Free energy of a Holonomous Plasma

Y. Hidaka, C. Korthals-Altes, H. Nishimura, RDP, & V. Skokov 1905....

Holonomous Plasma: $A_0 \neq 0$. Polyakov loop $l < 1$

Necessary to describe the "semi" QGP, near T_c.

Holonomous Potential: *old* story at one and two loop order

@ 2 loop order, *need* a gauge invariant source

For weak holonomy ($g A_0 \sim m_{Debye}$), *need* a source with an infinite # loops

Free energy $F_3 \sim g^3$ are *not* continuous as l -> 1 for fixed source

Need *dynamical* fields: e.g., two dimensional massless ghosts

Strong constraints on effective theory by computing to $\sim g^3$!

Lattice, matrix models for a Holonomous Plasma

T^2 term in the free energy for pure glue: *deconfined strings*

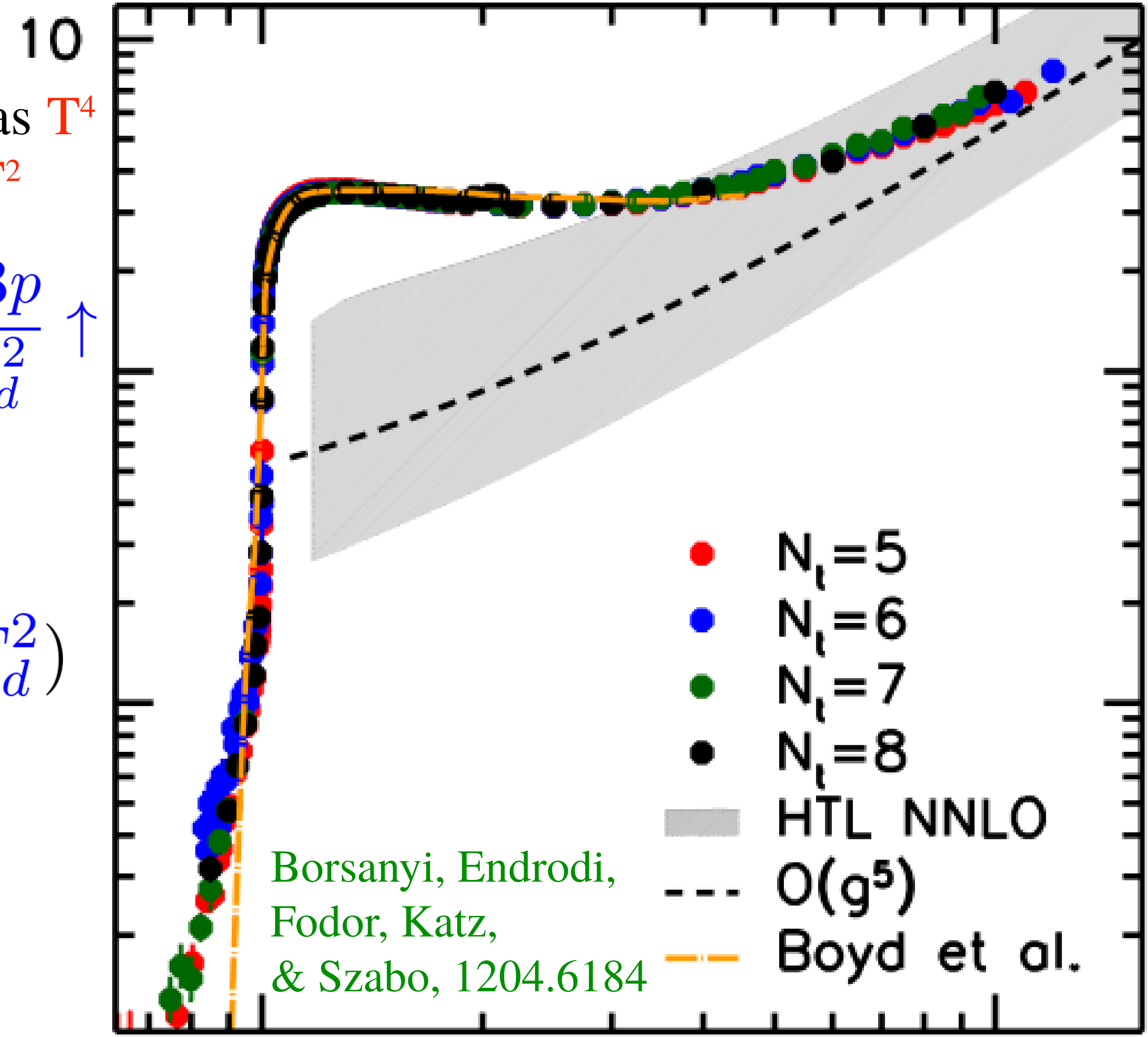
Polyakov loop's from the lattice: *broad* transition region
from confined to perturbative regime

Matrix models of a Holonomous Plasma: *narrow* transition region

Pure glue: *de*confined strings above T_d.

$T_d \rightarrow 4\,T_d$: for pressure, leading correction to ideal gas T^4 is *not* a bag constant, but ~ T^2

$$\frac{e - 3p}{T^2 T_d^2} \uparrow$$



$\uparrow T_d$ $10T_d \uparrow$

For T: 1.2 T_d -> 4 T_d,

$$p(T) \approx \#(T^4 - T^2 T_d^2)$$

T^2 term: *de*confined strings?

