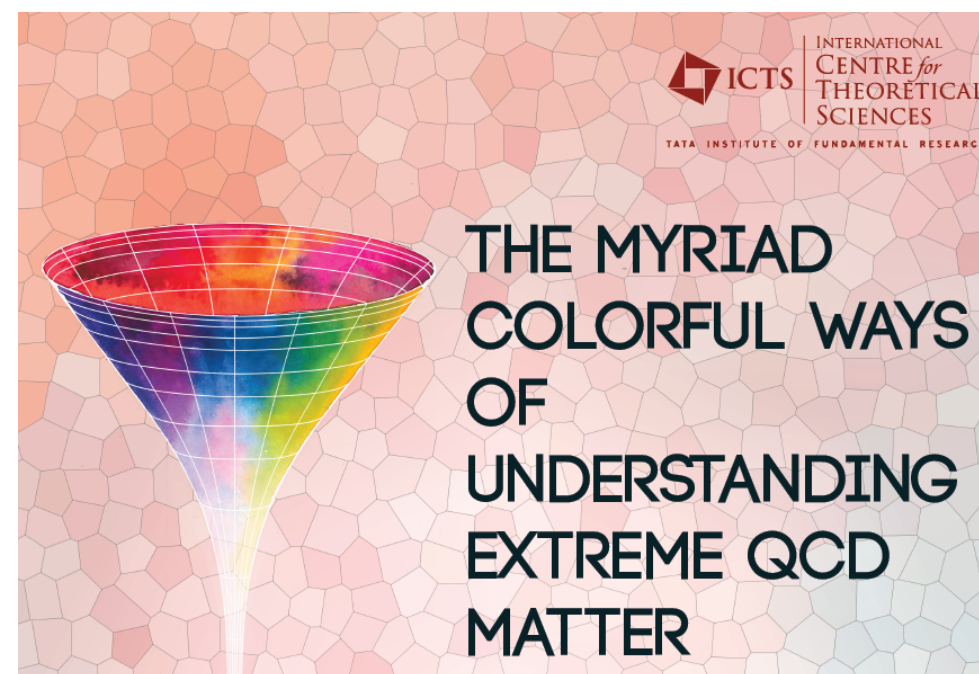


Inclusive prompt photon+dijet production in e+A DIS at small x as a probe of gluon saturation

Kaushik Roy

Stony Brook University and Brookhaven National Laboratory

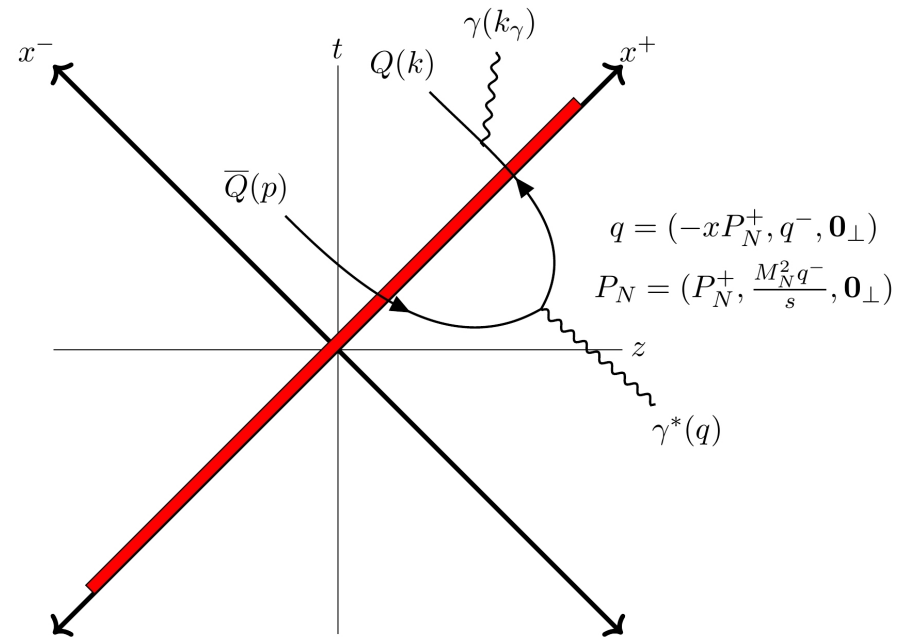


ICTS Bengaluru, April 1-17, 2019

KR, Venugopalan, JHEP 1805 (2018) 013 [arXiv:1802.09550]; and work in preparation

Inclusive photon production in small x DIS at LO

- Cleanest process after fully inclusive DIS
- Can be measured at the luminosities provided by a future **E**lectron **I**on **C**ollider (**EIC**)

