Jet evolution in a dense QCD medium

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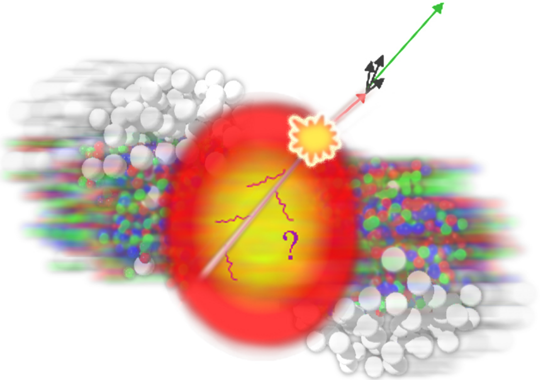
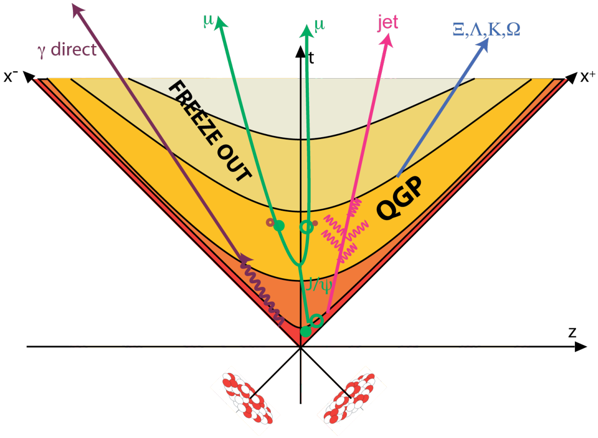


Outline

- Motivation: Hard Probes at RHIC and the LHC
  - observables related to jet quenching
  - the general physical picture
- Jets in the vacuum
  - radiation formation time
  - angular ordering
  - double logarithmic approximation
- Jet quenching in perturbative QCD
  - transverse momentum broadening
  - medium-induced radiation
  - multiple branching, energy loss, wave turbulence
- Adding vacuum-like radiation
  - the full, Markovian, picture ($\Rightarrow$ Monte-Carlo)
  - in-medium jet fragmentation

Hard probes in heavy ion collisions

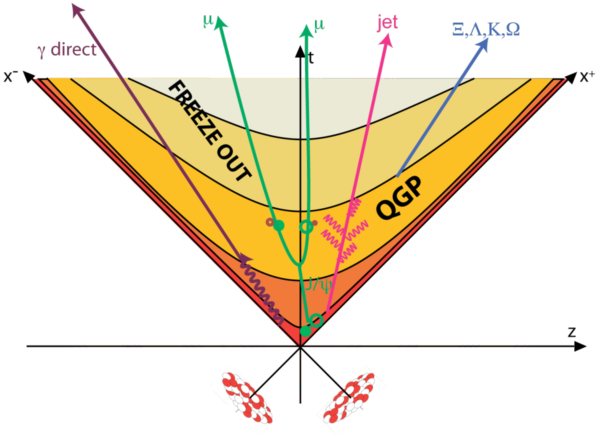
- The QGP phase "lives" for about 10 fm, that is, ... 10^{-23} seconds !



- How to measure such an ephemeral form of matter ?
- Use energetic partons, or jets as "thermometers"

Hard probes in heavy ion collisions

- The QGP phase "lives" for about 10 fm, that is, ... 10^{-23} seconds !



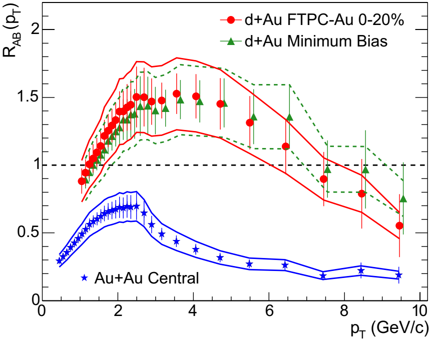
- Hard partons, photons, leptons created at early times : $\tau \lesssim 0.1$ fm/c
- Interact with the surrounding medium on their way to the detectors

Nuclear modification factor for hadrons

- Ratio of particle yield in AA and pp scaled by the number of binary collisions

$$R_{AA} \equiv \frac{1}{A^{4/3}} \frac{\mathrm{d}N_{pA}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{pp}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$

- Compare d+Au and Au+Au at RHIC at midrapidities ($\eta \sim 0$)

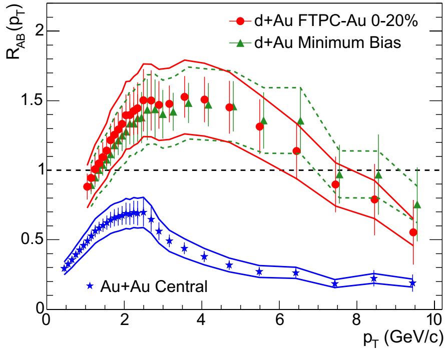


RHIC (STAR): $\sqrt{s_{\rm NN}} = 200$ GeV

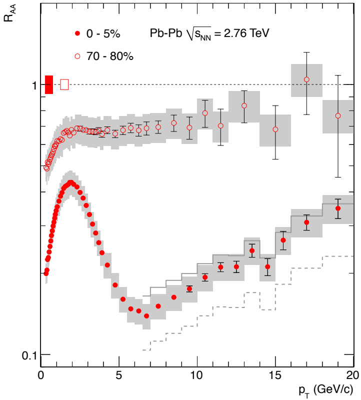
- d+Au: Cronin peak
- multiple scattering off the dense nuclear target
- "initial state effect": gluon saturation
- Au+Au: suppression at all p_T's
- "final state effect" : energy loss

Nuclear modification factor for hadrons

- A similar pattern emerges at the LHC energies



RHIC (STAR) $\sqrt{s_{NN}} = 200$ GeV

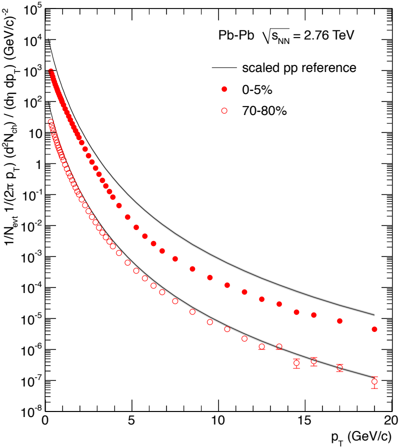


LHC (ALICE) $\sqrt{s_{NN}} = 2760$ GeV

- The amount of suppression increases with the centrality of the collision
  - central collisions (head-on Au+Au scattering) look denser

Energy loss

- Partons can lose energy via interactions in the plasma
- Hadrons measured with a given energy E have been produced with $E+\epsilon$



$$\frac{d\sigma^{\rm med}(E)}{dE} = \int d\epsilon\, \mathcal{P}(\epsilon)\, \frac{d\sigma^{\rm vac}(E+\epsilon)}{dE}$$

- $\mathcal{P}(\epsilon)$: probability density for losing ϵ

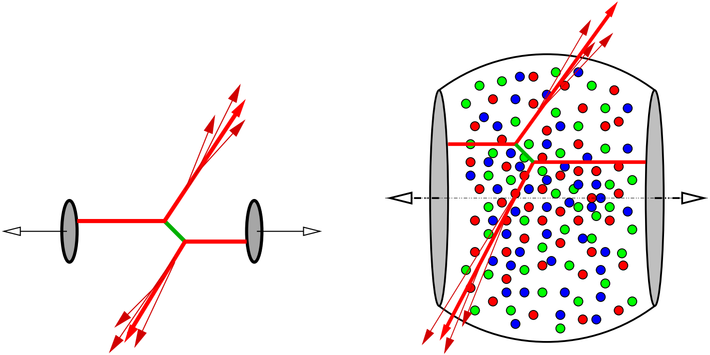
$$\frac{d\sigma^{\rm vac}(E)}{dE} \propto \frac{1}{E^n}\,, \qquad n = 7 \div 10$$

- Rapidly falling spectrum for the hard process
- Bias towards small values for ϵ

- Even a small ϵ may imply strong suppression

Jet quenching

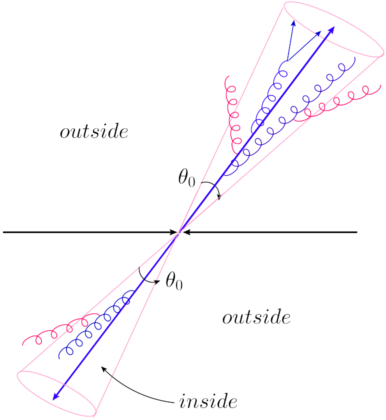
- Hard partons are typically created in pairs which propagate back–to–back in the transverse plane



- ‘Jet’: ‘leading particle’ + ‘products of fragmentation’
- AA collisions: jet propagation and fragmentation can be modified by the surrounding medium: ‘jet quenching’

Jets in practice *(cf. lectures by Michael Spira)*

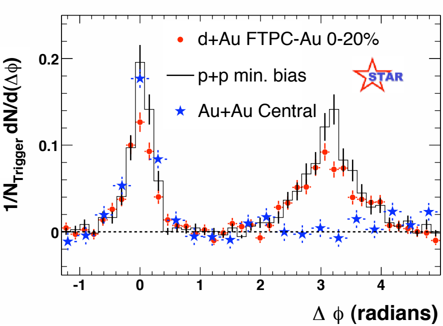
- Experimentally, jets are constructed by grouping together hadrons which propagate at nearby angles
- The jet opening angle θ_0 (a.k.a. R, $\theta_{\rm jet}$, $\bar{\theta}$ ...) is the same for both jets

