

Non-equilibrium QCD dynamics on the lattice

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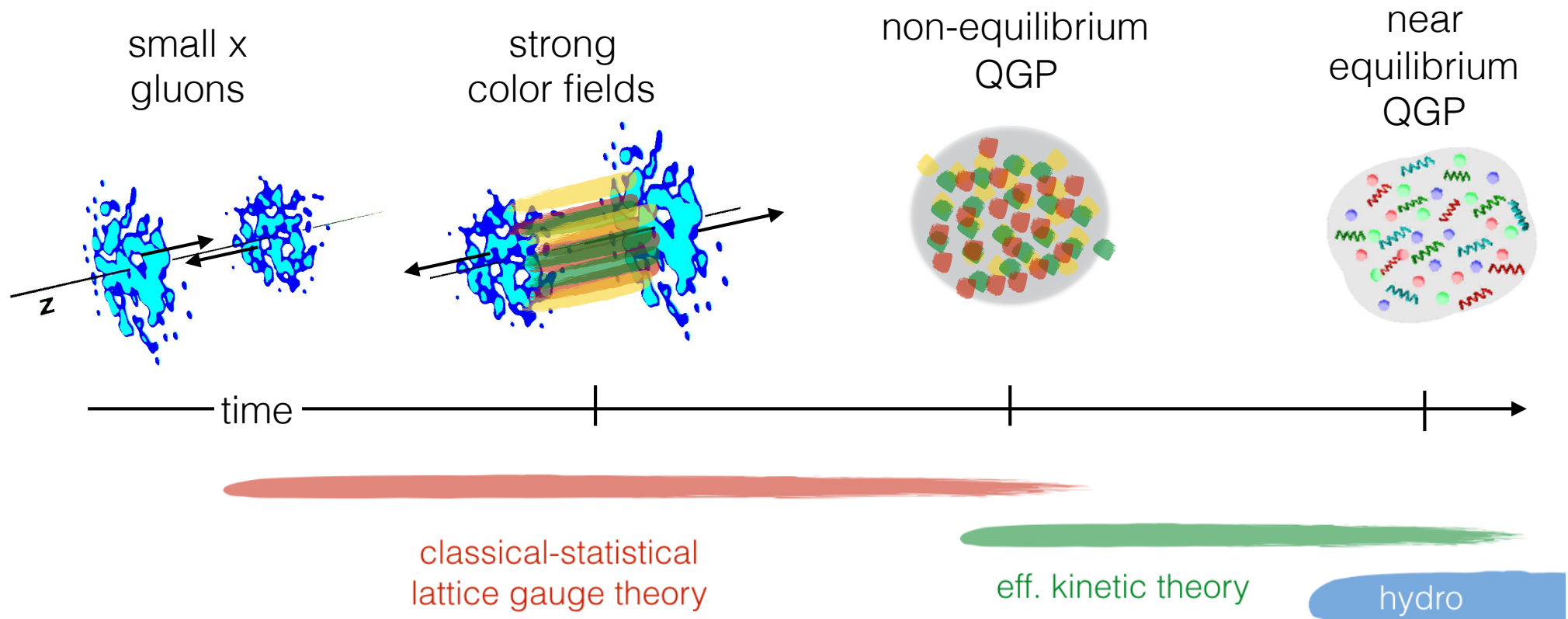
TIFR ICTS School

“THE MYRIAD COLORFUL WAYS OF UNDERSTANDING EXTREME QCD MATTER”

Bangalore, India April 2019

Early time dynamics of heavy-ion collisions

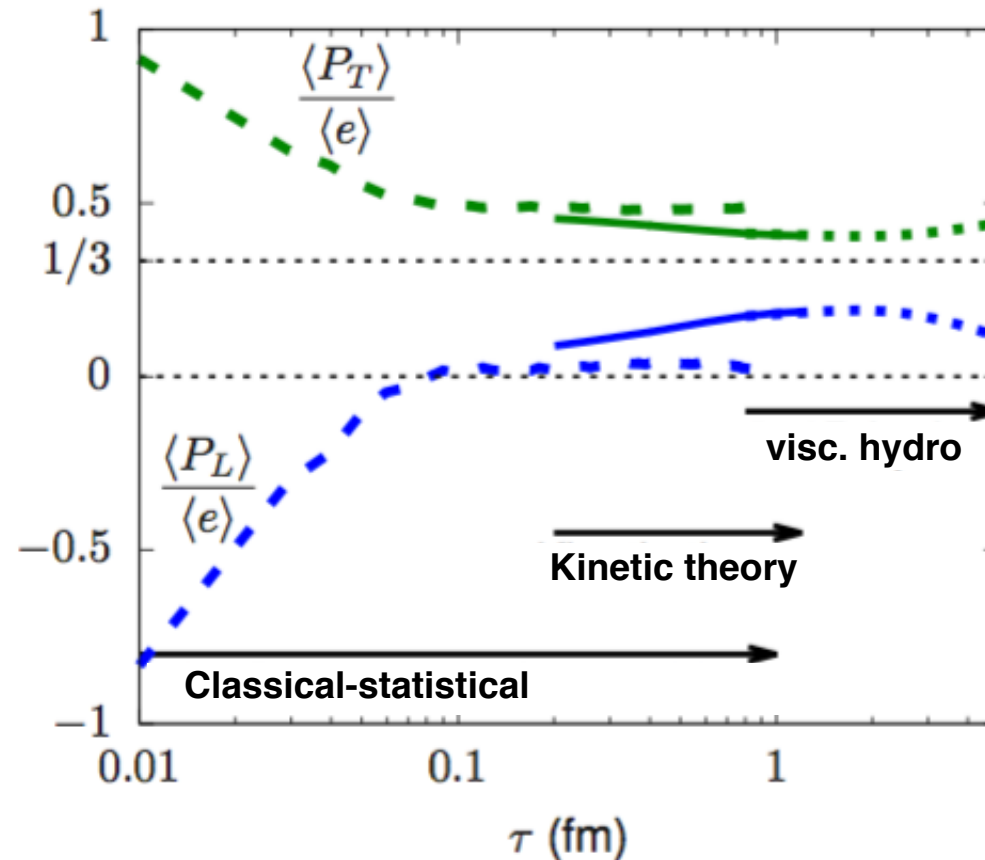
Different degrees of freedom relevant at different stages of the evolution



Need combination of non-equilibrium methods to describe early time dynamics & approach to equilibrium

Early time dynamics of heavy-ion collisions

Evolution of energy momentum tensor in central Pb+Pb event



Based on combination of non-equilibrium methods can describe the space-time dynamics of HIC from beginning to end on event-by-event basis

-> Overlap in the range of validity ensures consistent matching

Dynamical fermions

So far discussed classical-statistical approximation of boson fields

critterion for validity $n(t, p) \gg 1$ (strong Bose enhancement)

Not only interested in bosonic fields but also need to describe non-equilibrium dynamics of fermions

Pauli blocking $n(t, p) \leq 1$

However for theories of interest fermions appear quadratically in the action, can be integrated out exactly

$$\int_{\varphi_0^+}^{\varphi_0^-} D\varphi D\bar{\Psi} D\Psi e^{i[S_c^B[\varphi] + \int_{xy,c} \bar{\Psi}(x) iD^{-1}[\varphi](x,y) \Psi(y)]} \longrightarrow \int_{\varphi_0^+}^{\varphi_0^-} D\varphi e^{iS_c^B[\varphi] + \text{tr} \log D_c^{-1}[\varphi]}$$

expand $\text{tr} \log D^{-1}$ in powers of classical and quantum fields

$$\int_{\varphi_0^{cl}}^{\varphi_0^{cl}} D\varphi_{cl} e^{\text{tr} \log D_c^{-1}[\varphi_{cl}]} \int D\chi e^{i \int_{x,c} \left(\frac{\delta S^B[\varphi_{cl}]}{\delta \varphi_{cl}} - D[\varphi_{cl}] \frac{\delta iD^{-1}[\varphi_{cl}]}{\delta \varphi_{cl}} \right) \chi + h.o.}$$

Evolution of fermion
in classical field bg

Backreaction of fermions
via average current

Dynamical fermions

- Discretize theory on 3D spatial lattice using the Hamiltonian lattice formalism
- Solve operator Dirac equation in the presence of classical SU(N) and U(1) gauge fields

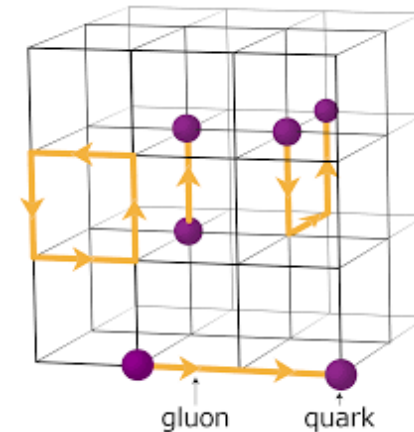
$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}_W^s + m)\hat{\psi}$$

- Compute expectation values of fermion currents

$$j_v^\mu(x) = \langle \hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x) \rangle$$

