

TRANSPORT COEFFICIENTS OF HOT AND DENSE QUARK MATTER IN EFFECTIVE MODELS OF QCD

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Myriad Colorful Ways of Understanding QCD Matter, ICTS, Bengaluru

OUTLINE

- **Introduction**
- **Boltzmann equation and transport coefficients**
- **Nambu-JonaLasinio model(NJL), Polyakov loop Quark Meson model(PQM)**
 - **Thermodynamics**
 - **Relaxation time estimation**
- **Heavy quark Drag and Diffusion coefficients**
- **Results**
- **Summary and Outlook**

INTRODUCTION

- Transport properties of hot/dense matter are important for heavy ion collision (HIC), cosmology and important for near equilibrium evolution of any thermodynamic system
- The most studied transport coefficient is perhaps shear viscosity η . In HIC spatial anisotropy of colliding nuclei gets converted to momentum anisotropy through a hydro evolvn. The equilibration is decided by η . ($\frac{\eta}{s} \sim \frac{1}{4\pi}$, the KSS bound)
- The bulk viscosity ζ - thought earlier to be not important for HIC hydro evolution. Argument: $\zeta \sim (\epsilon - 3p)/T^4$ that vanishes for ideal gas. However, lattice simulation \Rightarrow large $(\epsilon - 3p)/T^4$ near T_c . This, in turn, can give rise to different physical effects (Cavitation).

INTRODUCTION -CONTD. . . .

- The temperature and chemical potential dependence of transport coefficients may reveal the location of phase transition
- In principle, one can estimate transport coefficients using Kubo formulation but, QCD is strongly coupled for the energies accessible at HIC. Lattice QCD simulation is numerically challenging and also has problems in doing simulations at finite baryon densities.
- We shall approach the problem here within Boltzmann kinetic equation within relaxation time approximation
- The dynamics of chiral symmetry breaking in QCD with medium dependent masses are included in the formulation.
- Most earlier calculations are performed at zero baryon density ρ_B . Including finite density effects are relevant for upcoming HIC experiments, BES(Brookhaven), CBM at (GSI, Darmstadt), (NICA at Dubna).

BOLTZMANN EQUATION

Boltzmann equation describes the evolution of particle distribution function

$$\frac{df_a}{dt} = \frac{\partial f_a}{\partial t} + \frac{p^i}{E_a} \frac{\partial f_a}{\partial x^i} - \frac{\partial E_a}{\partial x^i} \frac{\partial f_a}{\partial p^i} = C^a$$

To estimate viscosity coefficients, consider small departure from equilibrium

$$f_a = f_a^0 + \delta f_a$$

The equilibrium distribution function

$$f_a^0 = \frac{1}{\exp \beta(u_\alpha p^\alpha \mp \mu) + 1}$$

The collision term C^a involve scattering processes.
Relaxation time approximation \Rightarrow in C^a , particle 'a' is out of equilibrium while all other particles are in equilibrium.

$$\frac{df_a}{dt} = \frac{p^\mu}{E_a} \frac{\partial f_a^0}{\partial x^\mu} - \frac{M}{E_a} \frac{\partial M}{\partial x^i} \frac{\partial f_a^0}{\partial p^i} = -\frac{\delta f_a}{\tau^a}$$

$T^{\mu\nu}$, J_μ and transport coefficients

Energy dependent relaxation time

$$\tau(E_a)^{-1} = \int d\Gamma_c d\Gamma_d d\Gamma_b f_b^0 W(ab, cd)$$

$$W(ab, cd) = \frac{(2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) |M|^2}{2E_a 2E_b 2E_c 2E_d}$$

LHS of Boltzmann Eqn. \Rightarrow

$$\partial_\mu f_a^a = -f_a^a (1 \mp f_a^a) \partial_\mu (\beta(E^a - \mu - \mathbf{p} \cdot \mathbf{u}))$$

Boltzmann Eq. relates non equilibrium part of distribution function to variation in fluid velocity, temperature and chemical potential.

Distribution function is related to the energy momentum tensor and the quark current

$$T^{\mu\nu} = \sum_a \int d\Gamma_a \frac{p^\mu}{E_a} p^\nu f_a + g^{\mu\nu} U(\sigma); \quad d\Gamma_a = v_a \frac{d\mathbf{p}}{(2\pi)^3}$$

$$J^\mu = \sum_a t_a \int d\Gamma_a \frac{p^\mu}{E_a} f_a$$

ζ, η, λ contd. . . .

