Recent progress in Hard Thermal Loop perturbation theory

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The Myriad Colorful Ways of Understanding Extreme QCD Matter, ICTS

Outline

1 Thermodynamics

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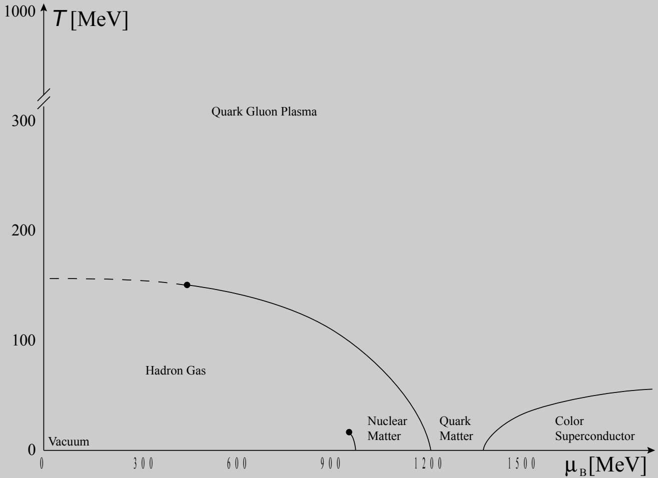


- The determination of the equation of state (EoS) of QCD matter is essential to QGP phenomenology.

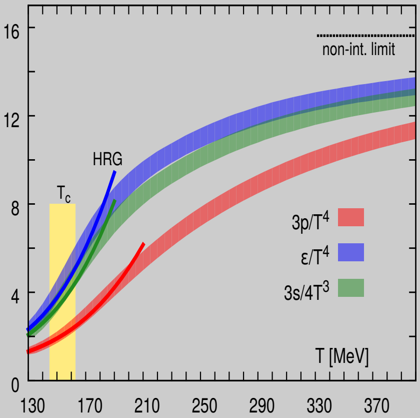
Thermodynamics using Lattice QCD

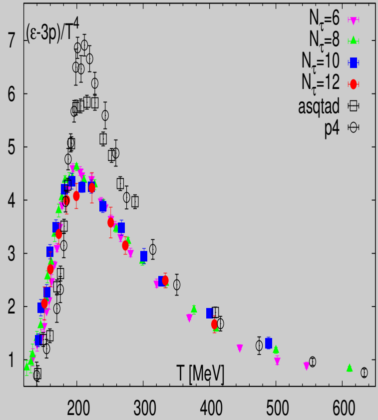
- The currently most reliable method for determining the equation of state at finite temperature is lattice QCD.
- Due to the sign problem, lattice QCD can not compute EoS at finite baryon chemical potential straightforwardly.
- It can compute thermodynamic functions at small chemical potential by making a Taylor expansion of the partition function around $\mu = 0$ and extrapolating the result as

$$P(T,\mu) = P(T,\mu=0) + \frac{\mu^2}{2}\left.\frac{\partial^2 P}{\partial\mu^2}\right|_{\mu=0} + \frac{\mu^4}{4!}\left.\frac{\partial^4 P}{\partial\mu^4}\right|_{\mu=0} + \cdots$$

Lattice EoS

(HotQCD Collaboration), Phys.Rev. D90 (2014) 094503





Limitations of LQCD EoS

- Limited to small chemical potential
- Numerical challenging
- High temperature computation is expensive as:
  Lattice spacing

$$a = \frac{1}{N_\tau T}$$

  For fixed N_τ, a should be smaller at large temperature $\Rightarrow$ More computational time.

$\Rightarrow$ It would be nice to have an alternative framework for calculating QCD thermodynamical quantities at finte T and μ.

Problems in normal perturbation theory

- Gauge dependent result

$$\gamma_g = a\frac{g^2 T}{8\pi},\ a = \begin{cases} 1 & \text{in Coloumb Gauge} \\ -5 & \text{in Feynman Gauge} \end{cases}$$

- Infrared singularities for mass $m \to 0$ though those diagrams are infrared finite at $T = 0$.
- Non-converging behavior the QCD thermodynamic quantities.

$\Rightarrow$ If one really wants to use perturbation theory, perturbation theory needs to be improved/resumed.

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Hard Thermal Loop perturbation theory(HTLpt)

- Hard Thermal Loop resummation originally proposed by Braaten and Pisarski in 1990 to cure serious problems of gauge theories at finite temperature using perturbation theory.
- They suggested that instead of using bare propagators (and vertices) effective propagators, constructed by resumming certain diagrams, the so-called HTL self energies, should be adopted.
- Hard Thermal Loop resummation initially applied to calculate quark and gluon damping rate, photon and dilepton production rate etc.
- In HTL approximation we define *Two Scales of Momentum*
  1. Hard momentum: $p_0, p \sim T$.
  2. Soft momentum: $p_0, p \sim gT$.

