Effects of magnetic field on plasma evolution in relativistic heavy-ion collisions

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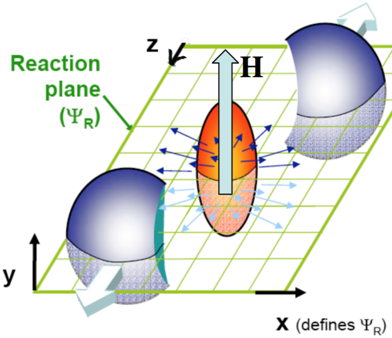
Outline of The Talk

1. Production and survival of magnetic field in relativistic heavy-ion collisions, and validity of ideal magneto-hydrodynamics approximations
2. RMHD equations and algorithm for solving them
3. RMHD simulations for relativistic heavy-ion collisions
4. RMHD simulations for deformed nucleus collisions
5. Conclusions

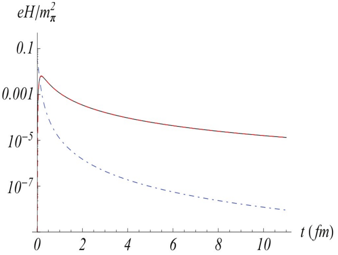
Production of Magnetic Field in HIC



- No magnetic field in the case of central collisions. Fluctuations??
- Magnetic field generates in the non-central collisions along the y-axis at the center.
- Magnitude of magnetic field at the center can be $\sim 10^{15}$ Tesla (~ 0.1 GeV^2) (10^4 times stronger than magnetic field of a magnetar).

Survival of Magnetic Field due to conducting plasma

- Medium forms at thermalization time $\tau_0 < 1$ fm (uncertain) in the presence of time varying magnetic field.
- Quick thermalization and large conductivity of the plasma may protect magnetic field (of high magnitude) from decay.



Time evolution of magnetic field created by a point unit charge at the center of system in vacuum and in plasma of conductivity 5.8 MeV.

Ref.: Kirill Tuchin, Phys. Rev. C, 88, 024911 (2013).

- For simplicity, we consider QGP in relativistic HIC as an ideal MHD fluid. Therefore conductivity of the fluid at each spacetime point is considered to be infinite.

Ideal Relativistic Magneto-hydrodynamics Equations

The basic equations of relativistic magnetohydrodynamics:

- Energy-momentum tensor for e.m. field in medium,

$$T^{\alpha\beta}_{em} = \frac{1}{4\pi}[F^{\alpha}_{\gamma} G^{\beta\gamma} - \frac{1}{4} F_{\gamma\delta} G^{\gamma\delta} \eta^{\alpha\beta}]. \quad (1)$$

  $F^{\alpha\beta}$: Field-strength tensor, $G^{\alpha\beta}$: EM induction tensor.

- Energy-momentum tensor for matter part of ideal fluid,

$$T^{\alpha\beta}_{pl} = (\epsilon + P) u^{\alpha} u^{\beta} + P \eta^{\alpha\beta}. \quad (2)$$

  Here $\eta^{\alpha\beta} =$ diag(-1,1,1,1).

- Energy-momentum tensor for a pefect fluid interacting with e.m. field,

$$T^{\alpha\beta} = T^{\alpha\beta}_{pl} + T^{\alpha\beta}_{em}, \quad (3)$$

  where

$$\partial_{\alpha} T^{\alpha\beta} = 0. \quad (4)$$

Ideal Relativistic Magneto-hydrodynamics Equations

Ideal RMHD Equations

- Energy-momentum conservation equation

$$\partial_\alpha\left((\epsilon + p_g + |b|^2)u^\alpha u^\beta - b^\alpha b^\beta + (p_g + \frac{|b|^2}{2})\eta^{\alpha\beta}\right) = 0 \qquad (5)$$

- Maxwell's equations

$$\partial_\alpha(u^\alpha b^\beta - u^\beta b^\alpha) = 0 \qquad (6)$$

- Four-vector b^α is related with the magnetic field and fluid velocity as,

$$b^\alpha = \gamma\left(\vec{v}.\vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v}.\vec{B})\right) \qquad (7)$$

- Total pressure of the fluid, $p = p_g + \frac{|\vec{B}|^2}{2\gamma^2} + \frac{(\vec{v}.\vec{B})^2}{2}$.

