Color screening in 2+1 flavor QCD, testing EQCD and spectral functions

Free energy of static quark anti-quark at $T>0$ and deconfinement transition in 2+1 flavor QCD

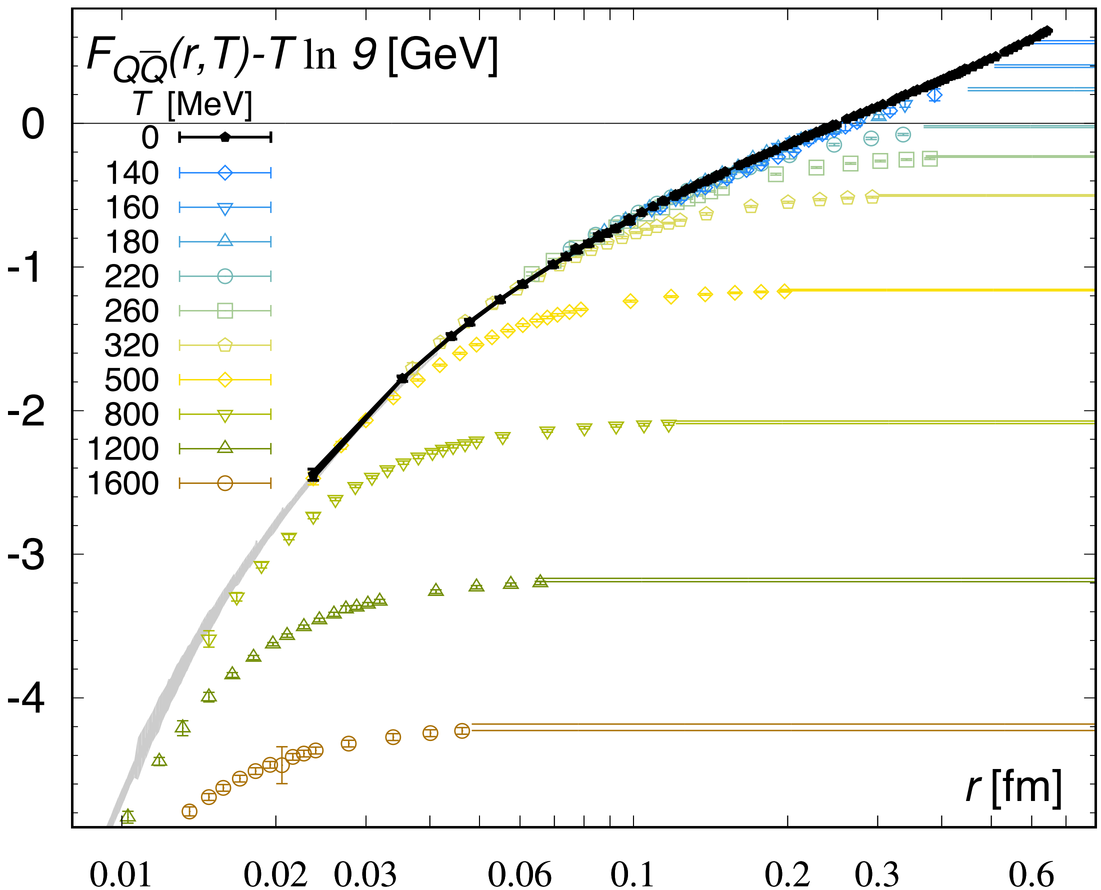
Onset of color screening and EQCD

Spatial string tension and further tests of EQCD

Spectral functions and heavy quark diffusion constant

Deconfinement and color screening in QCD

2+1 flavor QCD, continuum extrapolated, TUMQCD, PRD 98 (2018) 054511

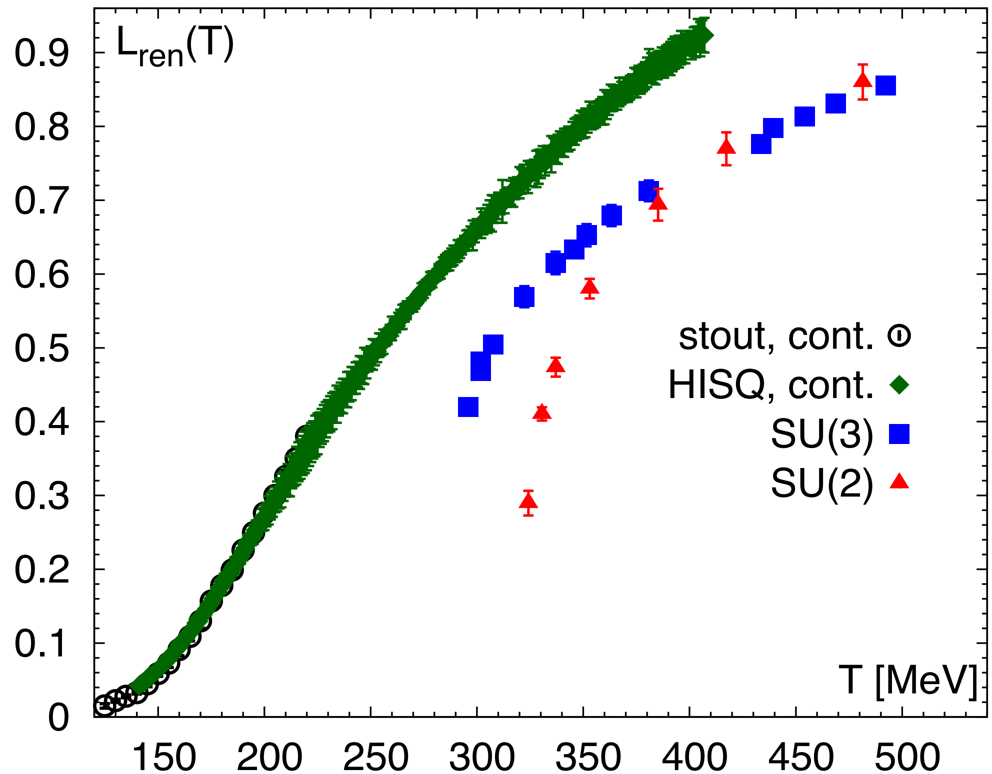


The free energy of static quark anti-quark pair agrees with the *T=0* potential for *r* << *1/T*

The free energy of static quark anti-quark pair is screened for $r>r_{scr}$ at any temperature

$r_{sc}r \sim 1/T$ => Debye screening

Deconfinement and color screening in QCD (cont'd)



Pure glue ≠ QCD !

Deconfinement transition happens at lower temperature but the Polyakov loop behaves smoothlyaround T_c , *Z(3) symmetry plays no apparent role*

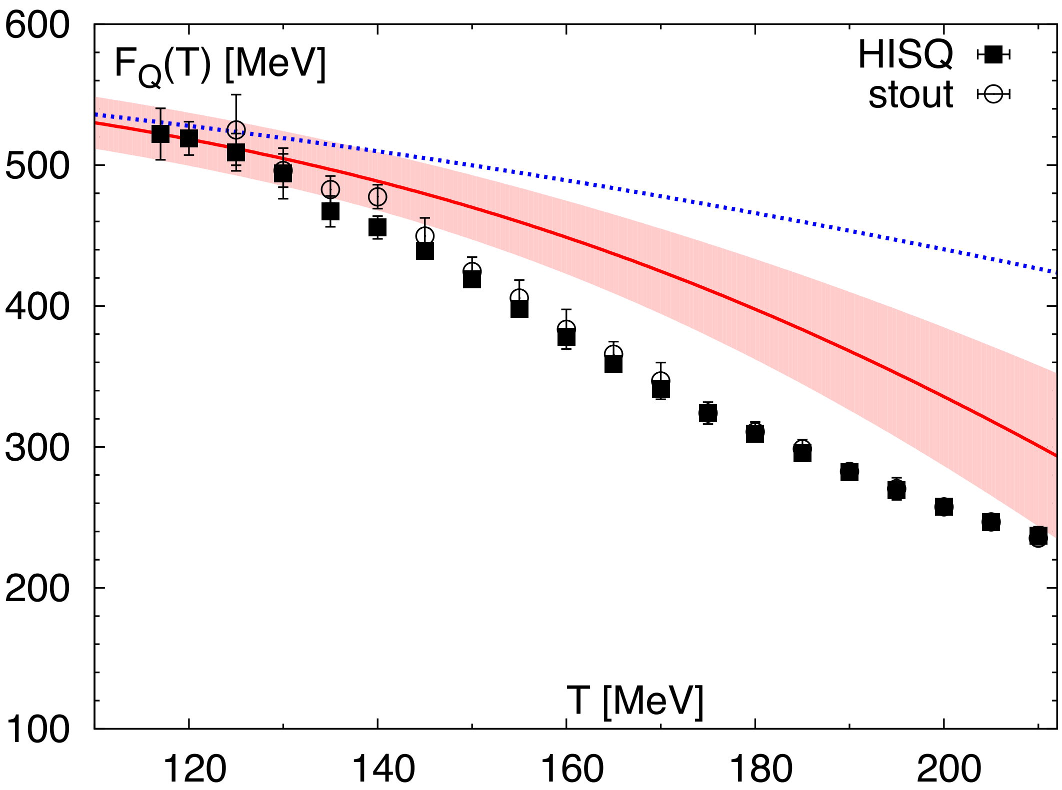
How to define deconfinement transition in QCD ?