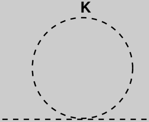
Hard Thermal Loop perturbation theory(HTLpt)

- In HTL approximation we are interested in high temperature limits, so one can take Loop Momentum $\gg$ External Momentum
- In 1999, HTLpt was developed by Andersen, Braaten and Strickland using the concept of HTL resummation.
- HTLpt is a gauge invariant reorganization of usual perturbation at finite temperature and finite chemical potential and higher order diagrams contribute to lower order.

HTL in scalar theory

$$\text{Lagrangian:} \qquad \mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - g^2\phi^4.$$

The one loop self energy:



After subtraction the vacuum contribution it gives

$$\Pi_1 = -12g^2\int\frac{d^4K}{(2\pi)^4}\frac{1}{K^2} = 12g^2T\sum_{n=-\infty}^{n=\infty}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_n^2+k^2} = g^2T^2.$$

Effective propagator

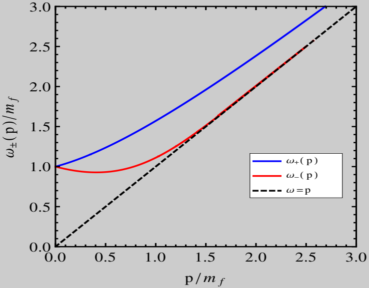
$$D^\star = \frac{1}{P^2-\Pi_1}$$

HTL in Gauge theory: Electron(Quark) Propagator

$$iS^\star(P) \;=\; \frac{1}{\not{P}-\Sigma(P)} = \frac{1}{2}\left[\frac{\gamma^0-\vec{\gamma}\cdot\hat{p}}{D_+(P)}+\frac{\gamma^0+\vec{\gamma}\cdot\hat{p}}{D_-(P)}\right].$$

Self-energy $\Sigma(P)$ and hence $D_\pm(P)$ is calculted in one-loop order within HTL approximation as

$$D_\pm(p_0,p) \;=\; -p_0 \pm p + \frac{m_q^2}{p}\left[\pm 1 + \frac{1}{2}\left(1 \mp \frac{p_0}{p}\right)\ln\frac{p_0+p}{p_0-p}\right]$$



General structure of photon/gluon self energy at finite T

- Four velocity u_μ of heat bath is introduced because of presence of medium. We choose rest frame of heat bath $u_\mu = (1, 0, 0, 0)$.
- Two independent Lorentz scalars are P^2 and $P \cdot u = \omega$.
- Space and time parts are not in same footing.
- Now we need to construct manifestly covariant structure of photon self energy.
- Now available tensors to construct $\Pi_{\mu\nu}$ are $\eta_{\mu\nu}, P_\mu P_\nu, u_\mu u_\nu, P_\mu u_\nu + u_\mu P_\nu$.

$$\text{Let, } \Pi^{\mu\nu} = \alpha\eta^{\mu\nu} + \beta P^\mu P^\nu + \gamma u^\mu u^\nu + \delta(P^\mu u^\nu + u^\mu P^\nu)$$

- $P_\mu \Pi^{\mu\nu} = 0$ gives two constraints.

Effective photon propagator at finite temperature

- Now, general structure of photon self energy at $T \neq 0$ can be written as,

$$\Pi_{\mu\nu} = \Pi_T(\omega, p) A_{\mu\nu} + \Pi_L(\omega, p) B_{\mu\nu}$$

- $\Pi_T = \frac{1}{2} A_{\mu\nu} \Pi^{\mu\nu}$.
- $\Pi_L = B_{\mu\nu} \Pi^{\mu\nu}$.
- Using this one can find the propagator

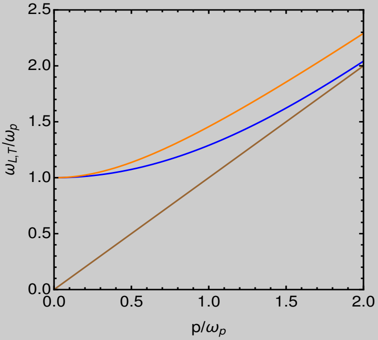
$$D_{\mu\nu} = \frac{\xi}{P^4} P_\mu P_\nu + \frac{1}{P^2 + \Pi_T} A_{\mu\nu} + \frac{1}{P^2 + \Pi_L} B_{\mu\nu}$$

Photon/Gluon Propagator

One loop photon/gluon self-energy can be calculated within HTL approximation and $\Pi_{L,T}$ become

$$\Pi_L(p_0,p) = -\frac{p_0^2-p^2}{p^2}m_D^2\left[1-\frac{p_0}{2p}\ \log\frac{p_0+p}{p_0-p}\right]$$

$$\Pi_L(p_0,p) = \frac{m_D^2}{2}\frac{p_0^2}{p^2}\left[1-\frac{p_0^2-p^2}{2p_0p}\ \log\frac{p_0+p}{p_0-p}\right]$$