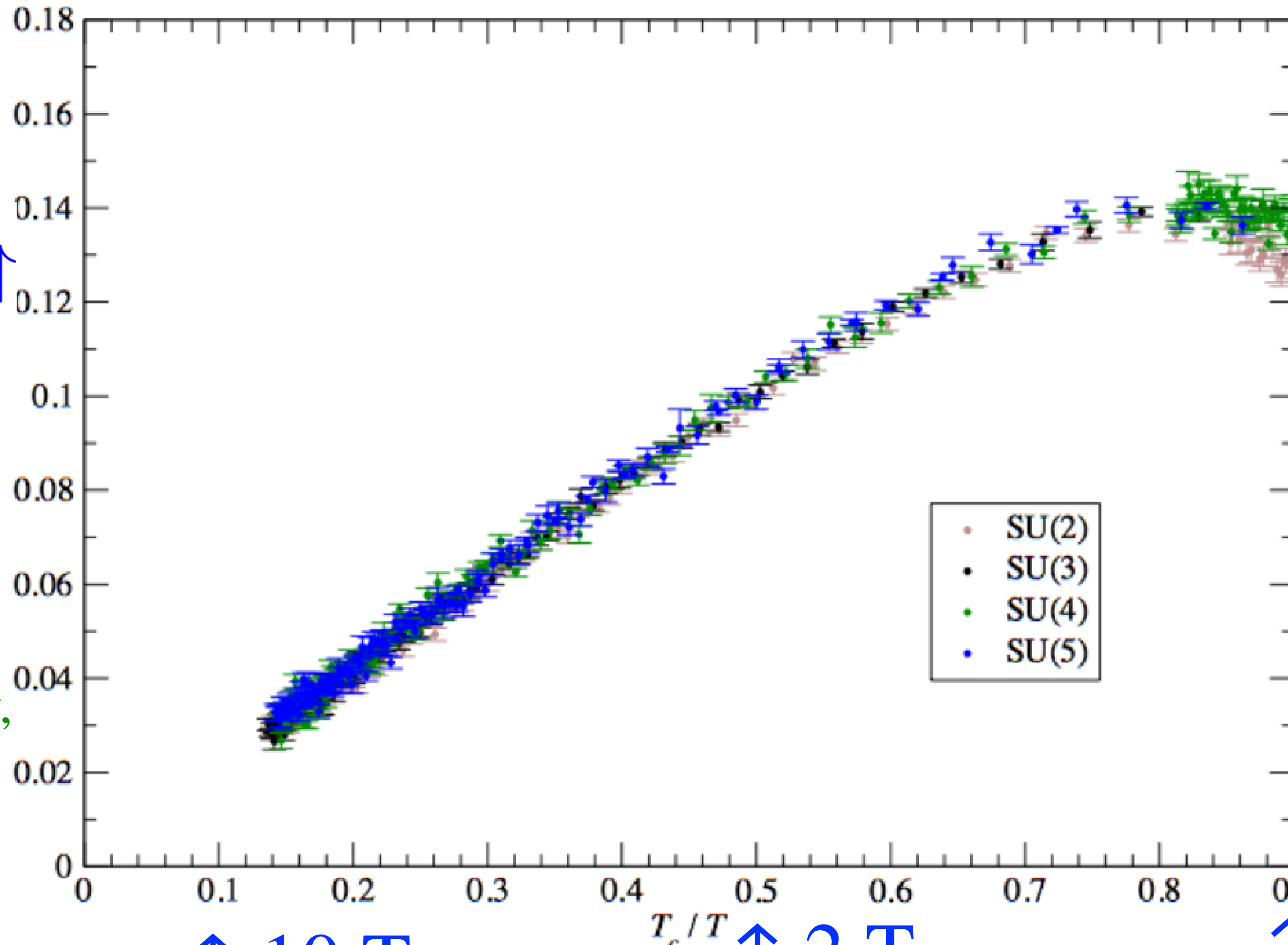
For T: T_d -> 1.2 T_d, involved transition. *Narrow region*

Pure glue: deconfined strings in 2+1 dim.'s

In 2+ 1 dimensions, leading correction to ideal gas T^3 is *again* T^2, N = 2,3,4,5

$$p(T) \sim \#(T^3 - T^2 T_d)$$

$$\frac{1}{N^2-1}\,\frac{e-2p}{T^3} \uparrow$$



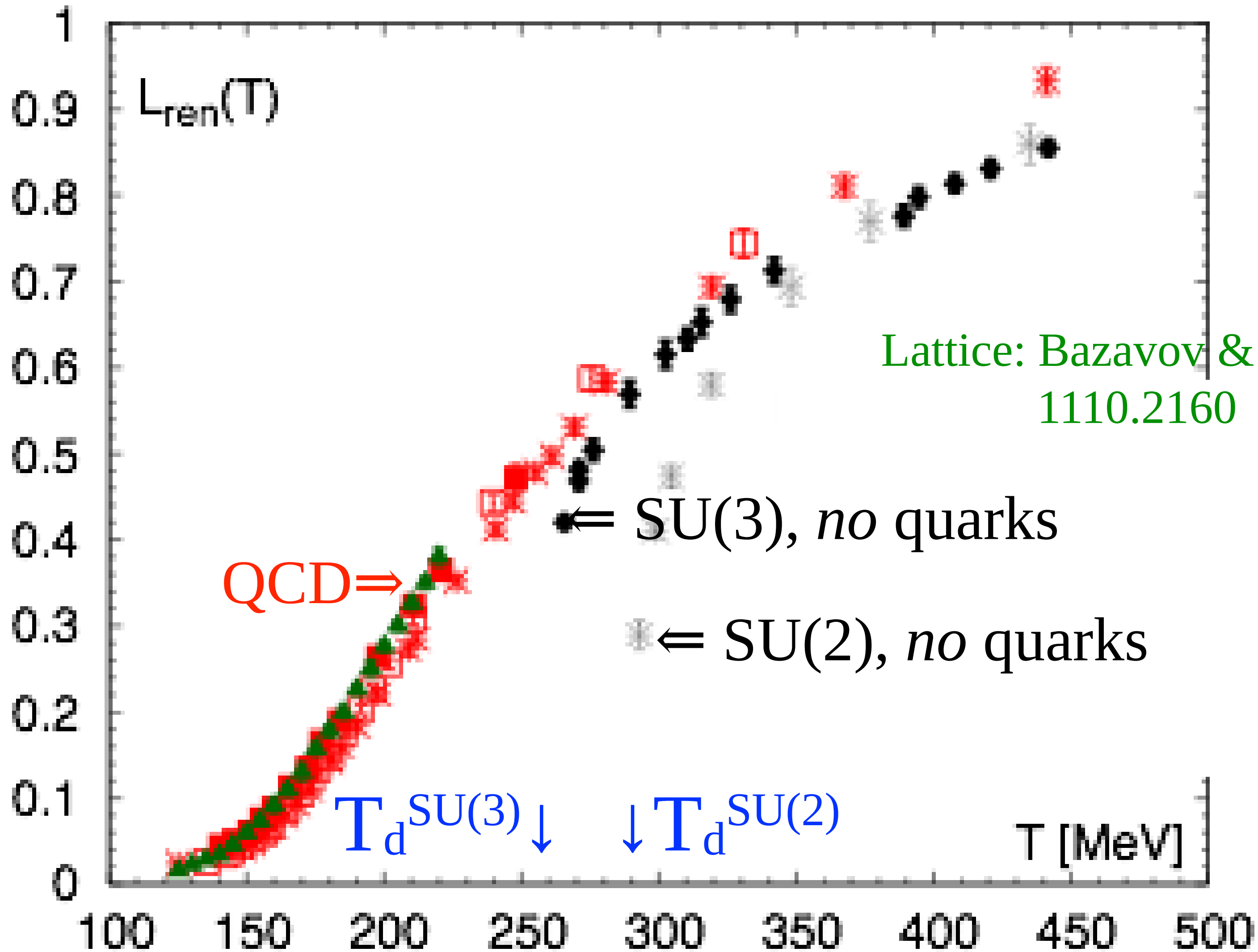
$T_d/T \rightarrow$

$\uparrow$ 10 T_d $\uparrow$ 2 T_d $\uparrow$ 1.1 T_d

Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580

Lattice: Polyakov Loop without and *with* quarks

Without quarks: *exact* order parameter for global Z(3) = Polyakov loop
Dynamical quarks a*lways* break Z(3). But in QCD, loop *small* at T_χ , ~0.1?
Broad transition from confined to deconfined phase