Polyakov and gas of static-light hadrons

$$Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_n^{Q\bar{Q}}(r \to \infty)/T)$$

Energies of static-light mesons: $\frac{1}{2} E_n^{Q\bar{Q}}(r \to \infty) = M_n - m_Q$

Free energy of an isolated static quark: $F_Q(T) = -\frac{1}{2}(T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T))$



Megias, Arriola, Salcedo,
PRL 109 (12) 151601

Bazavov, PP, PRD 87 (2013) 094505

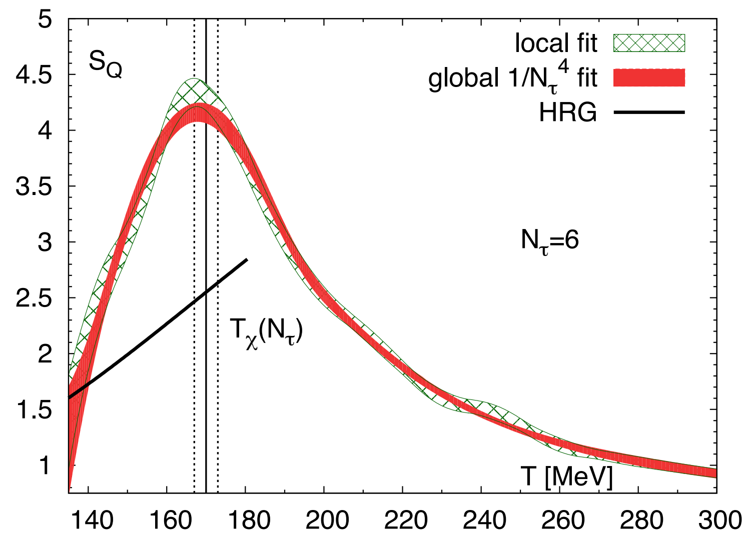
Ground state and first excited states are from lattice QCD
Michael, Shindler, Wagner,
arXiv1004.4235
Wagner, Wiese,
JHEP 1107 016,2011

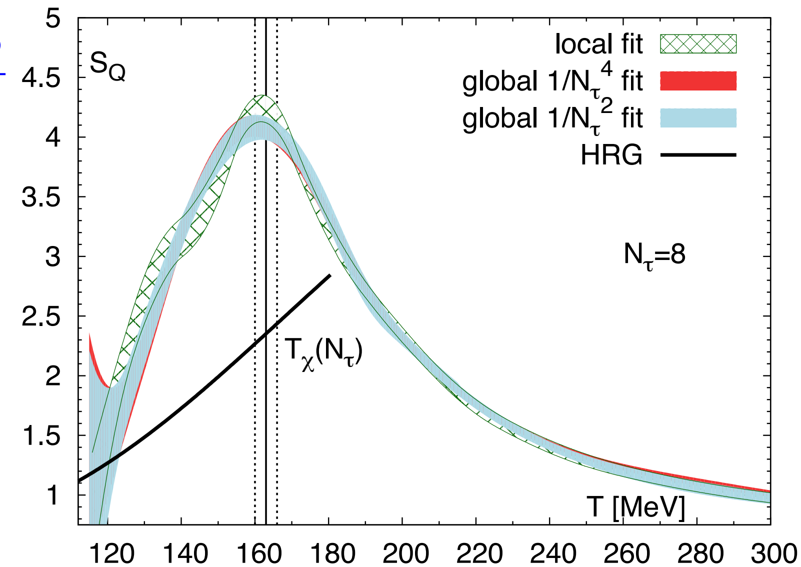
Higher excited state energies are estimated from potential model

Gas of static-light mesons
only works for $T < 145$ MeV

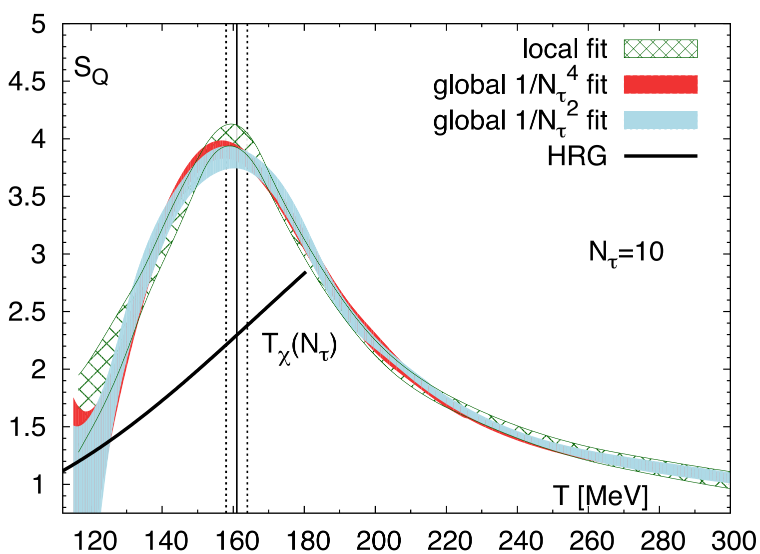
The entropy of static quark

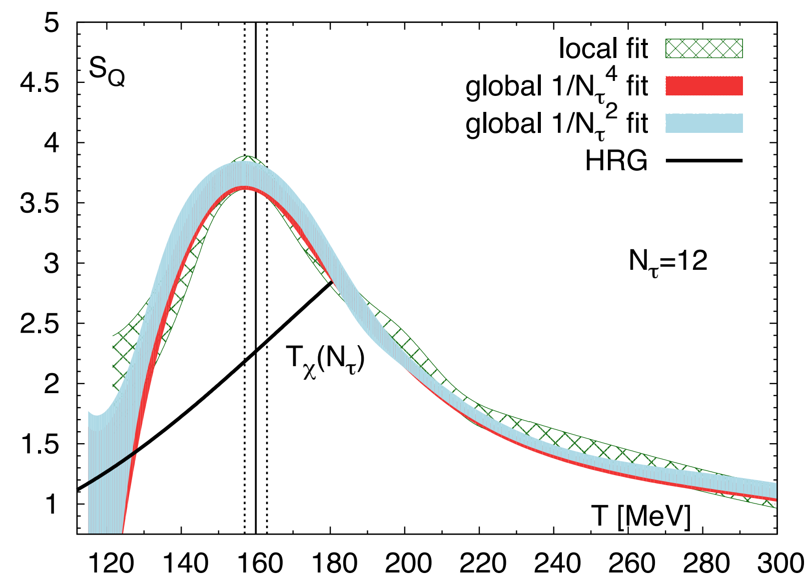
$$S_Q = -\frac{\partial F_Q}{\partial T}$$