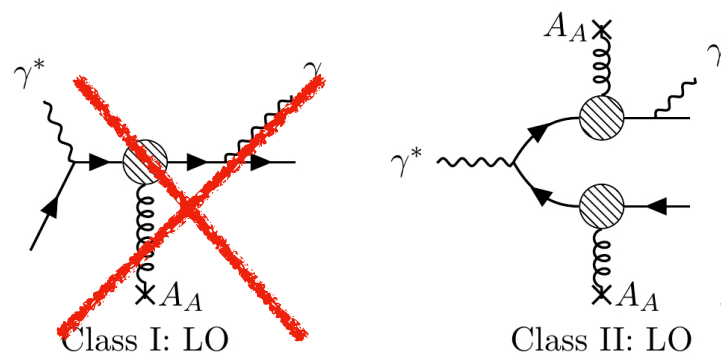


The right moving nucleus with large P_N^+ has its x^- extent Lorentz contracted

Compliments similar computations for pA collisions

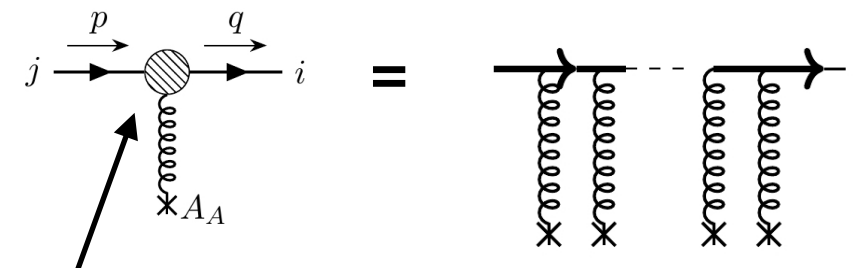
Gelis, Jalilian-Marian, hep-ph/0205037; Dominguez, Marquet, Xiao, Yuan, arXiv:1101.0715
Benic, Fukushima, arXiv:1602.01989;
Benic, Fukushima, Garcia-Montero, Venugopalan, arXiv:1609.09424, 1807.03806

Light cone coordinate convention: $V^\pm = \frac{V^0 \pm V^3}{\sqrt{2}}$



Suppressed at small x
Cross-section accompanied by
valence quark distribution
 $xf(x, Q^2) \ll xG(x, Q^2)$ at small x

Consider processes of this class



Quark must scatter off of the "shock wave"

CGC inputs:

Working gauge: $\partial_\mu A^\mu = 0$ (Lorenz gauge)

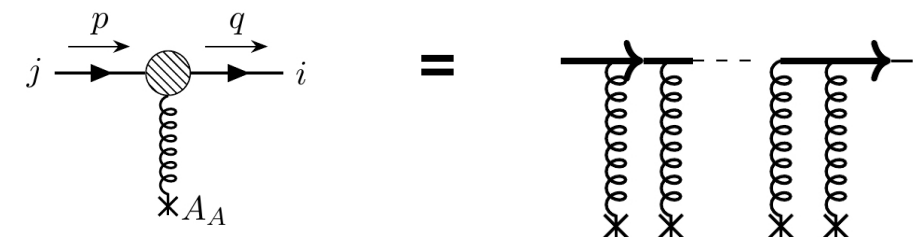
Background classical field: $A^{-,a} = A^{i,a} = 0$, $A^{+,a} = \int d^2 \mathbf{y}_\perp \langle \mathbf{x}_\perp | \frac{1}{-\nabla_\perp^2} | \mathbf{y}_\perp \rangle \rho_A^a(x^-, \mathbf{x}_\perp)$

Momentum space fermion propagator in this background field has a simple form

$$S_{Lor.}(q, p) = (2\pi)^4 \delta^{(4)}(q - p) S_0(p) + S_0(q) \mathcal{T}(q, p) S_0(p)$$

McLerran, Venugopalan, hep-ph/9402335, hep-ph/9809427

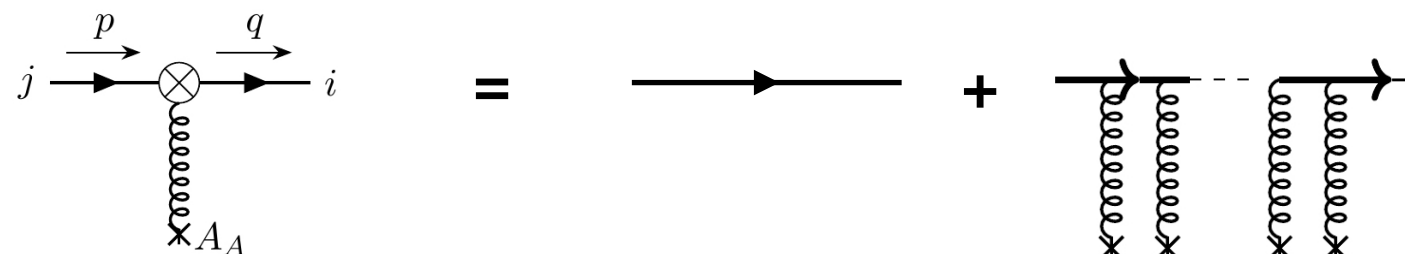
Diagrammatically represented by an effective vertex