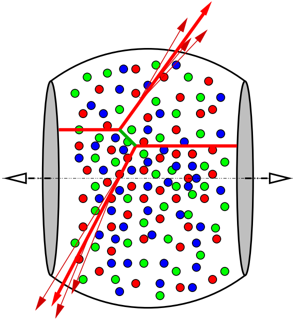


- Medium modifications refer both to the jets and to the outer regions

Di-hadron azimuthal correlations at RHIC

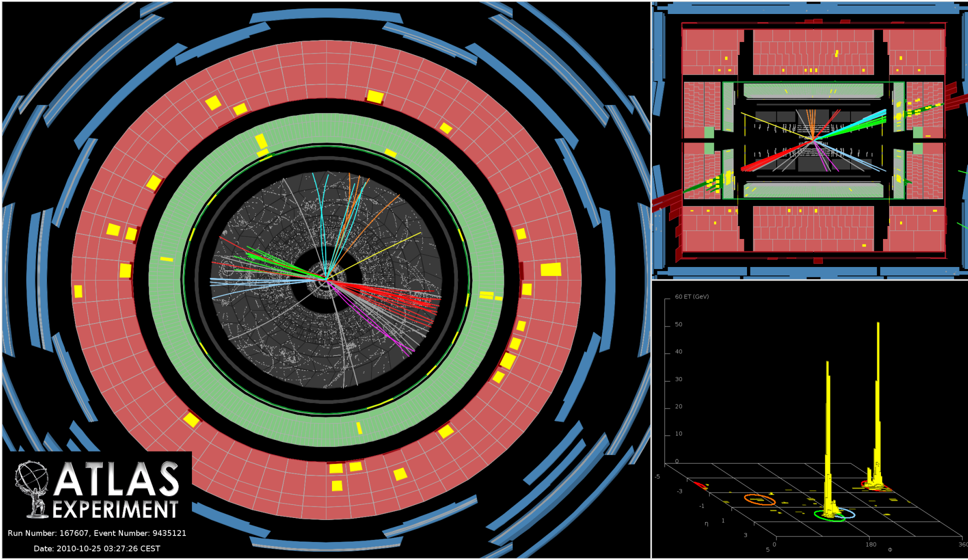
- Distribution of pairs of particles w.r.t. the relative azimuthal angle $\Delta\Phi$
- Compare p+p, d+Au and Au+Au, all at midrapidities ($\eta \sim 0$)



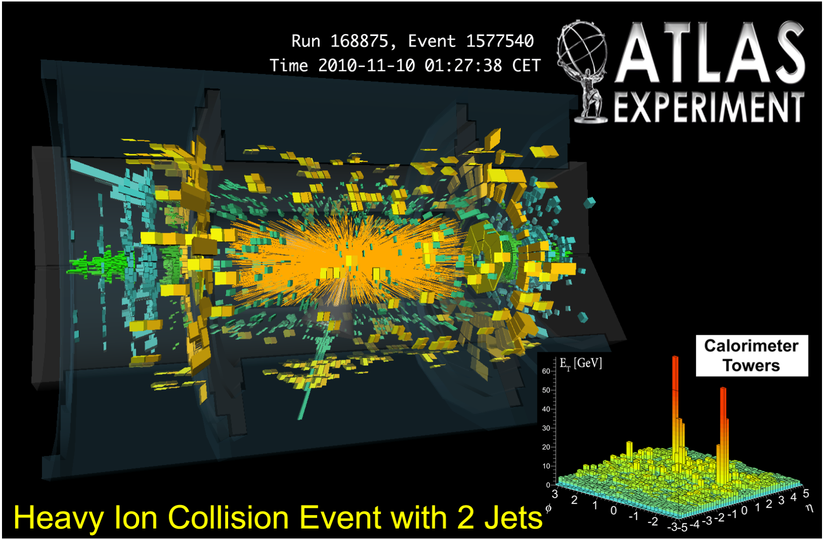


- Peak at $\Delta\Phi \sim 0$: both hadrons belong to a same jet
- Peak at $\Delta\Phi \sim \pi$: they belong to two back-to-back jets
- Au+Au: no peak at $\Delta\Phi \sim \pi$! One of the jets has "melted" into the medium

LHC: Di-jets in p+p collisions

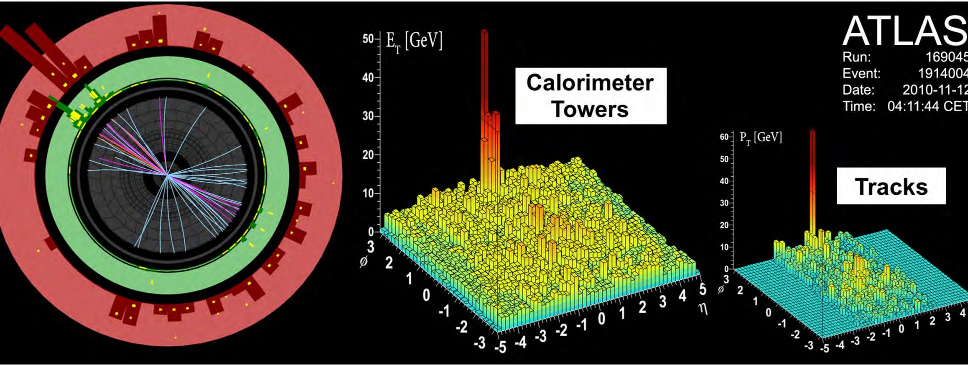


Jets in peripheral Pb+Pb collisions



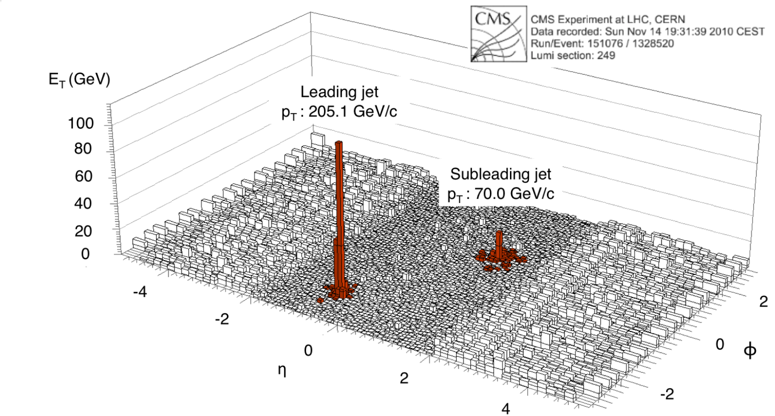
- Jets in peripheral AA collisions look very much like in pp collisions

"Mono-jets" in Pb+Pb collisions



- Central Pb+Pb: 'mono–jet' events
- The secondary jet can barely be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry at the LHC

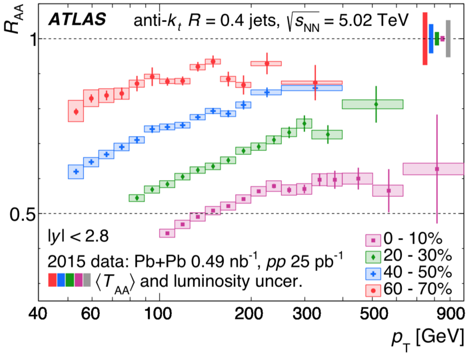


- Huge difference between the energies of the two jets
- The missing energy is found in the underlying event:
  - many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles
- Very different from the usual jet fragmentation pattern in the vacuum

The nuclear modification factor for jets

- The jet yield in Pb+Pb collisions normalized by p+p times the average nuclear thickness function $\langle T_{AA} \rangle$

$$R_{AA} \equiv \frac{\frac{1}{N_{\rm evt}} \frac{{\rm d}^2 N_{\rm jet}}{{\rm d}p_T {\rm d}y}\Big|_{AA}}{\langle T_{AA} \rangle \frac{{\rm d}^2 \sigma_{\rm jet}}{{\rm d}p_T {\rm d}y}\Big|_{pp}}$$



- different centrality bins
- stronger suppression for more central collisions

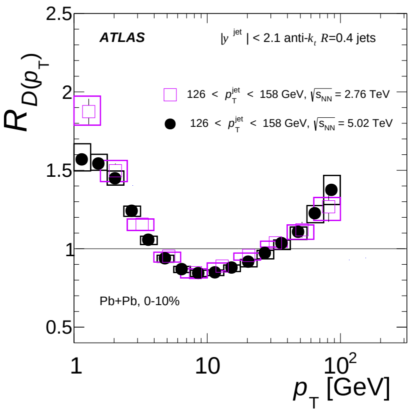
- Naturally interpreted as a consequence of energy loss inside the medium
- R_{AA} is almost flat at very high p_T: energy loss increases with p_T

Jet fragmentation function

- Energy distribution of the hadrons inside the jet

$$D(\omega) \equiv \omega \frac{\mathrm{d}N}{\mathrm{d}\omega}$$

$$= \int_0^R \mathrm{d}\theta \, \omega \frac{\mathrm{d}N}{\mathrm{d}\theta \mathrm{d}\omega}$$

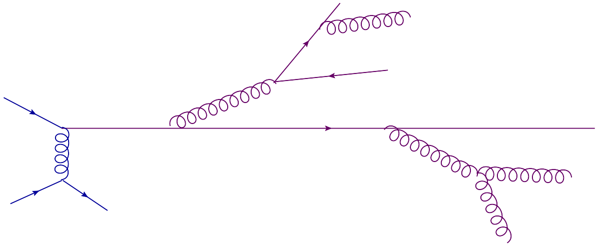


- $\omega \equiv p_T$ of a hadron inside the jet
- ratio of FFs in Pb+Pb and p+p
- enhancement at low energies: $p_T \ll p_T^{\text{jet}}$ ...
- ... and at relatively high ones: $p_T \sim p_T^{\text{jet}}$
- slight suppression at intermediate energies

- This is an example of intra-jet medium-induced modifications

Medium-induced jet evolution

- The leading particle (LP) is produced by a hard scattering
- It subsequently evolves via radiation (branchings) ...



