Equation of State and Fluctuations of Conserved Charges

Equation of state in SU(3) gauge theory and QCD at zero density

Taylor expansion and fluctuations of conserved charges: lattice vs. HRG and weak coupling expansion

Equation of State at non-zero baryon density

Integral method: equation of state in SU(3) gauge theory

In Monte-Carlo simulations ln *Z(T)* cannot be determined but only its derivatives

Boyd et al., NPB 496 (1996) 167

$$\frac{\partial \ln Z(\beta)}{\partial \beta} = \frac{1}{Z(\beta)}\frac{\partial}{\partial \beta}\int \mathcal{D}U e^{-\beta S_g(U)} = - < S_g >$$

$$\frac{p(T)}{T^4} = \int_{\beta_0}^{\beta(T)} d\beta' \left(\frac{\partial \ln Z(T)}{\partial \beta'} - \frac{\partial \ln Z(T=0)}{\partial \beta'}\right) = \int_{\beta_0}^{\beta(T)} d\beta' \left(\langle S_g\rangle_0 - \langle S_g\rangle_T\right)$$

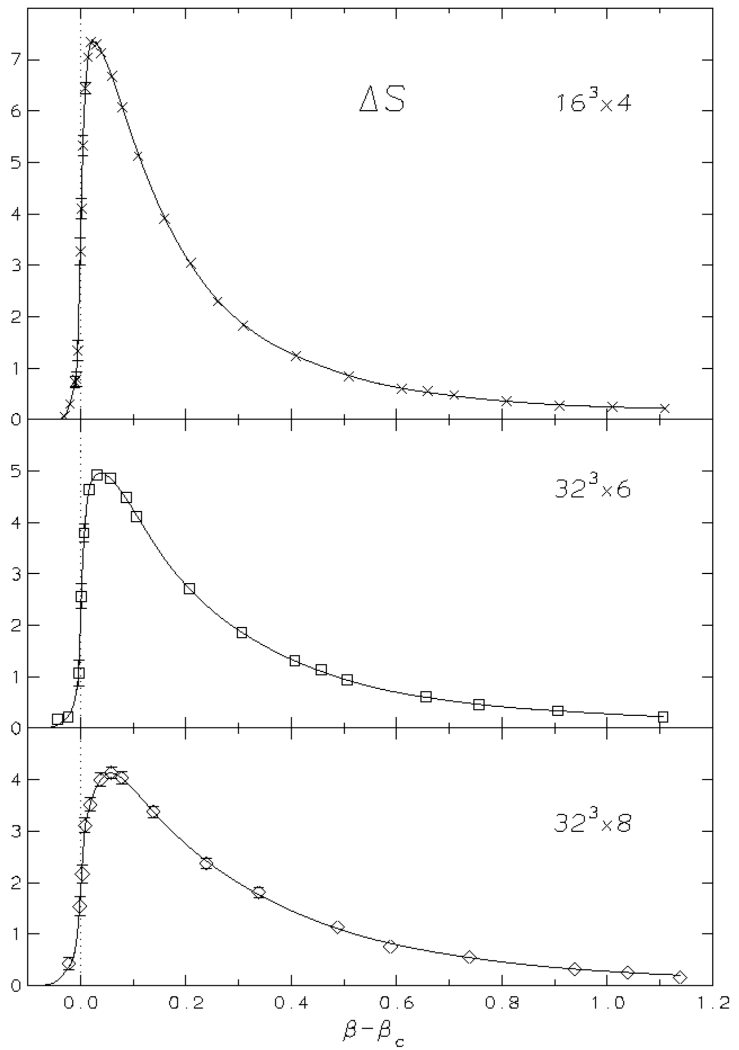
$$s = (\epsilon + p)/T = \frac{\partial p}{\partial T}$$

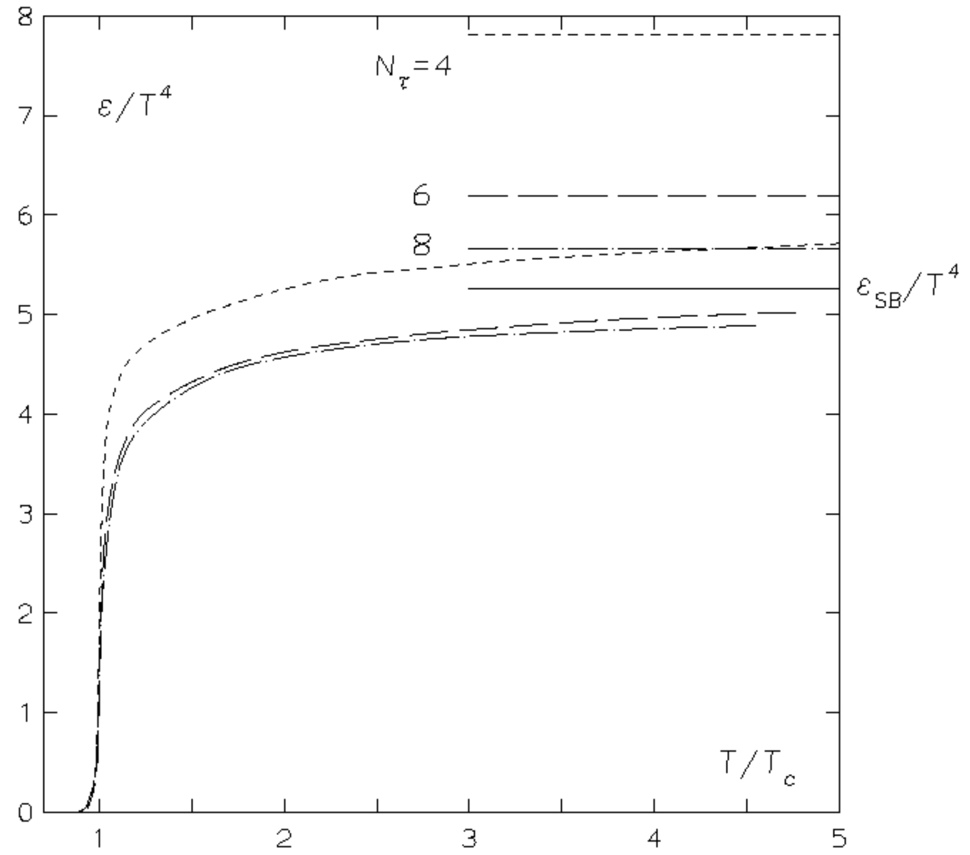
$$\frac{\epsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) = T\frac{d\beta}{dT}\frac{\partial p/T^4}{\partial \beta} = -\left(a\frac{d\beta}{da}\right)\frac{\partial p/T^4}{\partial \beta}$$

computational cost go as N_τ^4 because of the vacuum subtraction

large cutoff effects !

Boyd et al., NPB496 (1996) 167

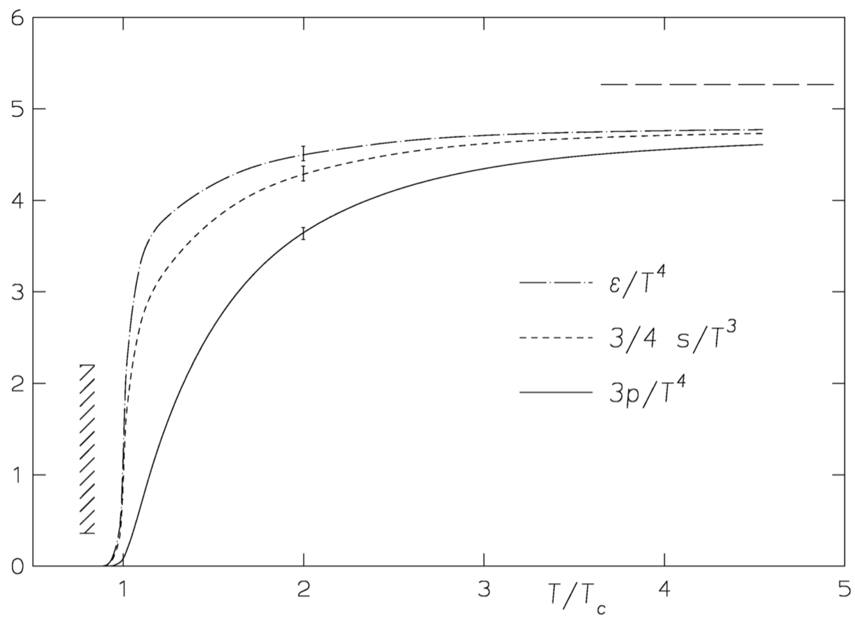




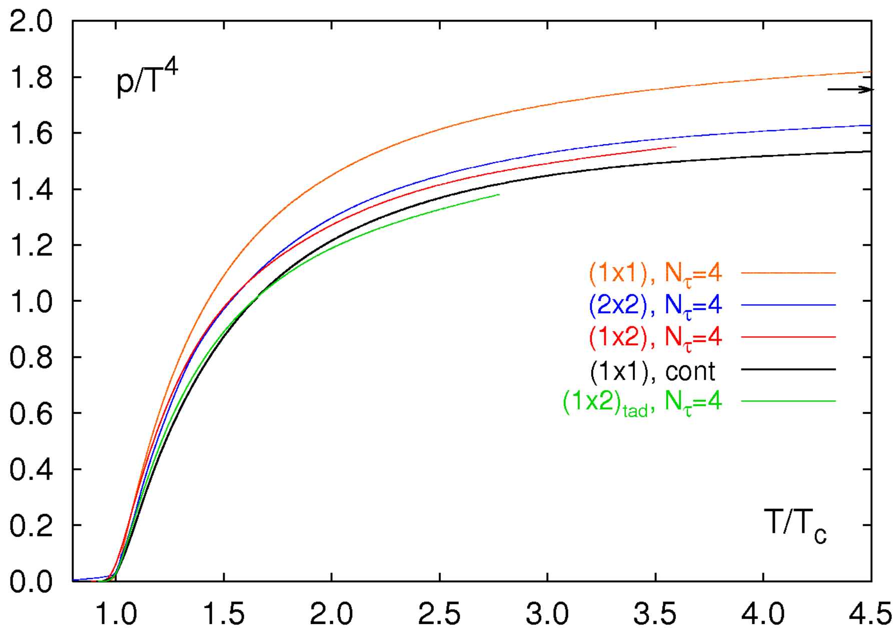
the free gas limit overestimates cutoff effects

