Deconfinement and chiral transitions

Center symmetry and deconfinement in SU(N) gauge theory

Polyakov loop correlators and the free energy of static quark anti-quark pair

Chiral crossover at physical quark masses

Chiral phase transition in 2+1 flavor QCD

Center symmetry and deconfinement transition

Above the phase transition temperature Z(N) (center) symmetry of SU(N) gauge theory is broken
Quarks transform non-trivially under Z(N) symmetry group
=> static charges in fundamental representations can be screened by gluons !

Lattice set-up:

$U_\mu(\tau,x) = e^{igA_\mu(\tau,x)},\ N_\sigma^3 \times N_\tau,\ T = 1/(N_\tau a)$

Thermodynamic limit: $N_\sigma/N_\tau \to \infty$; Continuum limit : $N_\tau \to \infty$, T-fixed Temperature is set by $a \leftrightarrow \beta = 2N_c/g^2$; allowable gauge transformations: $U_\mu(x) \to \Omega(x+\mu)U_\mu(x)\Omega^\dagger(x)$

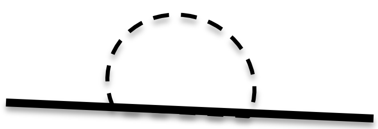
$$\Omega(0,\vec{x}) = \Omega(\beta,\vec{x})C, \quad C = e^{2\pi i n/N_c} I \quad \to Z(N) - \text{symmetry}$$

$$L(\vec{x}) = \frac{1}{N_c} \mathrm{tr} \prod_{\tau=1}^{N_\tau} U_0(\tau,\vec{x})$$

Polyakov loop is changed $L(\vec{x}) \to e^{2\pi n i/N_c} L(\vec{x})$

$< L > \neq 0 \ \to Z(N)$ spontaneously broken; $< L >= e^{-F_Q/T}$ -free energy of an isolated static quark is finite $\Rightarrow$ deconfinement

L is order parameter



The free energy of static quark is infinite in the Continuum limit due to linear *1/a* divergence => needs renormalization

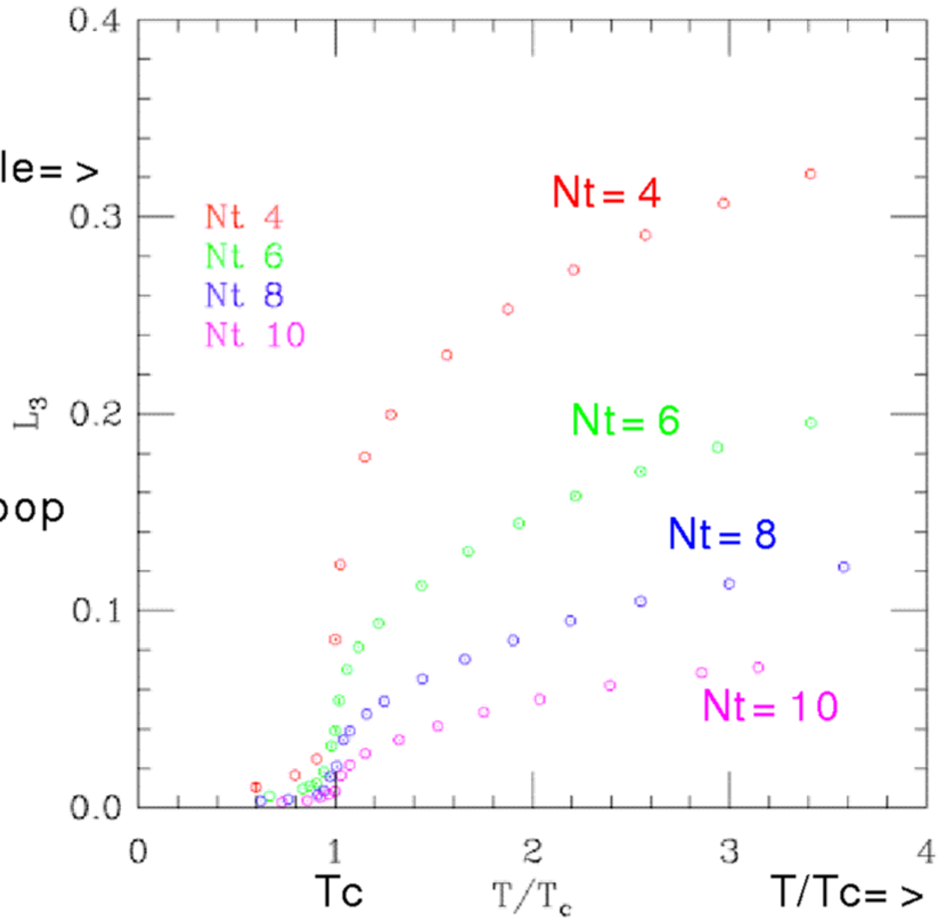
Continuum limit for *L* ?

Dumitru et al, hep-th/0311223

Bare triplet loop vs T, at different Nt

Note scale=>
~ .3

Triplet loop



Tc

T/Tc=>

Nt = #
time steps.

