# Coherent Diffractive Dijet Production in Deep Inelastic Scattering and Dipole Orientation

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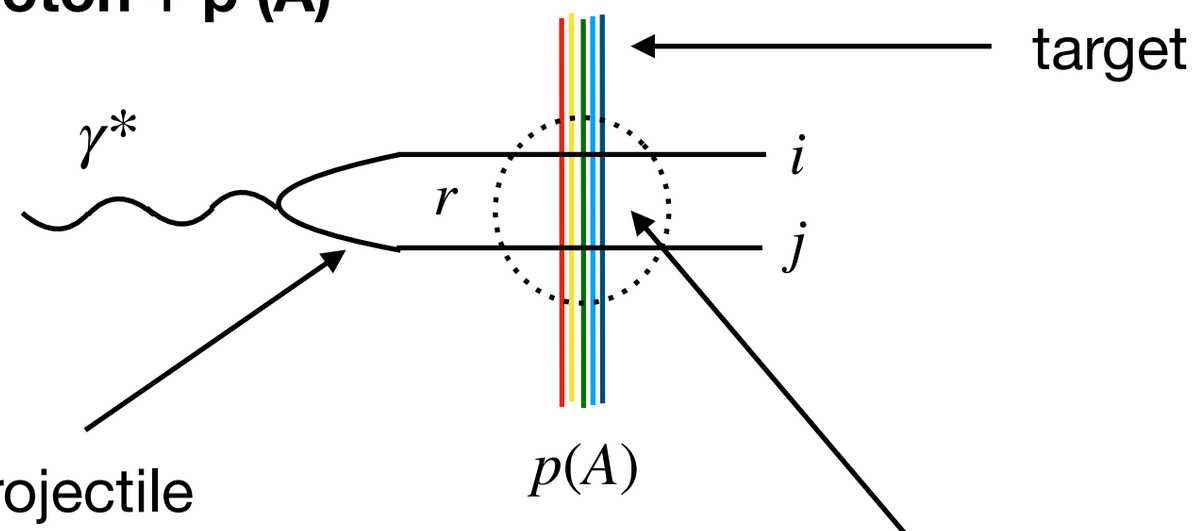
The Myriad Colorful Ways of Understanding Extreme QCD Matter
April 5th, 2019.

# Outline

- High energy scattering in QCD
  - Target, projectile and the scattering
  - From amplitude to cross section
- Coherent diffractive dijet
  - Target geometry, dipole spatial correlations and  cross section momentum correlations
  - Numerical approach
  - Semi-analytic approach
  - Conclusions/extensions

# Target, projectile and the scattering amplitude

**virtual photon + p (A)**




$$\Psi_L^{\gamma^*\to q\bar{q}}(\mathbf{r}) = \frac{eZ_f}{2\pi} z_0^{3/2} z_1^{3/2} 2Q(1-\delta_{\sigma_0\sigma_1}) K_0(\varepsilon_f r)$$

$$\Psi_{T\lambda}^{\gamma^*\to q\bar{q}}(\mathbf{r}) = \frac{eZ_f}{2\pi} z_0 z_1 \left[ \delta_{\sigma_0\sigma_1} \frac{m_f}{\sqrt{2}} (1+\sigma_0\lambda) K_0(\varepsilon_f r) + \delta_{\sigma_0-\sigma_1}(z_1 - z_0 - \sigma_0\lambda) i\varepsilon_f \frac{\epsilon_\lambda \cdot \mathbf{r}}{r} K_1(\varepsilon_f r) \right]$$

$$\varepsilon_f^2 = z_0 z_1 Q^2 + m_f^2$$

Strong classical fields

$$A^{\mu,a} = \delta^{\mu,+}\alpha(\mathbf{x}, x^-)$$

$$\nabla_{\mathbf{x}}^2 \alpha^a(\mathbf{x}, x^-) = -\rho^a(\mathbf{x}, x^-)$$

small-x (fields)

large-x (sources)

scattering

dipole eikonal scattering from strong color fields

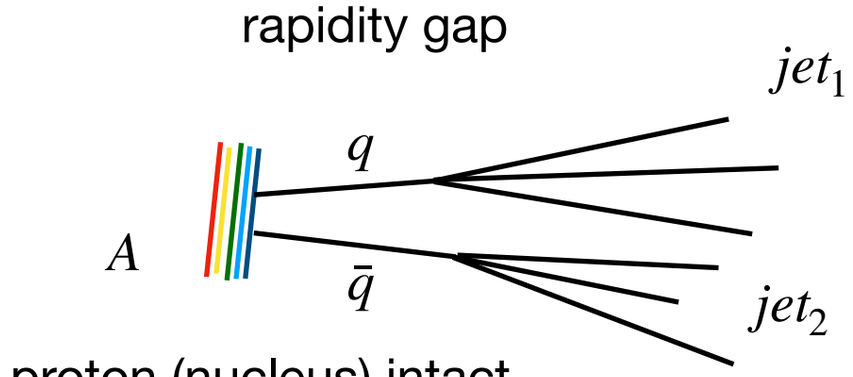
$$S_{ij}(\mathbf{x}, \mathbf{y}) = \left[ V(\mathbf{x}) V^\dagger(\mathbf{y}) \right]_{ij}$$

$$V(\mathbf{x}) = Pe^{-ig\int dz^- T^a \alpha^a(z^-, \mathbf{x})}$$

# From amplitude to cross section

$$\mathscr{M}(\mathbf{x}_1, \mathbf{x}_2) = \Psi(|\,\mathbf{x}_1 - \mathbf{x}_2\,|)\left[1_c - V(\mathbf{x}_1)V^\dagger(\mathbf{x}_2)\right] \qquad \tilde{\mathscr{M}}(\mathbf{p_1}, \mathbf{p_2}) = \int d^2\mathbf{x_1} \int d^2\mathbf{x_2} e^{i\mathbf{x}_1\cdot\mathbf{p}_1} e^{i\mathbf{x}_2\cdot\mathbf{p}_2} \mathscr{M}(\mathbf{x}_1, \mathbf{x}_2)$$