Wilson gauge action a^2 discretization errors => $1/N_\tau^2$ corrections to the pressure

Boyd et al., NPB 496 (1996) 167
Wilson gauge action
continuum extrapolation



Karsch et al, EPJ C 6 (99) 133
Luescher-Weisz gauge action:
large reduction of cutoff effects

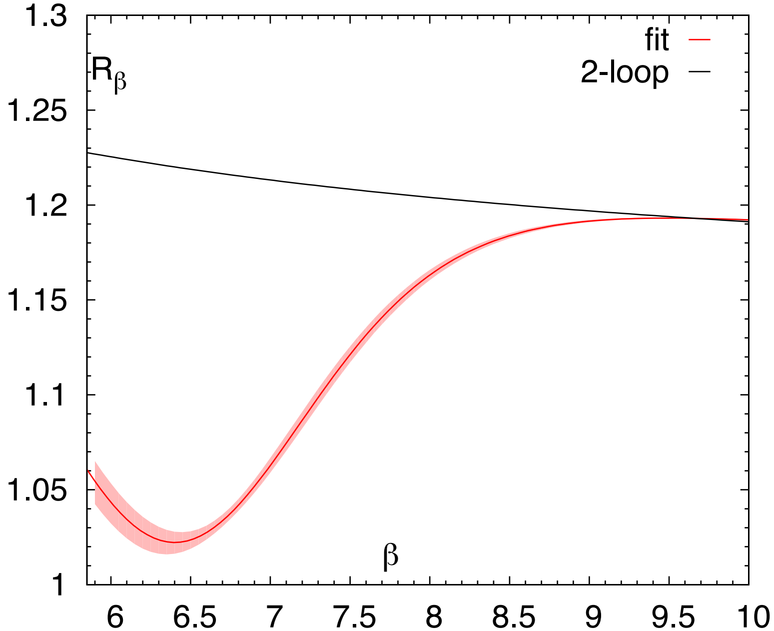


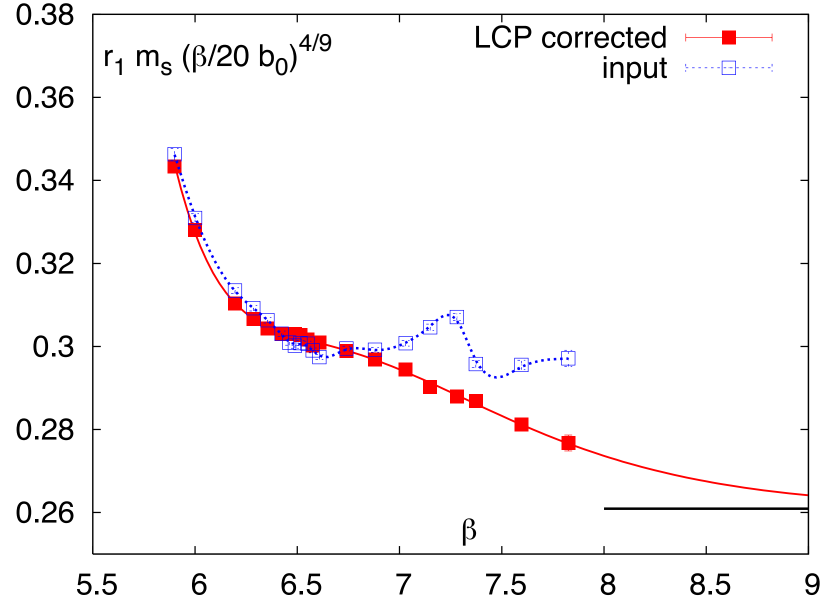
QCD trace anomaly and the integral method

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) \Rightarrow \frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = R_\beta \{\langle S_G\rangle_0 - \langle S_G\rangle_T\} - R_\beta R_m \{2m_l(\langle\bar{q}q\rangle_0 - \langle\bar{q}q\rangle) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)\}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da},\ R_m = \frac{1}{m_q(\beta)}\frac{dm_q(\beta)}{d\beta},\ \beta = 10/g^2$$





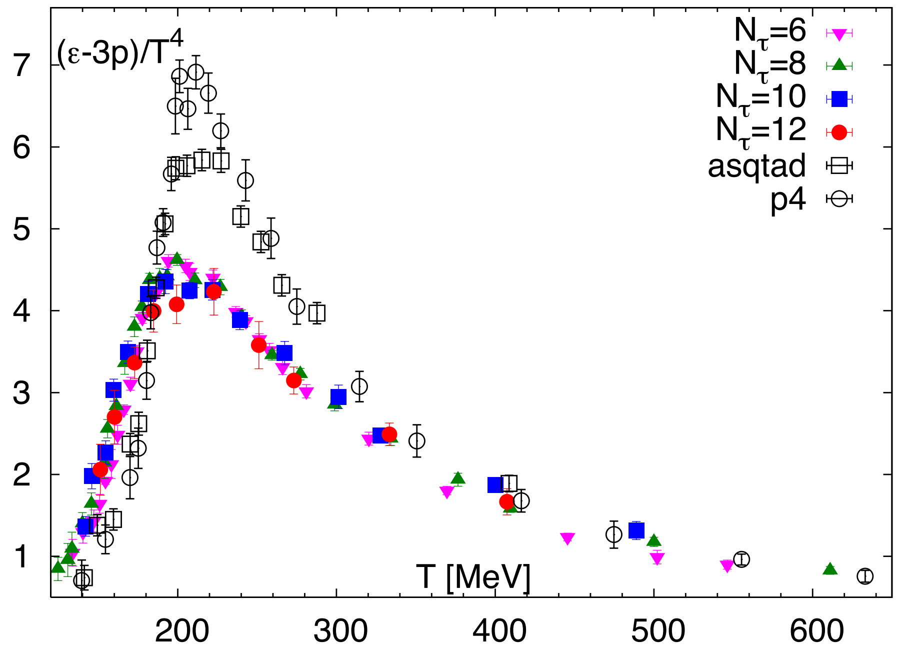
Bazavov, PP, Weber, PRD97 (2018) 014510

Bazavov et al, PRD 90 (2014) 094503

QCD results on the trace anomaly

2+1 flavor QCD calculations with almost physical light and strange quark masses

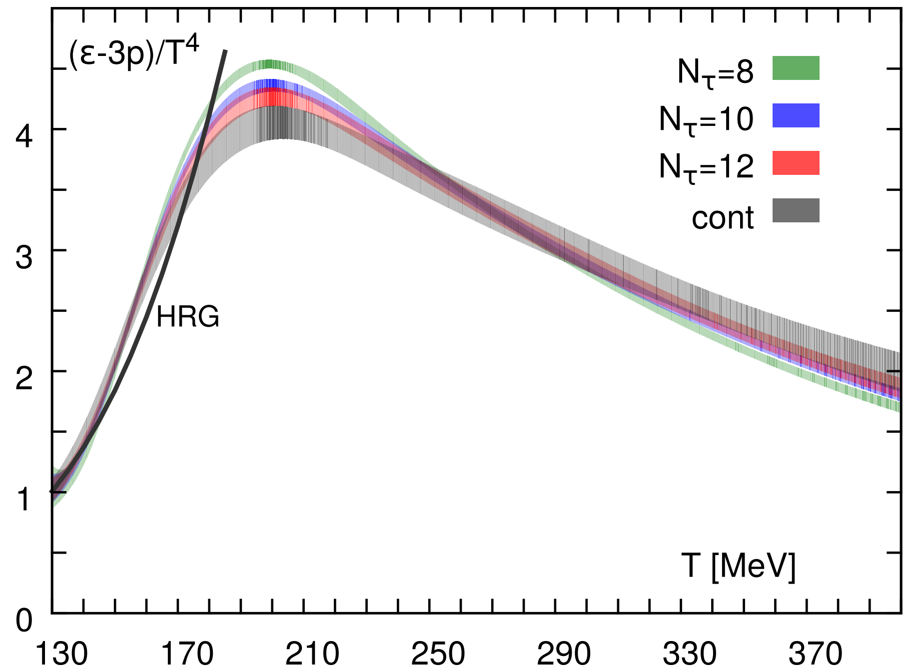
Bazavov et al, PRD 90 (2014) 094503



The peak height is much reduced compared to the asqtad and p4 $N_\tau=8$ calculations