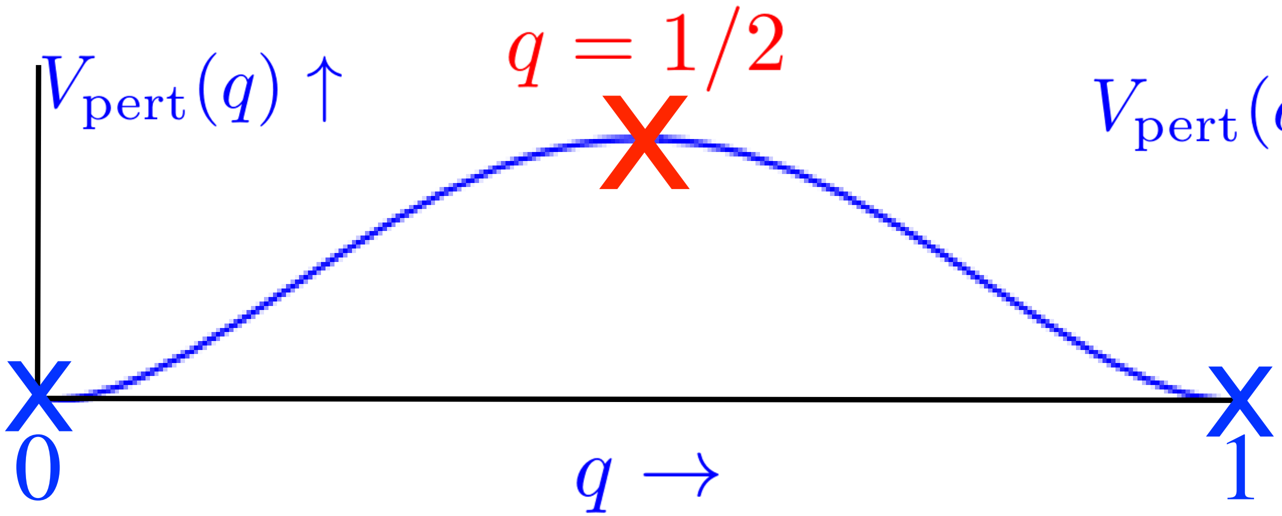


Holonomous Potential @ 1 loop order

Holonomy = constant A_0, . No potential classically, nonzero at 1 loop order.

Gross, RDP, Yaffe, '81; Weiss '82 For two colors:

$$A_0^{\rm cl} = \frac{\pi T}{g}\, q\, \sigma_3 \qquad \ell = \frac{1}{2}\, {\rm tr}\, \mathcal{P}\, {\rm e}^{ig\int_0^{1/T} A_0} = \cos(\pi q)$$



$$V_{\rm pert}(q) = \frac{4\pi^2}{3} T^4 q^2 (1-q)^2$$

Z(2) degenerate vacua: q = 0,1.

Confining vacuum, q = 1/2, is *maximum* of perturbative potential.

Non-perturbative Holonomous Potential

To model deconfinement, add - *by hand* - a non-perturbative potential for q:
Dumitru, Guo, Hidaka, Korthals-Altes & RDP 1011.3820; 1205.0137.

$$V_{\rm non}(q) = \frac{4\pi^2}{3} T^2 T_d^2 \left(-\frac{c_1}{5} q(1-q) - c_2 q^2 (1-q)^2 + \frac{c_3}{15} \right)$$

$V_{tot}(q) = V_{pert}(q) + V_{non}(q)$, determine <q> from minima, fit to $p(T) = - V_{tot}(<q>)$.

Find: $<q> \neq 0$ in *narrow* region, T: T_d -> 1.2 T_d .

Constant term, c_3 , gives $p(T) \sim T^2$ for T: 1.2 T_d -> 4 T_d.

Linear term at small q, ~ c_1 q, is *crucial* to ensure holonomy turns on *smoothly*.

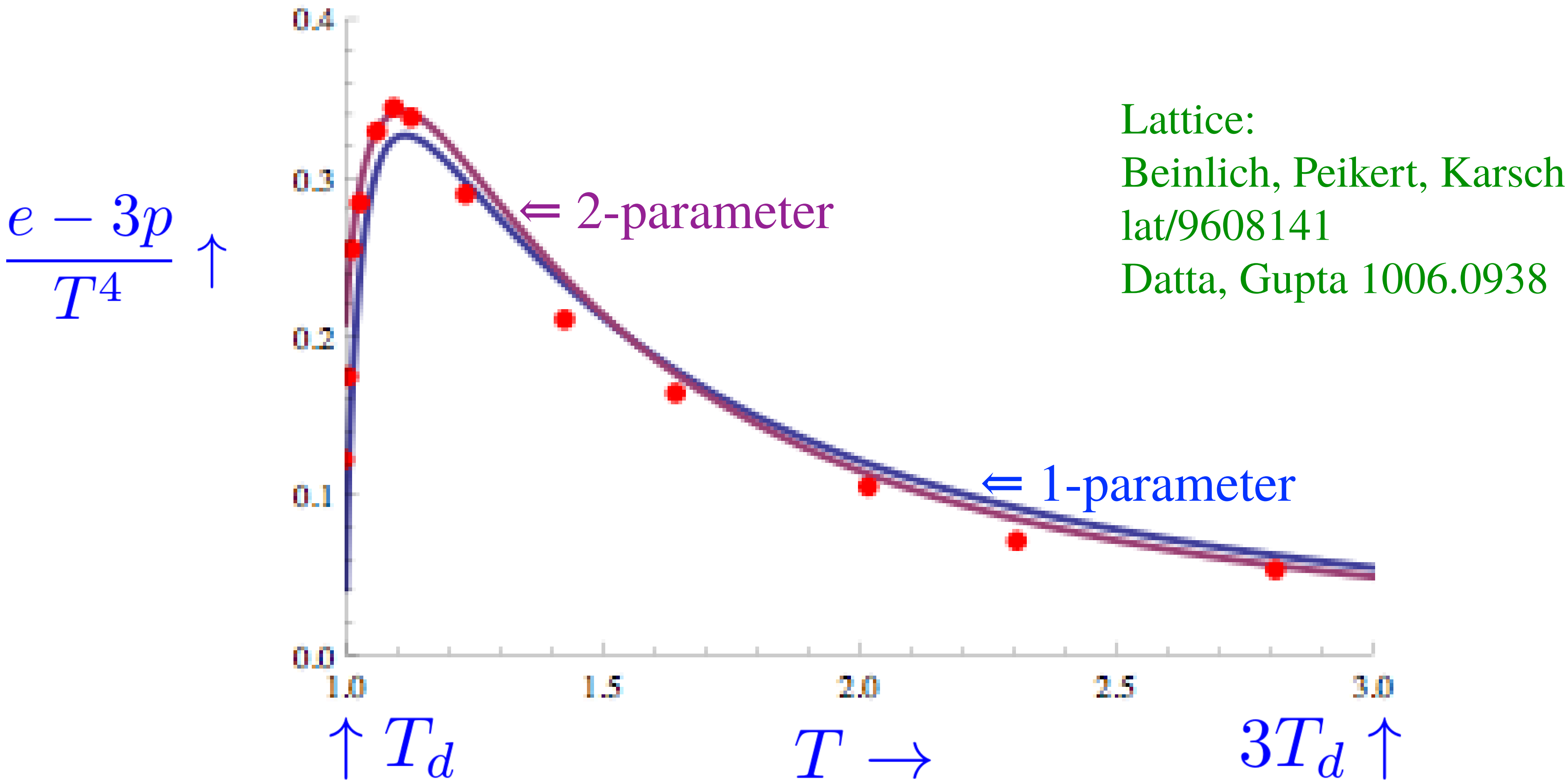
c_2 is like the $V_{pert}(q)$.