Ref.: A. Mignone and G. Bodo, Mon. Not. R. Astron. Soc.368, 1040 (2006).

Algorithm for solving RMHD Equations

- For computational purpose, the RMHD equations can be conveniently put in the following conservational form*,

$$\frac{\partial \mathbf{U}}{\partial t} + \sum_k \frac{\partial \mathbf{F}^k(\mathbf{U})}{\partial x^k} = 0, \tag{8}$$

where vector **U** is a collection of conservative variables,

$$\mathbf{U} = (m_x, m_y, m_z, B_x, B_y, B_z, E).$$

- m_k is the momentum density along k-th direction (using $p_g = \epsilon/3$),

$$m_k = (\frac{4}{3}\epsilon\gamma^2 + B^2)v_k - (\vec{v}.\vec{B})B_k. \tag{9}$$

- The total energy density,

$$E = \frac{4}{3}\epsilon\gamma^2 - p_g + \frac{B^2}{2} + \frac{v^2 B^2 - (\vec{v}.\vec{B})^2}{2}. \tag{10}$$

*Ref.: A. Mignone and G. Bodo, Mon. Not. R. Astron. Soc.368, 1040 (2006).

Algorithm for solving RMHD Equations

- $\mathbf{F}^k$ are the fluxes along the $x^k = (x, y, z)$ directions,

$$\mathbf{F}^x(\mathbf{U}) = \begin{bmatrix} m_x v_x - B_x \frac{b_x}{\gamma} + p \\ m_y v_x - B_x \frac{b_y}{\gamma} \\ m_z v_x - B_x \frac{b_z}{\gamma} \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ m_x \end{bmatrix}$$

- $\mathbf{F}^{y,z}(\mathbf{U})$ are similarly defined by appropriate change of indices.

Algorithm for solving RMHD Equations

- U evolve with time following the conservation equation.
- Independent variables, $V = (\vec{v}, p_g, \vec{B})$, are required when computing the fluxes.
- To recover V from U, define : $W = \frac{4}{3}\epsilon\gamma^2$ and $S = \vec{m}.\vec{B}$,

$$E = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|\vec{B}|^2 - \frac{S^2}{2W^2} \tag{11}$$

$$|\vec{m}|^2 = (W + |\vec{B}|^2)^2\left(1 - \frac{1}{\gamma^2}\right) - \frac{S^2}{W^2}(2W + |\vec{B}|^2) \tag{12}$$

- In the beginning of each time step, $\vec{m}$, $\vec{B}$ and S are known. γ in terms of W (only unknown) is,

$$\gamma = \left(1 - \frac{S^2(2W + |\vec{B}|^2) + |\vec{m}|^2 W^2}{(W + |\vec{B}|^2)^2 W^2}\right)^{-\frac{1}{2}} \tag{13}$$

Algorithm for solving RMHD Equations

- From EoS,

$$p_g(W) = \frac{W}{4\gamma^2} \tag{14}$$

- Unknown quantity W can be found out from,

$$f(W) = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|\vec{B}|^2 - \frac{S^2}{2W^2} - E = 0 \tag{15}$$

- This equation is solved using Newton-Raphson method to get W.
- Once W has been computed, one can get back γ and p_g . Velocities can be found by expression of m_k,

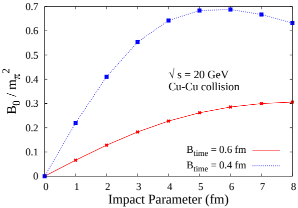
$$v_k = \frac{1}{(W + |\vec{B}|^2)}\left(m_k + \frac{S}{W}B_k\right) \tag{16}$$