Agreement with p4 and asqtad calculations for $T>350$ MeV

Small cutoff effects for HISQ except for $N_\tau=6$

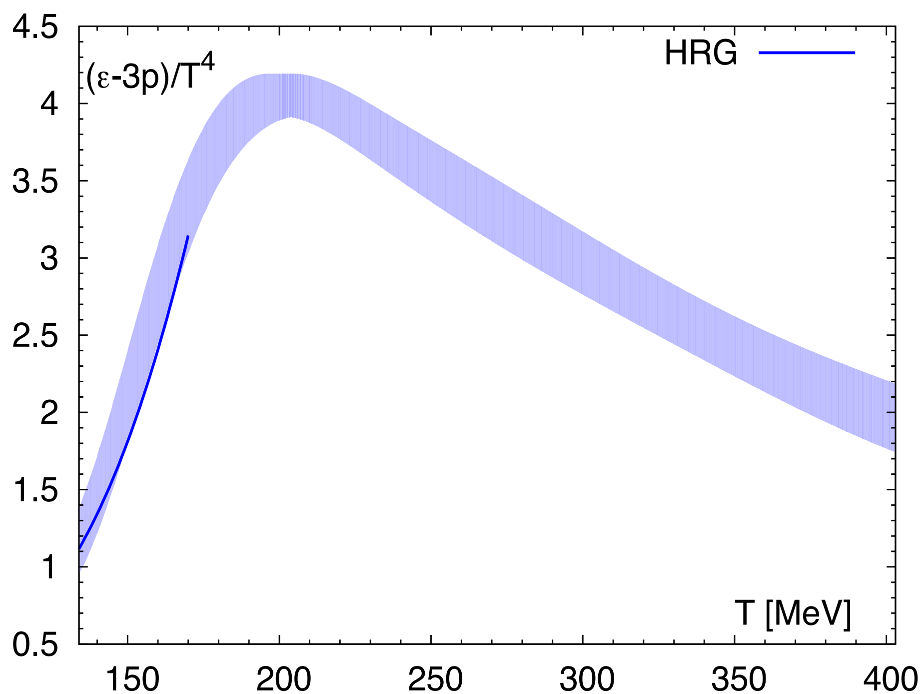


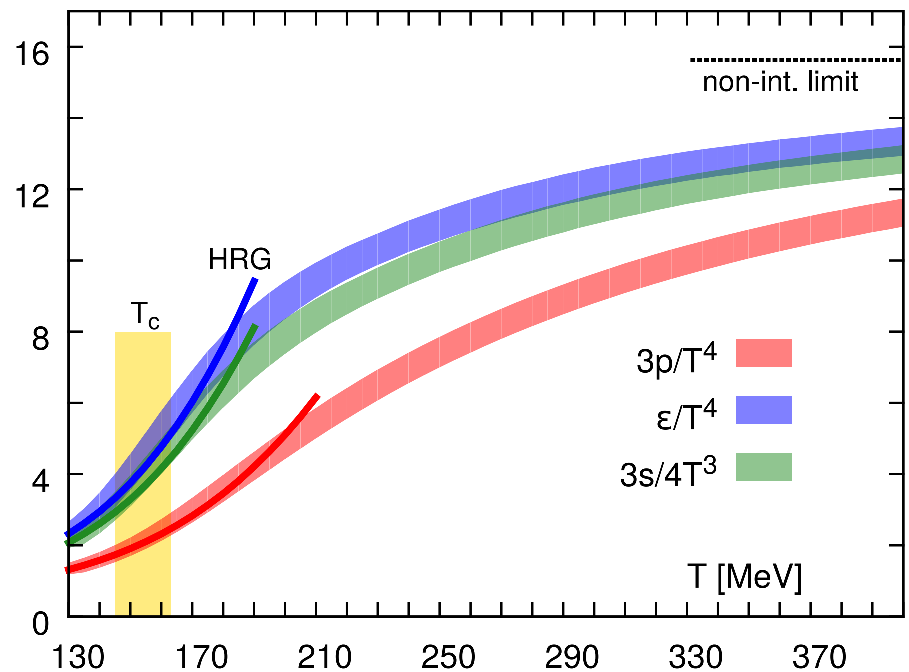
Perform spline interpolation of all the $N_\tau>6$ data with spline coefficients having $a+b/N_\tau^2$ *form*, stabilize the spline demanding that ϵ-$3p$ is given by HRG at $T=130$ MeV

QCD thermodynamics in the continuum limit

Set the lower integration limit to $T_0=130$ MeV and take $p_0=p^{HRG}(T=130\text{ MeV})$ → $p(T)$

Bazavov et al, PRD 90 (2014) 094503



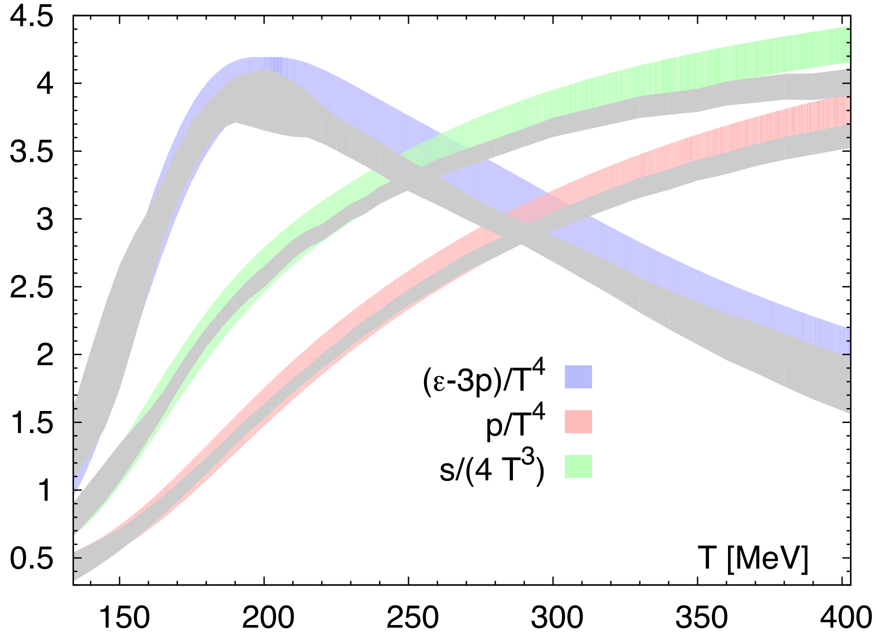


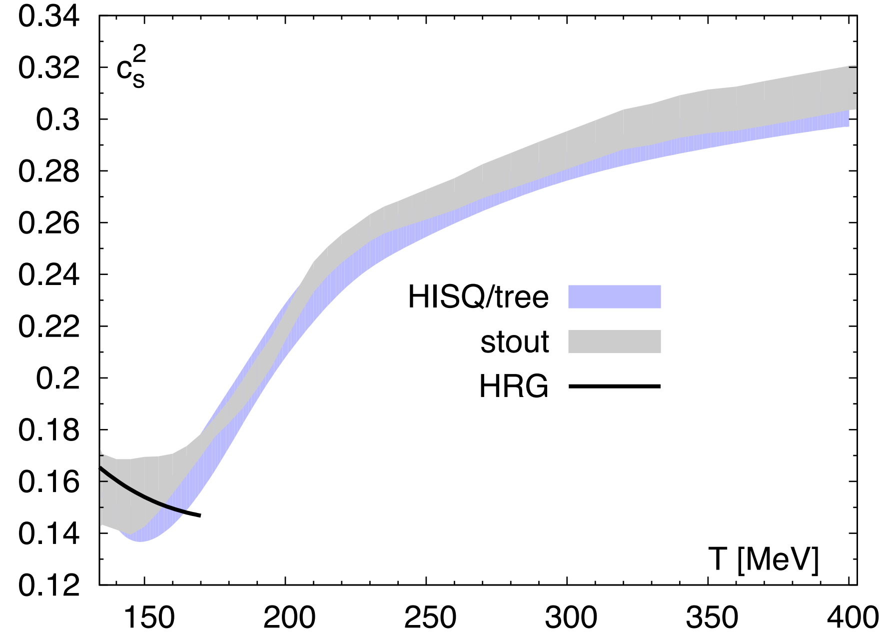
$$T_c = 156 \pm 1.5 \text{ MeV} \qquad \epsilon_{nucl} \simeq 150 \text{MeV/fm}^3$$

$$\epsilon_c = 420(60) \text{ MeV/fm}^3 \qquad \epsilon_{proton} \simeq 450 \text{MeV/fm}^3$$

HRG: all resonances from PDG treated as stable (zero width) particles in an ideal gas

Comparison of different continuum limit



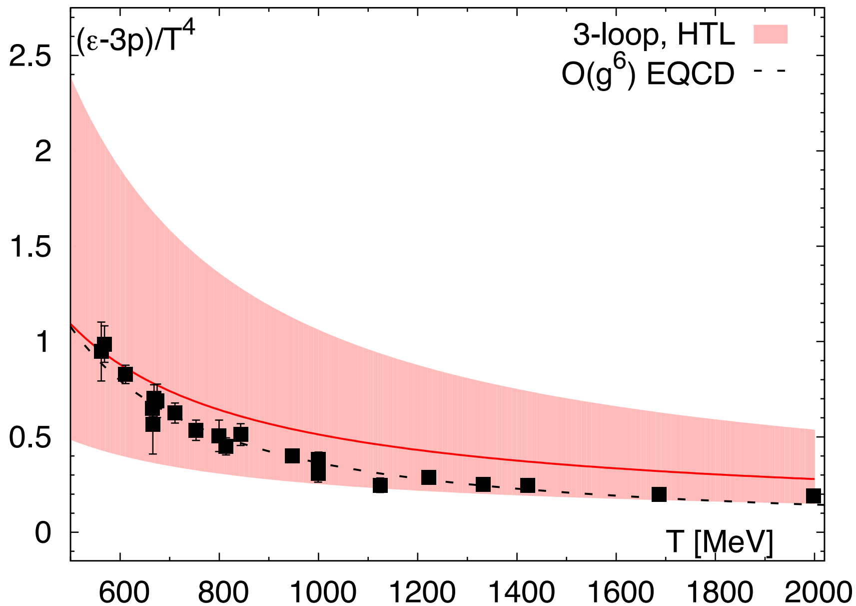


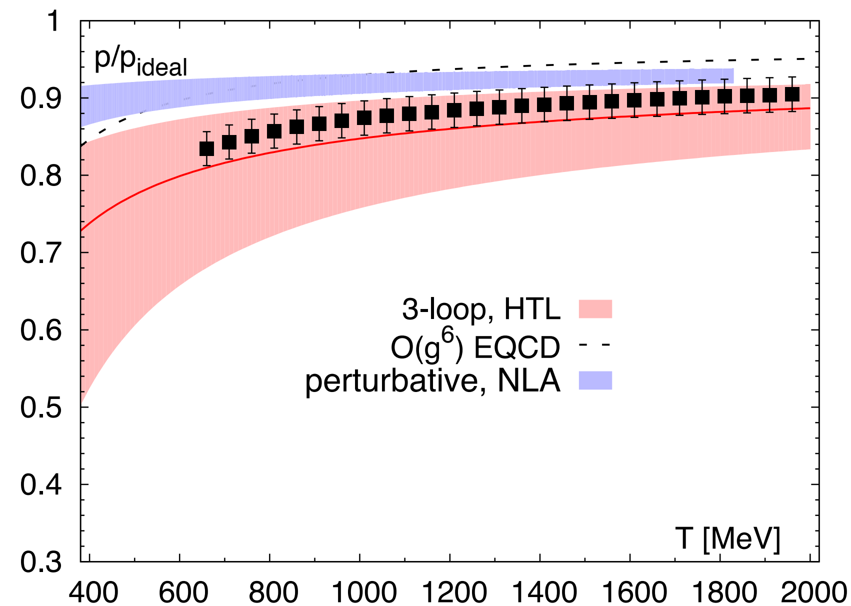
Continuum results obtained with stout and HISQ action agree reasonably well given their errors (some tension for the entropy density)

