# pQCD, kinetic theory, hydrodynamics and hadronic collisions

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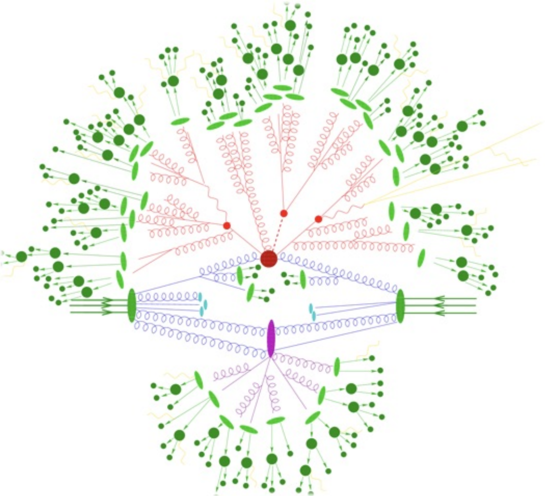
Bengaluru, April 2019

Phenomenological prelude:

# “Standard picture” in p-p collisions:

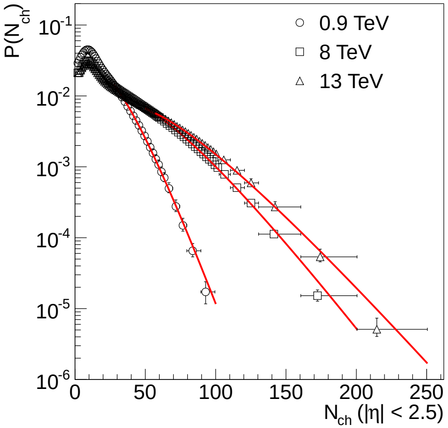


- Initial state radiation
- Hard processes
- Multi-parton interactions
- Fragmentation
- Hadronization
- ...

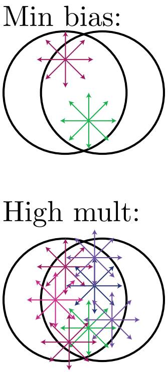
PYTHIA, HERWIG,...

- Hadronic collisions = superposition of individual partonic collisions
- No *final state interactions:* free streaming

# High-multiplicity collisions

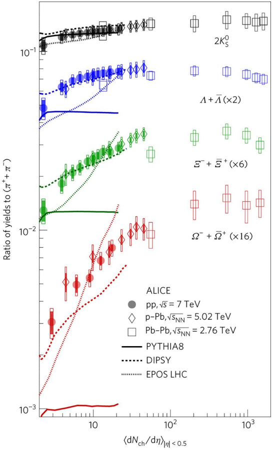



ATLAS EPJC76 (2016)




- Typical p-p collisions have $\mathcal{O}(10)$ final state hadrons
  in central rapidity region
- Very rarely a collisions results in $N_{ch} \sim \mathcal{O}(150)$

# Strangeness enhancement



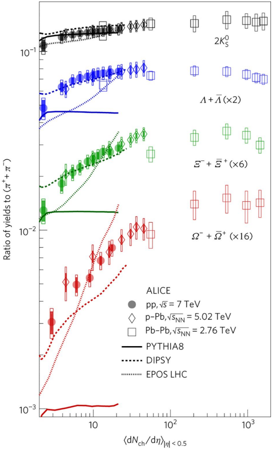

ALICE Nature Phys. 13 (2017) 535-539

- Something happens when the events get more active
- The *kind* of particles coming out changes as function of system size
- The bigger the system ($dN/d\eta$), more (multi-)strange particles

⇒ High multiplicity collisions not just more of the same

$K = us, \Lambda = uds, \Xi = uss, \Omega = sss$

# Strangeness enhancement

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$\Rightarrow$ High multiplicity collisions not just more of the same

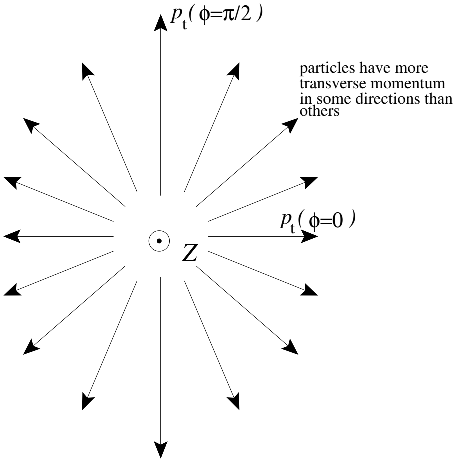
- Significant new physics needed to reproduce qualitative features

