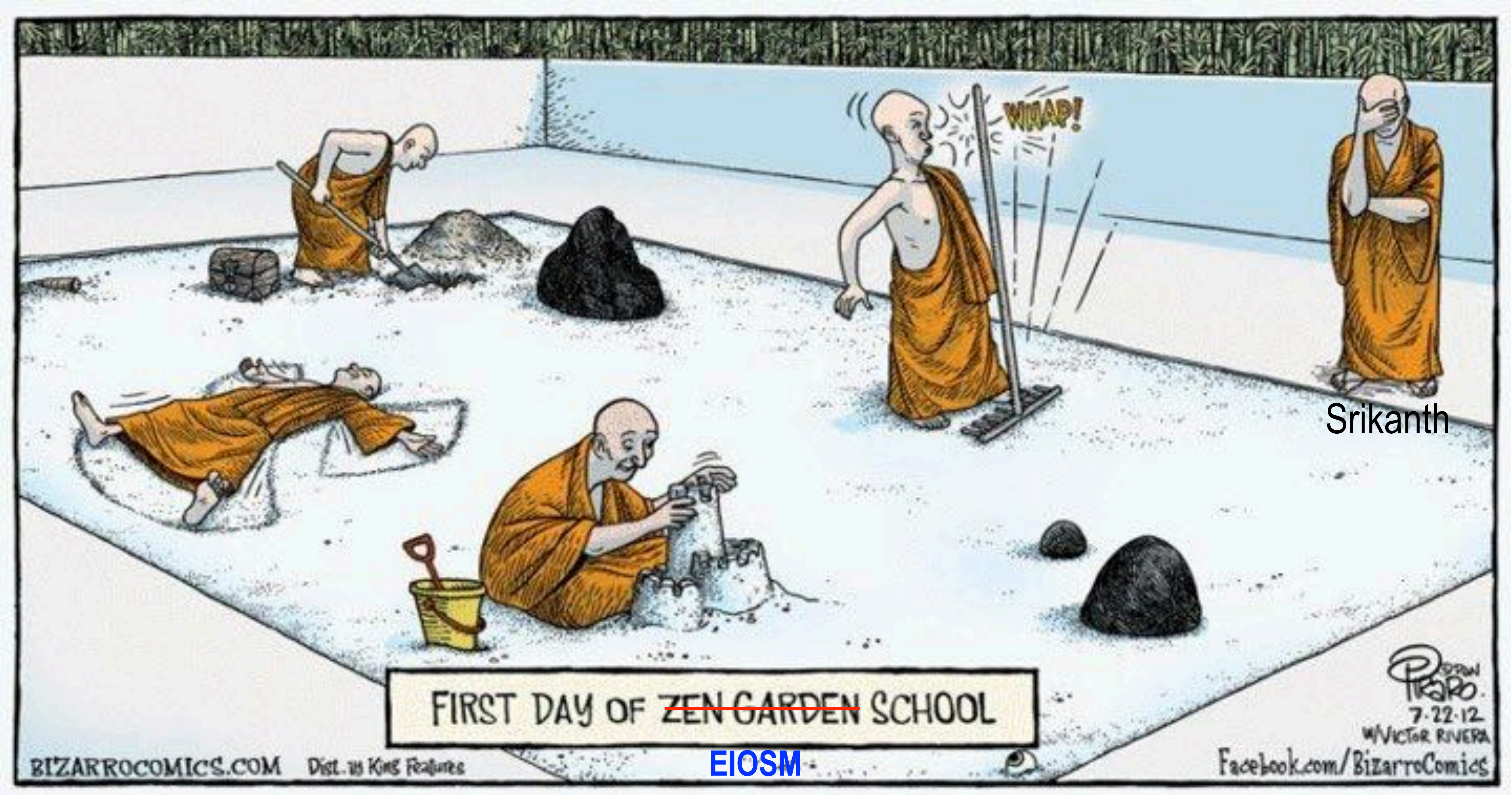
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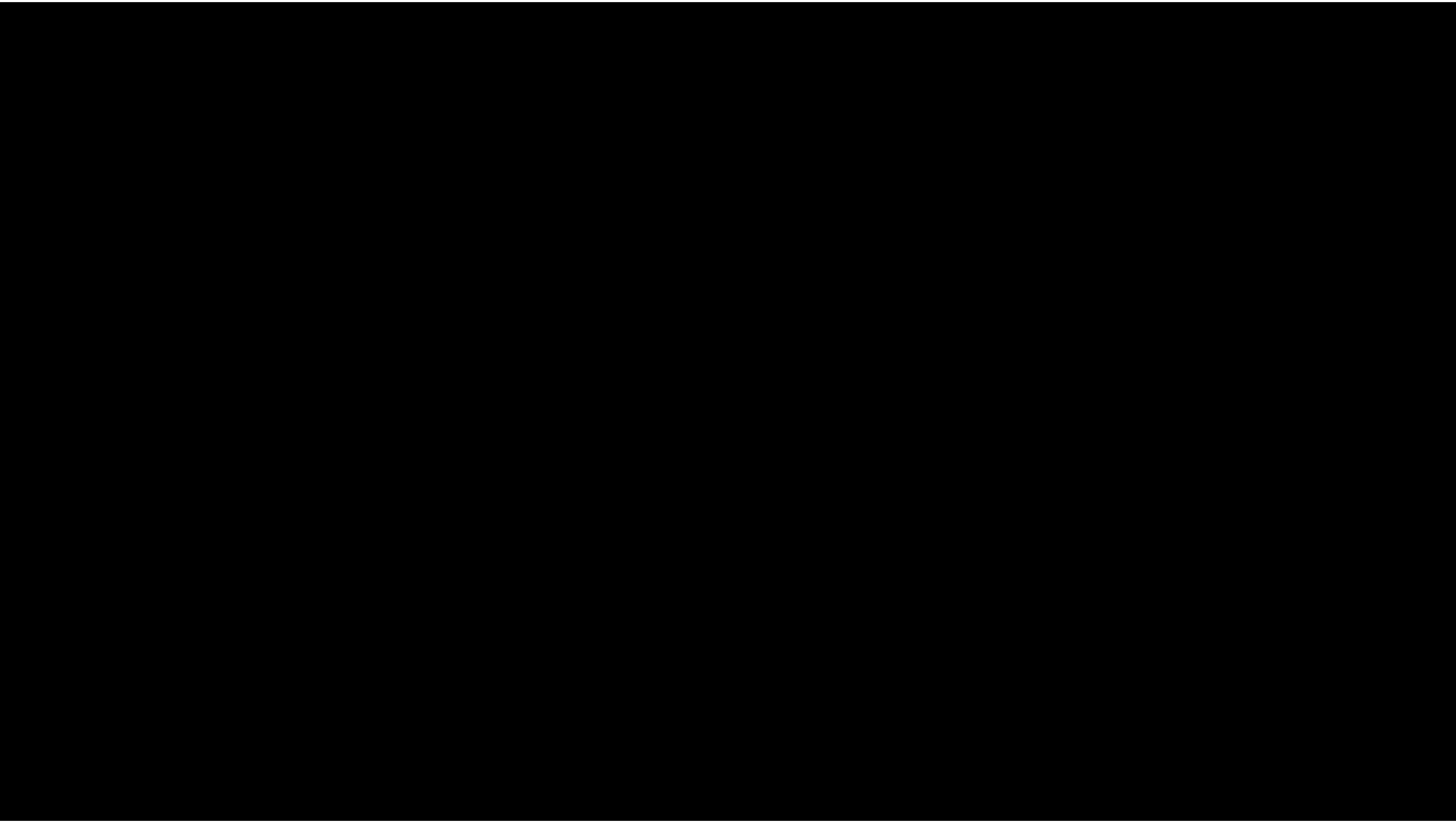


Young, fast growing place with ~65 faculty.
Hiring ~7 faculty/year aiming for 100 by 2021 & 200 by 2030.
Undergrad & MS internships, PhD & postdoc opportunities.

Nonlinear and Non-equilibrium Physics: groups.oist.jp/nnp

Hydrodynamics

- Turbulence
- Interfacial (Marangoni-driven) flows
- [Future] Evaporation/Condensation phenomena…

Statistical Physics of Energy & Sustainability

- Table-top experiments in Carbon geo-sequestration
- Fluctuations in Wind Energy
- [Ongoing] Fluctuations in Solar Photovoltaics
- [Ongoing] Fluctuations in flight (Drones/Quadcopters)

[Future] Random unrelated/unclassified questions

- Solar flares
- The delicate sound of thunder
- Geometric evolution of Scripts
- Stat. Mech. of Shredding
- *Nottuswaram*: Comparing Western & Indian musical systems
- Irreducible set of measurement methods?

Quantitative Life Sciences

- Red Blood Cell Elastohydrodynamics
- Collective effects in fungal spore ejection
- Birdsong prosody
- Evolution of stiffness in human feet & fish fins
- [Ongoing] Locomotory mechanics of amphibians
- [Ongoing] Colony behavior in Garden Eels

Mechanics of Amorphous Media

- Aggregates
- Granular Media
- Particle rafts

PhD and Postdoctoral opportunities available after Jan 2020

Higher-order Turbulence Spectra: Energy to UAV

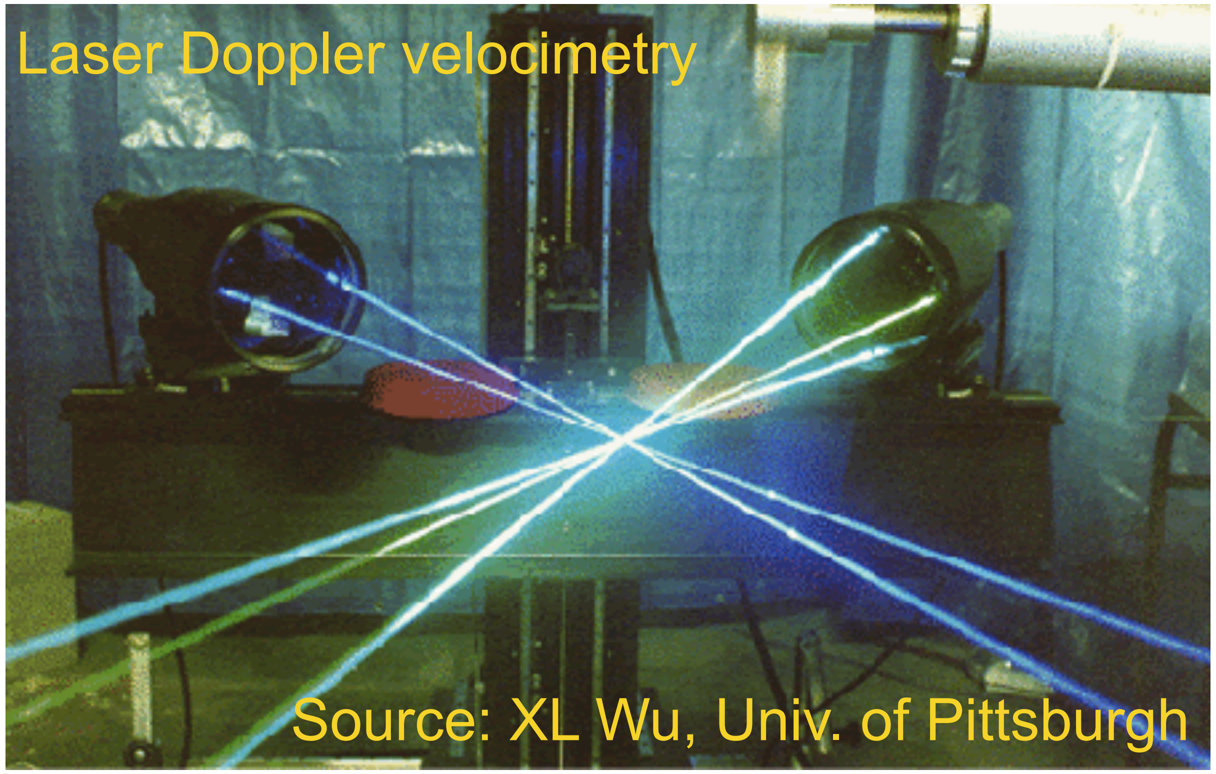
Leonardo da Vinci's illustration of the swirling flow of turbulence.
(The Royal Collection © 2004, Her Majesty Queen Elizabeth II)