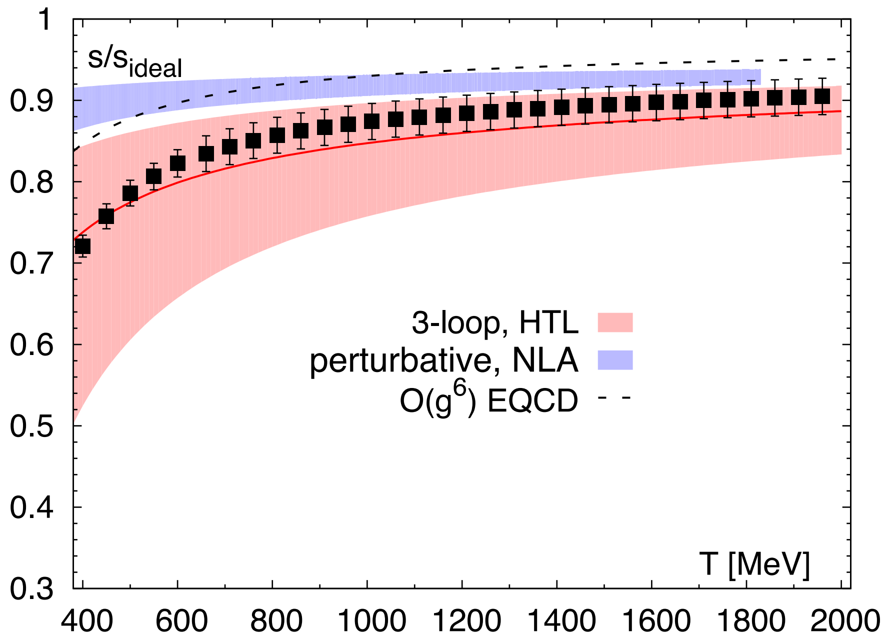
Even in the transition region the speed of sound is not much smaller than the HRG speed of sound (the EoS is never really soft)

HISQ: Bazavov et al, PRD 90 (2014) 094503
stout: Borsányi et al, PLB730 (2014) 99

Comparison of EoS with weak coupling results





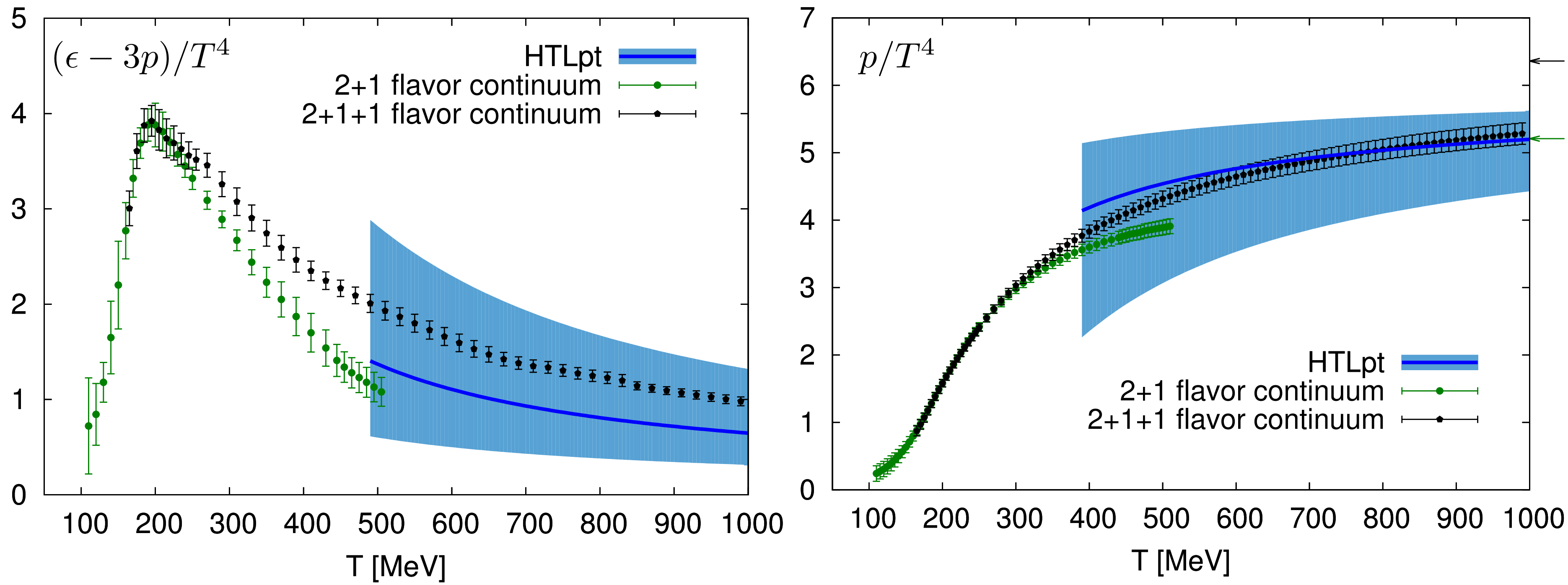


Reasonably good agreement
Between the lattice and the weak
coupling calculations
for *T>400* MeV

Bazavov, PP, Weber, PRD97 (2018) 014510

The role of charm quarks in QCD EoS

Borsányi et al, Nature 539 (2016) 69



The contribution of charm quarks becomes significant for *T>400* MeV

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{udsc} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l$$ quark

$$\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial \mu_b^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^i} \frac{\partial^l}{\partial \mu_d^l} \ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d) \,|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the susceptibilities, i.e. the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}\left(\langle X^2\rangle - \langle X\rangle^2\right) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}\left(\langle XY\rangle - \langle X\rangle\langle Y\rangle\right)$$

information about carriers of the conserved charges (hadrons or quarks)

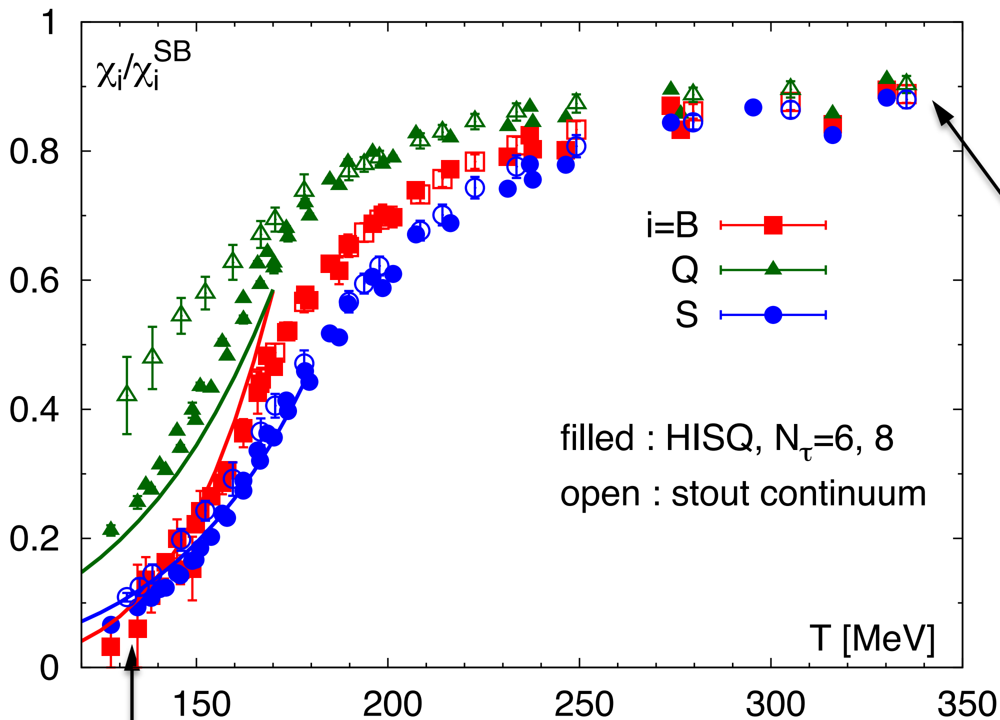
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