**Coherent diffractive dijet**



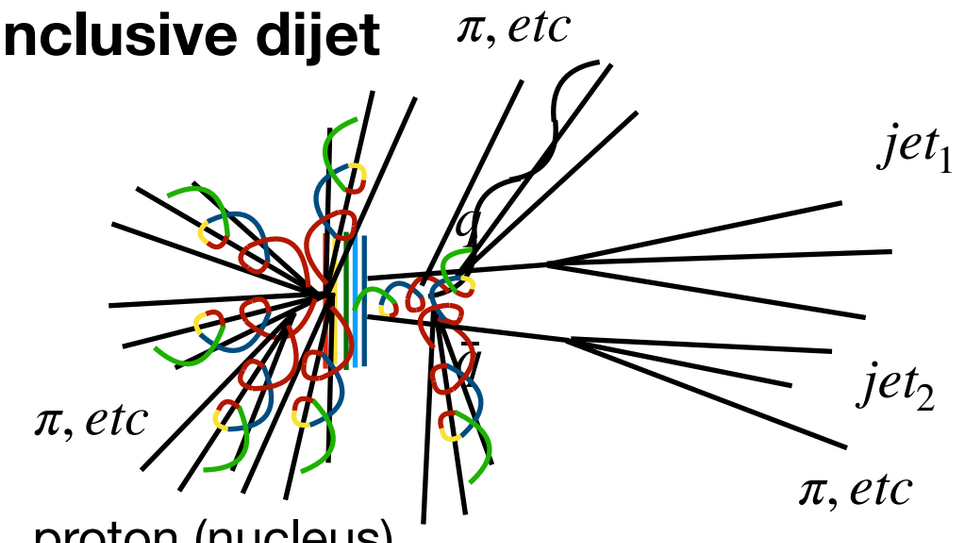

proton (nucleus) intact

$$\frac{d\sigma}{d\Omega}^{coh\ diff} \sim \left| \left\langle \mathrm{tr}\,\mathscr{M}(\mathbf{p_1}, \mathbf{p_2}) \right\rangle \right|^2$$

scattered dipole in color singlet

prescription for coherent process

**Inclusive dijet**




proton (nucleus) shattered

particles at different rapidities

$$\frac{d\sigma}{d\Omega}^{inclusive} \sim \mathrm{tr}\left\langle \left| \mathscr{M}(\mathbf{p_1}, \mathbf{p_2}) \right|^2 \right\rangle$$

sum over color in final states

prescription for inclusive process

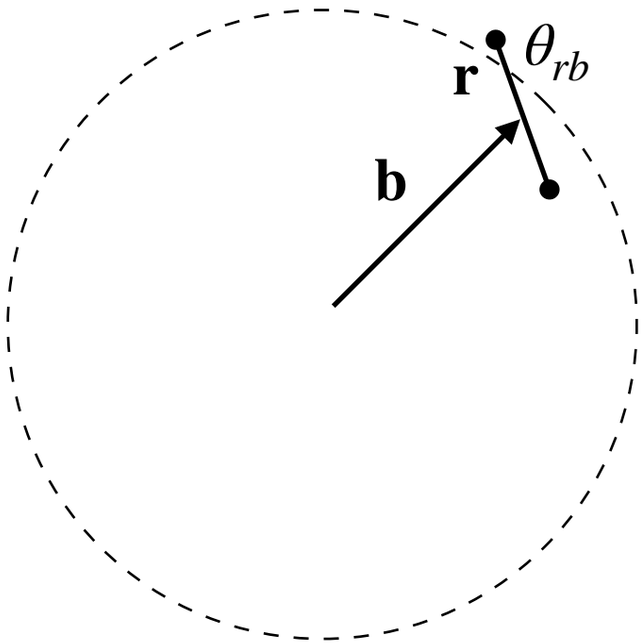
# Target geometry, dipole spatial correlations and cross section momentum correlations

$$p_0^+ p_1^+ \frac{d\sigma^{\text{coh diff}}}{dp_0^+ dp_1^+ d^2\mathbf{P} d^2\mathbf{\Delta}} = \frac{\delta(q^+ - p_0^+ - p_1^+)}{2q^+(2\pi)^5} \left| \int d^2\mathbf{r}\; d^2\mathbf{b}\; e^{-i\mathbf{P}\cdot\mathbf{r}} e^{-i\mathbf{\Delta}\cdot\mathbf{b}}\; \Psi(\mathbf{r}) D(\mathbf{r},\mathbf{b}) \right|^2$$

$$D(\mathbf{r},\mathbf{b}) = \left[ 1 - \frac{1}{N_c} \left\langle \text{tr}\left( \text{V}(\mathbf{b} + \frac{\mathbf{r}}{2}) \text{V}^\dagger(\mathbf{b} - \frac{\mathbf{r}}{2}) \right) \right\rangle \right]$$

$$\begin{aligned} \mathbf{r} &= \mathbf{x}_1 - \mathbf{x}_2 \\ \mathbf{b} &= \frac{1}{2}(\mathbf{x}_1 + \mathbf{x}_2) \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} \mathbf{P} &= \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) \\ \mathbf{\Delta} &= \mathbf{p}_1 + \mathbf{p}_2 \end{aligned}$$

Fourier conjugates




Transverse view of target

Target geometry

↓

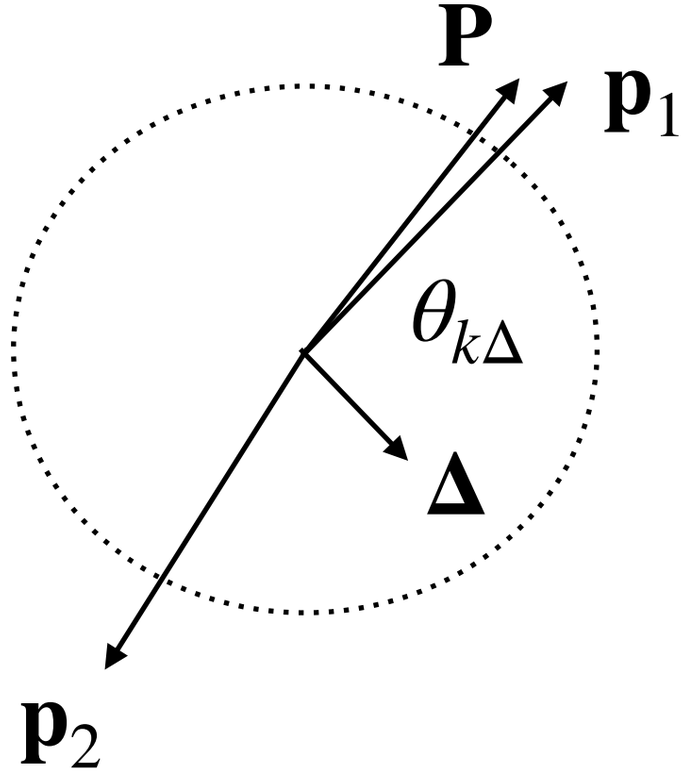
b dependence

$\theta_{rb}$ dependence (dipole orientation)