TUMQCD, PRDD93 (2016) 114502



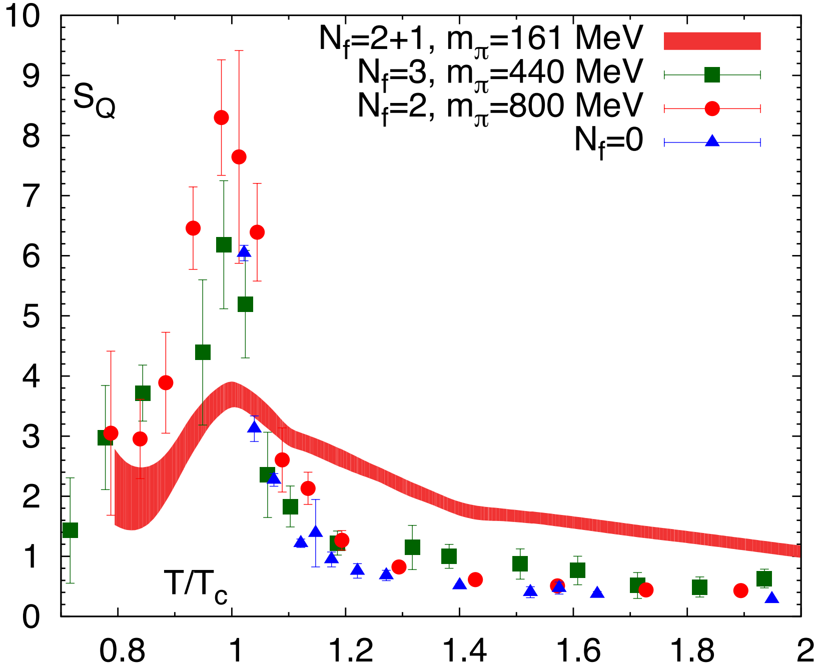


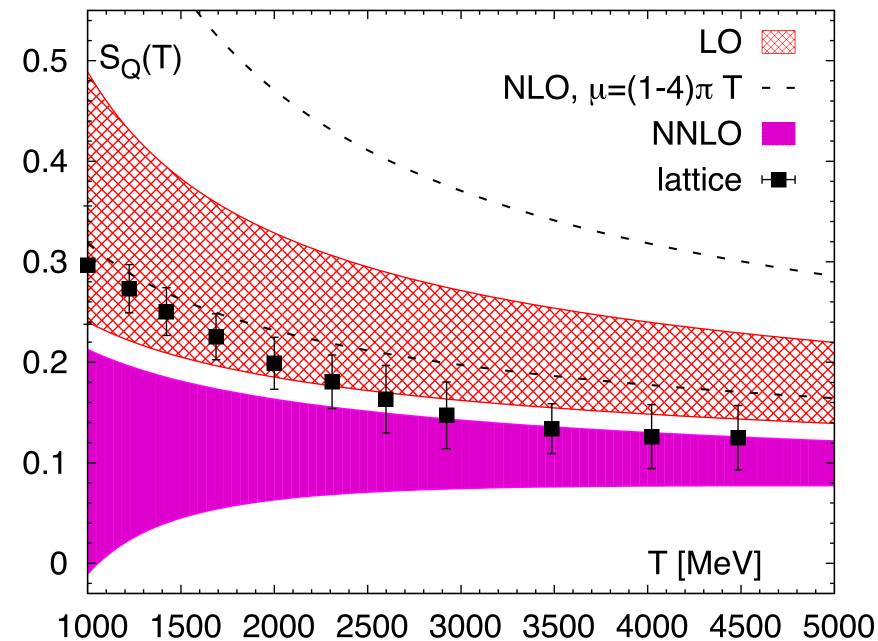
The onset of screening corresponds to peak is S_Q and its position coincides with T_c

The entropy of static quark

TUMQCD, PRDD93 (2016) 114502

$$S_Q = -\frac{\partial F_Q}{\partial T}$$





At low T the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to; at high temperature the static quark only "sees" the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

The peak in the entropy is broader and smaller for smaller quark mass

Weak coupling (EQCD) calculations work for $T > 1500$ MeV
Berwein et al, PRD 93 (2016) 034010

Free energy of a static quark anti-quark pair at high T

The work to separate the $Q\bar{Q}$ pair from distance r_1 to r_2 : $F_{Q\bar{Q}}(r_2) - F_{Q\bar{Q}}(r_1)$

Leading order in perturbation theory:

$$F_{Q\bar{Q}}(r,T) = -\frac{1}{9}\frac{\alpha_s^2}{r^2 T}\exp(-2m_D r) + F_\infty$$

In QED at leading order:

$$F_{Q\bar{Q}}(r,T) = -\frac{\alpha}{r}\exp(-m_D r) + F_\infty$$

Conjecture: in QCD the work is reduced due to cancelation of color singlet and octet contribution

Fierz idenity: $\delta_{ij}\delta_{lk} = \frac{1}{N_c}\delta_{ik}\delta_{lj} + 2T^a_{ik}T^a_{lj}$

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9}e^{-F_S(r,T)/T} + \frac{8}{9}e^{-F_O(r,T)/T}$$

$$e^{-F_S(r,T)/T} = \frac{1}{N_c}\langle \mathrm{tr} W(r) W^\dagger(0)\rangle,\ \ W(r) = \prod_{\tau=0}^{N_\tau-1} U_0(\mathbf{r},\tau)$$

LO:

$$F_S = -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r} + F_\infty,\ \ F_O = +\frac{1}{6}\frac{\alpha_s}{r}e^{-m_D r} + F_\infty$$

$\Rightarrow$ The LO result for $F_{Q\bar{Q}}$ is recovered

Free energy of a static quark anti-quark pair at high T (cont'd)

The definition of F_S requires gauge fixing. Is it possible to have a gauge invariant decomposition of $F_{Q\bar{Q}}$ into singlet and octet contributions ?

Yes, for $r \ll 1/T$ using pNRQCD

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \langle S(r,1/T)S(r,0)\rangle + L_A \langle O^a(r,1/T)O^a(r,0)\rangle$$

L_A is Polyakov loop in the adjoint representation

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9}e^{-f_s(r,T)/T} + \frac{8}{9}e^{-f_o(r,T)/T},$$

$$f_s = V_s(r) + \mathcal{O}(\alpha_s^2 r T^2),\ f_o = V_o(r) - \frac{N_c \alpha_s m_D}{2} + \mathcal{O}(\alpha_s^2 r T^2)$$

Brambilla et al, PRD 82 (2010) 074019

$$F_s = f_s + \mathcal{O}(\alpha_s^3 T) + \mathcal{O}(\alpha_s^2 r T^2),\ F_O = f_o + \mathcal{O}(\alpha_s^3 T) + \mathcal{O}(\alpha_s^2 r T^2)$$

Berwein et al, PRD 96 (2017) 014025

The naïve and the pNRQCD decomposition into singlet and octet agree

To calculate $F_{Q\bar{Q}}(r,T)$ and $F_S(r,T)$ for $r \sim 1/m_D$ another EFT, namely EQCD, should be used

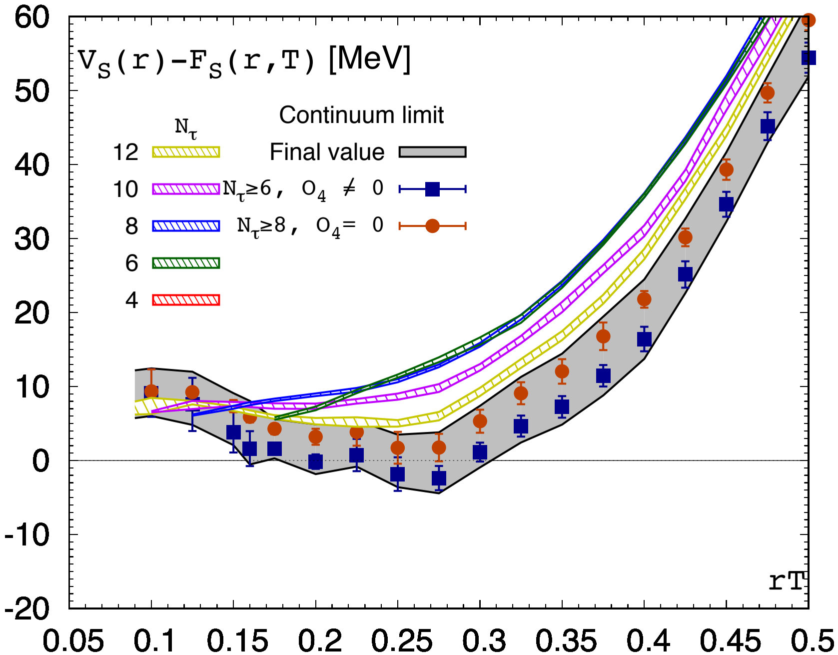
Free energy at short distances: lattice results vs. pNRQCD