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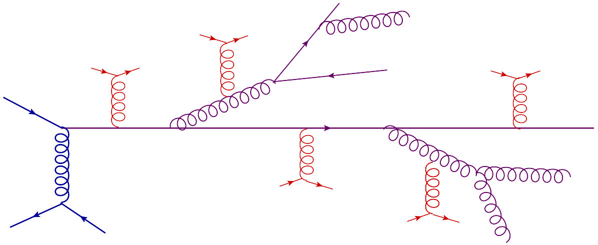

- ... and via collisions off the medium constituents

Medium-induced jet evolution

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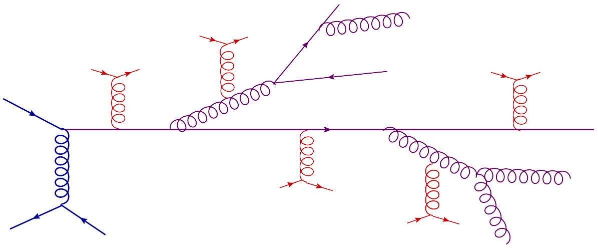
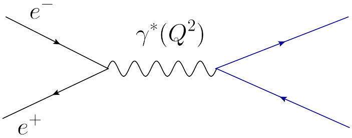


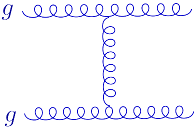
- ... and via collisions off the medium constituents
- Collisions can have several effects
  - transfer energy and momentum between the jet and the medium
  - trigger additional radiation ("medium-induced")
  - wash out the color coherence (destroy interference pattern)

- A hard scattering generates a time-like system (positive virtuality $Q^2 > 0$)

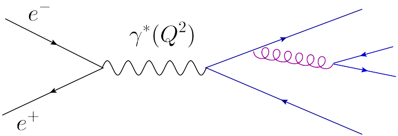
$$Q^2 \equiv (p_1 + p_2)^2 = 2p_1 \cdot p_2 = 2E_1E_2(1 - \cos\theta) \; = \; 4E^2_{\rm c.o.m}$$

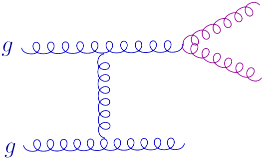
- ... which can decay into a pair of on-shell partons (at tree-level)





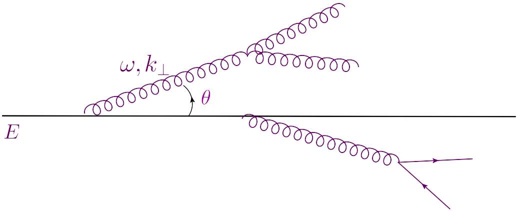
- ... or into other time-like partons, which will in turn decay ... and so on !





Jets in the vacuum (cont.)

- Multiple emissions ('fragmentation') leading to a jet structure
- The differential probability for one splitting is given by bremsstrahlung :



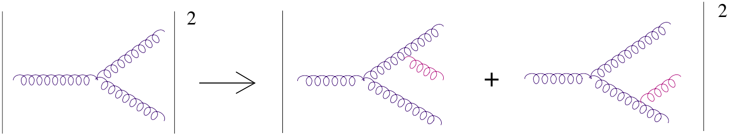
$$\mathrm{d}\mathcal{P} = \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

$$k_\perp = \omega \sin\theta \simeq \omega\theta$$

- It favors soft ($\omega \ll E$) and collinear ($\theta \ll 1$) splittings
  - many soft gluons ... but they carry very little energy
  - most of the energy remains in the few partons with large $x \equiv \omega/E$
  - small angle emissions $k_\perp \simeq \omega\theta$ with $\theta \ll 1 \Rightarrow$ jets are collimated
- The collimation property is further enhanced by angular ordering

Angular ordering in the vacuum

- Interference between emissions by several sources
  - successive quantum emissions are generally not independent
- One sums the amplitudes and only then one takes the modulus squared
  - the 'cross–terms' represent interference effects



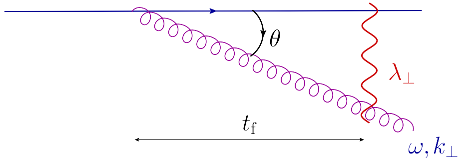
- Quantum interference complicates the "automatization" of the parton showers (e.g. Monte-Carlo event generators)
  - a priori, inconsistent with a classical probabilistic description
- Angular ordering "saves" the probabilistic picture and allows for Monte-Carlo

Radiation: Formation time

- Uncertainty principle: quantum particles are delocalized

$$\text{de Broglie wavelength:} \quad \lambda_\perp = \frac{2}{k_\perp} \simeq \frac{2}{\omega\theta}$$

- The gluon has been emitted when it has no overlap with its source

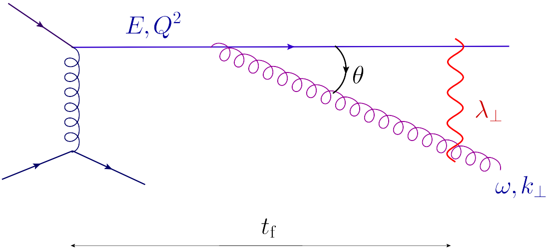


$$\Delta x_\perp \simeq \theta\,\Delta t \gtrsim \lambda_\perp \implies \Delta t \gtrsim t_f \equiv \frac{2\omega}{k_\perp^2} \simeq \frac{2}{\omega\theta^2}$$

- "Formation time" : the time it takes to emit a gluon
- This argument universally applies to radiation: in vacuum & in the medium

Formation time in the vacuum

- The emission is triggered by the hard scattering

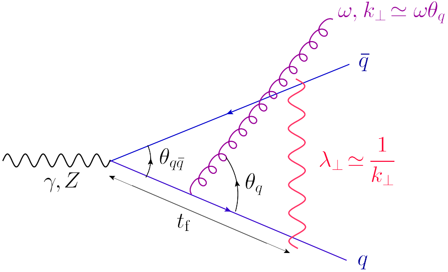


- The formation time is controlled by the parton virtuality: $t_{\rm f} \sim 2E/Q^2$
- $t_{\rm f}$ is measured from the hard scattering
  - the gluon is emitted within a distance $\sim t_{\rm f}$ from the scattering vertex
- In medium: additional decoherence introduced by collisions (see below)

A colorless antenna

- Interference: the gluon must overlap with both sources during its formation
- Simplest case: e^+e^- annihilation in a boosted frame $\Longrightarrow$

  ▷ colorless quark-antiquark "antenna" with opening angle $\theta_{q\bar{q}} \ll 1$



$$\lambda_\perp \simeq \frac{1}{\omega\theta_q} \gtrsim \theta_{q\bar{q}} t_f \simeq \frac{\theta_{q\bar{q}}}{\omega\theta_q^2}$$

$$\theta_q \gtrsim \theta_{q\bar{q}}$$

- "large angle" = out of cone

- Out-of-cone emissions "see" the total color charge — here zero — hence they are not permitted (destructive interference between q and $\bar{q}$)

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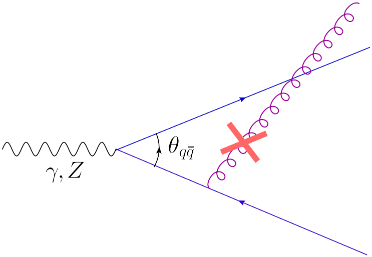

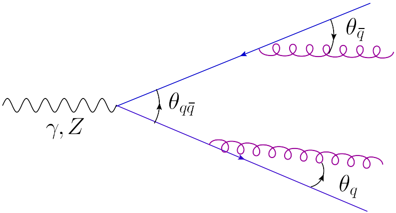

- Independent emissions by the quark and the antiquark at smaller angles

  $\theta_q, \theta_{\bar{q}} < \theta_{q\bar{q}} \Longrightarrow$ "angular ordering"

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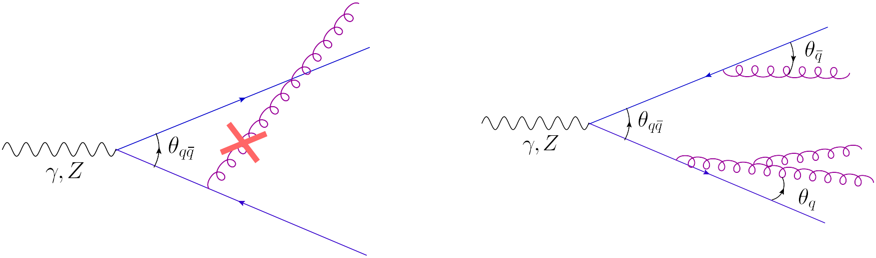


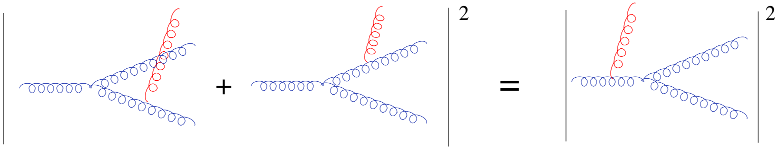
- Independent emissions by the quark and the antiquark at smaller angles

  $\theta_q, \theta_{\bar{q}} < \theta_{q\bar{q}} \Longrightarrow$ "angular ordering"

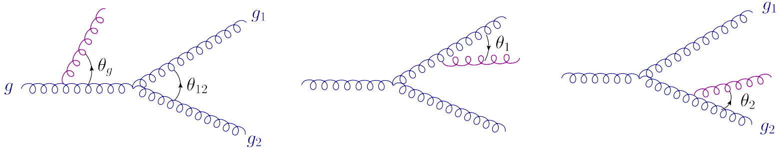
- This argument iterates to the subsequent emissions

A colored antenna

- E.g.: a color-octet antenna generated by the branching of a gluon
- A subsequent emission at $\theta > \theta_{gg}$ "sees" the overall color charge
  - it can be formally treated as an emission by the parent gluon



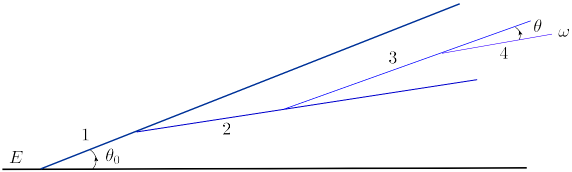
- Effectively: independent emissions with angular ordering



- Quantum emissions with interferences $\approx$ classical branchings with AO

Double-logarithmic approximation

- Parton showers in the vacuum: successive emissions are ordered in
  - energy ($\omega_i < \omega_{i-1}$), by energy conservation
  - angle ($\theta_i < \theta_{i-1}$), by color coherence



$$\mathrm{d}\mathcal{P} \simeq \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta^2}{\theta^2}$$

- Log enhancement for soft ($\omega \ll E$) and collinear ($\theta \ll 1$) gluons
- Double-logarithmic approximation (DLA): strong double ordering

$$E \gg \omega_1 \gg \omega_2 \gg \cdots \gg \omega \quad \& \quad \theta_0 \gg \theta_1 \gg \theta_2 \gg \cdots \gg \theta$$

- θ_0: the maximal angle as set by the first emission

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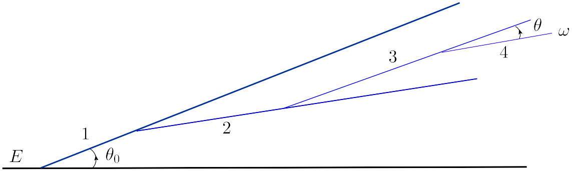

$$\mathrm{d}\mathcal{P} \simeq \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta^2}{\theta^2}$$