- Solve classical Yang-Mills equations in the presence of fermionic currents

$$D_\mu F^{\mu\nu} = j^\nu$$



Dynamical fermions

Solving the operator Dirac equation can be achieved by expanding the fermion field in operator basis at initial time

$$\hat{\psi}(x, t) = \sum_{p, \lambda} \hat{b}_{p, \lambda}(t = 0) \phi_u^{p, \lambda}(x, t) + \hat{d}_{p, \lambda}^\dagger(t = 0) \phi_v^{p, \lambda}(x, t)$$

and solving the Dirac equation for evolution of $4N_c N^3$ wave-functions

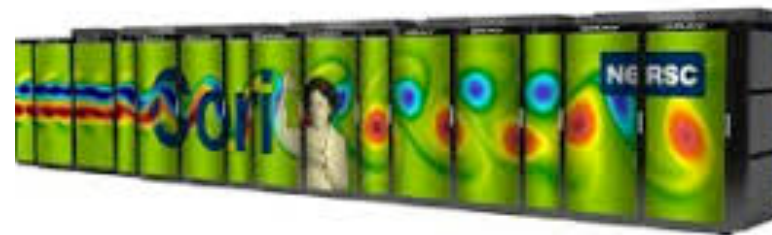
Numerical cost:

gauge fields $\sim N_c^3 N^3$



$\sim 10^2$ CPU hours

gauge fields + fermions $\sim N_c^2 N^6$



$\sim 10^6$ CPU hours

Example: Sphaleron transitions & Chiral transport

Chiral magnetic effect:

New kind of conductivity for systems with chiral fermions and chirality imbalance

$$\vec{j}_V \propto n_5 \vec{B}$$

n_5 : axial charge imbalance

B : magnetic field

Several possible manifestations of this effect
from high-energy QCD to Dirac/Weyl semi-metals

Example: Sphaleron transitions & Chiral transport

Magnetic field: Spectators in off-central heavy-ion collisions create a strong magnetic field $eB \sim m_\pi^2$ (although presumably only for a very short time)

Axial charge density: Expect space-time dependent fluctuations due to axial anomaly

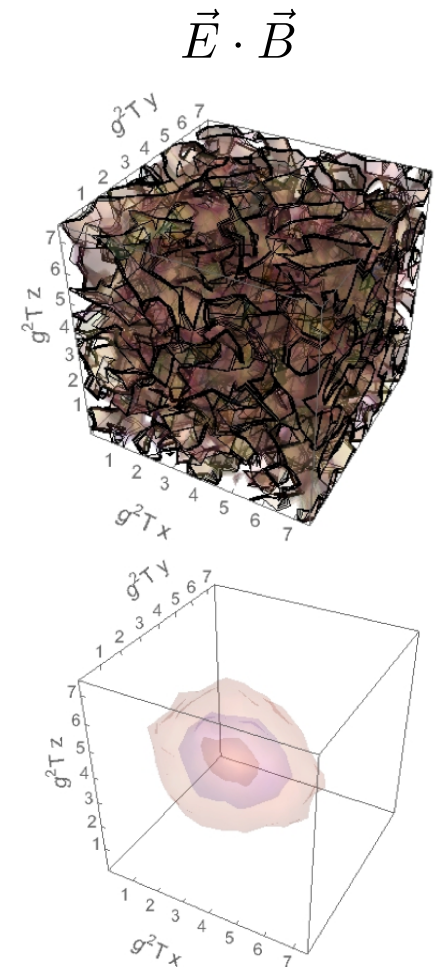
$$\partial_\mu j_{5,f}^\mu = 2m_f \bar{q} \gamma_5 q - \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} - \frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

axial current $j_5^\mu = (n_5, \vec{j}_5)$

quark mass

field-strength fluctuations $\propto \vec{E} \cdot \vec{B}$

- space-time dependent fluctuations of $\vec{E} \cdot \vec{B}$
- topological sphaleron transitions



Example: Sphaleron transitions & Chiral transport

Classical (3D) vacuum configurations physically equivalent and connected by gauge transformation

$$G : T_3 \rightarrow SU(N_c)$$

$$\pi_3(SU(N_c)) \simeq \mathbb{Z}$$

which can have topologically non-trivial properties

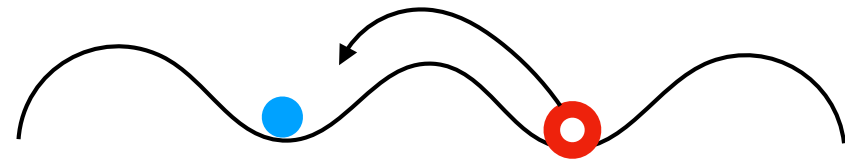
Sphaleron transitions:

real-time transition between topologically distinct sectors

Vacuum to vacuum transitions give rise to integer change of the Chern-Simons number

$$\Delta N_{CS} = \frac{g^2}{8\pi^2} \int d^4x \vec{E}_a \vec{B}_a$$

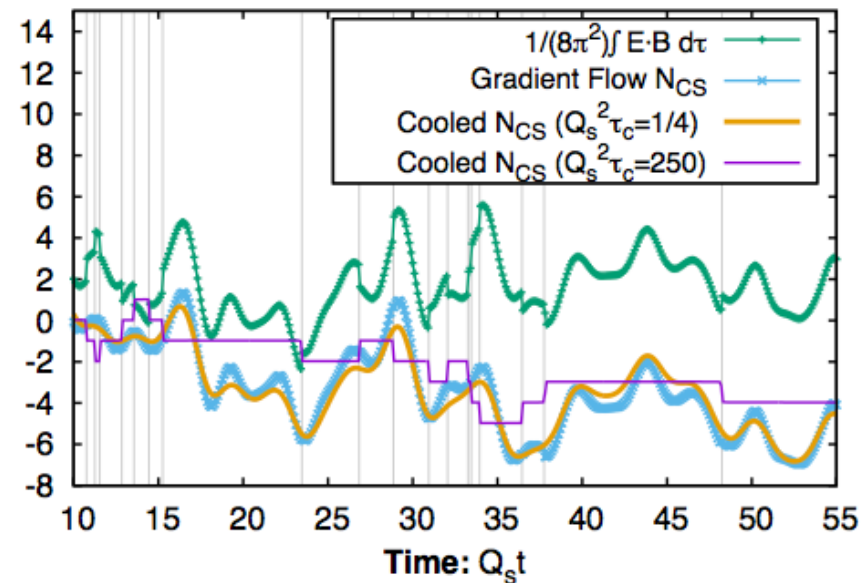
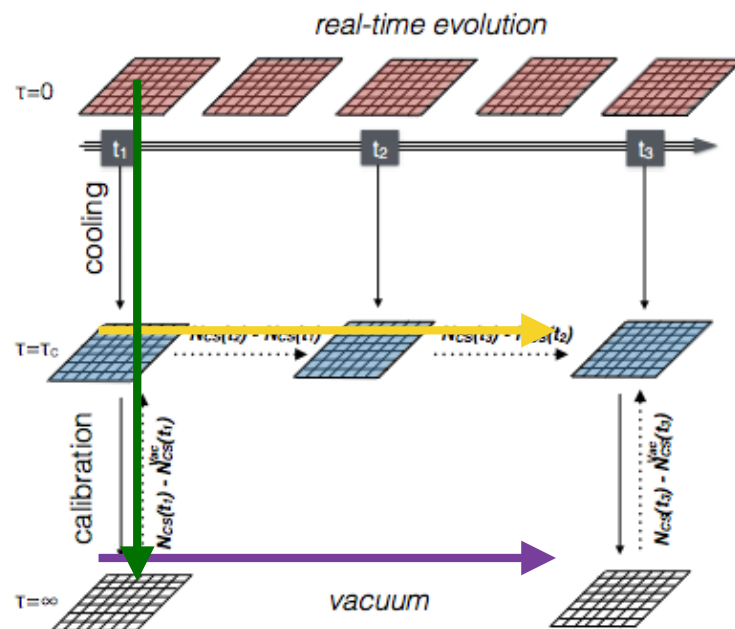
Generally $\Delta N_{CS}(t)$ is not integer valued and can be non-zero also in abelian gauge theories (e.g. parallel \vec{E}, \vec{B})



Example: Sphaleron transitions & Chiral transport

Definition of Chern-Simons number on the lattice can be problematic due to UV fluctuations