Vertex factor $\mathcal{T}_{ij}(q, p) = 2\pi \delta(q^- - p^-) \text{sign}(p^-) \gamma^- \int d^2 \mathbf{z}_\perp e^{-i(\mathbf{q}_\perp - \mathbf{p}_\perp) \cdot \mathbf{z}_\perp} [\tilde{U}^{\text{sign}(p^-)}(\mathbf{z}_\perp) - \mathbb{1}]_{ij}$

Fundamental Wilson line $\tilde{U}(\mathbf{x}_\perp) = \mathcal{P}_- \exp \left[-ig \int_{-\infty}^{+\infty} dz^- \frac{1}{\nabla_\perp^2} \rho_A^a(z^-, \mathbf{x}_\perp) t^a \right]$

What we show and utilize: Use modified vertices (same as above modulo identity)
Subtract “no-scattering” contribution at the end. Simple and equivalent



Hadron tensor: $X_{\mu\nu} = - \sum_{\text{spins}} \mathcal{M}_{\mu\alpha}^*(q, k, p, k_\gamma) \mathcal{M}_\nu^\alpha(q, k, p, k_\gamma)$

$$\mathcal{M}_{\mu\alpha}(q, k, p, k_\gamma) = \sum \left[\begin{array}{c} \text{(1)} \quad \text{(2)} \quad \text{(3)} \quad \text{(7)} \quad \text{(8)} \\ \text{(4)} \quad \text{(5)} \quad \text{(6)} \quad \text{(9)} \quad \text{(10)} \end{array} \right]$$

Equivalent results with vertex definitions modified slightly

$$\mathcal{M}_{\mu\alpha}(q, k, p, k_\gamma) = \sum \left[\begin{array}{c} \text{(7)} \quad \text{(8)} \\ \text{(9)} \quad \text{(10)} \end{array} \right] - \left[\begin{array}{c} \text{Diagram 1} \quad \text{Diagram 2} \end{array} \right]$$

Offers significant advantage for **NLO** computation

Key analytical results:

Triple differential cross-section

$$\frac{d\sigma}{dx dQ^2 d^6 K_\perp d^3 \eta_K} = \frac{\alpha^2 q_f^4 y^2 N_c}{512 \pi^5 Q^2} \frac{1}{(2\pi)^4} \frac{1}{2q^-} L^{\mu\nu} \tilde{X}_{\mu\nu} (2\pi) \delta(P^- - q^-)$$

Lepton tensor

$$L^{\mu\nu} = \frac{2e^2}{Q^4} \left[(\tilde{l}^\mu \tilde{l}'^\nu + \tilde{l}^\nu \tilde{l}'^\mu) - \frac{Q^2}{2} g^{\mu\nu} \right]$$

Hadron tensor

$$\tilde{X}_{\mu\nu} = \int_{x_\perp, y_\perp, x'_\perp, y'_\perp} \int_{l_\perp, l'_\perp} e^{-i(P_\perp - l_\perp) \cdot x_\perp - i l_\perp \cdot y_\perp + i(P_\perp - l'_\perp) \cdot x'_\perp + i l'_\perp \cdot y'_\perp} \tau_{\mu\nu}^{q\bar{q}, q\bar{q}}(l_\perp, l'_\perp | P_\perp) \times \Xi(x_\perp, y_\perp; x'_\perp, y'_\perp)$$

Complicated trace over gamma matrices

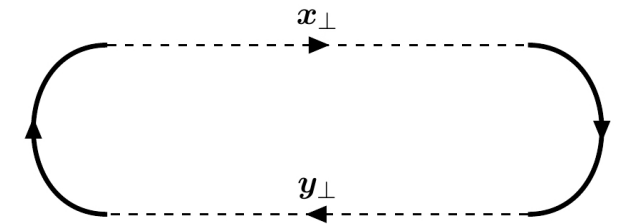


Non-perturbative input about strongly correlated gluons is contained in

$$\Xi(x_\perp, y_\perp; x'_\perp, y'_\perp) = 1 - D(x_\perp, y_\perp) - D(y'_\perp, x'_\perp) + Q(x_\perp, y_\perp; y'_\perp, x'_\perp)$$

