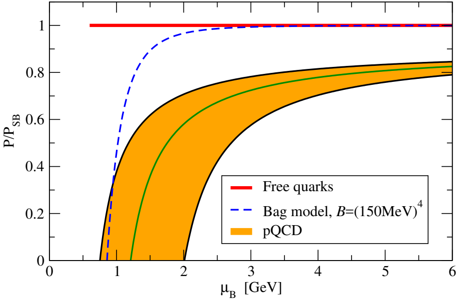


Full NNLO with $m_s \neq 0$: AK et al. Phys.Rev. D81 (2010)
$g^6 \log^2 g$: Gorda, AK, Vuorinen, Romatschke, Säppi, PRL 121 (2018)

$$P(\mu_B)/P_{\text{free}} \sim 1 + c_1 g^2 + c_2 g^4 + c_2' g^4 \log g + c_3' g^6 \log^2 g$$

Interactions are important!

Reliability on N^2LO pQCD:



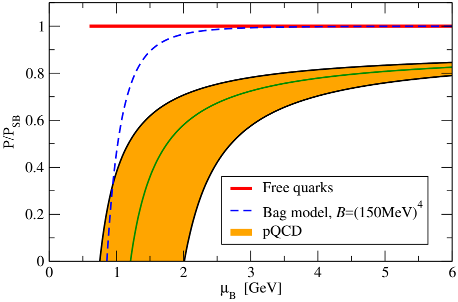
Coefficients depend on renormalization scale $\bar{\Lambda}$

$$P(\mu_B)/P_{\text{free}} \sim 1 + c_1 g^2[\bar{\Lambda}] + c_2[\bar{\Lambda}]g^4[\bar{\Lambda}] + c_2'[\bar{\Lambda}]g^4[\bar{\Lambda}]\log[g] + c_3' g^6 \log^2 g$$

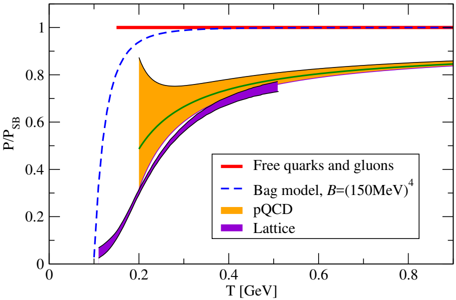
- Uncertainties though scale variation $\bar{\Lambda} = X\mu_q$ with $X = \{1, 2, 4\}$

Reliability of the errorbars

Dense matter:

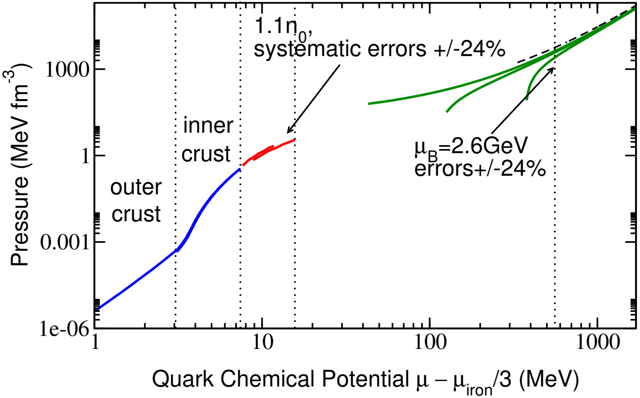


Hot matter:



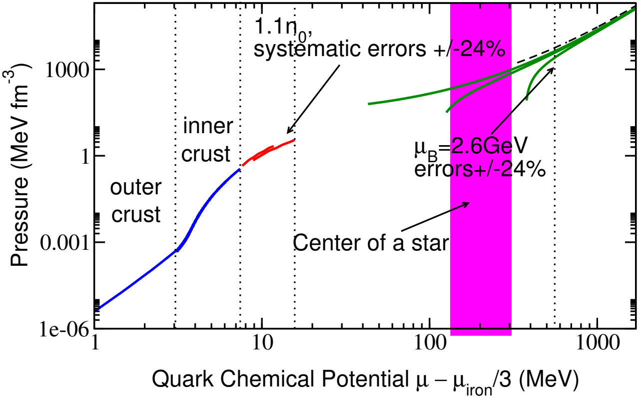
Fraga, AK, Vuorinen Astrophys.J. 781 (2014)
Lattice: Borsanyi et al. PLB 370 (2014)

State of the art in pQCD:



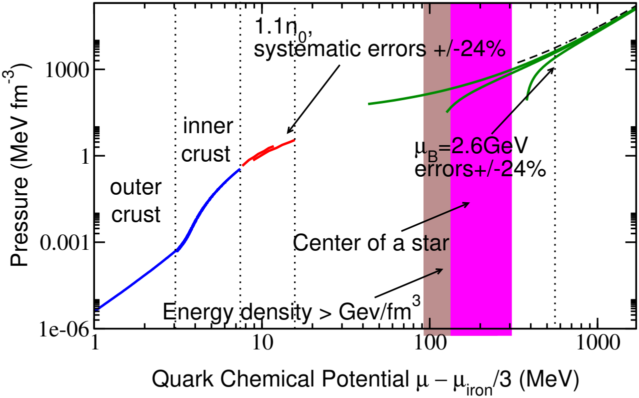
- Relative uncertainty $\pm 24\%$ at $\mu_B = 2.6\text{GeV}$, $n \approx 40 n_0$.

State of the art in pQCD:



- Relative uncertainty $\pm 24\%$ at $\mu_B = 2.6\text{GeV}$, $n \approx 40 n_0$.
- Cores lie in the poorly known no-man's-land

State of the art in pQCD:



- Relative uncertainty $\pm 24\%$ at $\mu_B = 2.6\text{GeV}$, $n \approx 40 n_0$.
- Cores lie in the poorly known no-man's-land
- Energy densities comparable to QGP

Interpolating: Polytropic EoS

Strategy:

- Interpolate where EoS not reliably known.
- Find all possible reasonable interpolations

Require:

- Thermod. consistency: $P(\mu_B)$ monotonic, match with nucleonic and pQCD EoS
- Subluminal: $c_s^2 < 1$ everywhere
- Smoothness: $P(\mu_B)$ and $\partial_{\mu_B} P = n(\mu_B)$ (mostly) continuous

allow also phase transitions

Interpolating: Polytropic EoS

Method:

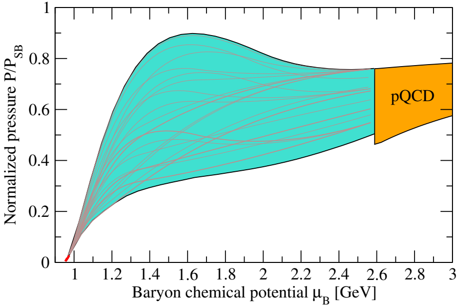
- Piecewise polytropes:

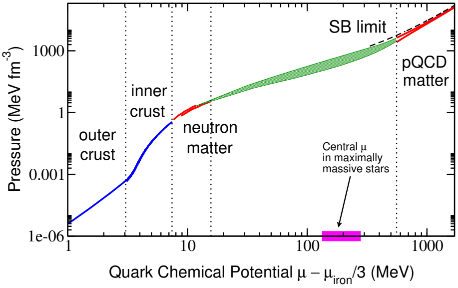
$$P_i(n) = \kappa_i n^{\gamma_i}, \text{ for } \mu_i < \mu_B < \mu_{i+1}$$

  - The larger γ, the stiffer the EoS is

γ	EoS
∞	incompressible matter
~ 2.5	nuclear matter at n_0
2	asymptotically $c_s(\mu_B) = 1$
5/3	Non-relativistic degenerate fermi gas
4/3	Ultrarelativistic fermi gas
1	Ideal gas
0	1st order phase transition

Complete set of interpolated EoSs

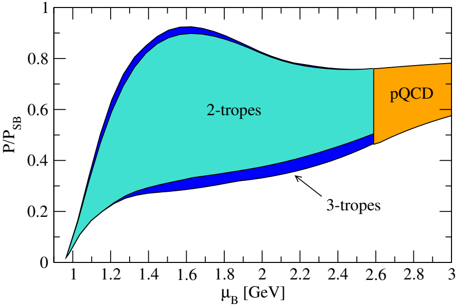




AK et al. Astrophys.J. 789 (2014) 127

- Complete set of EoS, quantifying our terrestrial understanding

Complete set of interpolated EoSs



AK et al. Astrophys.J. 789 (2014) 127

- Complete set of EoS, quantifying our terrestrial understanding
- Adding a third monotrope does not change qualitatively, quantitatively small effect.
- In the following: 4-tropes

