Non-perturbative study of heavy $Q\bar{Q}$ potential at finite temperature

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outline

- Quarkonia provides an important tool to probe quark-gluon plasma.
- One way to study quarkonia is to define an in medium potential.
- The potential is useful for the study of quarkonia states at finite temperature e.g. modification of quarkonia peak in dilepton production process.

$M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m) U(\vec{x}, \vec{y}; t_m) \psi(\vec{y}, t_m)$ for singlet channel.
The correlator $D(r, t_m) =< M^\dagger(r, t_m) M(r, 0) >$ in the heavy quark limit satisfy Schrodinger equation

$$[-\frac{\nabla^2}{m_Q} + V(r)] D(r, t_m) = i \frac{\partial D(r, t_m)}{\partial t_m}$$

The heavy quark potential $V(r)$ is defined by the following equation

$$i \frac{\partial w_m(r, t_m)}{\partial t_m} = V(r) w_m(r, t_m),$$

as $t_m \rightarrow \infty$. Here $w_m(r, t_m)$ is the Wilson loop given by

$$w_m(r, t_m) =< Tr(exp(-ig \oint_C A^\mu(x)\, dx_\mu)) >$$

M. Laine et al JHEP03(2007)054

- On the lattice Wilson loop is calculated in Euclidean time.
- In lattice Wilson loop is defined as

$$w(r,t) = \frac{1}{3}\,\mathrm{Tr}\left(\prod_{x} U_{x,x+\mu}\right)$$

- $w(r,t) = \int e^{-\omega t}\rho(r,\omega)$ and $w_m(r,t_m) = \int e^{-i\omega t_m}\rho(r,\omega)$

  Y. Burnier,O. Kaczmarek & A. Rothkopf, Phys. Rev. Lett, 114, 082001 (2015)

  P. Petreczky & J. Weber, Nucl Phys A 00 (2018) 1-4.

- We write the Wilson loop as

$$log(w(r,t)) = \frac{1}{2}log(\frac{w(r,t)}{w(r,\beta-t)}) + \frac{1}{2}log(w(r,t)w(r,\beta-t))$$

- From perturbation theory we see that

$$A(r,t) = \frac{1}{2}log(\frac{w(r,t)}{w(r,\beta-t)}) = (\frac{\beta}{2}-t)V_r(r)$$

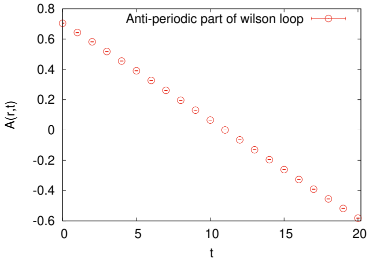
where $V_r(r) = (-\frac{g^2}{3\pi})(m_D + \frac{exp(-m_D r)}{r})$
$Y.Burnier\&A.Rothkopf, Phys.Rev.D87, 114019(2013)$

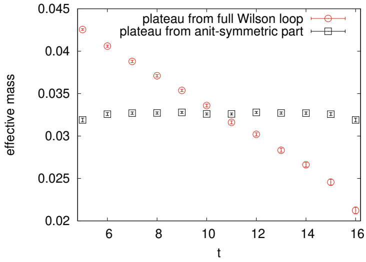
- We will try to see the behaviour of $A(r,t)$ non-perturbatively.

- The Wilson action for anisotropic lattice is given by

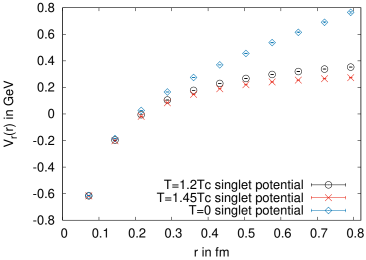
$$S_L = \frac{\beta_s}{3} \sum_{x} \sum_{\substack{i>j \\ i \neq 4}} \mathrm{Re}\ \mathrm{Tr}(1 - P_{ij}(x)) + \frac{\beta_t}{3} \sum_{x} \sum_{i \neq 4} \mathrm{Re}\,\mathrm{Tr}(1 - P_{4i}(x))$$

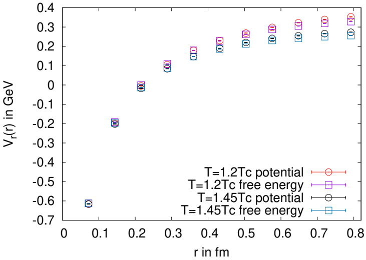
- We take
  $\beta_s = 2.53$
  $\beta_t = 15.95.$
  Lattice spacing
  $a_t = 0.024 fm$
  $a_s = 0.072 fm.$
- The spatial extension of our lattice is 1.7 fm.
- We change the temperature from 0.6Tc to 1.45Tc by varying the the temporal extent of the lattice.
- To remove the non-potential part of the Wilson loop we use smearing along the spatial direction.