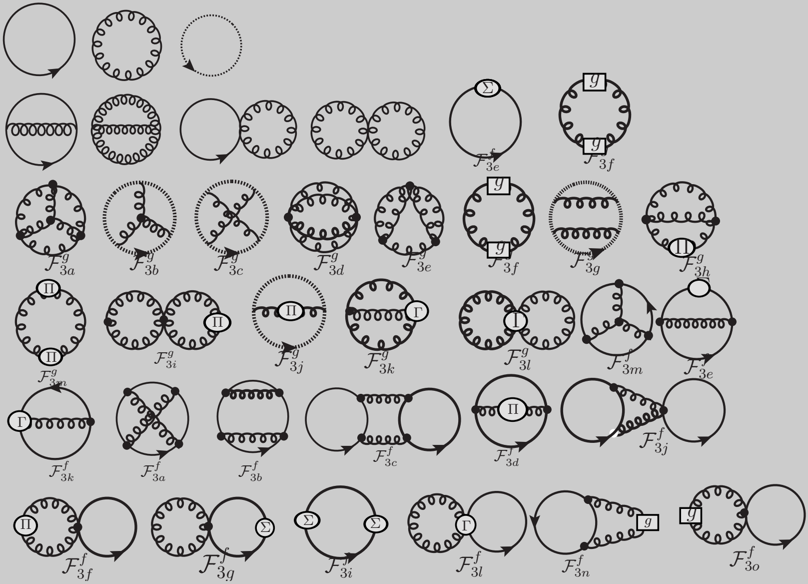
- Total Lagrangian density:

$$\mathcal{L} = (\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL})\big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL},$$

$$\mathcal{L}_{\rm HTL} = (1-\delta) i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{Y_\mu}{Y\cdot D}\right\rangle_{\hat{\mathbf{y}}} \psi - \frac{1}{2}(1-\delta) m_D^2 \,\mathrm{Tr}\left(F_{\mu\alpha}\left\langle \frac{Y^\alpha Y_\beta}{(Y\cdot D)^2}\right\rangle_{\hat{\mathbf{y}}} F^{\mu\beta}\right),$$

- The HTLpt Lagrangian reduces to the QCD Lagrangian if we set $\delta = 1$.
- Physical observables are calculated in HTLpt by expanding in powers of δ, truncating at some specified order, and then setting $\delta = 1$.
- m_D and m_q are two parameters will be treated as Debye mass and thermal quark mass respectively.

Three loop HTL thermodynamics



$$\mathcal{P}_{\rm NNLO} = \frac{d_A\pi^2T^4}{45}\Bigg[\!\!\Bigg[1+\frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\right)-\frac{15}{4}\hat m_D^3-\frac{s_F\alpha_s}{\pi}\bigg[\frac{5}{8}\left(5+72\hat\mu^2+144\hat\mu^4\right)$$

$$+90\hat m_q^2\hat m_D-\frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D-\frac{15}{2}\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\right)\hat m_D^3\bigg]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\bigg[-\frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right)$$

$$+\frac{15}{64}\bigg\{35-32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2+1328\hat\mu^4+64\Big(6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z)$$

$$-36i\hat\mu\aleph(2,z)\Big)\bigg\}\bigg]+\left(\frac{s_F\alpha_s}{\pi}\right)^2\bigg[\frac{5}{4\hat m_D}\left(1+12\hat\mu^2\right)^2+30\left(1+12\hat\mu^2\right)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\bigg\{\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right)$$

$$+\left(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\right)\ln\frac{\hat\Lambda}{2}+\frac{3\gamma_E}{5}\left(1+12\hat\mu^2\right)^2-\frac{8}{5}(1+12\hat\mu^2)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{72}{5}\Big[3\aleph(3,2z)$$

$$+8\aleph(3,z)-12\hat\mu^2\aleph(1,2z)-2(1+8\hat\mu^2)\aleph(1,z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu(1+12\hat\mu^2)\,\aleph(0,z)\Big]\bigg\}$$

$$-\frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\right)\hat m_D\bigg]+\frac{c_A\alpha_s}{3\pi}\frac{s_F\alpha_s}{\pi}\bigg[\frac{15}{2\hat m_D}\left(1+12\hat\mu^2\right)+90\frac{\hat m_q^2}{\hat m_D}$$

$$-\frac{235}{16}\bigg\{\left(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\right)\ln\frac{\hat\Lambda}{2}-\frac{24\gamma_E}{47}\left(1+12\mu^2\right)+\frac{319}{940}\left(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\right)-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)}$$

$$-\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D-\frac{44}{47}\left(1+\frac{156}{11}\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{72}{47}\Big[4i\hat\mu\aleph(0,z)+\left(5-92\hat\mu^2\right)\aleph(1,z)+144i\hat\mu\aleph(2,z)$$

$$+52\aleph(3,z)\Big]\bigg\}+\frac{315}{4}\bigg\{\left(1+\frac{132}{7}\hat\mu^2\right)\ln\frac{\hat\Lambda}{2}+\frac{11}{7}\left(1+12\hat\mu^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat\mu^2\right)+\frac{2}{7}\aleph(z)\bigg\}\hat m_D\bigg]$$

$$+\frac{c_A\alpha_s}{3\pi}\bigg[-\frac{15}{4}+\frac{45}{2}\hat m_D-\frac{135}{2}\hat m_D^2-\frac{495}{4}\left(\ln\frac{\hat\Lambda_g}{2}+\frac{5}{22}+\gamma_E\right)\bigg]+\left(\frac{c_A\alpha_s}{3\pi}\right)^2\bigg[\frac{45}{4\hat m_D}-\frac{165}{8}\bigg(\ln\frac{\hat\Lambda_g}{2}$$

$$-\frac{72}{11}\ln\hat m_D-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\bigg)+\frac{1485}{4}\left(\ln\frac{\hat\Lambda_g}{2}-\frac{79}{44}+\gamma_E-\ln 2-\frac{\pi^2}{11}\right)\hat m_D\bigg]\Bigg]\!\!\Bigg]$$

