Dilepton rate from a strongly magnetised hot and dense medium

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The Myriad Colorful Ways of Understanding Extreme QCD Matter

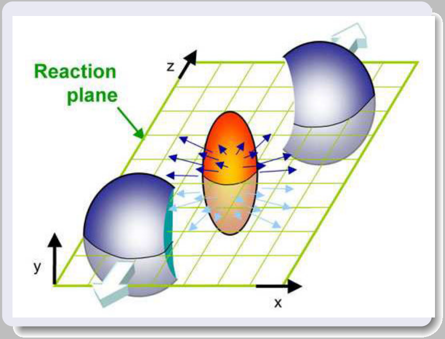
ICTS, Bengaluru

04.04.2019

Outline:

- Motivation
- Formalism
- Results
- Conclusion

Noncentral Heavy Ion collisions:



Pictorial representation of noncentral heavy ion collisions.

- A very strong magnetic field ($eB \approx m_\pi^2$ at RHIC) is generated in the direction perpendicular to the reaction plane, due to the relative motion of the ions themselves.

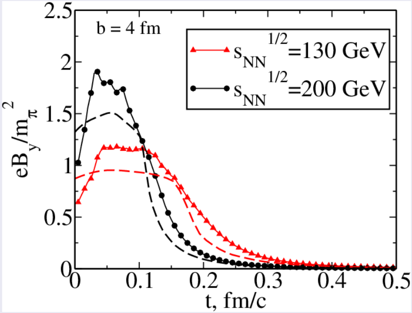
  ($m_\pi^2 = 1.96 \times 10^{-2}$ GeV$^2 \approx 10^{18}$ Gauss)

- The very high initial magnitude of this magnetic field then decreases very fast, being inversely proportional to the square of time(?).

  [A. Bzdak and V. Skokov, PRL,2013]

  [K. Tuchin, PRC,2013]

[V. Skokov et al, IJMP,2009]



- The presence of an external field in the medium subsequently requires modification of the present theoretical tools.

Regimes of study and our work:

- Strong magnetic field

  The initial magnitude of this magnetic field can be very high ($eB \approx 15 m_\pi^2$ at LHC) at the time of the collision.

- Weak magnetic field

  As the field strength decreases very fast, by the time the gluons and quarks thermalise, the temperature becomes the largest of the energy scales.

- Arbitrary magnetic field.

- We work in the strong magnetic field regime.
  [A. Bandyopadhyay, CAI and M. G. Mustafa, PRD,2016]
  [CAI *et al.*, arXiv:1812.10380]

Some generalities about correlation function:

- CF and its spectral representation are extensively used mathematical tools applied in almost every branch of physics.
- Generically it describes how microscopic variables co-vary with response to one another.
- Here, our main aim will be to calculate current-current CFs in a magnetised background.
- These CFs and their spectral representation reveal dynamical properties of many particle system and many of the hadronic properties.
- Here we will only talk about dilepton rate.

Euclidean current-current correlation function:

- Thermal Current-Current Correlator in Euclidean time τ:

$$\begin{aligned}\mathcal{G}^E(\tau,\vec{x}) &= \langle \mathcal{T}(J(\tau,\vec{x})J^\dagger(0,\vec{0}))\rangle_\beta \\ &= T\sum_{n=-\infty}^{\infty}\int \frac{d^3q}{(2\pi)^3}\; e^{-i(\omega_n\tau+\vec{q}\cdot\vec{x})}\;\mathcal{G}^E(\omega_n,\vec{q})\end{aligned}$$

- Currents: $J=\bar{\psi}(\tau,\vec{x})\Gamma\psi(\tau,\vec{x})$; $\Gamma=\mathbb{I},\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5$.

- The spectral function $\sigma_H(\omega,\vec{q})$ can be obtained through analytic continuation of $\mathcal{G}_H^E(\omega_n=\omega+i\epsilon)$ in full complex plane

$$\sigma_H(\omega,\vec{q}) = \frac{1}{\pi}\mathrm{Im}\;\mathcal{G}_H^E(\omega+i\epsilon,\vec{q})$$

- $H=(00,ii,V)$ denotes (temporal, spatial, vector).