Anti-periodic part of wilson loop
A(r,t)
t
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
5
10
15
20

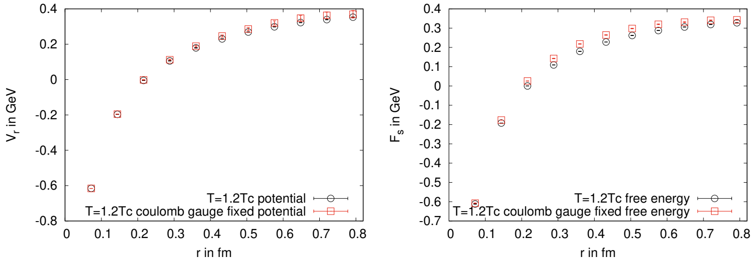

plateau from full Wilson loop
plateau from anit-symmetric part
effective mass
t
0.045
0.04
0.035
0.03
0.025
0.02
6
8
10
12
14
16

$$F_s(r, \beta) = -\frac{log(w_{smeared}(r, \beta))}{\beta}$$





Comparison with coulomb gauge fixed wilson line correalator.



$$P(r,t) = \frac{1}{2} log(w(r,t)w(r,\beta - t)) = \int_0^\infty (e^{-\omega t} + e^{-\omega(\beta - t)})\rho_p(r,\omega)$$

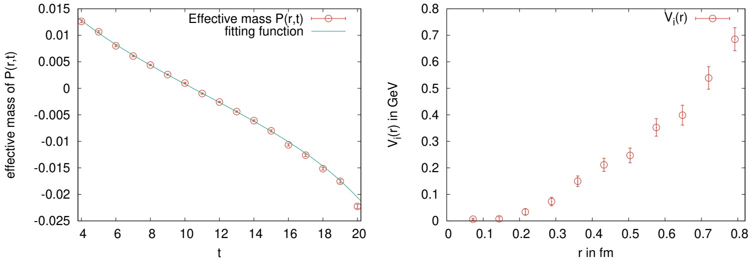
Perturbatively if

$$\rho_p(r,\omega) = \frac{V_i(r)}{\pi\omega^2} + \frac{b}{\omega}$$

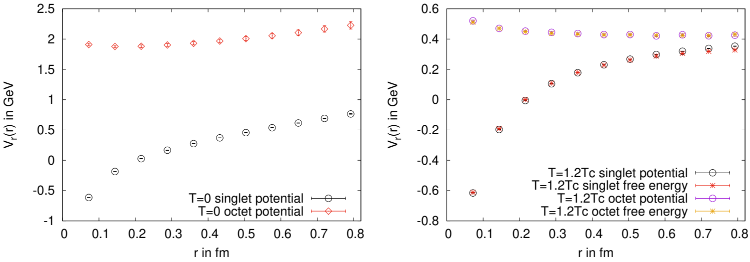
then

$$\frac{\partial P(r,t)}{\partial t} = \frac{V_i(r)}{\pi} log(\frac{t}{\beta - t}) + b\frac{\beta - 2t}{t(\beta - t)} + c$$

$Y.Burnier \& A.Rothkopf, Phys.Rev.D87, 114019(2013)$


Effective mass P(r,t)
fitting function
effective mass of P(r,t)
t
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
4
6
8
10
12
14
16
18
20
$V_i(r)$
$V_i(r)$ in GeV
r in fm
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

- In the QGP the quark antiquark pair can also be in an octet state.
- For octet state
  $M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m) U(\vec{x}, \vec{z}; t_m) T^a U(\vec{z}, \vec{y}; t_m) \psi(\vec{y}, t_m)$.
- We can not implement this operator directly in lattice.
- To make it gauge invariant we use the following operator
  $M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m) U(\vec{x}, \vec{z}; t_m) T^a B_a(z) U(\vec{z}, \vec{y}; t_m) \psi(\vec{y}, t_m)$.
- However the potential should not depend on any particular gluonic operator.


2.5
2
1.5
1
0.5
0
-0.5
-1
$V_r(r)$ in GeV
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
r in fm
T=0 singlet potential
T=0 octet potential
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
$V_r(r)$ in GeV
r in fm
T=1.2Tc singlet potential
T=1.2Tc singlet free energy
T=1.2Tc octet potential
T=1.2Tc octet free energy