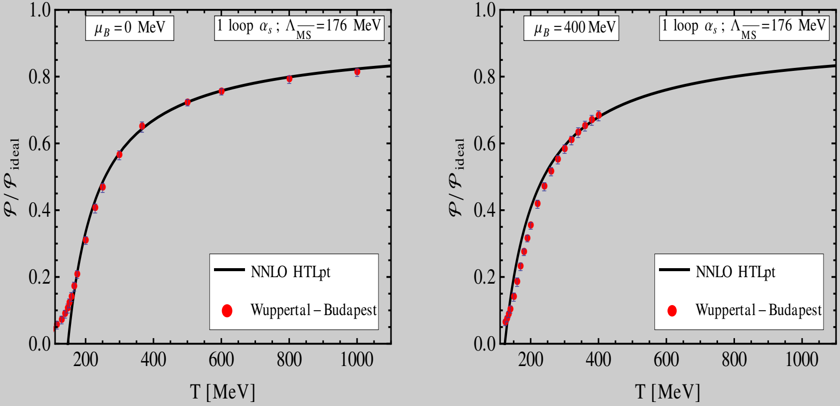
$$
\begin{aligned}
\mathcal{P}_{\text{NNLO}} = \frac{d_A \pi^2 T^4}{45} \Bigg[\!\!\Bigg[& 1 + \frac{7}{4}\frac{d_F}{d_A}\left(1 + \frac{120}{7}\hat{\mu}^2 + \frac{240}{7}\hat{\mu}^4\right) - \frac{15}{4}\hat{m}_D^3 - \frac{s_F \alpha_s}{\pi}\Bigg[\frac{5}{8}\left(5 + 72\hat{\mu}^2 + 144\hat{\mu}^4\right) \\
& + 90\hat{m}_q^2\hat{m}_D - \frac{15}{2}\left(1 + 12\hat{\mu}^2\right)\hat{m}_D - \frac{15}{2}\left(2\ln\frac{\hat{\Lambda}}{2} - 1 - \aleph(z)\right)\hat{m}_D^3\Bigg] + s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\Bigg[-\frac{45}{2}\hat{m}_D\left(1 + 12\hat{\mu}^2\right) \\
& + \frac{15}{64}\Bigg\{35 - 32\left(1 - 12\hat{\mu}^2\right)\frac{\zeta'(-1)}{\zeta(-1)} + 472\hat{\mu}^2 + 1328\hat{\mu}^4 + 64\Big(6(1 + 8\hat{\mu}^2)\aleph(1, z) + 3i\hat{\mu}(1 + 4\hat{\mu}^2)\aleph(0, z) \\
& - 36i\hat{\mu}\aleph(2, z)\Big)\Bigg\}\Bigg] + \left(\frac{s_F \alpha_s}{\pi}\right)^2\Bigg[\frac{5}{4\hat{m}_D}\left(1 + 12\hat{\mu}^2\right)^2 + 30\left(1 + 12\hat{\mu}^2\right)\frac{\hat{m}_q^2}{\hat{m}_D} + \frac{25}{12}\Bigg\{\frac{1}{20}\left(1 + 168\hat{\mu}^2 + 2064\hat{\mu}^4\right) \\
& + \left(1 + \frac{72}{5}\hat{\mu}^2 + \frac{144}{5}\hat{\mu}^4\right)\ln\frac{\hat{\Lambda}}{2} + \frac{3\gamma_E}{5}\left(1 + 12\hat{\mu}^2\right)^2 - \frac{8}{5}(1 + 12\hat{\mu}^2)\frac{\zeta'(-1)}{\zeta(-1)} - \frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)} - \frac{72}{5}\Big[3\aleph(3, 2z) \\
& + 8\aleph(3, z) - 12\hat{\mu}^2\aleph(1, 2z) - 2(1 + 8\hat{\mu}^2)\aleph(1, z) + 12i\hat{\mu}\left(\aleph(2, z) + \aleph(2, 2z)\right) - i\hat{\mu}(1 + 12\hat{\mu}^2)\aleph(0, z)\Big]\Bigg\} \\
& - \frac{15}{2}\left(1 + 12\hat{\mu}^2\right)\left(2\ln\frac{\hat{\Lambda}}{2} - 1 - \aleph(z)\right)\hat{m}_D\Bigg] + \frac{c_A \alpha_s}{3\pi}\frac{s_F \alpha_s}{\pi}\Bigg[\frac{15}{2\hat{m}_D}\left(1 + 12\hat{\mu}^2\right) + 90\frac{\hat{m}_q^2}{\hat{m}_D} \\
& - \frac{235}{16}\Bigg\{\left(1 + \frac{792}{47}\hat{\mu}^2 + \frac{1584}{47}\hat{\mu}^4\right)\ln\frac{\hat{\Lambda}}{2} - \frac{24\gamma_E}{47}\left(1 + 12\mu^2\right) + \frac{319}{940}\left(1 + \frac{2040}{319}\hat{\mu}^2 + \frac{38640}{319}\hat{\mu}^4\right) - \frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
& - \frac{144}{47}\left(1 + 12\hat{\mu}^2\right)\ln\hat{m}_D - \frac{44}{47}\left(1 + \frac{156}{11}\hat{\mu}^2\right)\frac{\zeta'(-1)}{\zeta(-1)} - \frac{72}{47}\Big[4i\hat{\mu}\aleph(0, z) + \left(5 - 92\hat{\mu}^2\right)\aleph(1, z) + 144i\hat{\mu}\aleph(2, z) \\
& + 52\aleph(3, z)\Big]\Bigg\} + \frac{315}{4}\Bigg\{\left(1 + \frac{132}{7}\hat{\mu}^2\right)\ln\frac{\hat{\Lambda}}{2} + \frac{11}{7}\left(1 + 12\hat{\mu}^2\right)\gamma_E + \frac{9}{14}\left(1 + \frac{132}{9}\hat{\mu}^2\right) + \frac{2}{7}\aleph(z)\Bigg\}\hat{m}_D\Bigg] \\
& + \frac{c_A \alpha_s}{3\pi}\Bigg[-\frac{15}{4} + \frac{45}{2}\hat{m}_D - \frac{135}{2}\hat{m}_D^2 - \frac{495}{4}\left(\ln\frac{\hat{\Lambda}_g}{2} + \frac{5}{22} + \gamma_E\right)\Bigg] + \left(\frac{c_A \alpha_s}{3\pi}\right)^2\Bigg[\frac{45}{4\hat{m}_D} - \frac{165}{8}\Bigg(\ln\frac{\hat{\Lambda}_g}{2} \\
& - \frac{72}{11}\ln\hat{m}_D - \frac{84}{55} - \frac{6}{11}\gamma_E - \frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\Bigg) + \frac{1485}{4}\left(\ln\frac{\hat{\Lambda}_g}{2} - \frac{79}{44} + \gamma_E - \ln 2 - \frac{\pi^2}{11}\right)\hat{m}_D\Bigg]\Bigg]\!\!\Bigg]
\end{aligned}
$$