Dipole Wilson line correlator

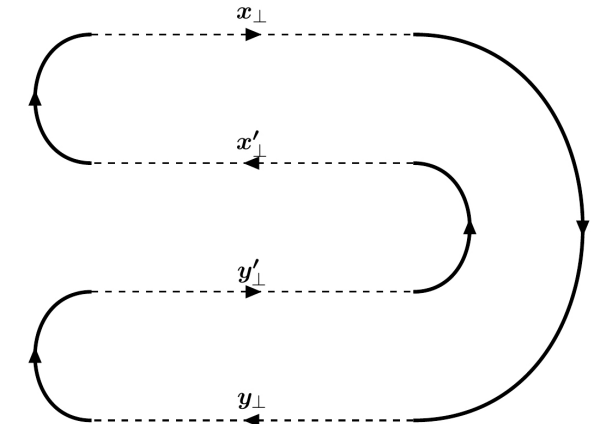
$$D(x_\perp, y_\perp) = \frac{1}{N_c} \langle \text{Tr} \left(\tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) \right) \rangle_{Y_A}$$



Quadrupole Wilson line correlator

$$Q(x_\perp, y_\perp; y'_\perp, x'_\perp) = \frac{1}{N_c} \langle \text{Tr} \left(\tilde{U}(y'_\perp) \tilde{U}^\dagger(x'_\perp) \tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) \right) \rangle_{Y_A}$$

$$\langle \mathcal{O} \rangle_{\Lambda^+(Y_A)} = \int [\mathcal{D}\rho_A] W_{\Lambda^+(Y_A)}[\rho_A] \mathcal{O}[\rho_A]$$



Ubiquitous building blocks of high energy QCD

Interesting limit:

In the soft photon limit of $k_\gamma \rightarrow 0$ the amplitude satisfies the Low-Burnett-Kroll theorem

$$\mathcal{M}_\mu(q, k, p, k_\gamma) \rightarrow -(eq_f)\epsilon_\alpha^*(k_\gamma, \lambda) \left(\frac{p^\alpha}{p \cdot k_\gamma} - \frac{k^\alpha}{k \cdot k_\gamma} \right) \mathcal{M}_\mu^{NR}(q, k, p)$$

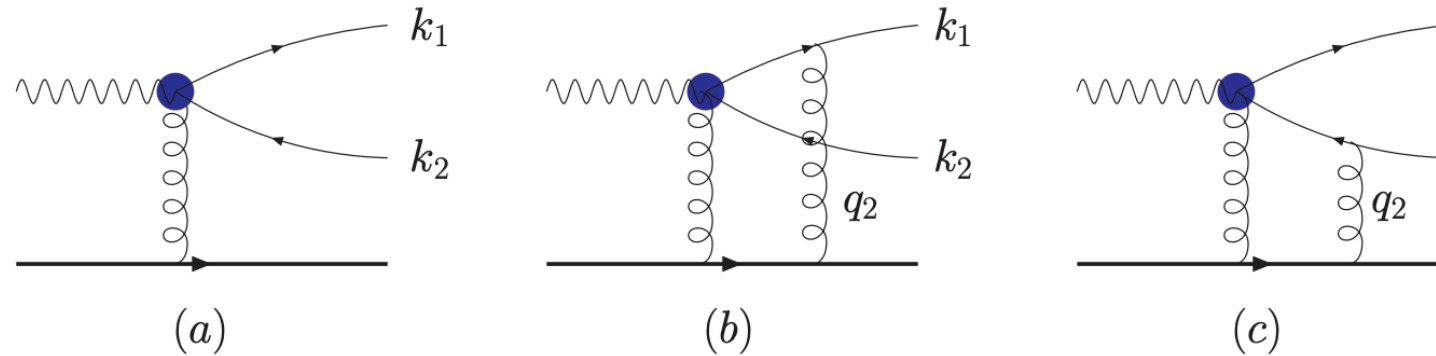
Polarization vector

\times

Vectorial structure depending only on momenta of emitted particles

\times

Non-radiative DIS amplitude



Recover existing results on inclusive dijet production in DIS

$$\begin{aligned} \frac{d\sigma^{L,T}}{d^3k d^3p} = & \alpha q_f^2 N_c \delta(q^- - p^- - k^-) \int \frac{d^2x_\perp}{(2\pi)^2} \frac{d^2x'_\perp}{(2\pi)^2} \frac{d^2y_\perp}{(2\pi)^2} \frac{d^2y'_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{-i\mathbf{p}_\perp \cdot (\mathbf{y}_\perp - \mathbf{y}'_\perp)} \\ & \times \sum_{\alpha, \beta} \psi_{\alpha\beta}^{L,T}(q^-, z, |\mathbf{x}_\perp - \mathbf{y}_\perp|) \psi_{\alpha\beta}^{L,T*}(q^-, z, |\mathbf{x}'_\perp - \mathbf{y}'_\perp|) \times \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \end{aligned}$$