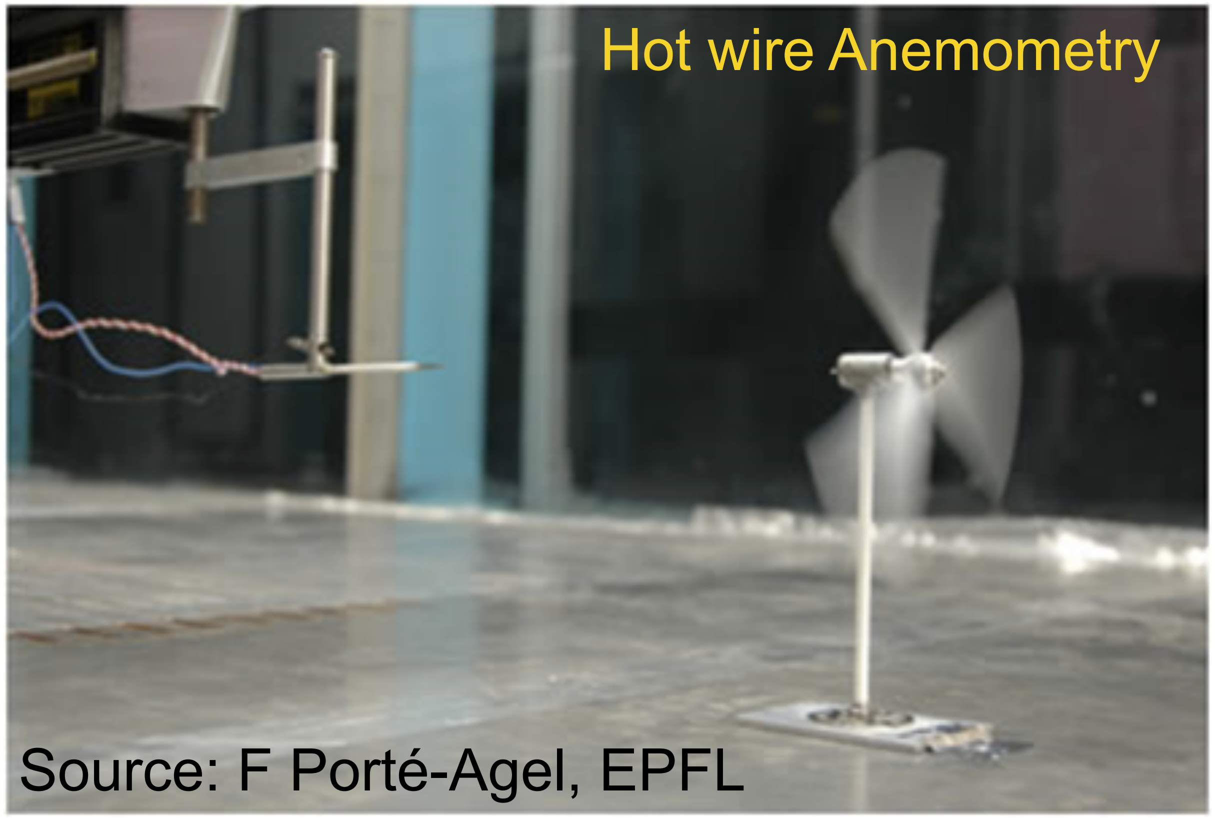
1510 CE

Point (Eulerian) vs. Field measurements in Turbulence

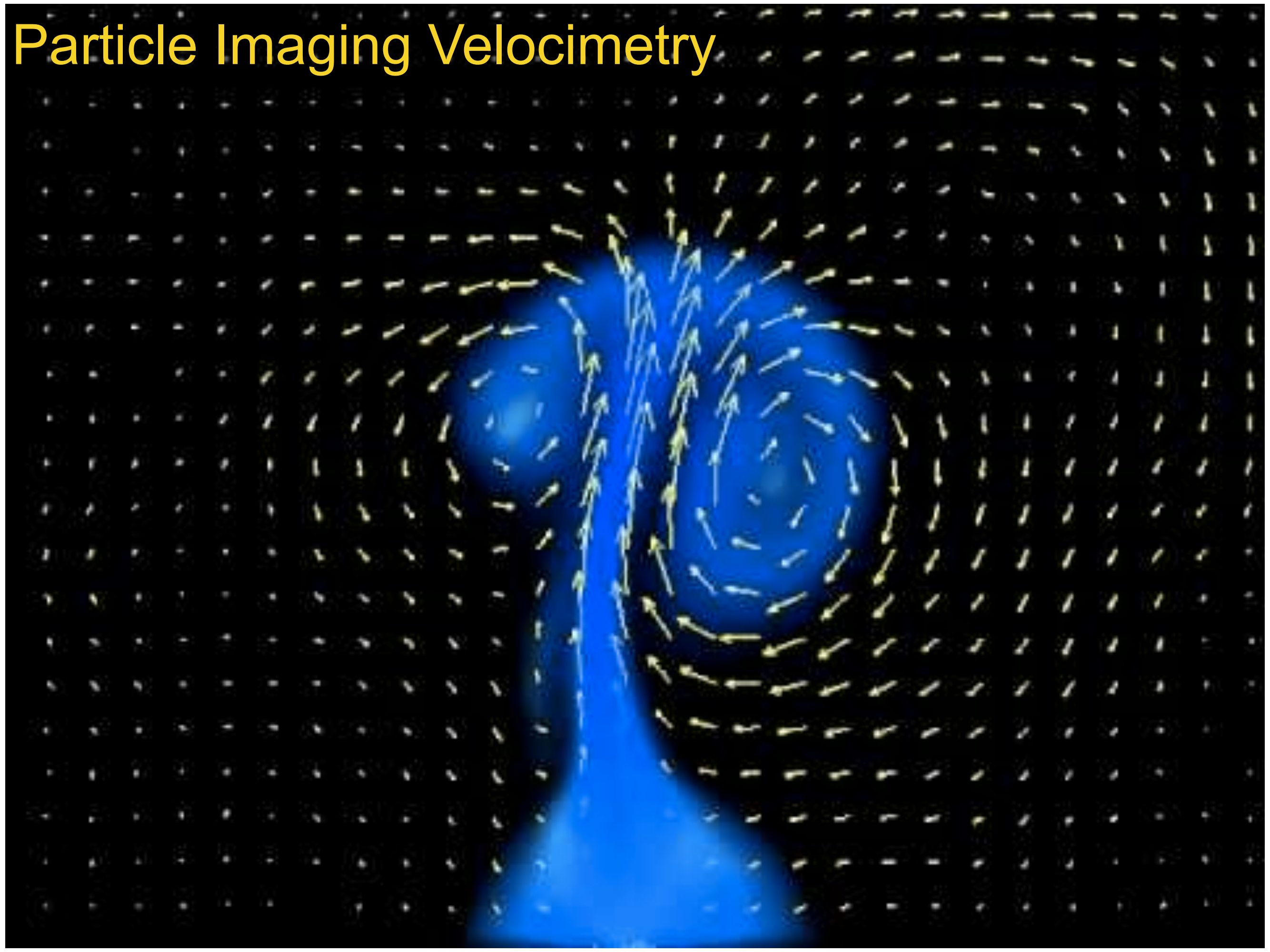
- 1941 Kolmogorov theory concerns field measurements.
- Early measurements at spatial points; PIV is relatively recent.

Point (Eulerian) measurements

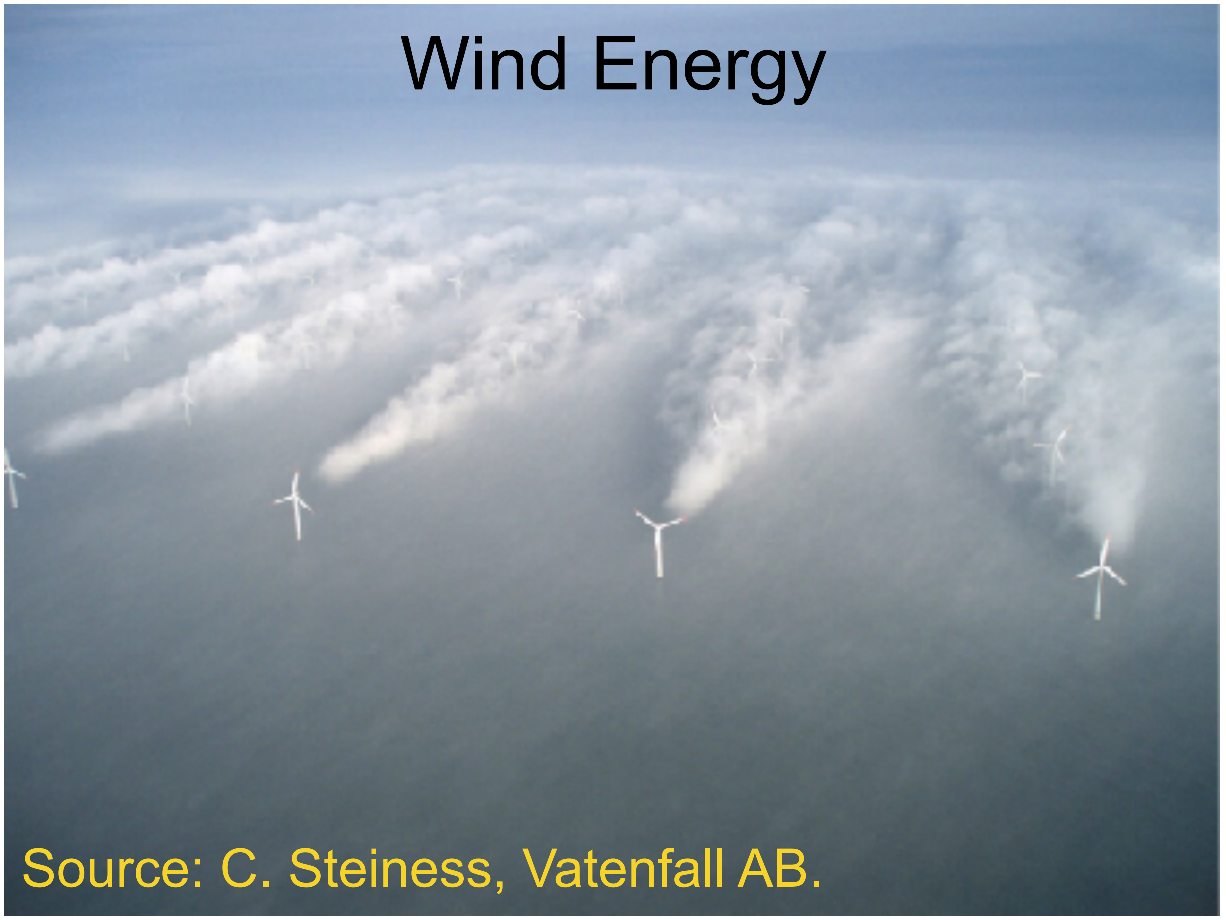


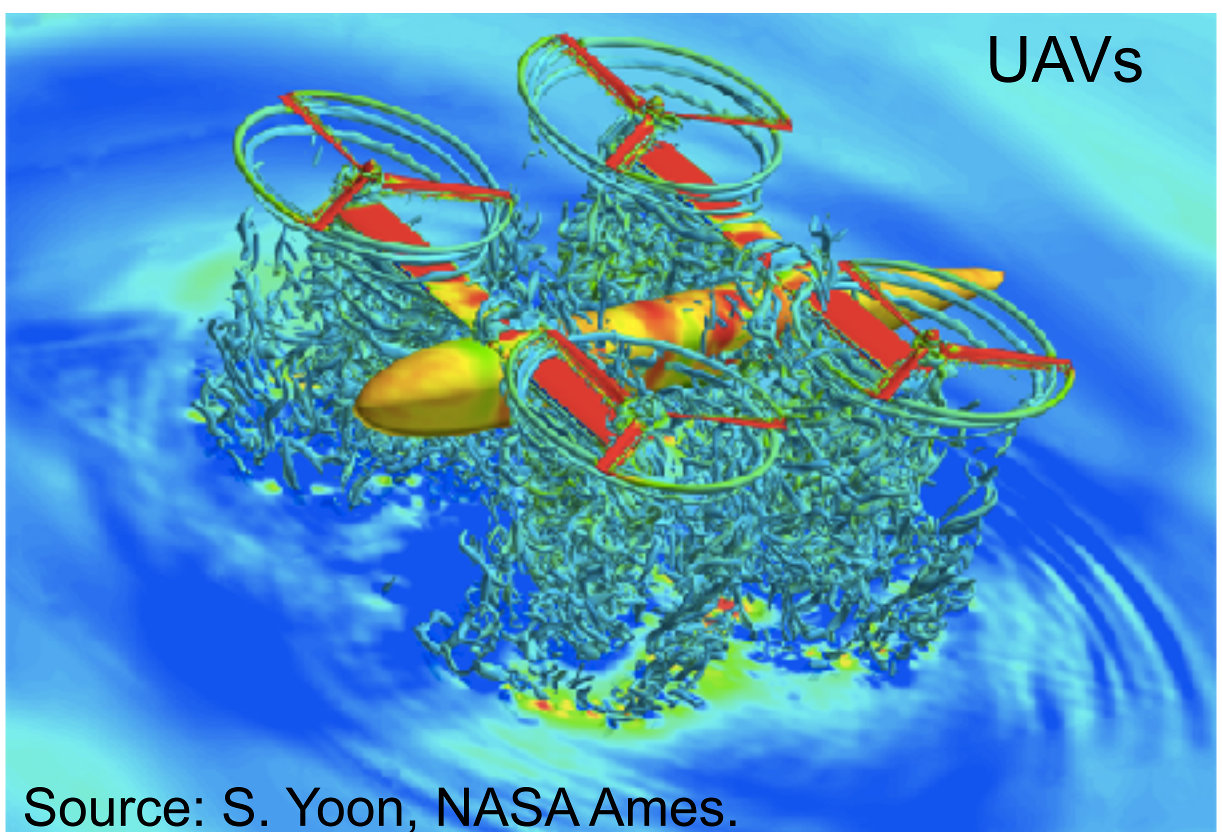


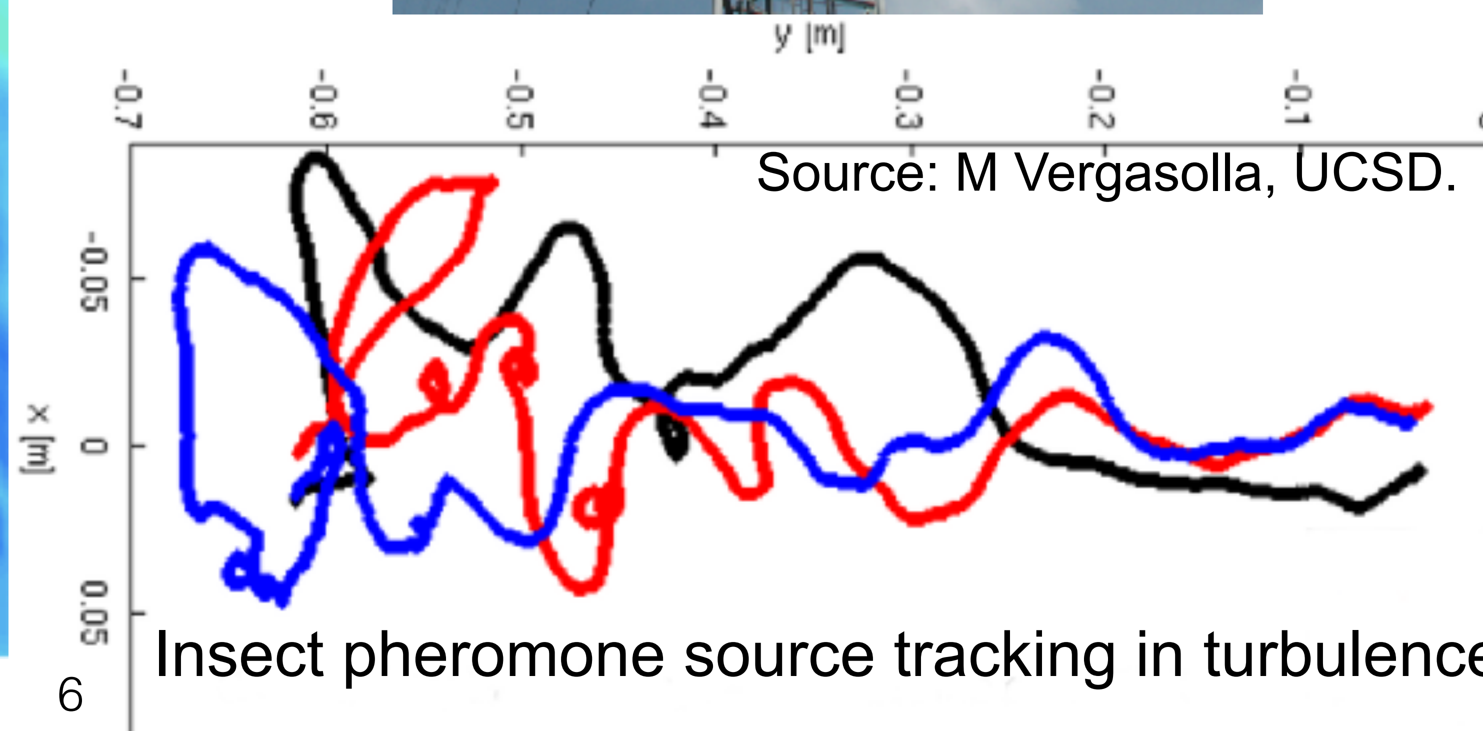
Field Measurements.



Static or moving (but Not Lagrangian) point measurements in time.







Outline

- Turbulence primer:
  - 3D Turbulence: 1941 Kolmogorov theory (K41).
  - 2D Turbulence: Kraichnan-Leith-Batchelor Theory (KLB).
  - Higher-order spectra.
- Experiments in Two (2D) & Three Dimensional (3D) Turbulence
- Applications:
  - Wind Energy: Fluctuations from Turbine to Grid scales.
  - Fluctuations in Flight (UAV) - Statistics & Spectra.
  - Atmospheric Flows: Are they 2D or 3D?

3D Turbulence

Turbulent kinetic energy transported in fluid parcels called 'eddies'.

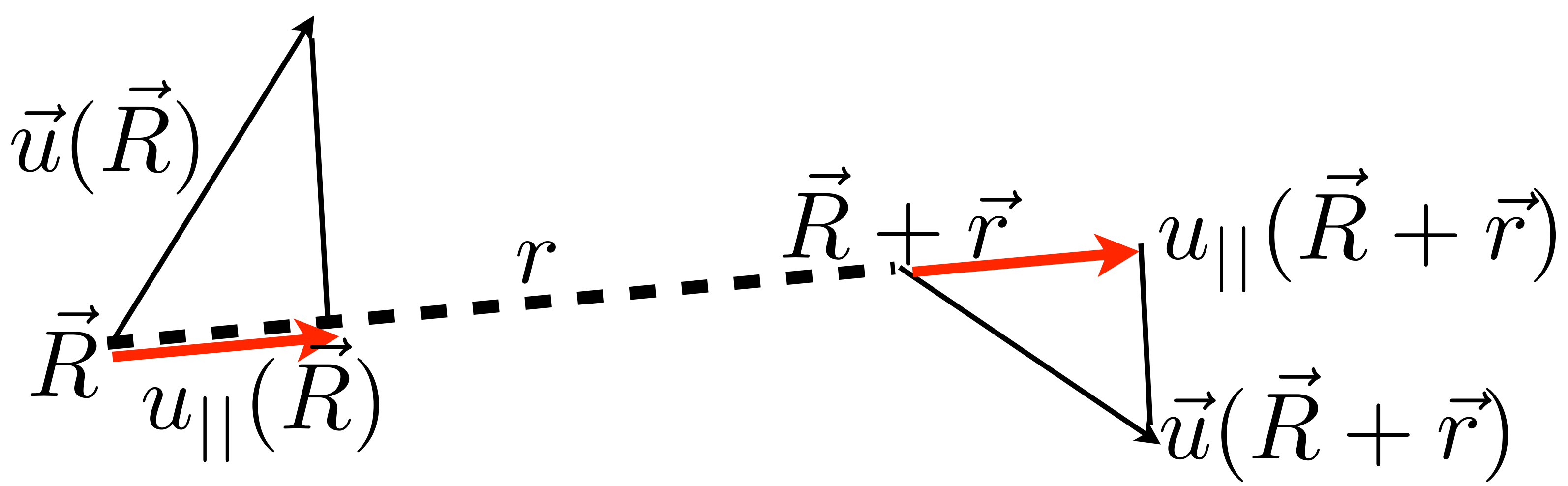
Eddy has spatial extent for a finite time period.