Use to compute Polyakov and ‘t Hooft loops.

Matrix model for three colors

Start with three parameters. Require transition occurs at T_d, and $p(T_d) \sim 0$.
Leave one free parameter, adjust to agree with $(e-3p)/T^4$.

$$T_d = 270 \text{ MeV} \;,\; c_1 = 0.315 \;,\; c_2 = 0.83 \;,\; c_3 = 1.13$$

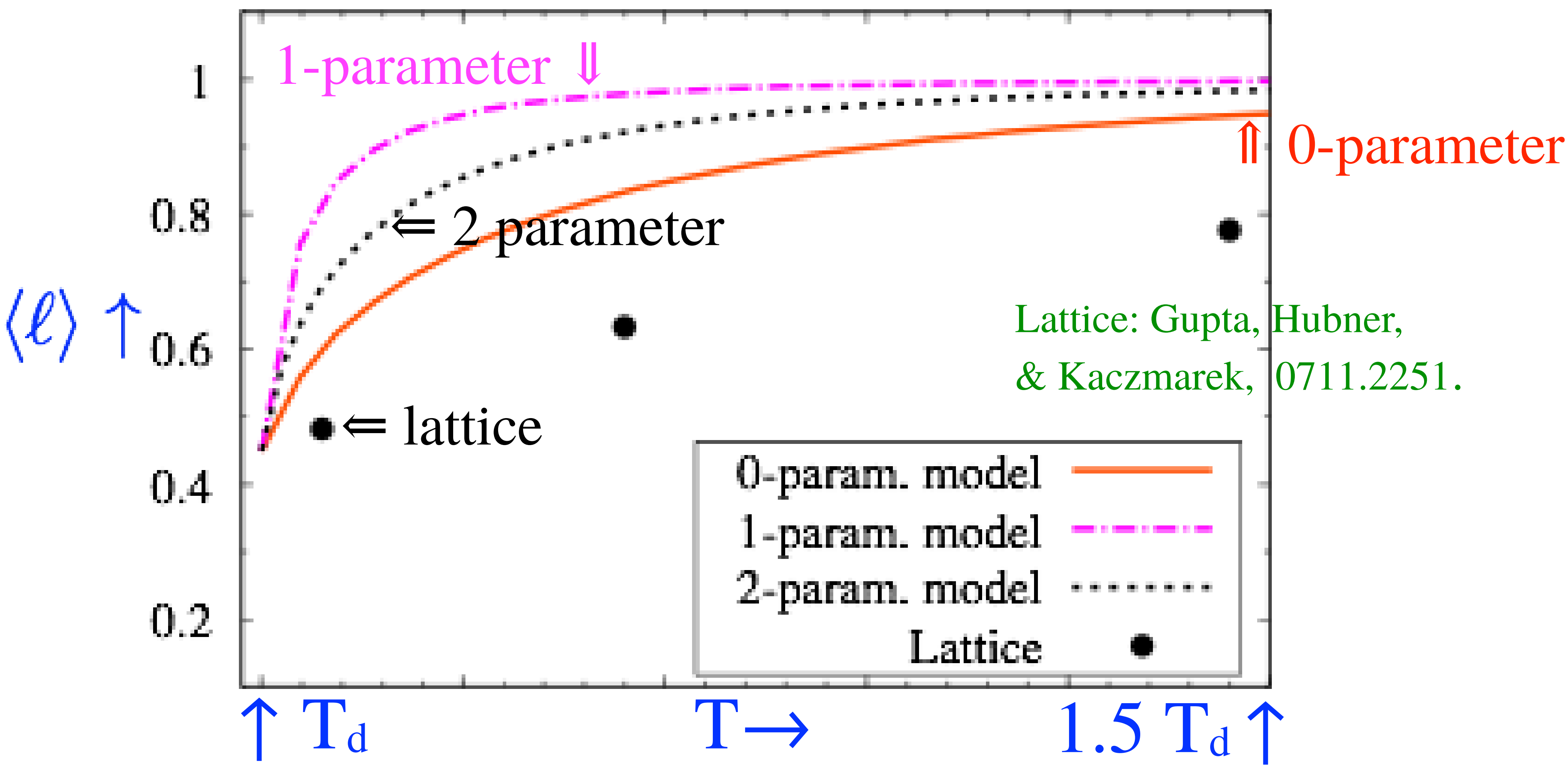


Polyakov loop: model vs lattice?

Polyakov loop *much* smaller than the matrix model

Transition region: matrix model *narrow*, to ~ 1.2 T_d.   Lattice *wide*, to ~ 4.0 T_d.

But: if one fits to lattice loop, 't Hooft loop is *much* too small.