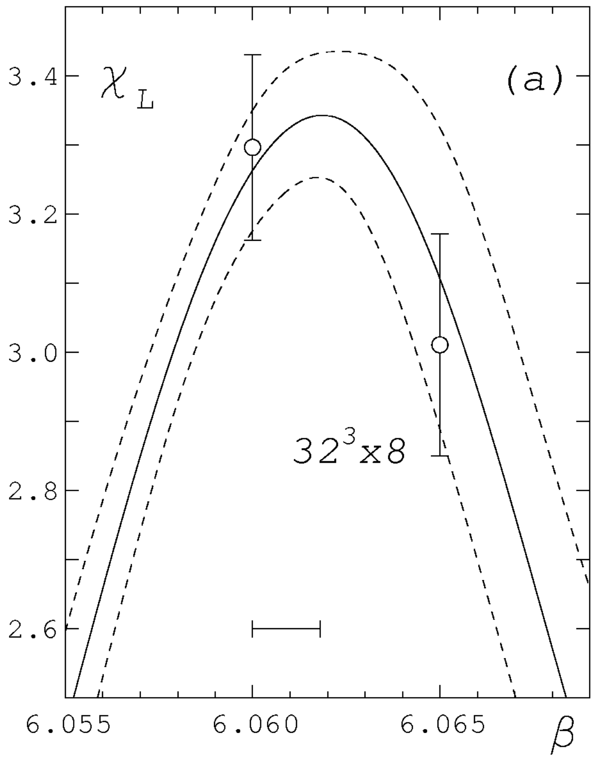
Bare loop vanishes as Nt -> ∞

needs renormalization !

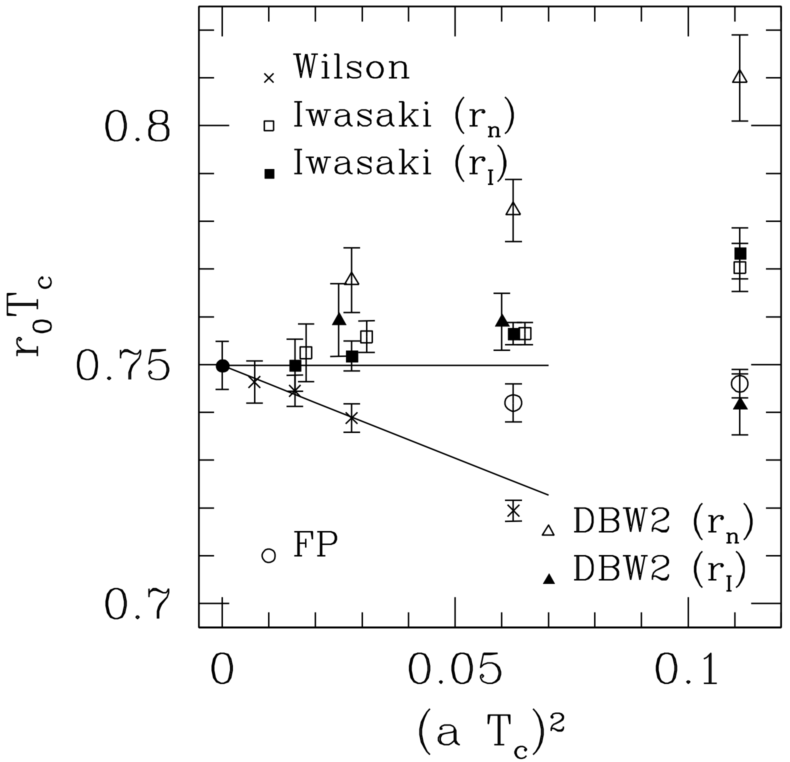
How to determine the deconfinement transition temperature ?

$$\frac{\chi_L}{T^2} = N_\sigma^3 \left(\langle L^2 \rangle - \langle L \rangle^2 \right) = \langle (\delta L)^2 \rangle \text{ has a peak at } \beta_c$$

Boyd et al., Nucl. Phys. B496 (1996) 167



Necco, Nucl. Phys. B683 (2004) 167



- Use different volumes and Ferrenberg-Swedsen re-weighting to combine information collected at different gauge couplings

Finite volume behavior can tell the order of the phase transition, e.g. for 1st order transition the peak height scales as spatial volume !

Correlator of Polyakov loops and deconfinement

The correlation function of Polyakov loops defines the free energy of static quark anti-quark pair (also an order parameter)

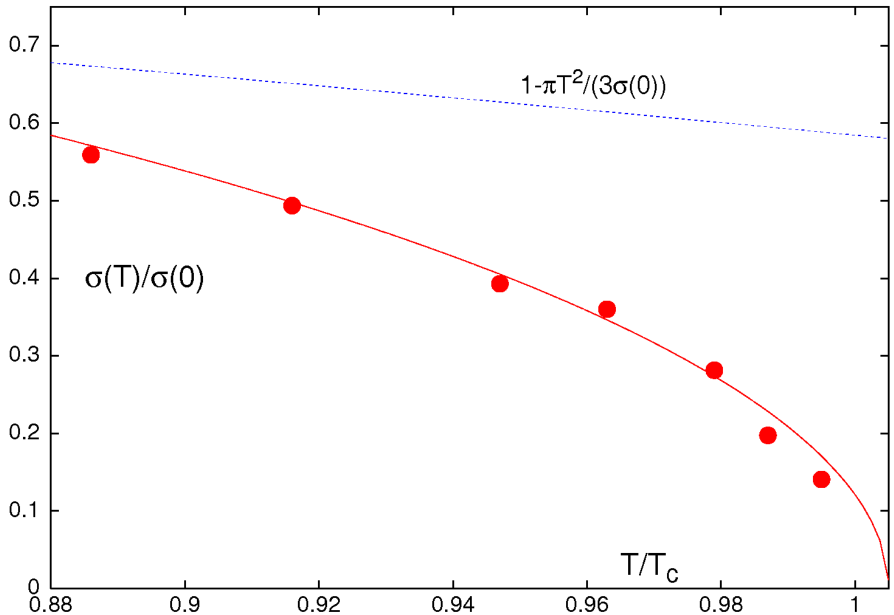
$T < T_c$:

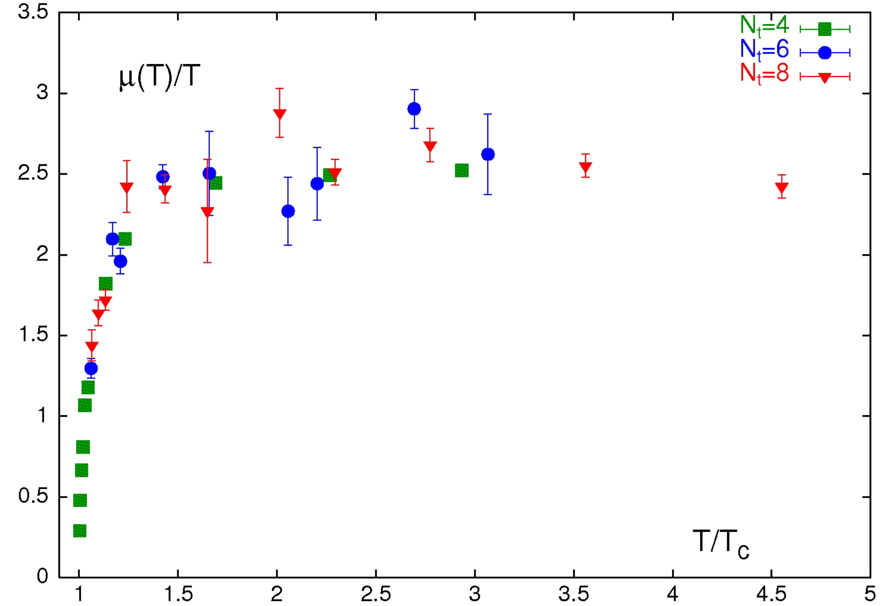
$$< L(r) L^\dagger(0) > \sim e^{-\sigma(T) r/T}$$

$T > T_c$:

$$\ln\left(\frac{< L(r) L^\dagger(0) >}{| < L > |^2}\right) \sim e^{-\mu(T) r}$$

Kaczmarek, Phys. Rev. D62 (00) 034021



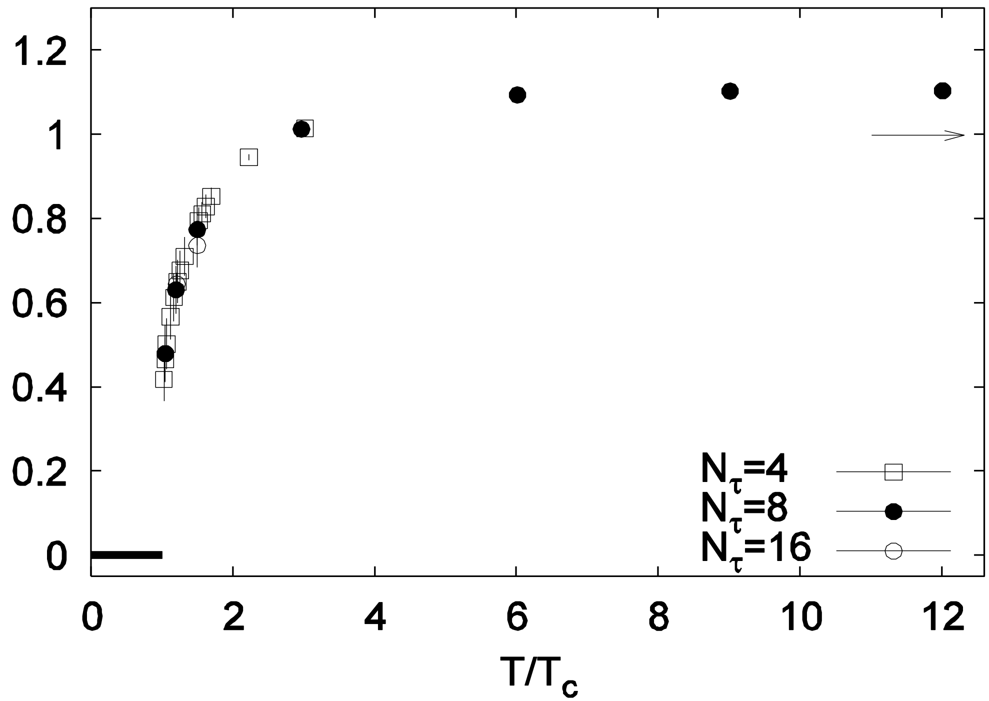


small inverse correlation length => weak 1st order phase transition SU(3) gauge theory is far from the large N-limit !

The renormalized Polyakov loop in pure glue theory

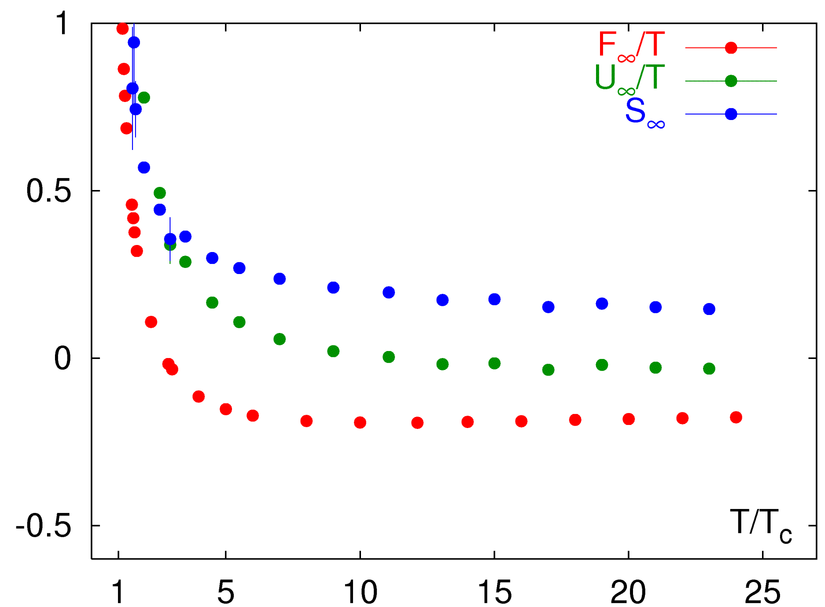
$$r \ll 1/T: \; F_{Q\bar{Q}}(r,T) = V(r, T=0) + T\ln 9$$
$$\Rightarrow \text{normalize the } Q\bar{Q} \text{ free energy to the } T=0 \text{ potential}$$

