ICTS, Bangalore, 4/2019

The myriad colorful ways of understanding extreme QCD matter

Holographic confinement and collective modes

Giuseppe Policastro

LPENS, Ecole Normale Supérieure
Paris

Outline

Hydrodynamics and linear response

Correlators from holography

Holographic confinement

QNM of the CR plasma

At finite temperature hydrodynamics is expected to be the effective description of a generic system

Length scales	m	l_F	ξ
Effective theory	microscopic physics	kinetic theory	hydrodynamics
Effective d.o.f.	particles	quasiparticles	conserved charges

The fluctation-dissipation theorem relates the approach to equilibrium of a small perturbation to correlators in the equilibrium state, computable using the holographic correspondence

Hydrodynamical Ansatz: effective degrees of freedom in the long range are the densities of conserved charges

The dynamics is captured by a small number of effective parameters (transport coefficients)

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\,\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\mu u^\mu\right) - \zeta\Delta^{\mu\nu}\partial_\lambda u^\lambda + O(\partial^2)$$

$$u^\mu = (1, \mathbf{v}^i) \qquad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$J^\mu = nu^\mu - T\sigma\Delta^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + O(\partial^2) \;\; = nu^\mu - D\Delta^{\mu\nu}\partial_\nu n + O(\partial^2)$$

$$\sigma = D\chi \qquad \chi = \frac{\partial n}{\partial \mu}$$

Linear response theory

Kadanoff, Martin, Ann. Phys. 24 (1963) 419

Forster, "Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions"

Kovtun, 1205.5040

$$\partial_\mu J^\mu = 0 \quad \Rightarrow \quad \partial_t n - D\boldsymbol{\nabla}^2 n = 0 \qquad n(t,\mathbf{k}) = e^{-D\mathbf{k}^2 t} n_0(\mathbf{k}) \qquad t \geq 0$$

Laplace transform $\quad n(z,\mathbf{k}) = \int_0^\infty dt\, e^{izt} n(t,\mathbf{k}) \;= \dfrac{\chi\, \mu_0(\mathbf{k})}{-iz + D\mathbf{k}^2}$

Response to external perturbation

$$\delta H = -\int d^d x\; \mu(t,\mathbf{x})\, n(t,\mathbf{x}) \qquad \delta\langle A(t,\mathbf{x})\rangle = -i\int_{-\infty}^{t} dt'\, \langle [A(t,\mathbf{x}), \delta H(t')]\rangle$$

$$\langle n(t,\mathbf{k})\rangle = -\int_{-\infty}^{0} dt'\; e^{\varepsilon t'} \mu(\mathbf{k})\, G^R_{nn}(t-t',\mathbf{k}) \;=\; -\mu_0(\mathbf{k}) \int \frac{d\omega}{2\pi}\, G^R_{nn}(\omega,\mathbf{k}) \frac{e^{-i\omega t}}{i\omega + \varepsilon}$$

$$G^R_{AB}(t,\mathbf{x}) = -i\theta(t)\langle [A(t,x), B(0,0)]\rangle$$

$$\langle n(z,\mathbf{k})\rangle = -\mu_0(\mathbf{k})\int\frac{d\omega}{2\pi}\,\frac{G^R_{nn}(\omega,\mathbf{k})}{(i\omega+\varepsilon)\,(i(\omega-z)+\varepsilon)} \quad = -\mu_0(\mathbf{k})\frac{G^R_{nn}(z,\mathbf{k})-G^R_{nn}(z{=}0,\mathbf{k})}{iz}$$

close the contour in UHP

$$\langle n(t{=}0,\mathbf{k})\rangle \;= -\mu_0(\mathbf{k})G^R_{nn}(z{=}0,\mathbf{k}) \qquad\qquad G^R_{nn}(z{=}0,\mathbf{k}) = -\chi$$

$$G^R_{nn}(z,\mathbf{k}) = \frac{\chi D\mathbf{k}^2}{iz - D\mathbf{k}^2} \qquad\qquad G^R_{ii} = \frac{\chi D\omega}{iz - D\mathbf{k}^2}$$

Density of conserved charge: vanishes $\sim \mathbf{k}^2$

Pole in the Lower Half-Plane $\quad \omega = -iD\mathbf{k}^2$

Kubo formula $\quad D\chi = -\lim_{\omega\to 0}\lim_{k\to 0}\frac{\omega}{\mathbf{k}^2}\mathrm{Im}\,G^R_{nn}(\omega,\mathbf{k}) \;= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}G^R_{ii}(\omega,\mathbf{k}=\mathbf{0})$

spectral function $\quad A(\omega,\mathbf{k}) \equiv -\mathrm{Im}G^R(\omega,\mathbf{k}) > 0$

Stress-energy tensor sector: fluctuations $\varepsilon, \pi_{\parallel}, \pi_{\perp}$ $\mathbf{k} = k\hat{\mathbf{x}}$

Decoupled from current at $\mu = 0$

Shear mode π_y $$G^R_{\pi_y\pi_y}(\omega, \mathbf{k}) = \frac{\bar{w}\gamma_\eta \mathbf{k}^2}{i\omega - \gamma_\eta \mathbf{k}^2}$$

Sound mode ε, π_x $$G^R_{ab}(\omega, \mathbf{k}) = \frac{\bar{w}}{\omega^2 - \mathbf{k}^2 v_s^2 + i\omega\gamma_s \mathbf{k}^2} \begin{pmatrix} \mathbf{k}^2 & \omega k_x \\ \omega k_x & \mathbf{k}^2 v_s^2 - i\omega\gamma_s \mathbf{k}^2 \end{pmatrix}$$

$$\bar{w} = \varepsilon + p \ = Ts \qquad v_s^2 = \frac{\partial p}{\partial \epsilon}$$

$$\gamma_\eta = \frac{\eta}{\bar{w}} \qquad \gamma_s = \frac{\frac{2d-2}{d}\eta + \zeta}{\bar{w}}$$

$$\eta = -\frac{\omega}{\mathbf{k}^2} \frac{1}{d-1} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \operatorname{Im} G^R_{\pi_i\pi_j}(\omega, \mathbf{k}\to 0)$$

$$\frac{2d-2}{d}\eta + \zeta = -\frac{\omega^3}{\mathbf{k}^4} \operatorname{Im} G^R_{\epsilon\epsilon}(\omega, \mathbf{k}\to 0)\,.$$

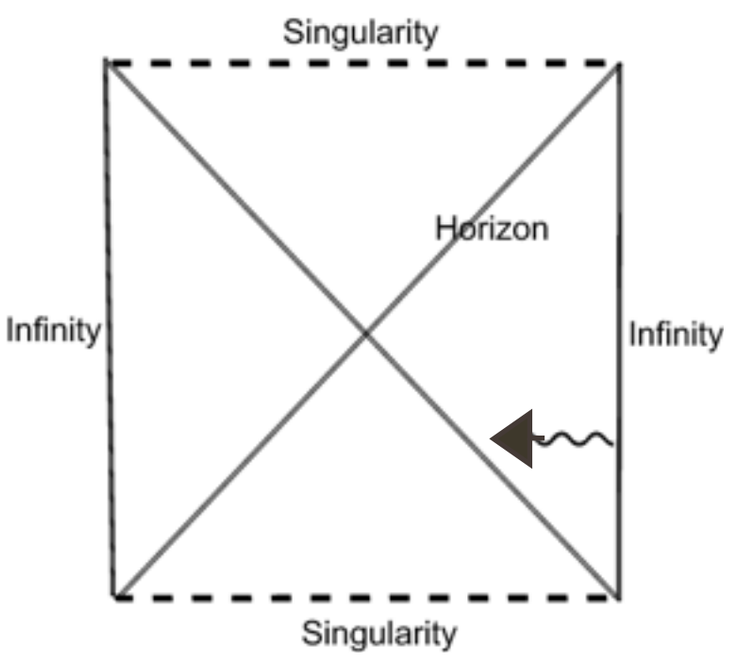
At $\mu \neq 0$ all fluctuations coupled

Correlators from holography $\quad Z_{grav}(\phi \to J) = \langle e^{J\mathcal{O}} \rangle_{CFT}$

$$\langle O_1 O_2 \rangle = \frac{\delta^2 S}{\delta J_1 \delta J_2} = \frac{\delta \langle O_1 \rangle}{\delta J_2} = \frac{\delta \langle O_2 \rangle}{\delta J_1}$$

In FG coordinates $\quad \phi(x,z) = J(x) z^{\Delta_-} + \langle O(x) \rangle z^{\Delta_+}$

$\langle O \rangle$ determined as function of J from IR boundary conditions $(z \to \infty)$



Correlators in Lorentzian AdS require a special prescription Son, Starinets 02

$$S = \int d^4k \, J(k) \mathcal{F}(k,z) J(-k) \Big|_{z=\epsilon}^{z=z_h}$$

$$G^R(k) = -2\mathcal{F}(k, z=\epsilon)$$

generalized by Skenderis, van Rees 0812.2909

$$G^R = \frac{\langle \mathcal{O} \rangle}{J} \qquad \text{has poles when} \qquad J = 0\,,\ \langle \mathcal{O} \rangle \neq 0$$

$\Longleftrightarrow$ Quasi-normal modes (normalizable fluctuations with incoming wave b.c. at the horizon)

Easier to compute using shooting methods, or Frobenius expansion, or many other methods Konoplya, Zhidenko 1102.4014

$$G^R(\omega, q) = \sum_n \frac{R_n(q)}{\omega - \Omega_n(q) + i\Gamma_n(q)} + P(\omega, q) \qquad \textit{divergent!}$$

$$\lim_{(t^2 - \mathbf{x}^2) \to 0} \langle \mathcal{O}(t, \mathbf{x})\, \mathcal{O}(0) \rangle = \frac{\mathcal{A}}{|t^2 - \mathbf{x}^2|^\Delta} + \cdots \qquad A(\omega, \mathbf{q}) \sim \mathcal{A}\, \omega^{2\Delta - 4}$$

In general, singularities in the LHP do not have to be simple poles; typically there can be branch cuts

Maxwell field QNM in AdS_5

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\mathbf{x}^2\right) + \frac{L^2}{r^2}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_5^2\right)$$

$$f(r) = 1 - \frac{r_0^4}{r^4} \qquad T = \frac{r_0}{\pi L^2} \qquad \bar{x} = 1 - \frac{r_0^2}{r^2}$$

Gauge invariant fields: longitudinal and transverse electric field

$$E_T'' + \frac{f'}{f}E_T' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{(1-\bar{x})f^2}E_T = 0\,,$$

$$E_L'' + \frac{\mathfrak{w}^2 f'}{f(\mathfrak{w}^2 - \mathfrak{q}^2 f)}E_L' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{(1-\bar{x})f^2}E_L = 0$$

$$\mathfrak{w} = \frac{\omega}{2\pi T}\,, \qquad \mathfrak{q} = \frac{q}{2\pi T}$$

$$\mathfrak{q} = 0 \qquad E_T = E_L \equiv E \qquad E'' + \frac{f'}{f}E' + \frac{\mathfrak{w}^2}{(1-\bar{x})f^2}E = 0$$

$$E(\bar{x}) = \bar{x}^{-i\mathfrak{w}/2}\,(2-\bar{x})^{-\mathfrak{w}/2}\,(1-\bar{x})^{\frac{(1+i)\mathfrak{w}}{2}}\,{}_2F_1\left(1 - \frac{(1+i)\mathfrak{w}}{2}, -\frac{(1+i)\mathfrak{w}}{2}; 1 - i\mathfrak{w}; \frac{\bar{x}}{2(\bar{x}-1)}\right)$$

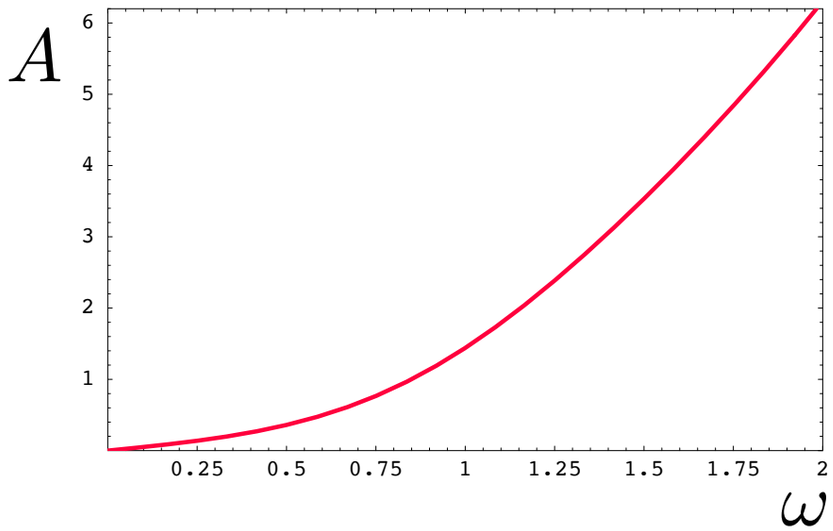
$$G_R(\omega) = \frac{N_c^2 T^2}{8} \lim_{\bar{x}\to 1} \frac{E'(\bar{x})}{E(\bar{x})} = \frac{N_c^2 T^2}{8} \left\{ i\,\mathfrak{w} + \mathfrak{w}^2 \left[\psi\left(\frac{(1-i)\mathfrak{w}}{2}\right) + \psi\left(-\frac{(1+i)\mathfrak{w}}{2}\right)\right]\right\}$$