Completely analytic

and

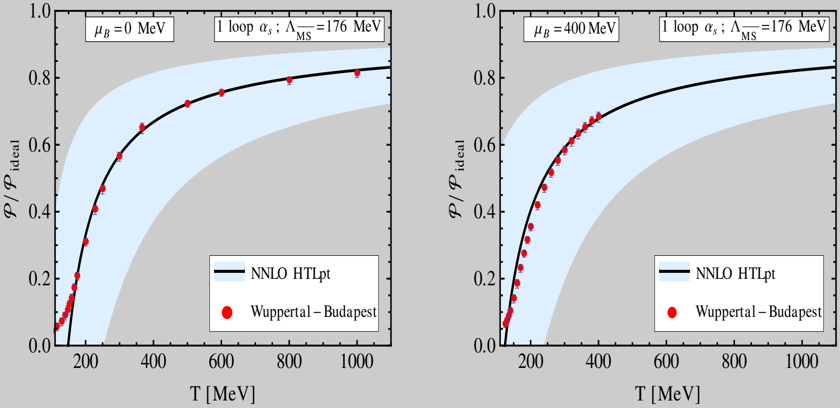
gauge independent

NNLO pressure for QCD HTL perturbation theory



- Thick Black Line: Renormalization Scale $\Lambda = 2\pi\sqrt{T^2 + \mu^2/\pi^2}$
- Band : Varying center value by factor of 2.

NNLO pressure for QCD HTL perturbation theory



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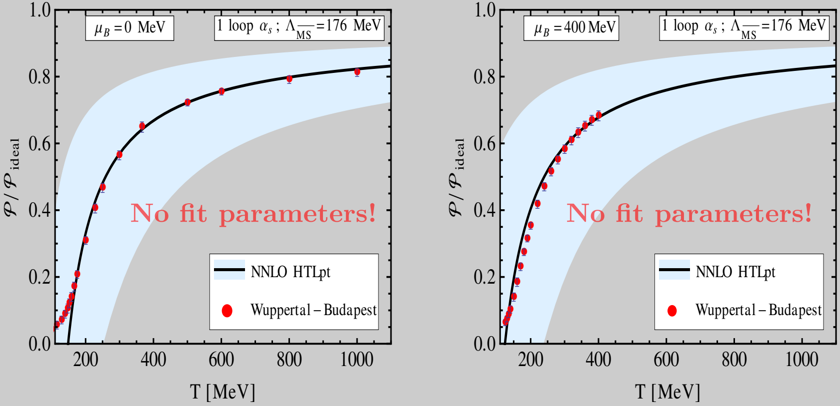
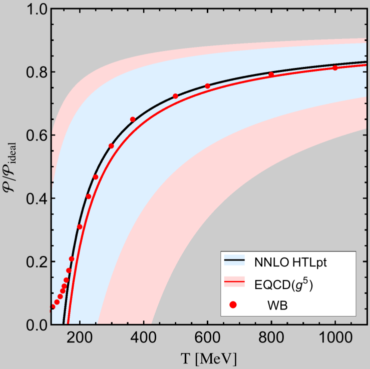


