Spin Polarization and Chiral Condensation in 2+1 flavor Nambu-Jona-Lasinio model at finite temperature and baryon chemical potential

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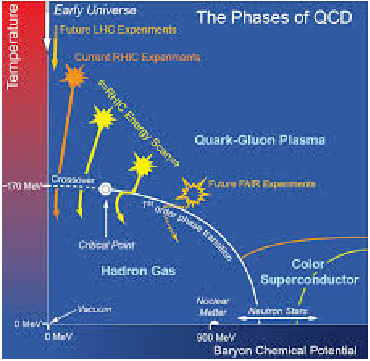
QCD phase diagram

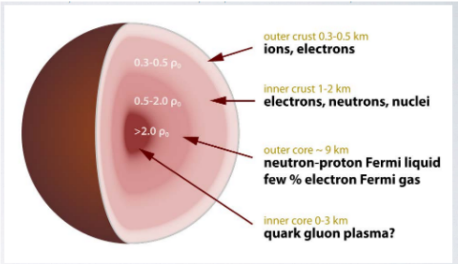
- Theory of strong interactions: Quantum Chromodynamics(QCD).
- Important properties: Asymptotic freedom and Color confinement.



- At high temperature and/ densities the hadronic matter should under go transition to a state of free quarks and gluons (QGP) [1].

[1] E.V.Shuryak, Phys. Lett. B 78 (1978) 150

Magnetized Neutron stars



- Neutron stars can be strongly magnetized.
- Magnetic field strength at the surface:$\sim 10^{15} - 10^{16}$ Gauss.
- In the core the magnetic field can be even stronger.
- Common understanding: magnetic field originated from the progenitor star.
- Can quark ferromagnetic phase exists inside the compact objects? [2].

[2] T. Tatsumi, Phys.Lett. B489 (2000) 280-286

- A collective spin polarization of charged quarks can give rise to ferromagnetic nature of quark matter at high density.
- For moderate densities near chiral phase transition density where perturbative QCD is not applicable, one can use QCD inspired low energy effective models e.g. NJL model $\rightarrow$ non perturbative effects.
- "Spin density" can be expressed in different ways: (1) spatial component of the axial vector (AV) mean field, $\psi^\dagger\Sigma^i\psi \equiv -\bar{\psi}\gamma_5\gamma^i\psi$, (2) tensor Dirac bilinear (T) $\psi^\dagger\gamma^0\Sigma^i\psi \equiv -\bar{\psi}\sigma^{12}\psi$.
- Using AV mean field within the framework of NJL model it has been shown that for one flavor, spin polarization is possible at finite density. [3].
- Fierz transformation: one can get AV channel interaction between quarks from one gluon exchange interaction.
- Tensor channel interaction does not appear in the Fierz transformation of the one gluon exchange interaction.
- We discuss the interplay between the spin condensate ($\langle\bar{\psi}\Sigma^i\psi\rangle$) and the scalar chiral condensate $\langle\bar{\psi}\psi\rangle$ in (2+1) flavor NJL model using only tensor(T) type interaction.

[3] S. Maedan, Progress of Theoretical Physics, Vol. 118, No. 4, 2007

Formalism: NJL model

- $(2+1)$ flavor and $SU(3)$ color quarks we start with the following NJL Lagrangian density [4],

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - \hat{m}\right)\psi + \mathcal{L}_{sym} + \mathcal{L}_{det} + \mathcal{L}_{tensor} + \mu\bar{\psi}\gamma^0\psi, \tag{1}$$

where $\psi = (u, d, s)^T$ is the three flavor quark field and the diagonal current quark matrix is $\hat{m} = \mathrm{diag}_f(m_u, m_d, m_s)$.