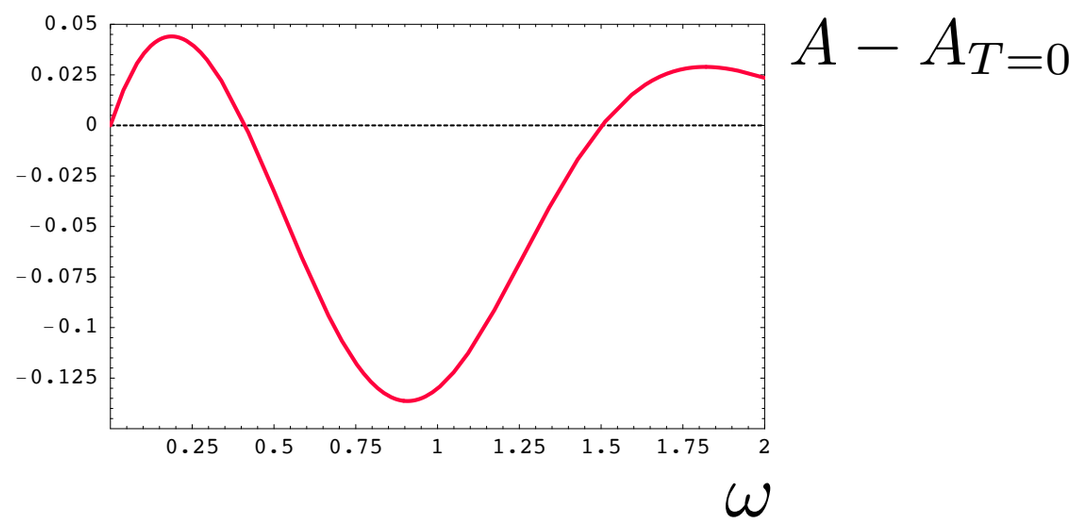
$$A(\omega) = \frac{N_c^2 T^2}{4} \frac{\pi \mathfrak{w}^2 \sinh \pi\mathfrak{w}}{\cosh \pi\mathfrak{w} - \cos \pi\mathfrak{w}}$$

$$A_{T=0} = \frac{N_c^2 \omega^2}{16\pi}$$

$$G^R_{T=0} \sim \omega^2 \ln \omega$$

poles at $\omega = 2\pi T n(\pm 1 - i)$





$$D = \frac{1}{2\pi T}$$

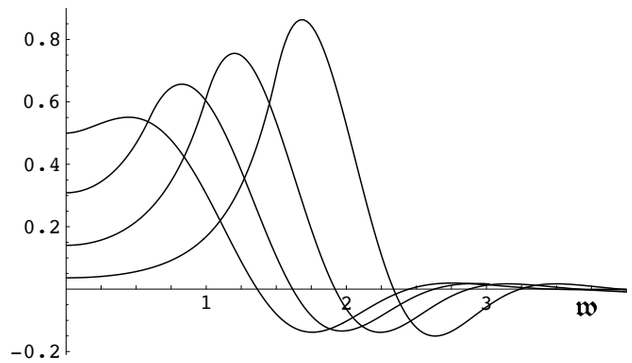
Stress-energy tensor

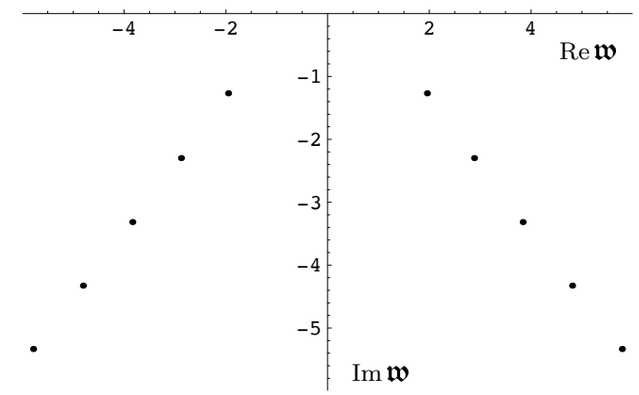
Kovtun, Starinets hep-th/0602059, hep-th/0506184

Thermal Spectral function

QNM

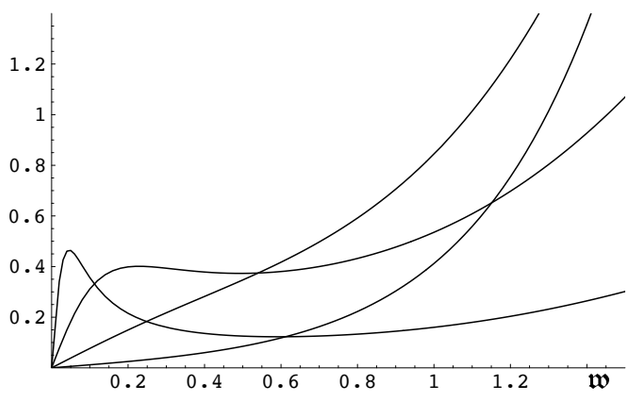
T_{xy}

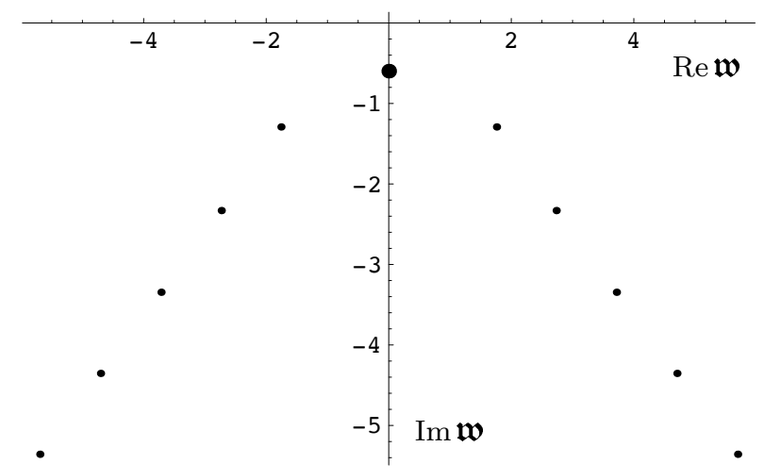




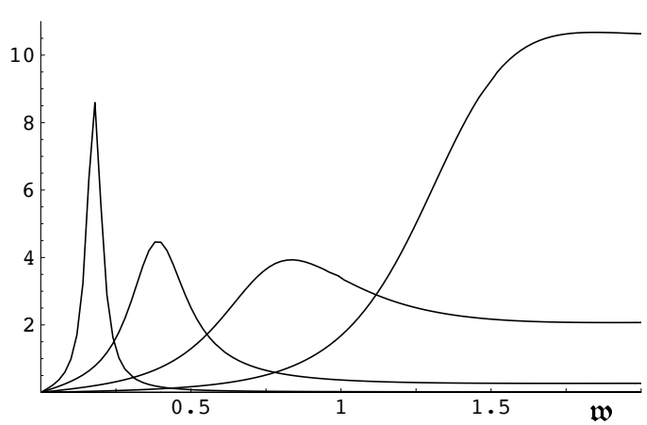
no hydro mode

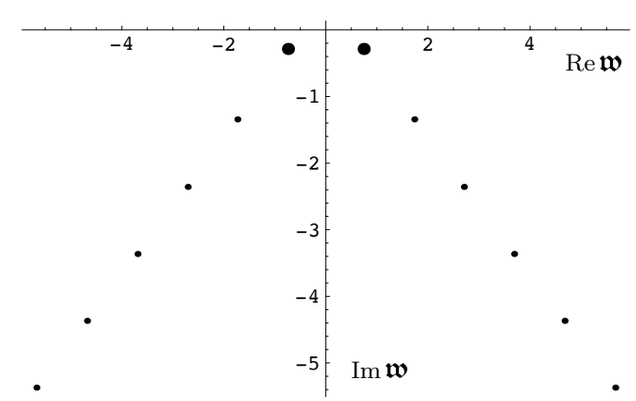
T_{xt}





T_{tt}





hydrodynamics in curved background

$$\delta T_{\mu\nu} = -\eta \Delta^{\mu\alpha} \Delta^{\nu\beta} \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{d} g_{\alpha\beta} \nabla \cdot u \right)$$

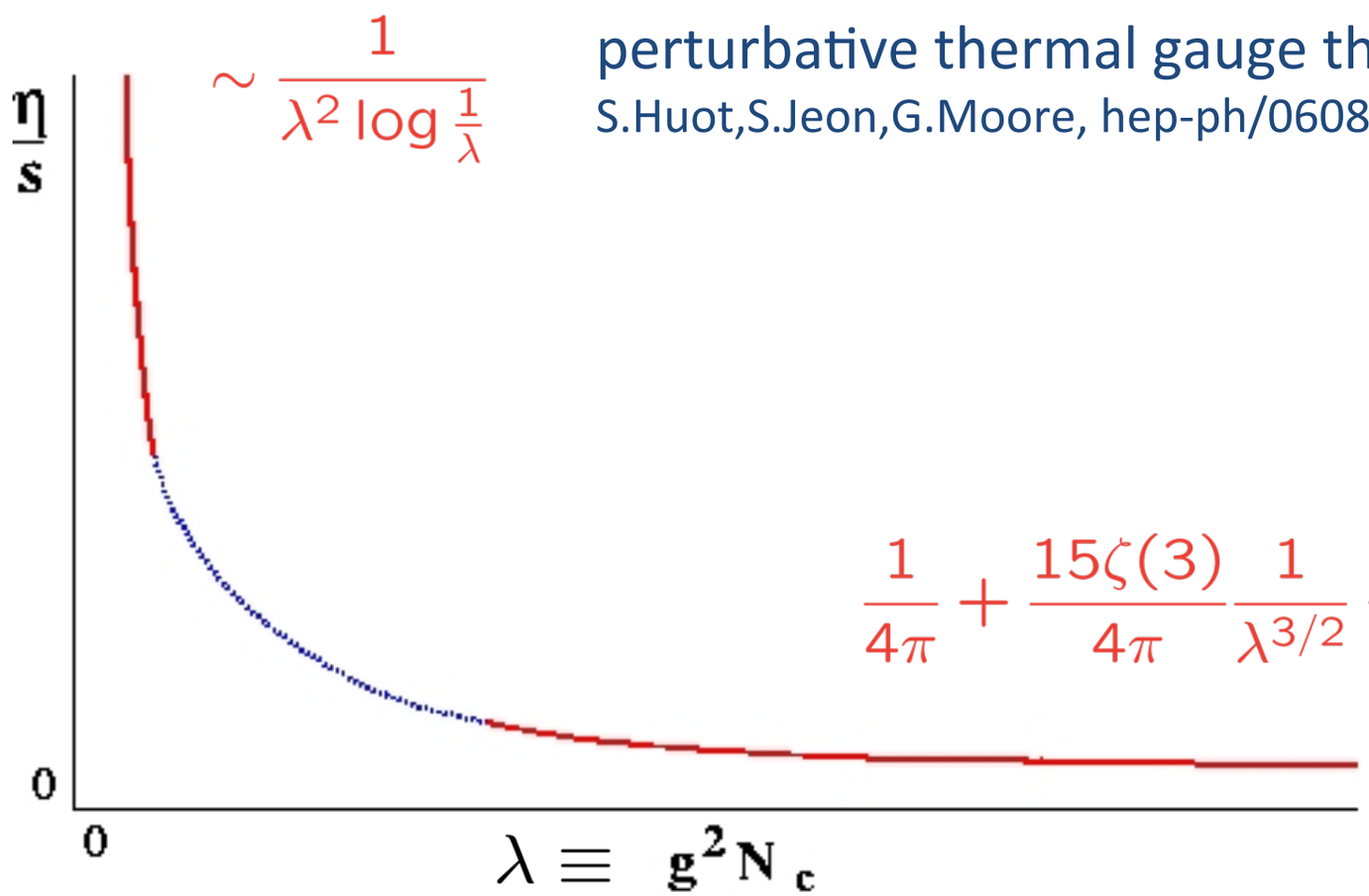
metric perturbation

$$g_{ij} = \delta_{ij} + h_{ij}(t) \qquad \delta g_{00} = \delta g_{0i} = 0$$

$$\delta T_{xy} = \eta \partial_0 h_{xy} \qquad G^R_{xy,xy} = -i\eta\omega + O(\omega^2)$$

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G^R_{xy,xy}(\omega, q = 0)$$

Kinetic theory $\eta = nk_BT\tau$



perturbative thermal gauge theory
S.Huot,S.Jeon,G.Moore, hep-ph/0608062

$$\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$$

Saturation at strong coupling

Conjectured universal bound $\frac{\eta}{s} \geq \frac{1}{4\pi}$ [Kovtun, Son, Starinets 03]