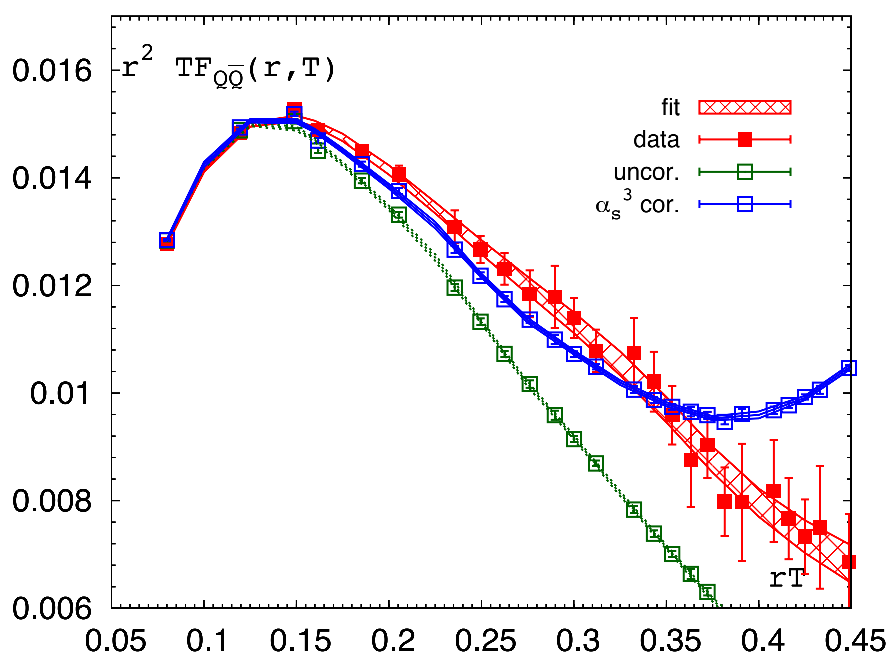
The difference between V_s and F_S is small as expected for $rT < 0.3$

Construct pNRQCD prediction for $F_{Q\bar{Q}}$ using the lattice data for F_S and V_s as proxy for f_s together with the relation:

$$f_o = -\frac{1}{8} f_s + \frac{3\alpha_s^3}{8r}\left(\frac{\pi^2}{4} - 3\right)$$

works !





T=407 MeV

TUMQCD, PRD 98 (2018) 054511

The interaction of static Q and $\bar{Q}$ is vacuum like for $rT < 0.3$

Free energy in the screening regime: lattice vs. weak coupling

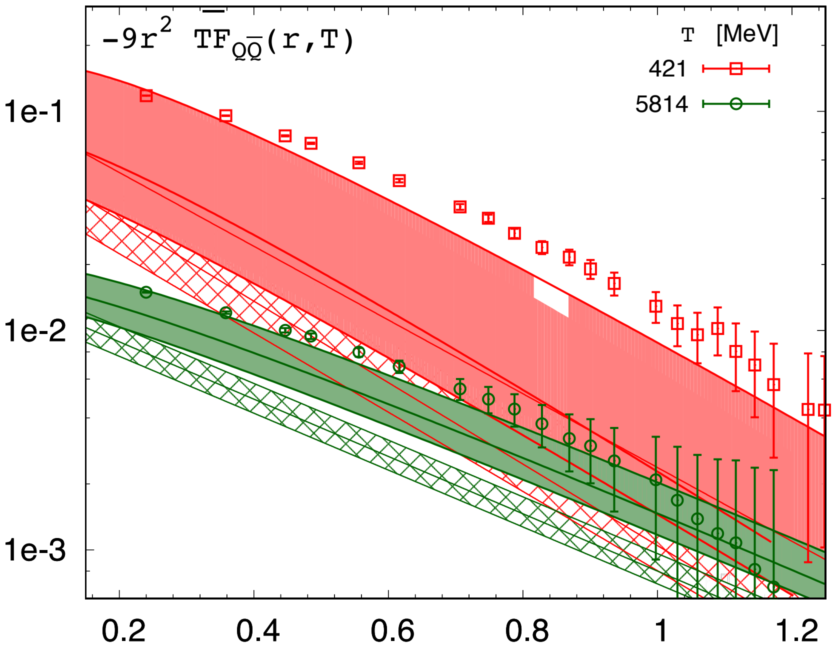
NLO in EQCD results are available for $\bar{F}_i(r,T) = F_i(r,T) - F_\infty(T)$

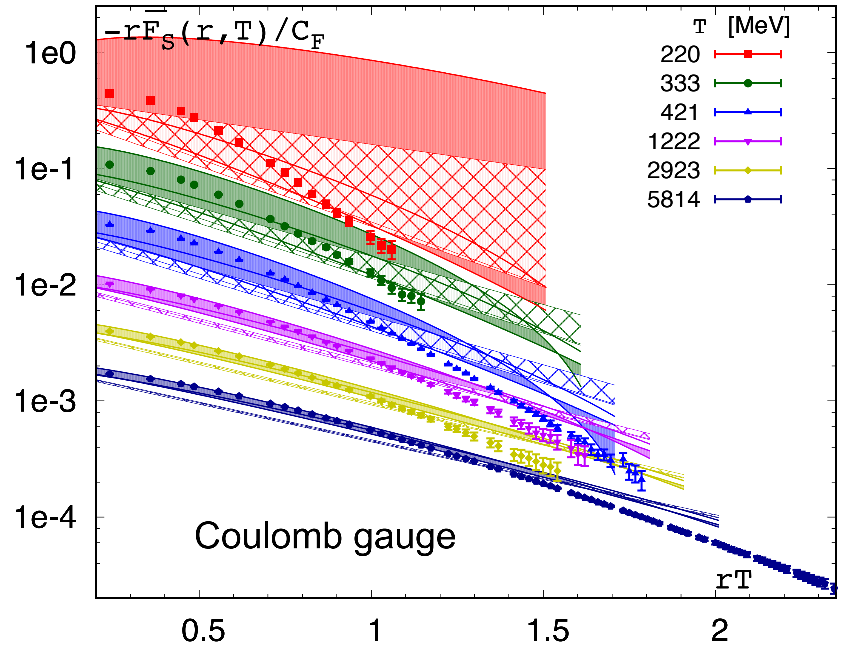
Nadkarni, PRD 33 (1986) 3738

$$F_{Q\bar{Q}}(r,T) = -\frac{\alpha_s^2 e^{-2m_D r}}{9r^2 T}\left(1 + \alpha_s(\delta Z_1(\mu) + rTf(rm_D))\right)$$

Burnier et al, JHEP 01 (2010) 054

$$F_S(r,T) = -\frac{4\alpha_s e^{-m_D r}}{3r}\left(1 + \alpha_s(\delta Z_1(\mu) + rTf_1(rm_D))\right)$$