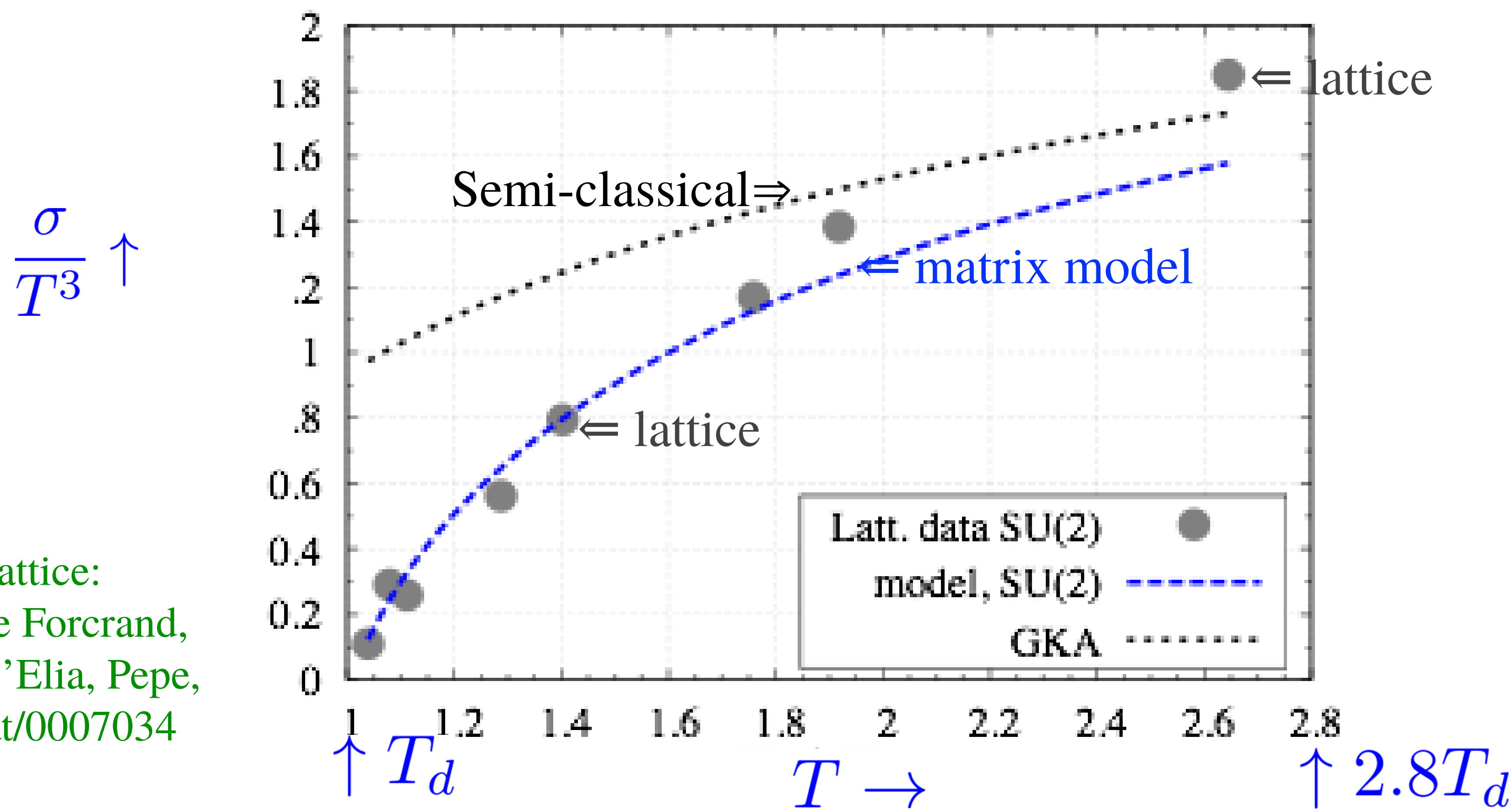
’t Hooft loop from Holonomous Potential

For pure gauge, ’t Hooft loop σ = $Z(N_c)$ interface tension.

Compute σ as tunneling problem in $V_{tot}(q)$, from q = 0 to 1.

Using $V_{pert}(q)$: Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

With loop as in matrix model, *excellent* agreement with lattice data.



Lattice:
de Forcrand,
D’Elia, Pepe,
lat/0007034

Holonomous Model with quarks

RDP & Skokov, 1604.00022. Add linear sigma model + quarks.

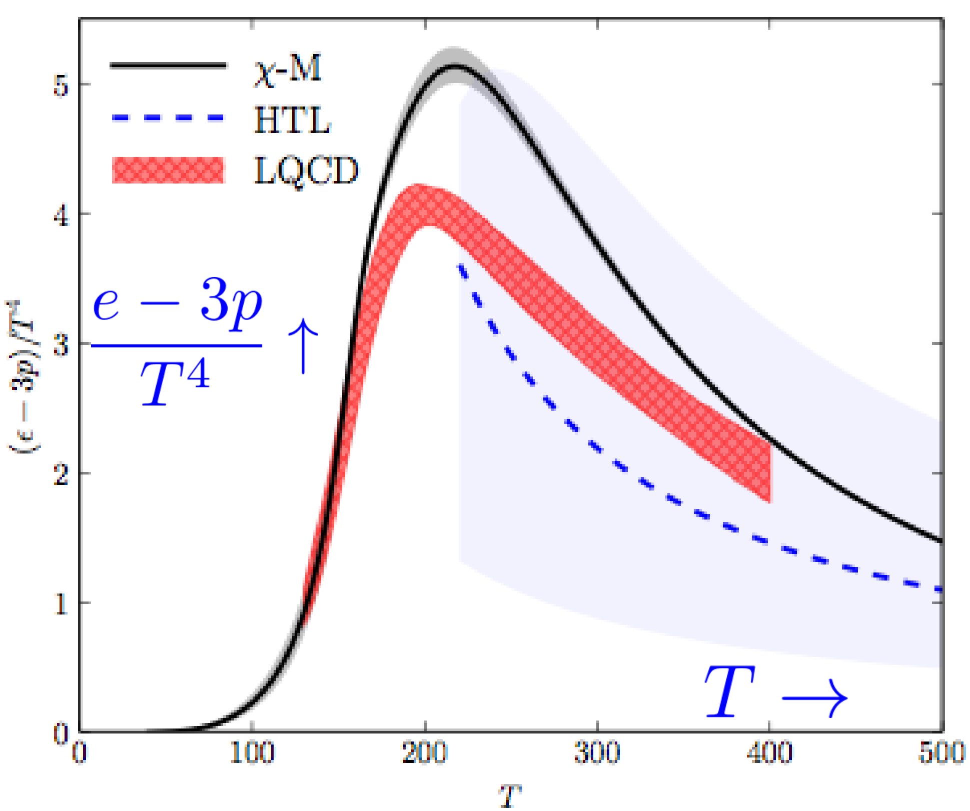
Quarks generate contributions to Holonomous Potential, break Z(3) symmetry.