Outline

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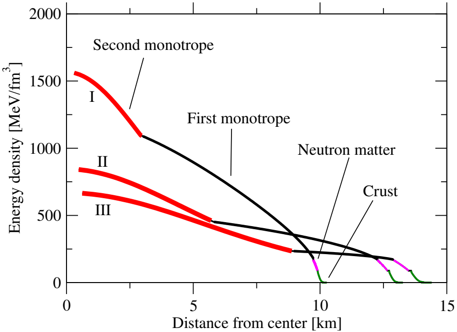
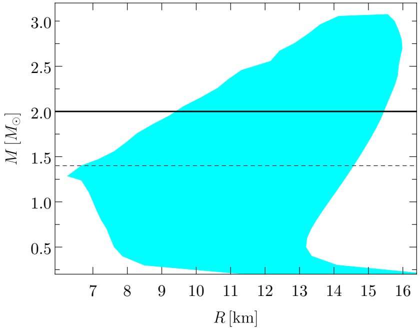
- Is there quark matter in neutron stars?

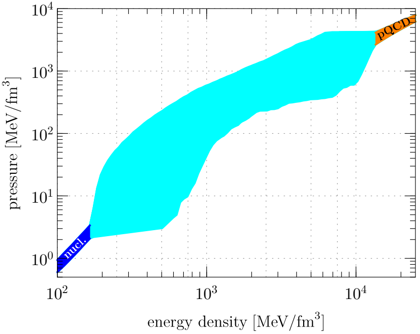
Building stars



- Nuclear EoS present only as thin crust:
  - Important as a boundary condition to interpolation, not the EoS itself!

Constraining neutron star properties using QCD

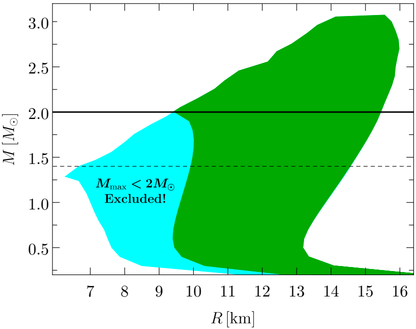


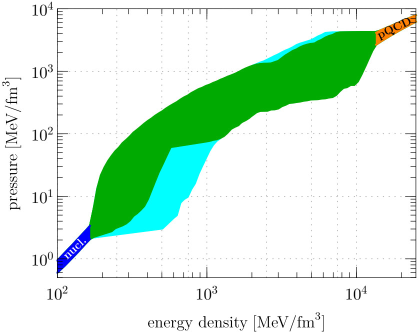


Annala et al. PRL 120 (2018)

- Most conservative set: 4-tropes, no constraints on phase transitions, no constraints on parameter values
- Maximal masses: $M_{\text{max}} \in [1.3, 3.0]$
  Radii: $R(1.4 M_\odot) \in [7, 14]$km.

Constraining QCD using neutron star properties

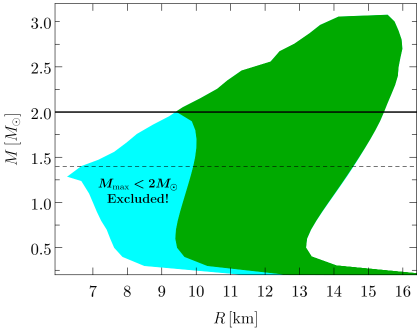


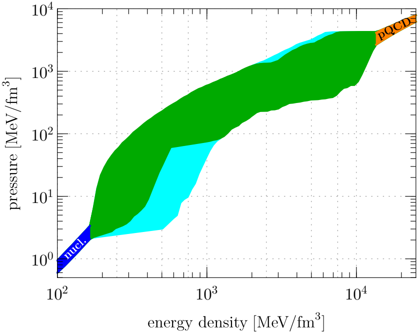


Annala et al. PRL 120 (2018)+update the new LIGO limits

- Requiring $2M_\odot$ implies that matter is stiff enough

Constraining QCD using neutron star properties

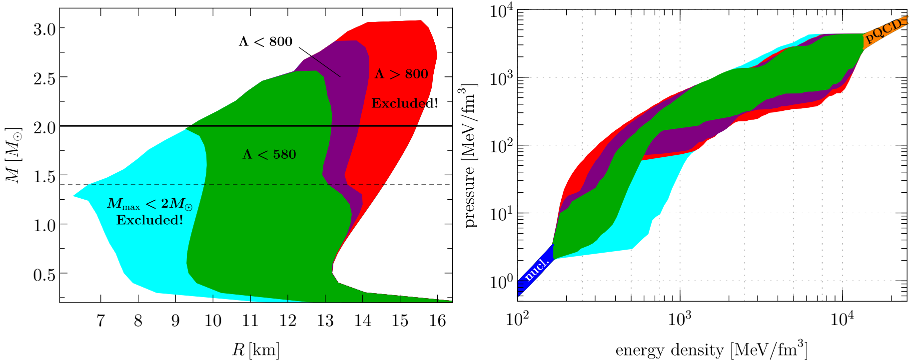




Annala et al. PRL 120 (2018)+update the new LIGO limits

- Requiring $2M_\odot$ implies that matter is stiff enough
- Lower limit for radius: $R(1.4M_\odot) > 10$km