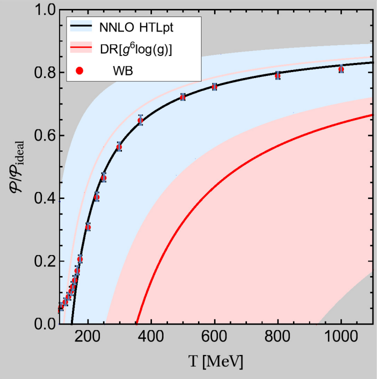
- Thick Black Line: Renormalization Scale $\Lambda = 2\pi\sqrt{T^2 + \mu^2/\pi^2}$
- Band : Varying center value by factor of 2.

HTL and EQCD/DR comparison

Braaten, Nieto, PRD53,3421(1996)

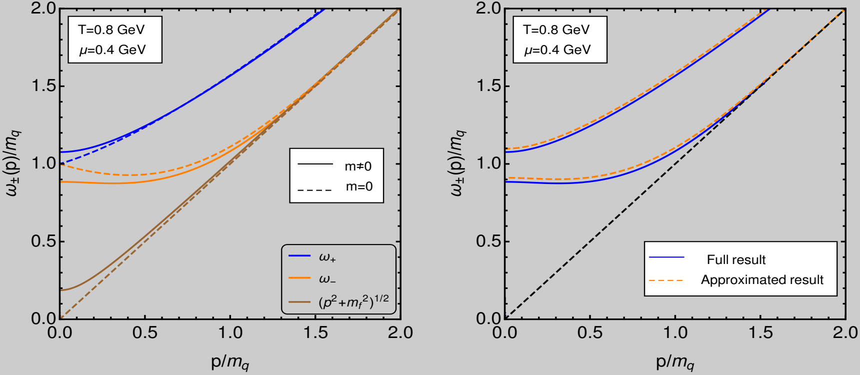


Vuorinen, PRD68,054017 (2003)



Strange-quark dispersion relation

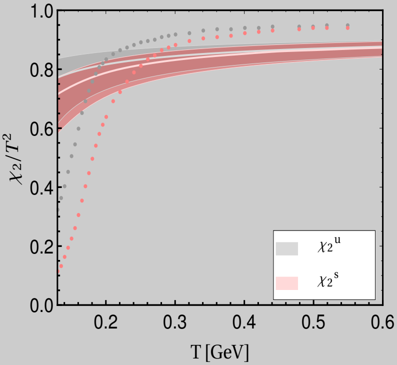
NH, PRD98, 014013(2018)

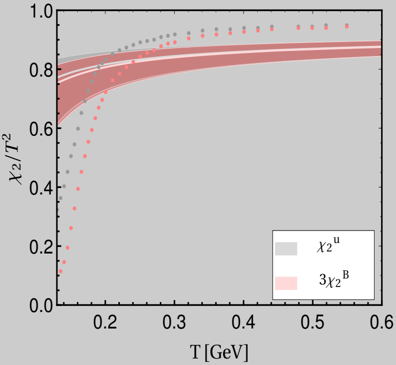


Approximated results are obtained considering finite quark masses only in the free propagator as quark masses are small compare to the temperature.

Quark/Baryon number Susceptibility

NH, PRD98, 014013(2018)





General structure of gluon self-energy at finite T & B

1. Now we take our z axis along the magnetic field, $n_\mu = \frac{1}{B} u^\nu \tilde{F}_{\mu\nu} = (0,0,0,1)$.
2. Presence of magnetic field breaks rotational symmetry of the system.
3. Available basis tensors to form manifestly covariant structure of photon self energy are $g^{\mu\nu}, P^\mu P^\nu, u^\mu u^\nu, n^\mu n^\nu, P^\mu u^\nu + u^\mu P^\nu, P^\mu n^\nu + n^\mu P^\nu, u^\mu n^\nu + n^\mu u^\nu$.
4. $P_\mu \Pi^{\mu\nu} = 0$ gives 3 constraints.
5. We need four independent parameters in the gluon self energy.