When passing stationary observer, eddy registers a fluctuation.

Fluctuations link length $r(k)$ & time scales $\tau(f)$ of turbulence.

Inertial range of turbulence:

- Largest: Integral scale $l_0(k_0)$ & large eddy turnover time $\tau_0\ (f_0)$.
- Smallest: Dissipative scale $\eta\ (k_\eta)$ & time scale $\tau_\eta\ (f_\eta)$.



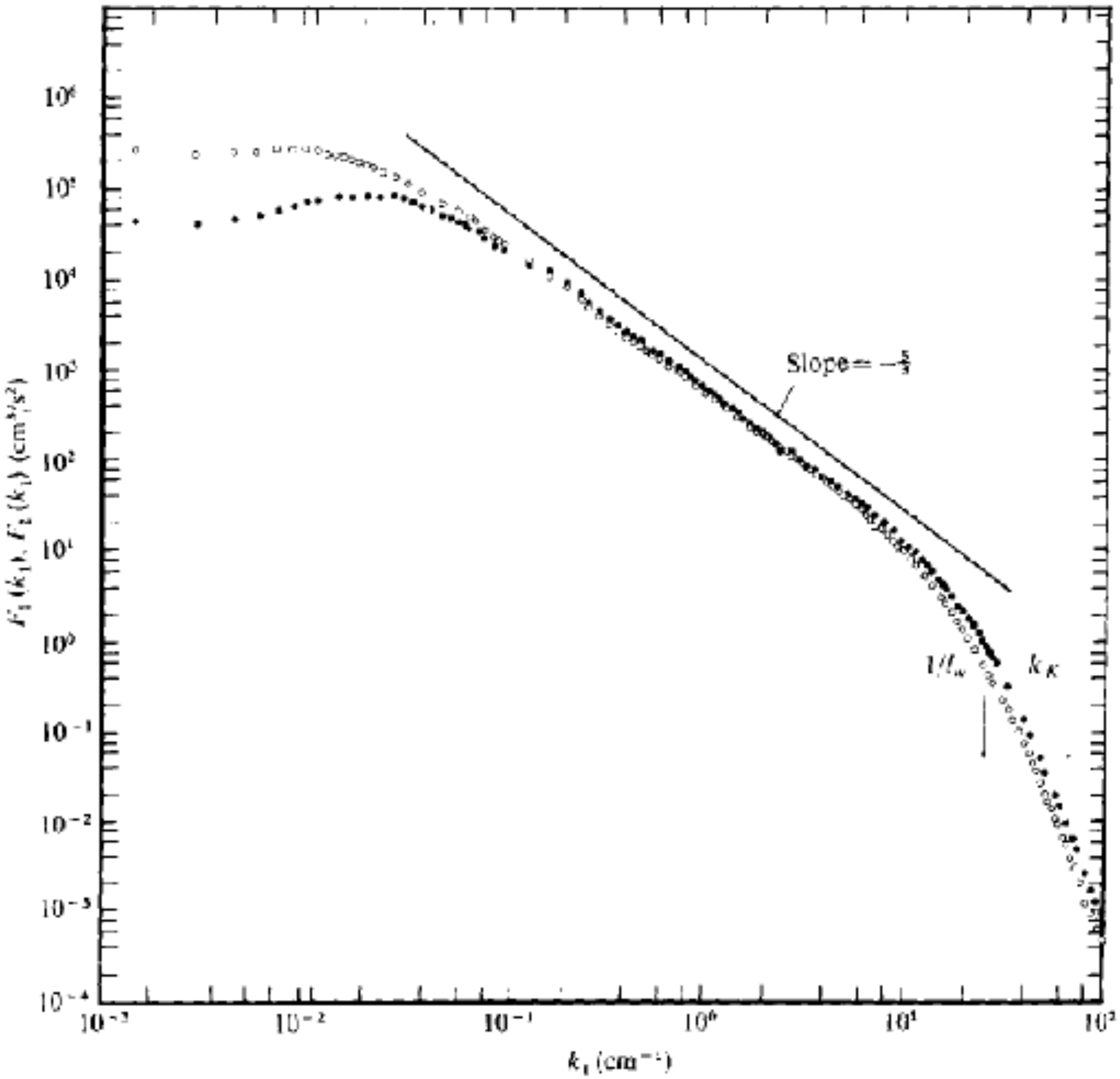
$$r \equiv |(\vec{R}+\vec{r}) - \vec{R}| \qquad \Delta u_{||}(r) \equiv (u_{||}(\vec{R}+\vec{r}) - u_{||}(\vec{R}))$$

A. N. Kolmogorov, *Dokl. Akad. Nauk SSSR* **30**, 299 (1941)

The second order structure function

$$S_2(r) \equiv \langle (\Delta u_{||}(r))^2 \rangle = \langle (u_{||}(\vec{R}+\vec{r}) - u_{||}(\vec{r}))^2 \rangle = C \ (\bar{\varepsilon} r)^{2/3}$$

C : Kolmogorov constant ; $\overline{\varepsilon}$: Avg. energy flux per unit mass



Velocity power spectrum:

$$\phi \ (k) = C \ \overline{\varepsilon}^{2/3} k^{-5/3}$$

Scaling correspondence ($r \Leftrightarrow k$):

$$S_2(r) \sim r^{2/3}$$

$$\Updownarrow$$

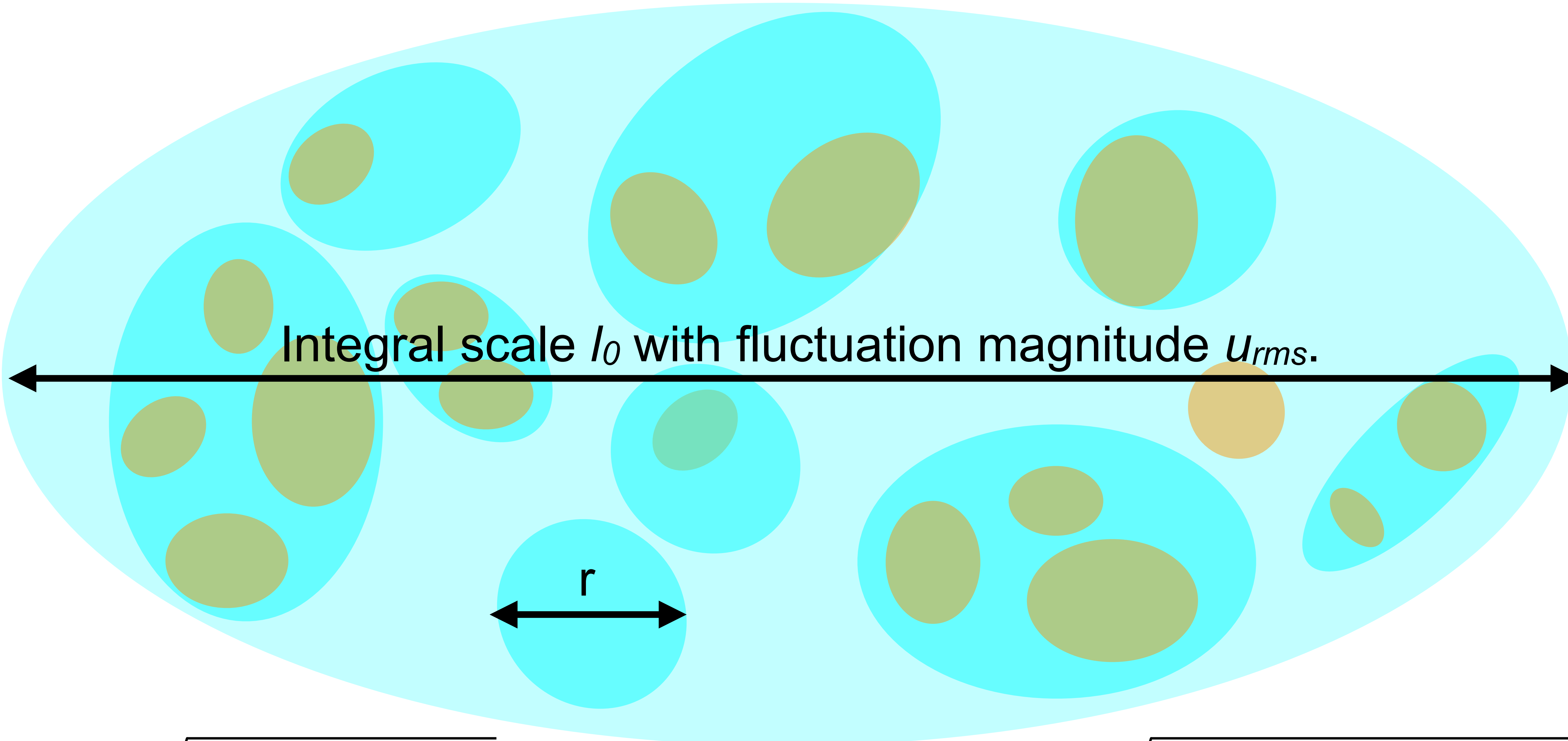
$$\phi \ (k) \sim k^{-5/3}$$

We will work in space/time domain for most part.

A. N. Kolmogorov, *Dokl. Akad. Nauk SSSR* **30**, 299 (1941)
F. H. Champagne, *J. Fluid. Mech.* **67**, 86 (1978)

Physical interpretation of Kolmogorov's result: $S_2(r) = C\ (\bar{\varepsilon} r)^{2/3}$

Large eddies contain smaller eddies, and so on in self-similar structure.

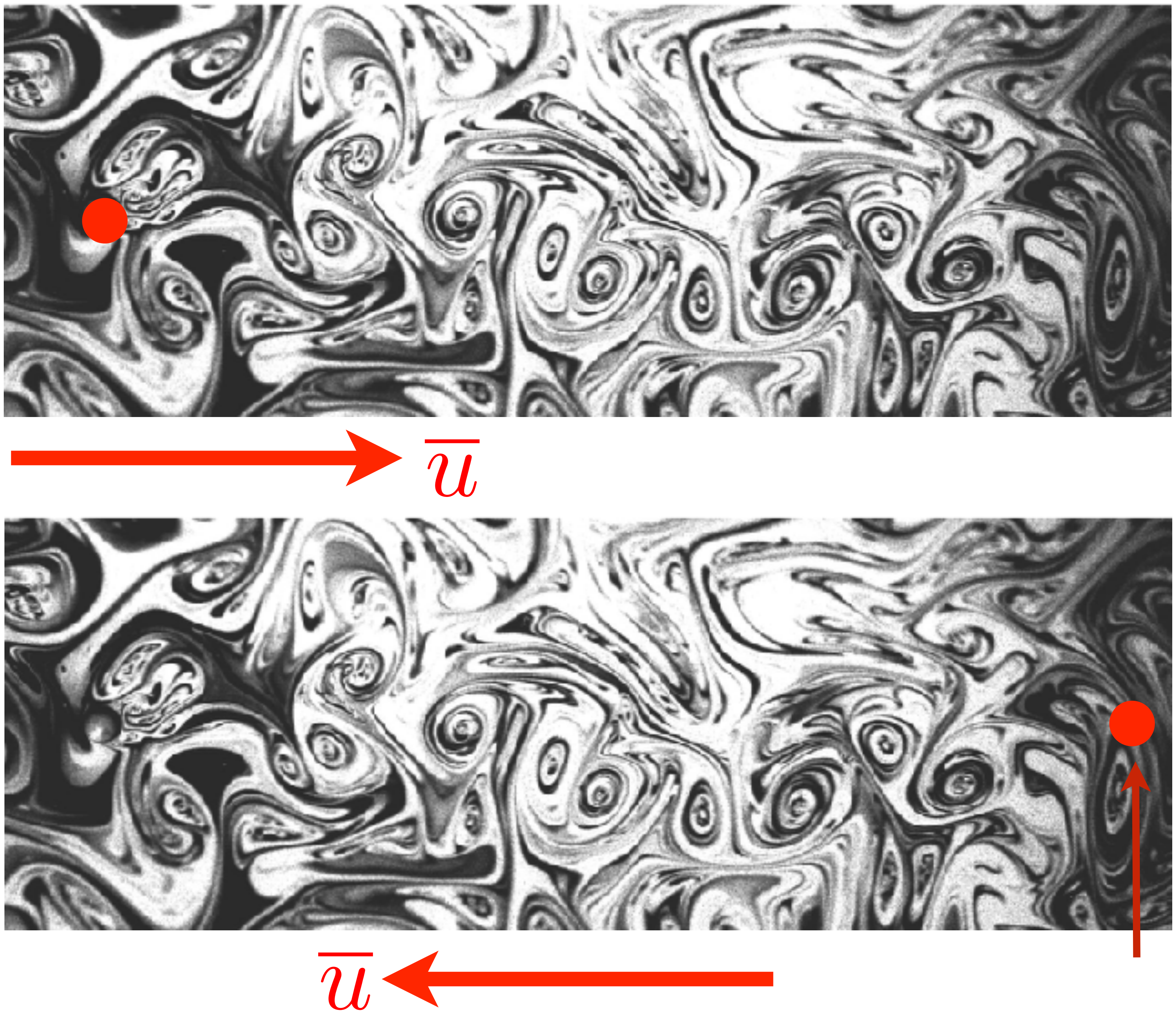


$$u_r = \sqrt{\langle (\Delta u_{||}(r))^2 \rangle} \qquad u_{rms} = \sqrt{\langle (\Delta u_{||}(r = l_0))^2 \rangle}$$

A. N. Kolmogorov, *Dokl. Akad. Nauk SSSR* **30**, 299 (1941)

Taylor's Hypothesis: Switching length & time scales.

Eddies swept past a stationary probe are static (structure frozen in time).



$$r = \overline{u}\tau$$

$$S_2(r) = C\,(\overline{\varepsilon}r)^{2/3}$$

$$\Updownarrow$$

$$S_2(\tau) = C\,(\overline{u\varepsilon}\tau)^{2/3}$$

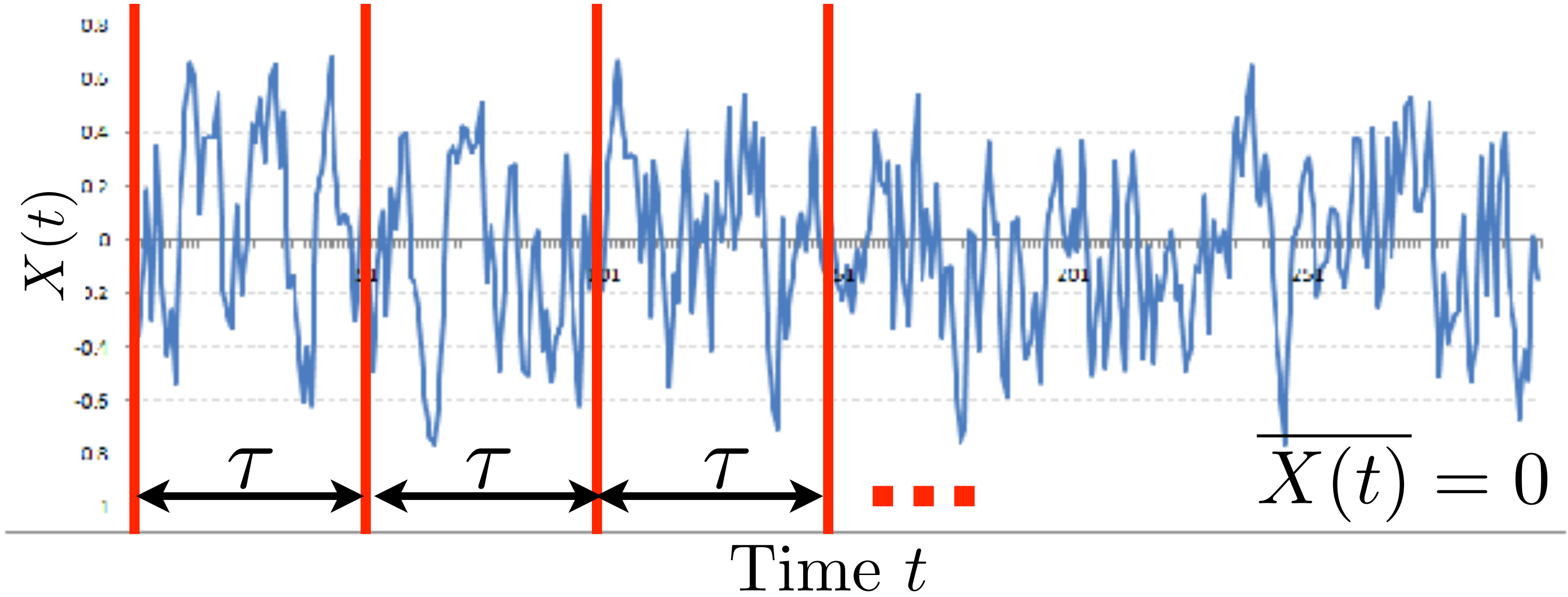
Nota Bene: Random Sweeping.