↓

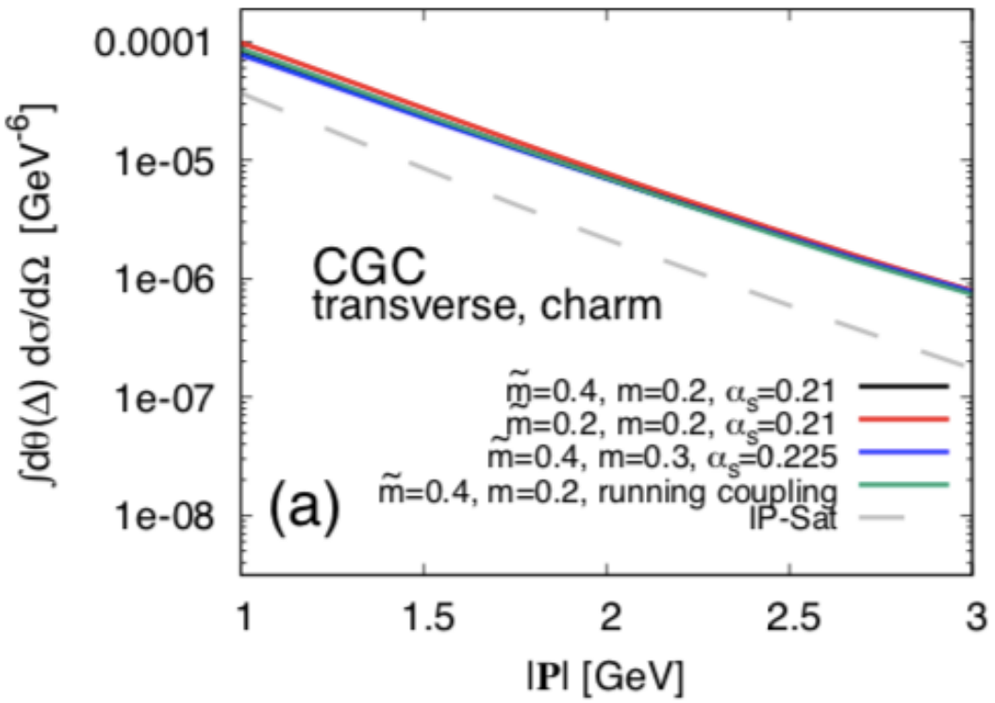
P dependence

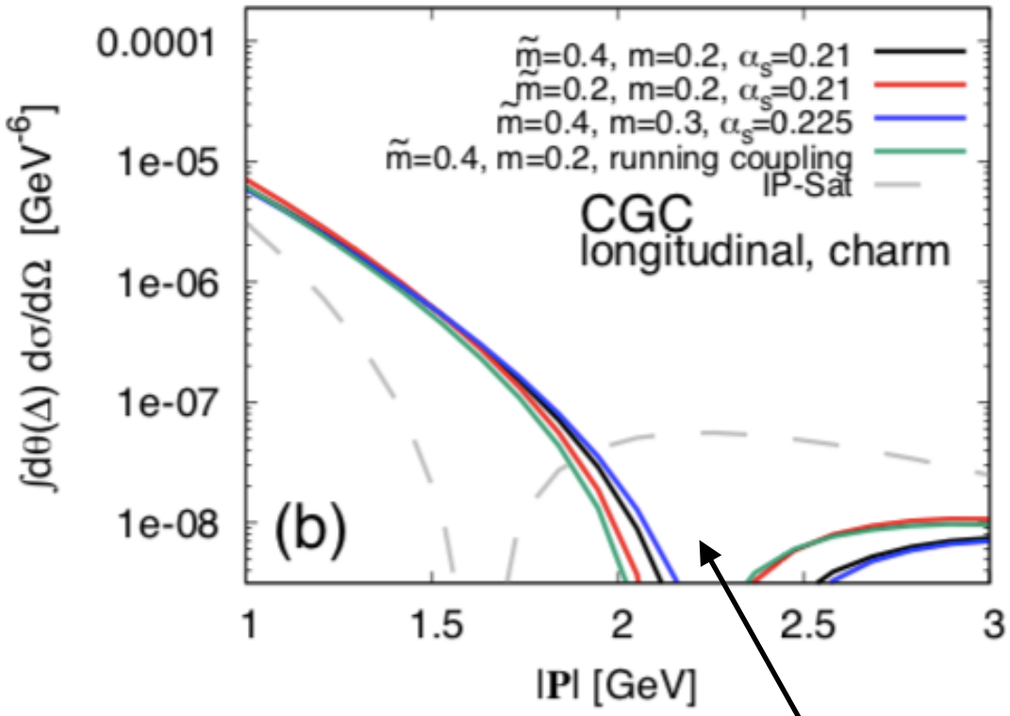
$\theta_{P\Delta}$ dependence (elliptic anisotropy)



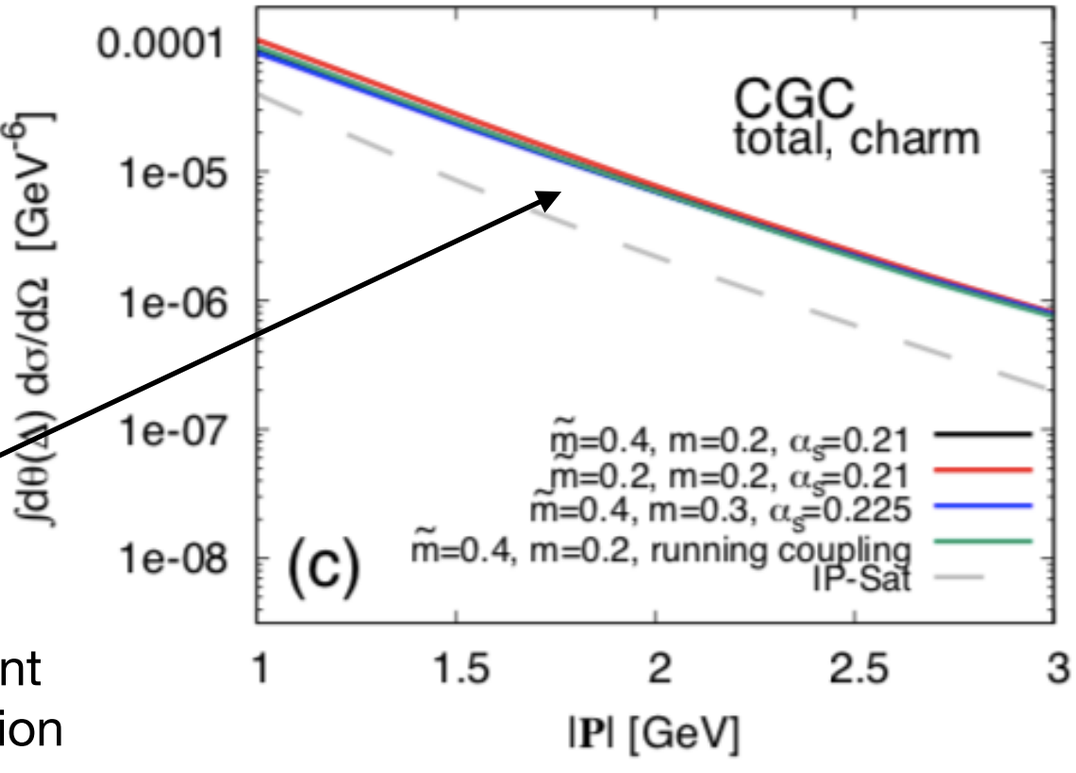

$$\frac{d\sigma}{d\Omega}(\mathbf{P},\mathbf{\Delta}) = \frac{d\sigma_0}{d\Omega}(P,\Delta) + 2\frac{d\sigma_2}{d\Omega}(P,\Delta)\cos 2\theta_{P\Delta} + \ldots$$