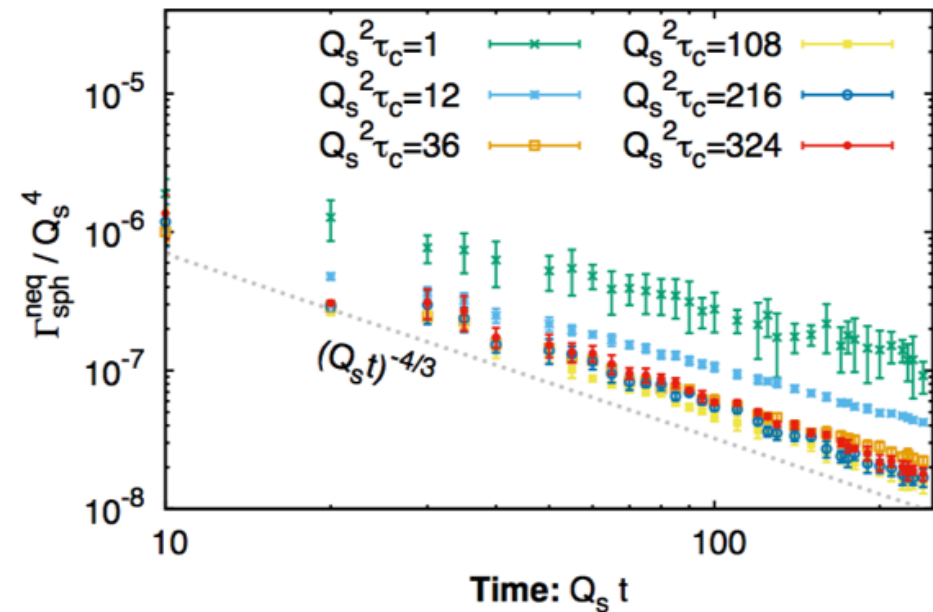
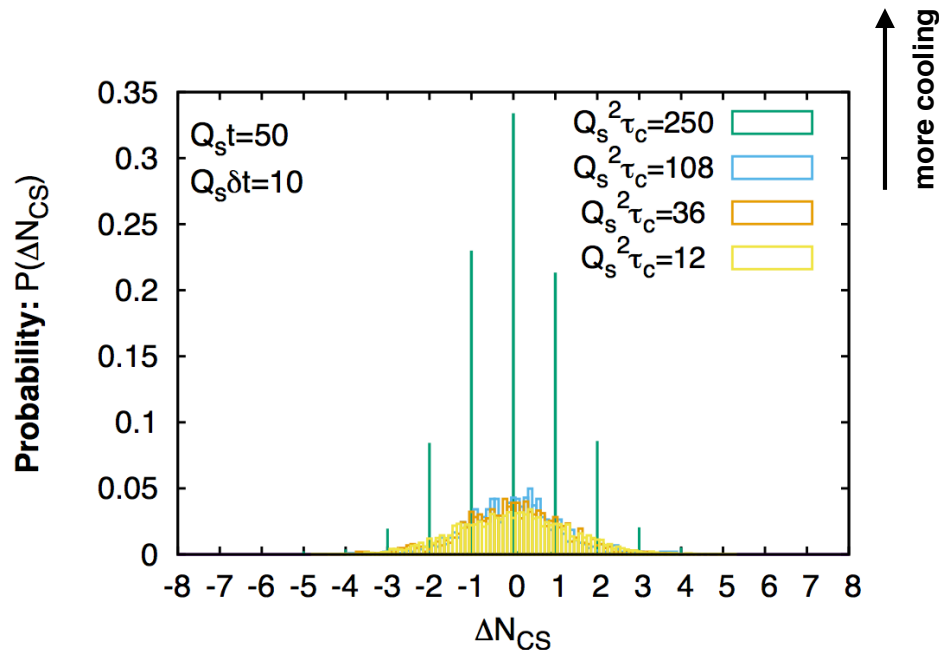
Generally strategy is to use cooling to define ΔN_{CS}



If only interested in topological information, can also measure winding number of gauge transformation to minimal Coulomb gauge

Example: Sphaleron transitions & Chiral transport

Histograms of Chern-Simons
number difference



Enhancement of the sphaleron transition
rate in out-of-equilibrium plasma

Example: Sphaleron transitions & Chiral transport

Very challenging to simulate dynamical fermions on top of noisy gauge field background

no fundamental problem
just need very large lattices

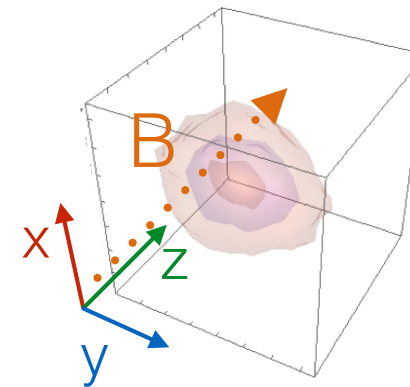


Will consider a clean theoretical setup

SU(N): Single sphaleron transition

U(1): constant magnetic field

Neglect back-reaction of fermions on dynamical gauge field in this study



Discretization of fermions & axial anomaly

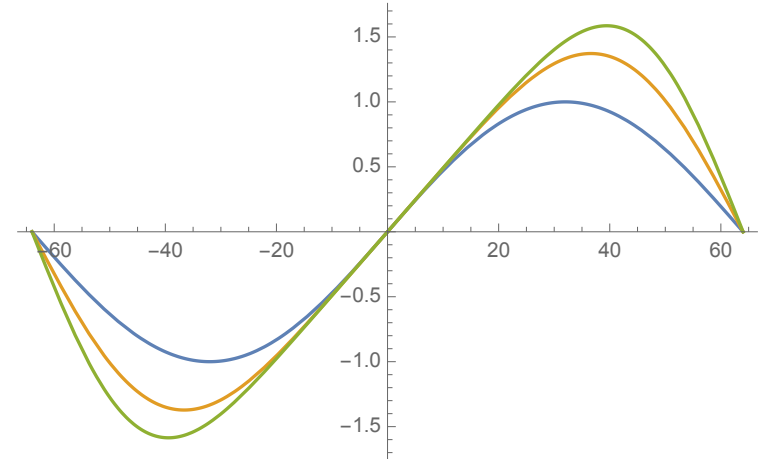
Definition of chiral properties (axial charge) of fermions on the lattice generally a tricky issue

Naive fermion discretization:

$$-i\not{D}_W^s\hat{\psi}_{\mathbf{x}} = -i\sum_{n,i}\frac{C_n}{2a_i}\left[\gamma^i U_{\mathbf{x},+ni}\hat{\psi}_{\mathbf{x}+n\mathbf{i}} - \gamma^i U_{\mathbf{x},-ni}\hat{\psi}_{\mathbf{x}-n\mathbf{i}}\right]$$

Cancellation of axial anomaly
due to Fermion doublers

$$\partial_\mu j_5^\mu(x) = 2m \langle \bar{\psi}(x) i\gamma_5 \psi(x) \rangle$$



Discretization of fermions & axial anomaly

Definition of chiral properties (axial charge) of fermions on the lattice generally a tricky issue

Wilson fermions: Explicit symmetry breaking term added to the Hamiltonian to decouple doublers (c.f. Aarts, Smit)

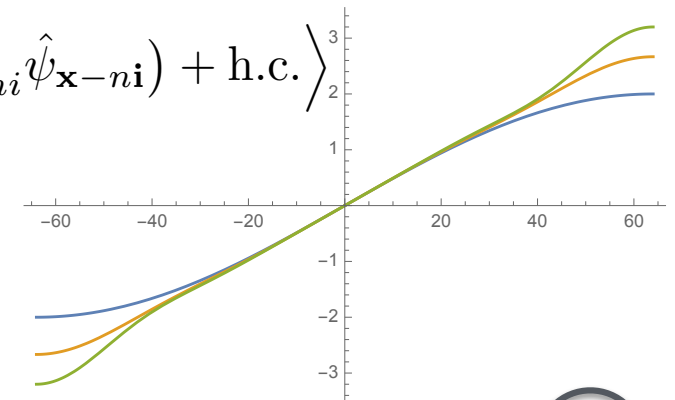
$$-i\not{D}_W^s \hat{\psi}_{\mathbf{x}} = \sum_{n,i} \frac{C_n}{2a_i} \left[\left(-i\gamma^i - nr_w \right) U_{\mathbf{x},+ni} \hat{\psi}_{\mathbf{x}+n\mathbf{i}} + 2nr_w \hat{\psi}_{\mathbf{x}} - \left(-i\gamma^i + nr_w \right) U_{\mathbf{x},-ni} \hat{\psi}_{\mathbf{x}-n\mathbf{i}} \right]$$

$$\partial_\mu j_5^\mu(x) = 2m \langle \bar{\psi}(x) i\gamma_5 \psi(x) \rangle + r_W \langle W(x) \rangle$$

$$W(x) = \sum_{n,i} \frac{n \cdot C_n}{2} \langle \hat{\psi}_{\mathbf{x}}^\dagger i\gamma_5 \gamma_0 (U_{\mathbf{x},+ni} \hat{\psi}_{\mathbf{x}+n\mathbf{i}} - 2\hat{\psi}_{\mathbf{x}} + U_{\mathbf{x}-n\mathbf{i},+ni}^\dagger \hat{\psi}_{\mathbf{x}-n\mathbf{i}}) + \text{h.c.} \rangle$$

cont. limit

$$\rightarrow -\frac{g^2}{8\pi^2} \text{Tr} F_{\mu\nu} F^{\mu\nu}$$



Example: Sphaleron transitions & Chiral transport

Non-trivial cross check of axial charge production (B=0)