G. I. Taylor, *Proc. Roy. Soc. Lond. A* **164**, 476 (1938)

Structure Function: Meaning

Stationary time series *X(t)*, with zero mean ($\overline{X(t)} = 0$).

Break into windows of duration τ.



$\Delta X(\tau) \equiv X(t+\tau) - X(t)$ is the fluctuation magnitude over time scale τ.

$\Delta X(\tau)$ is a random variable. Each τ gives a probability distribution (PDF) $\Pi[\Delta X(\tau)]$.

$S_n(\tau) \equiv \langle(\Delta X(\tau))^n\rangle$ are moments of PDF $\Pi[\Delta X(\tau)]$, for each value of τ.

$S_n(\tau)$ vs. τ contain structure of the time variation of moments, hence the name.

$S_2(\tau)$, 2nd order structure function (variance of $\Pi[\Delta X(\tau)]$) is particularly special:

$$S_2(\tau) \equiv \langle(X(t+\tau) - X(t))^2\rangle = \langle X(t+\tau)^2\rangle + \langle X(t)^2\rangle - 2\langle X(t)X(t+\tau)\rangle$$

$$= 2\langle X^2\rangle - 2\langle X(t)X(t+\tau)\rangle$$

Variance — Correlation function

Higher-order spectra: 2nd order structure functions for u^m.

Terminology: "Higher-order structure function" is $S_n^1(\tau) \equiv \langle(\Delta u^1(\tau))^n\rangle$

Higher-order spectra: 2nd order structure functions for u^m: $S_2^m(\tau) \equiv \langle(\Delta u^m(\tau))^2\rangle$

K41 is about Higher-order structure functions, says nothing about Higher-order spectra.

But if Higher-order spectra follow K41 dimensionality, they would scale as:

$$S_2^m(\tau) \equiv \langle(\Delta u^m(\tau))^2\rangle = C_m(\overline{u\varepsilon}\tau)^\gamma$$

$$\Updownarrow \qquad \Updownarrow$$

$$[L]^{2m}[T]^{-2m} = [L]^{3\gamma}[T]^{-3\gamma}$$

$$\Rightarrow \gamma = \frac{2m}{3}$$

$$\Rightarrow S_2^1(\tau) \sim \tau^{2/3}; \; S_2^2(\tau) \sim \tau^{4/3}; \; S_2^3(\tau) \sim \tau^2$$

...

These scalings are never observed.

DA Dutton & DG Deaven, *Statistical Mechanics and Turbulence. Lecture Notes in Physics, vol. 12.* Ed. Rosenblatt & Van Atta (Springer, 1972).

2nd order structure functions for u^m

Recall: $a^m - b^m = (a-b)((m-1)\text{ order term})$

$$S_2^m(\tau) \equiv \langle(\Delta u^m(\tau))^2\rangle \equiv \langle(u(t+\tau)^m - u(t)^m)^2\rangle$$
$$= \langle[u(t+\tau) - u(t)]^2[(m-1)\text{ order term}]^2\rangle$$
$$\sim \langle(\Delta u(\tau))^2\rangle\langle[(m-1)\text{ order term}]^2\rangle$$
$$\sim u_{rms}^{2(m-1)} S_2^1(\tau) + \text{ h.o. terms}$$

$S_2^1(\tau) \sim (\overline{u}\varepsilon\tau)^{2/3}$

$u_{rms}^{2(m-1)}$: Velocity fluctuation of largest (Integral scale) eddy.

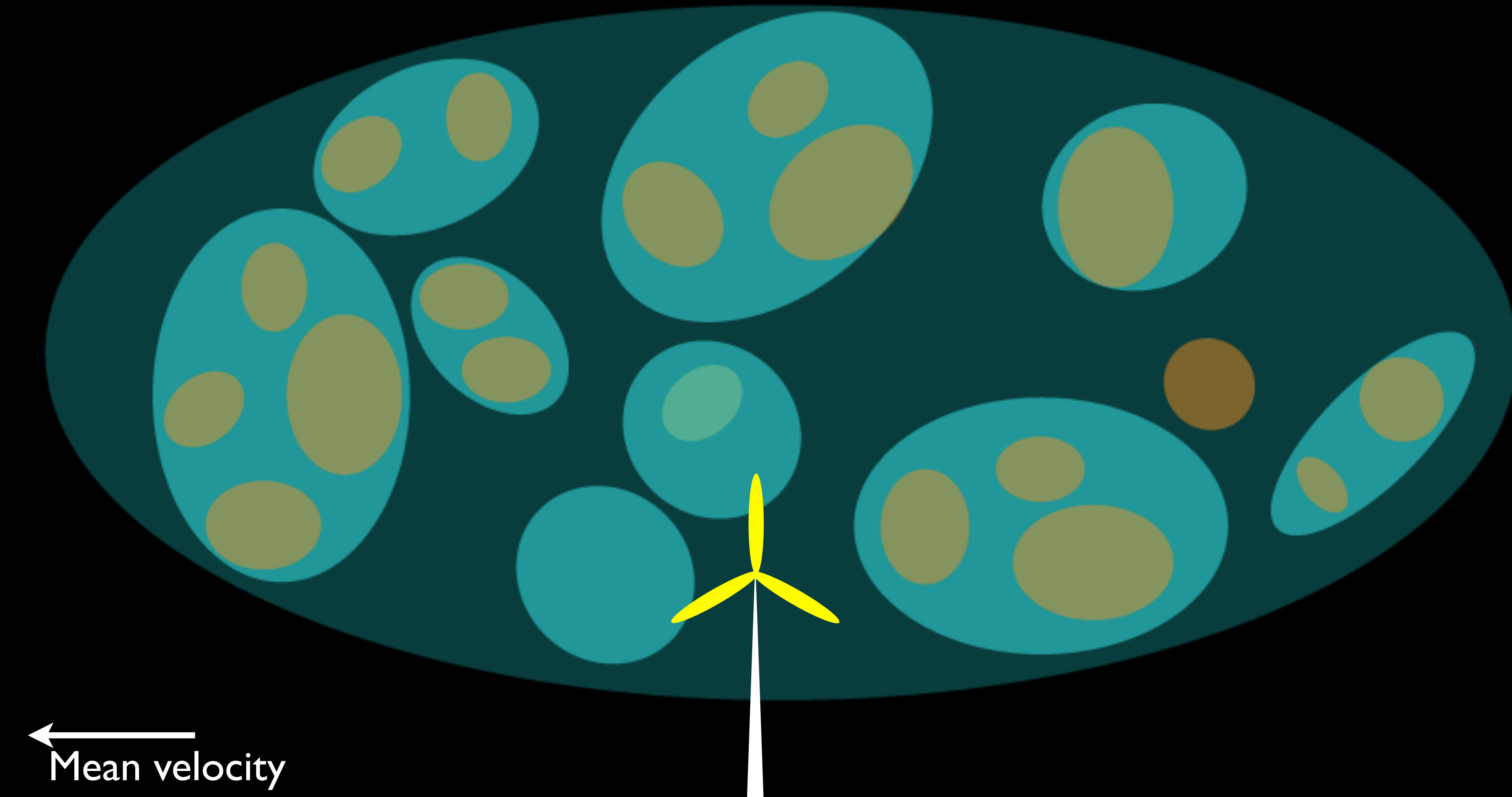
$$S_2^m(\tau) \sim \tau^{2/3}\ \forall\ m \geq 1$$

$\Delta u(\tau)$ is Galilean invariant, $\Delta u^m(\tau)\ (m>1)$ are not.

Crucial Kolmogorov assumption violated by Galilean invariance breakdown.

u_{rms} in $S_2^m(\tau)$: integral scale influences all small eddies.

Taylor's hypothesis regime



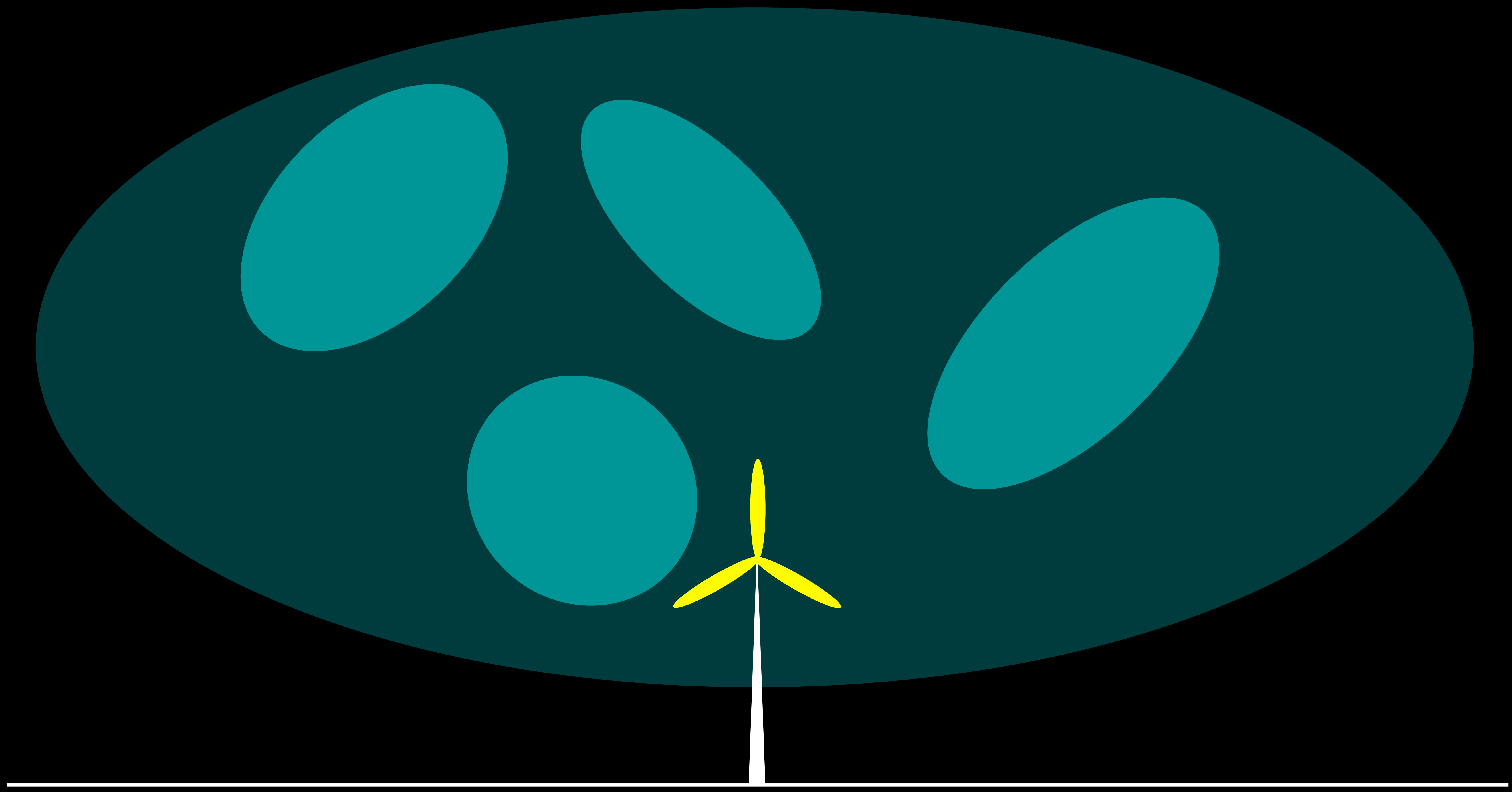
Small eddies not influenced by large eddies within which they nest.

All eddies swept by high mean velocity (no inherent timescale)

All eddies register shorter duration fluctuation: <u>same shift at all scales</u>.

Entire spectrum doppler shifted, but scaling not affected.

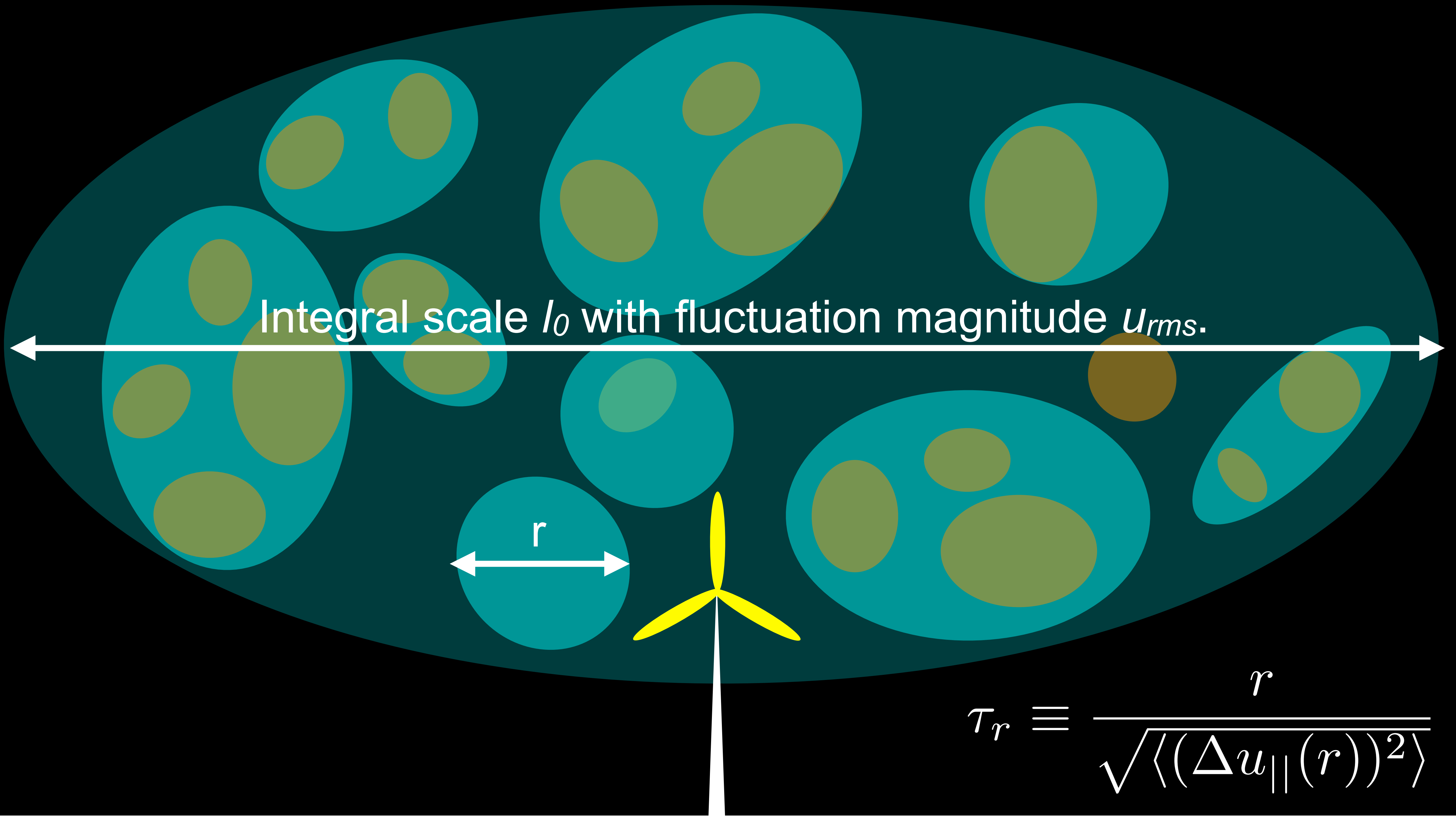
Large scale u_{rms} influence in Higher-order spectra



Galilean invariance breakdown: an artifact of what point probe registers.