Dilepton rate from the spectral function:

- The differential dilepton production rate in terms of spectral function:

$$\frac{dR}{d^4x d^4Q} = \frac{5\alpha^2}{54\pi^2}\frac{1}{M^2}\frac{1}{e^{\omega/T}-1}\sigma_V(\omega,\vec{q})$$

with, $\alpha = \frac{e^2}{4\pi}$; $Q \equiv (q_0 = \omega, \vec{q})$ and $q = |\vec{q}|$, invariant mass $M = \sqrt{\omega^2 - q^2}$.

Fermions in a constant magnetic field:

- The Lagrangian density of a fermion in a constant magnetic field ($B = B\hat{z}$) is given by,

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi$$
$$D_{\mu} = \partial_{\mu} - ieA_{\mu}^{ext}$$

- The orbital energy levels get discretized, which are known as the Landau Levels (LL).

$$k_{\shortparallel}^2 - m_f^2 - 2nq_fB = k_0^2 - k_3^2 - m_f^2 - 2nq_fB = 0$$
$$\Longrightarrow E_n = k_0 = \sqrt{k_3^2 + m_f^2 + 2nq_fB}.$$

$n = 0, 1, 2, \cdots$

- Each LL is degenerate.

LLL Fermion propagator and Dimensional reduction:

- For LLL approximation in the strong field limit the fermion propagator reduces to a simplified form as

$$iS_{ms}(k) = ie^{-k_\perp^2/q_f B} \; \frac{\not{k}_{\shortparallel} + m_f}{k_{\shortparallel}^2 - m_f^2}(1 - i\gamma_1\gamma_2),$$

- As $k_\perp^2 << q_f B$, an effective dimensional reduction from (3+1) to (1+1) takes place in the strong field limit.

- As a consequence the motion of the charged particle is restricted in the direction perpendicular to the magnetic field but can move along the field direction in LLL.

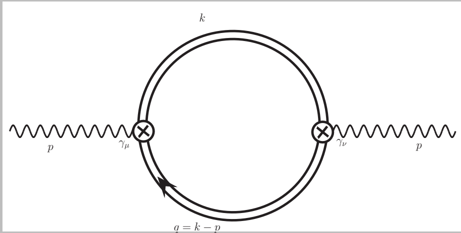
General expression for one loop correlator:

The electromagnetic spectral representation is extracted from the imaginary part of the two point correlation function $\Pi^{\mu}_{\mu}(p)$ as

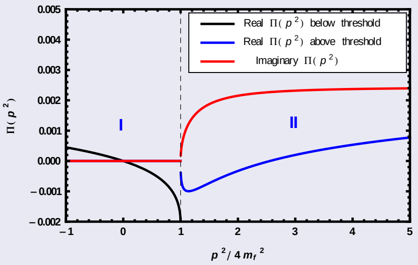
$$\sigma(p) \;=\; \frac{1}{\pi}\mathrm{Im}\ \chi(\mathrm{p}),$$

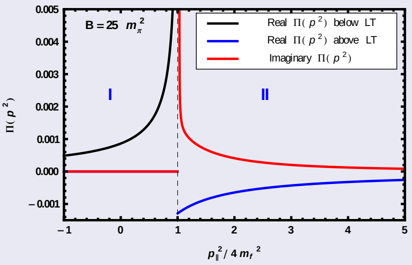
where $\chi(p) = \sum_f \ (1/q_f^2)\ \Pi^{\mu}_{\mu}$.

$$\Pi_{\mu\nu}(p)\Big|_{sfa} \quad = -i\sum_f q_f^2 \int \frac{d^4k}{(2\pi)^4}\mathsf{Tr}_c\left[\gamma_\mu S_{ms}(k)\gamma_\nu S_{ms}(q)\right]$$



Kinematic regions with respect to LT:





- Plot of $\Pi(p^2)$ as a function of scaled photon momentum square in kinematic regions I ($p_{||}^2 < 4m_f^2$) and II ($p_{||}^2 > 4m_f^2$), both in absence of a magnetic field (left panel) and in presence of a strong magnetic field (right panel).