- In this work we have assumed that due to isospin symmetry in the non strange quark sector $m_u = m_d$.
- Chiral symmetry invariant interaction term,

$$\mathcal{L}_{sym} = g\sum_{a=0}^{a=8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2\right]. \tag{2}$$

- $U(1)$ axial symmetry breaking interaction term,

$$\mathcal{L}_{det} = -K\mathrm{det}_f[\bar{\psi}(1+\gamma_5)\psi + h.c] \tag{3}$$

[4] H. Bohr et. al., Int. J. Mod. Phys. E 22, No.4, 1350019 (2013)

- The tensor interaction:

$$\mathcal{L}_{tensor} = \frac{G_T}{2} \sum_{a=3,8} \left(\bar{\psi}\Sigma_z\lambda_a\psi\right)^2, \qquad \Sigma_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}, \tag{4}$$

  where σ_z is the third Pauli matrix.
- Introducing mean fields, $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \sigma_{ud}$, $\langle \bar{s}s \rangle \equiv \sigma_s$, $F_3 = \langle \bar{\psi}\Sigma_z\lambda_3\psi \rangle$ and $F_8 = \langle \bar{\psi}\Sigma_z\lambda_8\psi \rangle$.
- In mean field approximation,

$$\begin{aligned} \mathcal{L} =& \bar{\psi}\left(i\partial\!\!\!/ - \hat{M} + G_T F_3 \Sigma_z \lambda_3 + G_T F_8 \Sigma_z \lambda_8 + \mu\gamma^0\right)\psi - 2g\left(\sigma_{ud}^2 + \sigma_{ud}^2 + \sigma_s^2\right) \\ &+ 4K\sigma_{ud}^2\sigma_s - \frac{G_T}{2}F_3^2 - \frac{G_T}{2}F_8^2, \end{aligned} \tag{5}$$

  where, $\hat{M} \equiv \mathrm{diag}(M_u, M_d, M_s)$, with effective masses,

$$\begin{aligned} M_u =& m_u - 4g\sigma_{ud} + 2K\sigma_{ud}\sigma_s \\ M_d =& m_d - 4g\sigma_{ud} + 2K\sigma_{ud}\sigma_s \\ M_s =& m_s - 4g\sigma_s + 2K\sigma_{ud}^2. \end{aligned} \tag{6}$$

- The thermodynamic potential :

$$\Omega(T,\mu,\sigma_{ud},\sigma_s,F_3,F_8) = -\frac{6}{4\pi^2}\sum_{f=u,d,s}\int dp_T \int p_T dp_z$$
$$\Bigg[\left(E_{f+}+E_{f-}\right) + T\ln\left(1+e^{-\beta(E_{f+}-\mu)}\right) + T\ln\left(1+e^{-\beta(E_{f+}+\mu)}\right)$$
$$+ T\ln\left(1+e^{-\beta(E_{f-}-\mu)}\right) + T\ln\left(1+e^{-\beta(E_{f-}+\mu)}\right)\Bigg]$$
$$+2g(\sigma_{ud}^2+\sigma_{ud}^2+\sigma_s^2) - 4K\sigma_{ud}^2\sigma_s + \frac{G_T}{2}F_3^2 + \frac{G_T}{2}F_8^2 \tag{7}$$

- The single particle energies are,

$$E_{u\pm} = \sqrt{p_z^2 + \left(\sqrt{p_T^2+M_u^2} \pm G_T\left(F_3+\frac{F_8}{\sqrt{3}}\right)\right)^2}$$
$$E_{d\pm} = \sqrt{p_z^2 + \left(\sqrt{p_T^2+M_d^2} \pm G_T\left(F_3-\frac{F_8}{\sqrt{3}}\right)\right)^2}$$
$$E_{s\pm} = \sqrt{p_z^2 + \left(\sqrt{p_T^2+M_s^2} \pm G_T\frac{2F_8}{\sqrt{3}}\right)^2}$$

- Thermodynamic behaviour of the condensates can be found by solving the gap equations, which can be found from the stationary conditions.