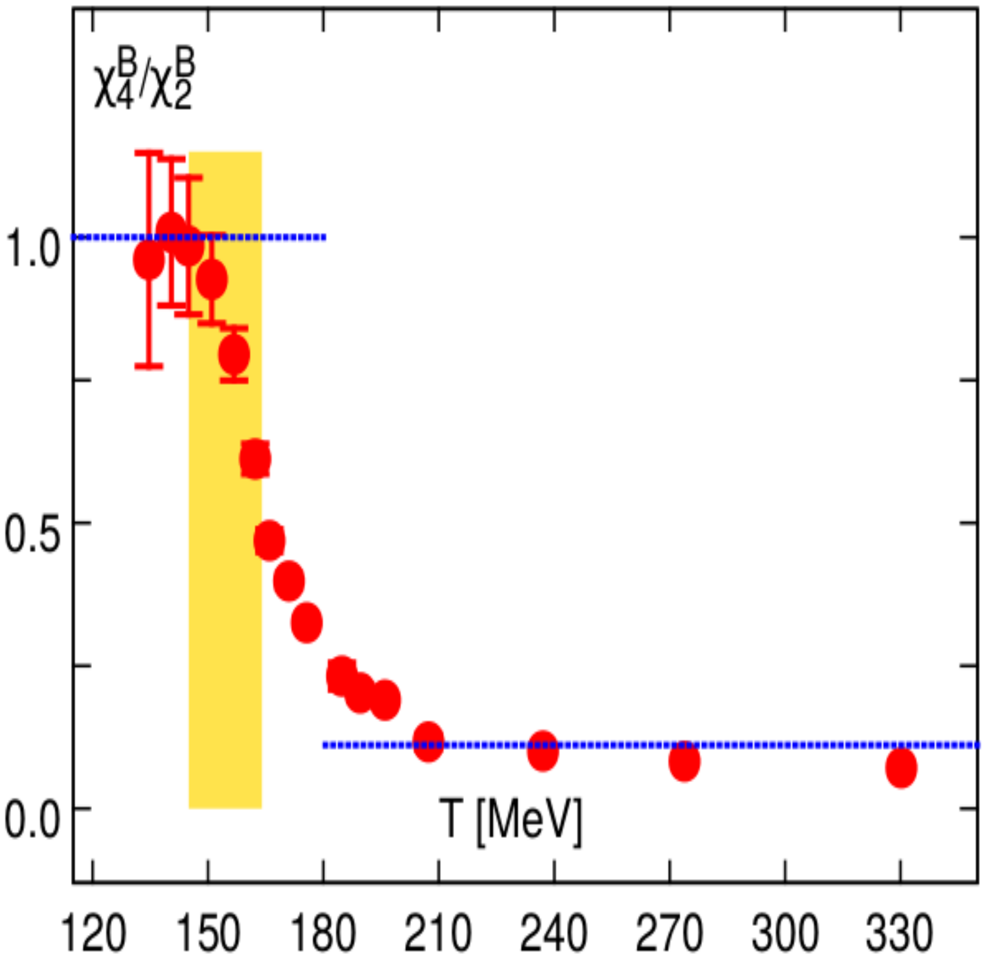
conserved charges carried by light quarks

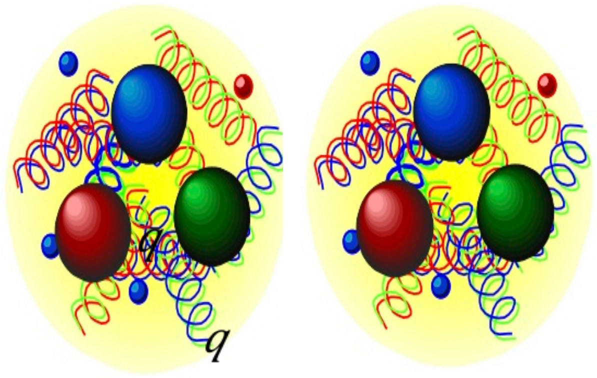
conserved charges are carried by massive hadrons

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

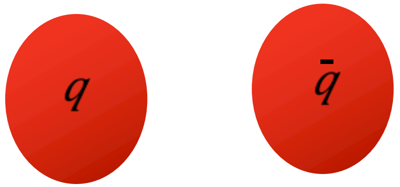
Deconfinement : fluctuations of conserved charges





B=1, -1

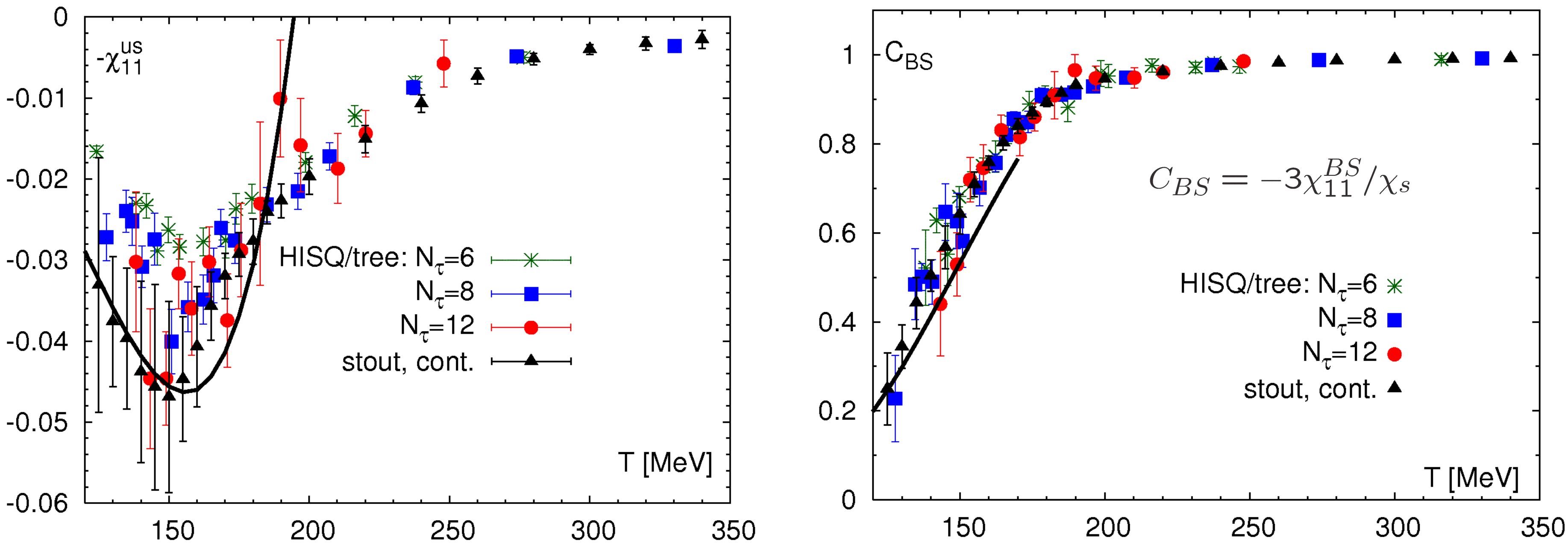
$$\chi_2^B = \langle B^2 \rangle = 1, \; \chi_4^B = \langle B^4 \rangle = 1$$



B=1/3, -1/3

$$\chi_2^B = \langle B^2 \rangle = \frac{1}{9}, \; \chi_4^B = \langle B^4 \rangle = \frac{1}{81}$$

Correlations of conserved charges



P.P. J.Phys. G39 (2012) 093002

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)* => weakly interacting quark gas

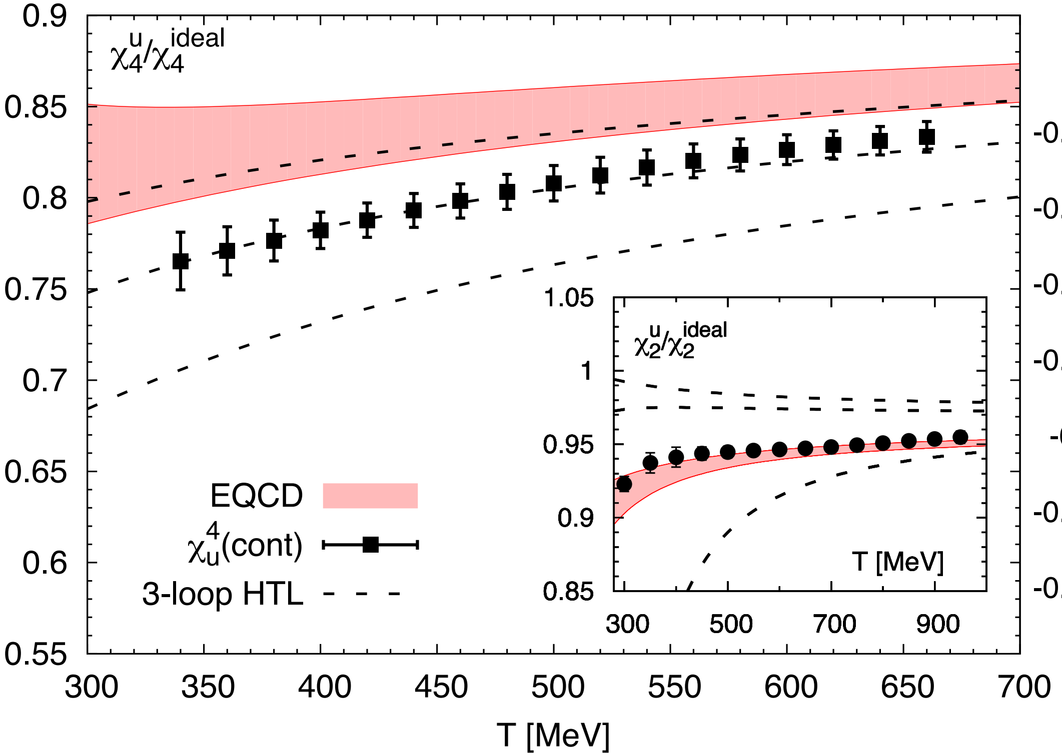
- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ (100-150) MeV