Change in nonequilibrium part \Rightarrow

$$\delta T^{ij} = \sum_a \int d\Gamma_a \frac{p^i p^j}{TE_a} \tau_a f_a^0 (1 - f_a^0) q_a(p, \beta, \mu)$$

$$\delta J^i = \sum_a t_a \int d\Gamma_a \frac{p^i}{E_a} \tau_a f_a^0 (1 - f_a^0) \left(t_a - \frac{nE_a}{\epsilon + p} \right) p^j \partial_j \left(\frac{\mu}{T} \right)$$

The non equilibrium contribution related to the velocity gradients can be reorganised as

$$q^a = Q^a \partial_i u_i - \frac{p^i p^j}{2E_a} W_{ij}$$

;

$$W_{ij} = \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k$$

Shear and bulk viscosities are defined through the dissipative part

$$\Delta T^{ij} = -\zeta \delta^{ij} \partial_k u_k - \eta W_{ij}$$

Thermal conductivity is defined through the dissipative part of the current

$$\Delta J_i = \lambda \left(\frac{nT}{w} \right)^2 \partial_i \left(\frac{\mu}{T} \right)$$

ζ, η, λ contd. . . .

$$\eta = \frac{1}{15T} \sum_a \int d\Gamma_a \frac{\mathbf{p}_a^4}{E_a} \left(\tau_a f_a^0 (1 - f_a^0) + \bar{\tau}_a \bar{f}_a^0 (1 - \bar{f}_a^0) \right)$$

$$\zeta = -\frac{1}{3T} \sum_a \int d\Gamma_a \frac{\mathbf{p}_a^2}{E_a} \left(\tau_a f_a^0 (1 - f_a^0) Q_a + \bar{\tau}_a \bar{f}_a^0 (1 - \bar{f}_a^0) \bar{Q}_a \right)$$

$$\lambda = \frac{1}{3} \left(\frac{w}{nT} \right)^2 \sum_a t_a \int d\Gamma_a \frac{\mathbf{p}_a^2}{E_a^2} f_a^0 (1 - f_a^0) \tau_a \left(t^a - \frac{nE_a}{w} \right)$$

In the bulk viscosity coefficient, the coefficient Q^a depends upon the equation of state

$$Q_a = - \left[\frac{\mathbf{p}_a^2}{3E_a} + \left(\frac{\partial P}{\partial n} \right)_\epsilon \left(\frac{\partial E}{\partial \mu} - 1 \right) - \left(\frac{\partial P}{\partial \epsilon} \right)_n \left(E_a - T \frac{\partial E_a}{\partial T} - \mu \frac{\partial E_a}{\partial \mu} \right) \right]$$

ζ contd

However, Q_a has to be supplemented by the conditions $u_\mu \delta J^\mu = 0$ and $u_\mu \delta T^{\mu\nu} u_\nu = 0$ corresponding to **baryon number** and **energy momentum** conservation. Within the relaxation time approximation, these Landau-Lifshitz conditions reduce to

$$\sum_a t_a \langle \tau_a Q_a \rangle = 0, \quad \sum_a \langle \tau_a E_a Q_a \rangle = 0$$

$$\langle \phi_a(p) \rangle = \int d\Gamma_a [\phi_a(p) f_a^0 (1 - f_a^0)]$$

If Landau Lifshitz conditions are not satisfied, replace

$$\tau_a Q_a \rightarrow \tau_a Q_a + \alpha t_a + \beta E_a$$

The unknown coefficients to be determined from the baryon number and energy momentum conservation equation. The expression for bulk viscosity consistent with the Landau Lifshitz condition is then given as

$$\zeta = -\frac{1}{T} \sum_a \langle (\tau_a Q_a + \alpha t_a + \beta E_a) \frac{\mathbf{p}^2}{3E_a} \rangle$$

η, ζ, λ contd.

The expressions for the transport coefficients become simpler when one realises that for ideal hydrodynamics the entropy per baryon (σ) is constant.

$$\eta = \frac{1}{15} \sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2} \tau_a f_a^0 (1 - f_a^0)$$
$$\zeta = \frac{1}{9T} \sum_a \int d\Gamma_a \frac{\tau_a f_a^0 (1 - f_a^0)}{E_a^2} \left[\mathbf{p}^2 + 3v_n^2 T^2 E_a \frac{\partial}{\partial T} \left(\frac{E_a - \mu_a}{T} \right) \right]^2$$
$$\lambda = \frac{1}{3} \left(\frac{w}{nT} \right)^2 \sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} \tau_a f_a (1 - f_a) \left(t_a - \frac{nE_a}{w} \right)^2$$

- Transport coefficients are nonnegative as they must be.
- It is important to include the Landa-Liftshitz conditions to obtain the above results.