$$\frac{\partial\Omega}{\partial\sigma_{ud}} = \frac{\partial\Omega}{\partial\sigma_s} = \frac{\partial\Omega}{\partial F_3} = \frac{\partial\Omega}{\partial F_8} = 0 \tag{9}$$

- Gap equations can have several roots, but the solution with the lowest value of thermodynamic potential is taken as the stable solution.
- NJL model Lagrangian in (3+1) dimension has operators which have mass dimension more than four, thus it can shown to be a non-renormalizable theory.
- Thus the divergence coming from the three momentum integral of the vacuum part can not be removed by the renormalization prescriptions.
- The model predictions inevitably depend on the regularization procedures.
- In this work we have considered the most frequently used 3D momentum cutoff regulation scheme to regularize the divergence in thermodynamic potential.

Parameter Set [5]	
Parameters and couplings	**Value**
Three momentum cutoff (Λ)	$\Lambda = 602.3 \times 10^{-3}$ (GeV)
u quark mass (m_u)	$m_u = 5.5 \times 10^{-3}$ (GeV)
d quark mass (m_d)	$m_d = 5.5 \times 10^{-3}$ (GeV)
s quark mass (m_s)	$m_s = 140.7 \times 10^{-3}$ (GeV)
Scalar coupling (g)	$g = 1.835/\Lambda^2$
Determinant interaction (K)	$K = 12.36/\Lambda^5$

- The parameter which plays the crucial role is the tensor channel interaction G_T.
- Fierz transformation from scalar and pseudo scalar interaction,

$$g\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2\right] = \frac{g}{4}\left[(\bar{\psi}\psi)^2 - \frac{1}{2}(\bar{\psi}\gamma^\mu\gamma^\nu\lambda_i\psi)^2 + ...\right], \tag{10}$$

which gives $|g/G_T| = 2$.

- In the present investigation we can take G_T as a free parameter.

[5] M. Buballa, Phys.Rept. **407**, 205-376 (2005).

Zero temperature results:

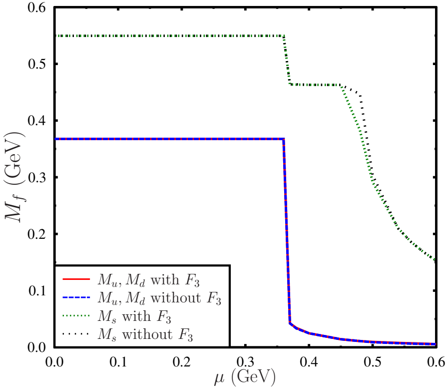
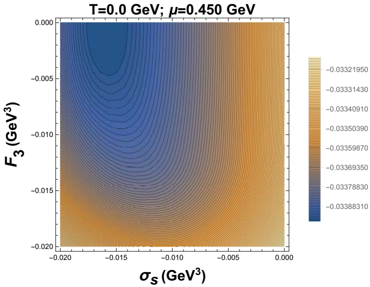
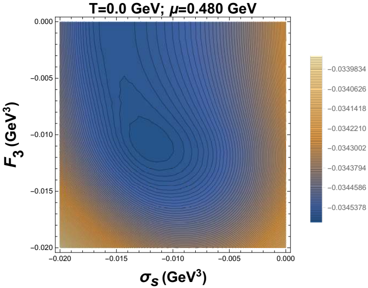
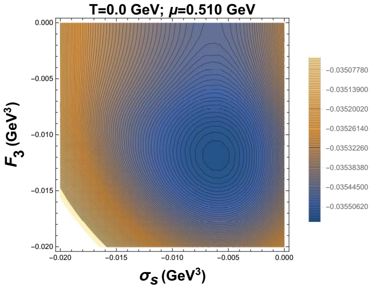
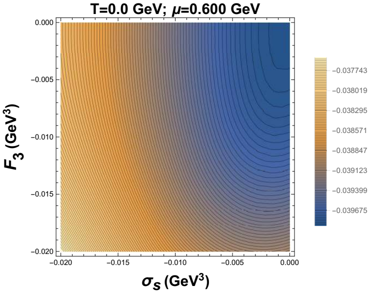


Figure: Constituent quark mass as a function of quark chemical potential at zero temperature in the presence and absence of spin polarization condensation, with $F_8 = F_3/\sqrt{3}$ and $G_T = 2g$.