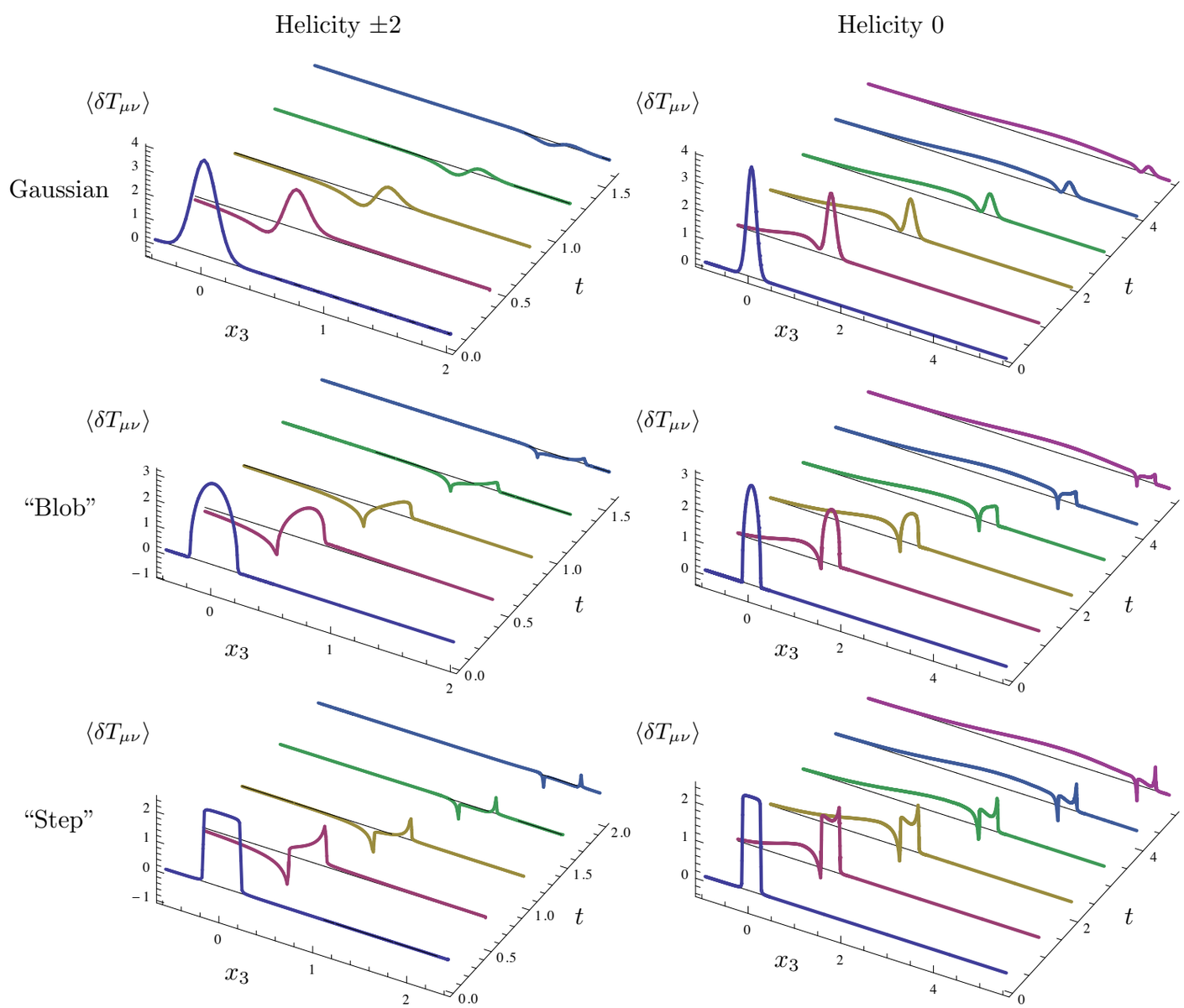
There are holographic counterexamples but not pushed arbitrarily low

$\zeta = 0$ because of conformal invariance

Weak damping of short-wavelength modes

Festuccia, Liu 9811.1033
Fuini, Uhlemann, Yaffe 1610.0349

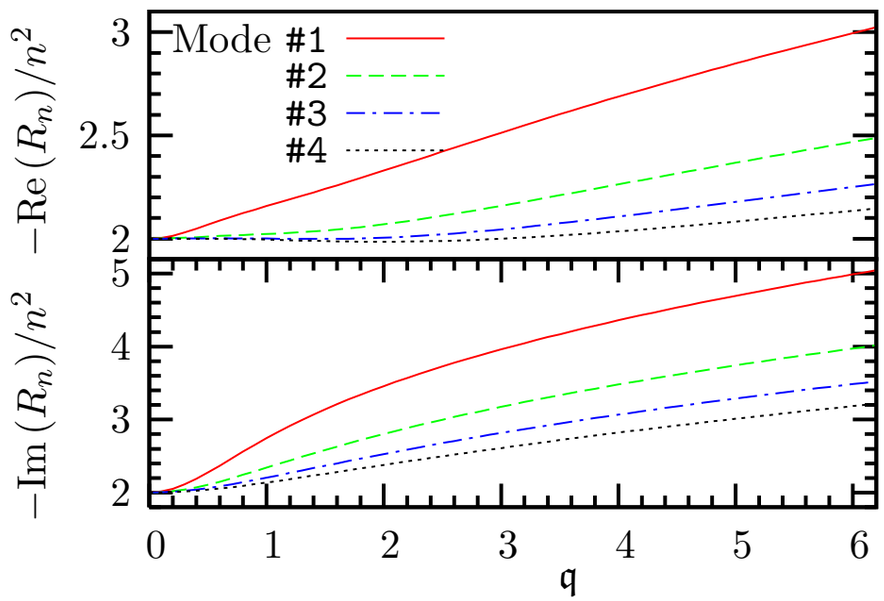
$$\omega_n^{\pm}(\mathbf{q})/|\mathbf{q}| \sim \pm\left[1 + c_n\, e^{\mp i\pi/3}\,(\pi T/|\mathbf{q}|)^{4/3}\right]$$



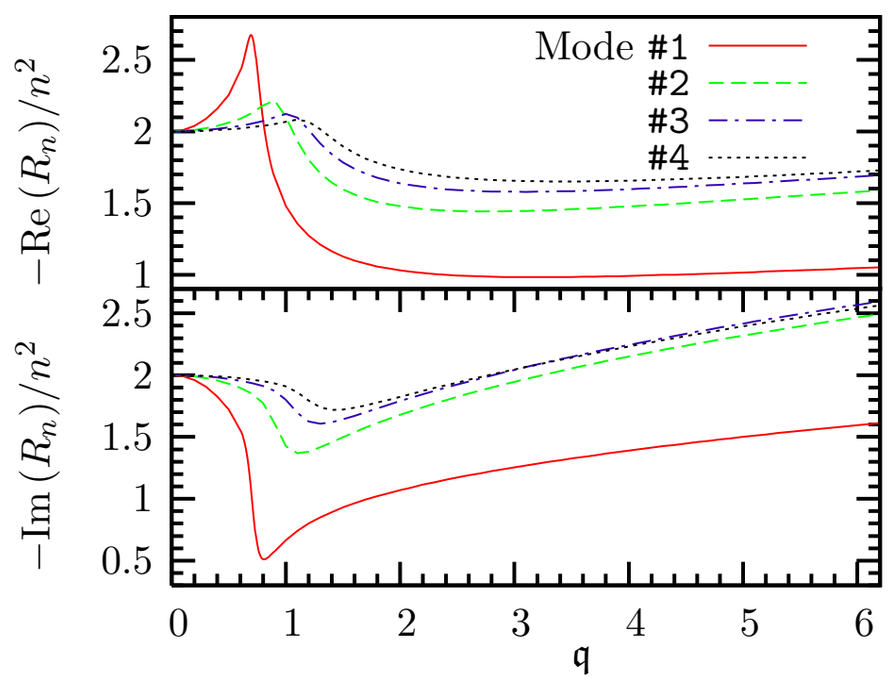
Residues of QNM

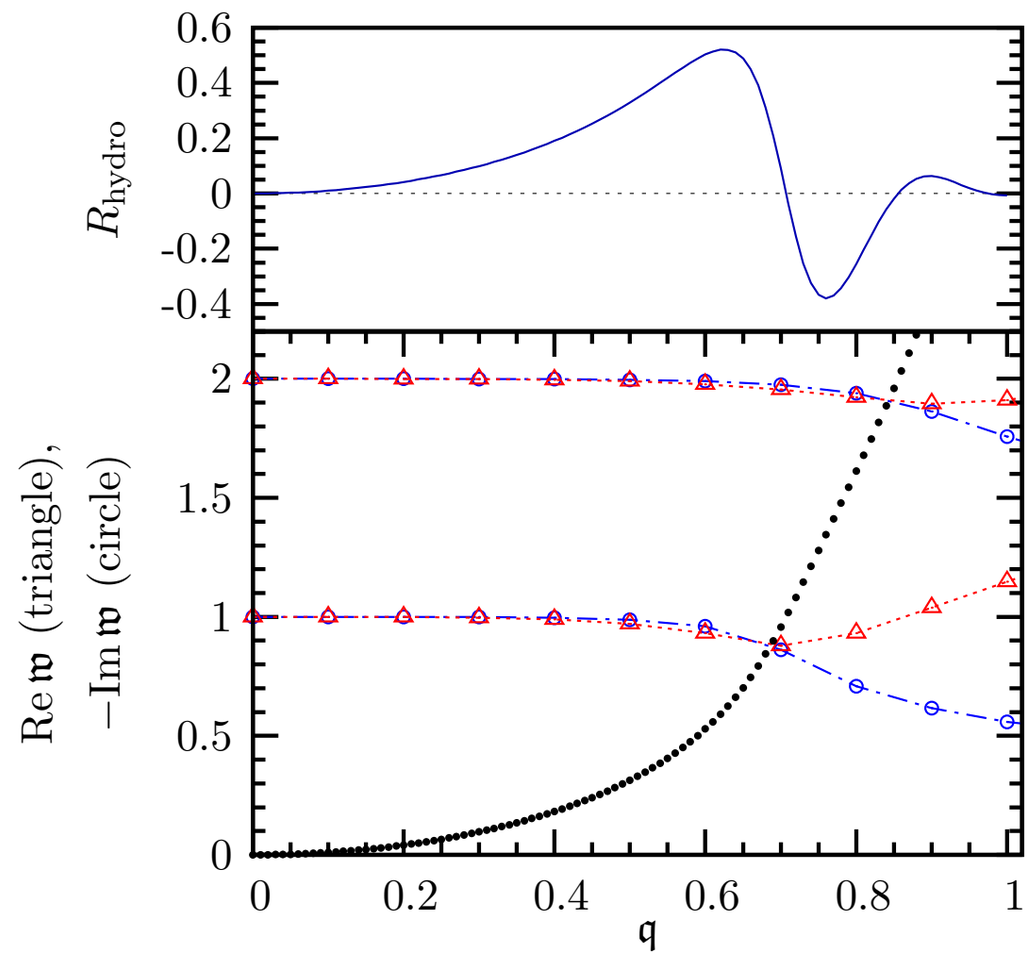
Amado, Hoyos, Landsteiner, Montero, 0710.4458

E_T



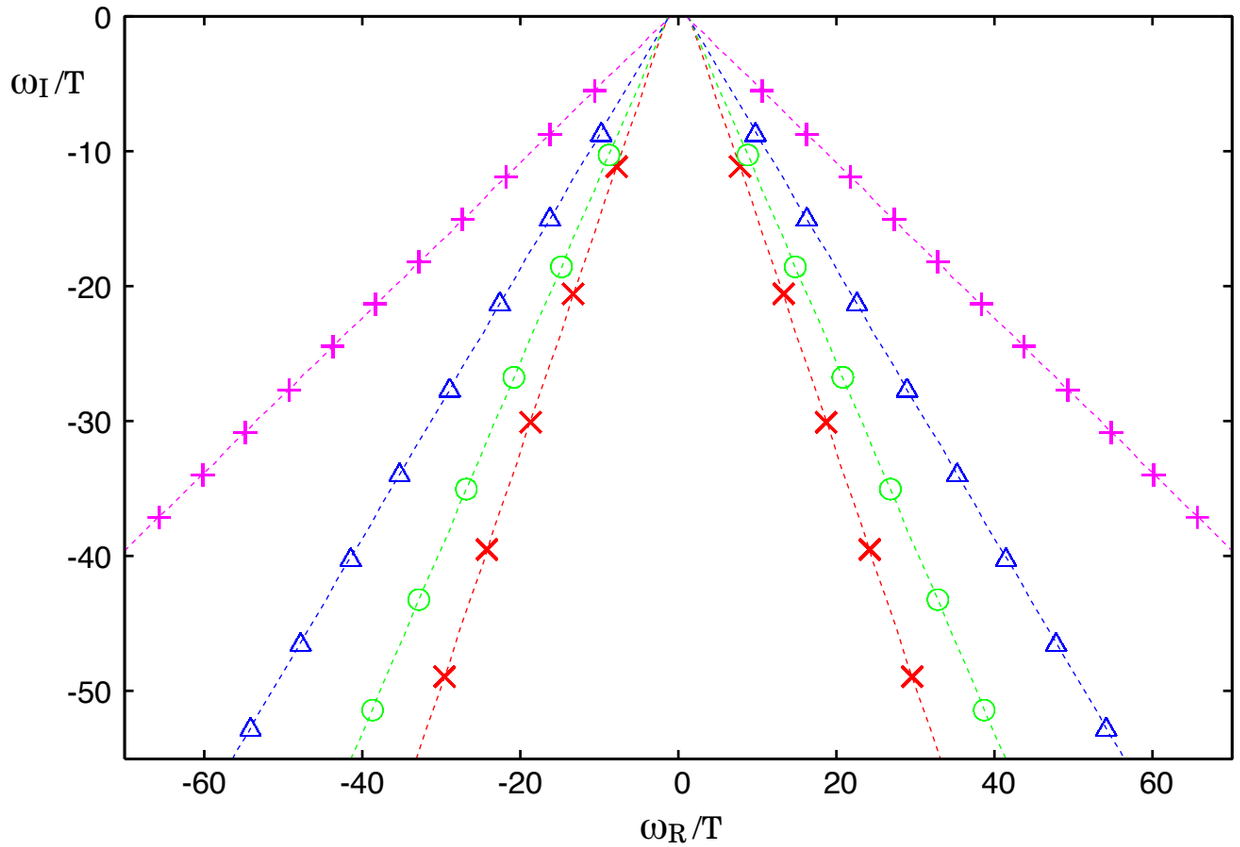
E_L





At weak coupling $\quad R \sim e^{-cq}$

Other D-branes QNM Maeda, Natsuume, Okamura hep-th/0509079

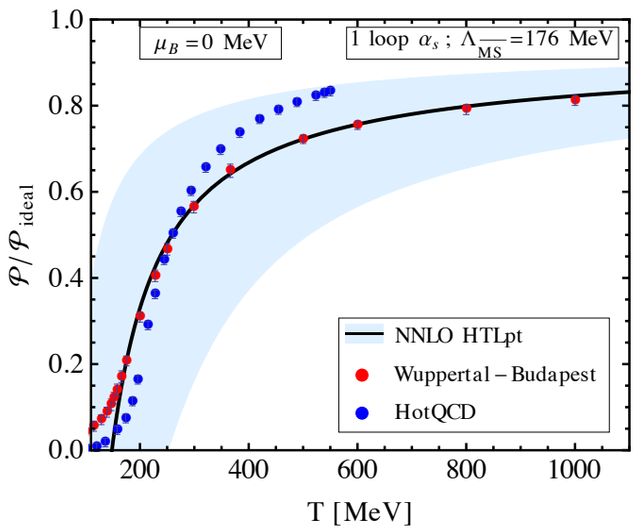


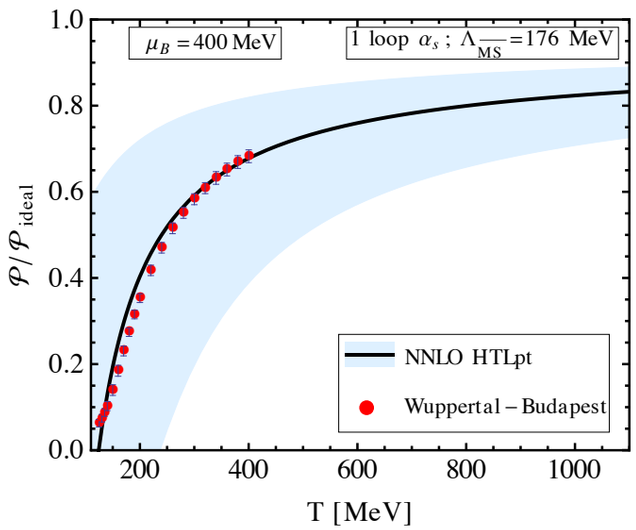
D1(×), D2(○), D3(△) and D4-branes(+).