$$\lim_{r\to\infty} \langle L(r) L^\dagger(0)\rangle = \exp(-F_{Q\bar{Q}}(r\to\infty, T)/T) = \exp(-F_\infty/T) = |\langle L\rangle|^2, F_Q = F_\infty/2$$



$$L_{ren} = \exp(-F_\infty(T)/(2T))$$

Kaczmarek et al, PLB 543 (02) 41,
PRD 70 (04) 074505, hep-lat/0309121



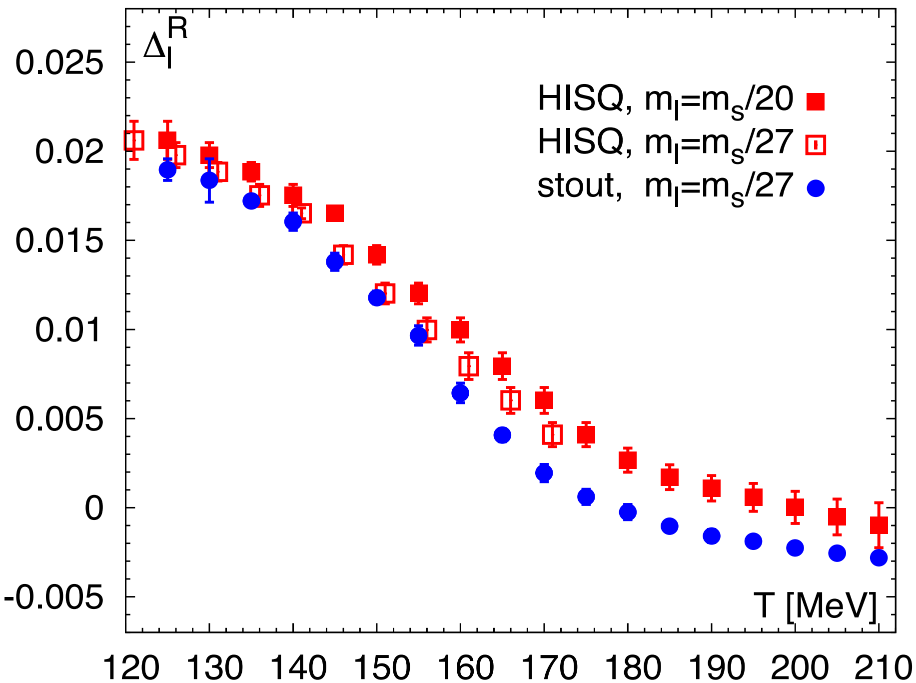
$$\text{LO}: F_\infty = -TS_\infty = -\frac{4}{3}\alpha_s m_D$$

The chiral transition at non-zero temperature

Renormalized chiral condensate

$$\langle\bar{\psi}\psi\rangle \Rightarrow \Delta_l^R(T) = \\ = m_s r_1^4 (\langle\bar{\psi}\psi\rangle_T - \langle\bar{\psi}\psi\rangle_{T=0}) + d, \\ d = m_s r_1^4 \langle\bar{\psi}\psi\rangle_{T=0}^{m_q=0}, \quad r_1 = 0.3106\text{fm}$$

Bazavov et al (HotQCD), PRD85 (2012) 054503;
Bazavov et al, PRD 87(2013)094505,
Borsányi et al, JHEP 1009 (2010) 073

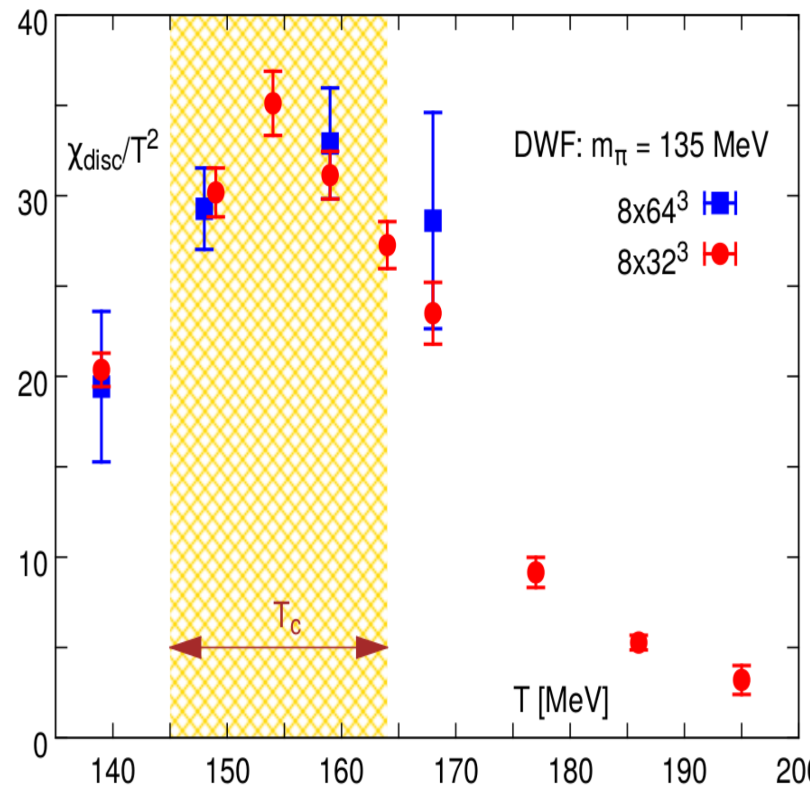


$T_c = (154 \pm 8 \pm 1(scale))$MeV

Fluctuations of the order parameter:

$$\chi_{disc} = VT^{-1}\left(\langle(\bar{\psi}\psi)^2\rangle - \langle\bar{\psi}\psi\rangle^2\right)$$