- The double-differential distribution of gluons in the $(\omega,\, \theta)$ phase-space:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}\omega \mathrm{d}\theta^2} \simeq \frac{\bar{\alpha}}{\omega\, \theta^2} \sum_{n\geq 0} \bar{\alpha}^n \left[\frac{1}{n!}\left(\ln \frac{E}{\omega}\right)^n\right] \left[\frac{1}{n!}\left(\ln \frac{\theta_0^2}{\theta^2}\right)^n\right]$$

  - $n \geq 0$ intermediate emissions that are not measured: additional sources for the measured gluon with $(\omega,\, \theta)$

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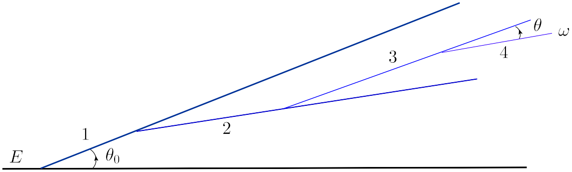

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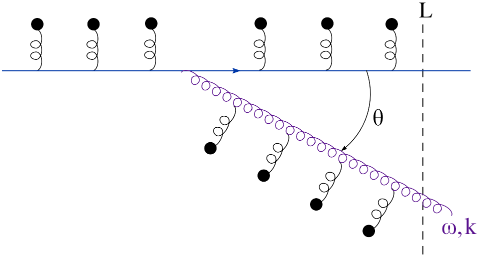
- The double-differential distribution of gluons in the $(\omega,\, \theta)$ phase-space:

$$\omega\theta^2 \frac{\mathrm{d}^2 N}{\mathrm{d}\omega \mathrm{d}\theta^2} = \bar{\alpha} \mathrm{I}_0 \left(2\sqrt{\bar{\alpha} \ln \frac{E}{\omega} \ln \frac{\theta_0^2}{\theta^2}} \right) \propto \exp\left\{ 2\sqrt{\bar{\alpha} \ln \frac{1}{x} \ln \frac{\theta_0^2}{\theta^2}} \right\}$$

  - exponential growth, like for the parton distribution, but no saturation: after being emitted, partons move away from each other

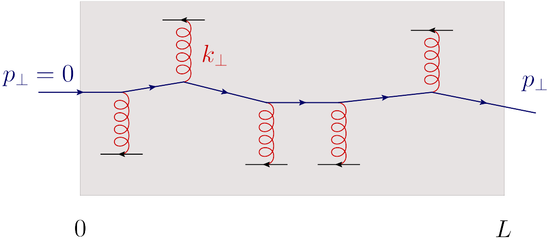
Jet quenching in pQCD

- Focus on the parton shower, use a simple description for the medium itself
- Weakly coupled quark-gluon plasma in thermal equilibrium at temperature T
  - jet quenching is biased towards (semi)hard momentum transfers
  - see also lectures by Peter Petreczky and Tony Rebhan for the QGP
- The basic physical mechanism for jet-medium interactions: elastic collisions
- Successive collisions are independent
  - mean free path between 2 successive collisions $\gg$ duration of a collision



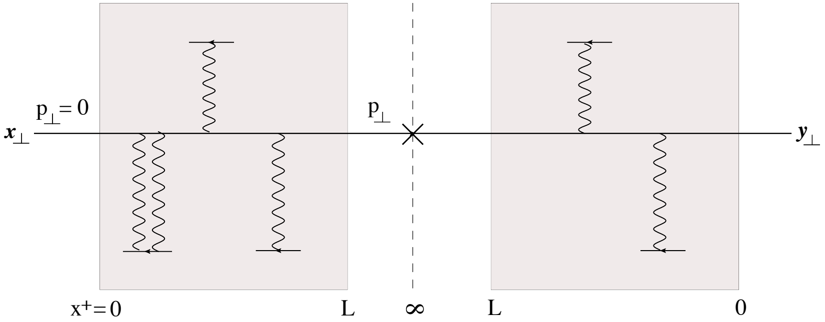
Transverse momentum broadening

- An energetic quark acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance L



- A random walk in transverse momentum: $\langle p_\perp^2 \rangle \simeq \hat{q}L$
- $\hat{q}$: the "jet quenching parameter" (a medium transport coefficient)
- $\hat{q}L$ plays exactly the same role as the saturation momentum $Q_s^2(L)$ for a "large nucleus" with longitudinal width equal to L
- Quasi-independent scattering centers in the plasma $\Longleftrightarrow$ MV model

- Direct amplitude (DA) $\times$ Complex conjugate amplitude (CCA)

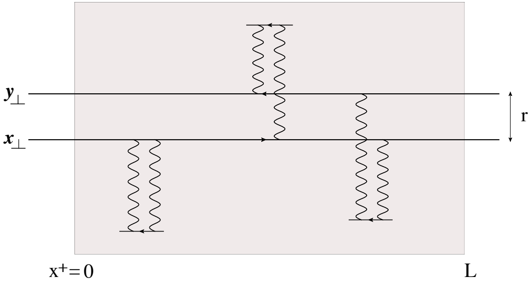


- Scattering can be computed in the eikonal approximation: Wilson lines

$$V^{\dagger}(\boldsymbol{x}) = \mathrm{P}\exp\left\{ig\int \mathrm{d}x^{+}\, A_a^{-}(x^{+},\boldsymbol{x})t^{a}\right\}$$

- Two such Wilson lines (DA $\times$ CCA) $\Longrightarrow$ a $q\bar{q}$ color dipole

Dipole picture (cont.)



$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\, \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}} \langle \hat{S}_{\boldsymbol{xy}} \rangle$$

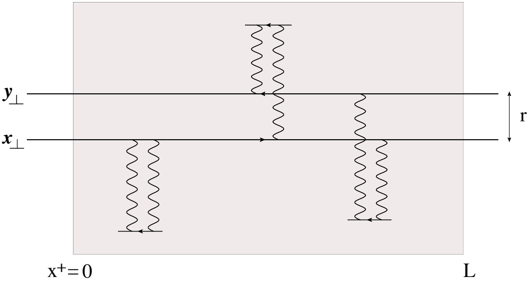
$$\hat{S}_{\boldsymbol{xy}} \equiv \frac{1}{N_c}\,\mathrm{tr}\big(V_{\boldsymbol{x}}^\dagger V_{\boldsymbol{y}}\big)$$

- Average over a Gaussian distribution of color charges representing the quasi-free thermal quarks and gluons:

$$\langle \rho_a(x^+, \boldsymbol{x}) \rho_b(y^+, \boldsymbol{y}) \rangle = g^2 \nu\, \delta^{ab} \delta(x^+ - y^+) \delta^{(2)}(\boldsymbol{x} - \boldsymbol{y})$$

- $\nu = (C_F n_q + N_c n_g)/N_g \sim T^3$: color-weighted density of quarks & gluons
- The same as the MV problem with $\mu^2 \to g^2 \nu L$

Dipole picture (cont.)



$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\,\mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\langle\hat{S}_{\boldsymbol{xy}}\rangle$$

$$\hat{S}_{\boldsymbol{xy}} \equiv \frac{1}{N_c}\,\mathrm{tr}\big(V_{\boldsymbol{x}}^{\dagger}V_{\boldsymbol{y}}\big)$$

- Classical Yang-Mills solution covariant gauge: $-\nabla_{\perp}^2 A_a^-(x) = \rho_a(x^+,\boldsymbol{x})$

$$\big\langle A_a^-(x^+,\boldsymbol{k})\,A_b^-(y^+,-\boldsymbol{k})\big\rangle_0 = \delta_{ab}\delta(x^+ - y^+)\,\frac{g^2\nu}{(\boldsymbol{k}^2+m_D^2)^2}$$

- Infrared behavior at small-$k_\perp$ is regulated by Debye screening

$$m_D^2 = \frac{N_c+N_f/2}{3}g^2T^2$$

(cf. lecture by Tony Rebhan)

The dipole S-matrix

- The same as the MV problem with $\mu^2 \to g^2\nu L$ and $\Lambda_{\rm QCD} \to m_D$

$$S(r) \,=\, \exp\left\{-\frac{1}{4}\,L\hat{q}(1/r^2)\,\boldsymbol{r}^2\right\}$$

- The scale-dependent "jet quenching parameter" $\hat{q}(Q^2)$:

$$\hat{q}(Q^2) \equiv g^4 C_F \nu \int^{Q^2} \frac{{\rm d}^2\boldsymbol{k}}{(2\pi)^2}\,\frac{\boldsymbol{k}^2}{(\boldsymbol{k}^2+m_D^2)^2}\;\simeq\; 4\pi\alpha_s^2 C_F \nu \ln\frac{Q^2}{m_D^2}$$

- A simple interpretation in kinetic theory: $\Gamma_{\rm el}$ = the elastic collision rate

$$\hat{q}(Q^2) = \int^{Q^2} {\rm d}^2\boldsymbol{k}\,\frac{{\rm d}\Gamma_{\rm el}}{{\rm d}^2\boldsymbol{k}}\,\boldsymbol{k}^2$$

  - the average $k_\perp^2$ acquired per unit time via elastic collisions
  - logarithmic ultraviolet divergence: what sets the upper scale Q^2 ?