# Numerical results: Differential Cross section (Proton)



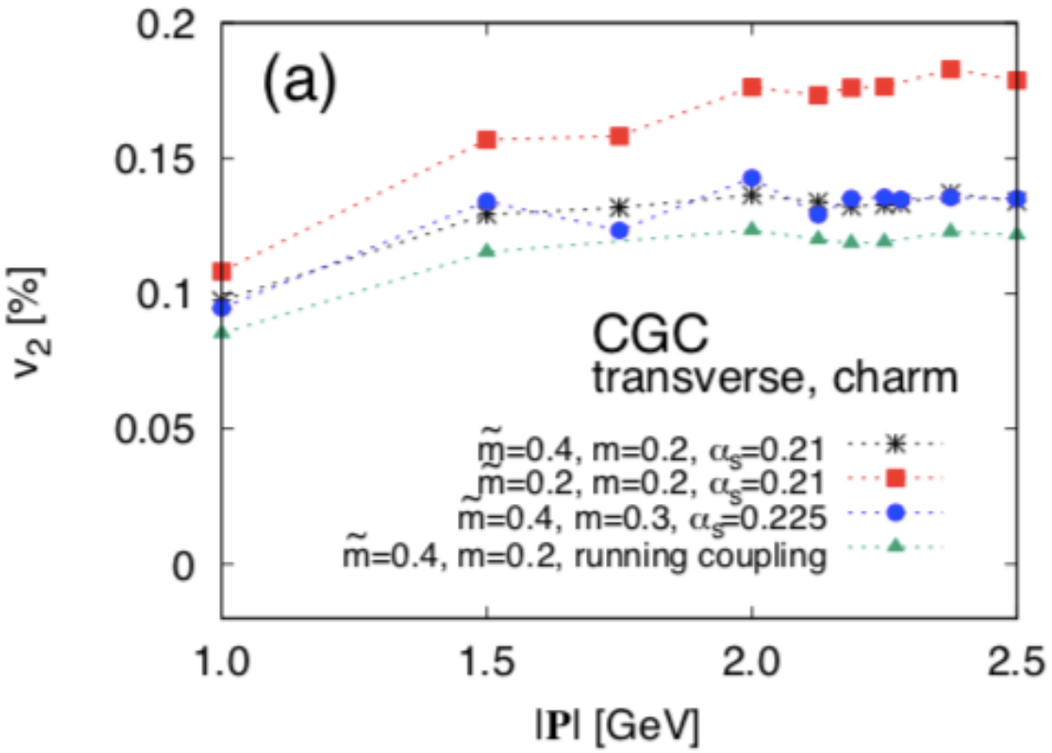

dip in P arising from projectile wave function and dipole amplitude r dependence

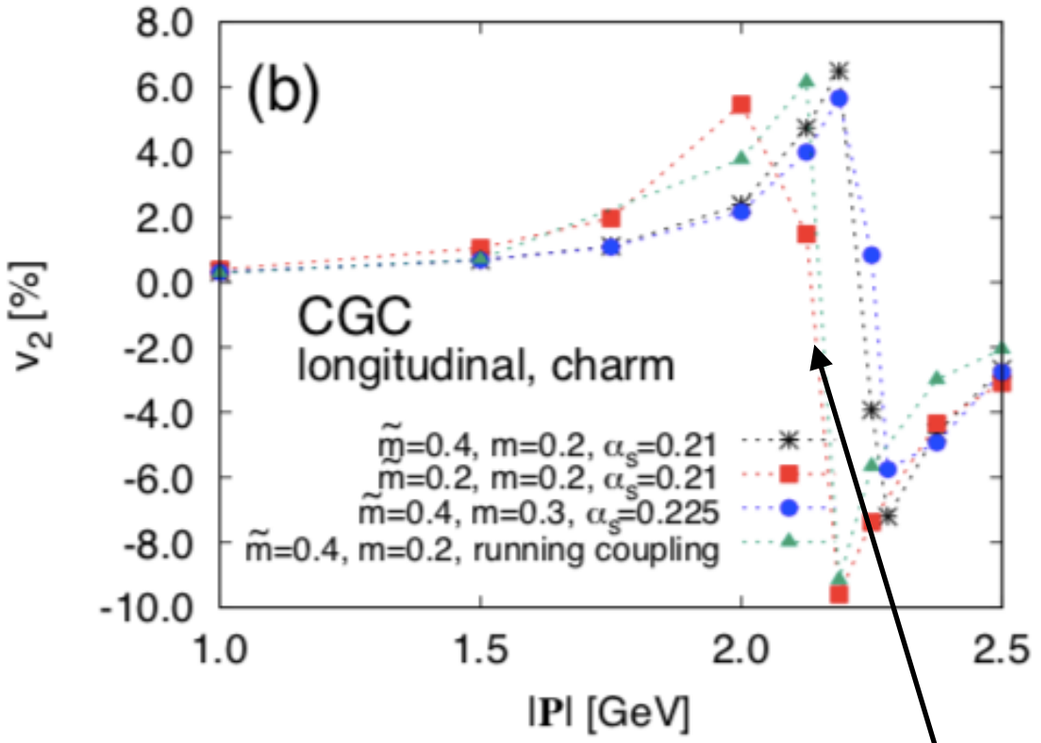



no dip in P transverse component dominates cross section

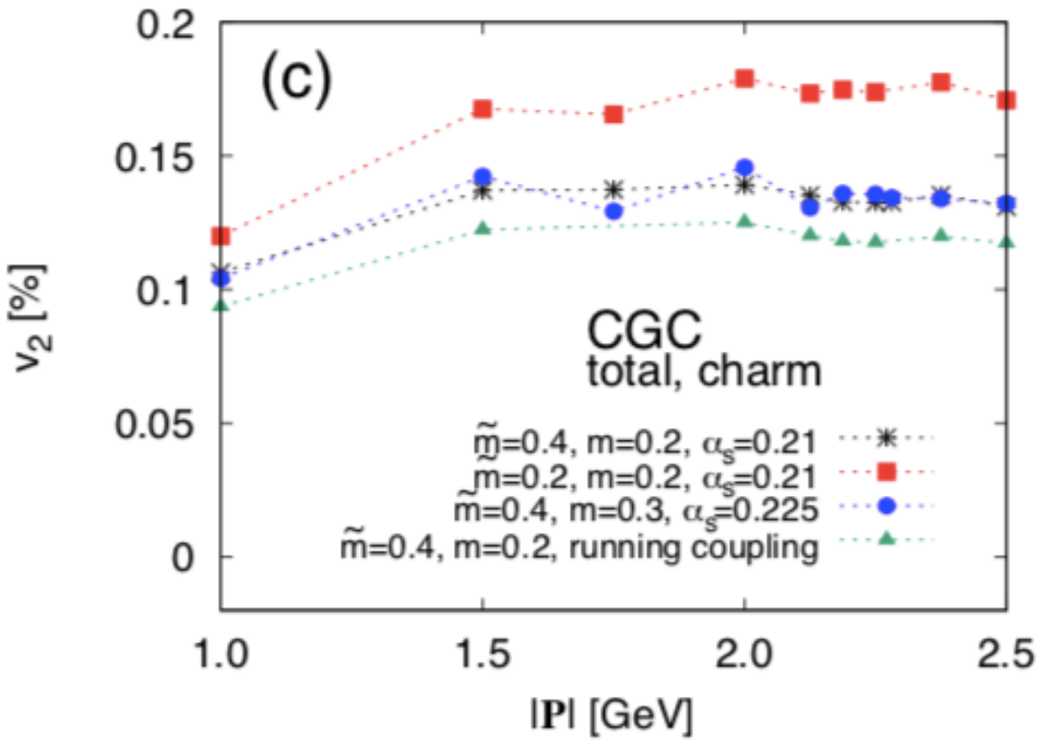
**Diffractive dijet production and Wigner distributions from the color glass condensate Mantysaari, Mueller, Schenke. arxiv: 1902.05087**

# Numerical results: Relative Elliptic Anisotropy (Proton)




change in sign of elliptic anisotropy at the location of dip




**Diffractive dijet production and Wigner distributions from the color glass condensate Mantysaari, Mueller, Schenke. arxiv: 1902.05087**

# Semi-analytic approach to coherent diffractive dijet

**Key ingredients for semi-analytic approach**

Classical Computation of Elliptic Flow at Large Transverse Momentum.
Teaney, Venugopalan. Phys.Lett.B539:53-58 (2002)

Elliptic flow from color-dipole orientation in pp and pA collisions
Iancu, Rezaeian. PhysRevD.95.094003 (2017)

- Dipole amplitude for extended MV with impact parameter:

$$\left\langle \rho^a(x^-,\mathbf{x})\rho^b(y^-,\mathbf{y})\right\rangle = g^2\mu^2\delta^{(2)}(\mathbf{x}-\mathbf{y})\,\delta(x^- - y^-)T(\mathbf{x})$$

$$D(\mathbf{r},\mathbf{b}) = 1 - e^{-(\mathcal{N}_0(r,b)+\mathcal{N}_2(r,b)\cos 2\theta_{rb})}$$

$$D_0(r,b) = 1 - e^{-\mathcal{N}_0(r,b)} I_0(\mathcal{N}_2(r,b))$$

$$D_2(r,b) = e^{-\mathcal{N}_0(r,b)} I_1(\mathcal{N}_2(r,b))$$

$$\mathcal{N}_0(r,b) = \frac{1}{4}Q_s^2 r^2 \log\left(\frac{1}{m^2r^2}+e\right)T(b)+\ldots \qquad \mathcal{N}_2(r,b) = \frac{1}{4}Q_s^2 r^2 \frac{1}{6m^2}\left[\frac{d^2}{db^2}-\frac{1}{b}\frac{1}{db}\right]T(b)+\ldots$$

- One-to-one relation between harmonics of amplitude and harmonics of Fourier transform of amplitude.