Note: Transport coefficients of the system is sum of the contributions of each species to the same. On the otherhand, the relaxation time of a given specie is added inversely arising from scattering of the given species with all other particles with which it scatters.

η, ζ, λ contd.

Knowing the equation of state and other thermodynamic quantities like velocity of sound etc. and the relaxation time one can estimate the viscosity coefficient.

This thermodynamics and estimation of relaxation time is done within two models - (Nambu-Jonalasinio model and Polyakov loop extended quark meson model)

Nambu JonaLasinio model : Thermodynamics

$$\mathcal{L} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m_0)\psi - G \left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\mathbf{t}^a\psi)^2 \right)$$

The thermodynamic potential (negative of pressure):

$$\Omega(\beta, \mu) = -\frac{\gamma}{(2\pi)^3} \int E(\mathbf{k})d\mathbf{k} - \frac{\gamma}{(2\pi)^3\beta} \int d\mathbf{k} (\ln(1 + \exp(-\beta(E - \mu))) + \mu \rightarrow -\mu) + \frac{(M - m_0)^2}{4G}$$

$\gamma = 2N_c N_f$ (degeneracy); $E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + M^2}$, M : Constituent quark mass get determined self consistently solving the mass gap equation

$$M = m_0 - 2G\langle\bar{\psi}\psi\rangle = m_0 + \frac{\gamma}{(2\pi)^3} \int \frac{M}{E(\mathbf{k})} (1 - f_-(\mathbf{k}, \beta, \mu) - f_+(\mathbf{k}, \beta, \mu)) d\mathbf{k}$$

MASSES ; NJL MODEL CONTD. . . .

Within RPA Meson propagators:

$$D = \frac{2iG}{1 - 2G\Pi_{\sigma/\pi}}$$

Mass of the meson determined by pole position of the Real part of meson propagator:

$$1 - 2G\text{Re}\Pi_M(m_M, \mathbf{0}) = 0,$$

For $m_M < 2M$, Π_M is real while for $m_M > 2M$, Π_M has imaginary part: Decay width of resonance $\Gamma_M = \text{Im}\Pi_M(m_M, \mathbf{0})$

This affects the quark scatterings through meson exchange and hence on the relaxation time.

MASSES ...

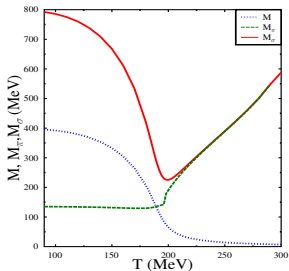


Fig. 1-a

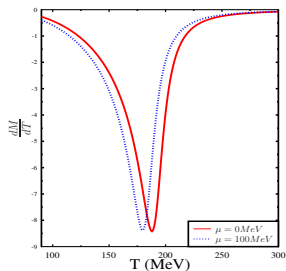


Fig. 1-b

Figure: Temperature dependence of the masses of constituent quark (M), and pions (M_π) and sigma mesons (M_σ) for $\mu = 0$ (Fig1-a) and temperature derivative of the constituent quark mass for $\mu = 0\text{MeV}$ and $\mu = 100\text{MeV}$ (Fig. 1-b).

$$T_\chi = 188\text{MeV}; \quad m_\pi(T_{Mott}) = 2M(T_{Mott}); \quad T_{Mott} \simeq 197\text{MeV}.$$

ESTIMATING THE AVERAGE RELAXATION TIME

Avg. relaxation time

$$\tau_a^{-1} = \sum_b n_b \bar{W}_{ab}$$

Thermally averaged transition rate

$$\bar{W}_{a,b} = \frac{1}{n_a n_b} \int f_a f_b W_{ab} d\pi_a d\pi_b$$

$$W_{ab}(s) = \frac{2\sqrt{s(s-4m^2)}}{1+\delta_{ab}} \int_{t_{min}}^0 dt \left(\frac{d\sigma}{dt} \right) (1 - f_c(\frac{\sqrt{s}}{2}, \mu))(1 - f_d(\frac{\sqrt{s}}{2}, \mu))$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s} \frac{1}{p_{ab}} |M|^2$$