T=0.0 GeV; μ=0.450 GeV
F_3 (GeV^3)
σ_s (GeV^3)
−0.03321950
−0.03331430
−0.03340910
−0.03350390
−0.03359870
−0.03369350
−0.03378830
−0.03388310


T=0.0 GeV; μ=0.480 GeV
F_3 (GeV^3)
σ_s (GeV^3)
−0.0339834
−0.0340626
−0.0341418
−0.0342210
−0.0343002
−0.0343794
−0.0344586
−0.0345378


T=0.0 GeV; μ=0.510 GeV
F_3 (GeV^3)
σ_s (GeV^3)
−0.03507780
−0.03513900
−0.03520020
−0.03526140
−0.03532260
−0.03538380
−0.03544500
−0.03550620


T=0.0 GeV; μ=0.600 GeV
F_3 (GeV^3)
σ_s (GeV^3)
−0.037743
−0.038019
−0.038295
−0.038571
−0.038847
−0.039123
−0.039399
−0.039675

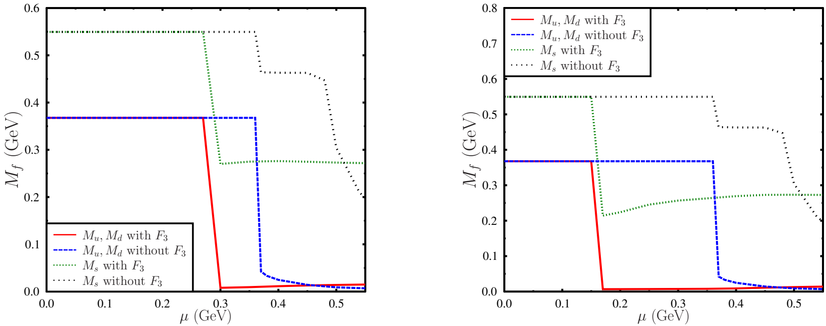
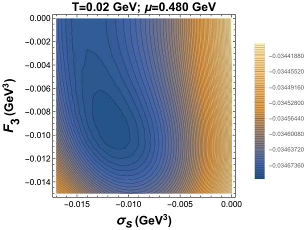
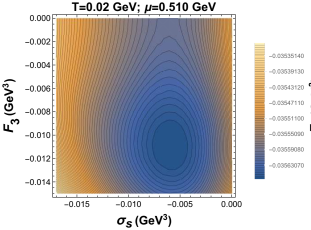
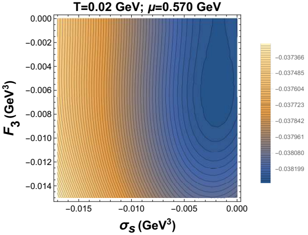


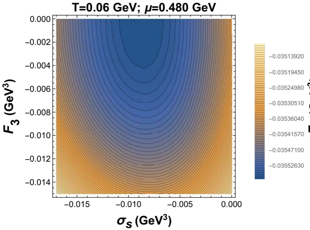
Figure: Dependence of constituent quark mass on the quark chemical potential at zero temperature in the presence as well as in the absence of spin polarization condensation for different values of tensor couplings $G_T = 4g$ (left plot) and $G_T = 4.3g$ (right plot) for $F_8 = F_3/\sqrt{3}$.