$$\mathscr{M}(\mathbf{r},\mathbf{b}) = \mathscr{M}_0(r,b) + 2\mathscr{M}_2(r,b)\cos 2\theta_{rb}+\ldots$$

$$\tilde{\mathscr{M}}(\mathbf{P},\mathbf{\Delta}) = \tilde{\mathscr{M}}_0(P,\Delta) + 2\tilde{\mathscr{M}}_2(P,\Delta)\cos 2\theta_{P\Delta}+\ldots$$

$$\tilde{\mathscr{M}}_k(P,\Delta) = \int rdrbdbJ_k(Pr)J_k(\Delta b)\mathscr{M}_k(r,b)$$

# Numerics vs Semi-analytics

$$\int d^2\mathbf{r}\; d^2\mathbf{b}\; e^{-i\mathbf{P}\cdot\mathbf{r}} e^{-i\mathbf{\Delta}\cdot\mathbf{b}}\; \Psi(\mathbf{r}) D(\mathbf{r},\mathbf{b})$$

MonteCarlo integration

Sample $\rho$ configurations.
Construct Wilson lines, then dipole.
JIMWLK evolution.
Average over configurations.

$$\left\langle \rho^a(x^-,\mathbf{x})\rho^b(y^-,\mathbf{y})\right\rangle = g^2\mu^2\delta^{(2)}(\mathbf{x}-\mathbf{y})\,\delta(x^- - y^-)T(\mathbf{x})$$

## Limitations

- From the numerical approach it is difficult to understand analytic properties of cross section and elliptic anisotropy.
- Furthermore, due to oscillating behavior in Fourier transform computation expensive for large $\mathbf{P}$ or $\mathbf{\Delta}$.

$$\tilde{F}_0(P,\Delta) = \int r dr\; b db\; J_0(Pr)J_0(\Delta b)K_0\left(\varepsilon_f r\right) D_0(r,b)$$

$$\tilde{F}_2(P,\Delta) = \int r dr\; b db\; J_2(Pr)J_2(\Delta b)K_0\left(\varepsilon_f r\right) D_2(r,b)$$

$$\tilde{G}_0(P,\Delta) = -\int r dr b db J_1(Pr)J_0(\Delta b)K_1(\varepsilon_f r)D_0(r,b)$$

$$\tilde{G}_2(P,\Delta) = -\int r dr b db \left[\frac{J_3(Pr)-J_1(Pr)}{2}\right] J_2(\Delta b)K_1(\varepsilon_f r)D_2(r,b)$$

$$\frac{d\sigma_{L,0}}{d\Omega} = \frac{8N_c\alpha_{EM}}{(2\pi)^6} z_0^3 z_1^3 Q^2 Z_f^2 \left|\tilde{F}_0\right|^2$$

$$\frac{d\sigma_{L,2}}{d\Omega} = \frac{8N_c\alpha_{EM}}{(2\pi)^6} z_0^3 z_1^3 Q^2 Z_f^2 \Re\left(2\tilde{F}_0\tilde{F}_2^*\right)$$