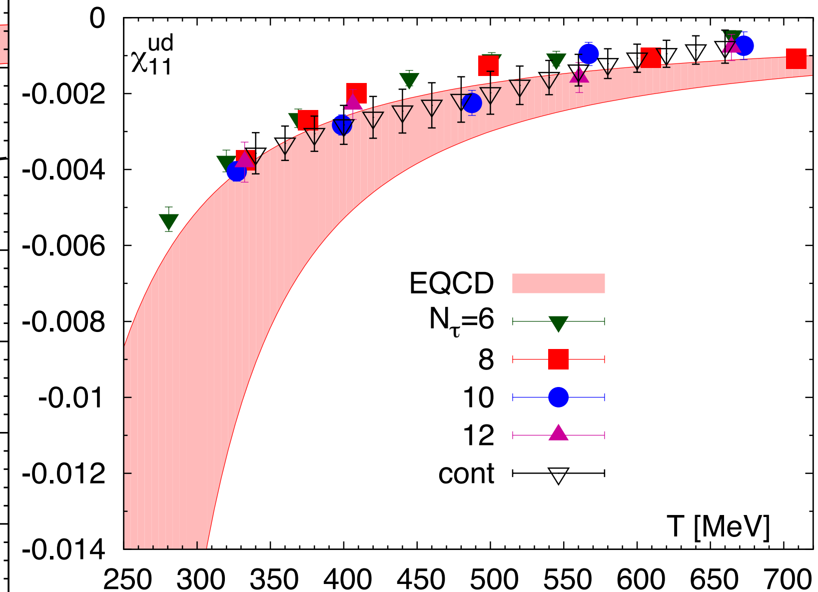
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations



- Good agreement between continuum extrapolated lattice results and the weak coupling approach
- Quark number correlations vanish at any loop order but can be calculated in EQCD and the EQCD calculations agree with the continuum extrapolated lattice results

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) +$$

$$B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

Strange hadrons are heavy treat them
As Boltzmann gas

$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

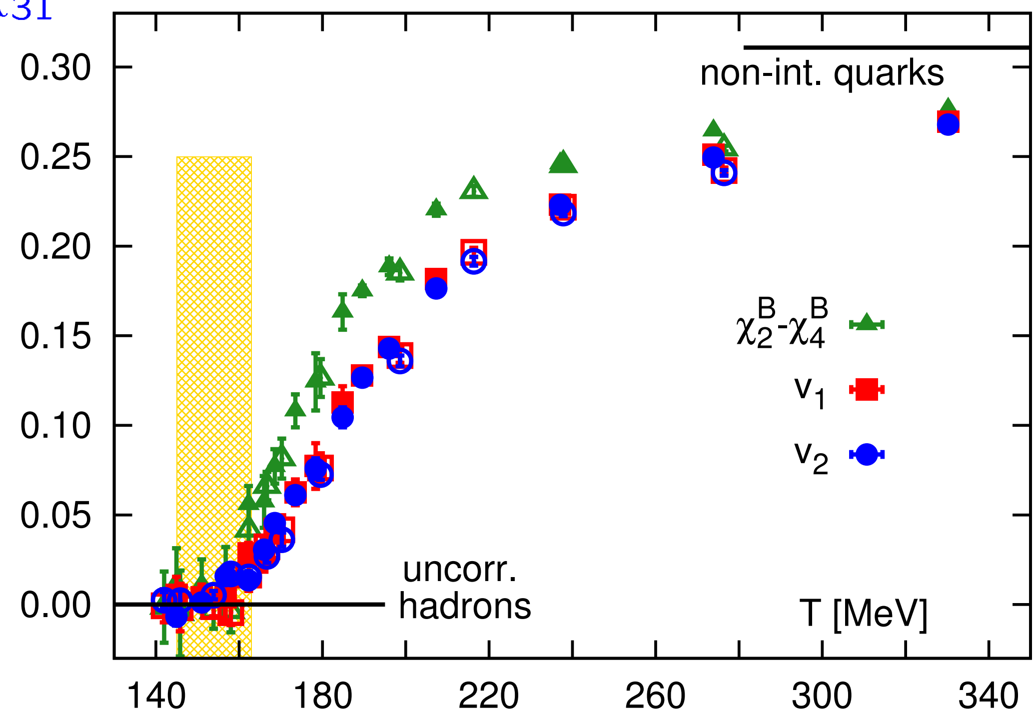
$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- v_1 and v_2 do vanish within errors at low T

- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Bazavov et al, PRL 111 (2013) 082301



Relativistic viirial expansion and Hadron Resonance Gas

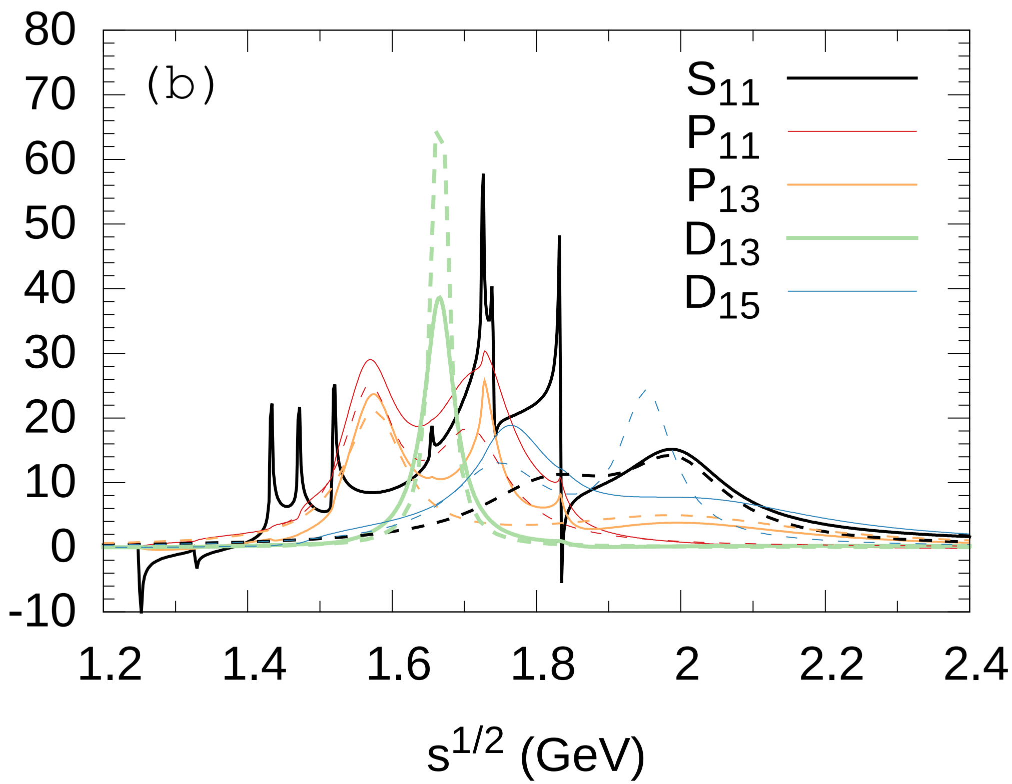
There are significant non-resonant meson-baryon interactions and overlapping resonances for example in the strange baryon sector

Fernandez-Ramirez, Lo, PP, PRC98 (2018) 044910

$$\Delta P_{int} = \sum_l \int dE E^2 T^2 K_2(E/T) \frac{d\delta_l(E)}{dE}$$

δ_l from coupled channel PWA analysis by JPAC

Fernandez-Ramirez et al, PRD 93 (2016) 034029

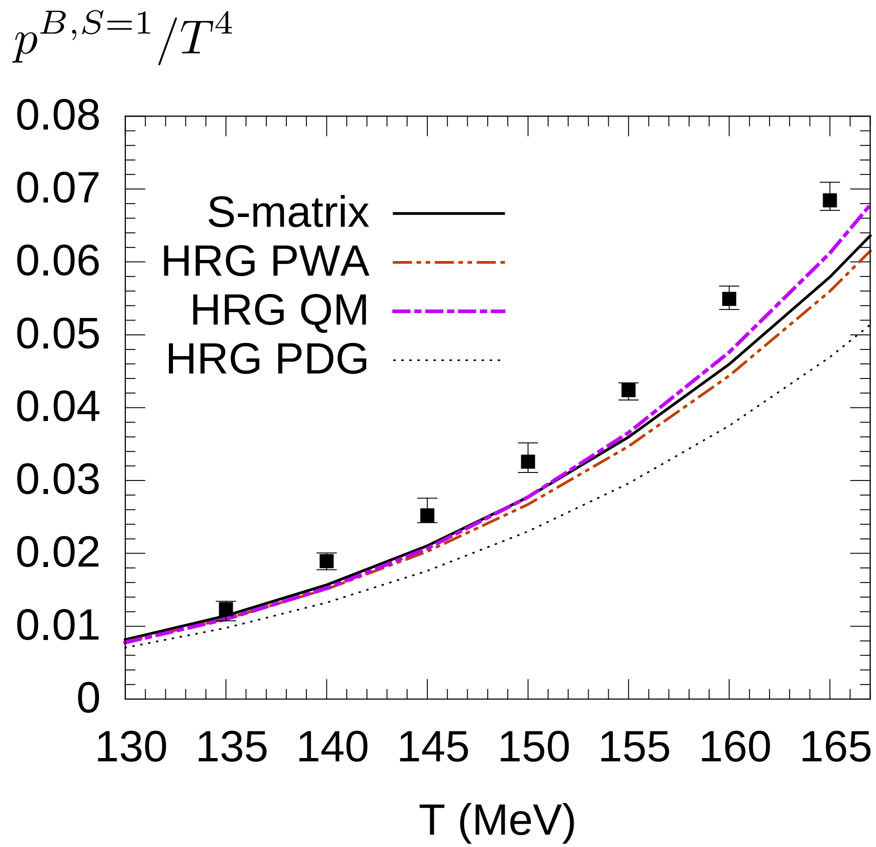


Partial pressures of *i=1* strange baryons in $10^{-3}\,T^4$

	$I = 1$		
	S-mat.	HRG	B-W
S_{11}	1.018	0.282	0.532
P_{11}	1.681	1.275	1.465
P_{13}	1.868	1.857	2.406
D_{13}	0.964	0.995	1.052
D_{15}	1.478	1.219	1.793
F_{15}	0.514	0.503	1.119
F_{17}	0.556	0.238	0.603
G_{17}	0.169	0.095	0.310