ESTIMATING THE AVERAGE RELAXATION TIME

- For two flavors we consider the following possible scatterings.

$$u\bar{u} \rightarrow u\bar{u}, \quad u\bar{d} \rightarrow u\bar{d}, \quad u\bar{u} \rightarrow d\bar{d},$$

$$uu \rightarrow uu, \quad ud \rightarrow ud, \quad \bar{u}\bar{u} \rightarrow \bar{u}\bar{u},$$

$$\bar{u}\bar{d} \rightarrow \bar{u}\bar{d}, \quad d\bar{d} \rightarrow d\bar{d}, \quad d\bar{d} \rightarrow u\bar{u},$$

$$d\bar{u} \rightarrow d\bar{u}, \quad dd \rightarrow dd, \quad \bar{d}\bar{d} \rightarrow \bar{d}\bar{d},$$

- Using i-spin symmetry, charge conjugation symmetry as well as the crossing symmetry to relate the matrix element square for the above 12 processes reduce to evaluating only two independent matrix elements $u\bar{u} \rightarrow u\bar{u}$ and $u\bar{d} \rightarrow u\bar{d}$
- Dominant contribution comes from propagation of pion and sigma mode in the s-channel.
- The temperature dependence of π and σ modes play an important role in these cross section evaluation.

RELAXATION TIME; η/s : T BEHAVIOR

$$\eta = \frac{1}{15} \sum_a \int d\Gamma_a \frac{\mathbf{p}_a^4}{E_a^2} \tau_a f_a^0 (1 - f_a^0)$$

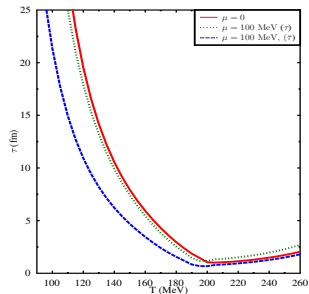


Fig. 2-a

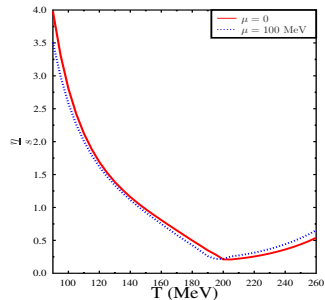
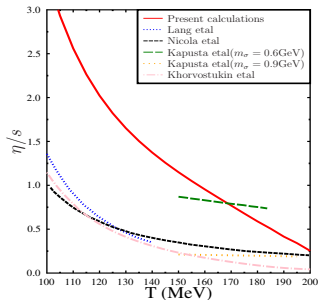


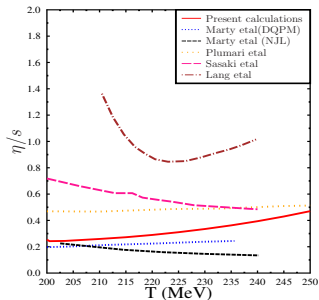
Fig. 2-b

Figure: Relaxation time as a function of temperature for $\mu = 0$ MeV and for $\mu = 100$ MeV (Fig 5-a). In Fig (5-b), shear viscosity to entropy density ratio is shown for $\mu = 0$ MeV and $\mu = 100$ MeV.

η/s : CONTD.



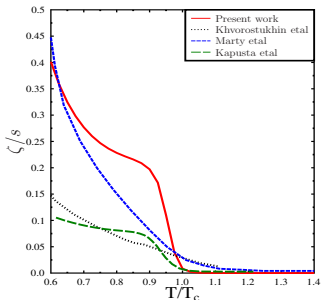
a



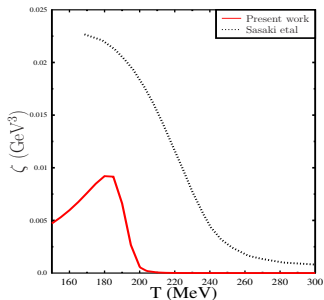
b

Figure: Shear viscosity to entropy ratio for $\mu = 0$ for low temperatures (Fig a). The present calculations is shown by the solid lines. The other results correspond to Lang etal (2012) of interacting pion gas, Fernandez-Fraile and Nicola (2009). The two curves by Kapusta etal (2011) correspond to different masses for the sigma mesons. The green dashed curve is for $m_\sigma = 600$ MeV while the orange dotted curve is with $m_\sigma = 0.9$ GeV. The pink dot dashed curve is for the SHMC model by Khorvostukhin etal(2010). In b is shown the ratio for higher temperatures. Present calculations is shown by solid red line, the two curves of Marty etal(2013)', correspond to dynamical quasi particle model (DQPM) and the 3 flavor NJL model, the orange dotted curve by Plumari etal(2012), the pink dashed curve by Sasaki etal(2010) is for two flavor NJL model while the brown dot dashed curve is from Lang etal