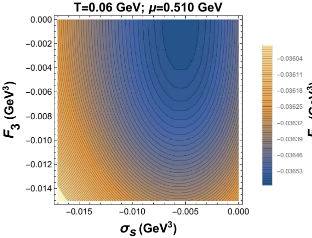
Finite temperature results

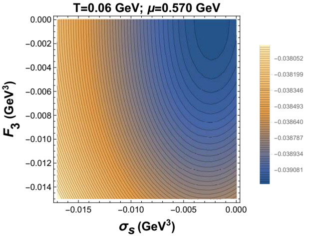




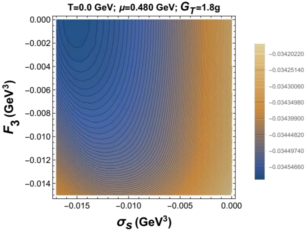


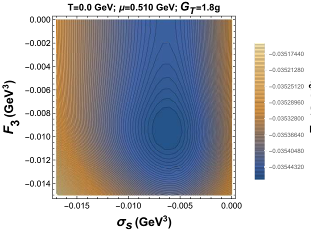


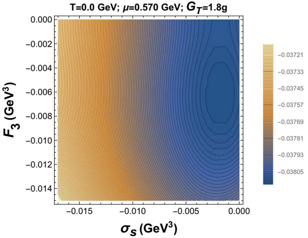


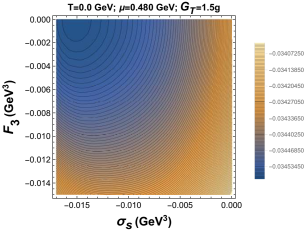


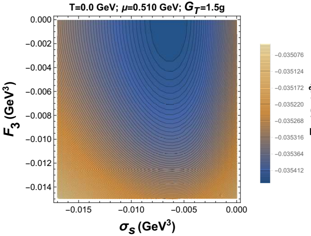
Threshold tensor coupling

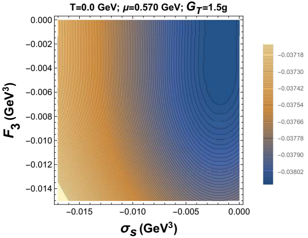




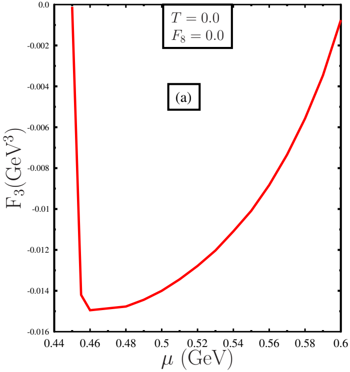








Behaviour of F_3 and F_8 independently



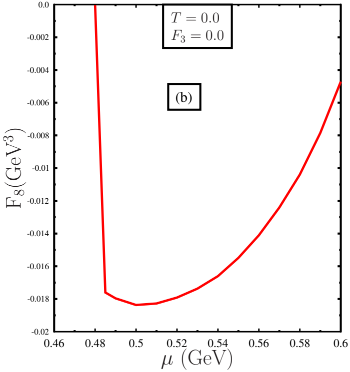
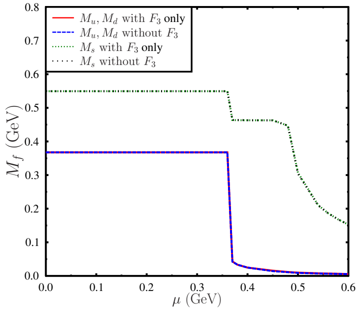


Figure: Variation of F_3 and F_8 with temperature, where F_3 and F_8 are considered independently, for $G_T = 2g$.

Effect of F_3 and F_8 on M_f



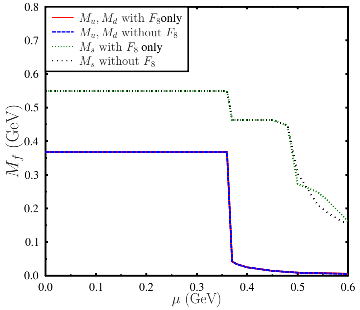


Figure: Dependence of constituent quark masses as a function of the quark chemical potential at zero temperature, when F_3 and F_8 considered independently, for $G_T = 2g$.