Constraining QCD using neutron star properties

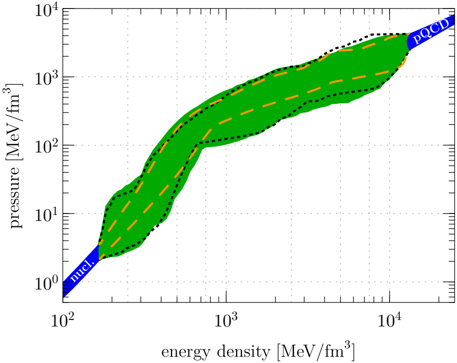


- Requiring $\Lambda(1.4M_\odot) < 580$ implies that the matter is soft enough

$$\Lambda(1.4M_\odot) < 580, \qquad R(1.4M_\odot) < 13\text{km}$$

LIGO+VIRGO, PRL 121 (2018): $10.5\text{km} < R(1.4M_\odot) < 13.3\text{km}$

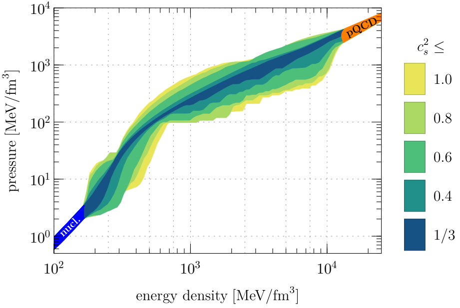
Robustness of the interpolation



- Three different interpolations agree quite well:
  - piecewise polytropic — up to 4 independent segments
  - Chebyshev polynomial polytropic index, $\gamma(p) = \exp(\sum_k T_k(p)\tilde{\gamma}_k)$ — up to degree 5
  - piecewise linear $c_s^2(p)$ — up to 5 independent segments

Annala et al. 1904.01354

Extremal EoSs are extreme



- The band is rather wide, but the boundaries are set by very extreme EoSs

- Almost no known first principles calculations with $c_s^2 > 1/3$

  Bedaque, Steiner, PRL 114 (2015)

  - QCD at high T (+ small μ)
  - pQCD at all temperatures and densities
  - Holography

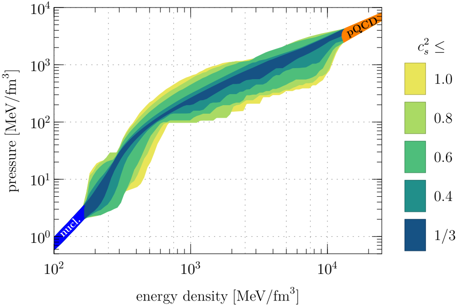
  Sakai-Sugimoto, D3/D7, $\mathcal{N} = 2^*$
  cf. C. Ecker et al JHEP 1711, 031 (2017).

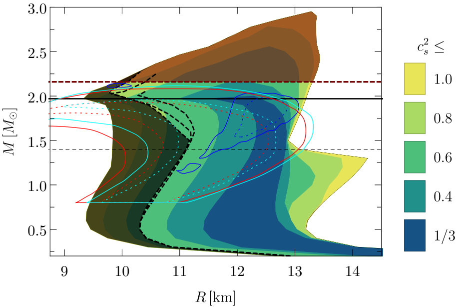
- However, few exceptions

  QCD at finite isospin desntiy, arXiv:1811.08698

- If $c_s^2 > 1/3$, the number of d.o.f's *shrinks* as a function of density
  $c_s^2 = \frac{1}{3}(1 + \frac{\mu}{3} n'(\mu)/n(\mu))^{-1}$

Extremal EoSs are extreme





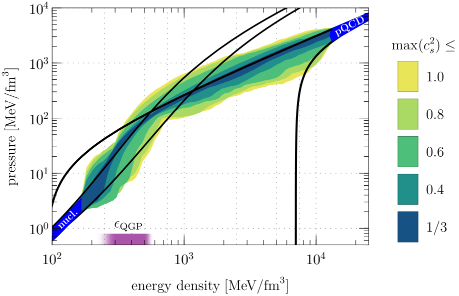
PRELIMINARY: Annala Gorda, AK, Nättilä, Vuorinen,
Measurements: Nättilä et al. Astron.Astrophys. 608 (2017)
Nättilä et al. Astron.Astrophys. A25 (2016)

The current best radius measurements seem to favour the low c_s^2

Outline

- Overview of neutron star observations
- What we know about the equation of state?
- Astrophysical constraint on the equation of state
- Is there quark matter in neutron stars?