- In coordinate space, this is fixed by the dipole size r: $Q^2 \sim 1/r^2$

The jet quenching parameter

- The transverse momentum distribution (typical $p_\perp$) :

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\,\mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\ \mathrm{e}^{-\frac{1}{4}L\hat{q}(1/r^2)\,\boldsymbol{r}^2} \simeq \frac{1}{\pi Q_s^2(L)}\,\mathrm{e}^{-p_\perp^2/Q_s^2(L)}$$

- The "plasma saturation momentum" $Q_s^2(L)$: exponent of $\mathcal{O}(1)$

$$Q_s^2(L) = L\hat{q}(Q_s^2) = 4\pi\alpha_s^2 C_F \nu L \ln\frac{Q_s^2(L)}{m_D^2} \propto L\ln L$$

- The physical parameter $\hat{q}$ is self-consistently determined by evaluating the function $\hat{q}(Q^2)$ for $Q^2 = \hat{q}L$:

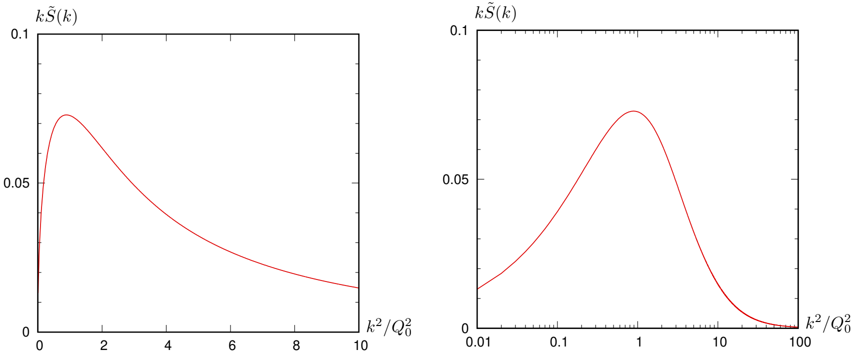
$$\hat{q} = 4\pi\alpha_s^2 C_F \nu\,\ln\frac{\hat{q}L}{m_D^2} \sim \alpha_s^2 T^3 \ln(\alpha_s^2 TL)$$

- Power-law tail at large $p_\perp \gg Q_s$: single hard scattering

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} \simeq \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\,\mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\ \left\{-\frac{1}{4}L\hat{q}(1/r^2)\,\boldsymbol{r}^2\right\} \simeq \frac{4\pi\alpha_s^2 C_F\nu}{p_\perp^4}$$

$p_\perp$-broadening: numerical results

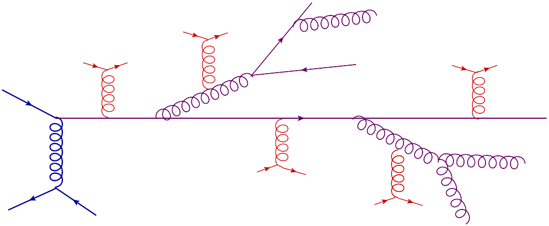
- Left: the dipole Fourier transform $k_\perp \tilde{\mathcal{S}}(k_\perp) \propto k_\perp (\mathrm{d}N/\mathrm{d}^2k_\perp)$
  - the probability distribution for $k_\perp$
  - "the dipole unintegrated gluon distribution"



- Right: the same function, but in logarithmic units
  - peaked at $k \simeq Q_s$, power-law tail at $k \gg Q_s$

Medium-induced radiation

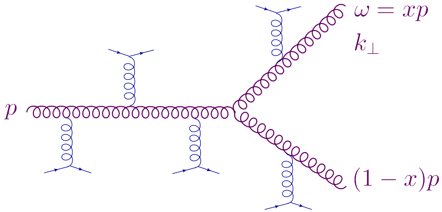
- Collisions in the medium can trigger additional radiation (similarly to the original hard scattering): they provide acceleration ("virtuality")



- A priori, 2 mechanisms for radiation ...
  - "vacuum-like" triggered by the original hard scattering
  - "medium-induced" associated rescattering in the medium
- ... but can one distinguish between them ?
- Look at the formation times: it takes a time $t_{\rm f} = \frac{2\omega}{k_\perp^2}$ to emit a gluon with energy ω and transverse momentum $k_\perp$

Medium-induced radiation (cont.)

- In the vacuum, ω and $k_\perp$ are independent kinematical variables
  - collinear radiation ($k_\perp \simeq \omega\theta \to 0$) has large formation times
- In the medium, collisions introduce a lower limit on $k_\perp$ ...



$$t_f = \frac{2\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \gtrsim \hat{q} t_f$$

$$t_f \lesssim \sqrt{\frac{2\omega}{\hat{q}}}$$

- ... hence an upper limit on the formation time !
  - vacuum-like emissions: $k_\perp^2 \gg \hat{q} t_f$, or $t_f \ll \sqrt{2\omega/\hat{q}}$
  - medium-induced emissions: $k_\perp^2 \simeq \hat{q} t_f$, or $t_f \simeq \sqrt{2\omega/\hat{q}}$
- In the medium, there is no genuinely collinear radiation: $k_\perp^2 \geq k_f^2 \equiv \sqrt{2\omega\hat{q}}$

Radiation from multiple soft scattering

(Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov; 1996-97)

- Medium-induced emissions can occur anywhere inside the medium
  - no correlation with the original hard process
- It is appropriate to work with the ("BDMPS-Z") emission rate :
  - differential probability for one medium-induced emission per unit phase-space per unit time

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\omega\,\mathrm{d}^2\boldsymbol{k}\,\mathrm{d}t} = \frac{\alpha_s C_R}{\pi}\,\frac{1}{\omega}\,\frac{1}{t_{\mathrm{med}}(\omega)}\,\frac{1}{\sqrt{2\hat{q}\omega}}\,\mathrm{e}^{-\frac{k_\perp^2}{\sqrt{2\hat{q}\omega}}}, \qquad t_{\mathrm{med}}(\omega) \equiv \sqrt{\frac{2\omega}{\hat{q}}}$$

  - transverse momentum broadening during formation: $\langle k_\perp^2 \rangle \simeq \sqrt{2\hat{q}\omega}$
  - $\omega t_{\mathrm{med}}(\omega) \propto \omega^{3/2} \Longrightarrow$ no soft or collinear logarithm
- This applies so long mean-free-path $< t_{\mathrm{med}}(\omega) \leq L$

$$T \;<\; \omega \;\leq\; \omega_c \equiv \frac{1}{2}\hat{q}L^2$$

Angular distribution

- Transverse momentum broadening during formation & after formation

- formation angle:

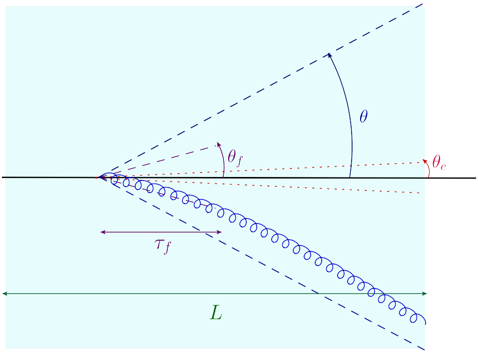
$$\theta_{\rm f}(\omega) \simeq \frac{(2\hat{q}\omega)^{1/4}}{\omega} \simeq \left(\frac{2\hat{q}}{\omega^3}\right)^{1/4}$$

- the minimal angle

$$\theta_c = \theta_{\rm f}(\omega_c) = \frac{2}{\sqrt{\hat{q}L^3}}$$

- final angle

$$\theta(\omega) \simeq \frac{\sqrt{\hat{q}L}}{\omega} = \theta_c \frac{\omega_c}{\omega}$$



- Soft gluons $\omega \ll \omega_c$: small formation times $(t_{\rm f} \ll L)$ & large angles $(\theta \gg \theta_c)$

- Gluons with angles larger than the jet opening angle θ_0 move outside the jet

Average energy loss by the LP

- Differential probability for one emission (integrated over t and over $k_\perp$)

$$\omega\,\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\omega} \simeq \bar{\alpha}\,\frac{L}{t_{\rm med}(\omega)} \simeq \bar{\alpha}\sqrt{\frac{\omega_c}{\omega}} \qquad (\omega < \omega_c \equiv \hat{q}L^2/2)$$

- The average energy loss by a particle with energy $E > \omega_c$

$$\langle \Delta E\rangle = \int^{\omega_c} \mathrm{d}\omega\,\omega\,\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\omega} \sim \alpha_s\omega_c \sim \alpha_s\hat{q}L^2$$

  - integral dominated by its upper limit $\omega = \omega_c$

- Hard emissions with $\omega \sim \omega_c$: probability of $\mathcal{O}(\alpha_s)$
  - rare events but which take away a large energy
  - small emission angle $\theta_c \Rightarrow$ the energy remains inside the jet

- Irrelevant for the di-jet asymmetry and also for the typical events

Multiple branchings

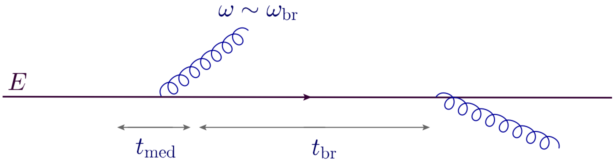
J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

$$\omega \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\omega} \simeq \bar{\alpha} \frac{L}{t_{\rm med}(\omega)} \simeq \bar{\alpha} \sqrt{\frac{\omega_c}{\omega}}$$

- When $\omega(\mathrm{d}\mathcal{P}/\mathrm{d}\omega) \gtrsim 1$, multiple branching becomes important

$$\omega \lesssim \omega_{\rm br}(L) \equiv \bar{\alpha}^2 \omega_c \quad \Longleftrightarrow \quad L \gtrsim t_{\rm br}(\omega) \equiv \frac{1}{\bar{\alpha}} t_{\rm med}(\omega)$$

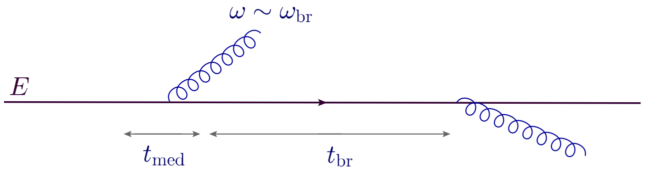
  - $\omega_{\rm br} \sim \bar{\alpha}^2 \hat{q} L^2$: characteristic energy for the onset of multiple branching



  - $t_{\rm br} = t_{\rm med}/\bar{\alpha}$: typical distance between 2 successive branchings
  - $t_{\rm br} \gg t_{\rm med}$: successive emissions do not overlap with each other

Multiple branchings

- LHC: the leading particle has $E \sim 100\,\text{GeV} \gg \omega_{\rm br} \sim 5\,\text{GeV}$



- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\rm br}$

Multiple branchings

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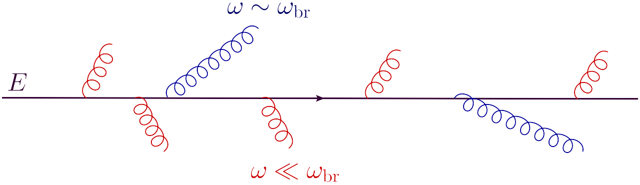

- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\rm br}$
  - a large number of softer gluons with $\omega \ll \omega_{\rm br}$
  - the energy loss is controlled by the hardest primary emissions