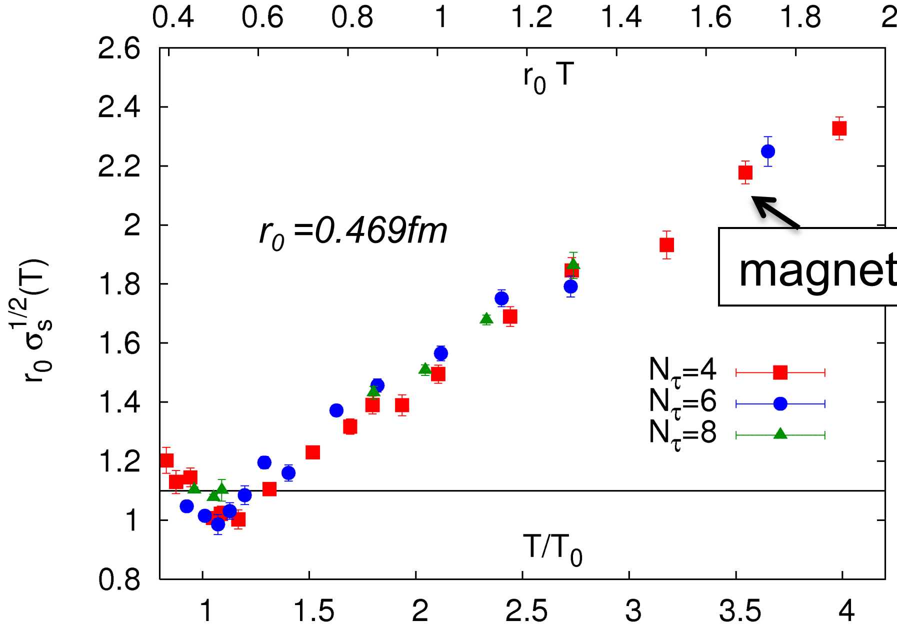
TUMQCD, PRD 98 (2018) 054511

Lattice results are in reasonable agreement with NLO weak coupling result for *rT<0.6*, at larger distances, non-pertubtative effects (due to chromo-magnetic sector) become important

Spatial string tension at $T>0$ and dimensional reduction

$$W(r(x,y),z) \sim \exp(-\sigma_s(T)\cdot r\cdot z)$$
$$\sigma_s(T) \simeq \sigma(T=0),\ T<T_c$$

Cheng et al., PRD 78 (2008) 034506



EQCD :

$$\sigma_s(T) = c_M \cdot g_3^4(T) \sim T^2,\ T \gg T_c$$

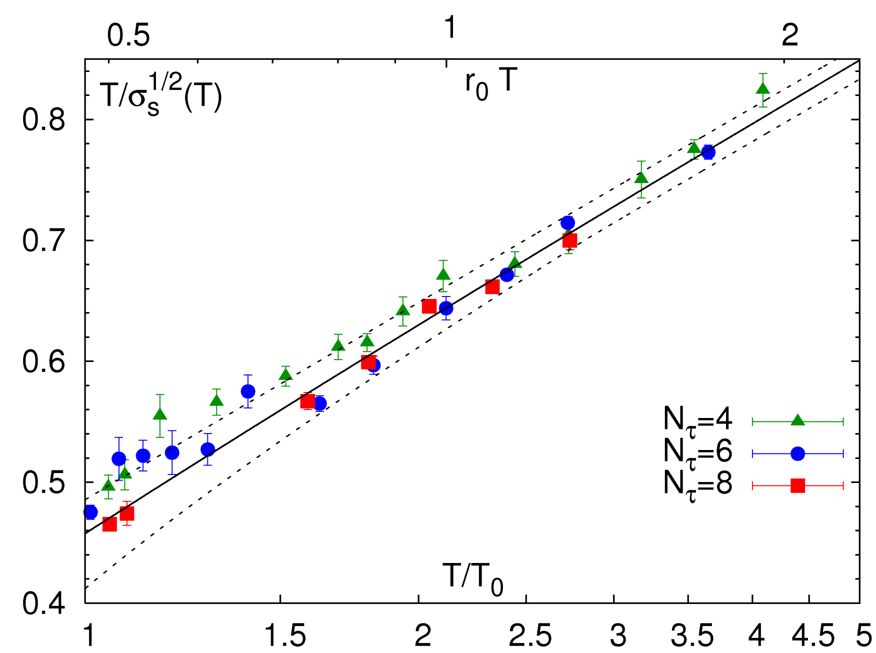
non-perturbative

$c_M = 0.55(1)$

$g_3^2(T) = g^2(T)T(1 + c_1(N_f,T)g^2(T) + ...)$

Laine, Schröder, JHEP0503(05) 067

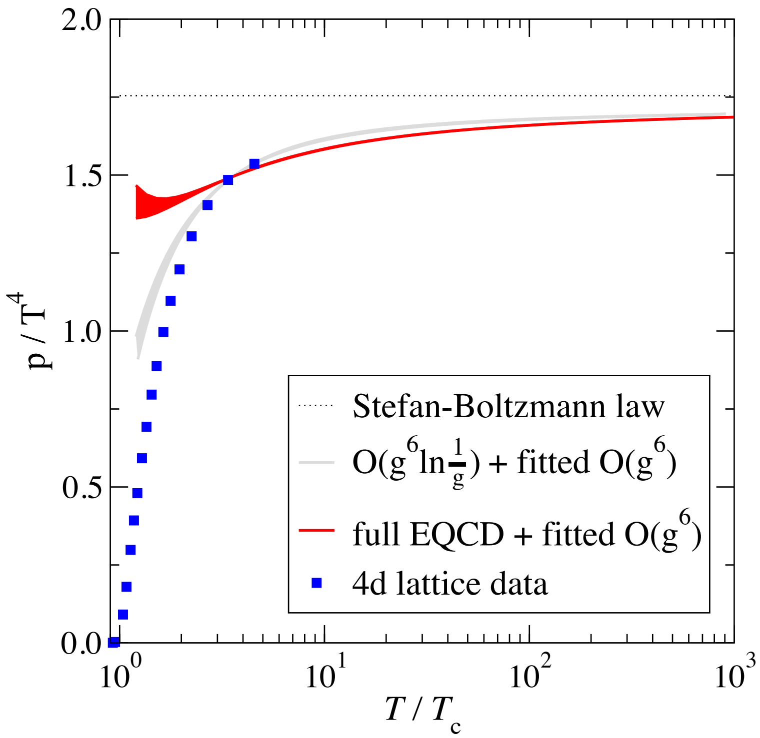
Calculated perturbatively



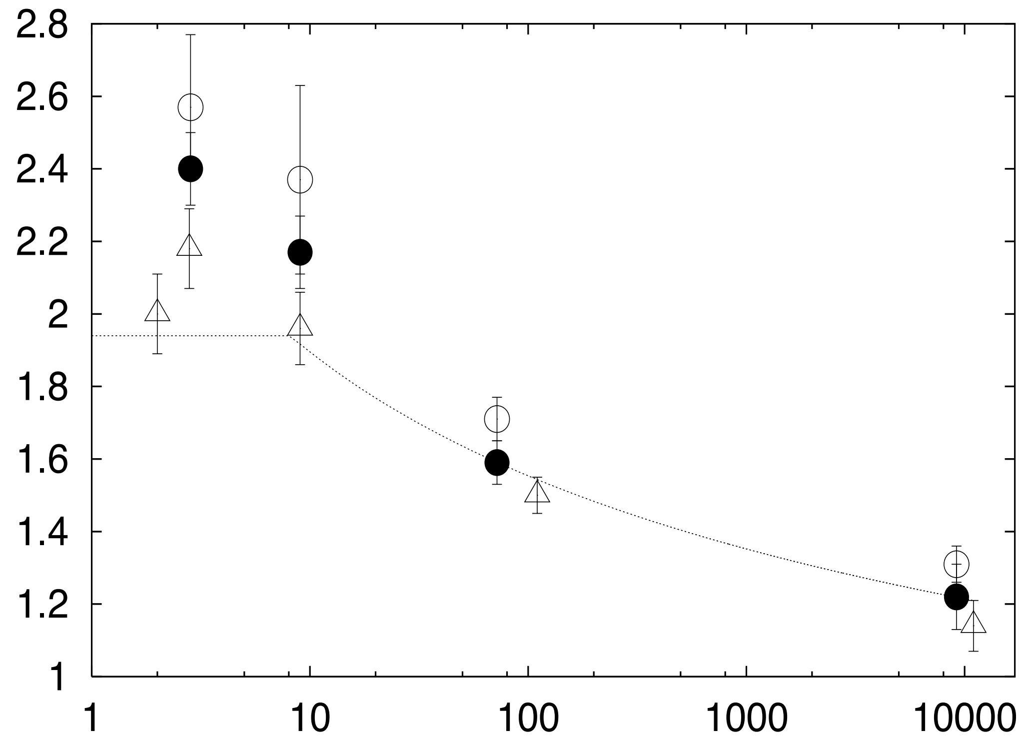
The T-dependence of spatial string tension is perturbative for $T > 1.5T_c$!