Deviation from conformality

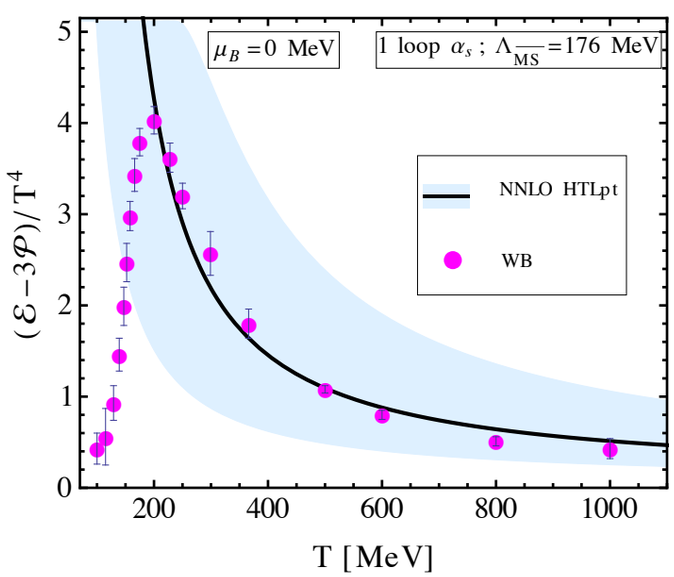
[arXiv:1402.6907]

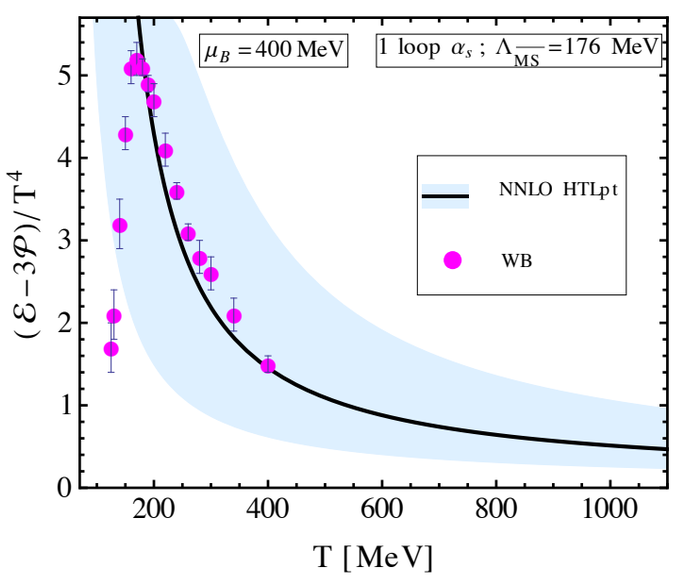
pressure





trace anomaly





Wilson loop test of confinement

Brandhuber, Itzhaki, Sonnenschein, Yankielowicz, hep-th/9803137

$$ds^2 = g_{uu}(u)du^2 - g_{tt}(u)dt^2 + g_{ab}(u)dx^a_{\parallel}dx^b_{\parallel} + g_{ij}(u)dy^i_{\perp}dy^j_{\perp}$$

Embedding $\quad u = u(x)$

$$S_{NG} = \int d^2\sigma\sqrt{\det g_{ind}} = \int dt \int dx \sqrt{f(u(x))^2 + g(u(x))^2 u_x^2}$$

$$f^2 = g_{tt}g_{xx} \qquad g^2 = g_{tt}g_{uu}$$

Integral of motion

$$H = pu_x - L = -\frac{f^2(u)}{\sqrt{f^2 + g^2 u_x^2}} = -f(u_0) \qquad \text{turning point}$$

$$u_x = \pm\frac{f}{g}\frac{\sqrt{f^2 - f_0^2}}{f_0}$$

length $$L = \int dx = 2\int_{u_0}^{u_1} \frac{g}{f}\frac{f_0}{\sqrt{f^2 - f_0^2}}du$$

$$S_{NG} = \int dt \int dx \frac{f^2}{f_0}$$

subtraction $$m_q = \int_0^{u_1} du\, g(u)$$

$$E = Lf_0 + 2\int_{u_0}^{u_1} \frac{g}{f}\left(\sqrt{f^2 - f_0^2} - f\right) - 2\int_0^{u_0} g du$$

confinement if $f_0 \neq 0$ and the other terms are bounded in L

- f has a minimum at u_0
- g diverges at u_0

Popular holographic models of confinement

Hard wall — Polchinski, Strassler hep-th/0109174

Soft wall — Karch, Katz, Son, Stephanov hep-th/0602229

Witten-Sakai-Sugimoto

Improved Holographic QCD

Witten-Sakai-Sugimoto model

Witten, hep-th/9803131

Sakai, Sugimoto hep-th/0412141, hep-th/0507073

Near-horizon limit of N_c $D4$ branes compactified on a circle with non-susy b.c. for fermions

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left[-dt^2+\delta_{ij}dx^i dx^j + f(u)dx_4^2\right] + \left(\frac{R_{D4}}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

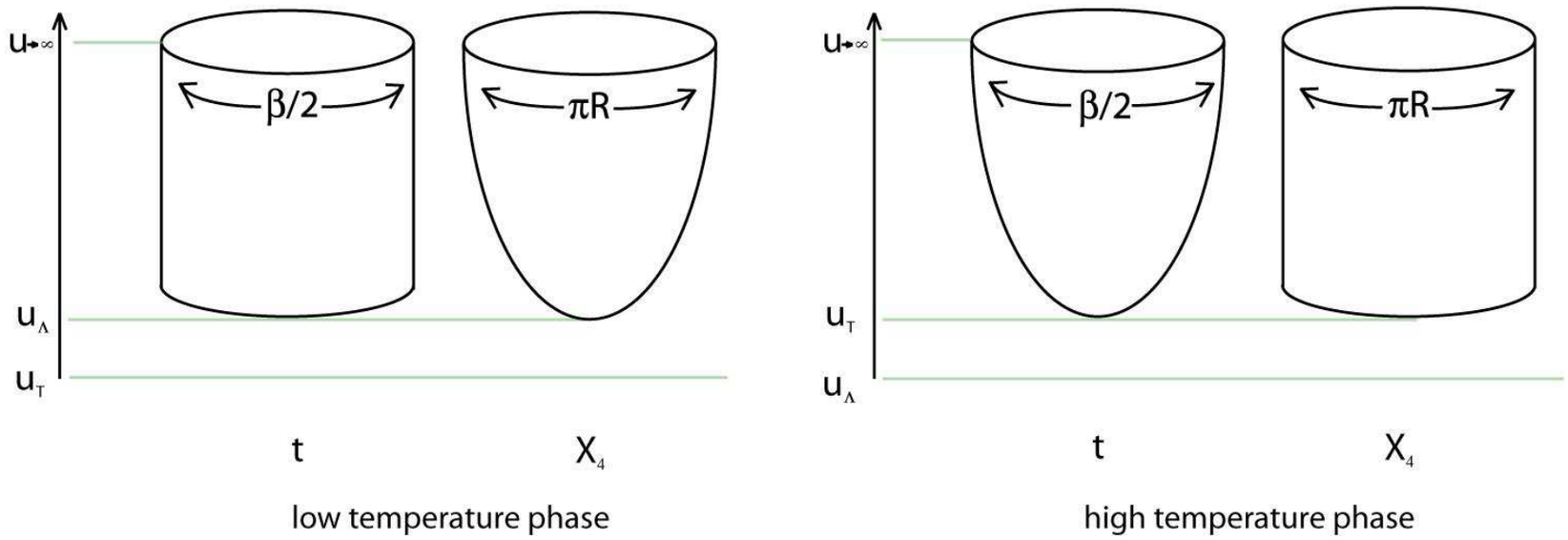
$$e^{\phi} = g_s\left(\frac{u}{R_{D4}}\right)^{3/4}, \quad R_{D4}^3 \equiv \pi g_s N_c l_s^3, \quad f(u) \equiv 1 - \left(\frac{u_\Lambda}{u}\right)^3$$

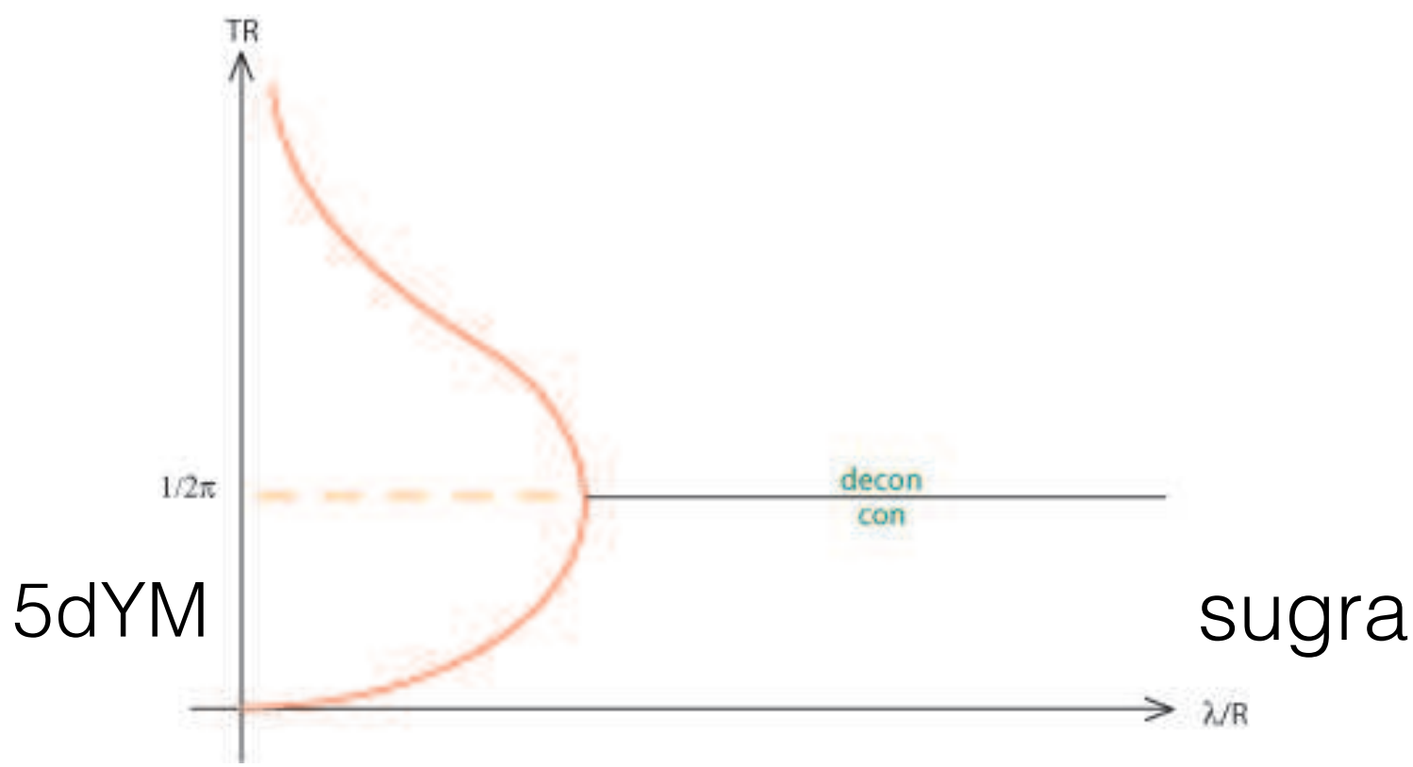
$$\delta x_4 = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_\Lambda}\right)^{1/2} = 2\pi R$$