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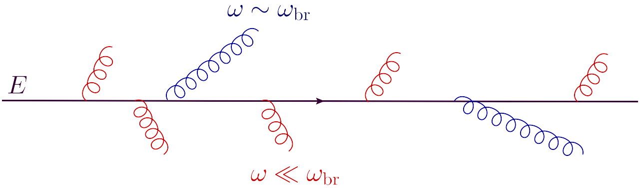
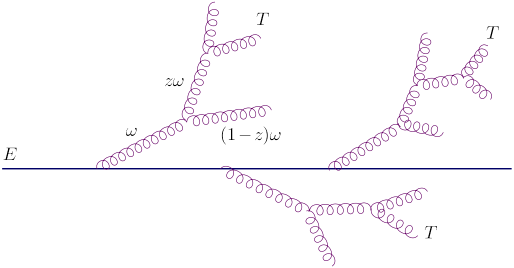


- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\rm br}$
  - a large number of softer gluons with $\omega \ll \omega_{\rm br}$
  - the energy loss is controlled by the hardest primary emissions
- In a typical event, the LP loses an energy $\Delta E \sim \omega_{\rm br}$
  - albeit smaller than the average energy loss, $\Delta E \sim \bar{\alpha}\langle \Delta E\rangle$, this typical energy loss is more important for our purposes
- This is also the energy lost by the jet

Democratic branchings

- The primary gluons generate 'mini-jets' via democratic branchings
  - daughter gluons carry comparable energy fractions: $z \sim 1 - z \sim 1/2$



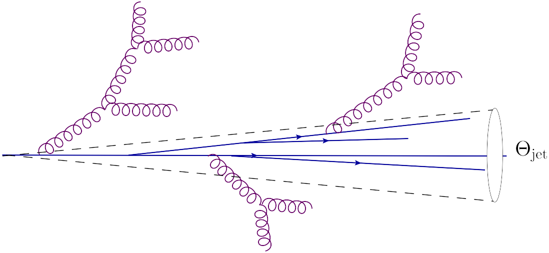
  - the probability for the splitting $\omega \to (z\omega,\ (1-z)\omega)$:

$$\mathcal{P}(z\omega, L) \simeq \frac{L}{t_{\rm br}(z\omega)} \simeq \bar{\alpha}\, L \sqrt{\frac{\hat{q}}{z\omega}}$$

  - when $\omega \sim \omega_{\rm br}$, $\mathcal{P}(z\omega, L) \sim 1$ independently of the value of z

Energy loss by the jet

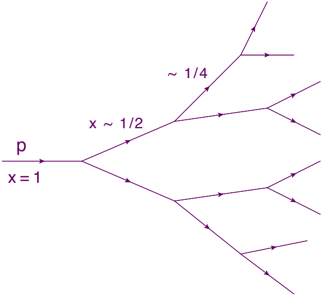
- Democratic branchings are very unusual in the context of gauge theories
  - recall: bremsstrahlung favors asymmetric splittings with $z \ll 1$
- Extremely efficient in redistributing the energy among softer quantas
  - a mini-jet with $\omega \lesssim \omega_{\rm br}$ "disappears" in a time $t_{\rm br}(\omega) \lesssim L$
  - its energy is successively transmitted to softer and softer gluons
- Soft gluons are easily deviated outside the jet cone by the elastic collisions

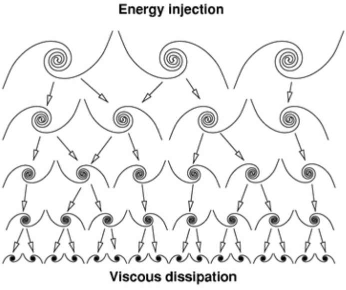


- The energy appears in many soft quanta propagating at large angles ✓

Wave turbulence

- Democratic branchings lead to wave turbulence
  - energy flows from one parton generation to the next one, at a rate which is independent of the generation
  - formally, it accumulates into a condensate at $x = 0$
  - physically, it dissipates into the medium, via thermalization
  - similar to Kolmogorov turbulence ... but much simpler: 1+1 dimensions, inverse cascade, exact solutions





Probabilistic picture for medium-induced radiation

Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv:1311.5823)

- Medium-induced jet evolution $\approx$ a Markovien stochastic process
  - successive branchings are non-overlapping: $t_{\rm br} \sim \frac{1}{\alpha_s} t_{\rm med}$
  - interference phenomena could complicate the picture ...
    (in the vacuum, they lead to angular ordering)
  - ... but they are suppressed by rescattering in the medium (see below)
- Hierarchy of equations for n-point correlation functions ($x \equiv \omega/E$)

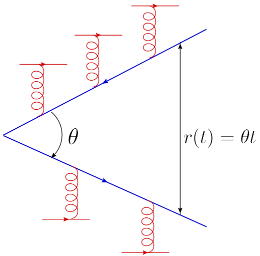
$$D(x,t) \equiv x \frac{\mathrm{d}N}{\mathrm{d}x}(t)\,, \qquad D^{(2)}(x,x',t) \equiv xx' \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}x\,\mathrm{d}x'}(t)$$

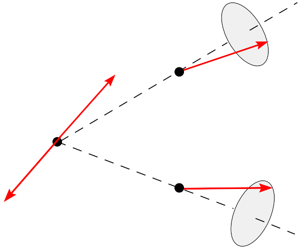
- Analytic solutions *(Blaizot, E.I., Mehtar-Tani, '13; Escobedo, E.I., '16)*
- New phenomena: wave turbulence, KNO scaling, large fluctuations...
- Recent Monte-Carlo implementation *(see the talk by Paul Caucal next week)*
  (P. Caucal, E.I., G. Soyez, A.H. Mueller, to appear)

Color antenna in the medium

Mehtar-Tani, Salgado, Tywoniuk (2010-11); Casalderrey-Solana, E. I. (2011)

- The two quarks undergo independent color precessions due to rescattering
- The instantaneous color state of each quark: the respective Wilson line
- Their color correlation is measured by the dipole S–matrix





$$S(t,\theta) \simeq \exp\left\{-\frac{1}{4}\hat{q}\int_0^t \mathrm{d}t'\, r^2(t')\right\} \simeq \exp\left\{-\frac{1}{12}\hat{q}\theta^2 t^3\right\}$$

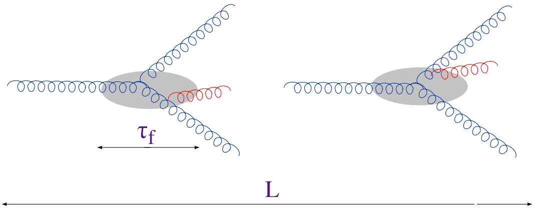
- Color coherence is lost for $t \gtrsim t_{\rm coh} \sim 1/(\hat{q}\theta^2)^{1/3}$

Color decoherence for medium induced emissions

- Consider an antenna generated via a medium-induced gluon splitting:

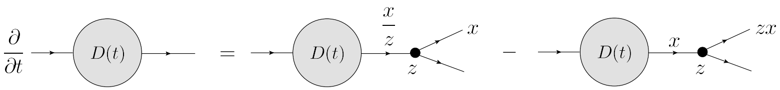
$$\theta_{\rm f}(\omega) \simeq \left(\frac{2\hat{q}}{\omega^3}\right)^{1/4} \Longrightarrow t_{\rm coh} \sim \frac{1}{(\hat{q}\theta_{\rm f}^2)^{1/3}} \sim \sqrt{\frac{2\omega}{\hat{q}}} = t_{\rm med}(\omega)$$

  - the two daughter gluons lose their mutual color coherence already during the formation time
  - not a "coincidence": decoherence via rescattering is what triggers the medium-induced radiation



J.-P. Blaizot, E. I., Y. Mehtar-Tani, 2013

- Kinetic equation for $D(x,t) = x(\mathrm{d}N/\mathrm{d}x)$: 'gain' - 'loss'

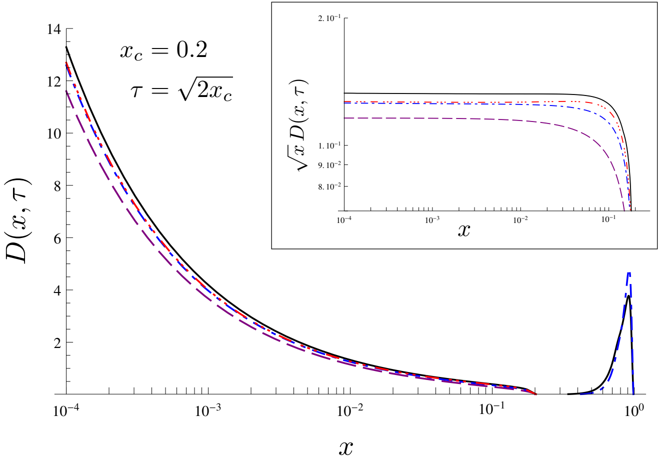


- Exact solution with initial condition $D(x, t=0) = \delta(x-1)$

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-2\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- $t_{\rm br}(E)$: the lifetime of the LP until its first democratic branching
  - power-law spectrum $D \propto \frac{1}{\sqrt{x}}$ at $x \ll 1$ for any τ
  - Kolmogorov fixed point: wave turbulence

- Numerical results for $D(x,t) = x(\mathrm{d}N/\mathrm{d}x)$: $E = 5\omega_c$ and $\bar{\alpha} = 0.3$



- pronounced LP peak near $x = 1$
- power-law spectrum $D \propto \frac{1}{\sqrt{x}}$ at $x \ll 1$

Energy loss at (very) large angles

- The energy leaks into the condensate at $x = 0 \Longrightarrow$ the energy fraction contained in the spectrum decreases with time: $\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-2\pi\tau^2}$
- The energy lost at very large angles = the energy missing from the spectrum

$$\Delta E \,=\, E\big(1-\mathrm{e}^{-2\pi\tau^2}\big) \,=\, E\left[1-\mathrm{e}^{-2\pi\frac{\omega_{\rm br}}{E}}\right]$$

- When $E \gg \omega_{\rm br}$, this is independent of E: $\Delta E \,\simeq\, 2\pi\omega_{\rm br} \,=\, \pi\bar{\alpha}_s^2\hat{q}L^2$
  - LHC: $E \sim 100\,{\rm GeV} \,\gg\, \omega_{\rm br} \sim 5 \div 10\,{\rm GeV}$
- Additional, angle-dependent, contribution from soft modes in the spectrum

$$\theta(\omega) \,\simeq\, \frac{\sqrt{\hat{q}L}}{\omega} \gtrsim \theta_0 \;\Longrightarrow\; \omega \lesssim \frac{\sqrt{\hat{q}L}}{\theta_0}$$