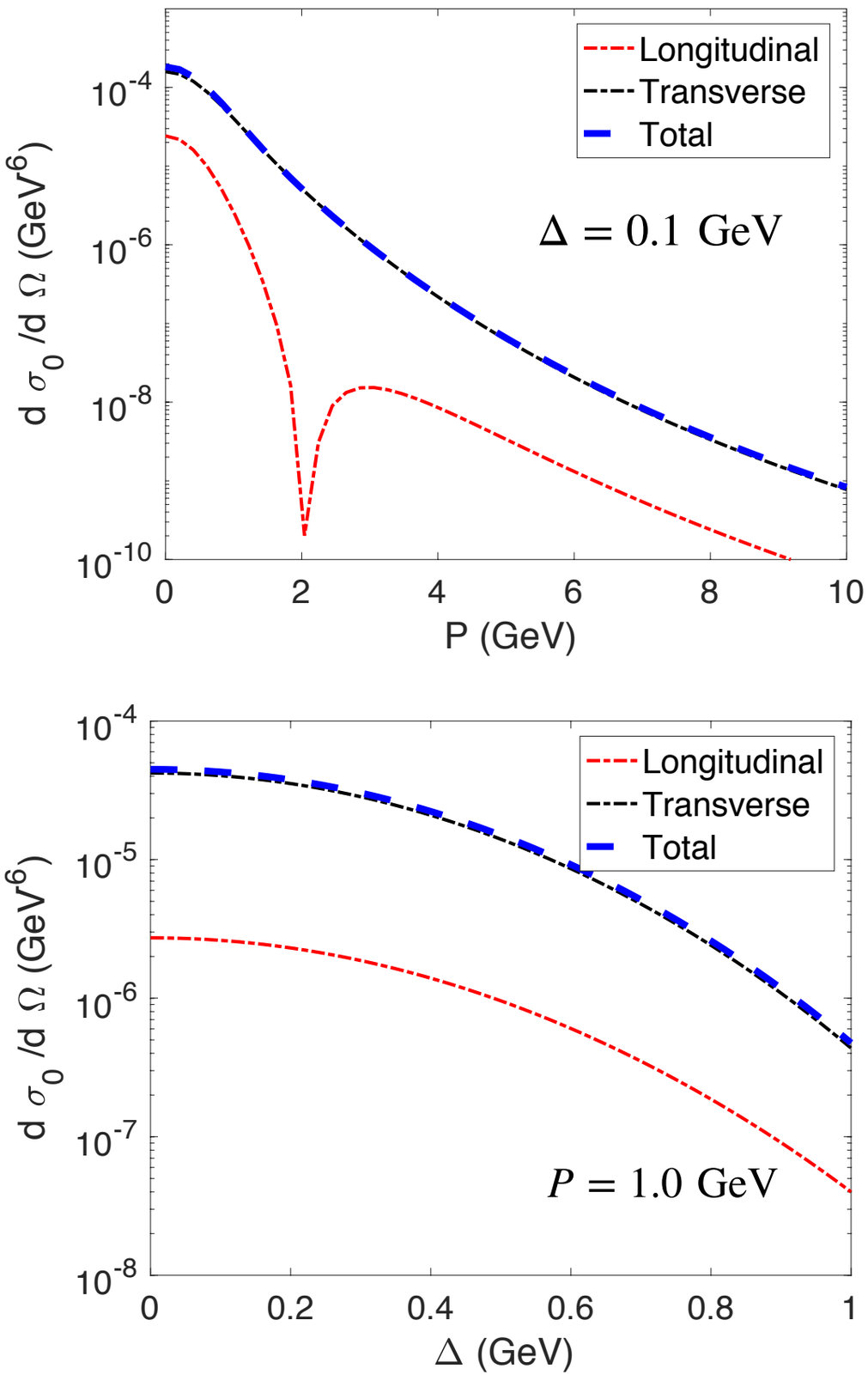
$$\frac{d\sigma_{T,0}}{d\Omega} = \frac{2N_c\alpha_{EM}}{(2\pi)^6} Z_f^2 z_0 z_1 \left\{\varepsilon_f^2(z_0^2+z_1^2)\left|\tilde{G}_0\right|^2 + m_f^2\left|\tilde{F}_0\right|^2\right\}$$

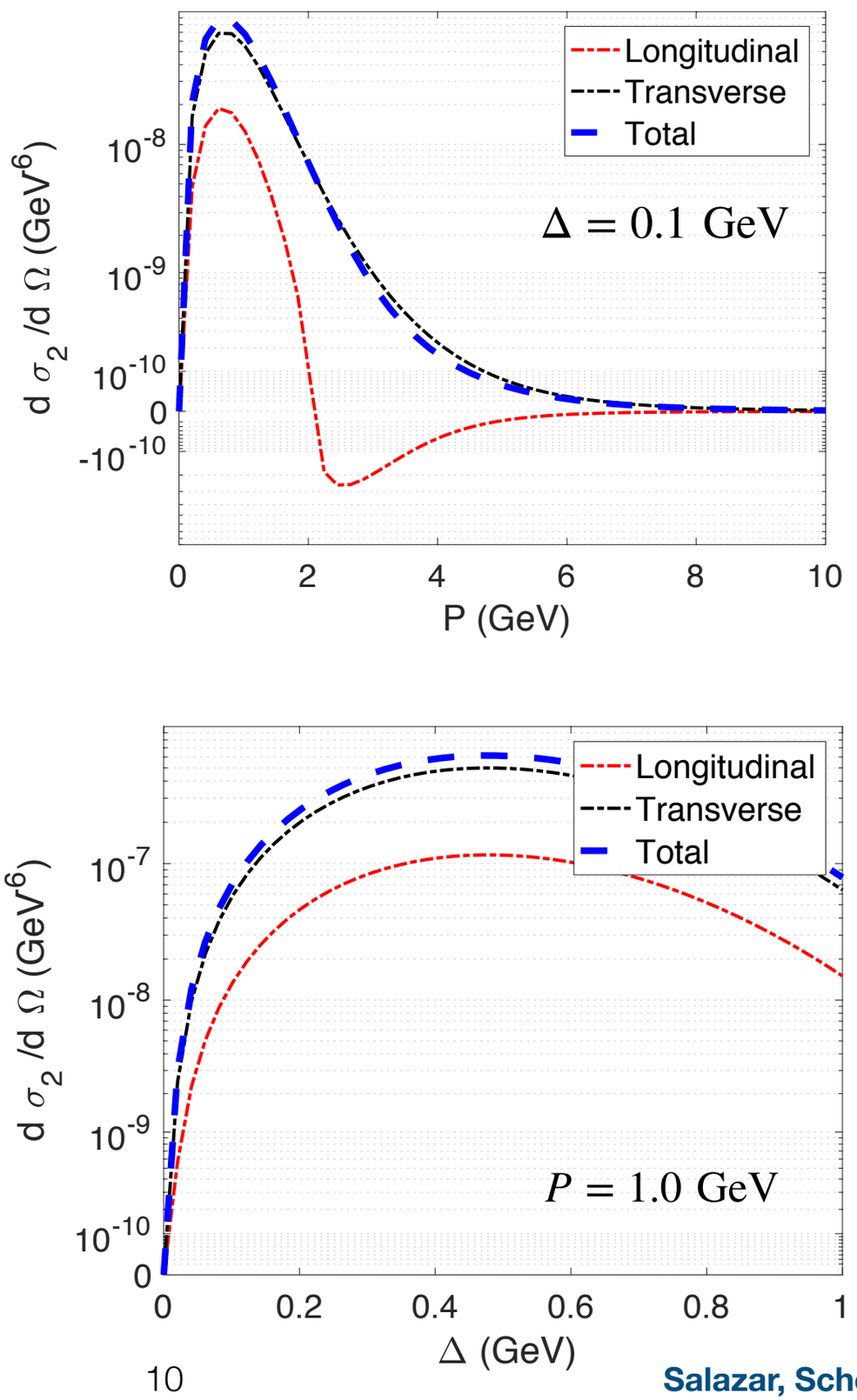
$$\frac{d\sigma_{T,2}}{d\Omega} = \frac{2N_c\alpha_{EM}}{(2\pi)^6} Z_f^2 z_0 z_1 \left\{\varepsilon_f^2(z_0^2+z_1^2)\Re\left(2\tilde{G}_0\tilde{G}_2^*\right) + m_f^2\Re\left(2\tilde{F}_0\tilde{F}_2^*\right)\right\}$$