EM Spectral function in medium:

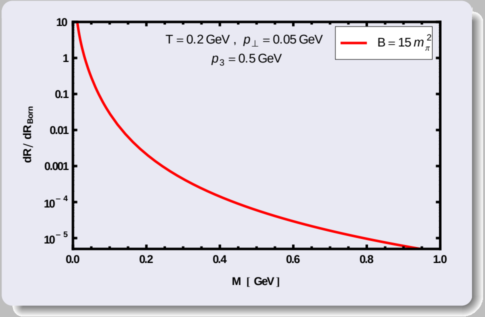
- The spectral function in strong field approximation is obtained as, [A. Bandyopadhyay, CAI and M. G. Mustafa, PRD,2016]

$$
\begin{aligned}
\sigma\Big|_{sfa} &= \frac{1}{\pi}\mathcal{I}m\ \chi(p)\Big|_{sfa} \\
&= N_c \sum_f \frac{q_f B m_f^2}{\pi^2 p_{\shortparallel}^2}\ e^{-p_\perp^2/2q_f B}\ \Theta\left(p_{\shortparallel}^2 - 4m_f^2\right)\left(1 - \frac{4m_f^2}{p_{\shortparallel}^2}\right)^{-1/2} \\
&\quad \times\Big[1 - n_F(p_+^s) - n_F(p_-^s)\Big]
\end{aligned}
$$

where

$$
p_\pm^s = \frac{\omega}{2} \pm \frac{p_3}{2}\sqrt{\left(1 - \frac{4m_f^2}{p_{\shortparallel}^2}\right)}.
$$

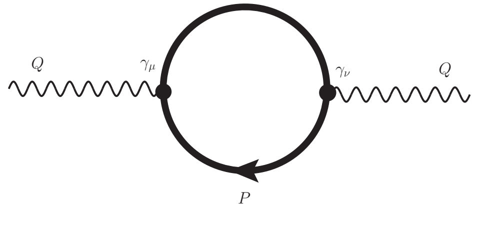
Dilepton rate in strong magnetic field:



- The LLL dynamics in strong field approximation enhances the dilepton rate as compared to the Born rate for a very low invariant mass ($\leq$ 100 MeV).
- At high invariant mass it falls off very fast; however such low mass enhancement is beyond the scope of the present detectors involved in HIC experiments.

[A. Bandyopadhyay, CAI and M. G. Mustafa, PRD,2016]

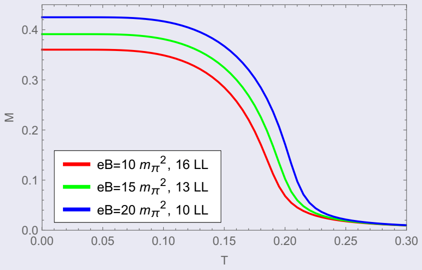
One loop correlator in light of mean field model:



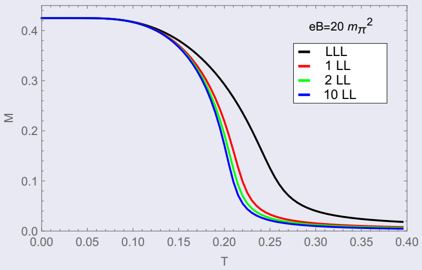
- Dynamical generation of quark mass in presence of background magnetic field is taken into account through effective mean field models.[CAI *et al.*, arXiv:1812.10380]

Dynamical mass generation in presence of B (NJL):

Effective mass



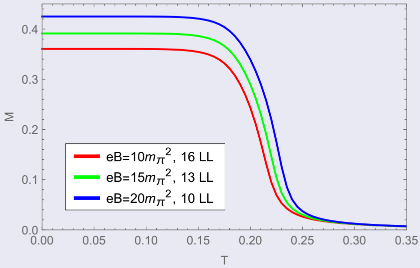
Validity of LLL



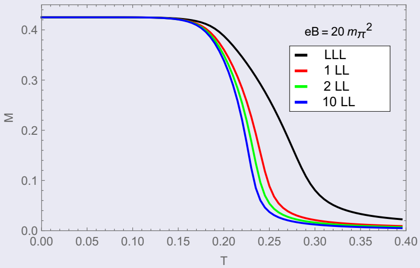
- The constituent mass increases with the increase of magnetic field, showing the MC effect.
- As in our main results of spectral properties, we will be using LLL approximation
- We have also shown the validity of the LLL approximation as a function of T for a given magnetic field.