Bulk viscosity: T behavior



a



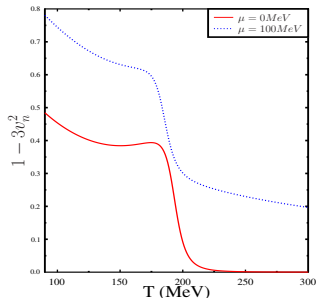
b

Figure: Bulk viscosity to entropy ratio as a function of temperature in units of T_c for zero baryon density (a). Also shown results from different models, the SHMC model of Khvorostukhin et al (2010), Kapusta and Chakraborty (2009), the three flavor NJL results of Marty et al. Bulk viscosity in units of GeV^3 as a function of temperature is shown in Fig. b. Solid red curve corresponds to the present calculations while dotted curve correspond to the results by Sasaki and Redlich (2010)

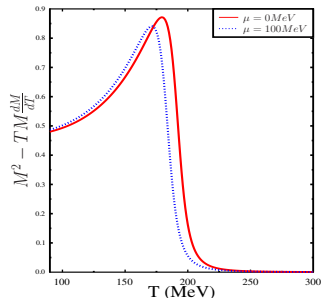
Bulk viscosity: T behavior

For zero chemical potential

$$\zeta = \frac{1}{9T} \sum_a d\Gamma^a \frac{\tau_a}{E_a^2} \left[\mathbf{p}^2(1 - 3v^2) - 3v^2(M^2 - TM \frac{dM}{dT}) \right]^2$$



a



b

Figure: The violation of conformality measure $C_1 = 1 - 3v^2$ (Fig 5a) and $C_2 = M^2 - TM \frac{dM}{dT}$ (Fig 5b) as a

$\lambda(T)$

$$\lambda = \frac{1}{3} \left(\frac{w}{nT} \right)^2 \sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} \tau_a f_a (1 - f_a) \left(t_a - \frac{nE_a}{w} \right)^2$$

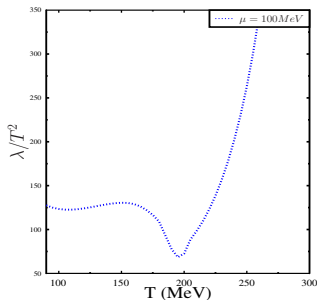


Figure: Thermal conductivity(λ) in units of T^2 for $\mu = 100$ MeV.

POLYAKOV LOOP QUARK MESON MODEL

PQM model : captures features of chiral symmetry breaking and confinement properties of strong interaction.

$$\mathcal{L} = \bar{\psi}(i\gamma_{\mu}D^{\mu} - m_0)\psi + (\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\pi\partial^{\mu}\pi) - U_{\chi} - U_{\phi}$$

$$U_{\chi} = \frac{\lambda}{4} (\sigma^2 + \pi^2 - v^2) - c\sigma$$

$$\phi = \frac{1}{N_c} \langle \text{Tr}_c P e^{i \int_0^{\beta} dx_0 A_0(x_0, \mathbf{x})} \rangle$$

Polyakov gauge : A_0 - time independent

Functional form of U: Choose the form so as to reproduce pure gauge lattice simulation results for thermodynamics

$$U(\phi, \bar{\phi}) = T^4 \left[-\frac{b_2(T)}{2} \bar{\phi}\phi - \frac{b_3}{2} (\phi^3 \bar{\phi}^3) + \frac{b_4}{4} (\bar{\phi}\phi)^2 \right]$$

Thermodynamics:PQM model

Thermodynamic potential (negative of pressure)

$$\Omega(T, \mu) = -2N_f T \int d\mathbf{p} \left[\ln \left(1 + 3(\phi + \bar{\phi} e^{-\beta\omega_-}) e^{-\beta\omega_-} + e^{-3\beta\omega_-} \right) \right. \\ \left. + \ln \left(1 + 3(\phi + \bar{\phi} e^{-\beta\omega_+}) e^{-\beta\omega_+} + e^{-3\beta\omega_+} \right) \right] + U_X + U_P$$

The quark

excitation energies are $\omega_{\mp} = \sqrt{\mathbf{p}^2 + m_q^2} \mp \mu$ and the constituent quark masses are dependent on the mean fields as $m_q^2 = g^2(\sigma^2 + \pi^2)$.