Dominguez, Marquet, Xiao, Yuan, arXiv:1101.0715

Important check because it is sensitive to the Weizsäcker-Williams

UGD in the back-to-back correlation limit $|\mathbf{k}_\perp + \mathbf{p}_\perp| \ll |\mathbf{k}_\perp - \mathbf{p}_\perp|/2$.

Golden channel for probing and understanding the WW distribution at a future EIC or LHeC!!

Structure of higher order computations

CGC inputs: Shockwave gluon propagator

- Convenient to work in the “wrong” light-cone gauge $A^- = 0$ for the kinematics of this problem. (Gauge links appearing in PDF definitions are unity in the conventional LC gauge $A^+ = 0$.)
- Resulting momentum space expression is simple and similar to the shockwave fermion propagator.

McLerran, Venugopalan, hep-ph/9402335

Ayala, Jalilian-Marian, McLerran, Venugopalan, hep-ph/9501324

Balitsky, Belitsky, hep-ph/0110158

$$G^{\mu\nu;ab}(p, p') = (2\pi)^4 \delta^{(4)}(p - p') G_0^{\mu\nu;ab}(p) + G_0^{\mu\rho;ac}(p) \mathcal{T}_{\rho\sigma;cd}(p, p') G_0^{\sigma\nu;db}(p')$$

$G_0^{\mu\nu;ab}$: Free gluon propagator in $A^- = 0$ gauge

Vertex structures identical to quark-quark-reggeon and gluon-gluon-reggeon in Lipatov's Reggeon EFT

Bondarenko, Lipatov, Pozdnyakov, Prygarin, arXiv: 1708.05183
Hentschinski, arXiv: 1802.06755

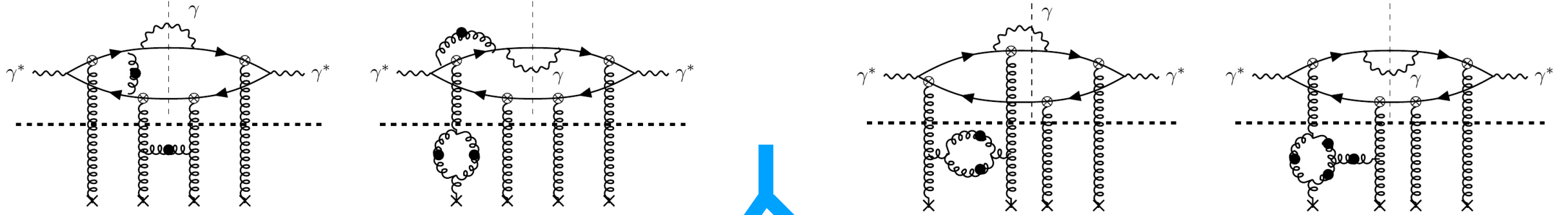
	$\mathcal{T}_{ij}(p, p') = (2\pi) \delta(p^- - p'^-) \gamma^- \text{sign}(p^-) \int d^2 \mathbf{z}_\perp e^{-i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} \tilde{U}_{ij}^{\text{sign}(p^-)}(\mathbf{z}_\perp)$
	$\mathcal{T}_{\mu\nu;ab}(p, p') = -2\pi \delta(p^- - p'^-) \times (2p^-) g_{\mu\nu} \text{sign}(p^-) \int d^2 \mathbf{z}_\perp e^{-i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} U_{ab}^{\text{sign}(p^-)}(\mathbf{z}_\perp)$

DIS dijet+photon at NLO+NLLx

KR, Venugopalan, in preparation

NNLO:1

NNLO:2



Formally NNLO(=O((α_s)²) in CGC power counting

Collect leading log in x (LLx) pieces $\sim \alpha_s \ln(\Lambda_1^-/\Lambda_0^-)$
+
NLO pieces $\sim \alpha_s$ in the photon+dijet impact factor