Bhattacharya et al (HotQCD), PRL 113 (2014)082001



$T_c = (155 \pm 8 \pm 1)$MeV

No increase with the volume
⇒ Crossover transition

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l) + f_s(t,h),\; t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa\frac{\mu_q^2}{T^2}\right),\; H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

governed by universal *O(4)* scaling $\quad M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\; z = t/h^{1/\beta\delta}$

$$\langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

$SU_A(2) \sim O(4)$

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities) :

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta-1} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \qquad \chi_{t,t} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial t^2} \sim |t|^{-\alpha}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$T_{m,l} \quad = \quad T_{t,l} \quad = \quad T_{t,t} \quad = \quad \boldsymbol{T_c^0}$$

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1}f_\chi(z) + reg.\right)$$

universal scaling function has a peak at $z=z_p$

$$\Downarrow$$

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0\frac{z_p}{z_0}H^{1/(\beta\delta)} + \ldots$$

Caveat : staggered fermions O(2)

$m_l \rightarrow 0$, $a > 0$,

proper limit $a \rightarrow 0$, before $m_l \rightarrow 0$

The chiral cross-over temperature for physical masses

Chiral order parameter:

$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle \bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle \bar{s}s\rangle\right] \qquad \langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f$$

and the corresponding susceptibilities:

$$\chi^\Sigma = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma \qquad \chi = \frac{m_s^2}{f_K^4}\left[\langle(\bar{u}u + \bar{d}d)^2\rangle - (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)^2\right]$$

For non-zero chemical potential we use Taylor expansion

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \quad \chi(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad C_0^\Sigma = \Sigma, \quad C_0^\chi = \chi$$

Derivatives in μ_X^2 are similar to derivatives in T $\qquad$ $e.g.\ \partial_T C_0^\chi \sim C_2^\chi$

$\Rightarrow$ the following quantities will peak at T_c

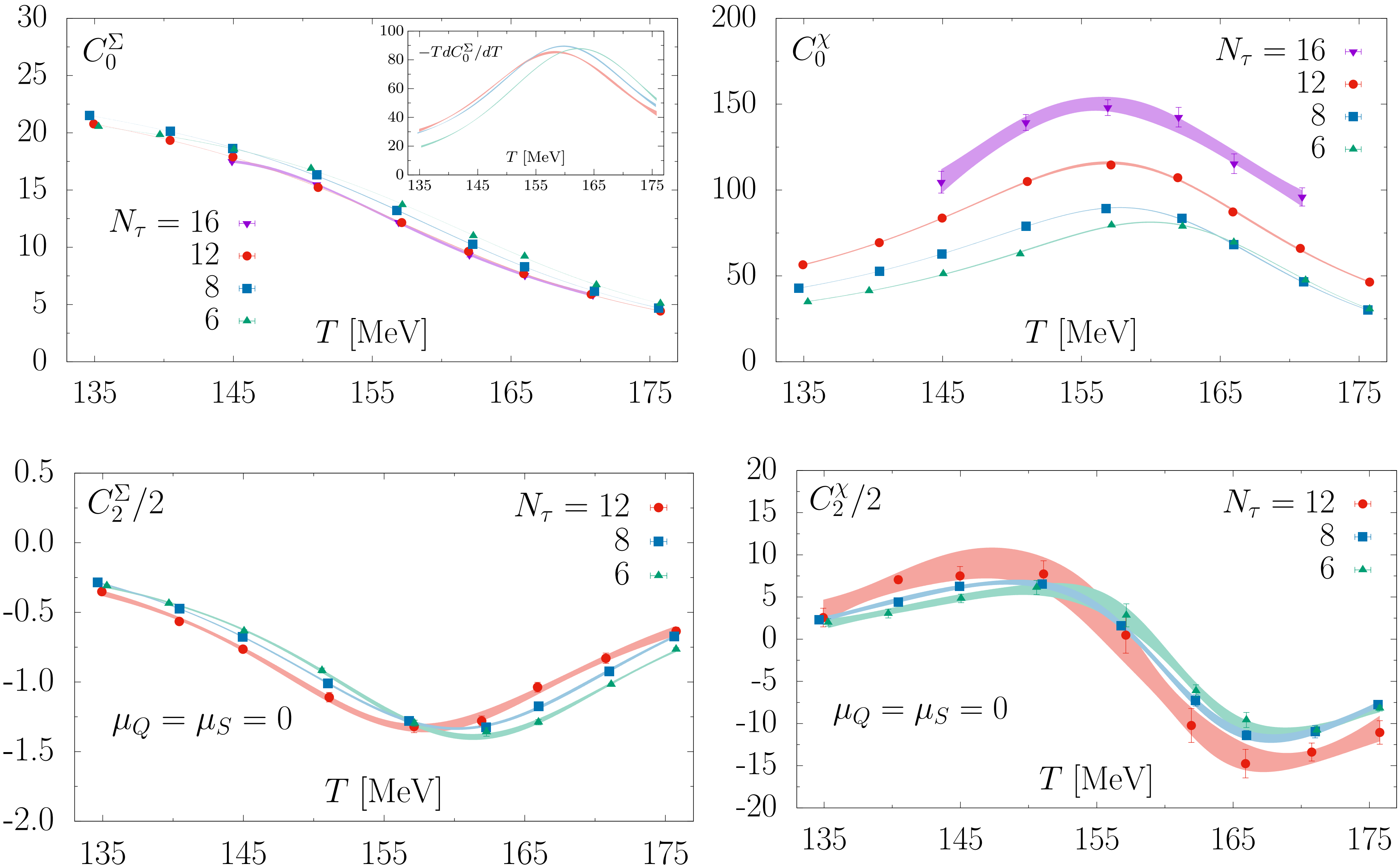
$\chi^\Sigma,\ C_0^\chi(T) \sim \chi_{l,m} \qquad \partial_T C_0^\Sigma,\ C_2^\Sigma(T) \sim \chi_{t,m}$ $\qquad$ HotQCD, arXiv:1812.08235