Probe registers small eddies being distorted by larger eddies.

Integral scale (largest eddy) with u_{rms} registers most influence.



$$\tau_r \equiv \frac{r}{\sqrt{\langle (\Delta u_{||}(r))^2 \rangle}}$$

u_{rms} induces oscillation (r/u_{rms}) in eddy (size r) & doppler shifts it.

Doppler shift: measurement artifact, not true time scale τ_r.

Scale dependent doppler shifting of spectrum from $\tau^{2m/3} \to \tau^{2/3}$.

Spectrum undergoes doppler broadening.

From Point to Field Limit: Spatially averaged temporal signal

Galilean invariance breakdown for point measurement, restored in field limit.

$\Rightarrow S_2^m(\tau) \equiv \langle(\Delta u^m(\tau))^2\rangle \sim \tau^\zeta$ & $\zeta : 2/3 \rightarrow 2m/3$ from point to field limit.

Sum *N* distributed points into composite signal: $\mathcal{U}^m(t) \equiv \sum_{i=1}^{N} u_i^m(t)$

Probe $S_2^m(N,\tau) \equiv \langle(\Delta\mathcal{U}^m(\tau))^2\rangle \equiv \langle(\mathcal{U}^m(t+\tau) - \mathcal{U}^m(t))^2\rangle$; where $S_2^m(N,\tau) \sim \tau^{\zeta_m(N)}$

Two-point correlator controls evolution of $\zeta_m(N)$ vs. N:

$$\langle\mathcal{U}^m(t)\mathcal{U}^m(t+\tau)\rangle = \sum_{i,j=1,j\neq i}^{N} \langle u_i^m(t)u_i^m(t+\tau)\rangle + \langle u_i^m(t)u_j^m(t+\tau)\rangle$$

self-corr x-corr

x-corr: spatial interaction between temporal signals separated by distance r_{ij}.

Fluctuations by eddy of size $r < r_{ij}$ decorrelate but remain correlated for $r > r_{ij}$.

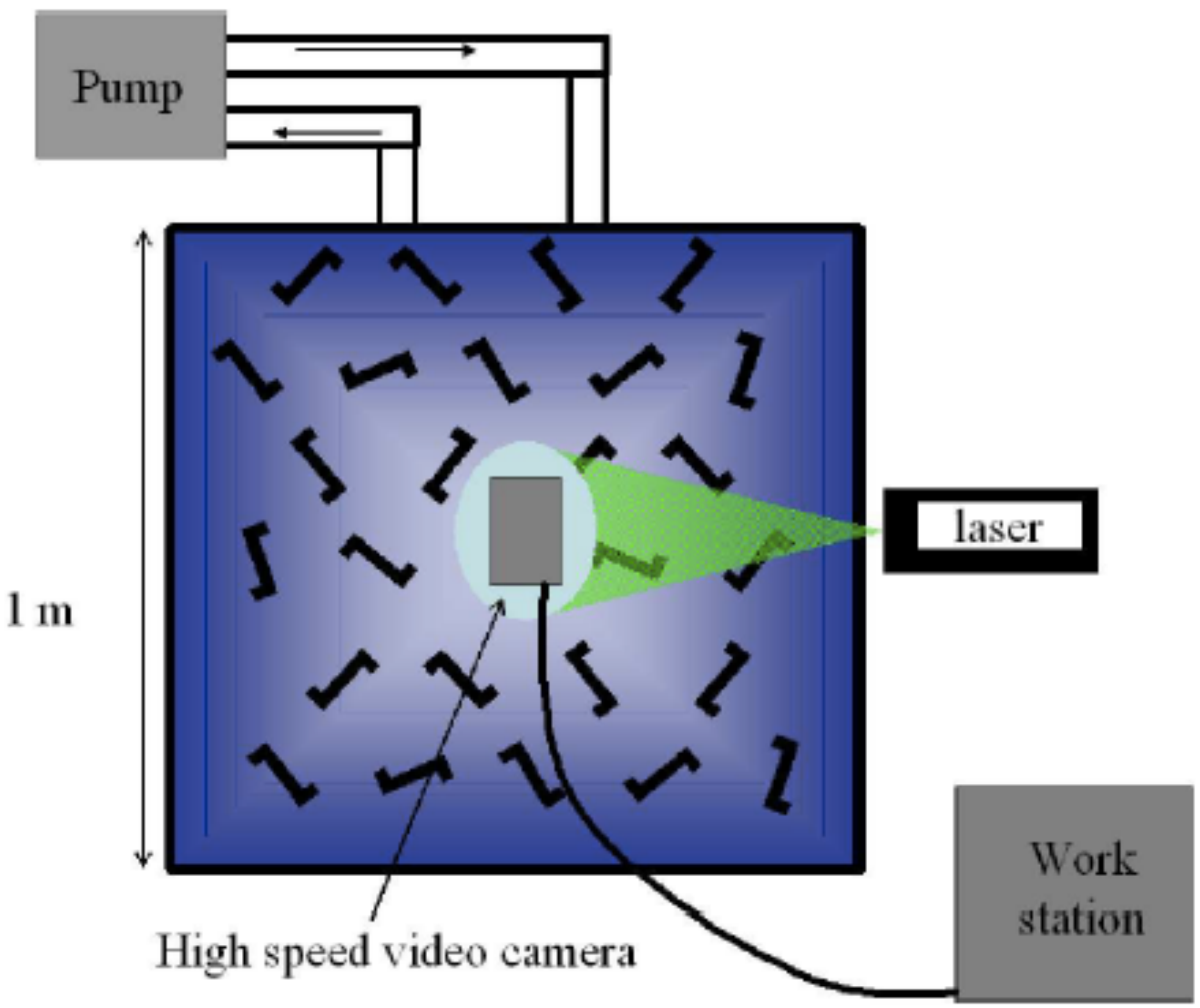
x-corr $\rightarrow 0$ as $r_{ij} \rightarrow l_0$; self-corr remains yielding $\zeta_m(N) = 2m/3$.

x-corr captures inter-scale couplings: local & decay exponentially in 3D.

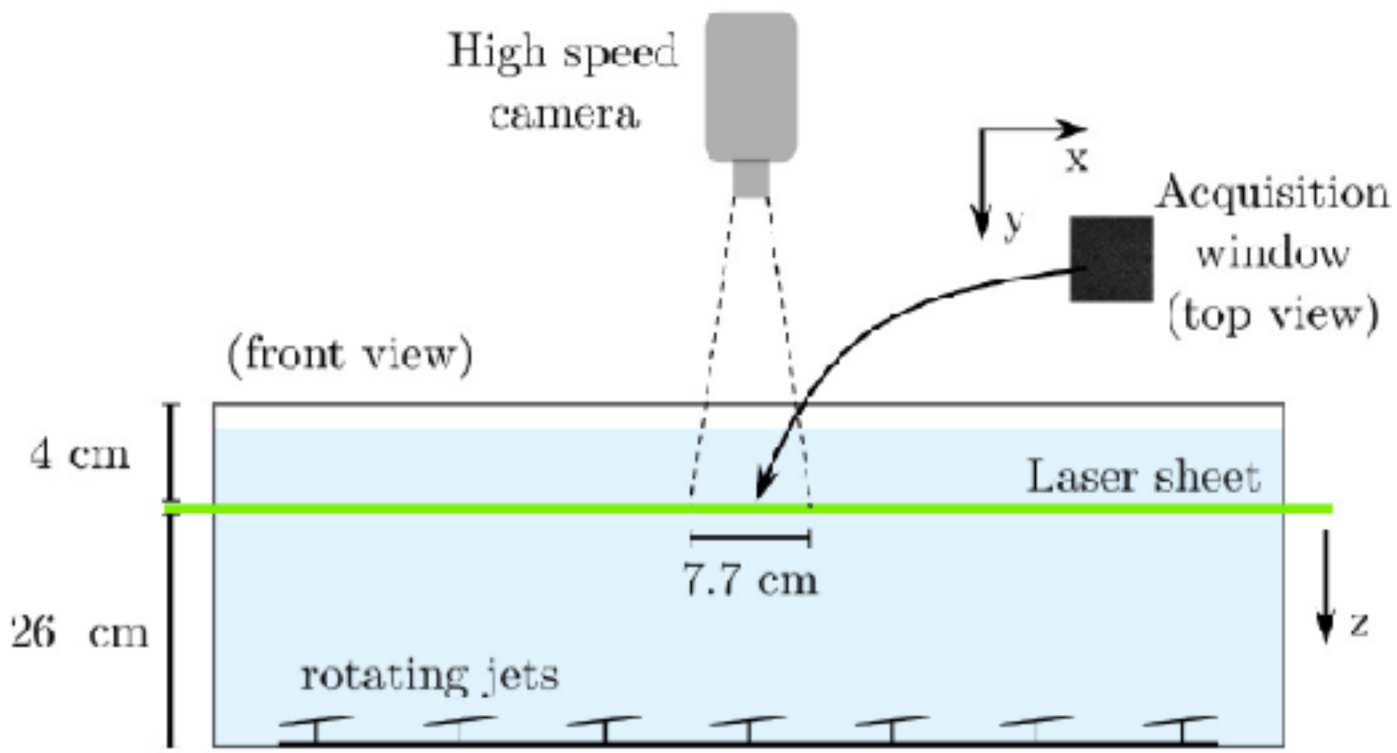
If correct, we expect $\zeta_m(N)$ vs. N will converge **exponentially** as $N = 1 \rightarrow \infty$.

3D Experimental Setup

Top view

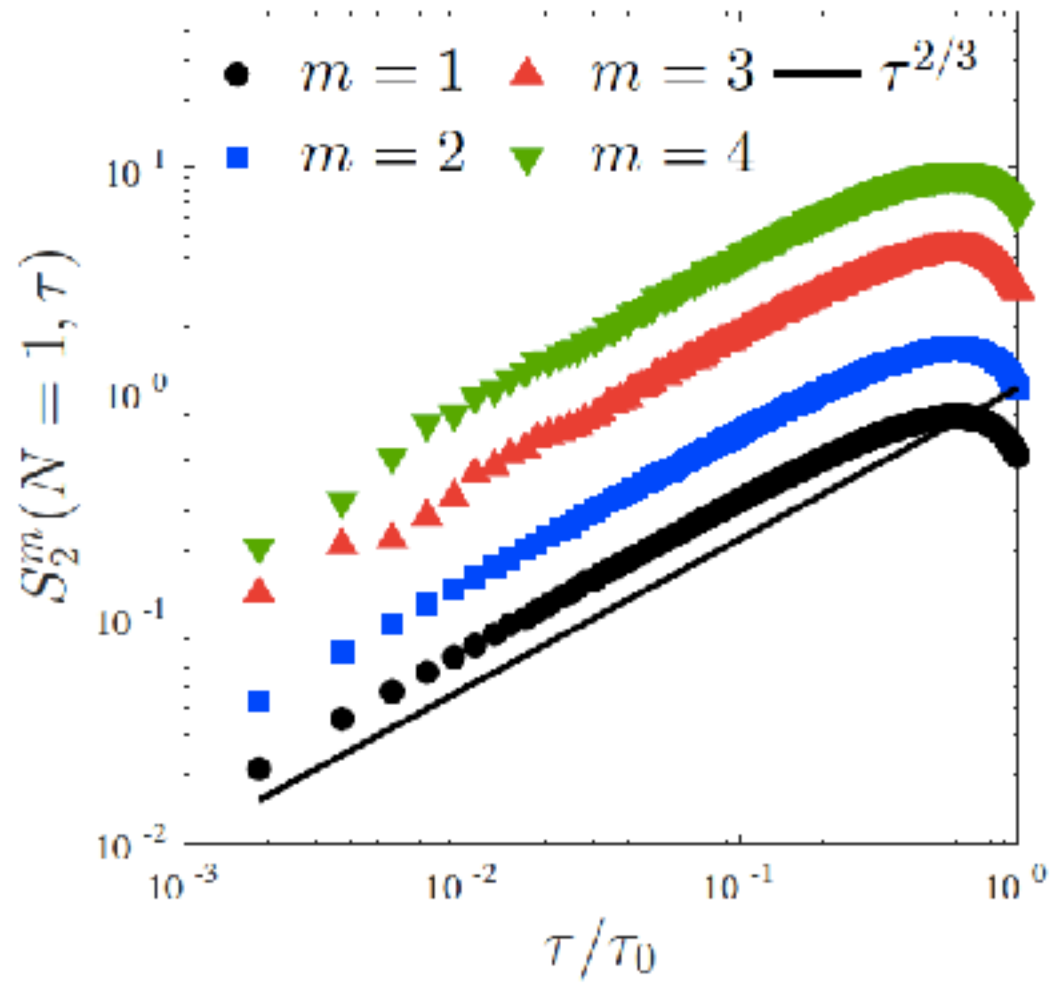


Side view

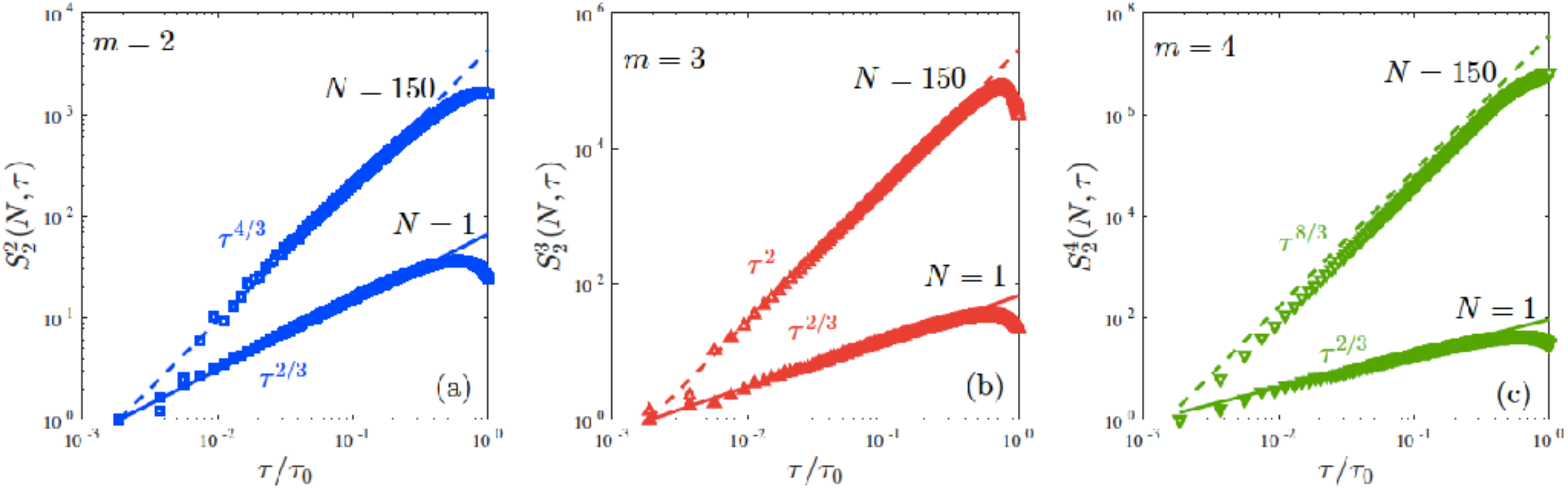


- 1 m x 1 m x 0.4 m tank filled with water; forcing: 8hp pump with rotating jets on floor.
- Turbulence vertically inhomogeneous but homogeneous along any horizontal plane.
- Laser sheet illuminates cut through bulk flow 4 cm below surface.
- TiO_2 tracers (dia. 10 microns) image flow at 500 fps with Phantom v641 camera.
- Each experimental run ~ 11 turnover times; total 20 runs.
- In-house PIV codes process images and construct velocity field.
- Taylor Microscale Reynolds: 140; u_{rms}: 3.5 cm/s; no mean velocity.