## Limitations

- Only includes Gaussian correlations of sources.
- Does not account for growth of the proton, and possibly change of gradients.

# Semi-analytic results: Cross section and Elliptic (Proton)




$Q_s = 0.7$ GeV

$m = 0.4$ GeV

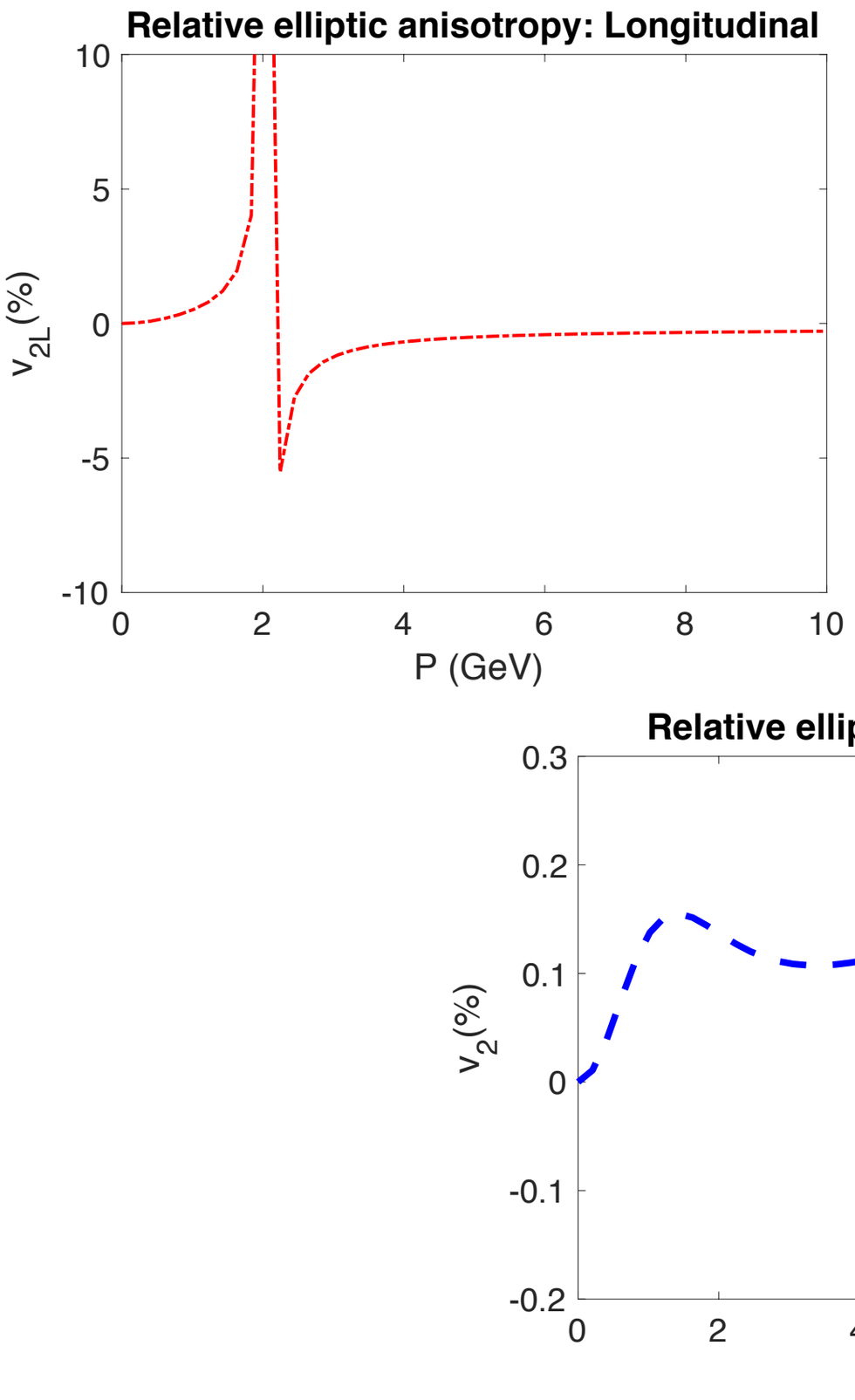
$R = 2.$ GeV$^{-1}$

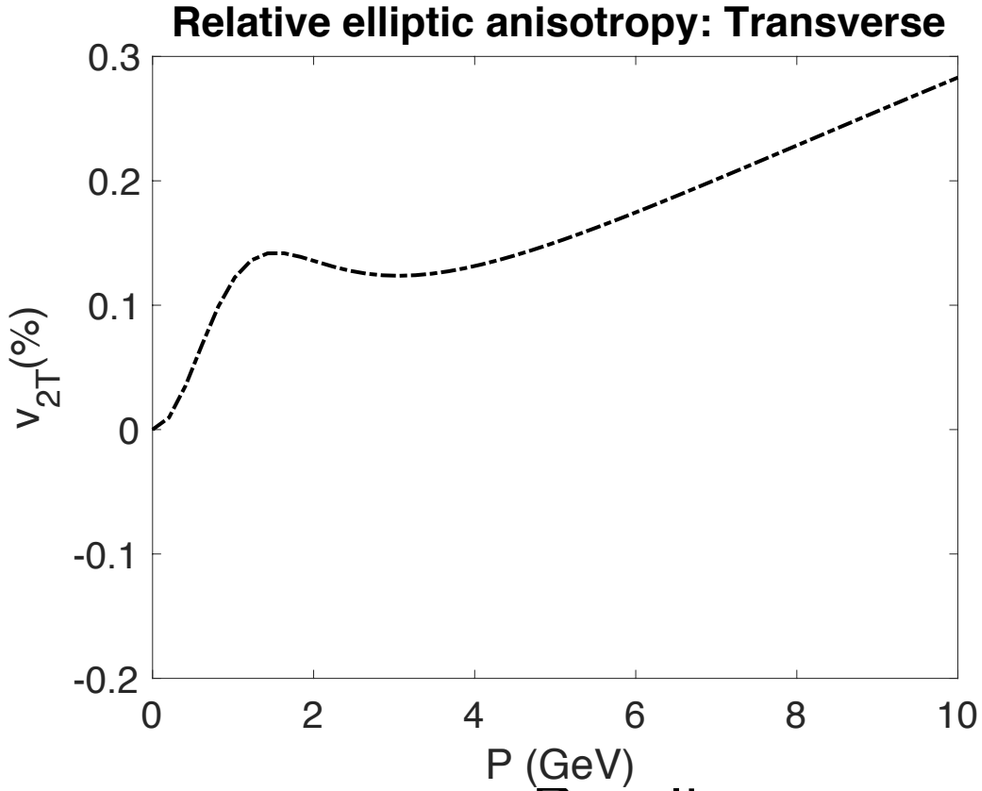
$Q^2 = 1.0$ GeV

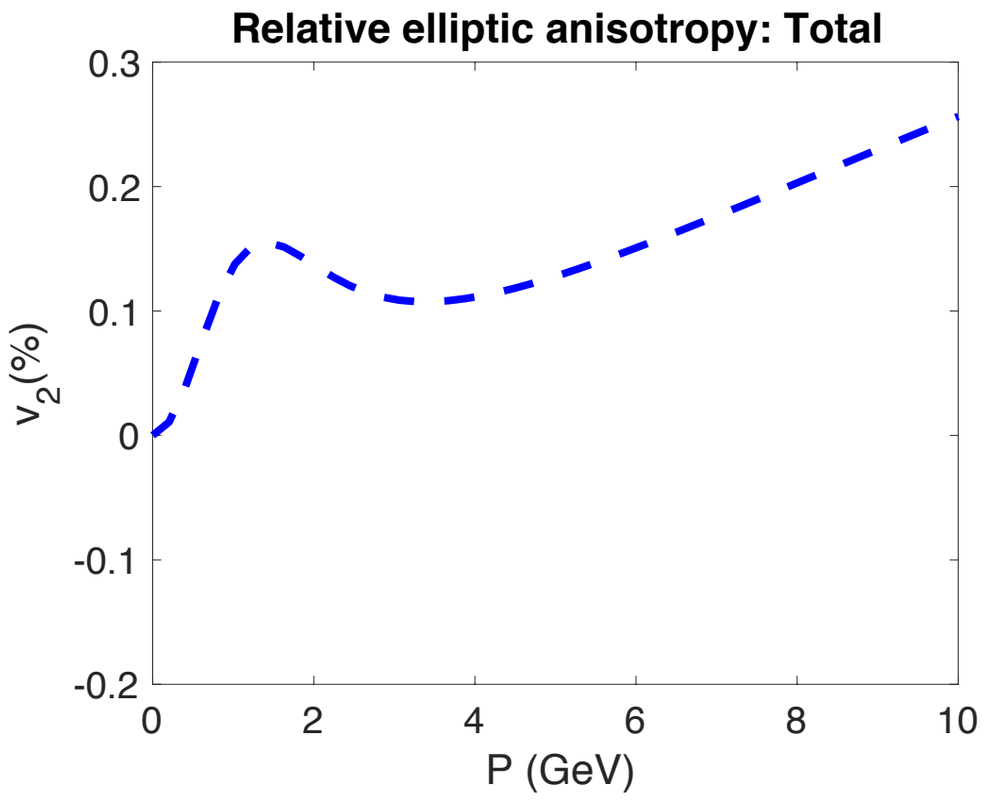


Salazar, Schenke (under preparation)

# Semi-analytic results: Relative elliptic anisotropy (Proton)




Results are consistent with fully numerical results.

Magnitude of differential cross section, $v_2$ diffractive dip, etc.

Except for maximum observed at $P = 2$ GeV

**Salazar, Schenke (under preparation)**

# Qualitative analysis

For large virtuality/massive quarks $\varepsilon_f \gtrsim Q_s^2$ , one can obtain analytic expressions*.
Example: longitudinal polarization (similar analysis for transverse):

$$\frac{d\sigma_{L,0}(P,\Delta)}{d\Omega} = \frac{8N_c\alpha_{EM}}{(2\pi)^4} Z_f^2 Q^2 z_0^3 z_1^3 Q_s^4 \frac{\left(P^2-\varepsilon_f^2\right)^2}{\left(P^2+\varepsilon_f^2\right)^6} |\tilde{T}(\Delta)|^2$$

dip at $P = \varepsilon_f$

change in sign in $v_2$