5 different definitions of T^{pc}:

$$\partial_T C_0^\chi = 0,\ \partial_T C_0^\Sigma = 0\ , C_2^\chi = 0 \qquad \partial_T^2 C_0^\Sigma = 0,\ \partial_T C_2^\Sigma = 0$$

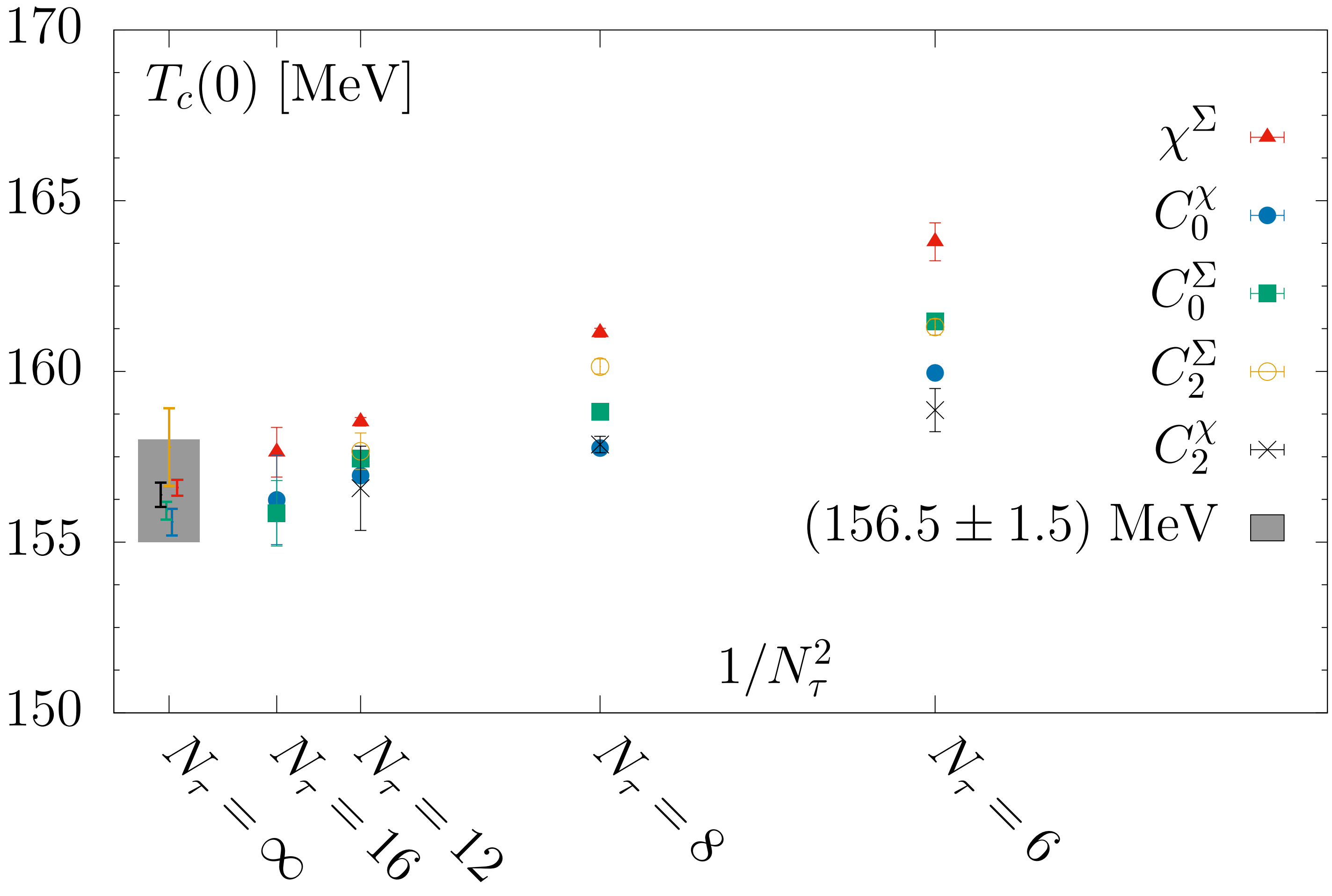
The 5 different T_c values reduce to $T_{l,m}$ and T_{let} if regular part is zero

Lattice calculations based on 100K - 500 K configurations, $N_\tau = 6 - 12$, and 4K configurations for $N_\tau = 16$