General structure of photon self energy can be written as,

$$\Pi^{\mu\nu} = b\, B^{\mu\nu} + c\, R^{\mu\nu} + d\, Q^{\mu\nu} + a\, N^{\mu\nu}$$

with

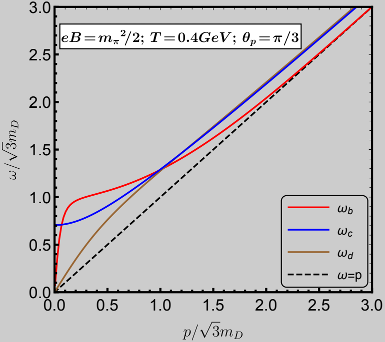
$$\begin{aligned} R^{\mu\nu} &= g_{\perp}^{\mu\nu} - \frac{P_{\perp}^{\mu} P_{\perp}^{\nu}}{(P_{\perp}^{\mu} P_{\perp\mu})}; \quad B^{\mu\nu} = \frac{\bar{u}^{\mu}\bar{u}^{\nu}}{(\bar{u})^2} \\ Q^{\mu\nu} &= g^{\mu\nu} - \frac{P^{\mu}P^{\nu}}{P^2} - B^{\mu\nu} - R^{\mu\nu} \\ N^{\mu\nu} &= \frac{\bar{u}^{\mu}\bar{n}^{\nu} + \bar{u}^{\nu}\bar{n}^{\mu}}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}} \end{aligned}$$

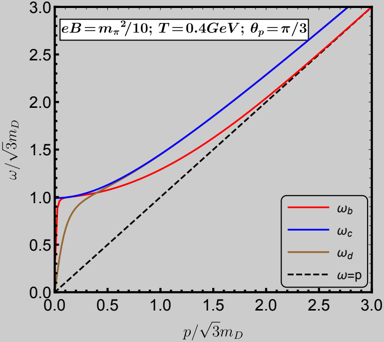
Using Dyson Schwinger equation one can calculate photon propagator as,

$$\begin{aligned} \mathcal{D}_{\mu\nu} &= \frac{\xi P_{\mu}P_{\nu}}{P^4} + \frac{(P^2-d)B_{\mu\nu}}{(P^2-b)(P^2-d)-a^2} + \frac{R_{\mu\nu}}{P^2-c} \\ &\quad + \frac{(P^2-b)Q_{\mu\nu}}{(P^2-b)(P^2-d)-a^2} + \frac{aN_{\mu\nu}}{(P^2-b)(P^2-d)-a^2} \end{aligned}$$

Gluon dispersion plot at $eB < T^2$

B Karmakar et al., 2018





Quark propagator in presence of magnetic field

- $$S^*(K) = \mathcal{P}_- \frac{\not{L}}{L^2}\mathcal{P}_+ + \mathcal{P}_+ \frac{\not{R}}{R^2}\mathcal{P}_- \qquad \text{with} \ \ \mathcal{P}_\pm = \frac{1}{2}\left(\mathbb{1} \pm \gamma_5\right),$$

$$\begin{aligned} \not{L} &= \left[(1+a)k_0 + b + b'\right]\gamma^0 - k(1+a)(\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}}) - c'\gamma^3 \\ \not{R} &= \left[(1+a)k_0 + b - b'\right]\gamma^0 - k(1+a)(\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}}) + c'\gamma^3 \end{aligned}$$

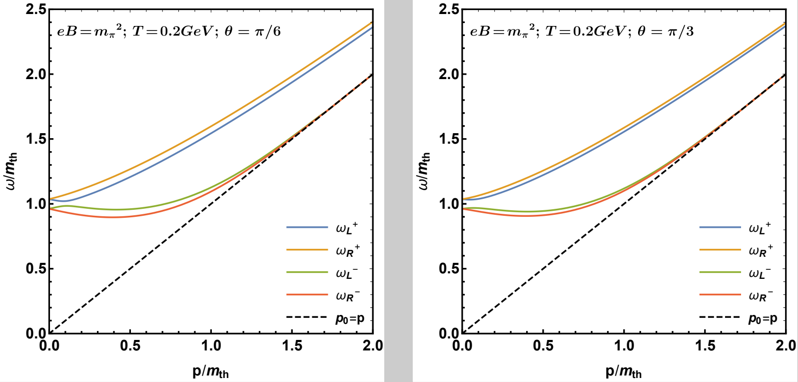
- a, b, c, b', c' are the structure functions of the quark self-energy as

$$\Sigma(P) = -a\not{P} - b\not{u} - c\not{n} - b'\gamma_5\not{u} - c'\gamma_5\not{n}$$

- $\Sigma(P)$ and hence a, b, c, b', c' can be obtained calculating one loop quark self-energy diagram at finite temperature and magnetic field.

Quark dispersion plot at $eB < T^2$

A Das et al., 2018



Conclusions and outlook

- Hard Thermal Loop perturbation theory and bulk thermodynamical quantities using HTLpt have been discussed
- NNLO pressure is completely analytic and does not depend on any free parameter except the choice of the renormalization scale.
- NNLO thermodynamical quantities are in good agreement results down to temperature ~ 250 MeV but have large dependency on the choice of renormalization scale.
- Finite quark masses effect on HTLpt formalism and LO QNS/BNS has been discussed. We have found that band witdth at finite quark mass is being reduced.
- Collective excitations for quarks and gluons in presence of magnetic filed has been discussed.

Thank you for your attention.

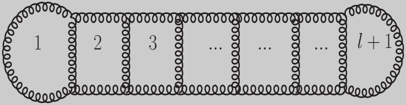
Back up slides.