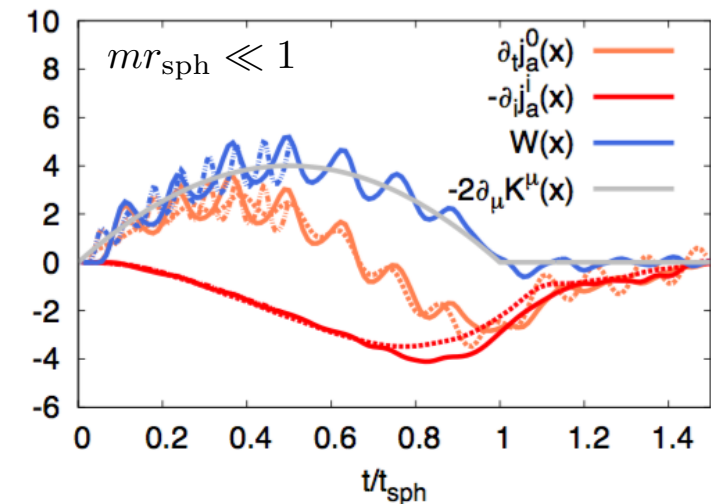
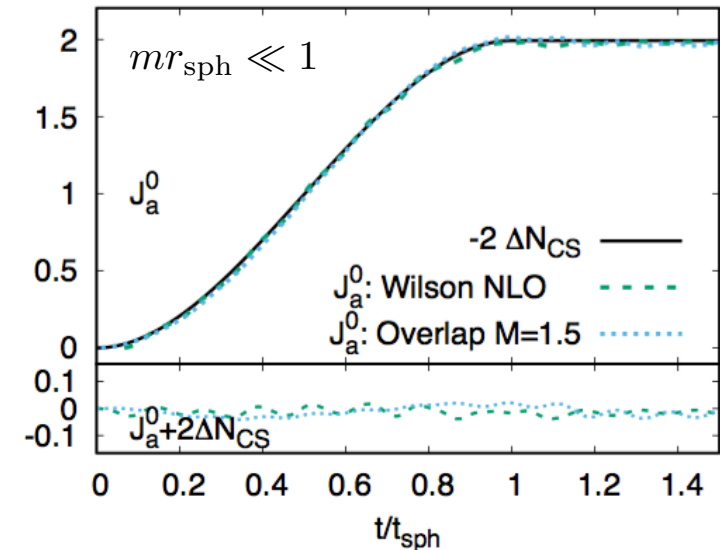
Over the course of the sphaleron transition Chern-Simons number

$$\Delta N_{CS} = \frac{g^2}{8\pi^2} \int d^4x \vec{E}_a \vec{B}_a$$

changes by an integer amount leading to an imbalance of axial charge

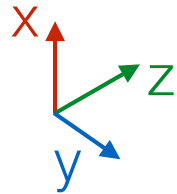
$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x \langle \bar{\psi} i \gamma_5 \psi \rangle$$

Excellent agreement for (almost) massless fermions from simulations with improved Wilson fermions and Overlap fermions



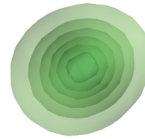
Example: Sphaleron transitions & Chiral transport

Axial charge j_5^0



Sphaleron transition induces local imbalance of axial charge density

Vector current j_V^z



Non-zero magnetic field B_z leads to vector current j_V^z in z-direction

Vector charge j_V^0



Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

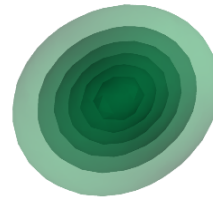
Example: Sphaleron transitions & Chiral transport

Axial charge j_5^0



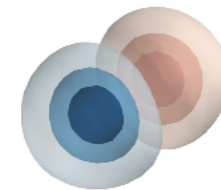
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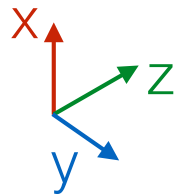
Vector charge j_V^0



Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

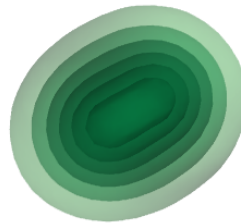
Example: Sphaleron transitions & Chiral transport

Axial charge j_5^0



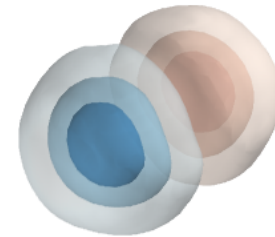
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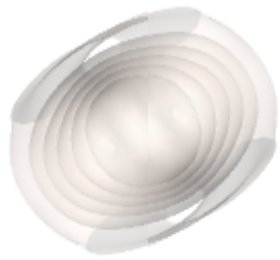


Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

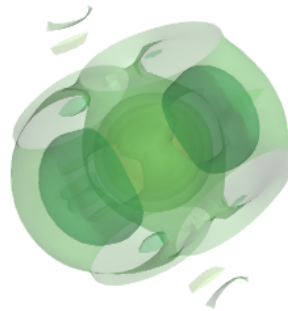
Example: Sphaleron transitions & Chiral transport

Vector charge imbalance j_V^0 generates an axial current j_5^z so that axial charge also flows along the B-field direction

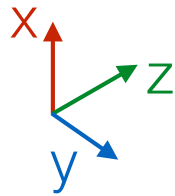
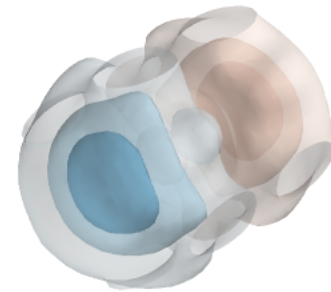
Axial charge j_5^0



Vector current j_V^z



Vector charge j_V^0

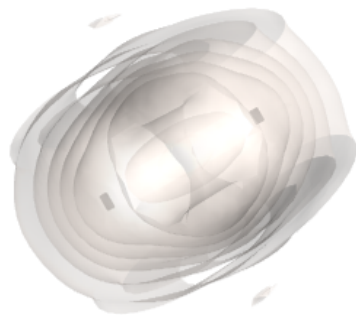


Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

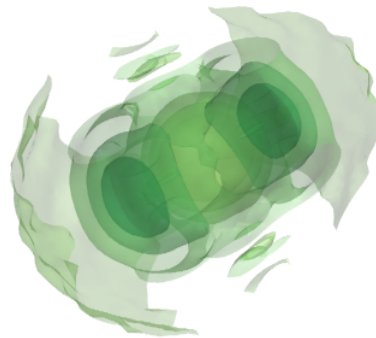
Example: Sphaleron transitions & Chiral transport

Vector charge imbalance j_V^0 generates an axial current j_5^z so that axial charge also flows along the B-field direction

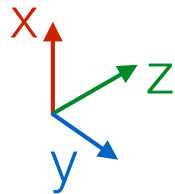
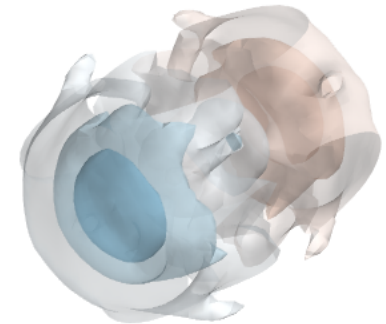
Axial charge j_5^0



Vector current j_V^z



Vector charge j_V^0

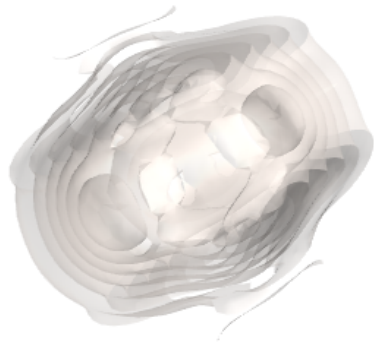


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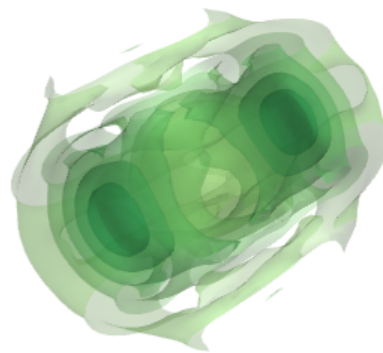
Example: Sphaleron transitions & Chiral transport

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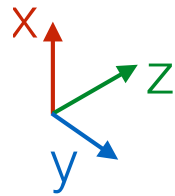
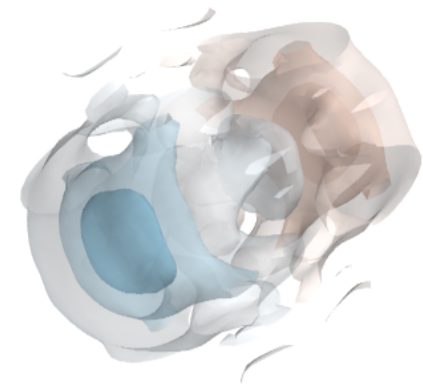
Axial charge j_5^0