Pressure and screening mass in the 3d effective theory

Pressure in EQCD
Kajantie et al, PRD79 (2009) 045018



Electric screening mass from gluon propagator
Cucchieri et al, PRD64 (2001) 036001



Line is the fit to the 4d lattice results. Different symbols correspond to different choices of the 3d mass parameter

Euclidean correlators and spectral functions

Lattice QCD is formulated in imaginary time

$$G(\tau,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})J_H^{+}(0,0)\right\rangle,$$

$$J_H(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H=1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$$

Physical processes take place in real tim

$$D^{>}(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(t,\vec{x})J_H\ (0,0)\right\rangle,$$

$$D^{<}(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(0,\vec{0})J_H\ (t,\vec{x})\right\rangle$$

$$\frac{D^{>}(\omega)-D^{<}(\omega)}{2\pi}=\frac{1}{\pi}\mathrm{Im}D_R(\omega)=\sigma(\omega)$$

$$R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\frac{\sigma(\omega)}{\omega^2}$$

$$\boxed{G(\tau,T)=D^{>}(-i\tau)}$$

$$\Downarrow$$

$$\boxed{G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}}$$

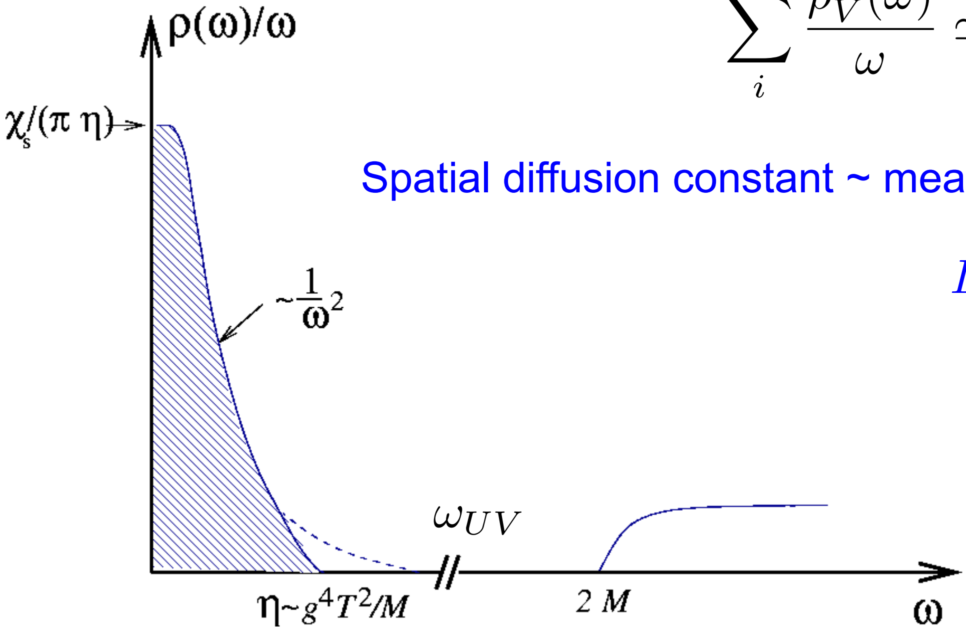
if $T=0$ and $\sigma(\omega)=\sum_n A_n\delta(\omega-E_n)$ $\Rightarrow$ $G(\tau)=A_0e^{-E_0\tau}+A_1e^{-E_1\tau}+\ldots$

fit the large distance behavior of the lattice correlation functions

This is not possible for $T>0$, $\tau_{max}=1/T$ $\Rightarrow$ Maximum Entropy Method (MEM)

Current-current correlators and heavy quark diffusion

$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t} \int \mathrm{d}^3 x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$



$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q D \frac{\eta^2}{\eta^2+\omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D}$$

Spatial diffusion constant ~ mean free path (weak coupling)

drag constant

$$D \sim \frac{1}{g^4 T}$$

Momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_2^q} \sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Bigg|_{\eta \ll \omega < \omega_{UV}}$$

$$\kappa^{(M)} = 2T^2/D$$

area under the peak ~ χ_2^q

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

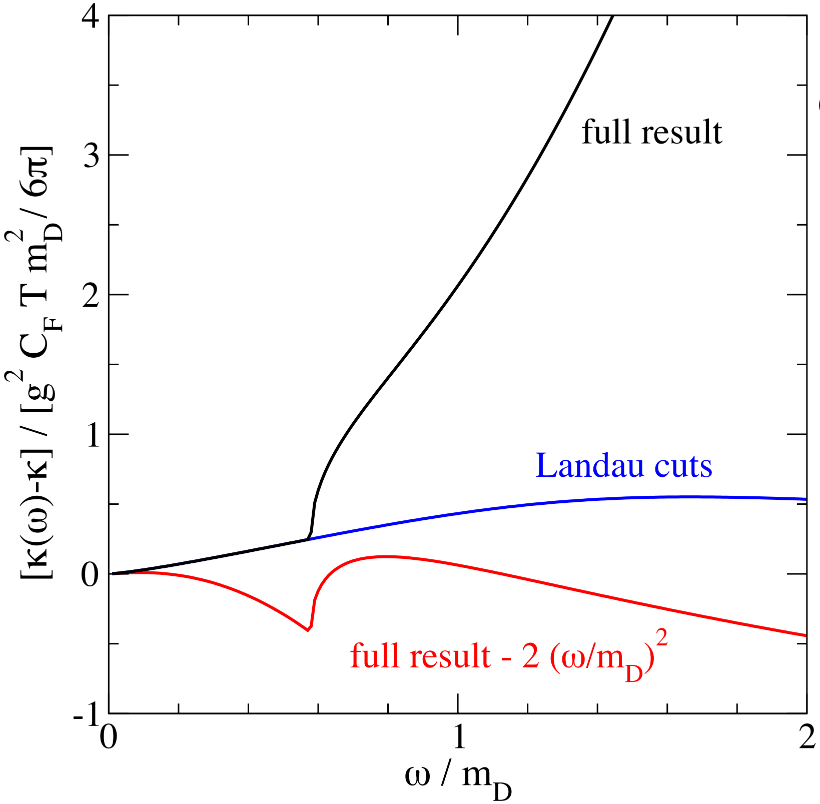
Current-current correlators in the heavy quark limit

$$\kappa = \frac{1}{3T}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_2^q}\int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\left\langle \frac{1}{2}\left\{\frac{\mathrm{d}\hat{J}^i(t,\vec{x})}{\mathrm{d}t},\frac{\mathrm{d}\hat{J}^i(t',\vec{0})}{\mathrm{d}t'}\right\}\right\rangle\right]$$

$$\frac{\mathrm{d}\hat{J}^i}{\mathrm{d}t} = \frac{1}{M}\left\{\hat{\phi}^\dagger g E^i \hat{\phi} - \hat{\theta}^\dagger g E^i \hat{\theta}\right\} + \mathcal{O}\left(\frac{1}{M^2}\right)$$

$$t \to i\tau$$

$$G_E(\tau) = \frac{1}{3\chi_2^q T}\sum_i \int \mathrm{d}^3x \left\langle \left[\phi^\dagger g E_i \phi - \theta^\dagger g E_i \theta\right](\tau,\vec{x})\left[\phi^\dagger g E_i \phi - \theta^\dagger g E_i \theta\right](0,\vec{0})\right\rangle$$



Integrate out ϕ, θ

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gE_i(\tau,\vec{0})\, U(\tau,0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle}$$

$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\tau - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