Low-energy theory is 3+1d YM, but when sugra approx is valid

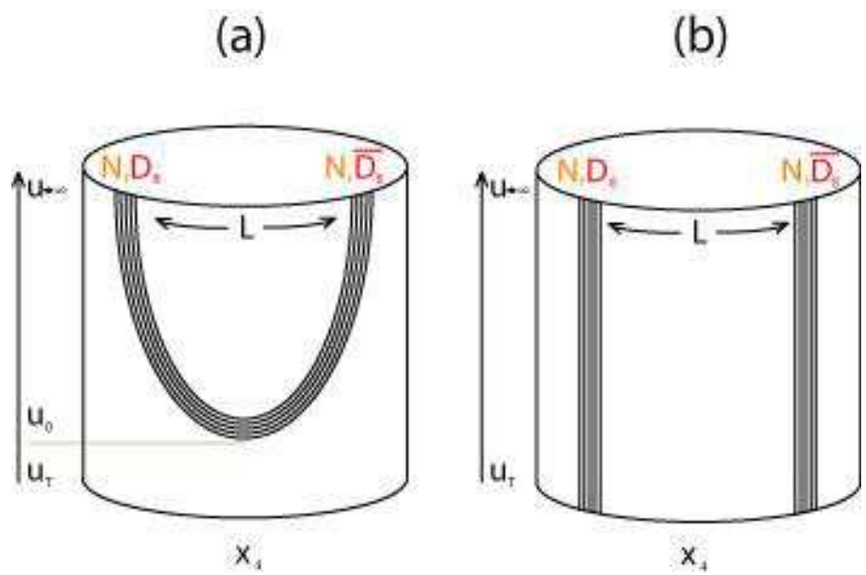
Kaluza Klein scale = glueball mass $= \dfrac{1}{R}$ *no decoupling of scales*

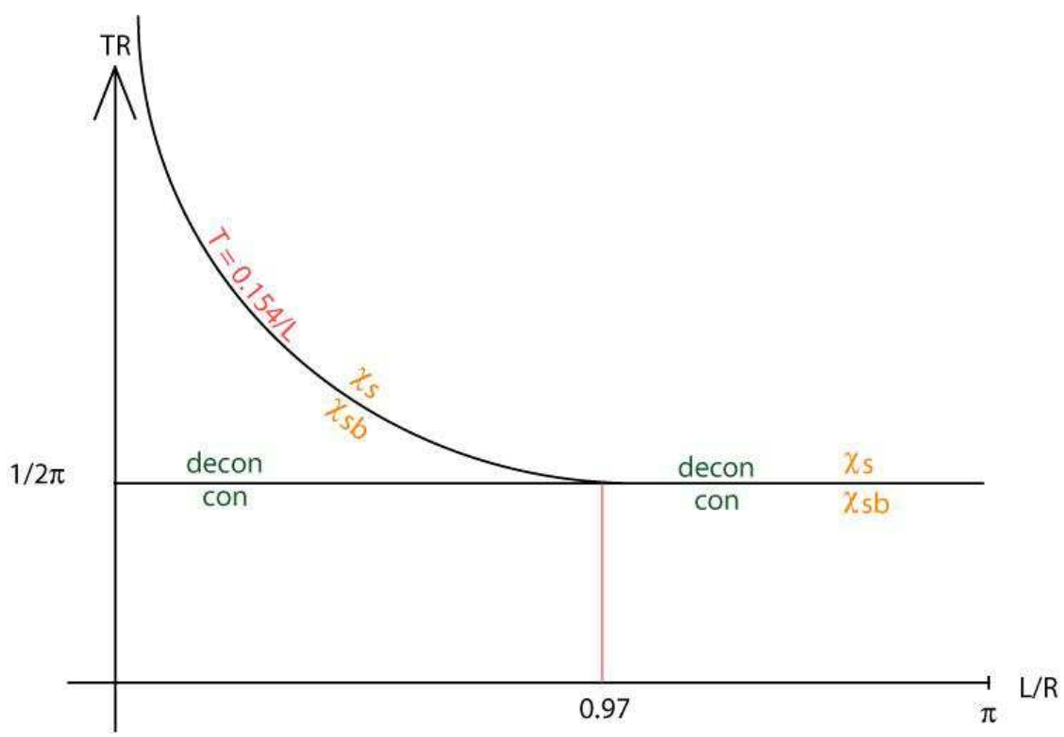
Deconfinement phase transition (1st order)



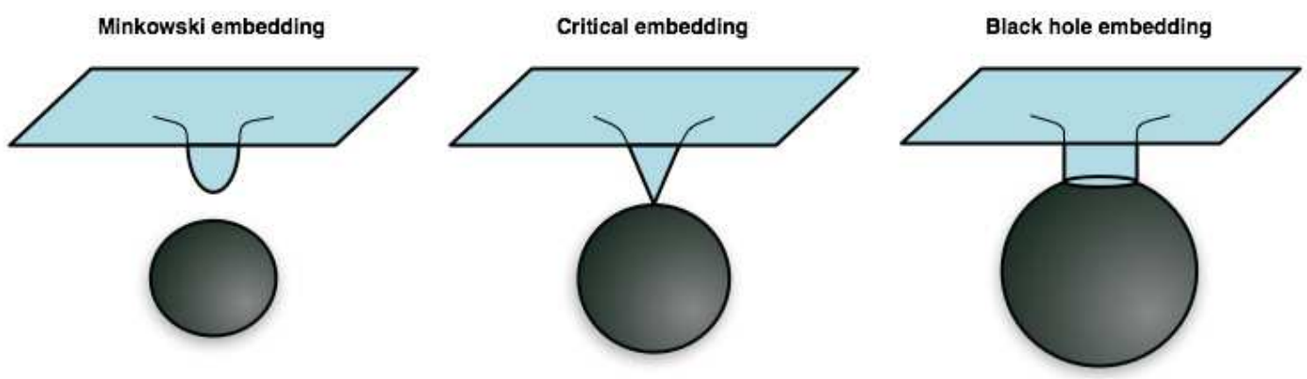


$D8 - \overline{D8}$ probes : chiral symmetry breaking



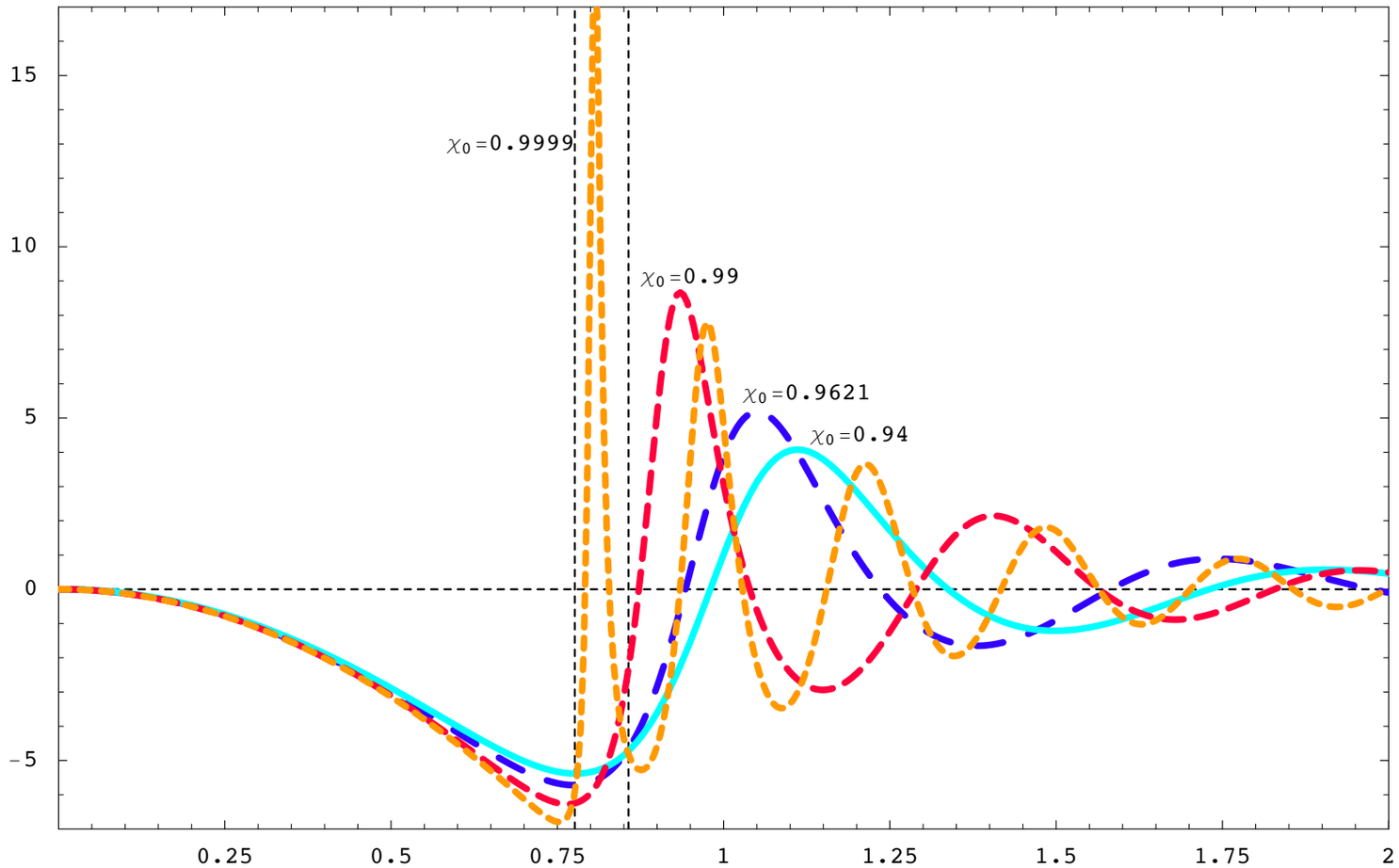


Is there a continuous connection between the mass spectrum in the confined phase and the resonance spectrum in the deconfined phase?