Collect next-to-leading-log (NLLx) pieces $\sim \alpha_s^2 \ln(\Lambda_1^-/\Lambda_0^-)$
+
LO pieces $\sim \alpha_s^0$ in the photon+dijet impact factor

$$\begin{aligned} \langle d\sigma_{NLO+NLLx} \rangle &= \int [\mathcal{D}\rho_A] \left\{ W_{\Lambda_0^-}^{NLLx}[\rho_A] d\hat{\sigma}_{LO}[\rho_A] + W_{\Lambda_0^-}^{LLx}[\rho_A] d\hat{\sigma}_{NLO}[\rho_A] \right\} \\ &= \int [\mathcal{D}\rho_A] \left(W_{\Lambda_0^-}^{NLLx}[\rho_A] \left\{ d\hat{\sigma}_{LO}[\rho_A] + d\hat{\sigma}_{NLO}[\rho_A] \right\} + O(\alpha_s^3 \ln(\Lambda_1^-/\Lambda_0^-)) \right) \end{aligned}$$

NLLx resummed weight functional

$$W_{\Lambda_0^-}^{NLLx}[\rho_A] = \left\{ 1 + \ln(\Lambda_1^-/\Lambda_0^-) (\mathcal{H}_{LO} + \mathcal{H}_{NLO}) \right\}$$

NLO photon+dijet impact factor

Correction terms are higher order compared to the relevant accuracy of the problem

\mathcal{H}_{NLO} = NLO JIMWLK Hamiltonian

NLO BK [Balitsky, Chirilli, arXiv:0710.4330
Kovchegov, Weigert, hep-ph/0609090
Kovner, Lublinsky, Mulian, arXiv:1310.0378
Grabovsky, arXiv:1307.5414
Caron-Huot, arXiv:1309.6521
Lublinsky, Mulian, arXiv: 1610.03453]

NLO JIMWLK

NLO impact factor for photon+dijet in eA DIS

Extant NLO results in literature

Fully Inclusive DIS

Balitsky, Chirilli, arXiv: 1009.4729

Beuf, arXiv: 1606.00777, 1708.06557

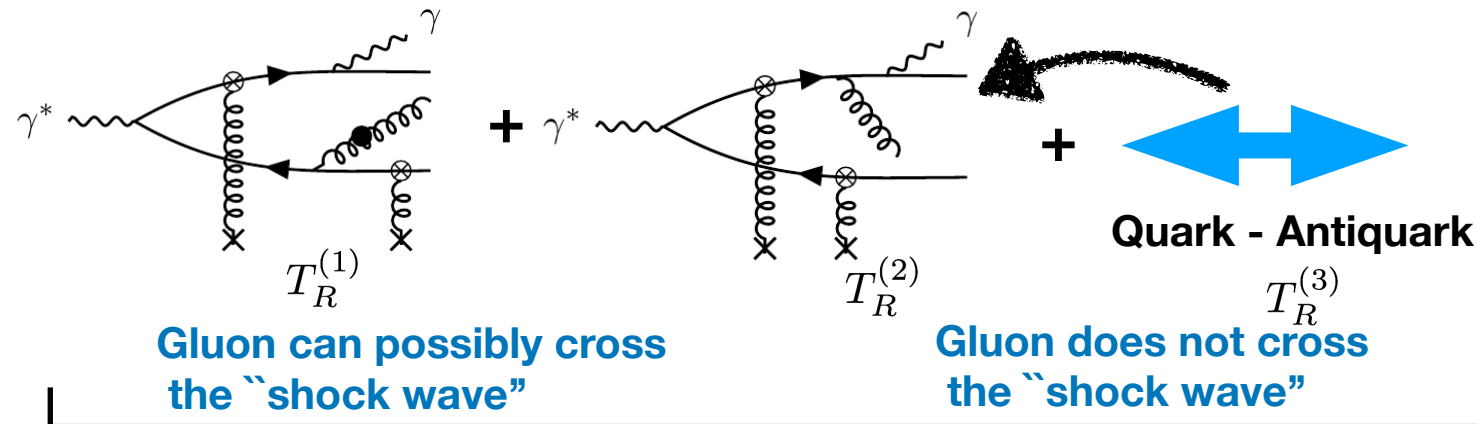
Hanninen, Lappi, Paatelainen, arXiv: 1711.08207