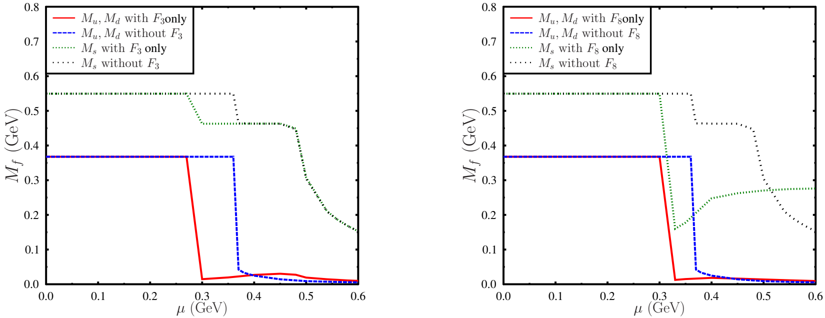


Figure: Dependence of constituent quark masses as a function of the quark chemical potential at zero temperature, when F_3 and F_8 considered independently. Left plot is for $G_T = 3.5g$ and the right plot is for $G_T = 4g$.

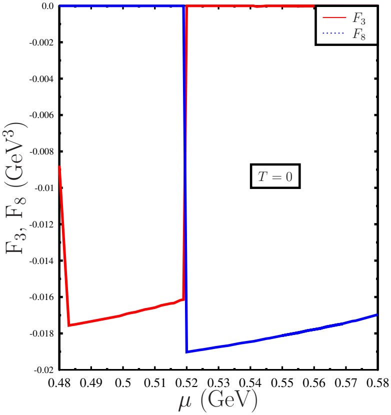


Figure: Variation of F_3 and F_8 with chemical potential where F_3 and F_8 considered simultaneously in the thermodynamic potential at zero temperature for $G_T = 2g$.

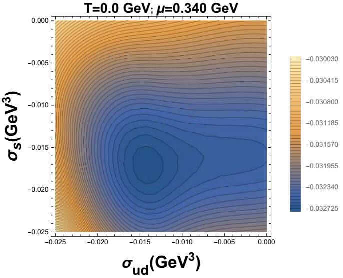
Conclusions

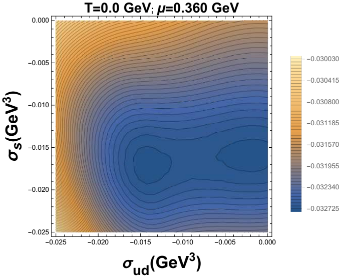
- Tensor condensate F_3 and F_8 exist in the chiral symmetric phase.
- With increasing temperature and chemical potential scalar condensate and tensor condensate both vanishes.
- Tensor condensate affects the single particle energies as well as the chiral symmetry restoration.
- Tensor condensate acts as the catalyst of chiral restoration.
- At zero temperature the spin polarization transition is first order in nature.

Thank You

Magnetic field in magnetars

- Scale: 1 GeV^2=10^{20} Gauss.
- Quantum critical magnetic field: When the rest mass is of same order as the cyclotron frequency. $B_{cr} = \frac{m^2c^3}{e\hbar}$, For electrons $B_{cr} = 10^{13}$ Gauss and for protons $B_{cr} = 10^{20}$ Gauss.
- For standard neutron stars: B= 10^{12} Gauss.
- For magnetars: B= 10^{15} Gauss.
- Core collapse supernova: magnetic flux conservation, $B^{(0)} = (\frac{R_{core}}{R_{NS}})^2 B_{*SN}$
- Supernova core $R_{core} \sim 10^6$ Km.
- Neutron star radius $R_{NS} \sim 10$ Km.
- Seed magnetic field inside the Supernova $B_{*SN} \sim 10^3$ Gauss.