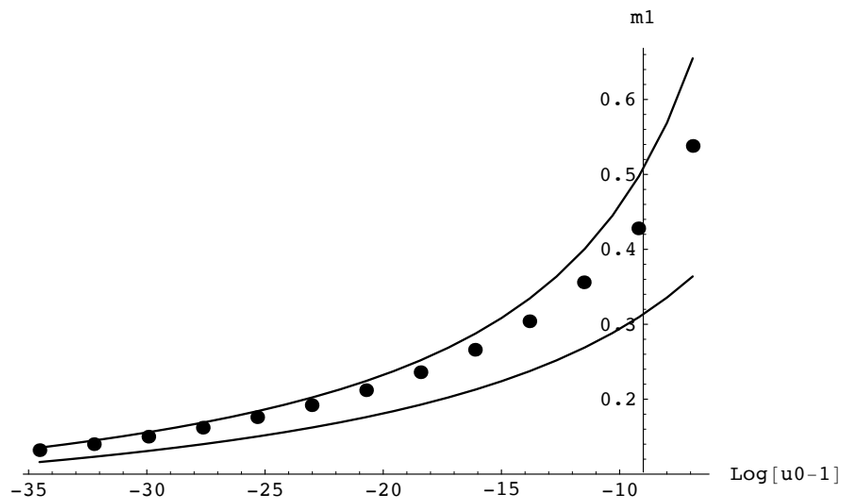
D3-D7

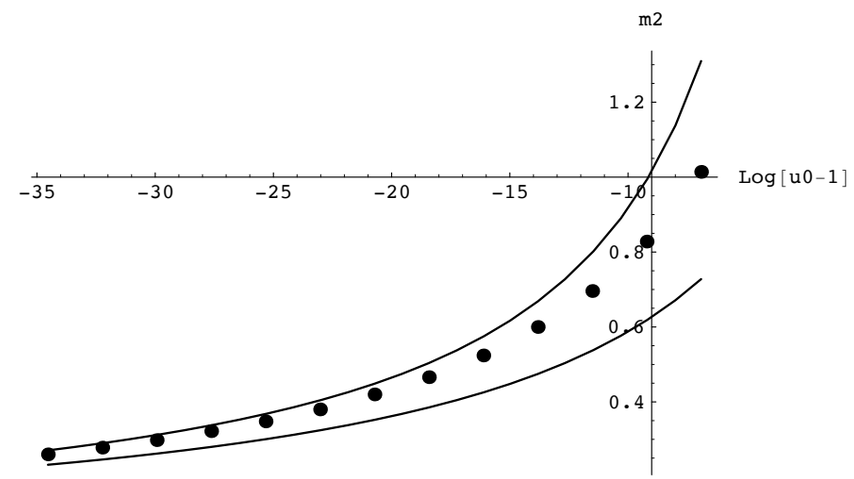
Myers, Starinets, Thompson, 0706.0162



Mesons vs QNM in SS

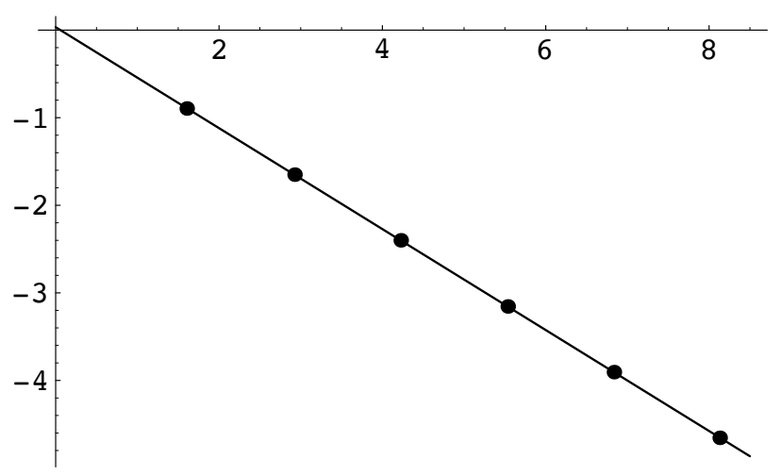
Paredes, Peeters, Zamaklar, 0803.0759





Vector and axial meson masses in chirally broken phase

Mass vanishes at the critical (overheated) embedding



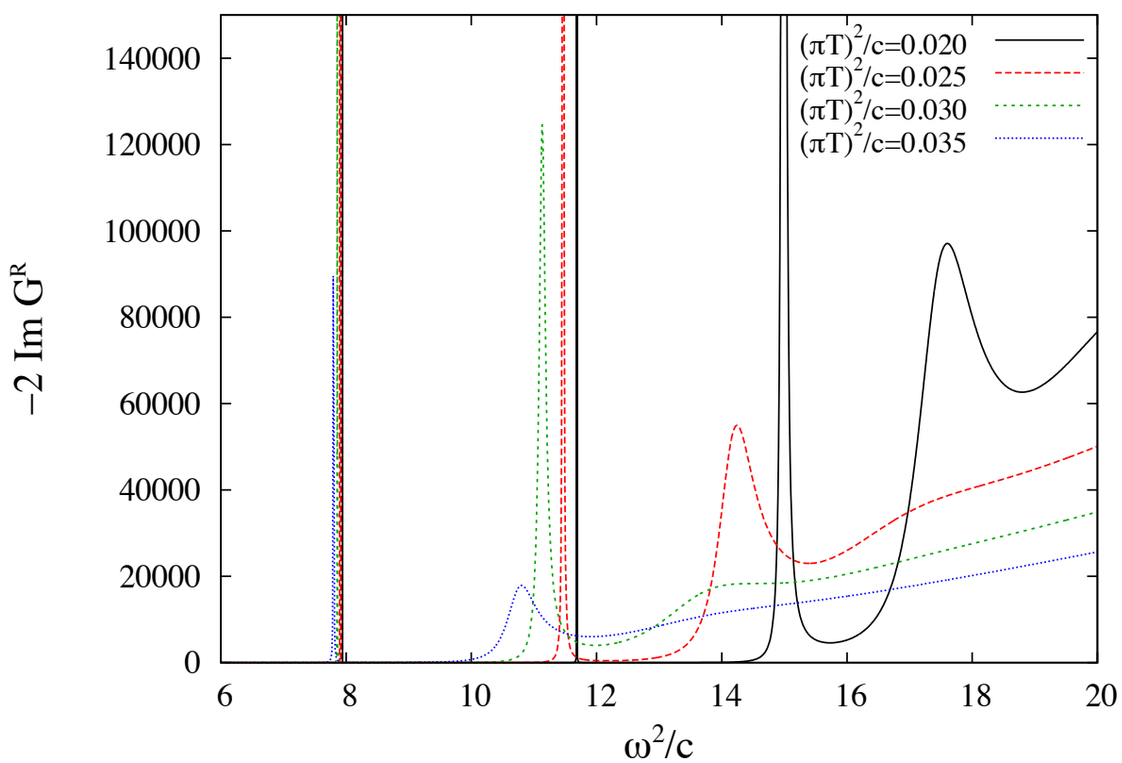
n	ω
1	$\pm 1.61 - 0.90i$
2	$\pm 2.93 - 1.65i$
3	$\pm 4.23 - 2.40i$
4	$\pm 5.54 - 3.16i$
5	$\pm 6.84 - 3.91i$
6	$\pm 8.13 - 4.66i$

QNM in chirally symmetric phase (doubly degenerate)

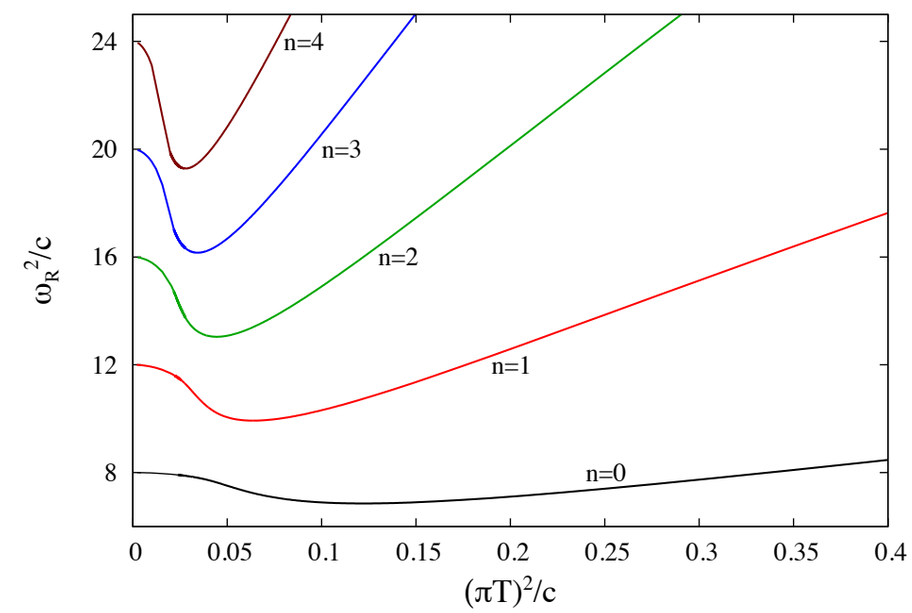
Soft-wall model

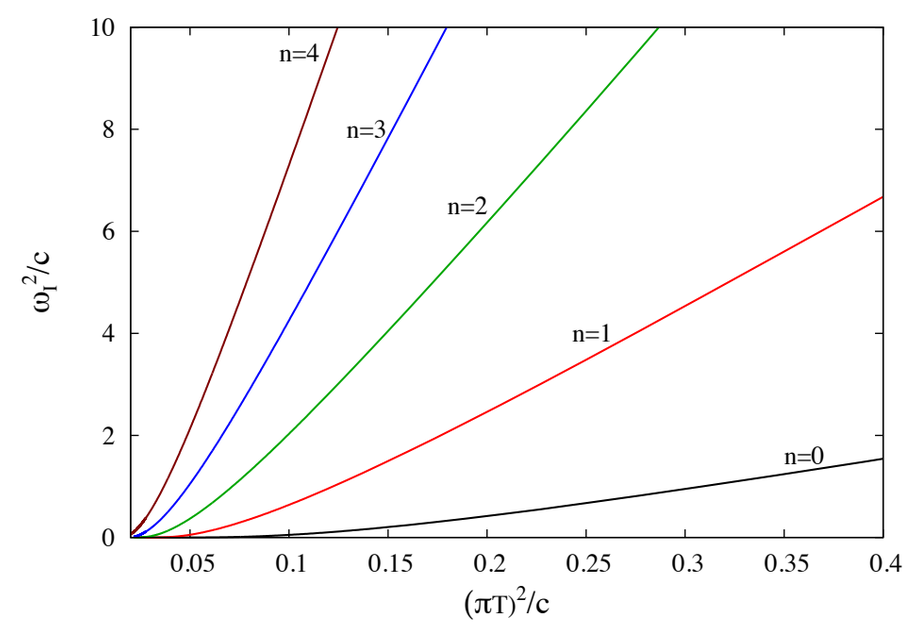
Miranda, Bayona, Boschi-Filho, Braga 0909.1790

Scalar glueball spectral function at q=0 and low T



QNM in BH phase



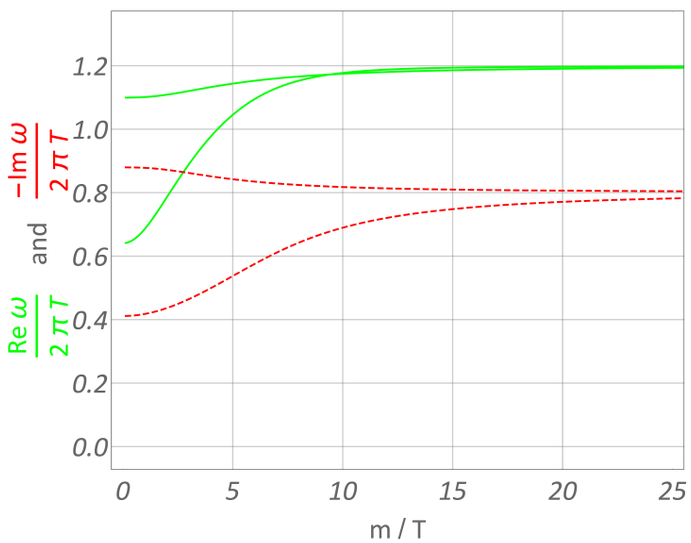


At T=0 the glueball spectrum is reproduced, but at Tc the connection is lost

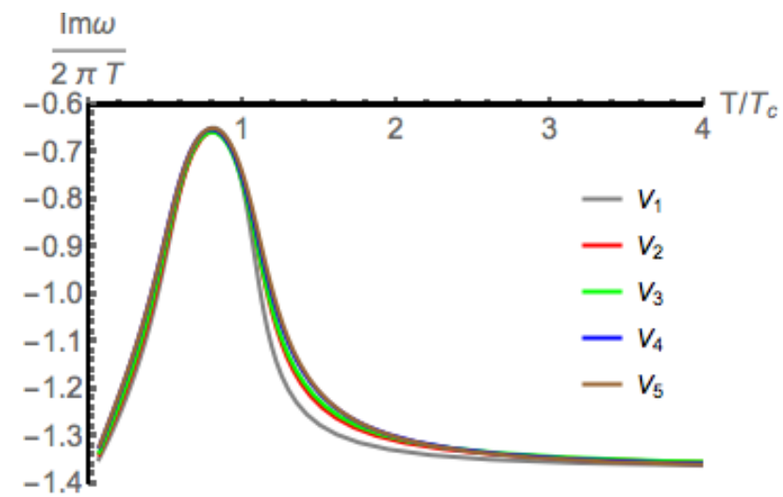
QNM in non-conformal models

Buchel, Heller, Myers 1503.07114

Janik. Plewa, Soltanpanahi, Spalinski 1503.07149



N=2*

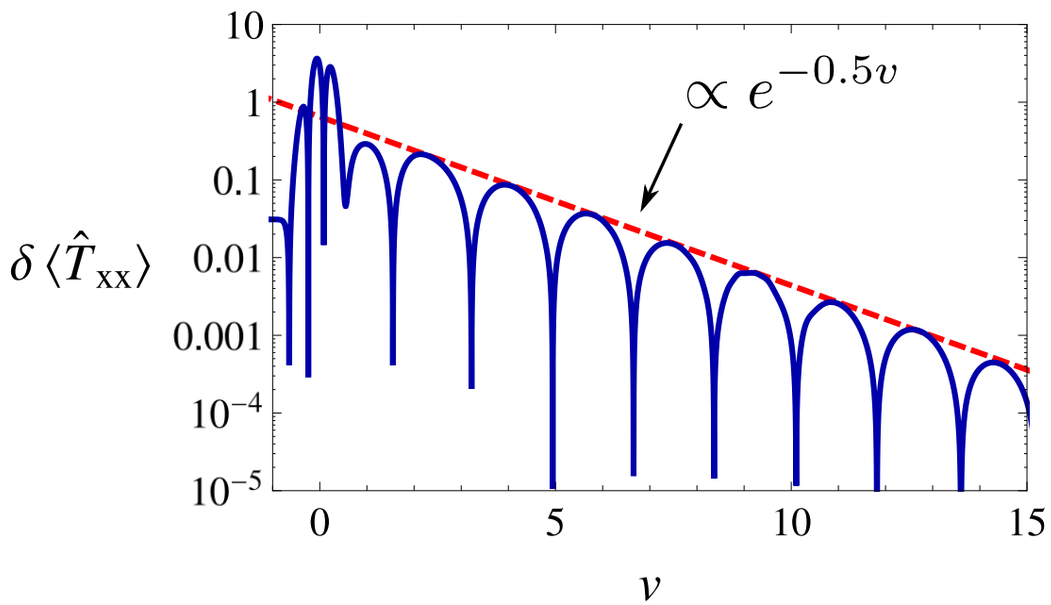


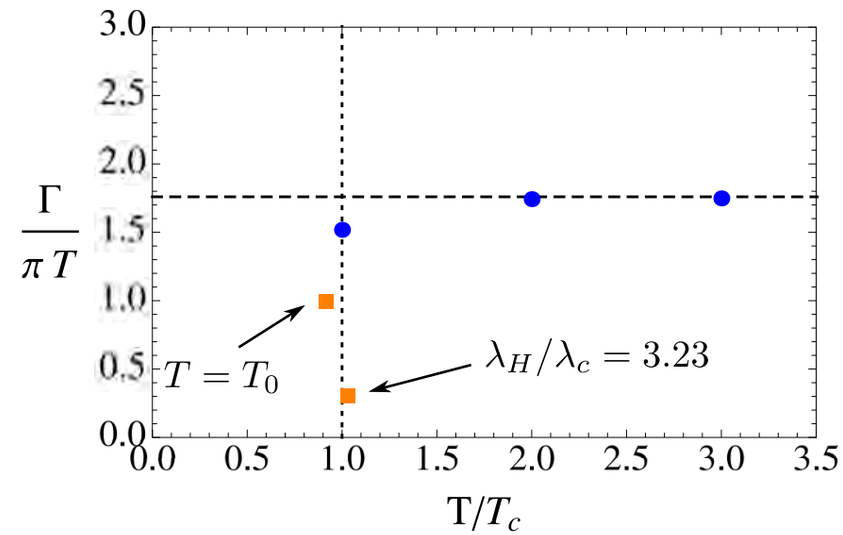
Einstein-scalar with

$$V(\phi) = cosh(\phi) + \phi^2 + \phi^4 + \phi^6$$

Variation of the imaginary part = attenuation rate by factor of ~ 2

Ishii, Kiritsis, Rosen, 1601.01947 consider thermalization after a quench





Relaxation time determined by lowest QNM, weak dependence on Λ

Improved Holographic QCD

Gursoy, Kiritsis, 0707.1324

Gursoy, Kiritsis, Nitti 0707.1349

Gursoy, Kiritsis, Mazzanti, Nitti, 0812.0792, 0903.2859

Bottom-up model using Einstein - dilaton

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\varphi)^2 - V(\varphi)\right) + S_{GH} + S_{ct}$$

$$g_{\mu\nu} \leftrightarrow T_{\mu\nu} \qquad\qquad \varphi \leftrightarrow \mathrm{tr}F^2$$

potential corresponds to beta-function of the dual theory

$$V(\phi) \sim V_0 + \frac{1}{2}m^2\varphi^2 \qquad\qquad \Delta(\Delta - 4) = m^2L^2$$