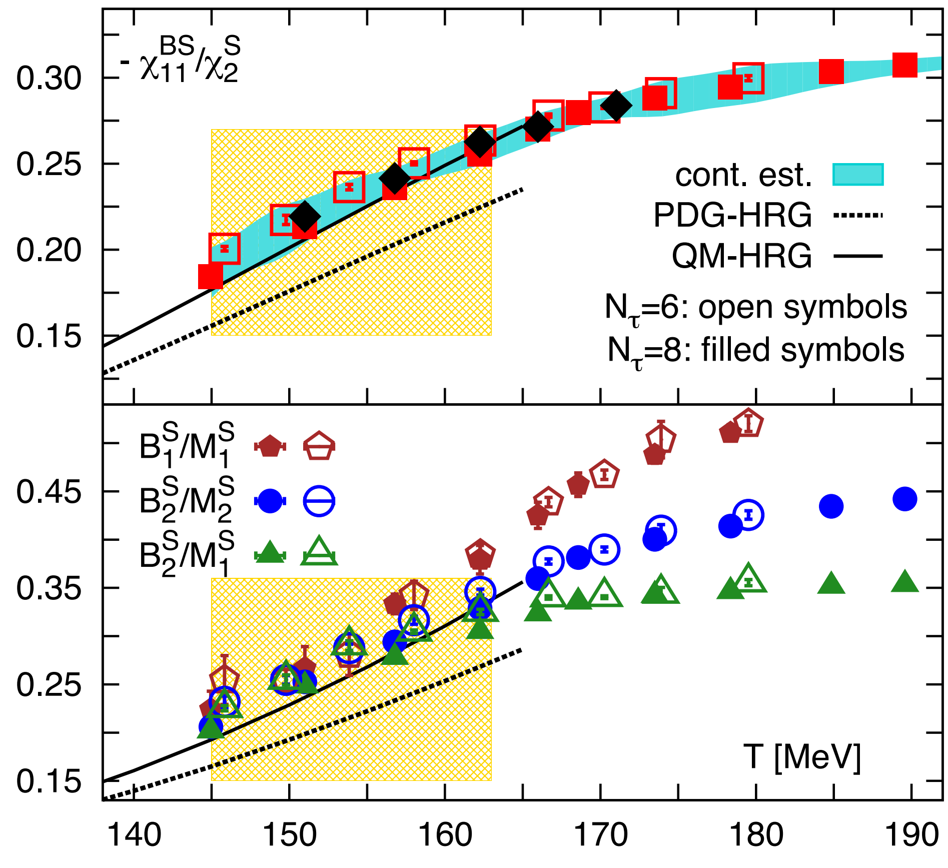
Yet the total strange baryon pressure is well approximated by HRG

Thermodynamics of strange hadrons and missing states



Partial pressure of S=1 baryons from S-matrix based virial expansion agrees with HRG that includes additional hyperon resonances in addition to PDG

Use excited strange hadrons from quark model to calculate the pressure from HRG



Lattice results agree with HRG that includes the "missing" states

Bazavov, PRL 113 (2014) 072001

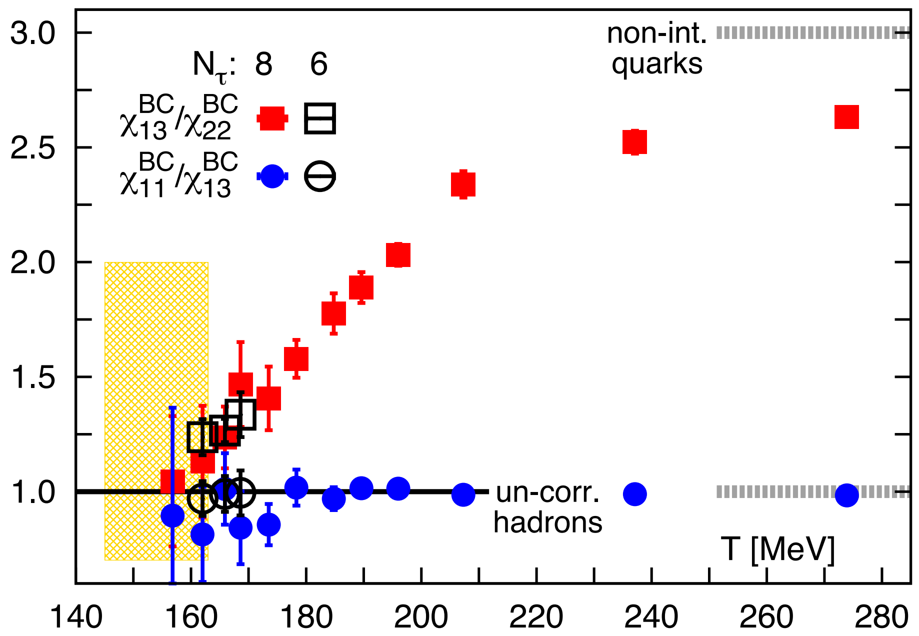
Charm fluctuations and correlations

$$\chi^{XYC}_{nml} = T^{m+n+l} \frac{\partial^{n+m+l} p(T,\mu_X,\mu_Y,\mu_C)/T^4}{\partial\mu_X^n \partial\mu_Y^m \partial\mu_C^l}$$

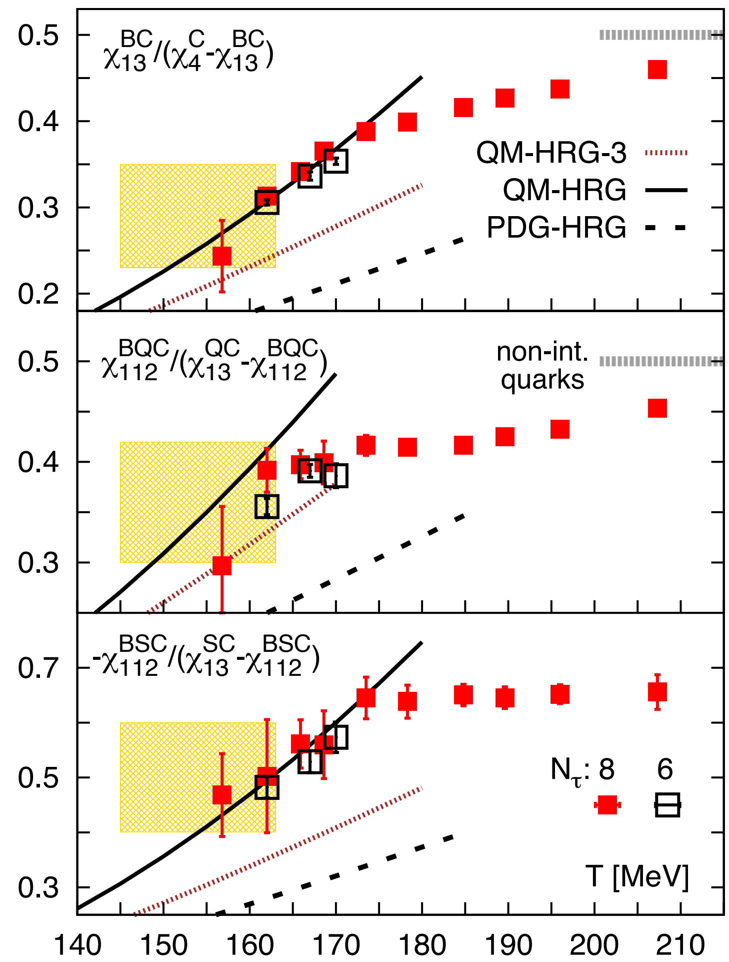
Bazavov et al, PLB 737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