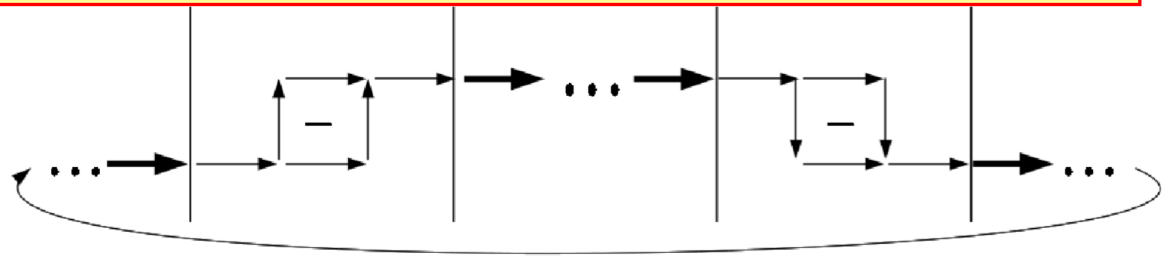
Transport coefficient ~ intercept of the spectral function not its width

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$

Caron-Huot, Laine, Moore, JHEP 0904 (2009) 053

Calculating the electric field strength correlator on the lattice

Straightforward to discretize by deforming the path of the Wilson lines to spatial direction



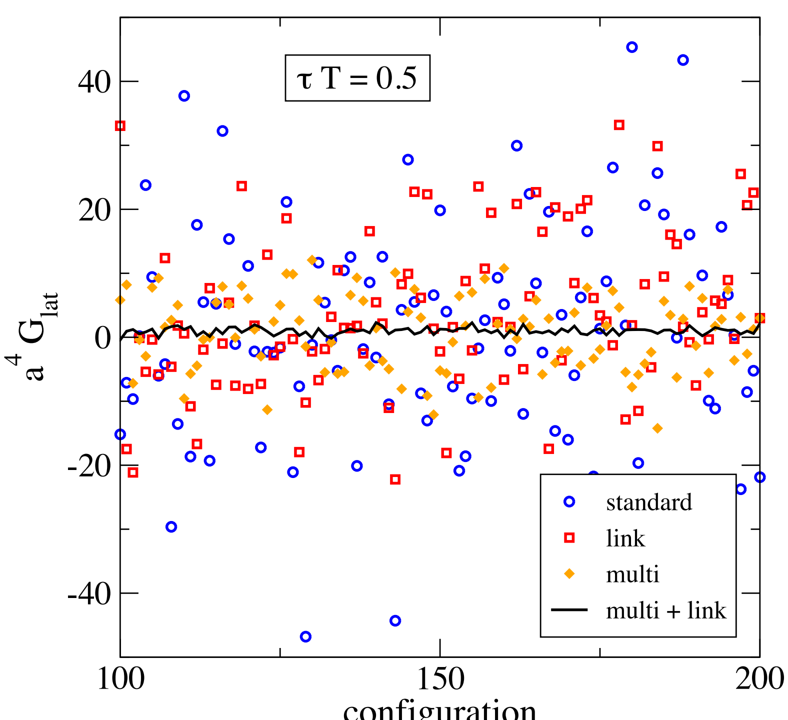
Challenge : MC noise

➡ multilevel algorithm + link integration (only works for pure glue theory)

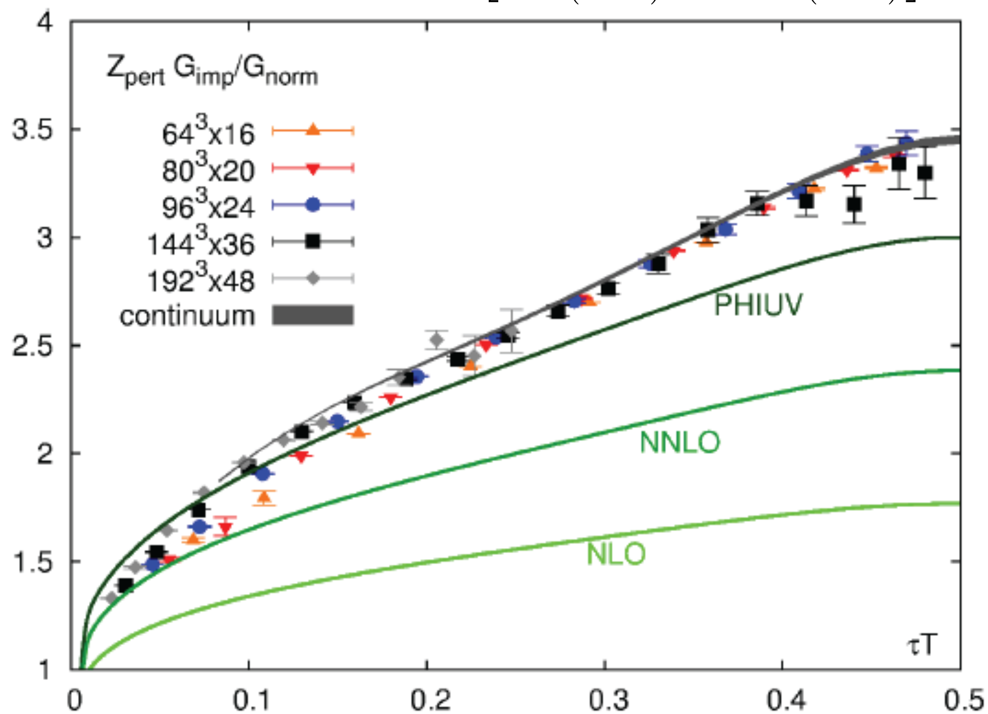
Luscher, Weisz, JHEP 0109 (2010), 010; Parisi, Petronzio, Rapuano, PLB 128 (1983) 418

Francis, Kaczmarek, Laine, et al, arXiv:1109.3941, arXiv:1311.3759, PRD 92 (2015) 116003

$T = 2.25\, T_c,\ \beta = 7.457,\ 64^3 \times 24$



$$G_{norm}(\tau) = g^2 C_F\, \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