$m = 0$ UV theory is CFT deformed by a marginally relevant operator

asymptotic freedom: minimum at $\lambda = 0\,,\ \varphi = -\infty$

$$b_0\lambda(r) = -\frac{1}{\log\Lambda r} + O(\frac{\log\log r}{\log^2 r})$$

string frame $\qquad e^{-2\Phi}\sqrt{g_s}\,R_s = \sqrt{g_E}\,R_E$

$$ds^2 = e^{2A(r)}(dr^2 - dt^2 + dx_i dx^i) \qquad A_s = A_E + \frac{2}{3}\Phi$$

$$f^2 = g^2 = e^{4A_s}$$

scale factor

IR $\qquad A(r) \sim -Cr^\alpha + \dots\,, \quad r \to \infty$

UV $\qquad A(r) \sim -\ln r\,, \quad r \to 0$

$$\dot\Phi^2 = -\frac{9}{4}(\ddot A - \dot A^2) \qquad e^{-2A}(3\ddot A + 9\dot A^2) = V(\Phi)$$

$$\Phi \sim -\frac{3}{2}A + \frac{3}{4}\ln|\dot A| + \Phi_0 \qquad A_s \sim \frac{\alpha - 1}{2}\ln\left(\frac{r}{R}\right)$$

$\alpha \geq 1 \qquad$ minimum at $\quad 0 < r_* < \infty \qquad$ Confinement

$$V(\Phi) \sim e^{\frac{4}{3}\Phi}\,\Phi^P\,, \quad P = 2\frac{\alpha - 1}{\alpha} \geq 0$$

Confinement = mass gap

Fluctuations $$S[\xi] = \int dr d^4x \, e^{2B(r)} \left((\partial_r \xi)^2 + (\partial_i \xi)^2 + M^2(r) \xi^2 \right)$$

$$\xi = e^{-B} \psi(r) \eta(x) \qquad \partial_i \partial^i \eta = m^2 \eta$$

$$-\frac{d^2}{dr^2} \psi + V_{eff}(r) \psi = m^2 \psi \qquad V_{eff}(r) = \ddot{B} + \dot{B}^2 + M^2$$

Transverse graviton $$B = \frac{3}{2} A \,, \quad M = 0$$

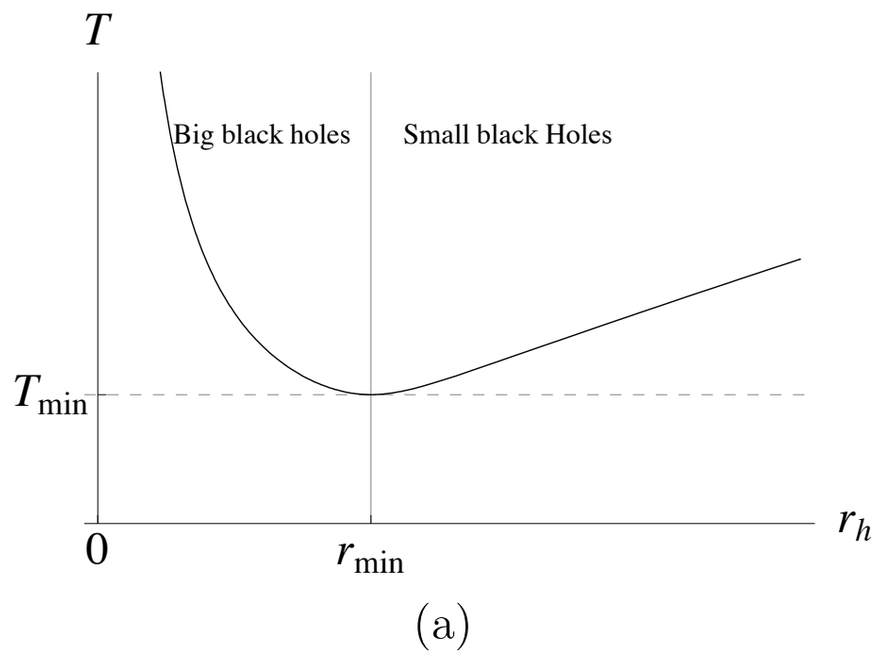
Scalar mode $$\zeta = h^i_i - \frac{1}{3X} \delta\Phi \qquad X = \frac{\Phi'}{3A'}$$

$$B = \frac{3}{2} A + \ln X \,, \quad M = 0$$

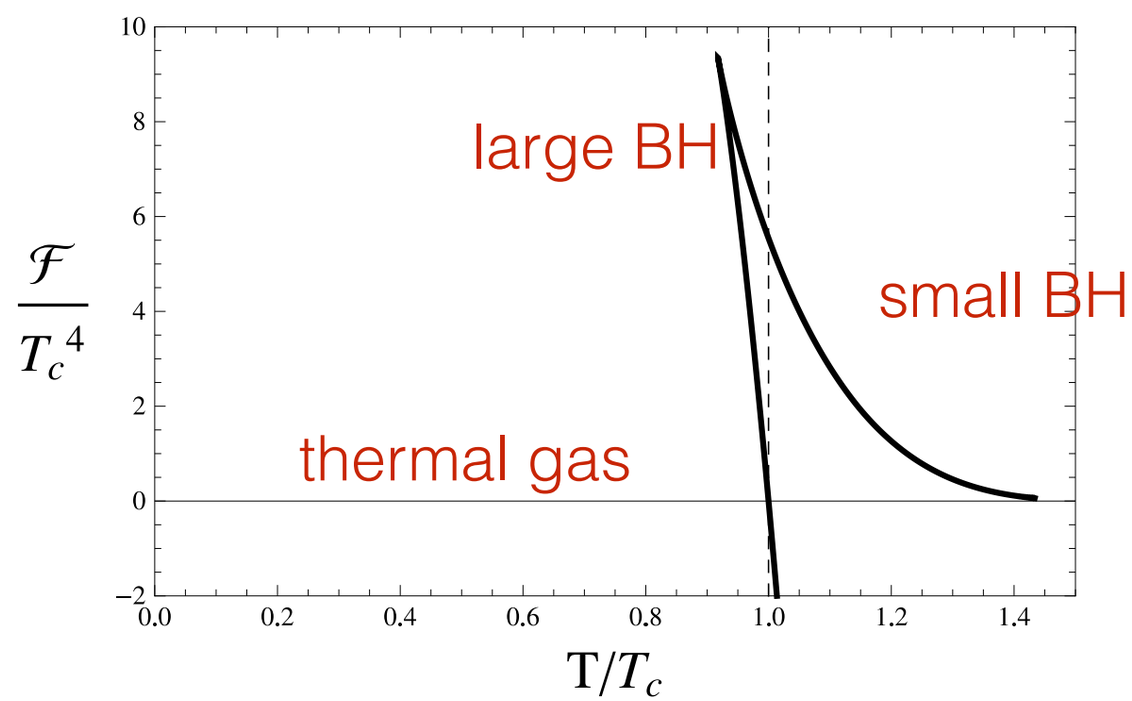
$$V_{eff}(r \to 0) \sim \frac{c}{r^2} \qquad V_{eff}(r \to \infty) \sim r^{2(\alpha - 1)}$$

Gapped spectrum for $\alpha \geq 1$

Confinement = thermal Hawking-Page phase transition



(a)



Chamblin-Reall plasma

Chamblin, Reall, 9903225

$$V(\phi) = \frac{12(1-X^2)}{(4X^2)^2\,\ell^2} e^{-\frac{8X}{3}\phi} \qquad -1 < X < 0$$

Model for IR properties of YM $\qquad \beta = X\lambda$

$$ds^2 = e^{2A_0}\hat{r}^{-\frac{2}{1-4X^2}}\left[-2\ell' d\hat{r}dv - f(r)dv^2 + \delta_{ij}dx^i dx^j\right]$$

$$\lambda = \hat{r}^{-\frac{3X}{1-4X^2}} = \hat{r}^{-X(\xi-1)}$$

$$f(r) = 1 - \left(\frac{r}{r_h}\right)^{\xi} \qquad \xi \equiv \frac{4(1-X^2)}{1-4X^2}$$ = # of extra dimensions

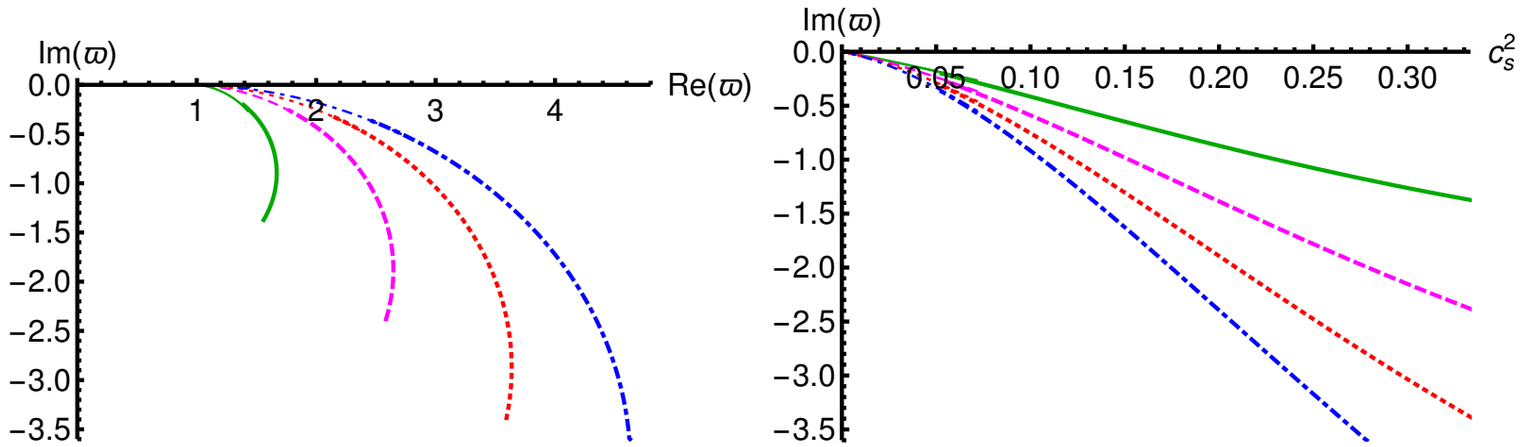
Hyperscaling-violating solution $\qquad S \sim T^{\frac{3}{1-4X^2}}$

$$\frac{p}{\varepsilon} = \frac{1-4X^2}{3} \qquad \frac{\zeta}{s} = -\frac{1}{4\pi}\sqrt{\frac{8}{3}}X$$

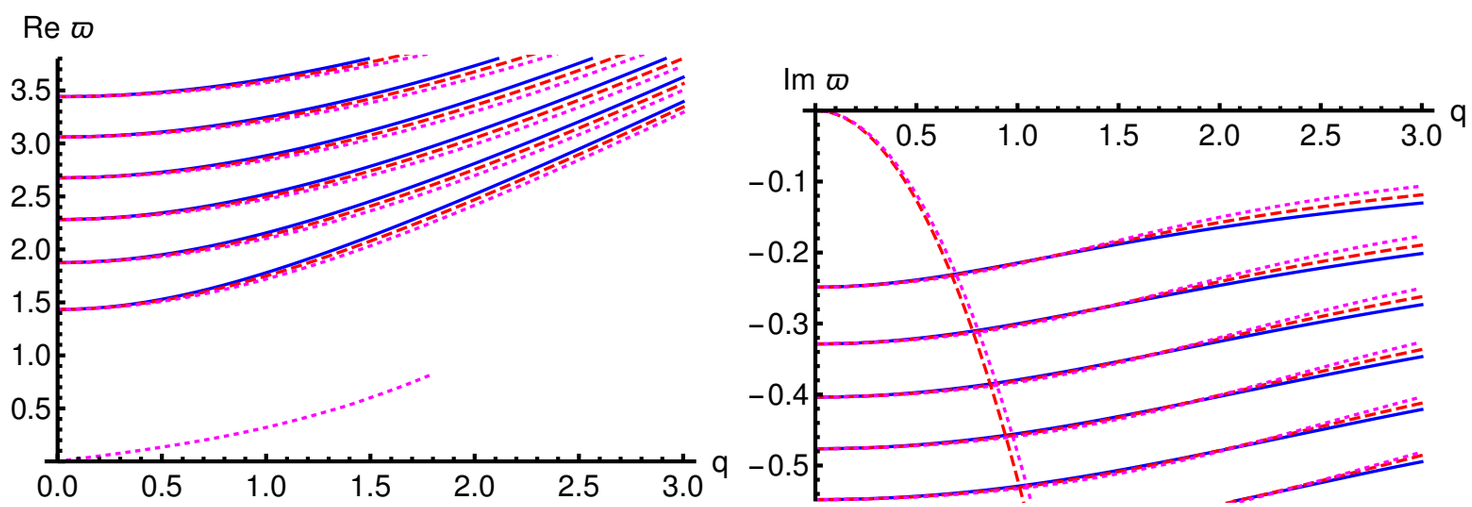
Bjorken flow $\qquad T \sim \tau^{-\frac{1}{3}(1-4X^2)}\log^P \tau$ $\qquad$ Gursoy, Jarvinen, GP, 1507.08628