Simulation details

- We have performed (3+1)-d simulation on $200 \times 200 \times 200$ lattice with 0.1 fm lattice spacing.
- We have performed low energy collisions of $\sqrt{s} = 20$ GeV with Cu nuclei.
- Because of computational limitations we have taken radius of copper nuclei as 4.0 fm with skin width 0.4 fm.
- Optical Glauber and Glauber Monte-Carlo like initial energy density are used for the simulations.
- We have taken EOS of ideal relativistic gas $p_g = \rho/3$ and zero chemical potential for simplicity.
- Initial central temperature is set to be ~ 180 MeV.

Simulation details

- Magnetic field is produced by considering two oppositely moving, uniformly charged, spheres and by taking appropriately Lorentz γ factor for their motion. The initial magnetic field profile is calculated at the thermalization time τ_0 of the system.



- We use Leap-Frog 2nd order method to solve ideal RMHD equations numerically in (3+1)-d, with total system size 20 fm.
- Initial fluid velocity in the transverse plane is taken to be zero.
- We have taken longitudinal velocity profile $\propto z$ with suitable maximum velocity at the edge of the plasma.
- We have performed our calculations in the central rapidity bin.

Flow Study by Fourier Analysis of fluid momentum distributions in the Transverse Plane

- The Fourier analysis of the azimuthal distribution function,

$$r(\phi) = \frac{\delta P(\phi)}{\overline{P}} = \frac{P(\phi) - \overline{P}}{\overline{P}} = \sum_{n} \Big(a_n \cos(n\phi) + b_n \sin(n\phi) \Big) \quad (17)$$

where, $a_n = \frac{1}{\pi} \int_0^{2\pi} r(\phi) \cos(n\phi)\, d\phi, \quad b_n = \frac{1}{\pi} \int_0^{2\pi} r(\phi) \sin(n\phi)\, d\phi.$

- The flow is characterized by the magnitude of flow coefficients, $v_n = \sqrt{a_n^2 + b_n^2}$ and by direction of flow ψ_n $(0 \leq \psi_n < 2\pi/n)$, where, $a_n = v_n \cos(n\psi_n)$ and $b_n = v_n \sin(n\psi_n)$.

The Azimuthal Distribution Function

$$r(\phi) = v_0 + \sum_{n=1}^{\infty} v_n \cos[n(\phi - \psi_n)] \quad (18)$$

Speed of sound in ideal MHD fluid

- Evolution of small perturbations (from the equilibrium value) in energy density, velocity and magnetic field in ideal MHD fluid provide three sound velocities for plane wave solution of perturbations with wave vector $\vec{k}$,
    1. When $\vec{k} \perp \vec{B}$, MHD equations gives magnetosonic wave of velocity,

$$c_\perp^2 = c_s^2 + v_A^2 \tag{19}$$

    2. When $\vec{k} \| \vec{B}$, MHD equations gives magnetosonic wave of velocity,

$$c_\|^2 = c_s^2 \tag{20}$$

    3. When $\vec{k} \| \vec{B} \perp \vec{v}$, then transverse wave called *Alfvén* wave moves with velocity v_A^2.

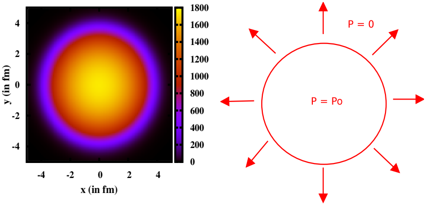
where,

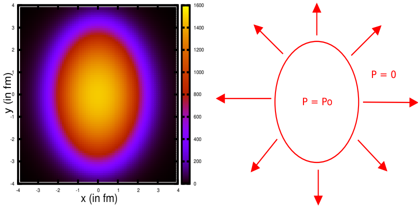
$$c_s = \left(\frac{\partial p}{\partial \epsilon}\right)^{1/2}, v_A \sim \left(\frac{B_0^2}{8\pi\epsilon}\right)^{1/2} \tag{21}$$