The mean fields are obtained by extremization of Ω with respect to $\sigma, \phi, \bar{\phi}$ and π .

Meson masses are determined by the curvature of Ω at the global minimum:

$$M_\sigma^2 = \frac{\partial^2 \Omega}{\partial \sigma^2} \quad M_\pi^2 = \frac{\partial^2 \Omega}{\partial \pi^2}$$

MESON MASSES ; ORDER PARAMETERS...

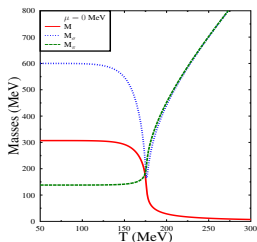


Fig. 1-a

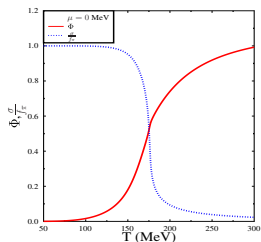


Fig. 1-b

Figure: Temperature dependence of the masses of constituent quark (M), and pions (M_π) and sigma mesons (M_σ) for $\mu = 0$ (Fig1-a) and the order parameters σ and ϕ as a function of temperature for $\mu = 0$ MeV.

PQM Thermodynamics contd....

Aman Abhishek, HM, S. Ghosh, PRD 97, 014005 (2018)

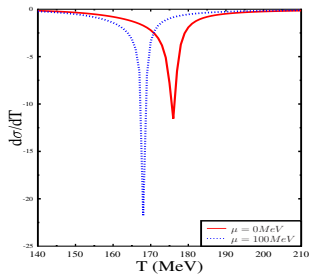


Fig. 2-a

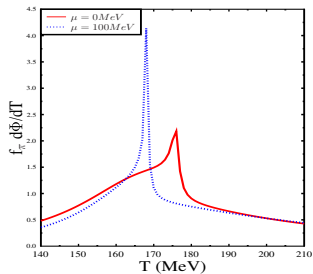


Fig. 2-b

Figure: Temperature derivative of the chiral order parameter (fig 2 a) and Polyakov loop order parameter.

$T_c \simeq 176$ MeV ($\mu = 0$); critical point $(T_c, \mu_c) = (165, 163)$
MeV

PQM Thermodynamics contd. . . .

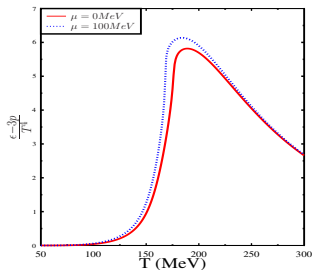


Fig. 3-a

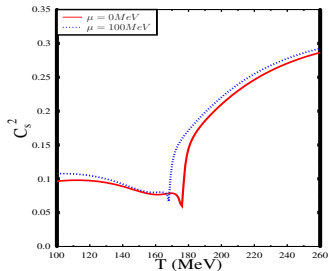


Fig. 3-b

Figure: Temperature dependence of the scaled trace anomaly $\frac{\epsilon - 3p}{T^4}$ (Fig 3a) and square of velocity of sound.

Conformal symmetry is broken maximally at the critical temperature

ESTIMATING THE RELAXATION TIME: MESON SCATTERING

Energy dependent relaxation time $\tau_a(E_a)$ for a scattering process

$$a, b \rightarrow c, d \quad (d\Gamma_i = \frac{d\mathbf{p}}{(2\pi)^3})$$

$$\tau_a^{-1}(E_a) = \omega(E_a) = \sum_b \int d\Gamma_b f_b^0 W_{ab}(s)$$

$$W_{ab}(s) = \frac{1}{1 + \delta_{ab}} \int d\Gamma_c d\Gamma_d (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) |M|^2 (1 \pm f_c) + (1 \pm f_d)$$