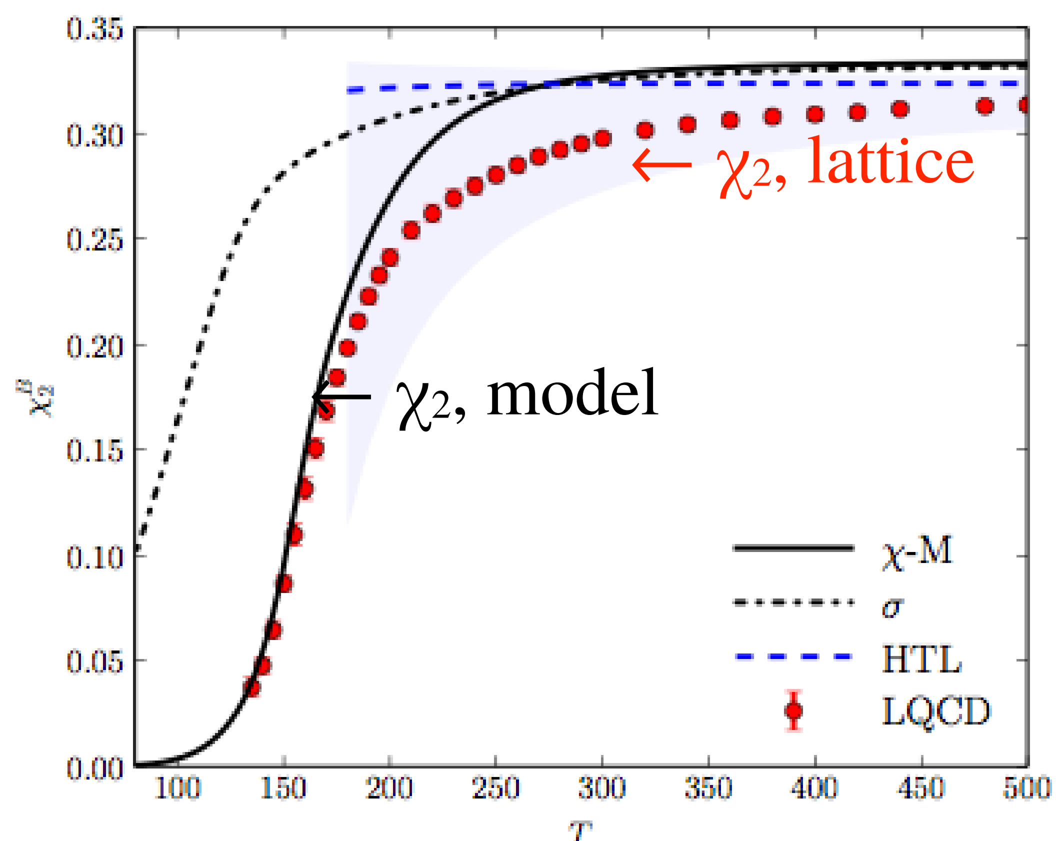
Keep T_d = 270 MeV, tune Yukawa interaction to get T_{chiral} = 154 MeV.

Non-trivial: pressure remains positive for $T < T_d$.

Polyakov loop *much* narrower than lattice, but: baryon suscep.'s ~ agree.

How broad is the transition regime with quarks? What's up c Polyakov loop?





Perturbative computations in a Holonomous Plasma

One loop order, ~1 in free energy: easy peasy

Two loop order, ~ g^2 in free energy:

gauge dependent source => gauge variant free energy

gauge invariant source => gauge invariant potential

=> *transverse* gluon self energy

~ g^3 in free energy for soft Q

need gauge invariant source with infinite sum over loops

free energy for off-diagonal gluons *dis*continuous as Q -> 0?!

generating Holonomous Plasma with *dynamical* fields

massless, 2D ghosts (~ deconfined strings)

free energy for off-diagonal gluons continuous as Q -> 0

Holonomous potential to one loop order

For SU(N), take: $A_\mu = A_\mu^{\rm cl} + A_\mu^{\rm qu}\,,\; A_0^{\rm cl} = \frac{2\pi T}{g}\, q$ $\quad (q^{ab} = q^a \delta^{ab}\; \sum_{a=1}^{N} q^a = 0)$

Work in background field gauge. In momentum space, n = 0, ±1...

$$D_\mu^{\rm cl} = \partial_\mu - ig[A_\mu^{\rm cl}, *]\,,\; iD_0^{\rm cl} \to p_0^{ab} = -i2\pi T(n + q_a - q_b)$$

Easy to compute, just p_0 -> p_0^{ab}. Result *in*dependent of gauge fixing:

$$\mathcal{S}_{\rm pert,1} = -2\,{\rm tr}\;\log((p_0^{ab})^2 + p^2) = -\frac{T^4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^4}|{\rm tr}\mathbf{L}^n|^2$$

$$= \frac{2\pi^2 T^4}{3}\sum_{a,b=1} B_4(q_a - q_b)\,,\; B_4(x) = -\frac{1}{30} + x^2(1 - |x|)^2$$

The q_a respect Z(N) symmetry: Weyl chamber. Term ~ q_a^2 = Debye mass sq'd

Holonomous Potential, two loop order

Add gauge variant source ~ tr J_0 A_0 . Enqvist & Kajantie '90 .

Use background field gauge with gauge fixing parameter ξ

$$\mathcal{V}_{\text{pert},2}(q) = \frac{g^2 T^3}{4} \sum_{a,b,c=1}^{N} B_2(q_a - q_c) B_2(q_b - q_c) + (1-\xi) B_1(q_a - q_c) B_3(q_b - q_c)$$

Bernoulli polynomials B_1...,

s(x) = sign(x)

$$B_1(x) = -s(x)/2 + x \; ; \; B_2(x) = 1/6 - |x| + x^2$$

$$B_3(x) = x/2 - 3s(x)x^2/2 + x^3$$

Holonomous Potential is ξ-dependent!

Can also show that apparently $\langle q_a \rangle \sim \xi$: spontaneously breaks CP!?

But the source is gauge variant, so....

Holonomous potential, two loop order, redux

Belyaev '91 Under a gauge transformation Ω,

$$\mathbf{L}(x) = \mathcal{P} \exp(ig \int_0^{1/T} A_0(\tau, x) d\tau) \rightarrow \Omega(1/T, x)^\dagger \mathbf{L}(x) \Omega(0, x)$$

Thermal Wilson line **L** is gauge dependent; eigenvalues, q_a, are gauge *in*variant.