Vector current j_V^z

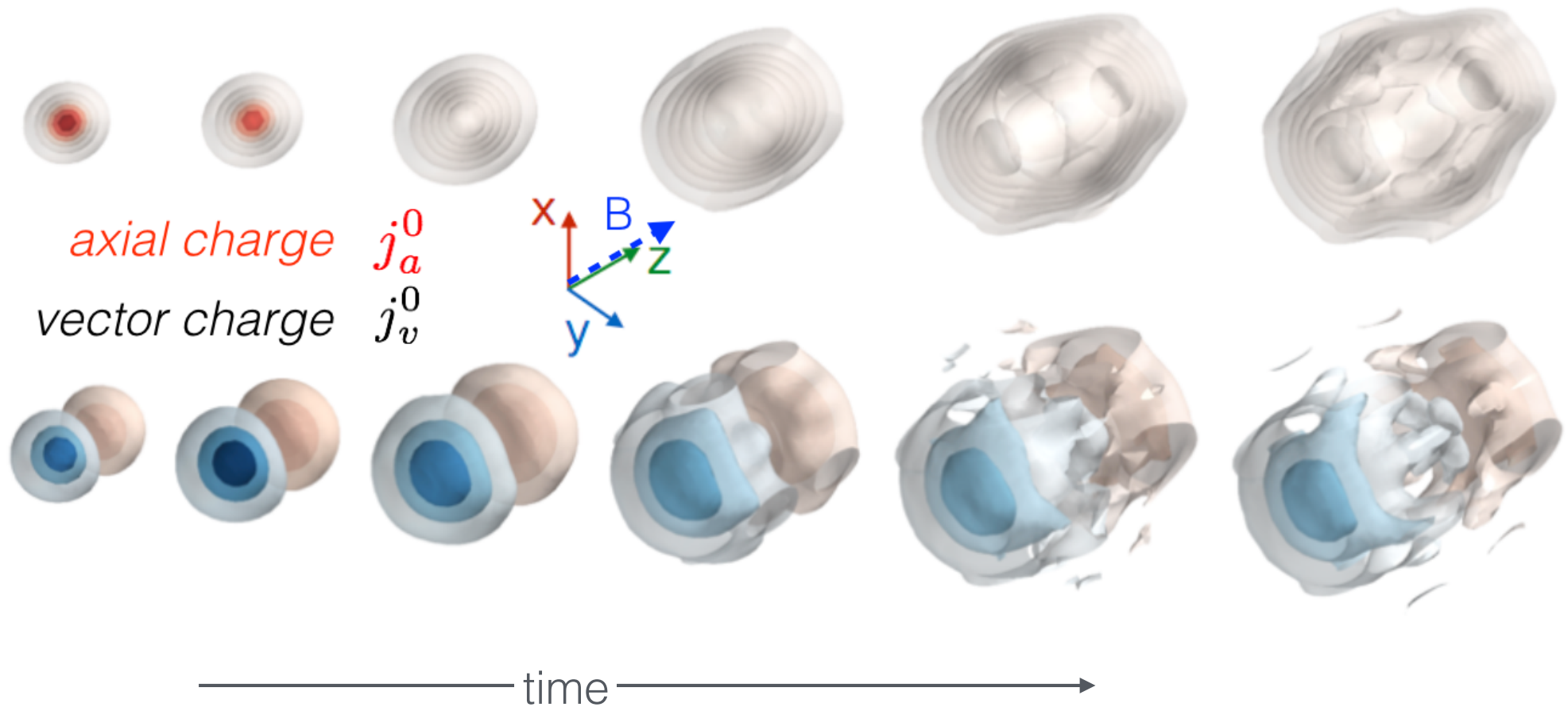


Vector charge j_V^0



Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

Example: Sphaleron transitions & Chiral transport



Clear separation of electric charge j_v^0 along the B-field direction

Example: Sphaleron transitions & Chiral transport

Comparison with anomalous hydro
(light quarks $mr_{\text{sph}} \ll 1$)

$$\partial_\mu j_a^\mu = S(x), \quad \partial_\mu j_v^\mu = 0$$

$$j_{v/a}^\mu = n_{v/a} u^\mu + \sigma_{v/a}^B B^\mu$$

Strong field limit ($B \gg r_{\text{sph}}^{-2}, m^2$)

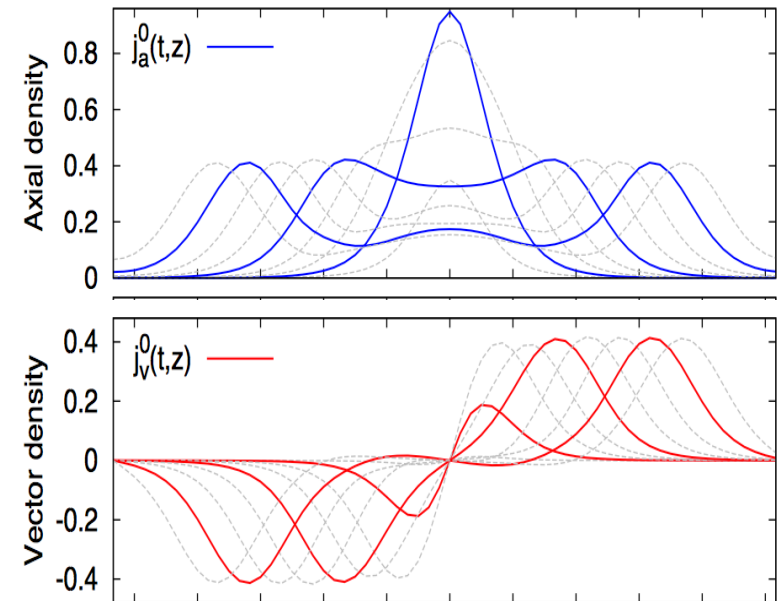
$$\partial_t \begin{pmatrix} j_v^0(t, z) \\ j_a^0(t, z) \end{pmatrix} = -\partial_z \begin{pmatrix} j_a^0(t, z) \\ j_v^0(t, z) \end{pmatrix} + \begin{pmatrix} 0 \\ S(t, z) \end{pmatrix}$$

Chiral magnetic shock-wave

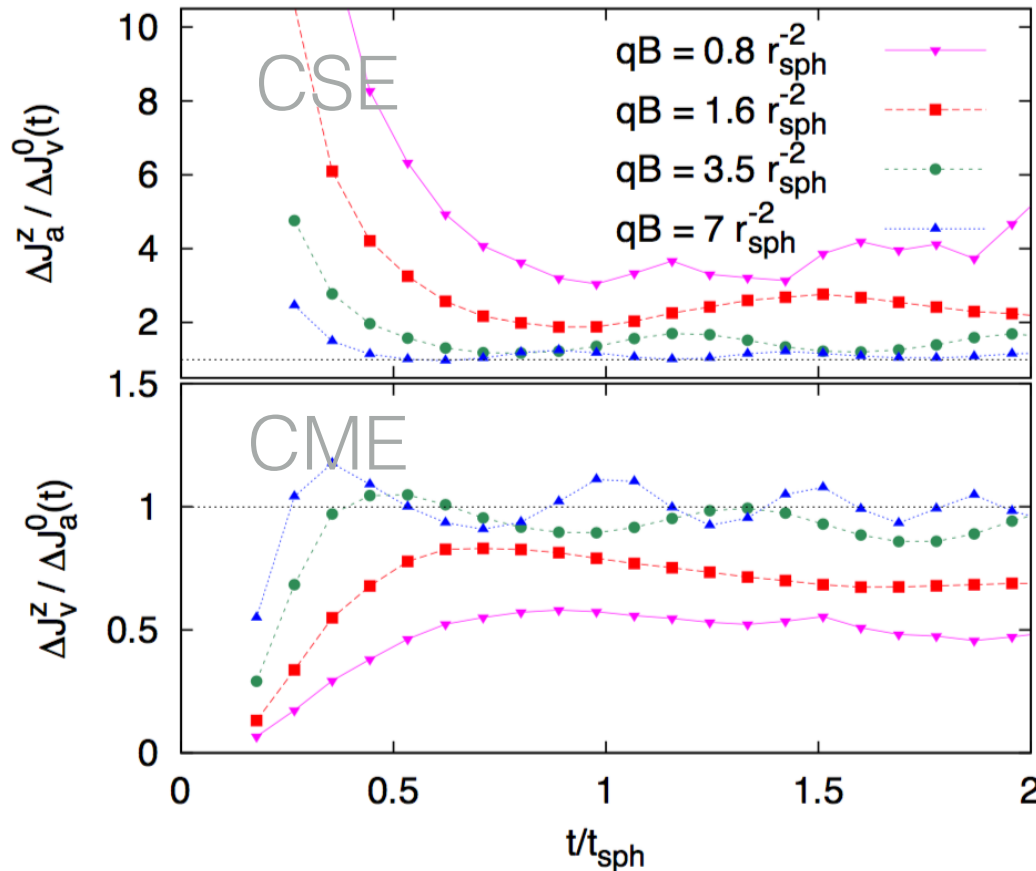
$$j_{v/a}^0(t > t_{\text{sph}}, z) = \frac{1}{2} \int_0^{t_{\text{sph}}} dt' \left[S(t', z - c(t - t')) \mp S(t', z + c(t - t')) \right]$$