Quark matter in neutron stars?



Robust feature:

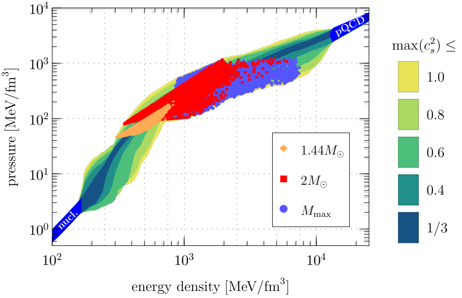
- Rapid change in stiffness around

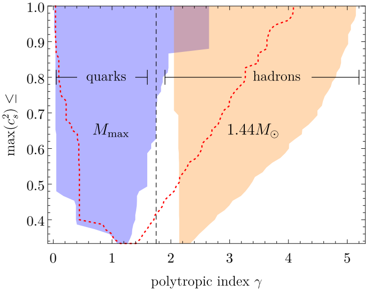
$$\epsilon \sim 500 - 750 \mathrm{MeV/fm}^3$$

cf. at finite T the chiral transition is around $\epsilon \lesssim 500 \mathrm{MeV/fm}^3$ HotQCD: PRD90 (2014)

Hadronic matter, slope $\gamma = \frac{d \log \epsilon}{d \log p} \approx 2.5$
Quark matter, slope $\gamma \approx 1$

Quark matter in neutron stars?

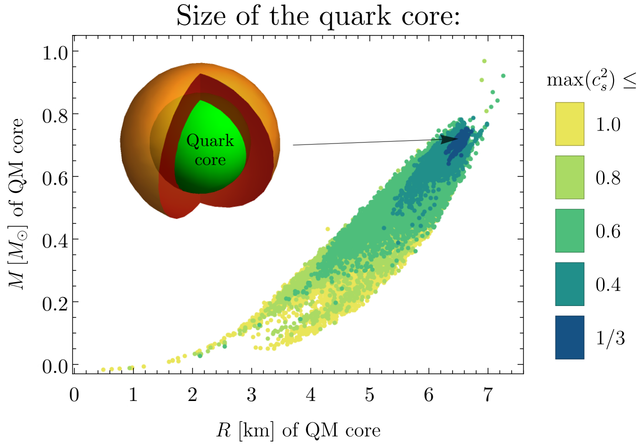




- Centers of $M = 1.44 M_\odot$ star is in the hadronic phase
- Centers of $M = 2.0 M_\odot$ either way
- Centers of maximally massive stars in quark matter phase

For purely hadronic stars need also a strongly first order transition with $\Delta\epsilon = 130 MeV/fm^3$

Quark core in maximally massive NSs



Amount of matter with $\gamma < 1.75$

Sizeable fraction of the mass of the star (25%) may be in the quark phase. If $c_s^2 < 0.4$, at least $0.4 M_\odot$ of quark matter.

Conclusions:

- The competition between gravity and pressure of strong interactions makes neutron stars unique *femtoscopes*
  Energy densities comparable to heavy-ion collisions

- The astronomical observations are advancing rapidly:
  - The observation of gravitational waves from binary neutron star mergers has started the era of multimessenger astronomy.
  - Current and future missions to measure radii of neutron stars will put stringent conditions on the propreties of neutron star matter.