T=0.0 GeV; μ=0.340 GeV
σ_s(GeV^3)
σ_{ud}(GeV^3)
−0.030030
−0.030415
−0.030800
−0.031185
−0.031570
−0.031955
−0.032340
−0.032725


T=0.0 GeV; μ=0.360 GeV
σ_s(GeV^3)
σ_{ud}(GeV^3)
−0.030030
−0.030415
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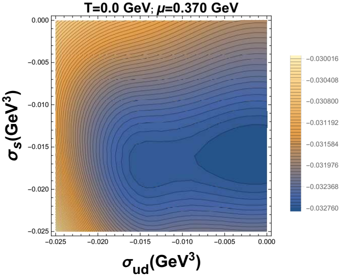
−0.031570
−0.031955
−0.032340
−0.032725


T=0.0 GeV; μ=0.370 GeV
σ_s(GeV^3)
σ_{ud}(GeV^3)
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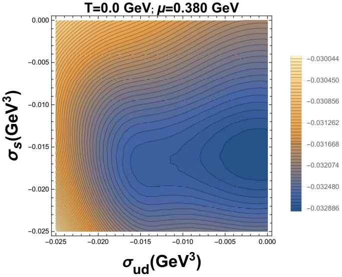
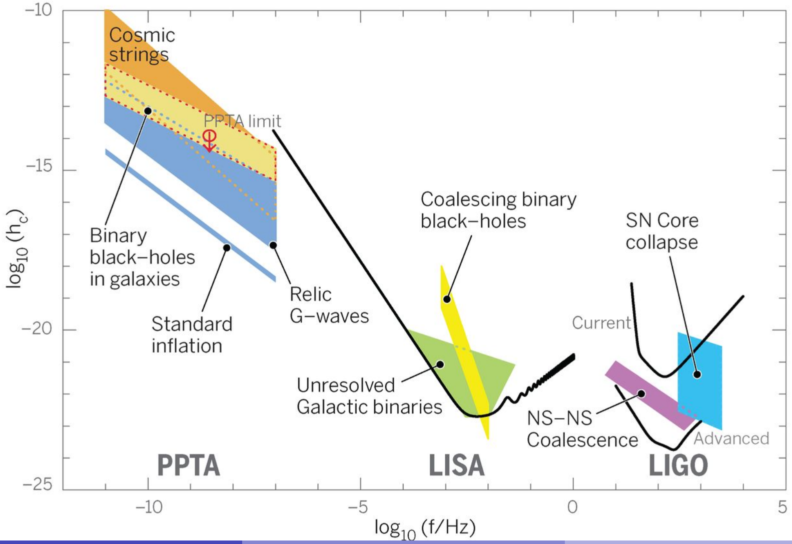
−0.030408
−0.030800
−0.031192
−0.031584
−0.031976
−0.032368
−0.032760


T=0.0 GeV; μ=0.380 GeV
σ_s(GeV^3)
σ_{ud}(GeV^3)
−0.030044
−0.030450
−0.030856
−0.031262
−0.031668
−0.032074
−0.032480
−0.032886

Techniques of gravitational radiation detection

Dimensionless strain (h_c) and wave frequency (f/Hz)



- Reduced magnetic field can be defined as [6]., $\overline{\mu_q} B_{red} = |G_T F|$
- $\overline{\mu_q} = \frac{\mu_u + \mu_d}{2}$, $\mu_u = \frac{2/3e}{2m_u}$, $\mu_d = \frac{-1/3e}{2m_d}$.
- $\overline{\mu_s} = \frac{-1/3e}{m_s s}$
- $B_{red} = \frac{|G_T F_8|}{\mu_s}$
- $\mu_s = \frac{-1/3e}{2m_s} \implies eB_{red} = 0.02 GeV^2 \sim 10^{18} Gauss.$

[6] Y. Tsue et.al., Progress of Theoretical Physics, 128, No.3, 2012