Sjöstrand, Fischer JHEP 1701 (2017)

$K = us, \Lambda = uds, \Xi = uss, \Omega = sss$

# Long-range azimuthal correlations

Individual events pick a preferred direction:



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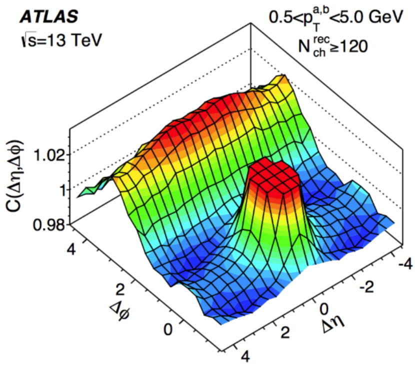
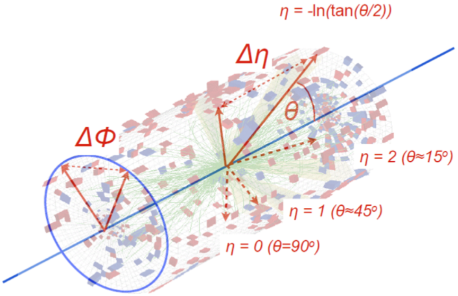
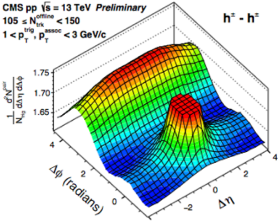


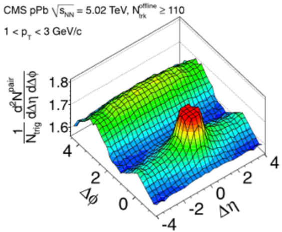

CMS cumulant analysis:PLB 765 (2017)

# Collectivity in nuclear collisions

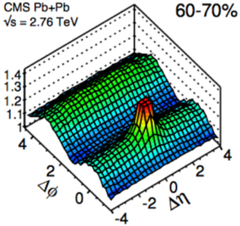
pp:




pA:

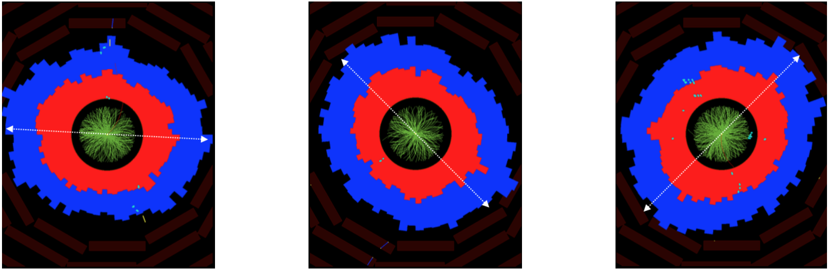



AA:




- Long range azimuthal correlations more prominent in larger systems

# Collectivity in nuclear collisions



G. Roland, Trento 2017

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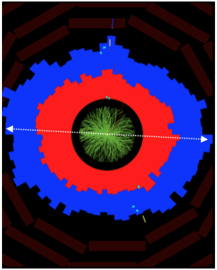

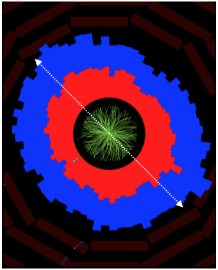

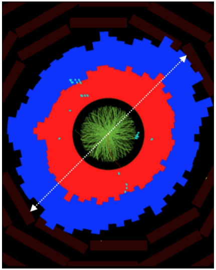

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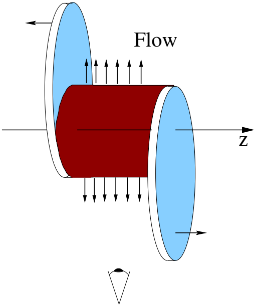
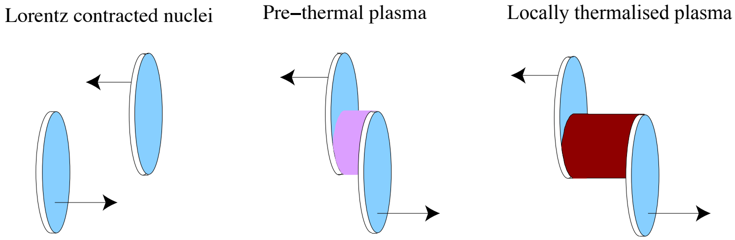