HotQCD, arXiv:1812.08235

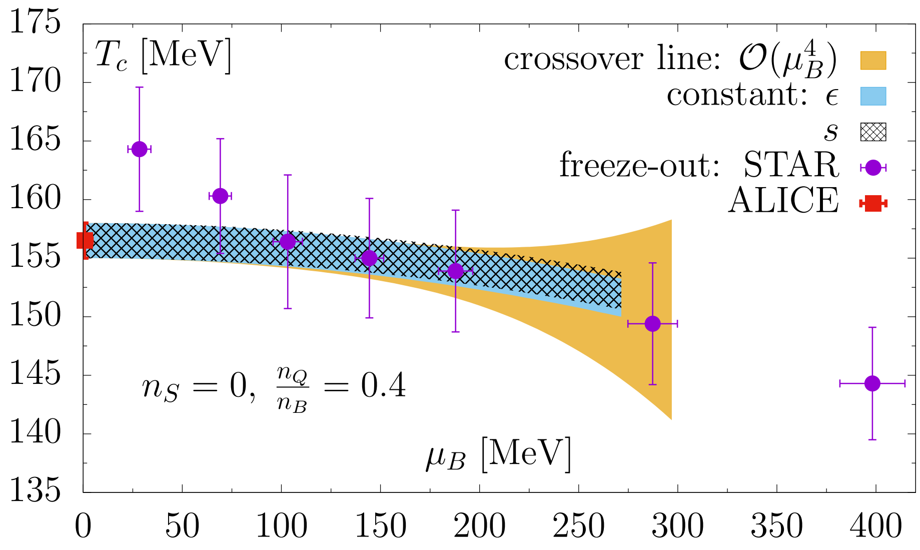
Different definitions of T_c surprisingly agree in the continuum limit and we for zero chemical potential we get $T_c = 156 \pm 1.5$ MeV



HotQCD, arXiv:1812.08235

The chiral cross-over temperature at non-zero density

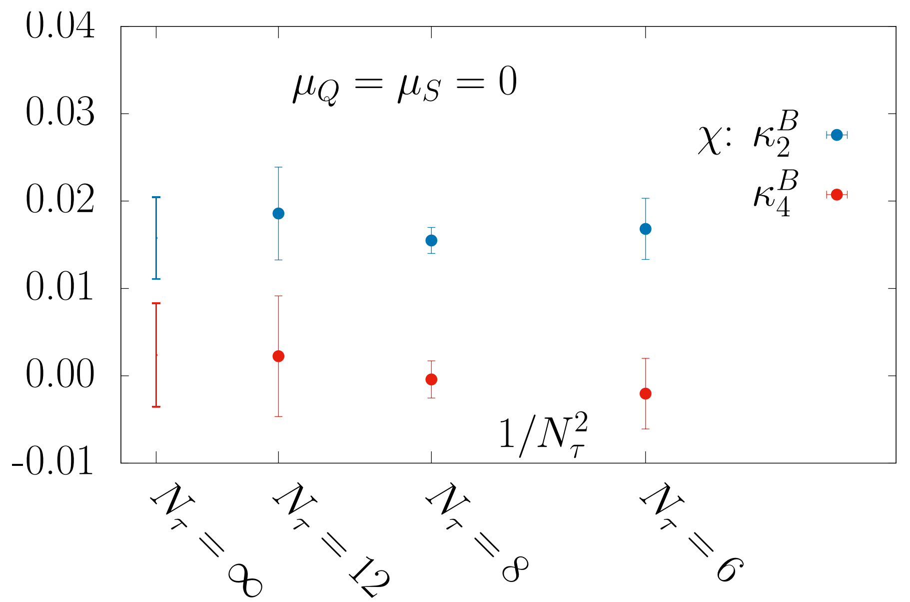
$$T_c(\mu_B) = T_c(0)\left[1 - \kappa_2^B\left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B\left(\frac{\mu_B}{T_c(0)}\right)^4\right]$$



The freeze-out condition corresponding to constant energy density or constant entropy density agrees with the crossover line within errors