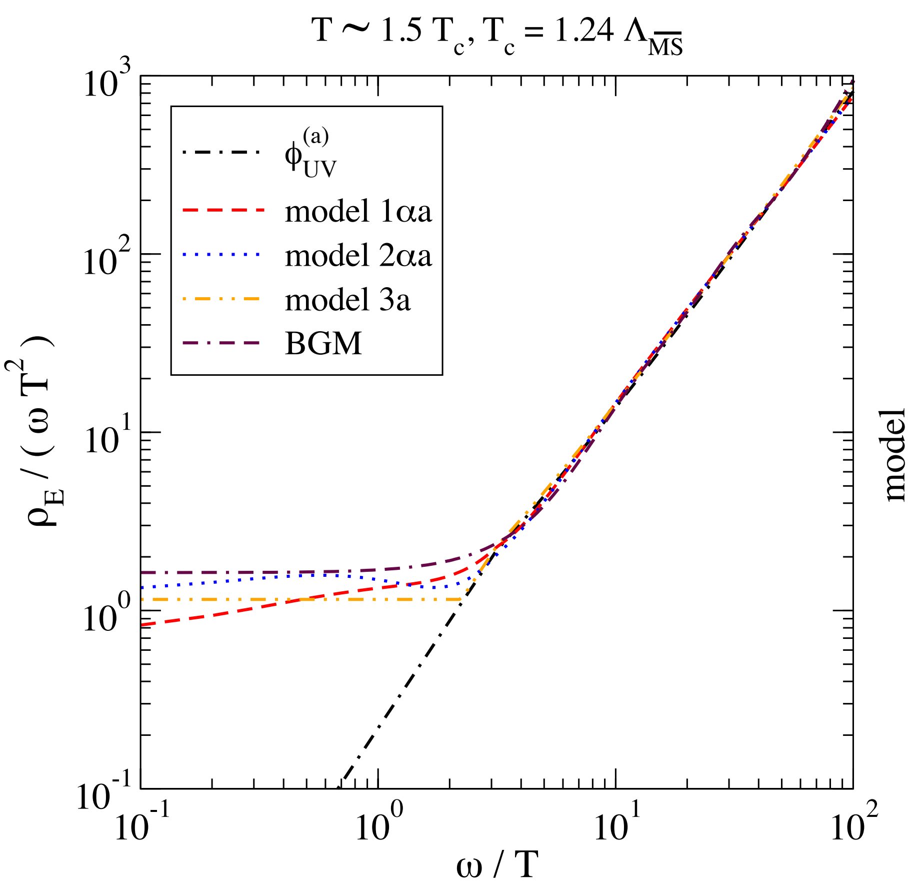


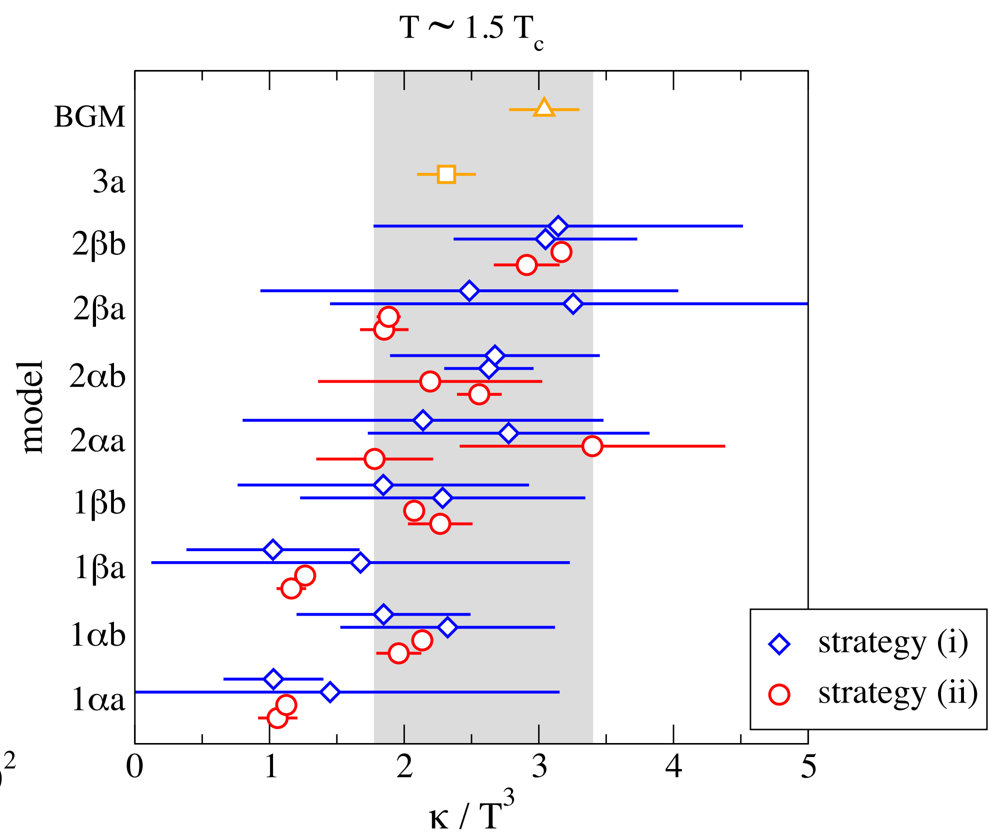
Extracting the spectral function and the diffusion constant

Fit the lattice using a forms of the spectral function constrained by low and high energy asymptotic behavior + corrections

$$\rho^{low}(\omega) = \frac{\kappa\omega}{2T}$$

$$\rho^{high}(\omega) = \frac{g^2(\mu_\omega)C_F}{6\pi}\omega^3, \mu_\omega = max(\omega, \pi T)$$





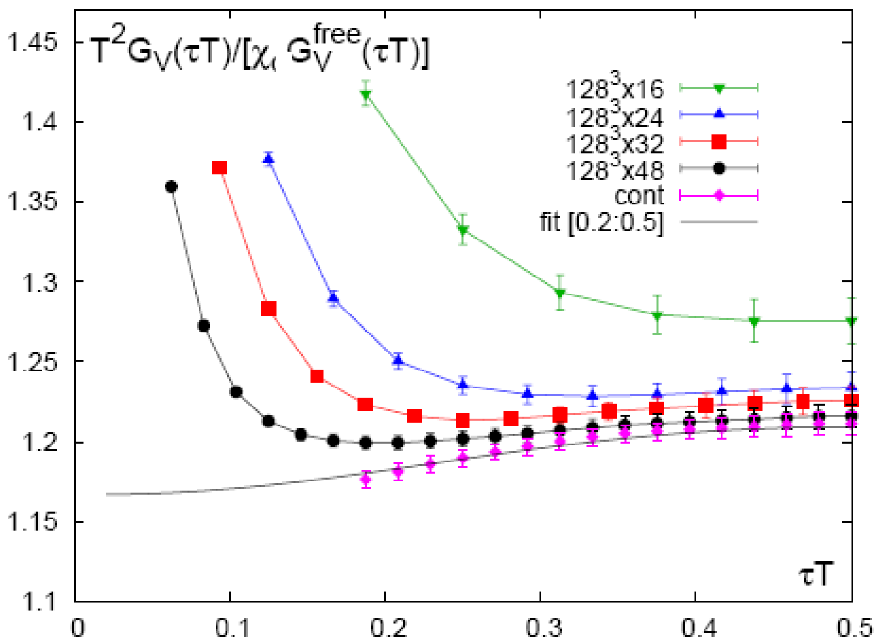
Francis, Kaczmarek, Laine, et al, PRD 92 (2015) 116003

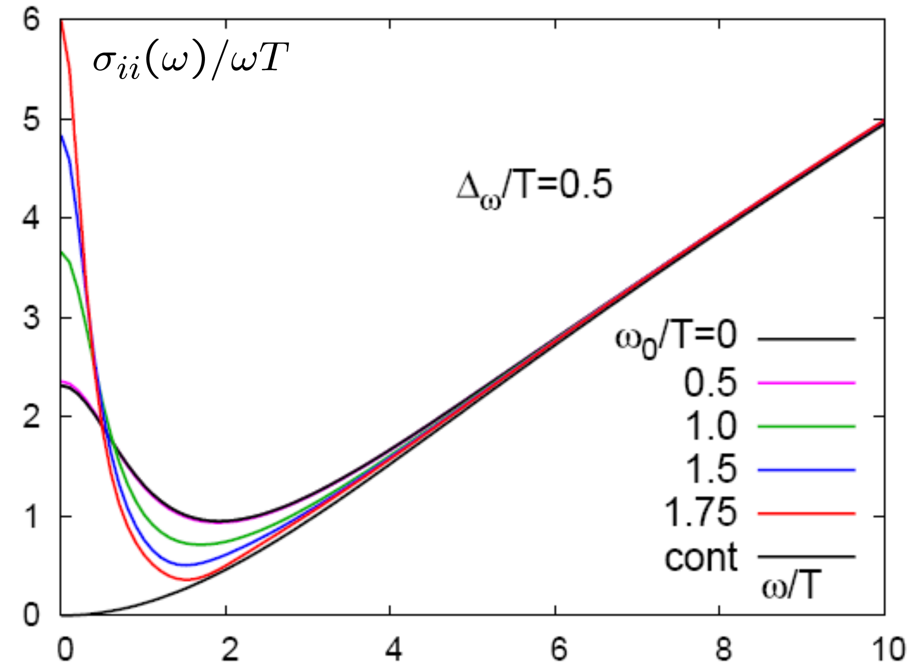
$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega) = (1.8 - 3.4)T^3$$

Lattice calculations of the vector spectral functions:

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45 T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$, $N_\tau = 24,\ 32, 48$ $(a^{-1} = 9.4 - 18.8\text{GeV})$





$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2}(1+k)\omega^2 \tanh(\omega/4T)\Theta(\omega_0, \Delta_\omega),$$

$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega \Delta_\omega})^{-1}$$

Fit parameters : $c_{BW},\ \Gamma,\ k$

Different choices of : ω_0, Δ_ω