- Theoretical computations advancing rapidly
  - N^3LO computation underway at high densities, results for $g^6 \log^2 g$
    Gorda, AK, Vuorinen, Romatschke, Säppi, PRL 121 (2018)

- Combining astronomical and theoretical inputs allows to empirically determine properties of strongly interacting matter in extreme conditions where no 1st principles calculations are available

Conclusions:

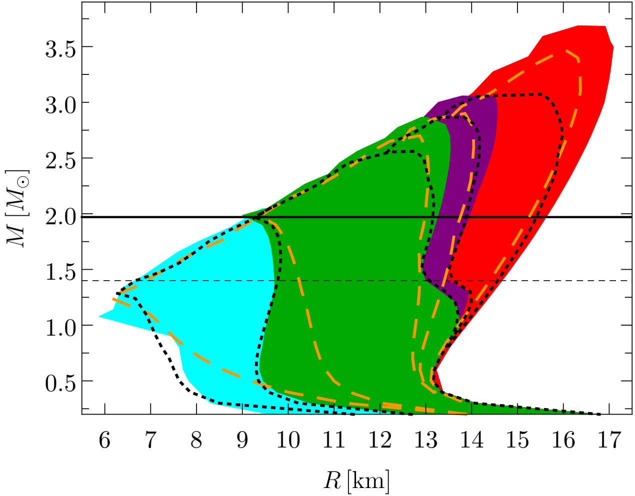
- Different observations are theoretical inputs are complementary, bracketing the allowed parameter regions from different sides.
- The matter softens significantly around hadronic density $\epsilon \sim 500 - 750\text{MeV}/\text{GeV}^3$

  Either:
- The most massive neutron stars should have sizeable core of soft quark matter

  or:
- the high density matter is so stiff that $c_s \sim 1$.

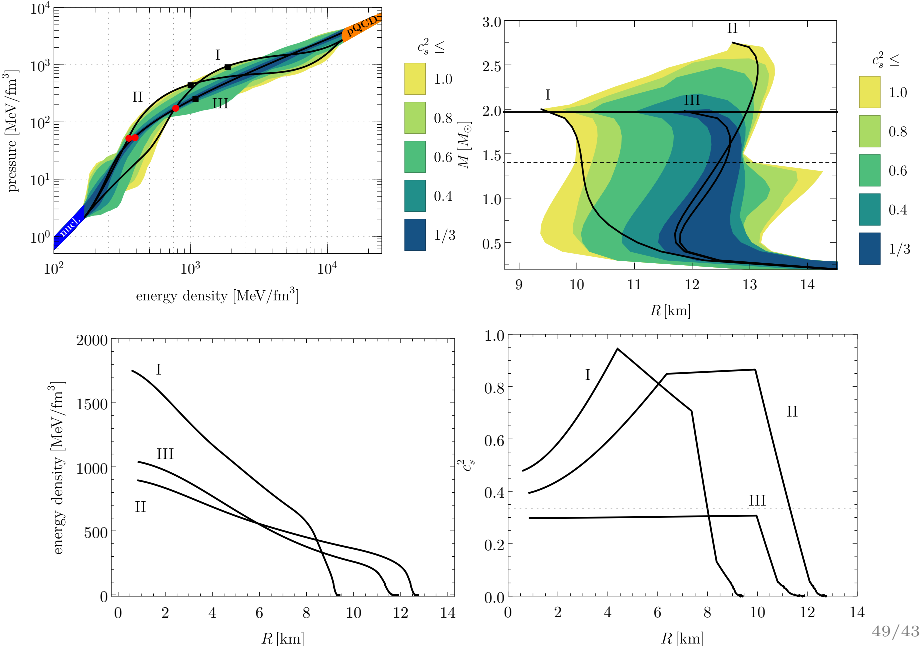
Extra slides

Comparison of the different interpolations:



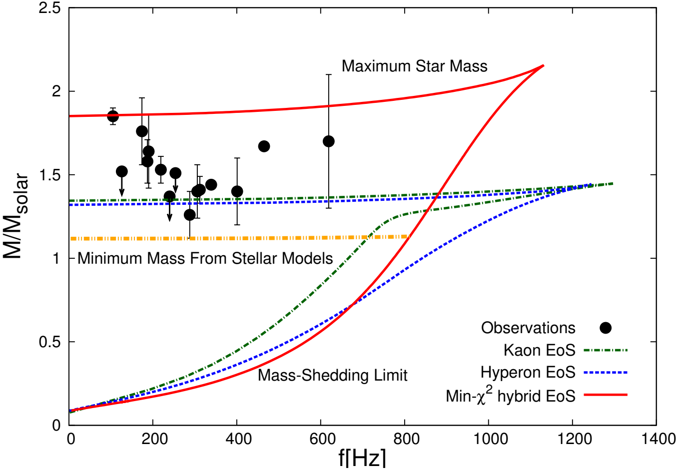
Solid: speed of sound. Dashed: Chebyshev. Dotted: polytropes

Extreme equations of states:



Rotating stars:

Upper and lower bounds on masses of rotating stars:



AK, romatschke, vuorinen, Wu, 1006.4062