3D Results

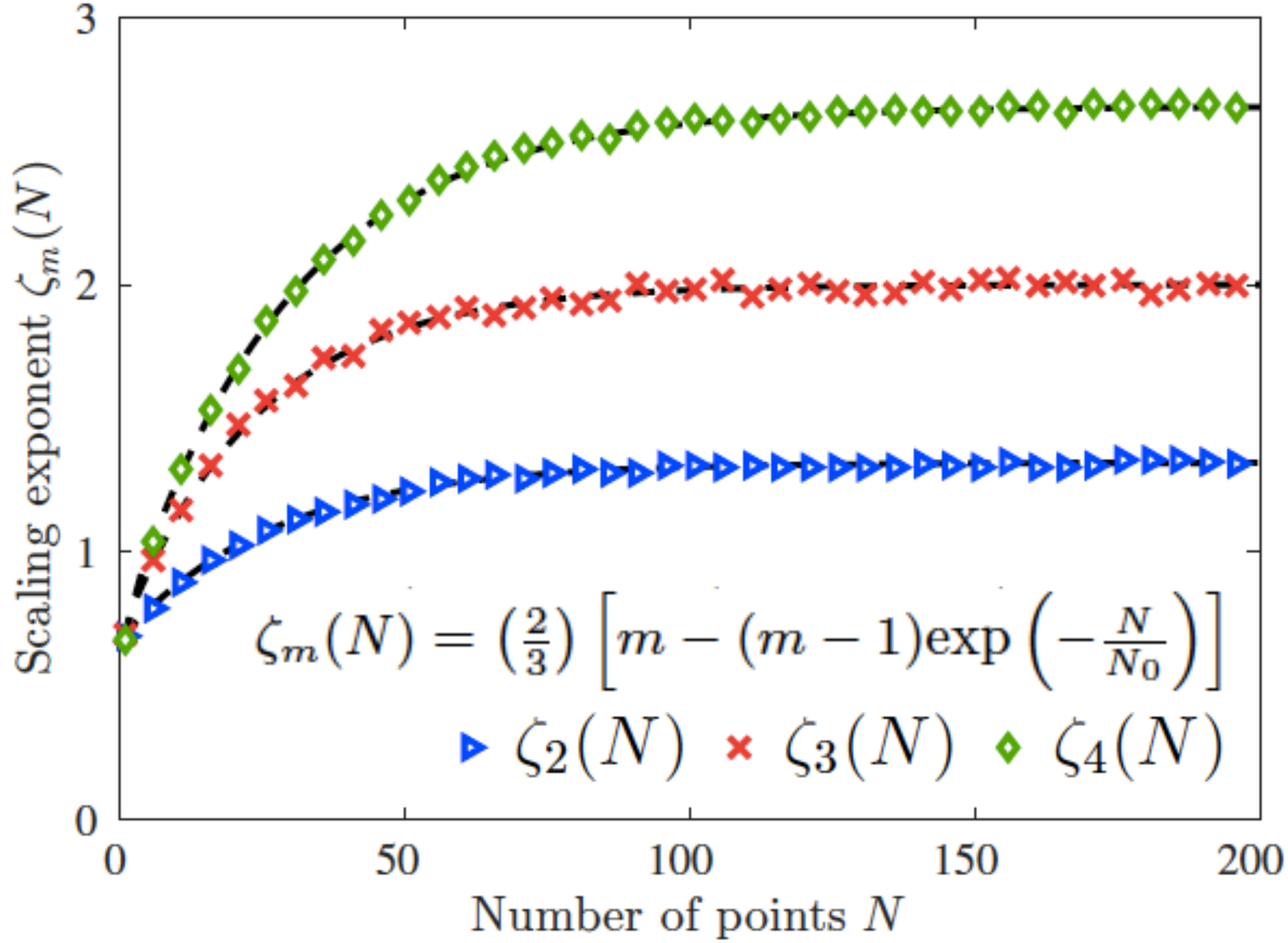


m = 1
m = 3
τ^{2/3}
m = 2
m = 4
S_2^m(N = 1, τ)
τ/τ_0



m = 2
N = 150
N = 1
τ^{4/3}
τ^{2/3}
(a)
S_2^2(N, τ)
τ/τ_0
m = 3
N = 150
N = 1
τ^2
τ^{2/3}
(b)
S_2^3(N, τ)
τ/τ_0
m = 4
N = 150
N = 1
τ^{8/3}
τ^{2/3}
(c)
S_2^4(N, τ)
τ/τ_0

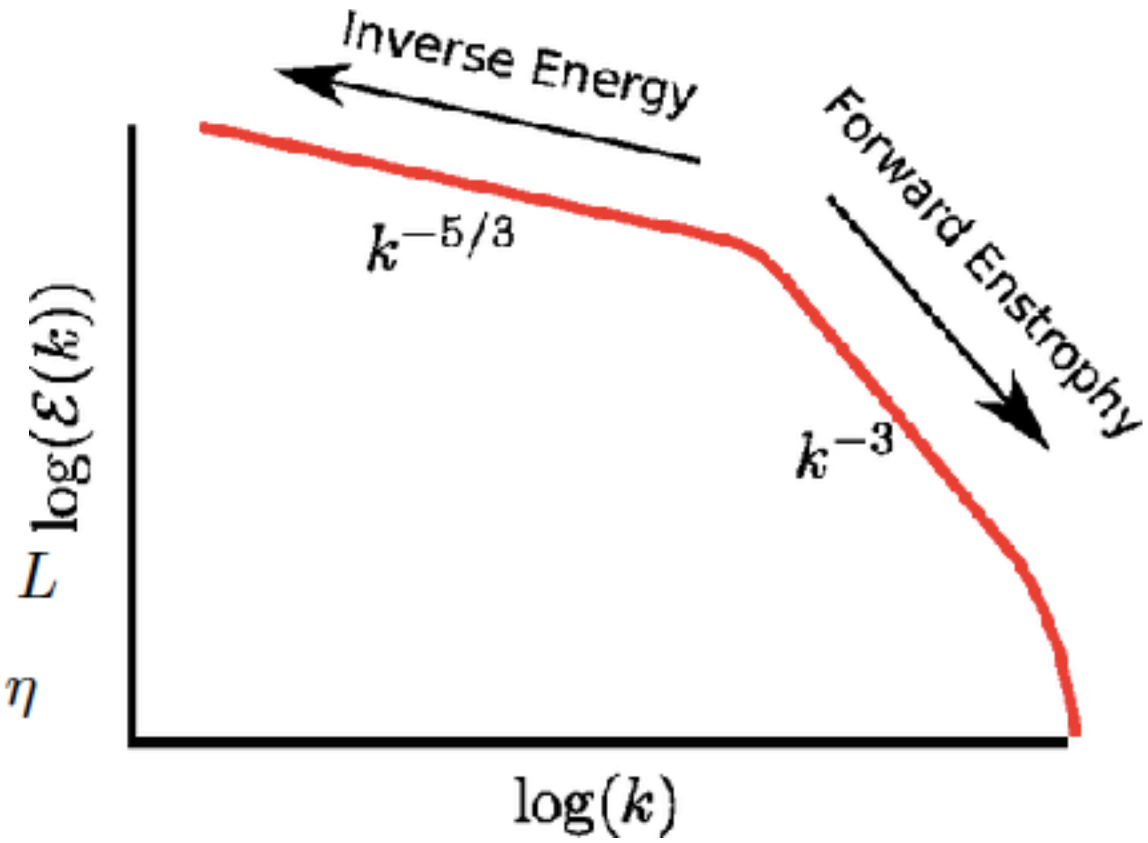
3D Results



- $S_2^m(\tau) \sim \tau^{2/3}\ \forall\ m$ for *N = 1* and $\zeta_m(N) : 2/3 \to 2m/3$ as $N : 1 \to \infty$.
- $\zeta_m(N)$ vs. N convergence from point to field limit is **exponential**.
- Best Fit: $\zeta_m(N) = \left(\frac{2}{3}\right)\left[m - (m-1)\exp\left(-\frac{N}{N_0}\right)\right]$. N_0 convergence rate constant.

2D Turbulence Primer

- 2D turbulence radically different from 3D.
- Energy & enstrophy are conserved.
- Primary scaling expectations:



Inverse cascade regime: $S_2^1(r) \sim (\bar{\varepsilon} r)^{2/3}, \quad r_{inj} < r < L$

Direct cascade regime: $S_2^1(r) \sim \beta^{2/3} r^2, \quad r_{inj} > r > \eta$

where $\beta \equiv \dfrac{d\langle(\vec{\nabla} \times \vec{u})^2\rangle}{dt}$

- In reality, Direct cascade sees infra red divergence, i.e. as $Re \to \infty; \ r_{inj} \to 0.$
- Constant enstrophy flux introduces log correction: $S_2^1(r) \sim \beta^{2/3} r^2 \ \ln\left(\dfrac{r_{inj}}{r}\right)$
- Log term implies non-local effects; not observed to date.
- Finally, no theory for higher-order spectra; we proceed through empirical evidence.

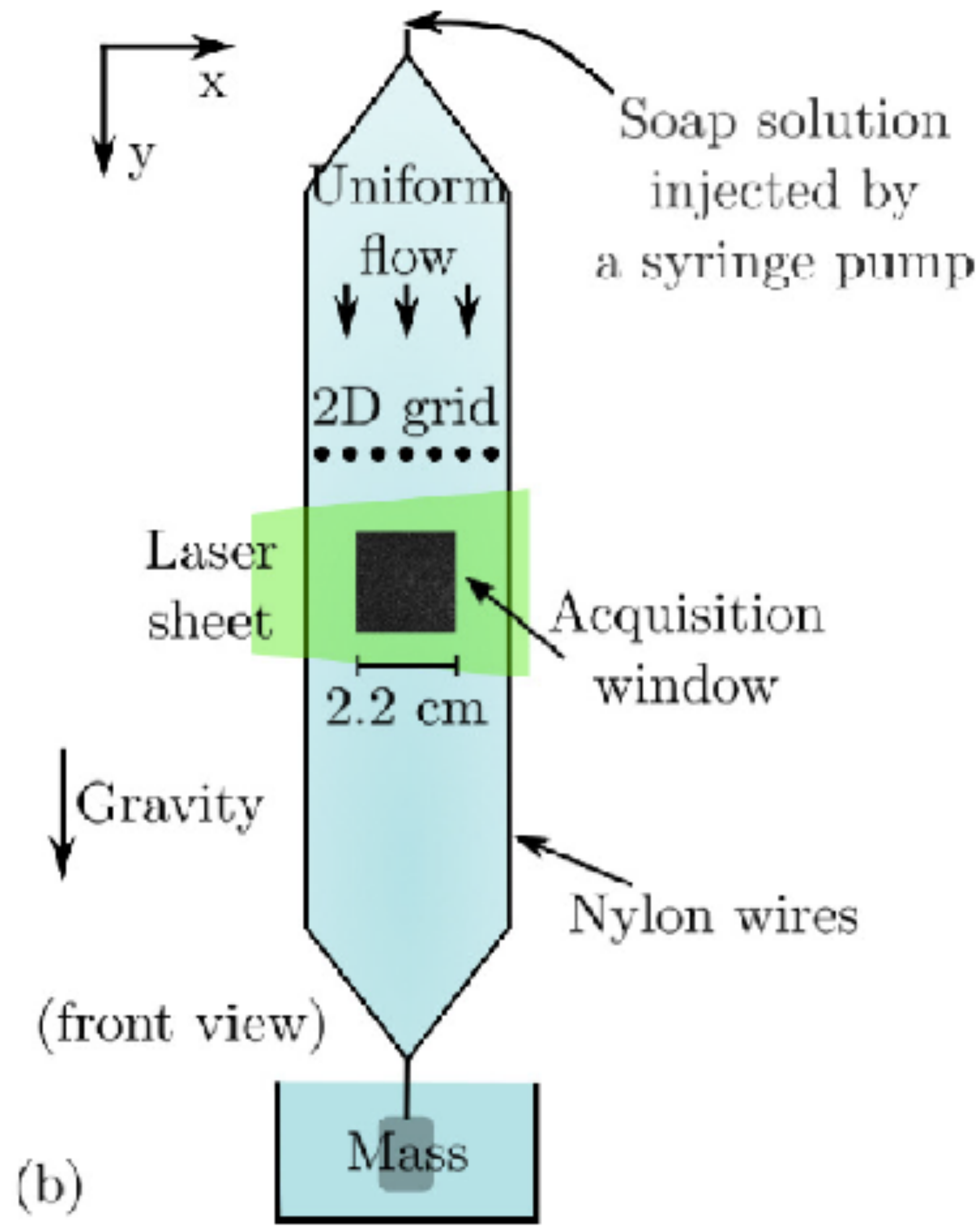
RH Kraichnan, *Phys. Fluids* **10**, 1417 (1967)
CE Leith, *Phys. Fluids* **11**, 671 (1968)
GK Batchelor, *Phys. Fluids* **12**, 233 (1969)
RH Kraichnan & D Montgomery, *Rep. Prog. Phys.* **65**, 845 (1980)
G Boffetta & RE Ecke, *Annu. Rev. Fluid Mech.* **44**, 427 (2012)

2D Experimental Setup

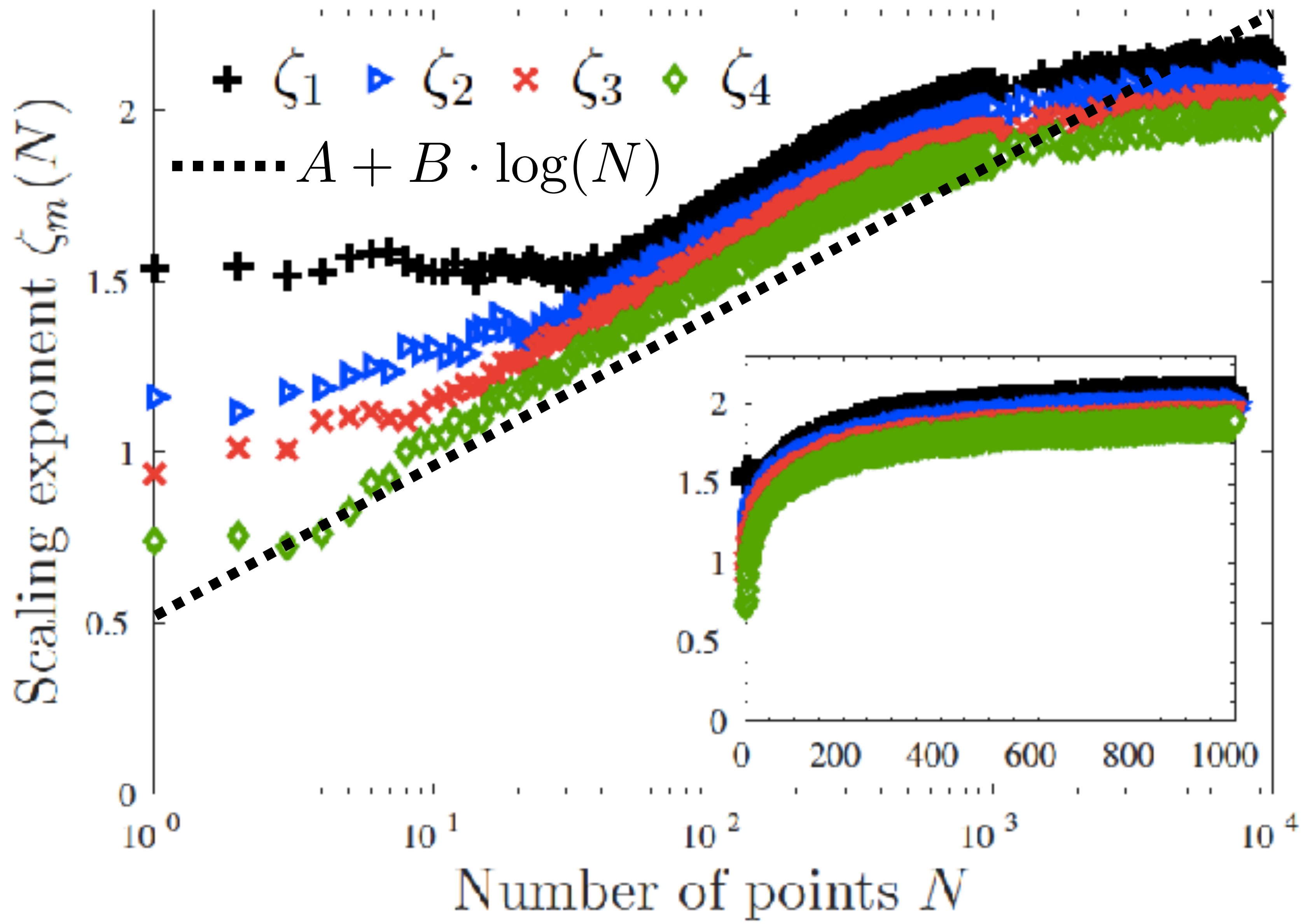
(Experiments by Dr. Florine Paraz, OIST)

- Soap soln. from syringe pump drains down nylon wire pair held by hanging mass.
- Stretch wires to generate film, grid (comb) generates turbulence.
- Direct cascade: grid normal to flow. Inverse cascade: grid parallel to flow.
- Laser sheet illuminates film section below grid, TiO_2 tracers image flow.
- Imaging at 5000 fps; in-house PIV codes construct velocity field.