Meson scatterings:

$$M_{\sigma, \sigma \rightarrow \sigma, \sigma} = -6\lambda - 36\lambda^2 f_\pi^2 \left(\frac{1}{s - m_\sigma^2} + \frac{1}{t - m_\pi^2} + \frac{1}{u - m_\pi^2} \right)$$

$$M_{\pi, \sigma \rightarrow \pi, \sigma} = -2\lambda - 4\lambda^2 f_\pi^2 \left(\frac{3}{t - m_\sigma^2} + \frac{1}{u - m_\pi^2} + \frac{1}{s - m_\pi^2} \right)$$

$$M_{\pi, \pi \rightarrow \pi, \pi} = -2\lambda \left(\frac{s - m_\pi^2}{s - m_\sigma^2} \delta_{ab} \delta_{cd} + \frac{t - m_\pi^2}{t - m_\sigma^2} \delta_{ac} \delta_{bd} + \frac{u - m_\pi^2}{u - m_\sigma^2} \delta_{ad} \delta_{bc} \right)$$

$$M_{\pi, \pi \rightarrow \sigma, \sigma} = -6\lambda - 4\lambda^2 f_\pi^2 \left(\frac{3}{s - m_\sigma^2} + \frac{1}{t - m_\pi^2} + \frac{1}{u - m_\pi^2} \right)$$

Poles in s and u channels in the propagators— approximate by taking $s, t, u \rightarrow \infty$ limit.

ESTIMATING VISCOSITIES: MESON SCATTERINGS

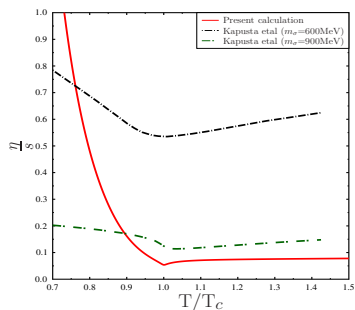


Fig. 4-a

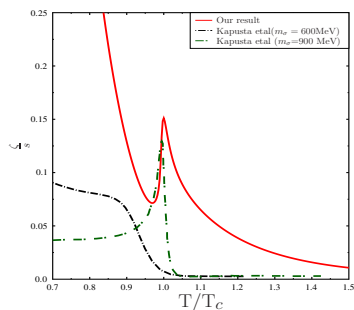


Fig. 4-b

Figure: Shear viscosity to entropy ratio (Fig4-a) and bulkviscosity to entropy ratio (Fig4 b)

ESTIMATING VISCOSITY COEFFICIENTS-QUARK SCATTERINGS

- For two flavors we consider the following possible scatterings through meson exchanges.

$$u\bar{u} \rightarrow u\bar{u}, \quad u\bar{d} \rightarrow u\bar{d}, \quad u\bar{u} \rightarrow d\bar{d},$$

$$uu \rightarrow uu, \quad ud \rightarrow ud, \quad \bar{u}\bar{u} \rightarrow \bar{u}\bar{u},$$

$$\bar{u}\bar{d} \rightarrow \bar{u}\bar{d}, \quad d\bar{d} \rightarrow d\bar{d}, \quad d\bar{d} \rightarrow u\bar{u},$$

$$d\bar{u} \rightarrow d\bar{u}, \quad dd \rightarrow dd, \quad \bar{d}\bar{d} \rightarrow \bar{d}\bar{d},$$

- Using i-spin symmetry, charge conjugation symmetry as well as the crossing symmetry to relate the matrix element square for the above 12 processes reduce to evaluating only two independent matrix elements $u\bar{u} \rightarrow u\bar{u}$ and $u\bar{d} \rightarrow u\bar{d}$
- Dominant contribution comes from propagation of pion and sigma mode in the s-channel.
- The temperature dependence of π and σ modes play an important role in these cross section evaluation.

$\eta/s, \zeta/s$: QUARK SCATTERING

$$\eta = \frac{1}{15} \sum_a \int d\Gamma_a \frac{\mathbf{p}_a^4}{E_a^2} \tau_a f_a^0 (1 - f_a^0)$$

With Polyakov loop dependent quark distribution functions,

$$f_a^0 (1 - f_a^0) \rightarrow \left[\frac{\phi e^{-\beta\omega} + 4\bar{\phi} e^{-2\beta\omega} + 3e^{-3\beta\omega}}{1 + 3\phi e^{-\beta\omega} + 3\bar{\phi} e^{-2\beta\omega} + 3e^{-3\beta\omega}} - 3f_a^{02} \right]$$

ESTIMATING THE VISCOSITIES-QUARK MESON SCATTERINGS

- Quark pion scattering: ($q, \pi \rightarrow q, \pi$) Matrix element:

$$T_{ba} \sim \frac{g^2}{2} (\gamma^\mu q_{1\mu} + \gamma^\mu q_{2\mu}) [B^+ \delta_{ab} + B^{(-)} i \epsilon_{abc} \tau^c]$$

$$B^{(+)} = \left[\frac{1}{u - m_q^2} - \frac{1}{s - m_q^2} \right], \quad B^{(-)} = - \left[\frac{1}{u - m_q^2} + \frac{1}{s - m_q^2} \right]$$

- Poles can arise in the u-channel
Include in medium quark width

$$\Sigma = m \Sigma_0 + \gamma \cdot \mathbf{p} \Sigma_3 - \gamma_0 p_0 \Sigma_4$$

Relevant imaginary part is included in the propagator.