Charm baryon to meson pressure



Hadronic description breaks down just above T_c

⇒ open charm deconfines above T_c

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included

Quasi-particle model for charm degrees of freedom

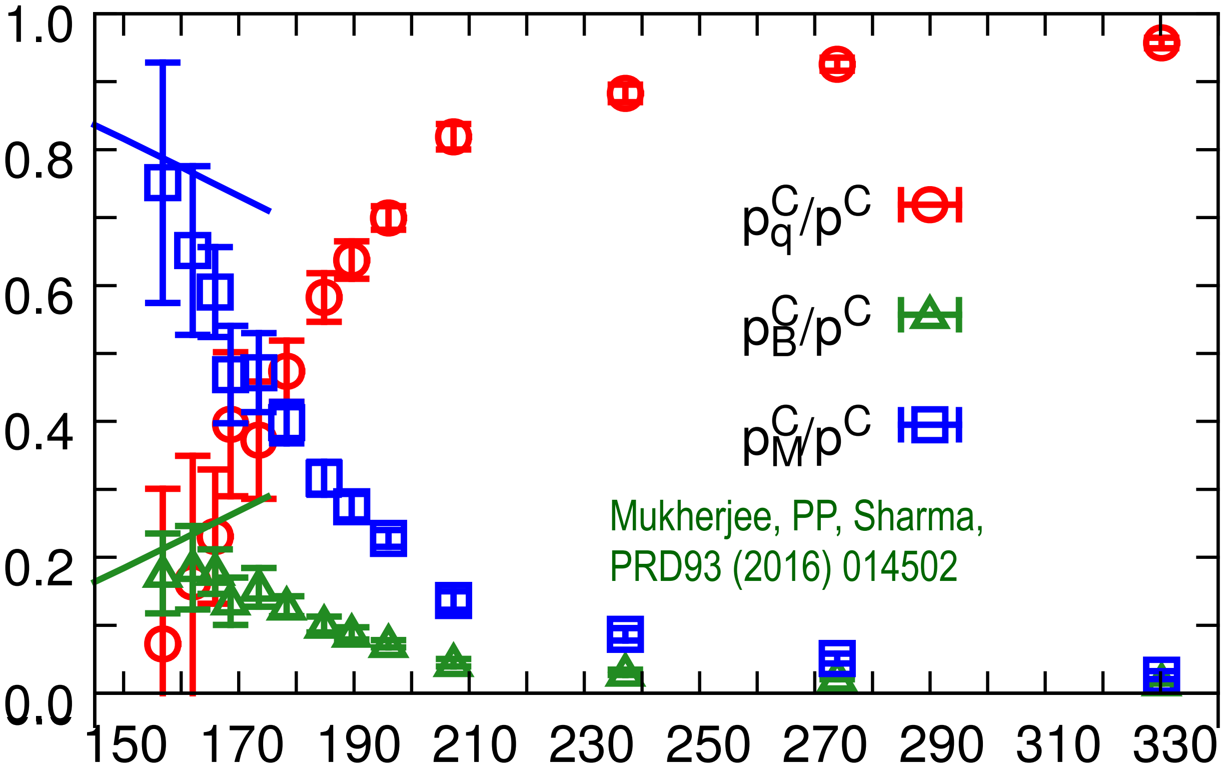
Charm dof are good quasi-particles at all *T* because $M_c >> T$ and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T)\cosh(\hat{\mu}_C)$$

$$\hat{\mu}_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at *T>200* MeV



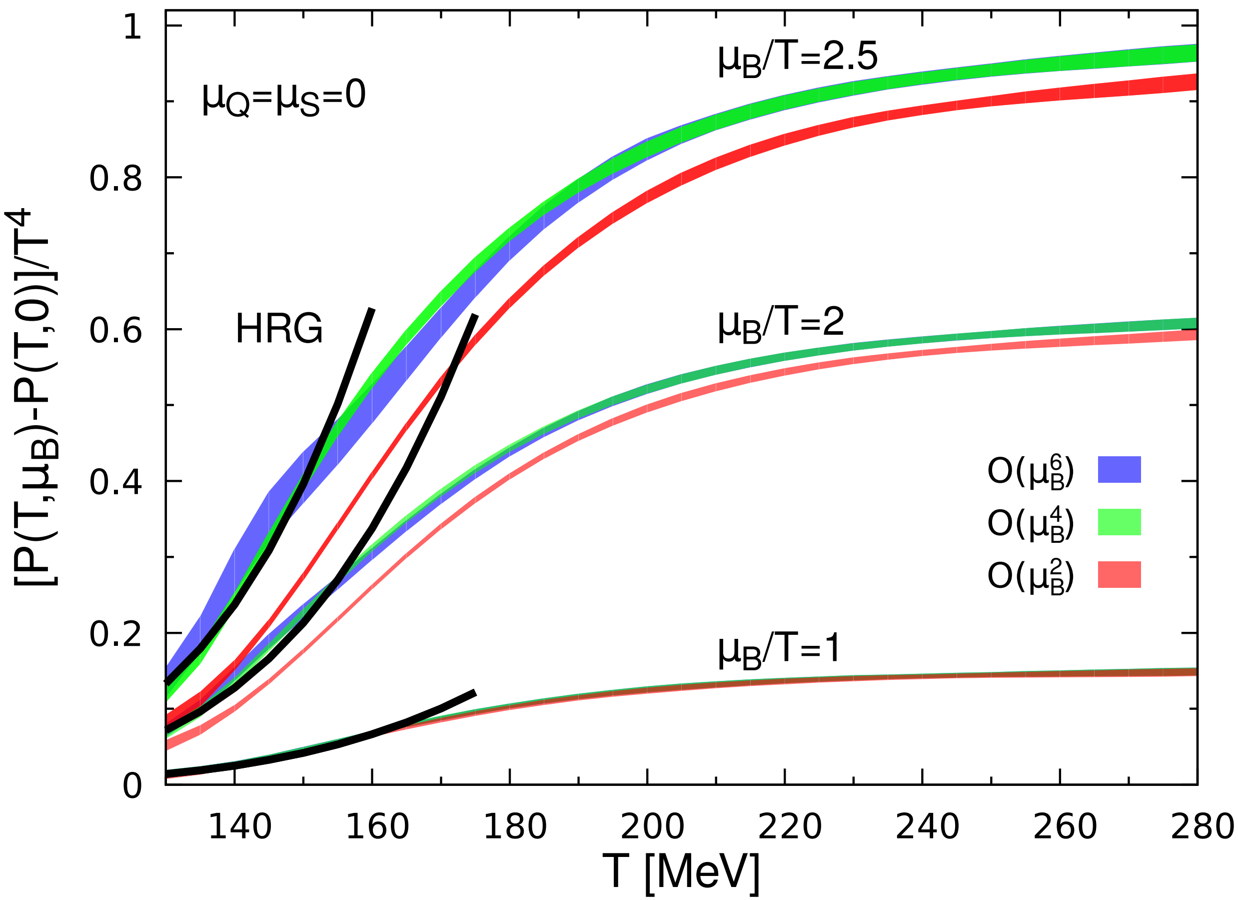
Partial pressures drop because hadronic cxcitations become broad at high temperatures (bound state peaks merge with the continuum)

See
Jakovác, PRD88 (2013), 065012
Biró, Jakovác, PRD(2014)065012

Vice versa for quarks

Thermodynamics at non-zero net baryon density

6[th] order Taylor expansion, BNL-Bielefeld-CCNU Coll., PRD 95 (2017) 054504

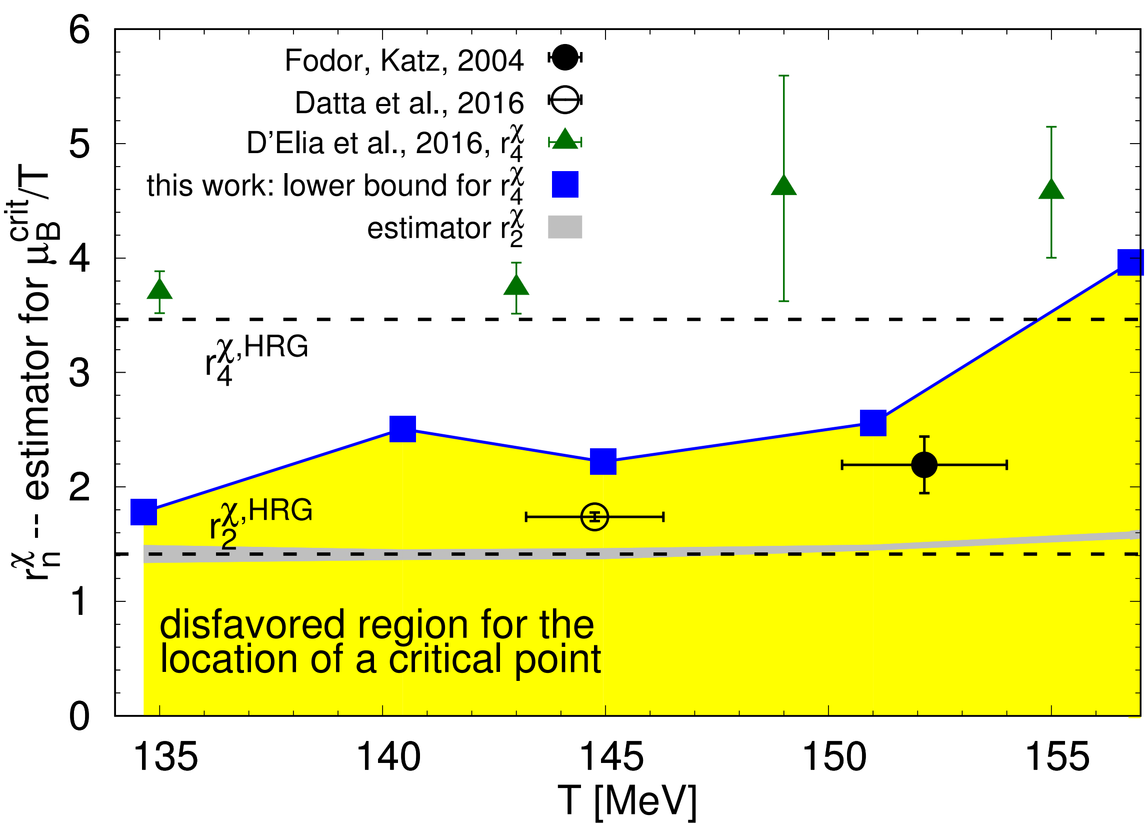


Truncation errors of the 6[th] order Taylor expansions are small for $\mu_B/T<2.5$

Radius of convergence of Taylor series and critical point

$$\frac{P(T,\mu_B)-P(T,0)}{T^4}=\sum_{n=1}^{\infty}\frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_2^B(T,\mu_B)=\sum_{n=1}^{\infty}\frac{\chi_{2n+2}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$

$T<T_c$: $\chi_n^B>0 \Rightarrow$ convergence of the Taylor expansion
is limited by a similarity on the real axis $\mu_B=\mu_B^c$ (critical point).



Estimator for radius of convergence:

$$r_n^\chi=\left|\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^2$$

BNL-Bielefeld-CCNU Coll., PRD 95 (2017) 054504

existence of a critical point for $\mu_B<2T$ is strongly disfavored