Formation of Quark-gluon plasma

- Azimuthal asymmetry from anisotropic explosion of quark-gluon plasma
- Strangeness content of plasma in chemical equilibrium

# Hydrodynamics



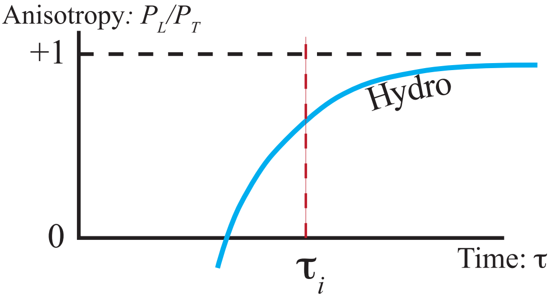

- Relativistic fluid dynamics =

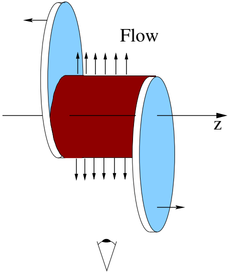
  i) conservation of currents and

  ii) gradient expansion around local thermal equilibrium

$$\partial_\mu T^{\mu\nu} = 0$$
$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} - \eta \nabla^{<\mu} u^{\nu>} + \ldots$$

# Hydrodynamization

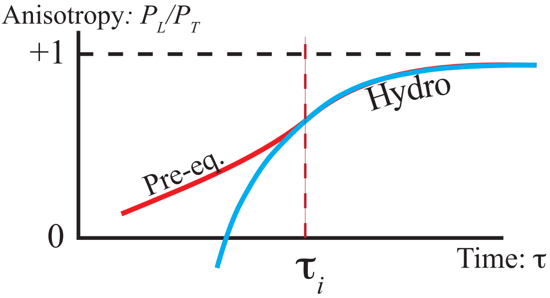


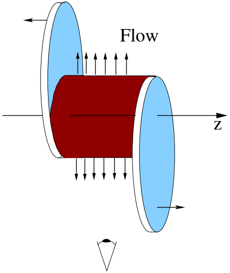




- Strong anisotropy $P_L/P_T \ll 1$, sign of large correction
- At early times *pre-equilibrium* evolution

Hydro: EoS, $\eta/s$, etc., Pre-equilibrium: need microscopic description

# Hydrodynamization




- Strong anisotropy $P_L/P_T \ll 1$, sign of large correction
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  Hydro: EoS, $\eta/s$, etc., Pre-equilibrium: need microscopic description
- hydro evolution maybe not reached at all in small systems
  - Even if there is no hydro, the same microscopic final state interactions are present

# Motivation:

Existential questions:

- How does collectivity arise from microscopic interactions?
- What signs of collectivity are really signs of fluid like behaviour? Which come from just final state interactions? Are there maybe other confounding effects that mimic those of final state interactions?
- How does the "perfect fluid" melt into free streaming particles in pp?

What is the microscopic structure of quark-gluon plasma?

To answer these questions, must understand far-from-equilibrium physics

# Outline:

- Simple controlled example
- pQCD kinetic theory
  - 2-2 scattering
  - 1-2 induced splitting
  - Overoccupied cascade
  - Landau-Pomeranchuk-Migdal Suppression
  - Underoccupied cascade
- Bottom-up thermalization and applications to hadronic collisions

Simple controlled example:

- Far-from equilibrium $\Rightarrow$ Near-equilibrium
- QFT $\rightarrow$ KT $\rightarrow$ Hydro

- Start with a system that is global thermal equilibrium
- Push the system out of equilibrium by an external source
  - Couple to external EM-field: $A^\mu$
  - Couple with gravity: $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}(t)$

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- Follow time evolution of the conserved currents in the system

$$T^{\mu\nu}(t) \quad J^\mu(t) \tag{1}$$

  - Expect that at late times $t \gg t_{\mathrm{micro}}$, evolution given by hydrodynamics
  - At earlier times something else, non-hydrodynamical evolution

For a small perturbation, linear response is given by the retarded correlation function

$$J^{\mu}(x,t) = \int d^3x' dt G_J^{\mu\nu}(\mathbf{x},t;\mathbf{x}',t')\delta A_{\nu}(\mathbf{x}',t')$$

$$T^{\mu\nu}(x,t) = \int d^3x' dt G_T^{\mu\nu,\alpha\beta}(\mathbf{x},t;\mathbf{x}',t')\delta h_{\alpha\beta}(\mathbf{x}',t')$$

The retarded response functions carry all information on how small perturbations propagate