RELAXATION TIME: QUARK MESON SCATTERING

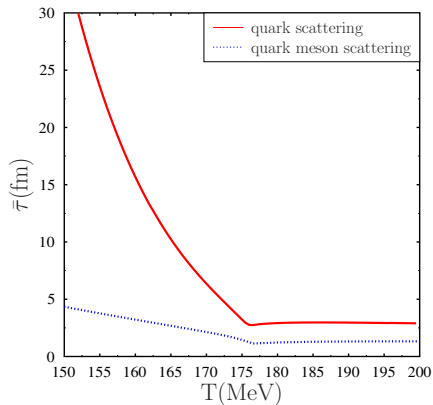


Figure: average relaxation time for quarks. Solid line corresponds to quark-quark and quark-antiquark scattering with meson exchange. Dashed curve includes the contribution of quark meson scatterings.

$\eta/s, \zeta/s$: QUARK MESON SCATTERING

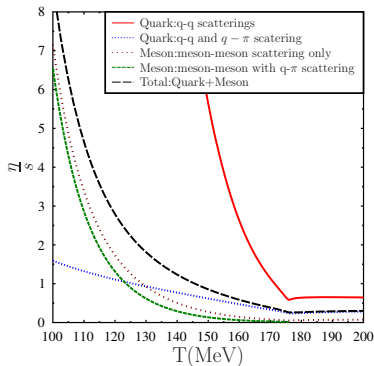


Fig. 6-a

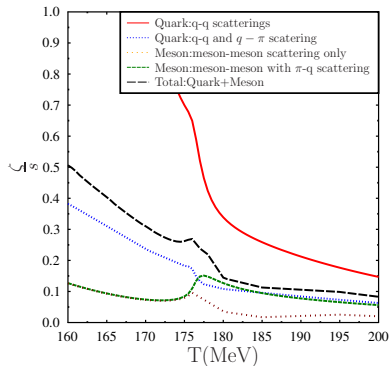


Fig. 6-b

Figure: Different contributions as well as the total contribution to $\frac{\eta}{s}$ (Fig 6 -a) and $\frac{\zeta}{s}$ (Fig. (6-b) as a function of temperature . ($\mu = 0$).

$\lambda(T)$

$$\lambda = \frac{1}{3} \left(\frac{w}{nT} \right)^2 \sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} \tau_a f_a (1 - f_a) \left(t_a - \frac{nE_a}{w} \right)^2$$

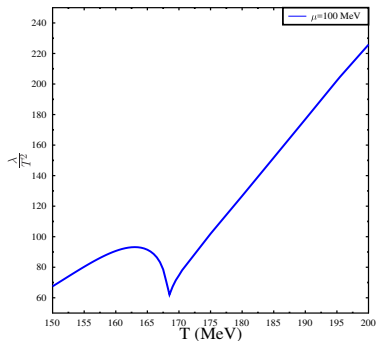


Figure: Thermal conductivity(λ) in units of T^2 for $\mu = 100$ MeV.

SUMMARY, CONCLUSIONS AND OUTLOOK

- We tried to derive the viscosity coefficients using Boltzmann kinetic equation with relaxation time approximation with mean field and medium dependent masses.
- While η depends only on the behaviour of relaxation time and the medium dependent masses, ζ depends on other thermodynamic quantities and the equation of state.
- The deviation from equilibrium should be consistent the Landau Lifshitz conditions.
- The thermodynamics of hot and dense matter is estimated within PQM model.
- The transport coefficients are non negative in the relaxation time approximation which is a consequence of Landau-Lifshitz conditions of fit.
- Relaxation times are estimated using meson meson scattering, quark scattering through meson exchange as well as quark meson scattering.
- Medium dependence of meson masses and widths affect the relaxation time and hence the transport coefficients.

Thank you