Diffractive DIS dijet

→ Boussarie, Grabovsky, Szymanowski, Wallon, arXiv:1606.00419

First computation of photon+dijet in eA DIS at small x

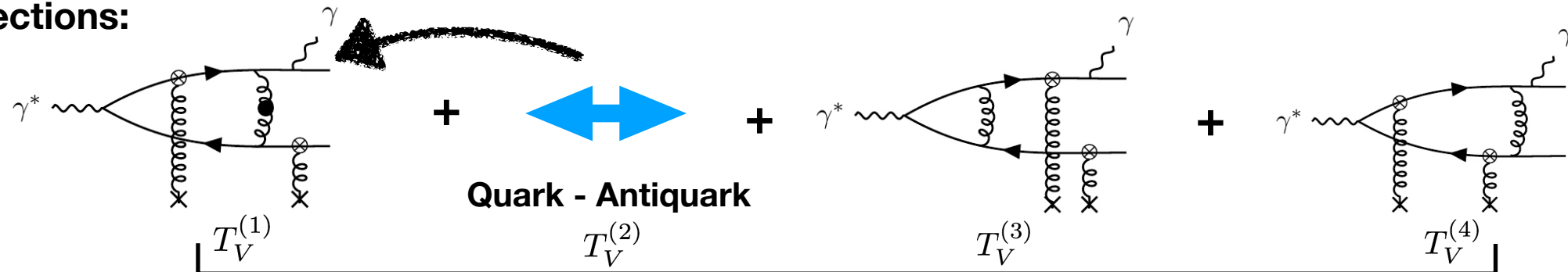
Real gluon emission:



20x20=400 contributions in the squared amplitude

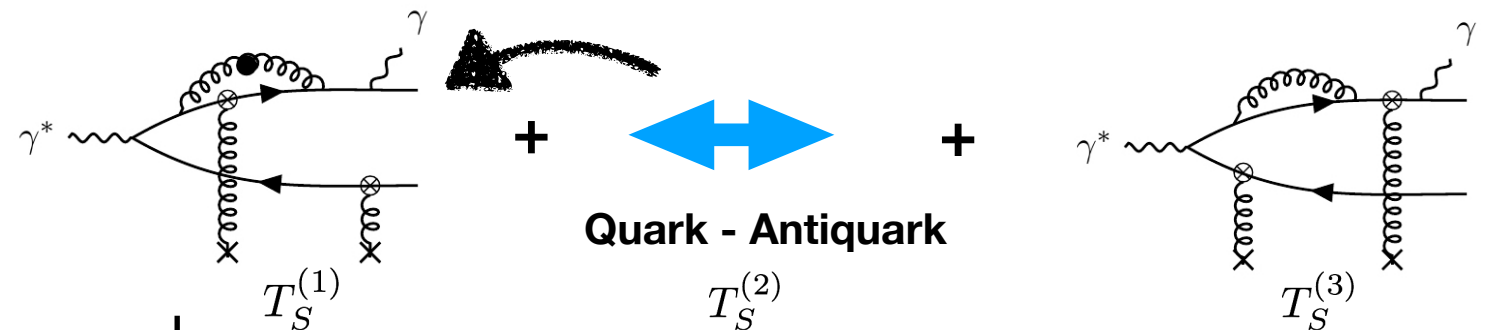
Virtual emission:

a. Vertex corrections:



6x4=24 diagrams in total interfering with LO processes

b. Self-energy corrections:



(6x2=12)+24=36 diagrams in total interfering with LO processes

Assembling different contributions in the amplitude squared:

KR, Venugopalan, in preparation

- Novel and rich structure in terms of 2-point and 4-point Wilson line correlators obtained.

$$|\mathcal{M}^R|^2 \quad \longleftrightarrow \quad \mathcal{M}_{NLO} \mathcal{M}_{LO}^* + c.c$$

Wilson line factor	Real emission	Virtual: Vertex	Virtual: Self-energy
$\frac{N_c^2}{2} \left(1 - D_{xz} D_{zy} - D_{y'z} D_{zx'} + D_{y'y} D_{xx'} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(1)*} T_R^{(1)}$		
$C_F N_c \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(2)*} T_R^{(2)} + T_R^{(3)*} T_R^{(3)}$	$T_{LO}^* T_V^{(3)} + c.c$	$T_{LO}^* T_S^{(3)} + c.c$
$\frac{N_c^2}{2} [(1 - D_{xy})(1 - D_{y'x'})] - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(2)*} T_R^{(3)} + c.c$	$T_{LO}^* T_V^{(4)} + c.c$	
$\frac{N_c^2}{2} \left(1 + (Q_{zy;y'x'} - D_{zy}) D_{xz} - D_{y'x'} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(2)*} T_R^{(1)}$	$T_{LO}^* T_V^{(1)}$	$T_{LO}^* T_S^{(1)}$
$\frac{N_c^2}{2} \left(1 + (Q_{y'x';xz} - D_{xz}) D_{zy} - D_{y'x'} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(3)*} T_R^{(1)}$	$T_{LO}^* T_V^{(2)}$	$T_{LO}^* T_S^{(2)}$

Collinear divergences cancel between real and interference contributions

No collinear singularities due to transverse momentum kicks from nucleus

+conjugates of last two rows
Wilson line factors obtained by permutations of coordinates

Rapidity and UV divergent pieces: absorb into the NLLx JIMWLK expressions using a suitable subtraction scheme.