Particularly nice to analyse in fourier space:

$$J^{\mu}(\omega, k) = G_J^{\mu\nu}(\omega, k) A_{\nu}(\omega, k)$$
$$T^{\mu\nu}(\omega, k) = G_T^{\mu\nu,\alpha\beta}(\omega, k) h_{\alpha\beta}(\omega, k)$$

- No mode-mixing in the linear level
- Corresponds to a plane wave perturbation $k||z$
  - $A^{\mu}$ electric charge diffusion mode
  - $h_{00}, h_{xx}, h_{yy}, h_{zz}, h_{0z}$, sound mode
  - $h_{0x}, h_{0y}, h_{zy}, h_{zx}$, shear mode
  - $h_{xy}$, tensor mode

Non-analytic structures in $G(\omega, k)$ describe how the system evolves towards equilibrium:

Example: Hydrodynamics

$$G_J^{0,0} = i\frac{\chi D}{\omega + iDk^2}$$
$$G_T^{00,00} = \frac{-k^2 sT}{-\omega^2 + c_s^2}$$
$$G_T^{0x,0x} = i\frac{k^2\eta}{\omega + i\gamma_\eta}$$
$$G_T^{xy,xy} = -i\eta\omega$$

$\gamma_\eta = \frac{4\eta}{3sT}$

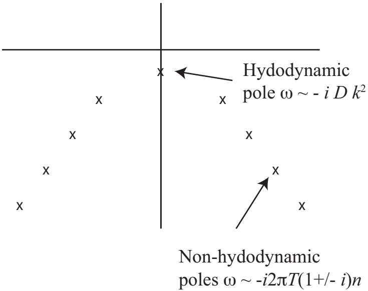
In time-domain:

$$\int \frac{d\omega}{2\pi} e^{i\omega t} G_T^{0x,0x}(\omega, k) = i\mathrm{Res} G_T^{0x,0x}(\omega = \gamma_\eta, k) = e^{-\gamma_\eta t} k^2 \eta$$

- Note, stability requires that all the non-analyticities are in the lower half of the complex plane

Non-analytic structures in $G(\omega, k)$ describe how the system evolves towards equilibrium:

Example AdS/CFT:




Son, Starinets JHEP 0209 (2002), Starinets PRD 66 (2002)

$$G_T^{0x,0x}(t,k) = \mathrm{Res}(\omega_{\mathrm{hydro}})e^{-i\omega_{\mathrm{hydro}}t} + \sum_n \mathrm{Res}(\omega_n)e^{-i\omega_n t}$$

- Note: the structures must always come in pairs for the response to be real in time-domain

What happens in free kinetic theory?

$$\underbrace{p^\mu \partial_\mu f(\mathbf{x}, \mathbf{p}, t)}_{\text{free streaming}} + \underbrace{F^i \nabla_i^{(p)} f(\mathbf{x}, \mathbf{p}, t)}_{\text{external force}} = 0$$

- On-shell particles: $p^0 = |\mathbf{p}|$ ($m = 0$ for simplicity)

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- On-shell particles: $p^0 = |\mathbf{p}|$ ($m = 0$ for simplicity)
- Force:

$$F^i = gF^{i\beta} p_\beta = g(\mathbf{E}\cdot\mathbf{v} + \mathbf{B}\times\mathbf{v}) \quad \text{for electro-mag}$$
$$F^i = -G\Gamma^i_{\beta\gamma} p^\beta p^\gamma \quad \text{for gravity}$$

with

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})$$
$$\approx \frac{1}{2}\eta^{\mu\nu}(\partial_\alpha h_{\nu\beta} + \partial_\beta h_{\nu\alpha} - \partial_\nu h_{\alpha\beta})$$

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Fourier transform:

$$f(\omega, \mathbf{k}, p) = \int \frac{d\omega}{2\pi} \frac{d^3k}{2\pi} e^{-i\omega t + i\mathbf{k}\cdot x} f(t, \mathbf{x})$$

$$(-i\omega + i\mathbf{k}\cdot\mathbf{v}) f + \frac{1}{p} F^i \nabla_i^{(p)} f = 0$$

With $\mathbf{v} = \mathbf{p}/p$.