Dynamical mass generation in presence of B (PNJL):

Effective mass



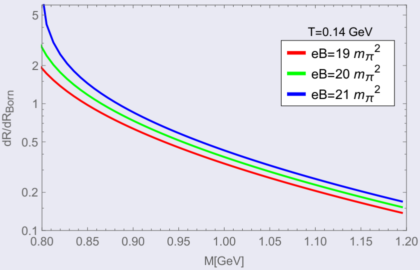
Validity of LLL



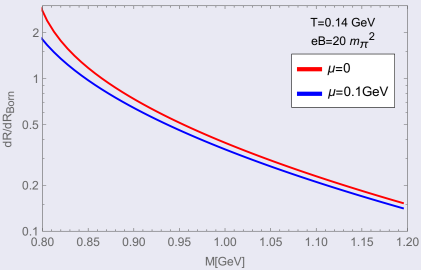
- Here also, expectedly, we observe MC effect.
- For a given magnetic field the validity of LLL approximation in PNJL model is up to a higher value of T as compared to NJL one.

Dilepton rate in NJL:

$\mu = 0$



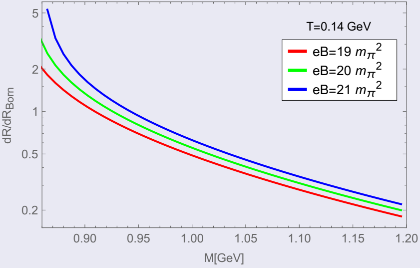
$\mu \neq 0$



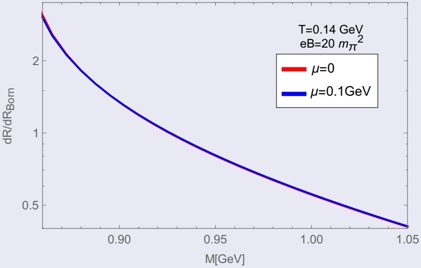
- The enhancement is observed for a range of invariant mass starting at $M \sim 800$ MeV as compared to ~ 100 MeV in the previous case.
- For a given temperature it increases with the increase of magnetic field and decreases with the increase of μ.

Dilepton rate in PNJL:

$\mu = 0$



$\mu \neq 0$



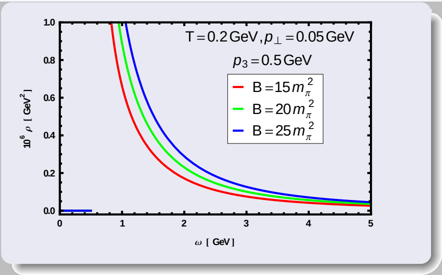
- In PNJL the enhancement is observed for a range of invariant mass starting at $M \sim 850$ MeV.
- Similar tendencies as those in NJL are observed except the change for μ is milder.

Conclusion:

- The dilepton rate for a strongly magnetised hot nuclear matter have been investigated.
- The LLL dynamics in strong field approximation enhances the dilepton rate as compared to the Born rate for a very low invariant mass (≤ 100 MeV).
- Dynamical mass generation plays a crucial role in the spectral properties like dilepton rate.
- It (dilepton rate) gets enhanced as compared to the Born rate in presence of such magnetic field dependent effective mass.
- For NJL and PNJL models the enhancement is observed for a range of invariant mass starting at $M \sim 800$ and 850 MeV, respectively.

Thank You

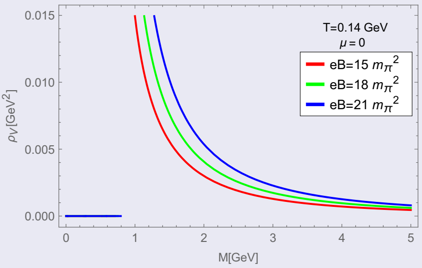
Plot of EM spectral function:



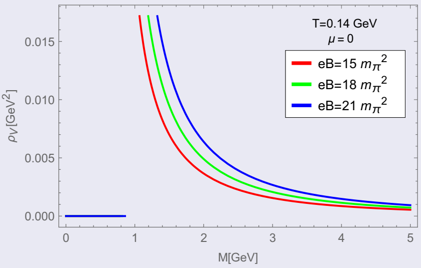
- It vanishes in the massless limit of quarks because of the dimensional reduction.
- Like vacuum case here also it vanishes below the threshold because the polarization tensor is purely real below the threshold.
- When $p_{\shortparallel}^2 = 4m_f^2$, the spectral strength diverges because of the factor $\left(1 - 4m_f^2/p_{\shortparallel}^2\right)^{-1/2}$ that appears due to the dimensional reduction.

SF in light of mean field model:

NJL



PNJL



- The enhancement is observed for a range of invariant mass starting at $M \sim 800$ MeV as compared to ~ 100 MeV in the previous case.
- For a given temperature it increases with the increase of magnetic field and decreases with the increase of μ.