The μ_B dependence of T_c is small

$$\kappa_2^{B,\chi} \simeq \kappa_2^{B,\Sigma}$$

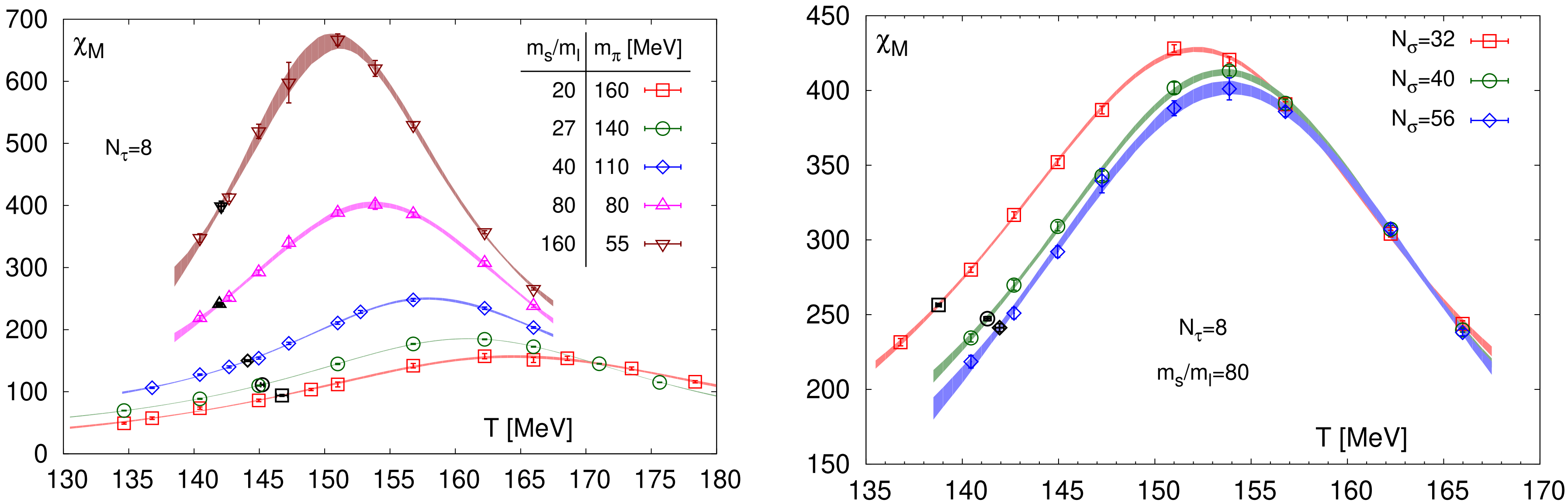


HotQCD, arXiv:1812.08235

Chiral phase transition in 2+1 flavor QCD

What is the nature of the chiral transition in 2+1 flavor QCD for fixed m_s and $m_l \to 0$?

HotQCD, arXiv:1903.04801

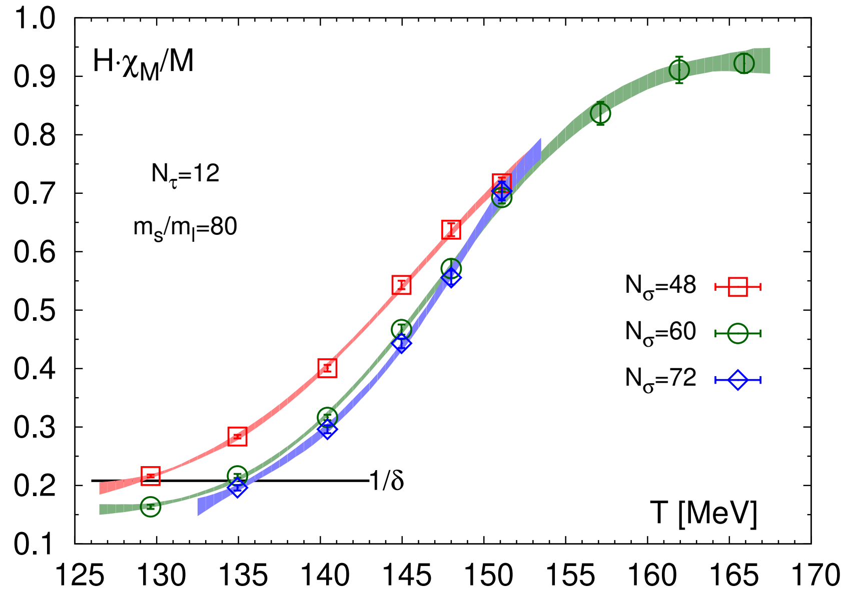


$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L) \ ,$$

$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L) \ .$$

$$T_p(H, L) = T_c^0 \left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta} \right) \ + \ \text{sub leading}$$

$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0} \right) H^{1/\beta\delta} \right)$$

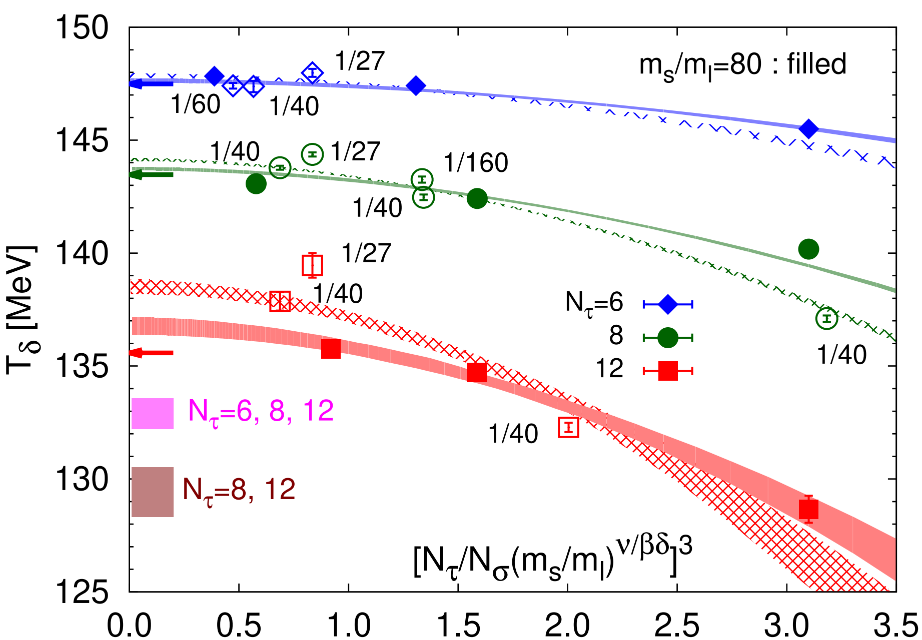


$$\frac{H\chi_M(T_\delta,H,L)}{M(T_\delta,H,L)} = \frac{1}{\delta}\ ,$$

$$\chi_M(T_{60},H) = 0.6\chi_M^{max}\ .$$

$$T_X(H,L) = T_c^0\left(1+\left(\frac{z_X(z_L)}{z_0}\right)H^{1/\beta\delta}\right)$$
$$+c_X H^{1-1/\delta+1/\beta\delta}\ ,\quad X=\delta,\ 60$$

$$z_{60}\simeq z_\delta\simeq 0$$



Use $O(4)$ fits for m_l and volume dependence

Continuum extrapolations:

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

HotQCD, arXiv:1903.04801