- A natural explanation for the di-jet asymmetry observed at the LHC

Energy loss at (very) large angles

- The energy leaks into the condensate at $x = 0 \Longrightarrow$ the energy fraction contained in the spectrum decreases with time: $\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-2\pi\tau^2}$
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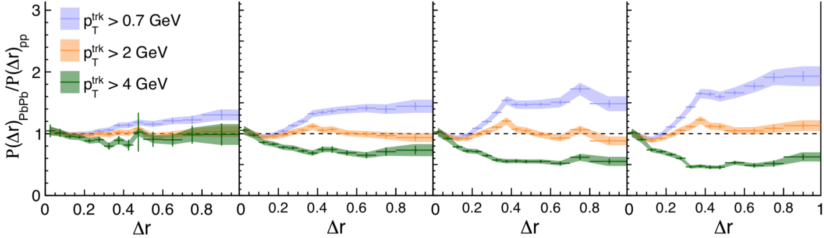
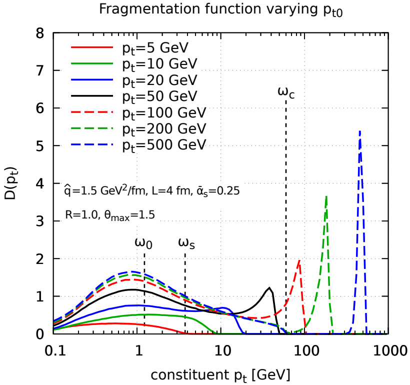
$$\Delta E \;=\; E\big(1-\mathrm{e}^{-2\pi\tau^2}\big) \;=\; E\left[1-\mathrm{e}^{-2\pi\frac{\omega_{\rm br}}{E}}\right]$$

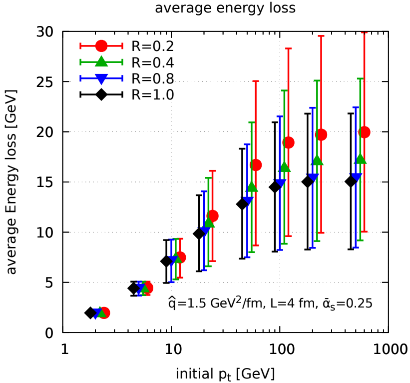


- CMS: The jet transverse momentum profile $P(\Delta r)$ (p_T in annular rings)
  - Pb+Pb vs. p+p: excess of soft particles at large angles outside the jet

Monte Carlo: Medium-induced radiation only

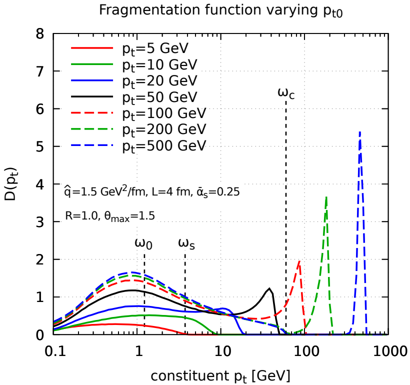
- Initial parton with energy $E \equiv p_{t0}$, jet with opening angle R
- Left: fragmentation function (or spectrum) $D(\omega) = \omega \frac{\mathrm{d}N}{\mathrm{d}\omega}$
  - leading-particle peak so long as $E \gg \omega_{\rm br}\ (\equiv \omega_s)$
  - medium-induced radiation: $D(\omega) \propto \frac{1}{\sqrt{\omega}}$ at $\omega < \omega_c$

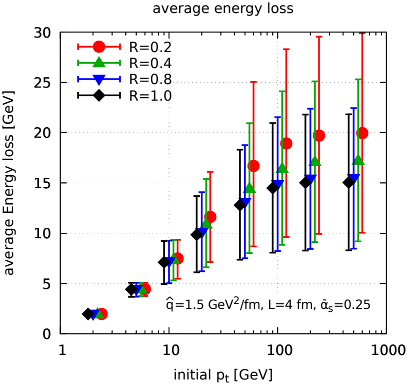




Monte Carlo: Medium-induced radiation only

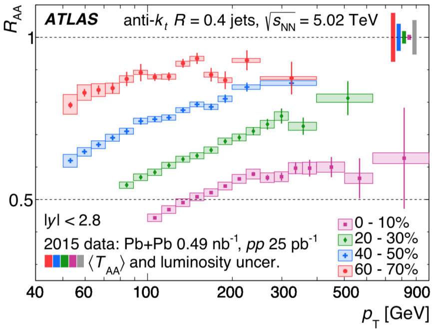
- Initial parton with energy $E \equiv p_{t0}$, jet with opening angle R
- Right: average energy loss by the jet + its fluctuations
  - ΔE becomes independent of E when $E \gg \omega_{\rm br}$
  - energy is recovered when increasing R ... but very slowly

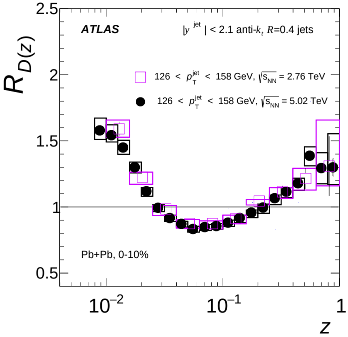




With due respect to the vacuum

- So far: energy loss at large angles via medium-induced emissions
  - energy loss predicted to approach a constant value at high energy
- The data for R_{AA} suggest that the energy loss should increase with p_T





- Medium-induced modifications of the hadron distribution inside the jet cone
- Features that are naturally explained after adding the vacuum-like radiation

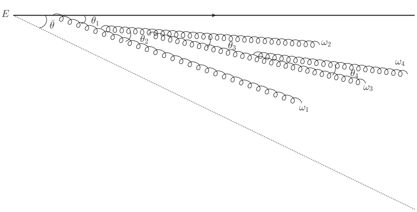
Vacuum-like emissions inside the medium

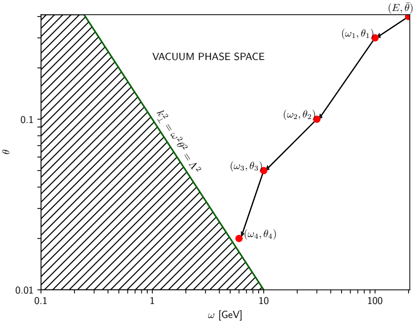
- "What do you mean by that ?!?"
- Emissions triggered by parton virtualities, not by collisions in the medium
- Can they be different from the standard parton showers in the vacuum (say, as measured in pp collisions) ?
- YES, THEY CAN ! They have a restricted phase-space ...
  - no collinear singularity, prompt formation times

$$t_{\rm f} = \frac{2}{\omega\theta^2} < t_{\rm med} = \sqrt{\frac{2\omega}{\hat{q}}} \qquad \text{or} \qquad \theta > \left(\frac{2\hat{q}}{\omega^3}\right)^{1/4}$$

- They can suffer energy loss & $p_\perp$-broadening via medium-induced emissions
  - they do so ... but only after formation !
- They can be affected by color decoherence due to in-medium scattering
  - is angular ordering still there ?

- Each emission: a point $(\omega_i,\, \theta_i)$ in the energy–emission angle phase-space
  - logarithmic units: convenient for the double logarithmic approximation
  - $\bar{\theta}$: the maximal angle allowed for the first emission
  - E: the energy of the leading parton, hence the maximal energy allowed to the first emission



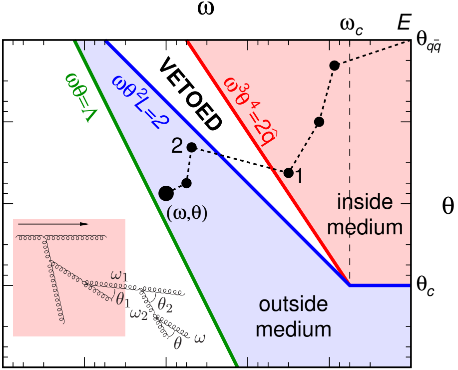


- Evolution stopped by hadronisation: $k_\perp \simeq \omega\theta \gtrsim \Lambda_{\rm QCD}$

Vacuum-like emissions inside the medium

P. Caucal, E. I., A. Mueller, G. Soyez, `arXiv:1801.09703`, PRL 120 (2018)

- Vacuum-like parton cascades occur relatively fast and can be factorized in time from the medium-induced cascades



- inside-medium cascades:

$$\frac{2}{\omega\theta^2} < \sqrt{\frac{2\omega}{\hat{q}}}$$

- outside-medium emissions:

$$\frac{2}{\omega\theta^2} > L$$

- forbidden region in between

- End point of VETOED at $\omega_c = \frac{1}{2}\hat{q}L^2$, $\theta_c = 2/\sqrt{\hat{q}L^3}$
  - typical values: $\hat{q} = 1\,\mathrm{GeV}^2/\mathrm{fm}$, $L = 4\,\mathrm{fm}$, $\omega_c = 50\,\mathrm{GeV}$, $\theta_c = 0.05$

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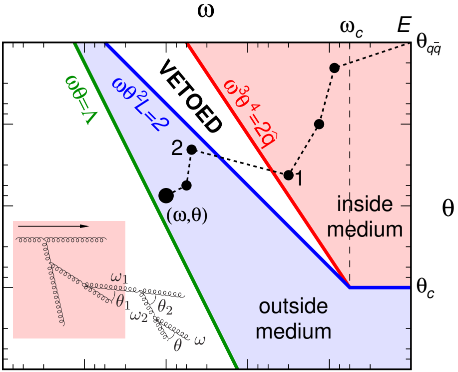

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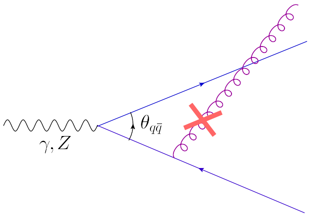
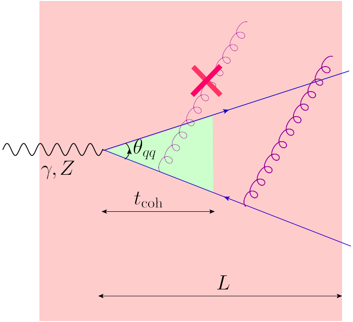
- forbidden region in between