Effect of sound speed on fluid evolution in ideal MHD fluid

$$v_x = c_s^2 \left(\frac{xt}{\sigma_x^2} \right), \qquad (22)$$

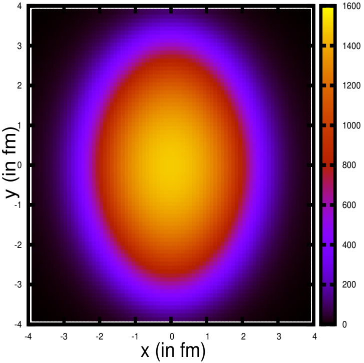
$$v_y = c_s^2 \left(\frac{yt}{\sigma_y^2} \right) \qquad (23)$$

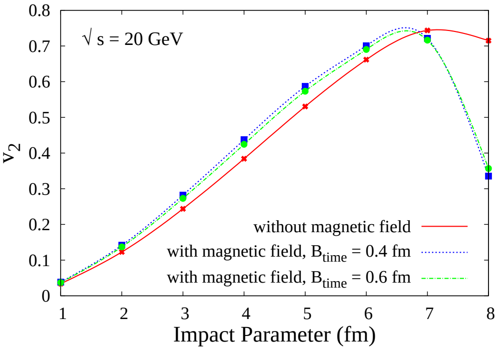




Enhancement of elliptic flow due to magnetic field

Ideal Relativistic Magnetohydrodynamics Simulation result :

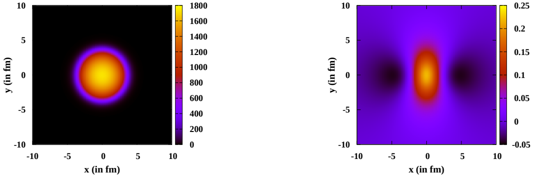




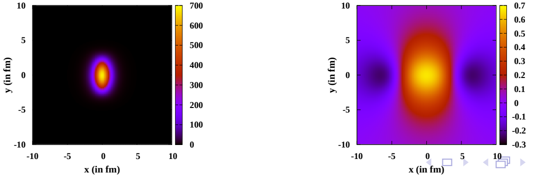
At low impact parameter, magnetic field is well inside the plasma region, hence argument of sound speed holds true.

At high impact parameter, extension of magnetic field much outside the plasma region. Therefore according to the Lenz's law, magnetic field opposes the expansion of the conducting fluid in x-direction.

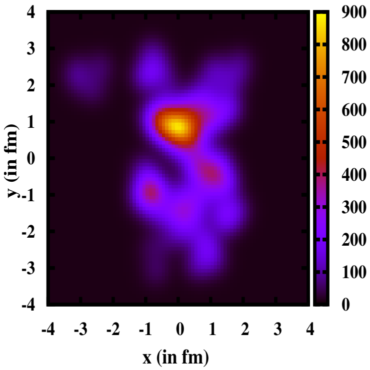
$b = 1\ fm$:

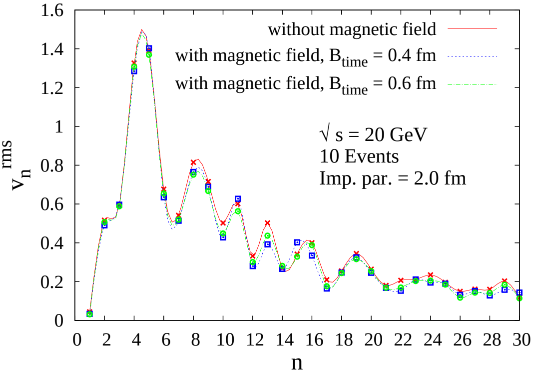


$b = 7\ fm$:

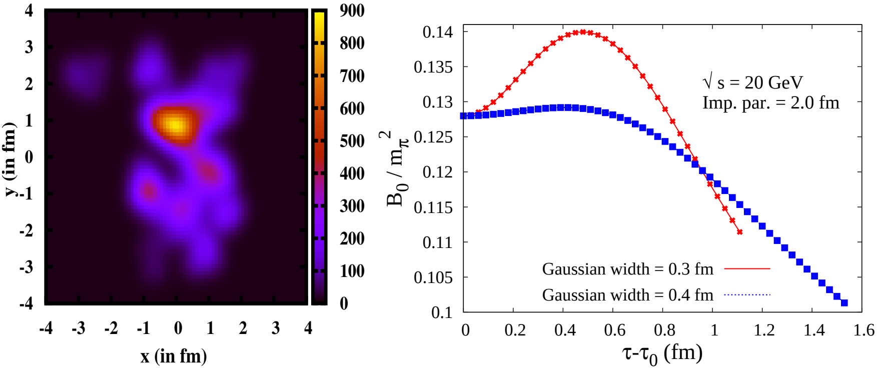


Power spectrum



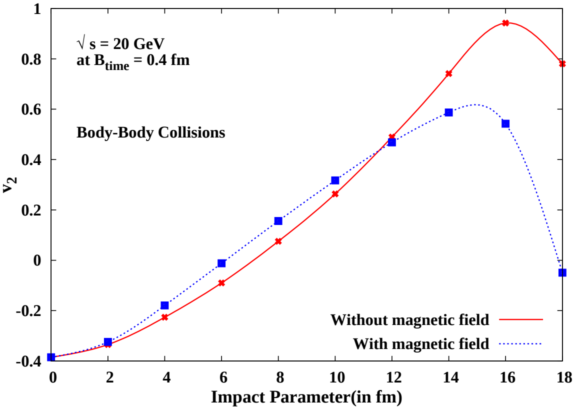


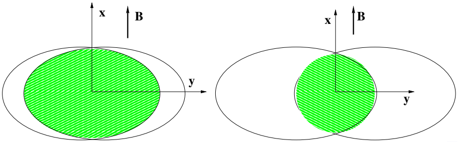
Enhancement of magnetic field due to fluctuation



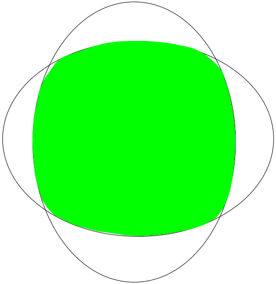
- When fluctuations evolve, magnetic flux can get reorganized. This can lead to local and temporary enhancement of magnetic field.

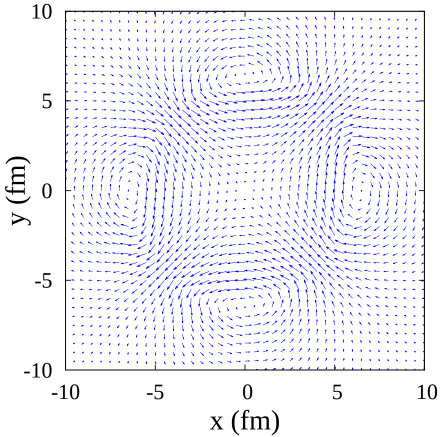
Deformed Nucleus collisions





Deformed Nucleus collisions





- The quardrupolar magnetic field profile suppresses the expansion rate of the fluid in the transverse plane in the central rapidity region. Therefore QGP can survive for a longer time in such a collisions.

Conclusions

- Magnetic field in the fluid can change the ellipic flow depending upon the impact parameter of the collision. It can be very important in the study of viscosity of QGP and can provide signal of the presence of magnetic field.
- We have found that magnetic field can be temporarily enhanced in the fluid due to the evolution of fluctuations.
- In the case of body-body collisions of deformed nuclei, we get suppression in the elliptic flow at very low impact parameter. While at higher impact parameter we get the similar qualitative results as in the case of spherical nuclei collisions.
- Deformed nucleus cross collision can give very interesting possibility of qudrupolar magnetic field in HIC which may suppress the expansion rate of the medium and QGP may survive for a longer time.

Thank You !!