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With $\mathbf{v} = \mathbf{p}/p$. Linearize in force around thermal equilibrium:

$F^i = \delta F^i,\ f = f_{\text{eq}} + \delta f$

$$\delta f = -g^2 i \frac{\frac{1}{p} F^i \nabla_i^{(p)} f}{\omega - \mathbf{k}\cdot\mathbf{v} + i\epsilon}$$

$i\epsilon$ to choose the retarded, not advanced solutions

Now we can compute the correlation function for currents:

here electric current for simplicity:

$$\delta J^\mu = g \int_p p^\mu \delta f$$
$$= -ig \int_p \frac{\frac{p^\mu}{p} F^i \nabla_i^{(p)} f}{\omega - \mathbf{k}\cdot\mathbf{v} + i\epsilon}$$

Use that in isotropic system $\frac{\partial}{\partial p^\alpha} f(p) = \frac{\partial p}{\partial p^\alpha} f'(p) = \frac{p^\alpha}{p} f'(p)$

$$\delta J^\mu = -g^2 \frac{4\pi}{(2\pi)^3} \int dp p^2 f'(p) \times \int \frac{d\Omega_{\mathbf{v}}}{4\pi} \frac{v_\alpha F^\alpha}{\omega - \mathbf{k}\cdot\mathbf{v} + i\epsilon}$$

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$$\delta J^{\mu} = \underbrace{g^2 \frac{8\pi}{(2\pi)^3} \int dp p f(p)}_{\text{Screening mass } m^2} \times i \underbrace{\int \frac{d\Omega_{\mathbf{v}}}{4\pi} \frac{v_{\alpha} F^{\alpha}}{\omega - \mathbf{k} \cdot \mathbf{v} + i\epsilon}}_{\text{angular structure}}$$

This is the famous HTL polarization tensor that Tony will talk about

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$$\delta T^{\mu\nu} = 3G^2(\epsilon + P) i \int \frac{d\Omega}{4\pi} \frac{-\Gamma^0_{\alpha\beta} v^\alpha v^\beta}{\omega - \mathbf{k}\cdot\mathbf{v} + i\epsilon} \tag{2}$$

$$\int \frac{d\Omega_{\mathbf{v}}}{4\pi} \frac{v_\alpha F^\alpha}{\omega - i\mathbf{k}\cdot\mathbf{v} + i\epsilon}$$

Ballistic propagator:

$$\int \frac{d\omega}{2\pi} \frac{d^3k}{(2\pi)^3} \frac{e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}}}{\omega - \mathbf{k}\cdot\mathbf{v} + \epsilon}$$

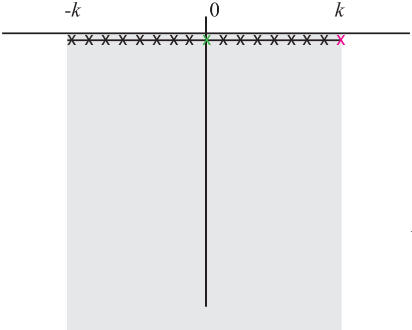
$$= \int \frac{d^3k}{(2\pi)^3} \theta(t) e^{i\mathbf{k}\cdot\mathbf{v}t - i\mathbf{k}\cdot\mathbf{x}}$$

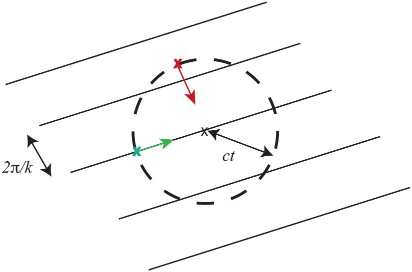
$$= \delta(\mathbf{v}t - \mathbf{x})\theta(t)$$

Master integral:

$$
\begin{aligned}
i\int \frac{d\Omega_{\mathbf{v}}}{4\pi}\frac{1}{\omega - i\mathbf{k}\cdot\mathbf{v}+\epsilon} &= i\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{-1}^{1}\frac{d\cos(\theta)}{2}\frac{1}{\omega - ik\cos(\theta)+i\epsilon} \\
&= -\frac{1}{2k}\int_{-1}^{1} d\log(\omega - ikx + i\epsilon) \\
&= -\frac{1}{2k}\log(\frac{\omega - k + i\epsilon}{\omega + k + i\epsilon})
\end{aligned} \tag{3}
$$

# Kinetic theory, noninteracting




$$\underbrace{p^\mu \partial_\mu f}_{\text{free streaming}} = \underbrace{\Gamma^i_{\alpha\beta} p^\alpha p^\beta \nabla_i^{(p)} f}_{\text{external source}} = S$$

$$G^{00,00} = 3sTi\omega \int \frac{d\Omega}{4\pi} \frac{1}{\omega - \mathbf{v}\cdot\mathbf{k}} = -sT\frac{3\omega}{2k} \log\left(\frac{\omega - k}{\omega + k}\right)$$