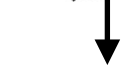
Deriving the JIMWLK evolution from the projectile side for a non-trivial process

Non-linear RG evolution equation, governs rapidity evolution of many-body gluon correlators

Hadron tensor at LO:

$$X_{\mu\nu}^{\text{LO}} = C_{\mu\nu}^{\text{LO}} \otimes \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-)$$

$$\Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) = 1 - D(\mathbf{x}_\perp, \mathbf{y}_\perp) - D(\mathbf{y}'_\perp, \mathbf{x}'_\perp) + Q(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$$



Hard coefficient

In the soft gluon limit which generates logarithms in x, we obtain the following for our hadron tensor at NLO

$$\begin{aligned} X_{\mu\nu;LLx}^{\text{NLO}} = C_{\mu\nu}^{\text{LO}} \otimes \ln(\Lambda_1^- / \Lambda_0^-) & \left[\frac{\alpha_S N_c}{2\pi^2} \left\{ \mathcal{K}_B(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{z}_\perp) \boxed{D_{xy}} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} - \frac{\alpha_S N_c}{(2\pi)^2} \mathcal{K}_1(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{y}'_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) \boxed{Q_{xy}} \right. \\ & - \frac{\alpha_S N_c}{2\pi^2} \left\{ \mathcal{K}_B(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{z}_\perp) D_{xz} D_{zy} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} + \frac{\alpha_S N_c}{(2\pi)^2} \left(\left\{ \mathcal{A}(\mathbf{x}_\perp, \mathbf{y}'_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) \boxed{D_{xx'} D_{y'y}} + \mathbf{x}_\perp \leftrightarrow \mathbf{y}'_\perp \right. \right. \\ & \left. \left. + \left\{ \mathcal{K}_2(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) \boxed{D_{xz} Q_{zy; y'x'}} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} + \left\{ \mathcal{K}_2(\mathbf{x}'_\perp, \mathbf{x}_\perp, \mathbf{y}'_\perp; \mathbf{z}_\perp) D_{zx'} Q_{y'z; xy'} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} \right) \right] \end{aligned}$$

Building blocks: Non-trivial combinations of **dipole** and **quadrupole** Wilson line correlators

Dominguez, Mueller, Munier, Xiao, arXiv: 1108.1752

$\mathcal{A}, \mathcal{K}_{1,2}$ are evolution kernels composed of several BFKL kernels $\mathcal{K}_B = \frac{(\mathbf{x}_\perp - \mathbf{y}_\perp)^2}{(\mathbf{x}_\perp - \mathbf{z}_\perp)^2 (\mathbf{y}_\perp - \mathbf{z}_\perp)^2}$

Remarkably, this whole thing can be simply written as

$$X_{\mu\nu;LLx}^{\text{NLO}} = C_{\mu\nu}^{\text{LO}} \otimes \ln \left(\frac{\Lambda_1^-}{\Lambda_0^-} \right) H_{\text{JIMWLK}}^{\text{LO}} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-)$$

Leads immediately to the JIMWLK evolution equation: $W_{\Lambda_1^-}[\rho_A] = \left(1 + \ln \left(\frac{\Lambda_1^-}{\Lambda_0^-} \right) H_{\text{JIMWLK}}^{\text{LO}} \right) W_{\Lambda_0^-}[\rho_A]$

$$\frac{\partial}{\partial (\ln \Lambda^-)} (d\sigma^{LO}) = \langle H_{\text{JIMWLK}} (d\sigma^{LO}) \rangle$$

Summary

- We present a first computation of inclusive photon production in deeply inelastic electron-nuclear scattering at small x in the CGC framework. Clean way of studying the emergent regime of saturation physics that has aspects of both weak and strong interactions.
- The LO result is proportional to universal 2-point and 4-point Wilson line correlators in the nucleus. Extant results on fully inclusive DIS dijet are obtained in the soft photon limit.
- The simple structure of the dressed quark and gluon propagators in the “wrong” light cone gauge enables higher order computations in momentum space using otherwise standard covariant perturbation theory (pQCD) techniques.
- The techniques employed in NLO calculation are also discussed, with emphasis on the high energy JIMWLK factorization.
- The computation of the NLO impact factor is the missing non-trivial piece and **COMING SOON** ON arXiv..

We thank Ian Balitsky, Renaud Boussarie, Al Mueller and Yair Mulian for useful discussions

Thank you...