For SU(2), eigenvalue q renormalizes at one loop order:

$$q_{\rm ren} = -(3-\xi) g^2/(8\pi^2)(q - 1/2)$$

Including this, for SU(N)

$$\mathcal{V}_{\rm pert,2}(q_a) = -5 g^2 T^3/24 \sum_{a,b=1}^{N} B_4(q_a - q_b)$$

Manifestly gauge invariant, perturbative vacuum $q_a = 0$ stable

Bhattacharya, Gocksch, Korthals-Altes, RDP '90, '92 : Z(N) interface tension at NLO

Dumitru, Guo, Korthals-Altes, 1305.6846; Guo 1409.6539. General result in SU(N)

Consistent analysis of Holonomous Potential

Korthals-Altes '93 Compute gluon self energy perturbatively to 1 loop:

$$P_\mu^{ab}\,\Pi^{ab;\mu\nu}_{\rm pert} = +\delta^{\nu 0}\,4\pi g^2 T^3/3 \sum_{a,b,c=1}^{N} \left(B_3(q_a - q_c) + B_3(q_c - q_b)\right)$$

With gauge *in*variant sources; consistent with BRST identities

Severe problem in computing to higher loop order, esp. ~ g^3 in free energy.

Resolution: B_3 = 4 d/dx $B_4(x)$ ~ derivative of the Holonomous Potential.

We show: expanding about a *consistent* stationary point,

the gluon self energy *is* transverse.

Gauge invariant sources

SU(N) sources for first N Polyakov loops:

$$\mathcal{S}_{\rm J} = 1/V \int d^3x \sum_{r=1}^{N} J_r \, {\rm tr}\, \mathbf{L}^r(x)$$

Terms linear in $A_\mu^{qu} = 0$
=> equations of motion

$$1/V \sum_{r=1}^{N} 2\pi i \, J_r \, r \, {\rm e}^{2\pi i r q_a} + 16\pi^2 T^3/3 \sum_{b=1}^{N} B_3(q_a - q_b) = 0$$

Only N-1 independent sources:

$$\sum_{a=1}^{N} \sum_{r=1}^{N} r \, J_r \, {\rm e}^{2\pi i r q_a} = 0$$

Sources ~ Polyakov loops *non*linear in A_μ^{qu}, so terms *quadratic* in A_μ^{qu} :

$$\Pi_{\rm J}^{ab;00} = -(1/p_0^{ab}) \, 4\pi g^2 T^3/3 \sum_{a,b,c=1}^{N} \left(B_3(q_a - q_c) + B_3(q_c - q_b)\right)$$

Using equations of motion. Valid for *arbitrary* sources. Self energy transverse

$$P_\mu^{ab} \left(\Pi_{\rm pert}^{ab;\mu\nu} + \Pi_{\rm J}^{ab;\mu\nu} \right) = 0$$

Gives same, gauge invariant free energy, to g^2.

Weak Holonomous Potential to g^3.

Consider small $q_a \sim g$. Then for A^{qu}, $q_a{}^2\, T^2 \sim m_{Debye}{}^2$. Weak Holonomous plasma

Perturbatively, need to resum "ring" diagrams. In Holonomous Plasma,

$$\mathcal{F}_3 = -T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \, \mathrm{tr} \, \log \left((P_\mu^{ab})^2 + (\xi^{-1} - 1) P_\mu^{ab} P_\nu^{ab} - \delta\Pi_{\mu\nu}^{ab} \right)$$

Only the static mode, $p_0 = 0$, contributes. Typical momenta are $p \sim g\, T$.

To be independent of ξ, gluon self energy *must* be transverse.

In Holonomous Plasma, diagonal and off-diagonal gluons contributions.

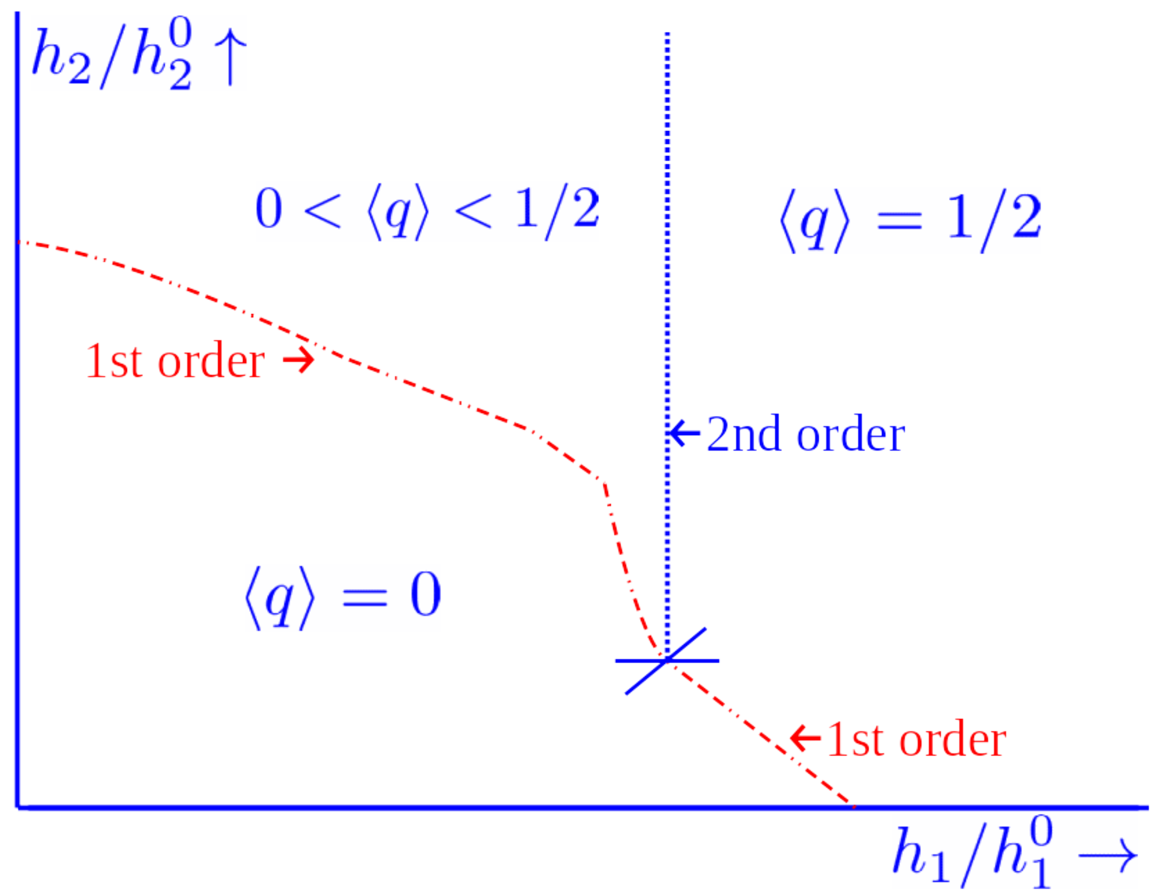
Find: mass^2 of diagonal gluons are *negative* with sources linear in loops!