$$\frac{d\sigma_{L,2}}{d\Omega} = \frac{8N_c\alpha_{EM}}{(2\pi)^4} z_0^3 z_1^3 Q^2 Z_f^2 Q_s^4 \frac{(\varepsilon_f^2-P^2)P^2}{(P^2+\varepsilon_f^2)^6} \frac{\Delta^2 |\tilde{T}(\Delta)|^2}{3m^2}$$

For $\varepsilon_f \approx Q_s^2$ , saturation shifts the dip in $P$ will shift to larger momentum, the scaling of cross section with $Q_s$ saturates ,among other effects.
The $P$ and $\Delta$ dependence does not factorize.

* We ignored the log in $\mathcal{N}_0$ ,adding the log increases the cross section, and shifts the dip to larger $P$ and softens the power law in $P$ .

# Conclusions/extensions

- Semi-analytic expression gives some insight into some properties of cross section and elliptic anisotropy.

- Semi-analytic expression is faster to compute than purely numerical approach. It seems to give analytic behavior and magnitude compared full numerics.

- Extend results to study semi-analytically elliptic anisotropy and cross section in inclusive dijet. Need a formula that incorporates geometry for quadrupole (Gaussian approximation?).

- Extend results to incoherent diffractive dijet. This will be sensitive to local fluctuations and gradients.

# Thanks for your attention