-> Evolution for light quarks and strong magnetic fields
well described by anomalous hydrodynamics at late times

Simulation results for light quarks



Example: Sphaleron transitions & Chiral transport



Check of constitutive relations:

Verify ratios vector/axial currents and axial/vector charge

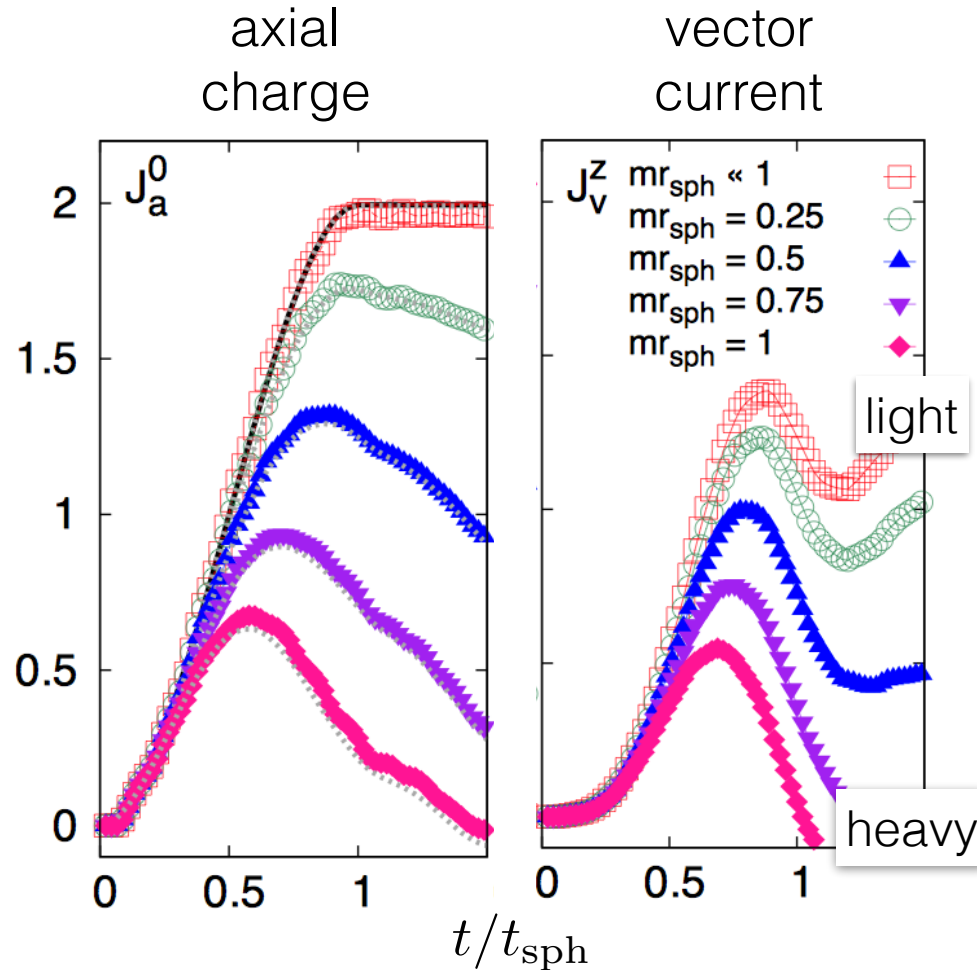
$$C_{\text{CME}}(t) = \frac{\Delta J_v^z(t)}{\Delta J_a^0(t)}, \quad C_{\text{CSE}}(t) = \frac{\Delta J_a^z(t)}{\Delta J_v^0(t)}.$$

In the strong field limit related to thermodynamic constitutive relations

$$C_{\text{CME}} = 1, \quad C_{\text{CSE}} = 1.$$

equal to time independent constants.

Example: Sphaleron transitions & Chiral transport



Quark mass dependence:

Explicit violation of axial charge conservation for finite quark mass

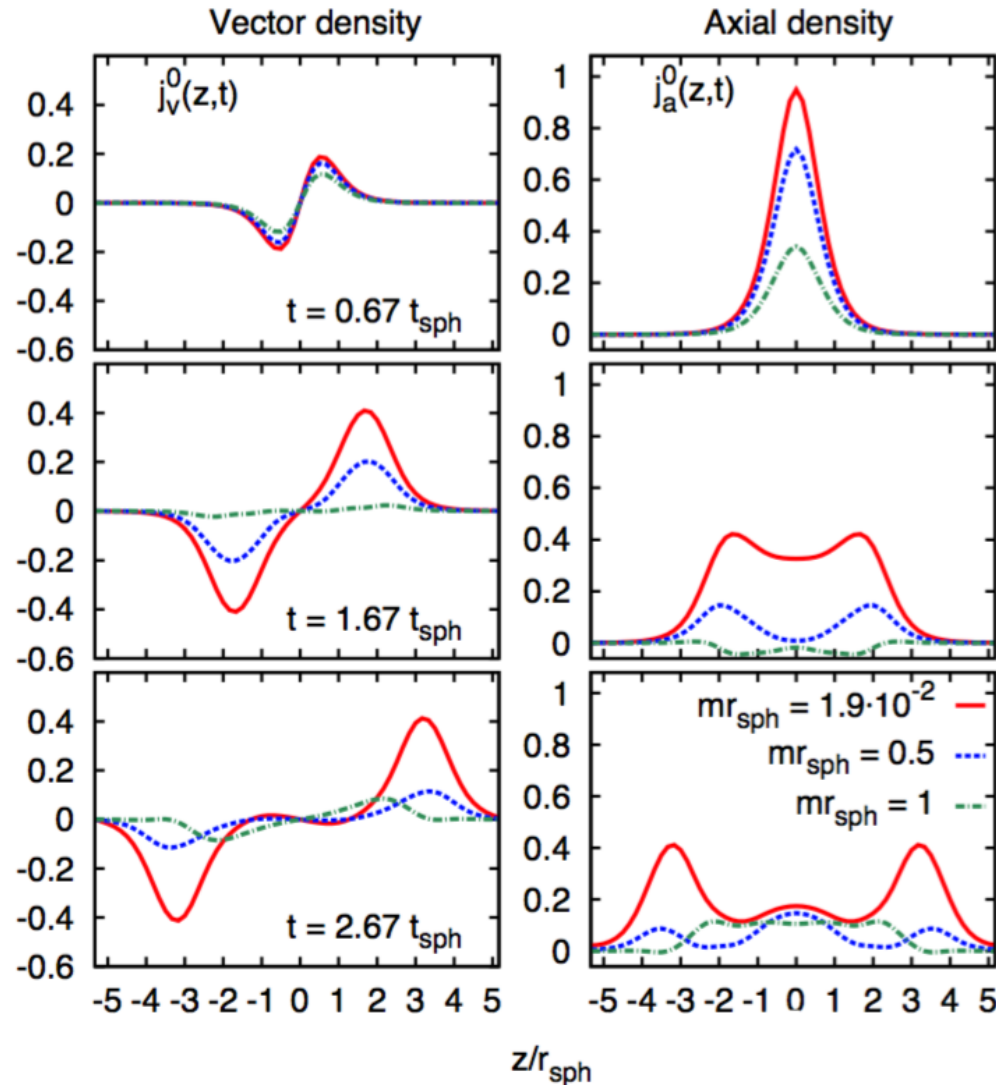
$$\partial_\mu j_a^\mu(x) = 2m \langle \hat{\psi}(x) i \gamma_5 \hat{\psi}(x) \rangle + S(x)$$

leads to damping of axial charge

Since chiral magnetic effect current is proportional to axial charge density it will also be reduced

$$\vec{j}_v \propto j_a^0 \vec{B}$$

Example: Sphaleron transitions & Chiral transport



Light quarks ($mt_{\text{sph}} \ll 1$)

Chiral magnetic wave leads to non-dissipative transport of axial and vector charges

Heavy quarks ($mt_{\text{sph}} \sim 1$)

Dissipation of axial charge leads to significant reduction of charge separation

Quo vadis non-equilibrium QFT?

We want to describe non-equilibrium QFT to study
Heavy-Ion collisions, Cosmology, Cold Atom experiments, ...

$$Z[J, R, \hat{\rho}_0] = \underbrace{\int [d\varphi_0^+] [d\varphi_0^-] \langle \varphi_0^+ | \hat{\rho}_0 | \varphi_0^- \rangle}_{\text{initial state}} \underbrace{\int_{\varphi_0^+}^{\varphi_0^-} D\varphi e^{\frac{i}{\hbar} \left[S_C[\varphi] + \int_{x_C} J(x) \varphi(x) + \frac{1}{2} \int_{x_C, y_C} \varphi(x) R(x, y) \varphi(y) \right]}}_{\text{non-equilibrium dynamics}}$$

Discussed different non-equilibrium methods based on
expansion

- semi-classical expansions (\hbar), weak-coupling expansions (λ), ...

No exact answers; generally need multiple methods to describe physics

Natural question: Can we do better?

Complexification: real-time stochastic quantization (Complex Langevin), deformed integration contours (Lefschetz thimbles, ...)