Direct (Enstrophy) Cascade Regime



2D Point to Field Scaling



$$\zeta_m(N) = A + B \cdot \log(N)$$

m value	Parameter A	Convergence rate constant B	N value for start of log behavior
1	0.5	0.42	32
2	0.4	0.4	18
3	0.28	0.45	8
4	0.12	0.5	4

Brief Summary

- 3D: All point spectra $S_2^m(\tau) \sim \tau^{2/3} \; \forall \; m \geq 1$
- 3D: Field limit $S_2^m(N \rightarrow \infty, \tau) \sim \tau^{2m/3}$.
- 3D: **Exponential** point-to-field convergence.
- 2D: **Logarithmic** field convergence in 2D Direct cascade. Some remarks are in order.
- 2D: Direct cascade field limit exponents exhibit: $\zeta_m(N \rightarrow \infty) = 2 \; \forall \; m \geq 1$
- 2D: Theory for higher-order spectra remains to be worked out.

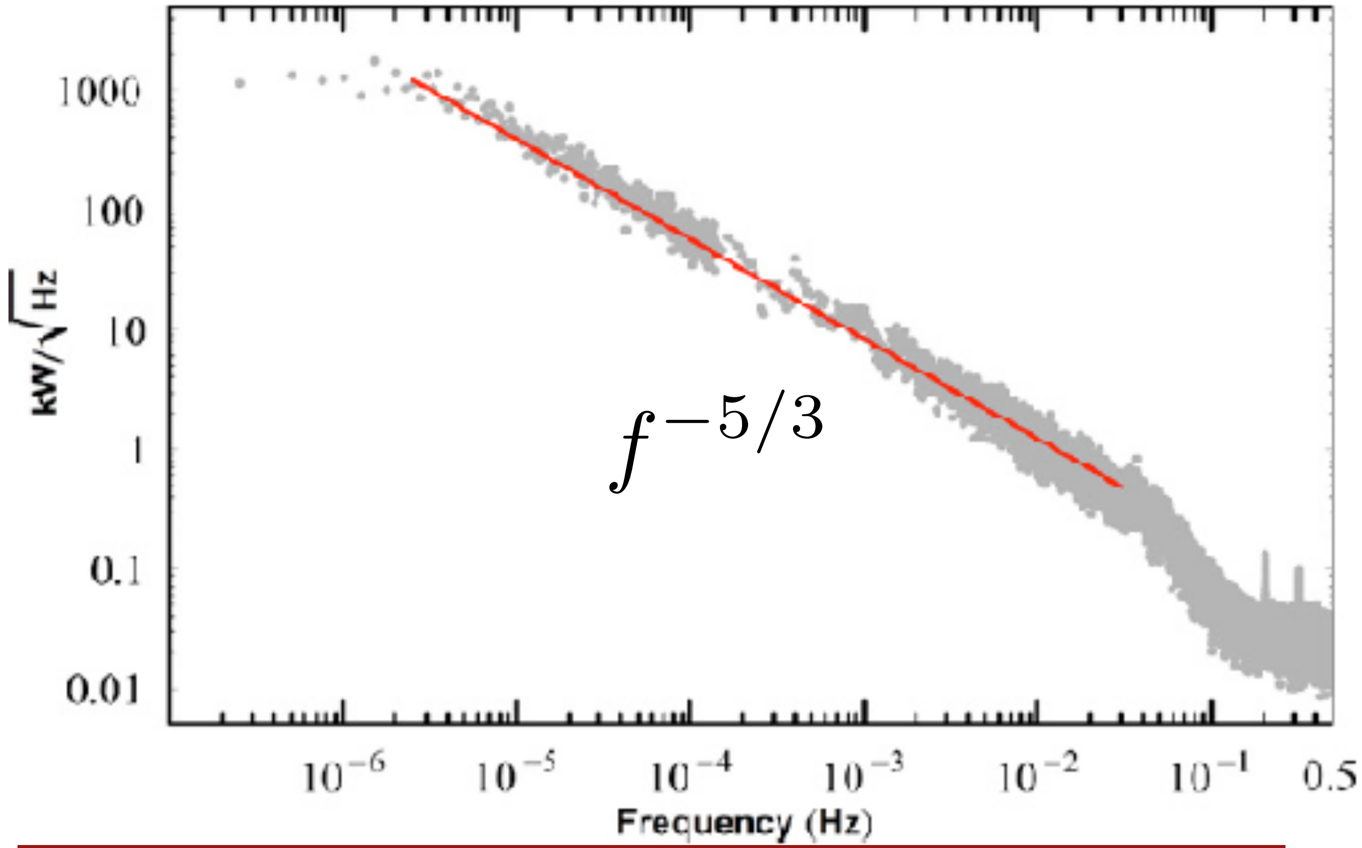
We're not done, there's more…

- Higher-order spectra toolkit applies to:
  - Wind Power Fluctuations.
  - Fluctuations in flight for UAVs.
  - Atmospheric flows are 2D or 3D?
  - etc.

"Are you not thinking what I'm not thinking?"

F Paraz & MM Bandi "Eulerian Point to field convergence in higher-order turbulence spectra" *In Review*.

Wind power fluctuates with wind speed



…follows a Kolmogorov spectrum over more than 4 orders of magnitude from 30 s to 2.6 days.

J Apt, *J. Power Sources* **169**, 369 (2007)

Power available in wind blowing past a turbine of area A:

$$P(t) = Ku(t)^3; \; K \leq \left(\frac{16}{27}\right)\left(\frac{1}{2}\right)\rho A$$

We seek the spectrum for P(t)

Relating 2nd order structure function of *P(t)* to that of *u(t)*^{m}, one can show:

$$D_2(\tau) \equiv \langle(\Delta P(\tau))^2\rangle \sim 9K^2\overline{u}^4 S_2^1(\tau) + 9K^2\overline{u}^2 S_2^2(\tau) + K^2 S_2^3(\tau)$$

Previously, we saw from theory and experiments that: $S_2^m(\tau) \sim \tau^{2/3} \; \forall \; m \geq 1$

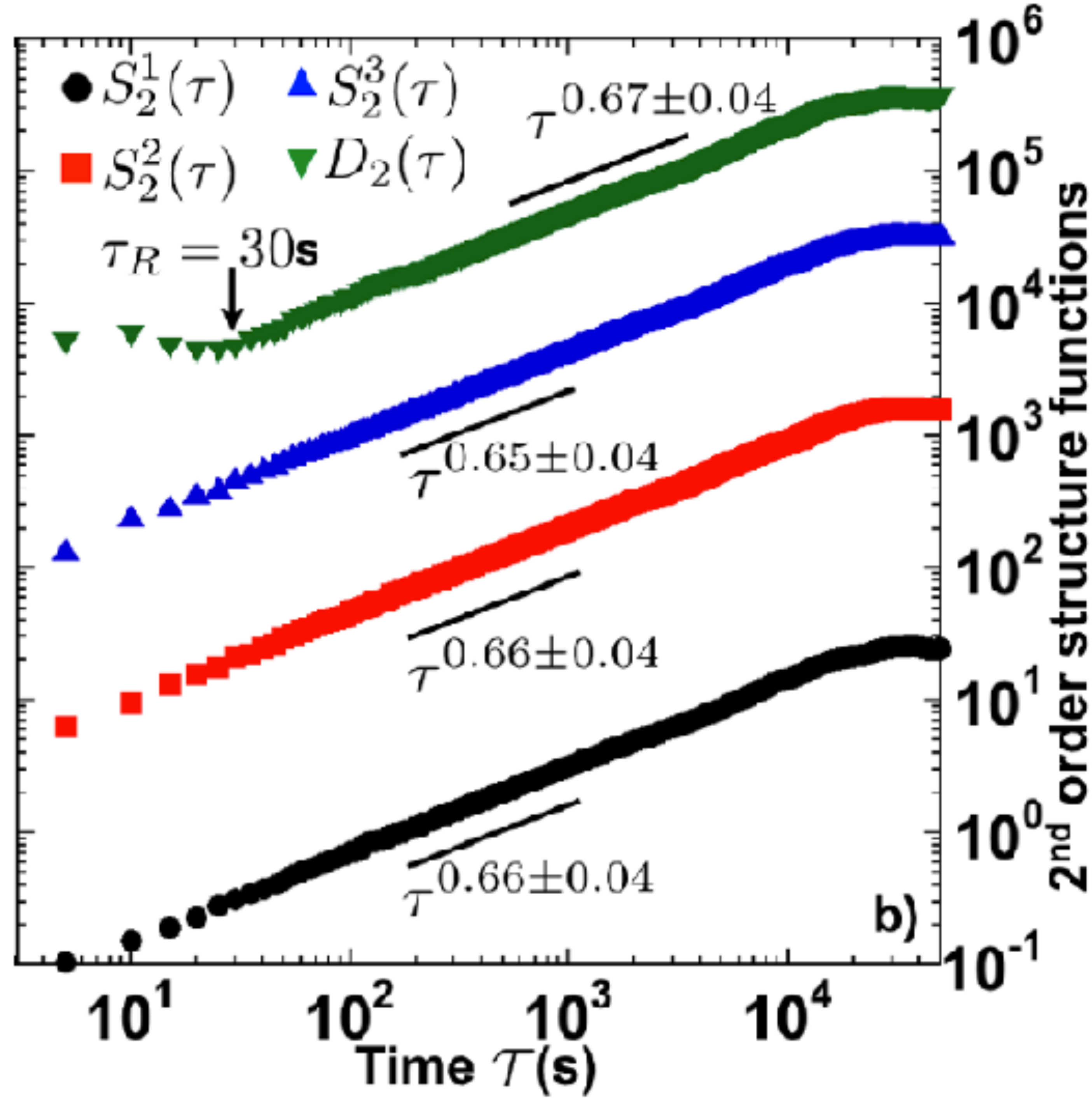
Single Turbine Measurements.

$$D_2(\tau) \equiv \langle (\Delta P(\tau))^2 \rangle \sim 9K^2\overline{u}^4 S_2^1(\tau) + 9K^2\overline{u}^2 S_2^2(\tau) + K^2 S_2^3(\tau)$$

- Speed & power data: Howard, NY.
- Time series sampled at 5 s for 20 days.
- Corr. time: 12.8 Hrs.
- Corr. time (τ_0): diurnal oscillations.

$$l_0 = u_{rms} \cdot \tau_0$$
$$= 4.75 \text{ m/s } \times 12 \text{ Hrs}$$
~ 200 km Correlation length

- Turbines within 200 km correlated.
- Grid level consequences.



Long range correlation is real

Our estimate $l_0 \sim 200$ km

Farm correlation ~$e^{(-d/D)}$

d: Inter-farm distance

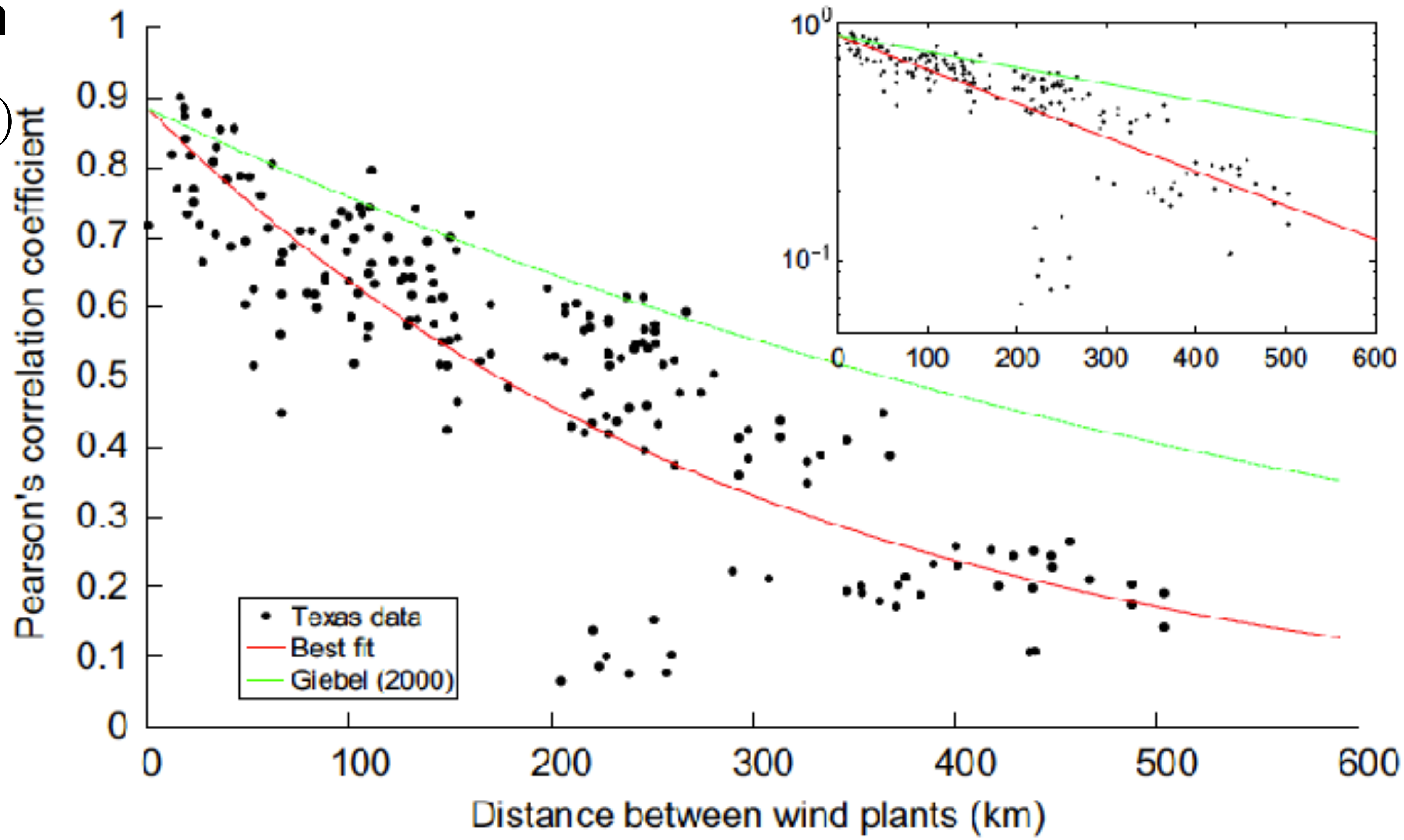
D: Decay length.