Holonomous Plasma for two colors

Consider two colors, add to the perturbative HP two non-perturbative terms:
Nishimura & Ogilvie, 1111.6101; DGHKP, 1205.0137; HKNPS, 1905...

$$\mathcal{V}(q) = (4\pi^2 T^3/3)\,(q^2 - |q|^3 + q^4) + 4 j_1\,\ell^2 + 16 j_2\,\ell^4\,,\ \ell = \cos(\pi q)$$

Add two non-pert. terms, ~ j_1 & j_2.



Find that there is *always* a
1st order transition from pert. vac.
to Holonomous Plasma, <q>≠0

Trivial reason: loop is always
even in q about q = 0

1-loop HP has cubic term - q^3!

- cubic => 1st order transition

Non-perturbative potentials for HP

Usual source ~Jφ: for *any* J, even infintesimal, $\varphi \neq 0$.

Here: need gauge invariant sources: any *finite* number of loops ~ q_a^2 , $q_a << 1$

For HP @ 1 loop, term *cubic* in q_a because sum over ∞ number of loops.

Source linear in q_a , q_a,<< 1, *need* sum over ∞ number of loops. We choose

$$S_{2D} = \sum_{n=1}^{\infty} \frac{1}{n^2} |\mathrm{tr}\mathbf{L}^n|^2 = \sum_{a,b=1}^{N} B_2(q_a - q_b) \, , \; B_2(x) = 1/6 - |x| + x^2$$

Like the free energy of a massless boson in 1+1 dimensions...

Many others possible: B_3, B_5...

With a source ~ $J_2 S_{2D}$, $q_a \neq 0$ for *any* infintesimal J_2.

For diagonal gluons, this source gives positive mass^2,

free energy ~ g^3 that is smooth as J_2 -> 0

Off-diagonal gluons for free energy ~g^3.

Need: self energy for off-diagonal gluons, $p_0 = 0$, $q_a \sim g$.

Can use Holonomous Hard Thermal Loop: Hidaka & RDP 0906.1751

$$\delta\Pi^{ab;ij} \sim \int_0^\infty d^3k \, \frac{k^i k^j}{E_k E_{p-k}} \int \frac{d\Omega}{4\pi} \mathcal{I} \, , \; \mathcal{I} = \frac{n(E_k - iQ_a) - n(E_{p-k} + iQ_b)}{ip_0^{ab} - E_k + E_{p-k}}$$

Usual HTL valid for soft $\omega = -ip_0 \sim p \sim gT$; Holonomous HTL for soft $\omega = -ip_0^{ab}$

$p_0^{ab} = p_0 - (Q_a - Q_b) = p_0 - 2\pi T (q_a - q_b)$: soft when $p_0 = 0$, $q_a \sim g$.

Dominated by hard k ~ T. Then

$$\mathcal{I} \approx \frac{1}{-\hat{k}\cdot\vec{p} + i(Q_a - Q_b)} \Big(n(k) - n(k - \hat{k}\cdot\vec{p} + i(Q_a - Q_b)) \Big) \approx -\frac{d}{dk} n(k)$$

So independent of Q! => constant. Could have been function of p/Q.

Implies (off-diagonal) self energy *vanishes*!

Free energy from off-diag ~g^3 *dis*continous: $\neq 0$ when $q_a = 0$; $= 0$ when $q_a \sim g$.

Makes *no* sense.

Two dimensional fields

Introduce 2-dimensional fields:

$$x^\mu = (\hat{x}, x_\perp)\,,\ \hat{x} = (x_0, \vec{x}\cdot\hat{n})\,;\ \hat{n}^2 = 1\,,\ x_\perp\cdot\hat{n} = 0$$

Embed isotropically by integrating over all directions of unit vector n.
Anistropic between along n and perp. to n.

$$\mathcal{S}_{\rm 2D} = \int_0^{1/T} d\tau \int \frac{d\Omega_{\hat{n}}}{4\pi} \int_0^\infty d\hat{x} \int_{1/T_d}^\infty d^2x_\perp \ {\rm tr}\left((\hat{D}\phi)^2 + (D_\perp\phi)^2 \right)$$

Adjoint scalar ϕ is two dimensional at short distances, < $1/T_d$, but four dimensional over large distances.

At one loop order, gluon self energy gauge invariant; scalar not, will patch up.

Two dimensional ghosts

Assume $T_d << T$: then momentum integral trivially reduces to 2D

$$\mathcal{S}_{\rm 2D} = {\rm tr}\;\log\,(-\hat{D}^2 - D_\perp^2) \approx T_d^2\;{\rm tr}\;\log\,(-\hat{D}^2) = -T^2 T_d^2 \sum_{a,b=1}^{N} B_2(q_a - q_b)$$

The ϕ field must be a *ghost* field for B_2 to have the proper sign.

Natural: one wants to decrease the pressure from physical gluons.

Find Holonomous HTL's in Euclidean space: $Q_{ab} = 2\,\pi\, T\,(q_a - q_b)$

$$\delta\Pi^{ab}_{\rm long} = -1 + \frac{Q_{ab}}{p}\;\arctan\left(\frac{p}{Q_{ab}}\right)\;,$$

$$\delta\Pi^{ab}_{\rm tr} = \frac{3}{2}\left(\left(\frac{Q_{ab}^2}{p^2} + 1\right)\frac{Q_{ab}}{p}\;\arctan\left(\frac{p}{Q_{ab}}\right) + \frac{Q_{ab}^2}{p^2}\right)$$

Can show: if holonomy generated by 2D ghosts, then free energy ~ g^3 is smooth as $q_a \to 0$.

Using two dimensional ghosts

Previously: constructed effective theory with B_2 to fit Euclidean pressure, etc.

Now: Generate B_2 dynamically from massless, 2D ghosts

To do: compute transport coefficients using gluons + 2D ghosts
in Holonomous Plasma in perturbation theory

Severe constraint: ghosts could drive pressure, transport coefficients negative!

Doesn't happen for the pressure.

Shear viscosity *suppressed* by loop^2 in Holonomous Plasma:

including *only* gluons, Hidaka & RDP, 0803.0453, 0912.0940

$$\eta \sim \frac{T^3}{g^4 \log(1/g)} \, |\ell|^2$$

Need to include 2D ghosts...