Hamiltonian simulation: Variational diagonalization (DMRG, TNS),
Quantum computing/simulation of Hamiltonian dynamics

Sign problem for real-time dynamics

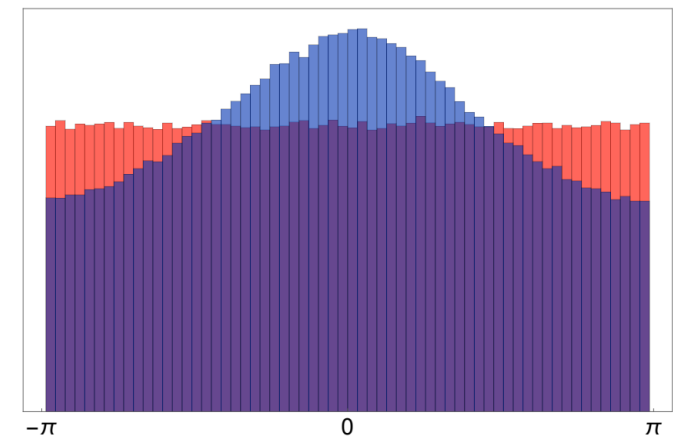
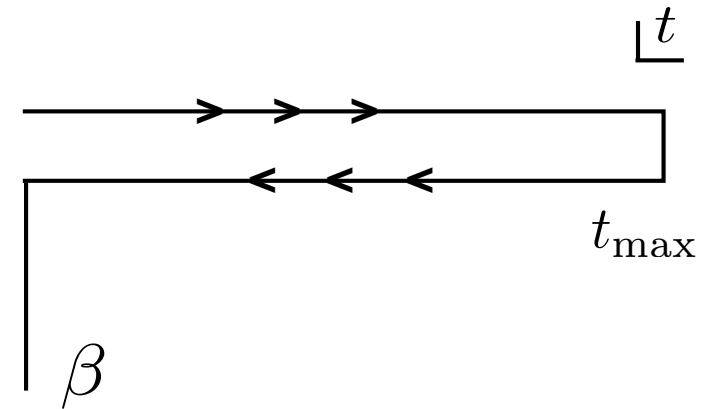
Consider for definiteness real-time dynamics in thermal equilibrium

$$Z[J] = \int_{\text{pbc}} D\varphi e^{i\left(S_{\mathcal{C}_\beta}[\varphi] + \int_{\mathcal{C}_\beta} J(x)\varphi(x)\right)}$$

Euclidean part of the contour:
action weight is real (<0)

Minkowski part of the contour:
action weight is purely imaginary (up to ie)

Strong oscillations of the phase factor
prohibit direct importance sampling

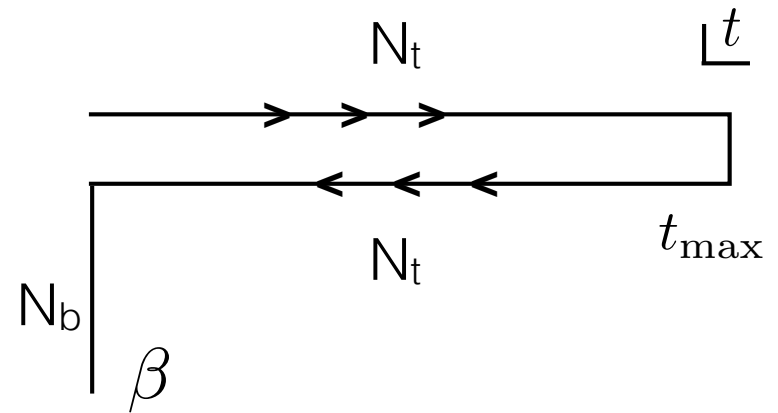


Complexification: Holomorphic gradient flow/Lefschetz thimbles

Discretize space on N_s sites and time path in N_t and N_b pieces

Need to perform $X = N_s^d \times (2N_t + N_b)$
dimensional integral over real variables

$$\phi(x_i, t_i)$$



Basic idea is to consider a deformation of the integration from
real domain R^X to a X -dimensional manifold embedded in C^X
where the sign problem is mild

calculate observables
via reweighting

generate field
configurations
with probability

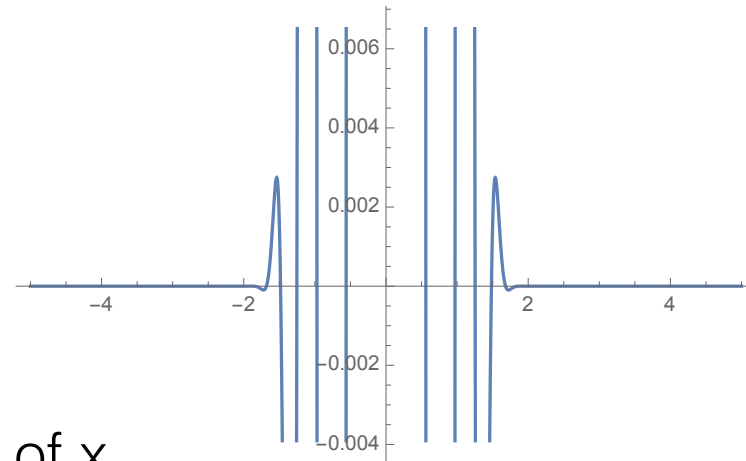
$$\propto e^{\text{Re}(iS_{c_\beta}[\varphi])}$$

$$\langle O \rangle = \frac{\langle O[\varphi] e^{i\text{Im}S_{c_\beta}} \rangle}{\langle e^{i\text{Im}S_{c_\beta}} \rangle}$$

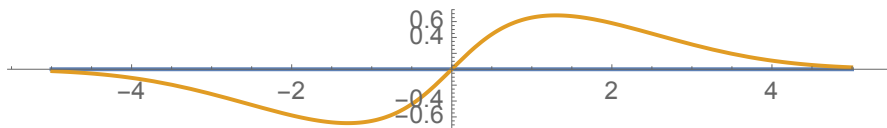
Complexification: Holomorphic gradient flow/Lefschetz thimbles

Consider one dimensional integral

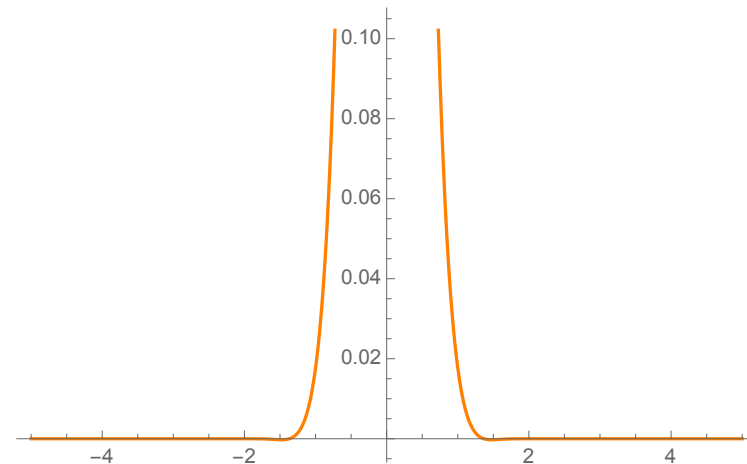
$$Z = \int_{\mathcal{R}} dx e^{5ix^2 - x^4}$$



Since action is holomorphic function of x
can use Cauchy theorem to deform
integration contour



$$Z = \int_{\mathcal{M}} dx e^{ix^2 - x^4}$$



Complexification: Holomorphic gradient flow/Lefschetz thimbles

General strategy

$$\int_{R^X} e^{iS[\varphi]} = \int_{\mathcal{M}} e^{iS[\varphi_{\mathcal{M}}]} = \int_{R^X} J[\varphi] e^{iS[\varphi_{\mathcal{M}}[\varphi]]} \quad J[\varphi] = \det \frac{\partial \varphi_{\mathcal{M}}}{\partial \varphi}$$

Need to find suitable integration manifolds \mathcal{M} and parametrize them

Lefschetz thimbles: Stationary phase contours attached to a critical point*

$$\left. \frac{\partial S}{\partial \varphi} \right|_{\varphi_c} = 0, \quad \text{Im}(S[\varphi]) = \text{Im}(S[\varphi_c])$$

Holomorphic Gradient flow:
Generated by flow equation**

$$\partial_{\tau} \varphi = \frac{\partial \overline{S}}{\partial \varphi} \quad \partial_{\tau} S[\varphi] = \frac{\partial S}{\partial \varphi} \frac{\partial \overline{S}}{\partial \varphi} \geq 0$$

Sign Optimized Manifolds:

Explicit parametrization or Neural networks***

*see e.g http://ribf.riken.jp/~tanizaki/thesis/yuya_phd.pdf

** Bedaque EPJ Web Conf. 175 (2018) 01020

***Ohnishi, Mori, Kashiwa arXiv:1812.11506

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Lefshetz thimbles: stationary phase contours

(+ mathematically clean, -efficient parametrization & sampling)

Holomorphic Gradient flow: generated by flow equation

(+ mathematically clean, - numerical cost, efficient sampling)