Spin 2 lowest QNM

Betzios, Gursoy, Jarvinen, GP, 1708.02252
1807.01718



$$q = 0\,,\ -.46 < X < 0$$

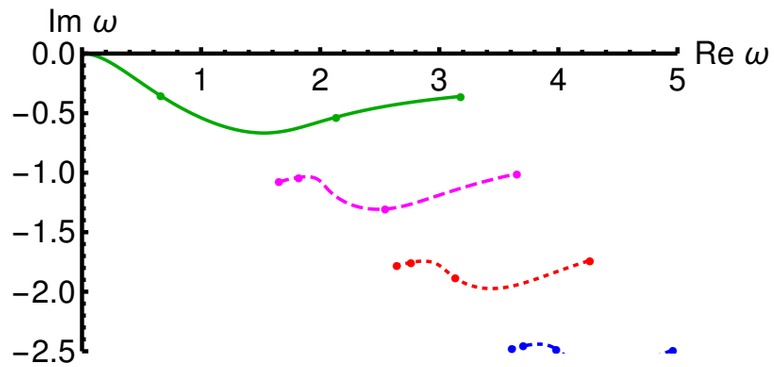


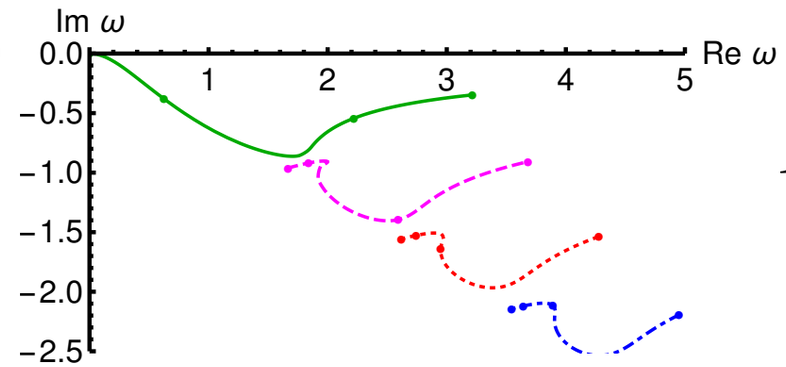
$$X = -.45$$

Crossover of hydro-non hydro modes at $\quad q \sim \dfrac{1}{\sqrt{\xi}}$

Sound modes

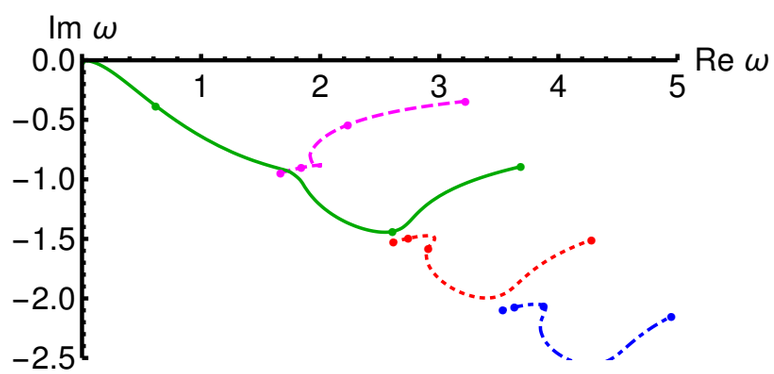
$X = -.25$

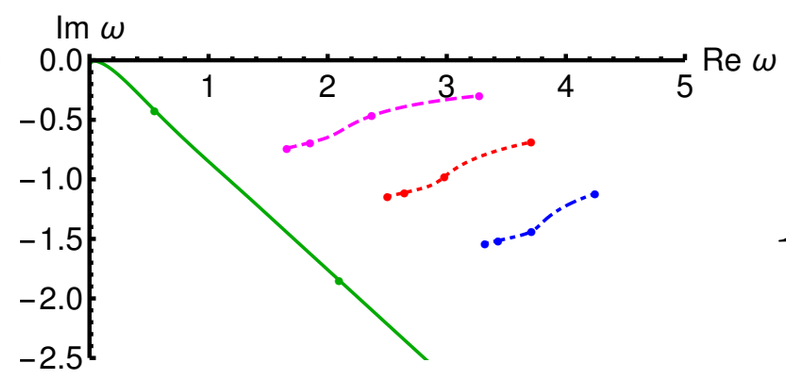




$X = -.29$

$X = -.295$

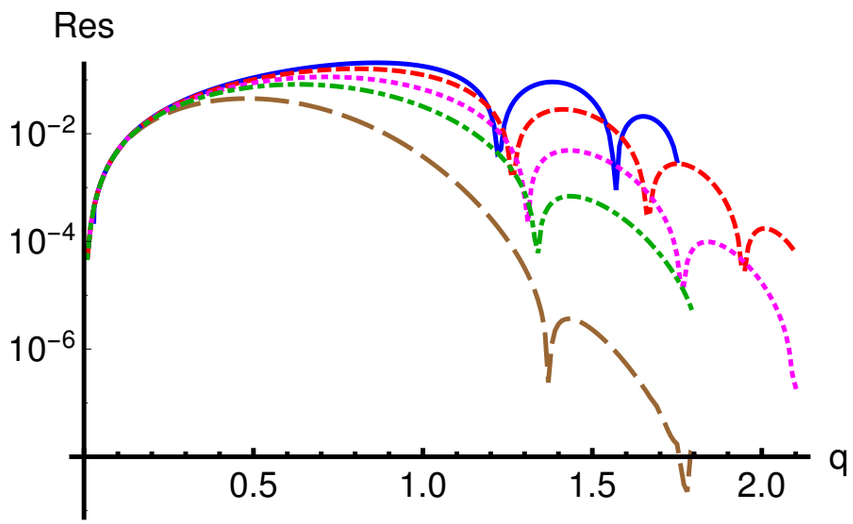


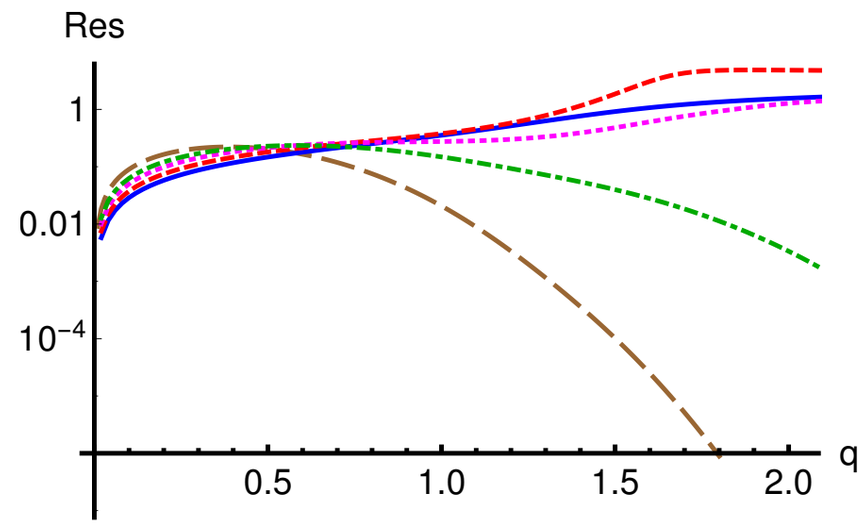


$X = -.35$

Residues

shear





sound

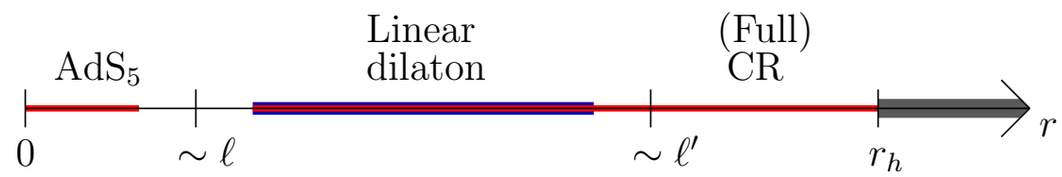
$X = 0$

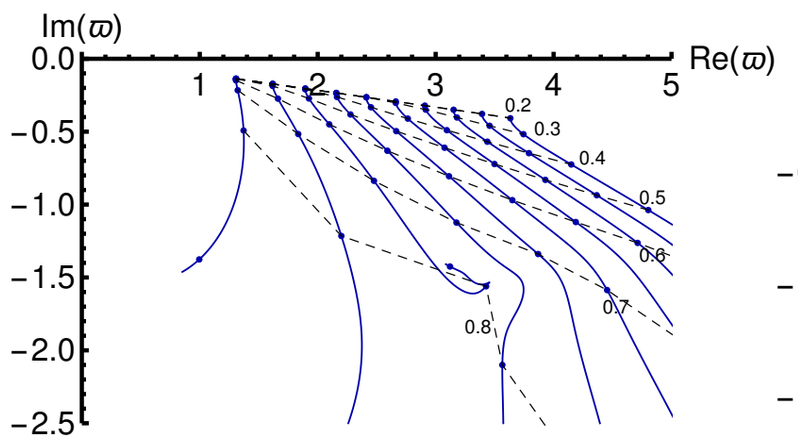
$X = -.45$

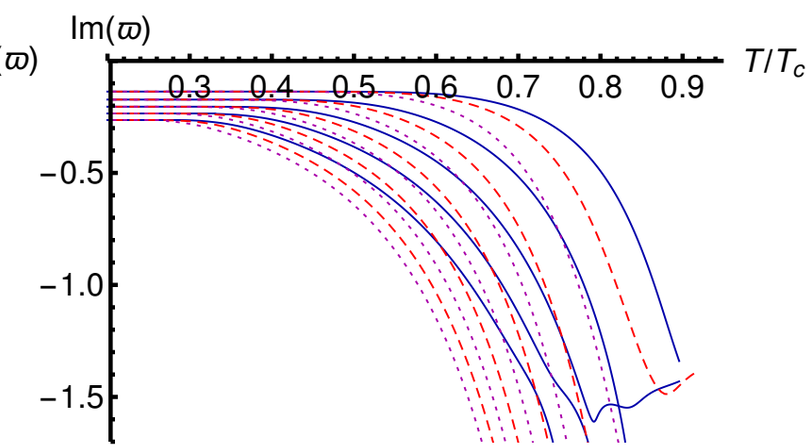
Short wavelength modes

$$\mathrm{Im}\ \frac{\omega}{q} \sim q^{-\frac{2\xi}{\xi+2}}$$

UV-completed geometry

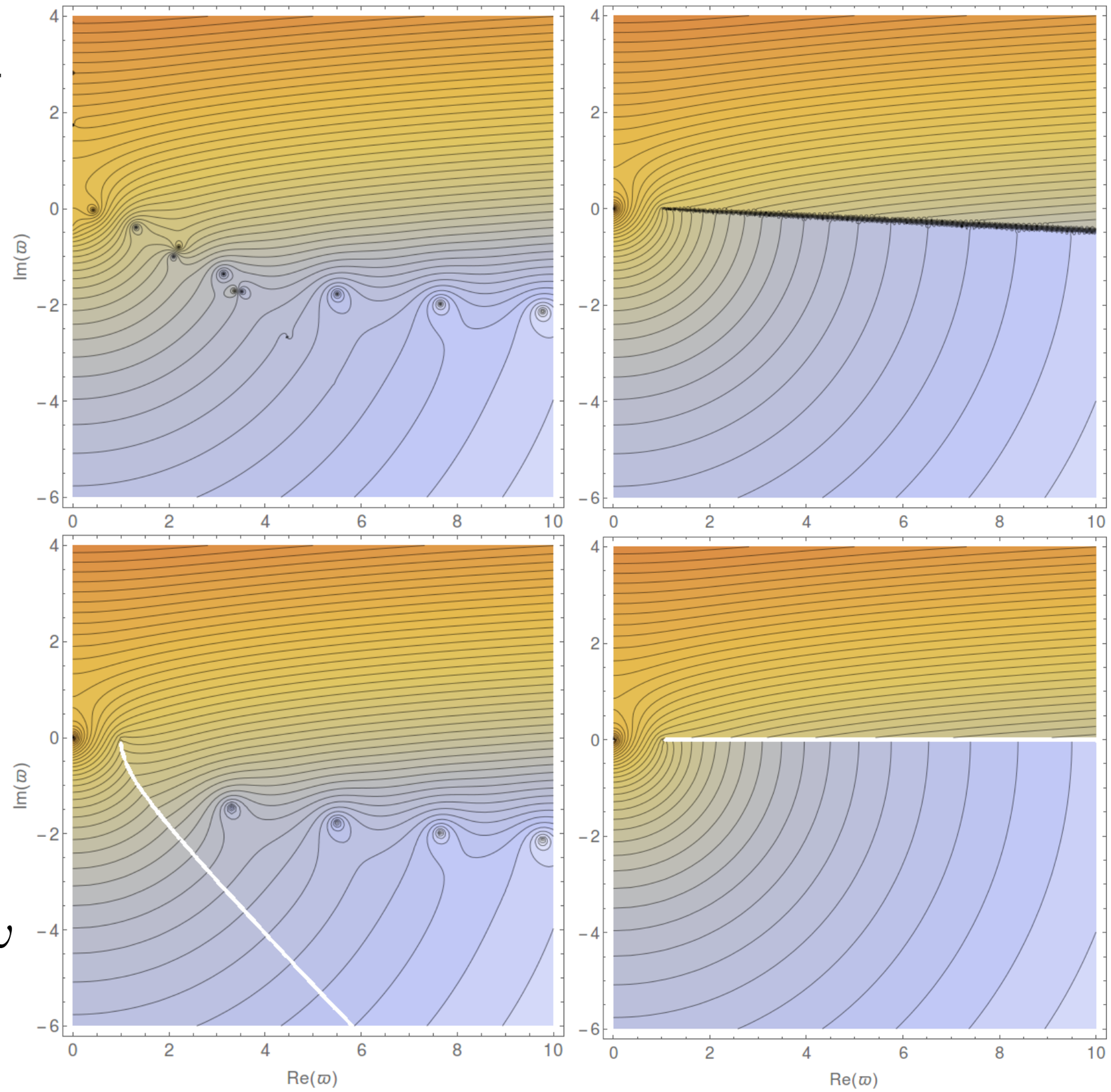






$\log|G(\omega)|$ $\qquad X = -\frac{1}{2}, q = 0$

high T

low T

large ω