- End point of VETOED at $\omega_c = \frac{1}{2}\hat{q}L^2\,,\ \theta_c = 2/\sqrt{\hat{q}L^3}$
- Energy loss & $p_\perp$-broadening during formation are negligible

Color (de)coherence

- In vacuum, wide angle emissions ($\theta > \theta_{q\bar{q}}$) are suppressed by color coherence



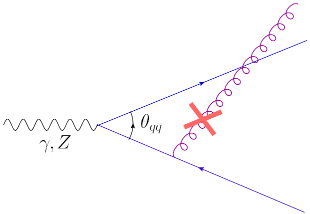


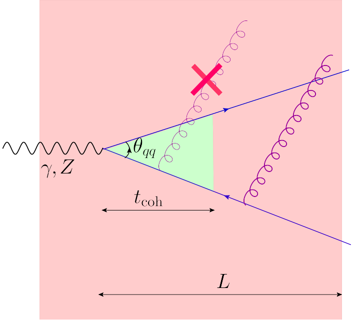
- In medium, color coherence is washed out by collisions after a time $t_{\rm coh}$

$$t_{\rm coh} = \frac{1}{(\hat{q}\theta_{q\bar{q}}^2)^{1/3}} \ll L \quad \text{if} \quad \theta_{q\bar{q}} \gg \theta_c \simeq 0.05$$

- Angular ordering could be violated for emissions inside the medium

- In vacuum, wide angle emissions ($\theta > \theta_{q\bar{q}}$) are suppressed by color coherence



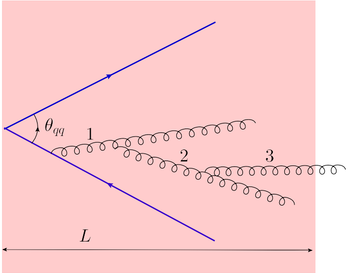


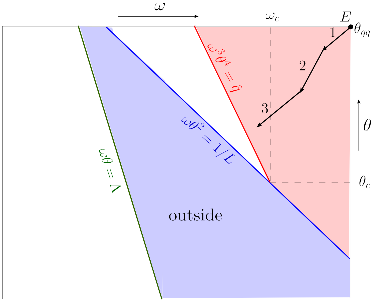
- But this is not the case for the VLEs !

$$\theta > \theta_{q\bar{q}} \quad \& \quad t_{\rm f} = \frac{2}{\omega\theta^2} \ll \sqrt{\frac{2\omega}{\hat{q}}} \implies t_{\rm f} \ll t_{\rm coh}$$

- Wide-angle VLEs ($\theta > \theta_{q\bar{q}}$) are still suppressed $\Longrightarrow$ angular ordering

- Previous arguments extend to a sequence of angular-ordered VLEs
  - angular ordering $\Longrightarrow$ formation times are strongly increasing
  - formation time for the whole cascade $\approx$ that of the last gluon

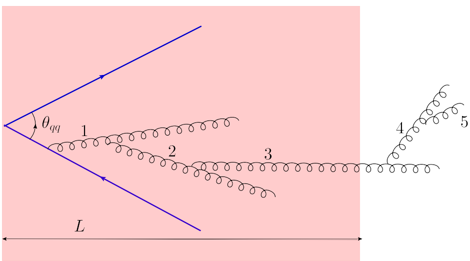


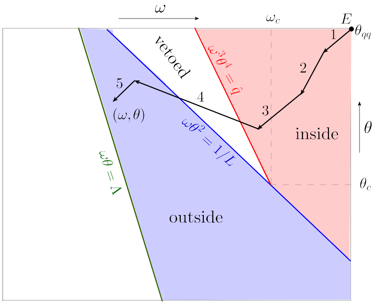


- After formation, partons propagate in the medium along a distance $\sim L$
  - additional sources for medium-induced radiation
  - ... and for vacuum-like emissions outside the medium

First emission outside the medium

- The respective formation time is necessarily large: $t_{\rm f} \gtrsim L$
- In-medium sources with $\theta \gg \theta_c$ lose coherence in a time $t_{\rm coh} \ll L$
- First outside emission can violate angular ordering
  - re-opening of the angular phase-space
  - enhanced radiation at low energies and large angles

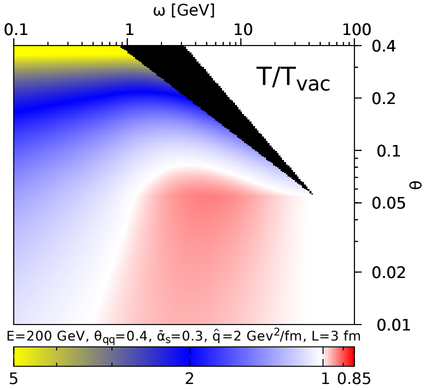


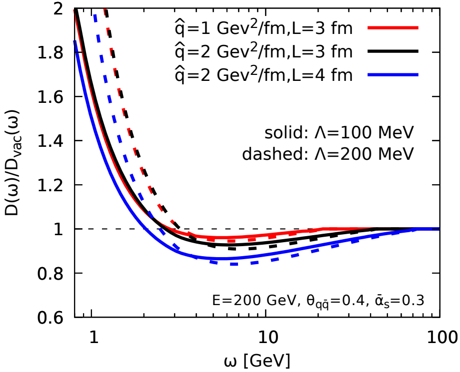


- Subsequent "outside" emissions obey angular-ordering, as usual

- The double-differential distribution & the fragmentation function:

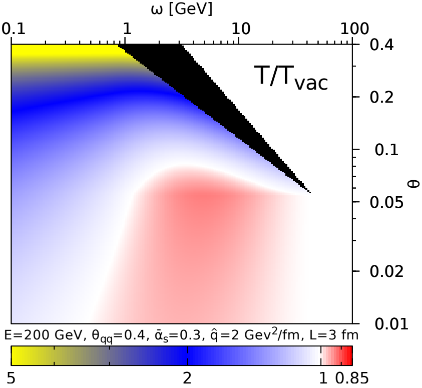
$$T(\omega,\theta) \equiv \omega\theta^2 \frac{\mathrm{d}^2 N}{\mathrm{d}\omega \mathrm{d}\theta^2} \quad \Longrightarrow \quad D(\omega) \equiv \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} = \int \frac{\mathrm{d}\theta^2}{\theta^2}\, T(\omega,\theta^2)$$

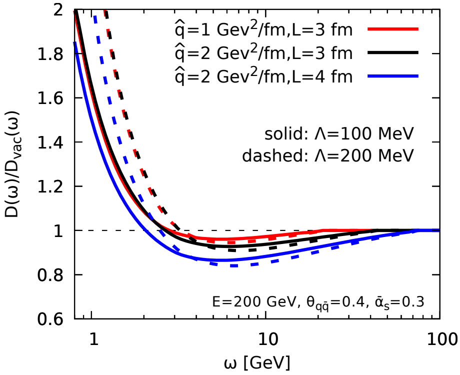




DLA as a warm up

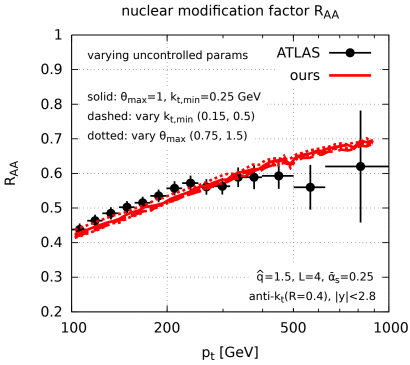
- Slight suppression at intermediate energies (from 3 GeV up to ω_c)
  - the phase-space is reduced by the vetoed region
- Significant enhancement at low energy (below 2 GeV)
  - lack of angular ordering for the first emission outside the medium

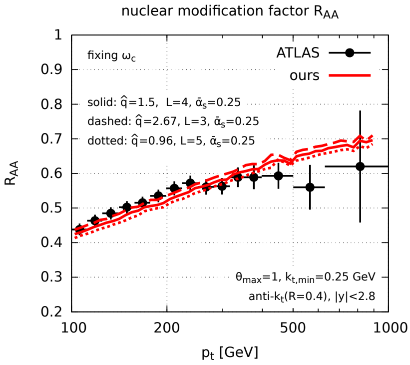




(P. Caucal, E.I., A. H. Mueller and G. Soyez, in preparation)

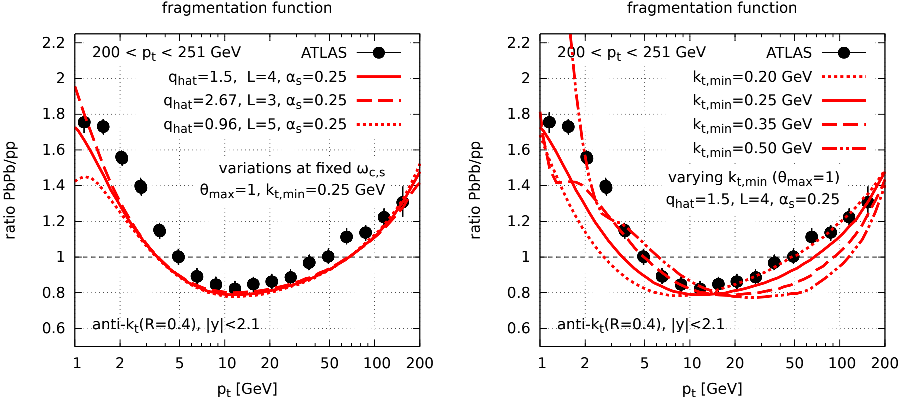
- Probabilistic (Markovian) picture $\Longrightarrow$ straightforward MC implementation
- Not a fit: just a choice of "reasonable" values for the physical parameters
- Left: varying kinematical cuts: $k_{t,\min} \equiv \Lambda$, $\theta_{\max} \equiv \theta_{q\bar{q}}$
- Right: R_{AA} depends upon the medium only via $\omega_c = \hat{q}L^2$





Monte Carlo: Fragmentation function

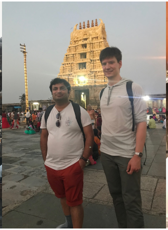
- The same "canonical" values for the medium parameters as in the "best" description of R_{AA} *(ATLAS data from arXiv:1805.05424)*
- Stronger variability with respect to the "uncontrolled parameters"



- *See the talk by Paul Caucal next Wednesday for more details !*

THANK YOU FOR YOUR ATTENTION !









Have a good end of stay at ICTS and a safe trip back home!