Sign Optimized Manifolds: Explicit parametrization

(+ numerically cheap, - need physical insight)

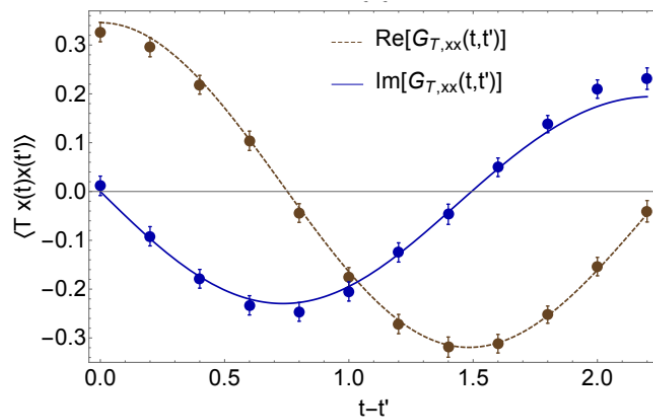
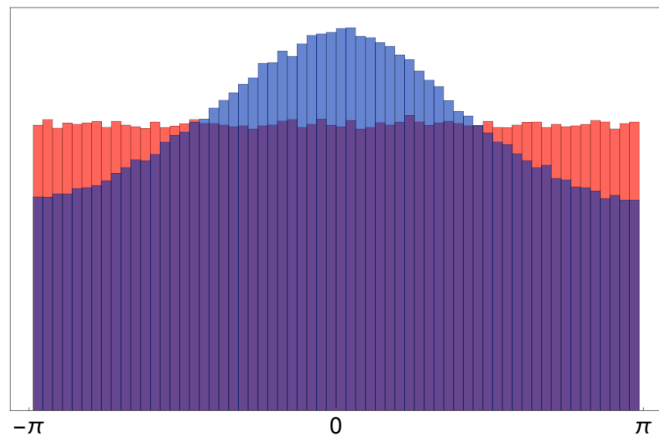
Challenge is to efficiently find suitable manifolds & perform efficient Monte-Carlo sampling in presence of large action barriers

Complexification: Holomorphic gradient flow/Lefschetz thimbles

Simulation results based on Holomorphic gradient flow simulations

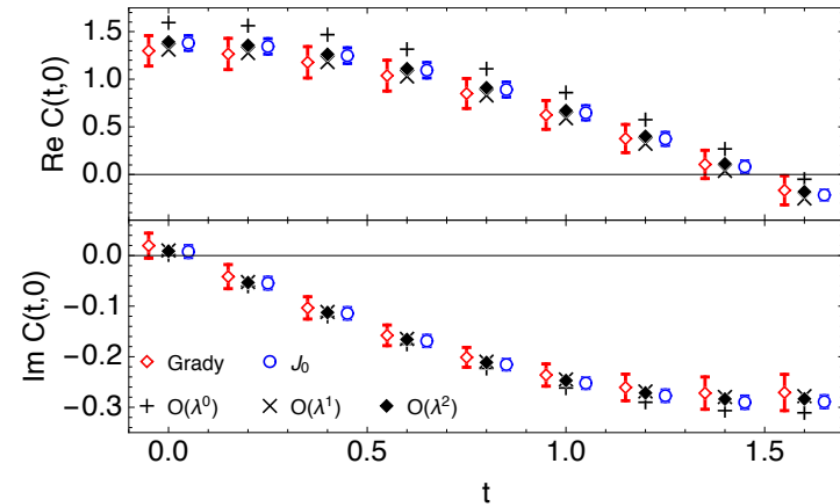
0+1D Φ^4

$N_t = 8$, $N_\beta = 2$, and $N_x = 8$



1+1D Φ^4

$N_t = 8$, $N_\beta = 2$, and $N_x = 8$



Alexandru et al. PRD 95 (2017) no.11, 114501

Alexandru et al. PRL 117 (2016) no.8, 081602

Hamiltonian simulation

Basic idea is to perform Hamiltonian evolution of the system

$$|\underbrace{000001}_{N_{\text{qubits}}}\rangle$$

Digital quantum computing

Quantum computer with universal set of instructions allows for any operation to be performed on spin system

Natural to perform time evolution by applying e^{iHt}

Need to map Hamiltonian of theory on set of spin 1/2s

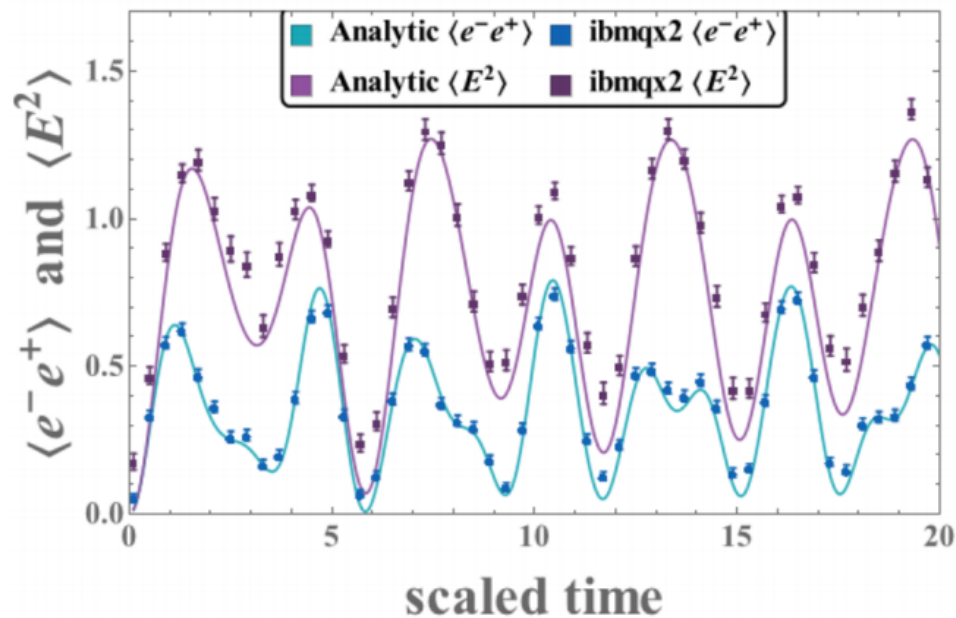
Discretize theory on lattice & discretize field space

$$\hat{\phi}_{\mathbf{x}} \approx \sum_{i=1}^N \phi_{i,\mathbf{x}} |\phi_{i,\mathbf{x}}\rangle \langle \phi_{i,\mathbf{x}}|$$

need to take limits $N \rightarrow \infty$ in addition to usual continuum and infinite volume limits

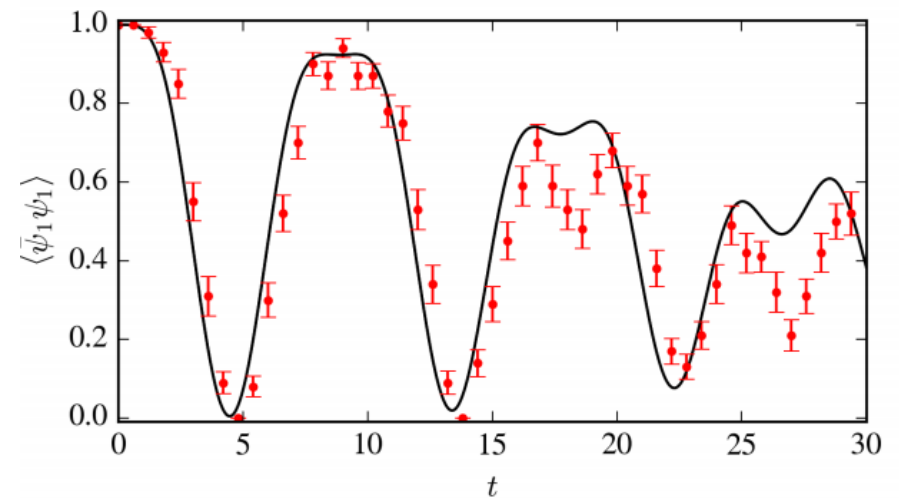
Examples of digital quantum computing

Schwinger model*
(2 sites; 3 qubits; 8192 meas)



*Klco et al. PRA98 (2018) no.3, 032331

Z_2 gauge theory
(4 sites; 8 qubits; 100 meas)



NuQS Collaboration
arXiv:1903.08807

Digital quantum computing

$$|\underbrace{000001}_{N_{\text{qubits}}}\rangle$$

Conceptual Challenges:

Discretization of (non-abelian) gauge theories &
reducing redundancies in the calculation

Computational Challenges:

Building and operating a large-scale quantum computer
Errors induced due to loss imperfect isolation

Near term: NISQ era Noisy IntermediateScale Quantum

~50-100s of qubits

~1000 two-qubit operations

Epilogue

Description of real-time/out-of-equilibrium dynamics in QCD remains a formidable challenge

Number of different approaches with different ranges of validity

Exciting non-equilibrium phenomena discovered with available methods

Currently no first principles methods available

Non-equilibrium QCD remains a largely unexplored area

Great opportunities for young scientists to have an impact!





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Thank you!!!