Other thermodynamic quantities

1. Energy density: $\mathcal{E} = T\frac{\partial \mathcal{P}}{\partial T} + \mu\frac{\partial \mathcal{P}}{\partial \mu} - \mathcal{P}$
2. Trace anomaly $\mathcal{I} = \mathcal{E} - 3\mathcal{P}$
3. Speed of sound $c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}}$
4. Entropy density $\mathcal{S} = \frac{\partial \mathcal{P}}{\partial T}$
5. Quark number density $\rho = \frac{\partial \mathcal{P}}{\partial \mu}$
6. Quark number susceptibilities $\chi^n(T) = \left.\frac{\partial^n \mathcal{P}}{\partial \mu^n}\right|_{\mu\to 0}$
7. Baryon number susceptibilities $\chi_B^n(T) = \left.\frac{\partial^n \mathcal{P}(\mu_\mathcal{B},\mathcal{T})}{\partial \mu_B^n}\right|_{\mu\to 0}$ by changing the chemical potential basis from (μ_u, μ_d, μ_s) to (μ_B, μ_I, μ_S) at $\mu_I = \mu_S = 0$.
8. Later in PRD93(2015) 054045, we have extended thermodynamic calculations in finite isospin chemical potential also.

Beyond three loop

One can try to compute next order (g^6) in coupling constant g.



$$Z_l \sim g^{2(l-1)} \left(T \int d^3k \right)^l k^{2(l-1)} (k^2 + m^2)^{-3(l-1)}$$

- $l < 4$: Z_l is IR regular.
- $l = 4$: $Z_l \sim g^6 T^4 \log\left(\frac{T}{m}\right)$
- $l > 4$: $Z_l \sim g^6 T^4 \left(\frac{g^2 T}{m}\right)^{l-4}$

- For longitudinal gluons, $m_{el} \sim gT$. So for $l > 4$, $Z_l \sim g^{l+4} T^4$.
- For transverse gluons, $m_{mag} \sim g^2 T$, So for $l > 4$, $Z_l \sim g^6 T^4$.

Conclusion $\longrightarrow$ To compute $\mathcal{O}(g^6)$, one needs to compute infinite number of diagrams.

Magnetic scale resummation in HTL

- No infrared (IR) cut off by screening in the magnetic sector in HTLpt
- Magnetic scale resummation can be taken into account via Gribov-Zwanziger (GZ) formalism, in which the IR behavior of QCD is regulated by fixing the residual gauge transformations that remain after the usual Faddeev-Popov procedure

$$D^{\mu\nu}(P) = \left[\delta^{\mu\nu} - (1-\xi)\frac{P^\mu P^\nu}{P^2}\right]\frac{P^2}{P^4+\gamma_G^4}$$

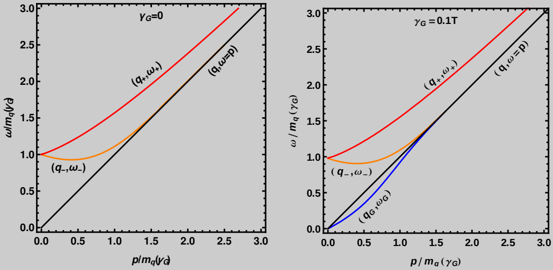
- The gap equation at one-loop order can be solved analytically at asymptotically high temperatures and gives

$$\gamma_G = \frac{D-1}{D}\frac{N_c}{4\sqrt{2}\pi}g^2T$$

- One can calculate quark self using this gluon propagator as

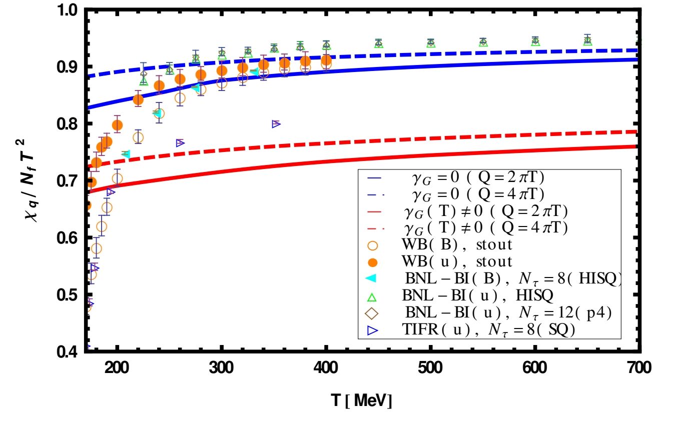
$$\Sigma(P) \;=\; (ig)^2 C_F \sum\!\!\!\!\!\!\!\!\int_{\{K\}} \gamma_\mu S_f(K)\gamma_\nu D^{\mu\nu}(P-K)$$

- Dispersion relation: PRL**114**, 161601(2015), N. Su & K. Tywoniuk



- We used the magnetic scale resummed quark propagator that results to calculate QNS and dilepton rate.

A Bandyopadhyay et al., 2016



Conclusion ⟶ Resummation of magnetic scales fails to improve existing results at one loop level.