D ~ 305 km (ERCOT, TX)

D ~ 375 km (Germany)

D ~ 500 km (Denmark)

D ~ 641 km (Europe)



Henceforth, we assume *D ~ l_0*, *i.e Decay length ~ integral scale*

W Katzenstein, E Fertig & J Apt, Energy Policy 38, 4400 (2010)
G Giebel, PhD Dissertation, Oldenburg University (2000)
R Steinberger-Willms, PhD Dissertation, Oldenburg University (1993)
L Landberg et al, Risø National Laboratory, Tech. Report (1997)

Aggregate plant fluctuations

Aggregate plant fluctuations ~ $\tau^{2/3}$ scaling.

3 turbulence sources contribute here.

Dominant contribution from correlation time.

1) Boundary layer turbulence:

$\tau_0 = l_0/v_{rms} = 1 \text{ km}/4 \text{ m}\cdot\text{s}^{-1} \sim 4 \text{ min}$

2) Plant generated (wake) turbulence:

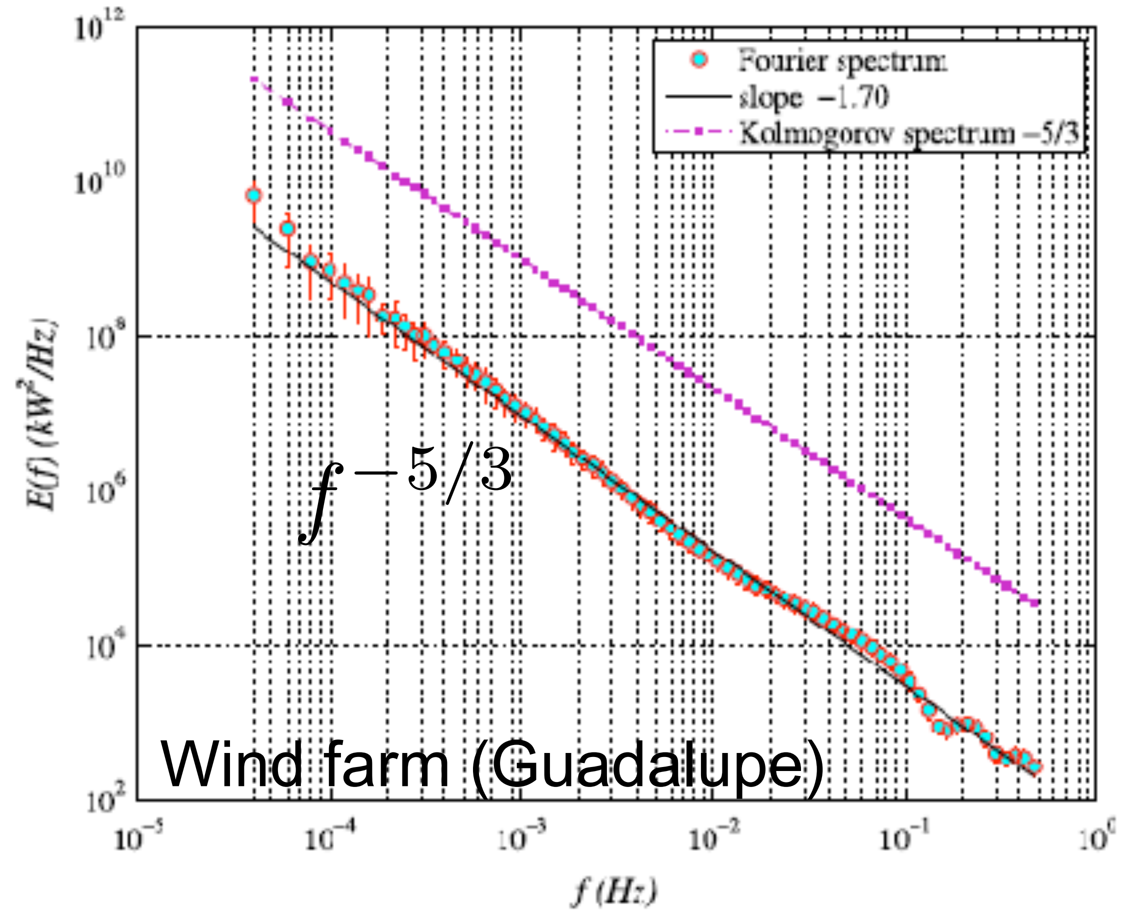
$\tau_0 = l_0/v_{rms} = 5 \text{ km}/4 \text{ m}\cdot\text{s}^{-1} \sim 21 \text{ min}$

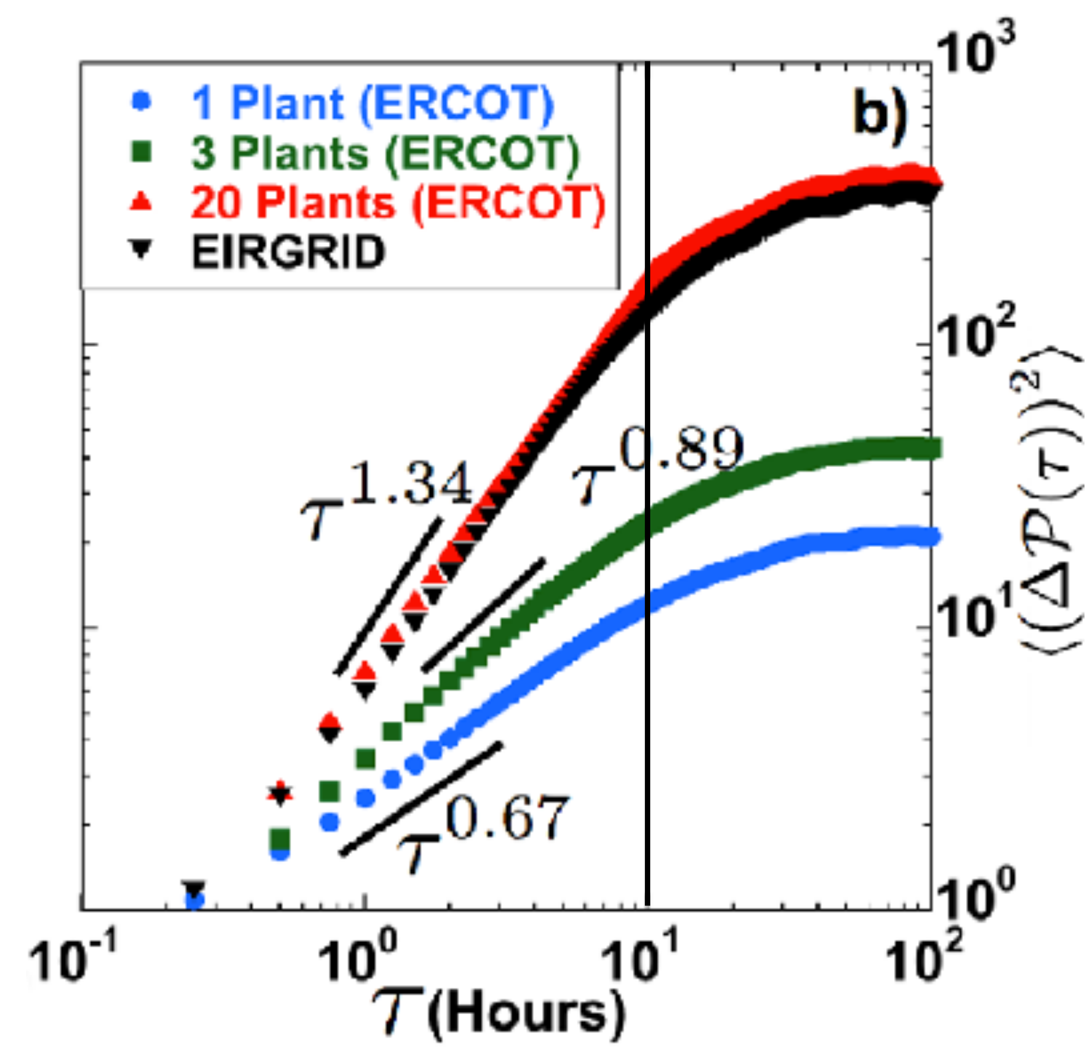
3) Atmospheric turbulence:

$\tau_0 \sim 12 \text{ hours}$

Wake & boundary layer turbulence are slave to atmospheric flow.

Plant behaves like an integral probe for eddies larger than itself.





Spatially averaged temporal signal

$$\mathcal{P}(t) \equiv \Sigma_{i=1}^{N} P_i(t) = \Sigma_{i=1}^{N} K_i u_i(t)^3$$

$$\langle(\Delta\mathcal{P}(\tau))^2\rangle \equiv \langle\mathcal{P}(t)^2\rangle + \langle\mathcal{P}(t+\tau)^2\rangle - 2\langle\mathcal{P}(t)\mathcal{P}(t+\tau)\rangle$$

$$\overline{\mathcal{P}(t)\mathcal{P}(t+\tau)} = \Sigma_{k,l}\overline{P_k(t)P_k(t+\tau)} + \overline{P_k(t)P_l(t+\tau)}$$

self-correlation cross-correlation

- Cross-correlation encoding long-range spatial correlation decays with distance.

$$\Sigma_{k,l}\overline{P_k(t)P_l(t+\tau)} \rightarrow 0 \text{ as inter-farm distance } \rightarrow l_0 \sim 200 \text{ km}$$

- Leaves only self-correlation representing field averaged, temporal fluctuations.

$$\Sigma_{k,l}\overline{P_k(t)P_k(t+\tau)}$$

- P(t) ~ $u(t)^1$, $u(t)^2$ & $u(t)^3$; odd terms minimize, even (quadratic) term amplifies.
- Recall, in the field limit, we expect:

$$S_2^m(\tau) \equiv \langle(\Delta u^m(\tau))^2\rangle \sim \tau^{2m/3}$$

$$S_2^2(\tau) \equiv \langle(\Delta u^2(\tau))^2\rangle \sim \tau^{4/3}$$

ERCOT (Texas) Data:

N = 1: $\langle(\Delta\mathcal{P}(\tau))^2\rangle \sim \tau^{2/3}$

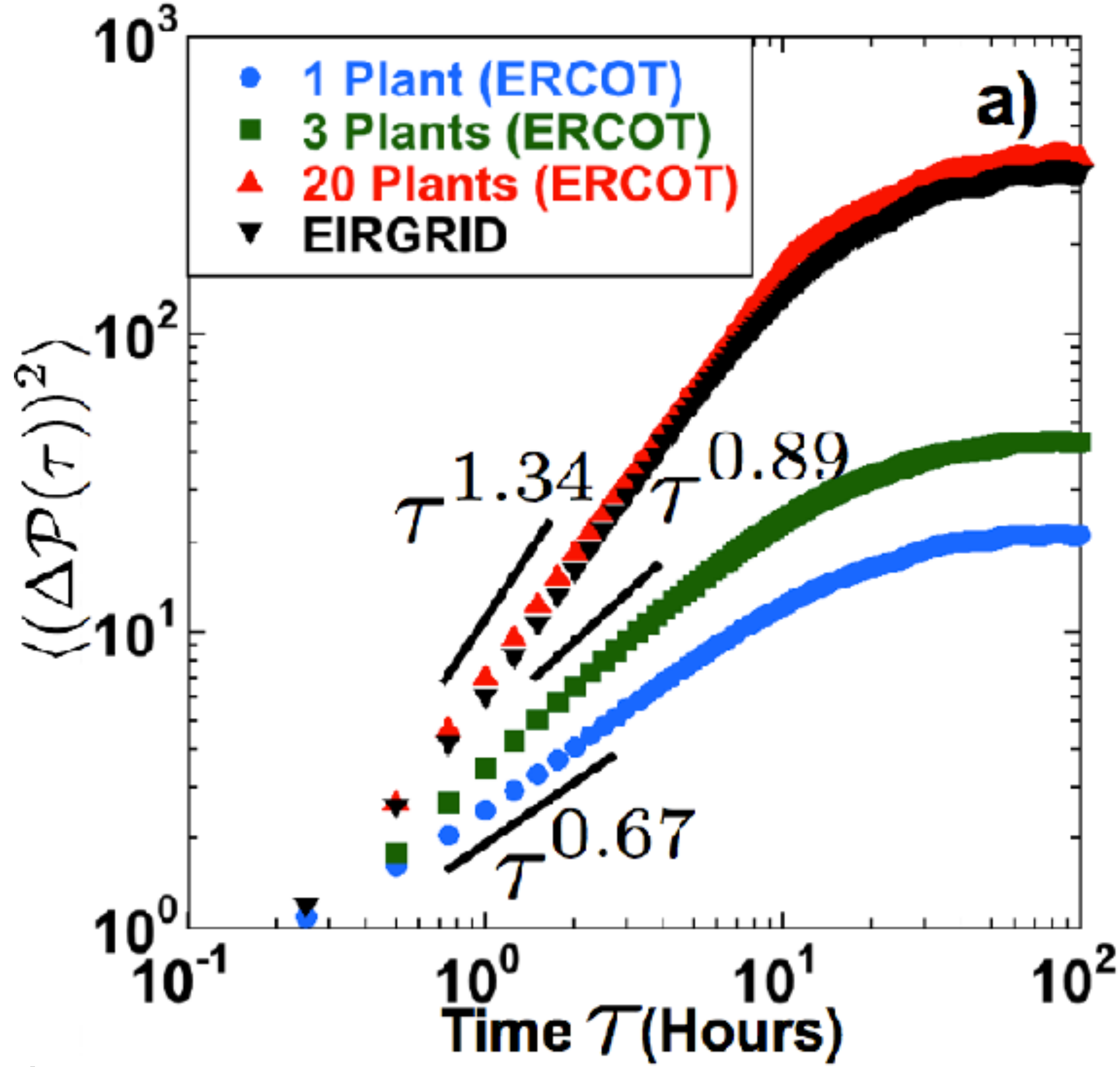
N = 3: $\langle(\Delta\mathcal{P}(\tau))^2\rangle \sim \tau^{0.89}$

N = 20: $\langle(\Delta\mathcal{P}(\tau))^2\rangle \sim \tau^{4/3}$

Overlaid on it, Irish Grid (EIRGRID) data; N = 224.

$$\langle(\Delta\mathcal{P}(\tau))^2\rangle \sim \tau^{4/3}$$

A spectral limit hit at $\tau^{4/3}$.



Adding farms within correlation length, won't smooth fluctuations any more.

MM Bandi, "Spectrum of Wind Power Fluctuations" *Phys. Rev. Lett.* **118**, 028301 (2017).

Propellor based UAV motion in unsteady medium (Ongoing work)

- Turbine & propellor equation same, operation reciprocal. $P(t) = Ku(t)^3$
- Higher-order spectra not limited to time measurement at fixed spatial point.
- Preliminary results for electrical power fluctuations in a quadcopter.

$$D_2(\tau) \equiv \langle (\Delta P(\tau))^2 \rangle \sim 9K^2\overline{u}^4 S_2^1(\tau) + 9K^2\overline{u}^2 S_2^2(\tau) + K^2 S_2^3(\tau)$$

Commercial DJI Matrice 100 Heavy lift quadcopter.
On-board custom electronics to measure velocity, acceleration, power.
But what do we compare power with?
Electrical Power: V(t)^2/R
Mechanical Power: Torque(t) x RPM(t)

All three forms of power correlated, will have similar spectra (may have varying cut-off).

Statistics of Power.

Consider random variable X with PDF $f_X(x)$ & CDF $F_X(x)$. By Defn.: $\frac{dF_X(x)}{dx} = f_X(x)$

Consider $Z = X^2$. Constructing CDF & differentiating:

$$\begin{aligned} F_Z(z) &= \mathbb{P}(Z \le z) \\ &= \mathbb{P}(X^2 \le z) \\ &= \mathbb{P}(-\sqrt{z} \le X \le \sqrt{z}) \\ &= \mathbb{P}(X \le \sqrt{z}) - \mathbb{P}(X \le -\sqrt{z}) \\ &= F_X(\sqrt{z}) - F_X(-\sqrt{z}). \end{aligned}$$

Differentiating w.r.t. z using chain rule gives PDF of Z

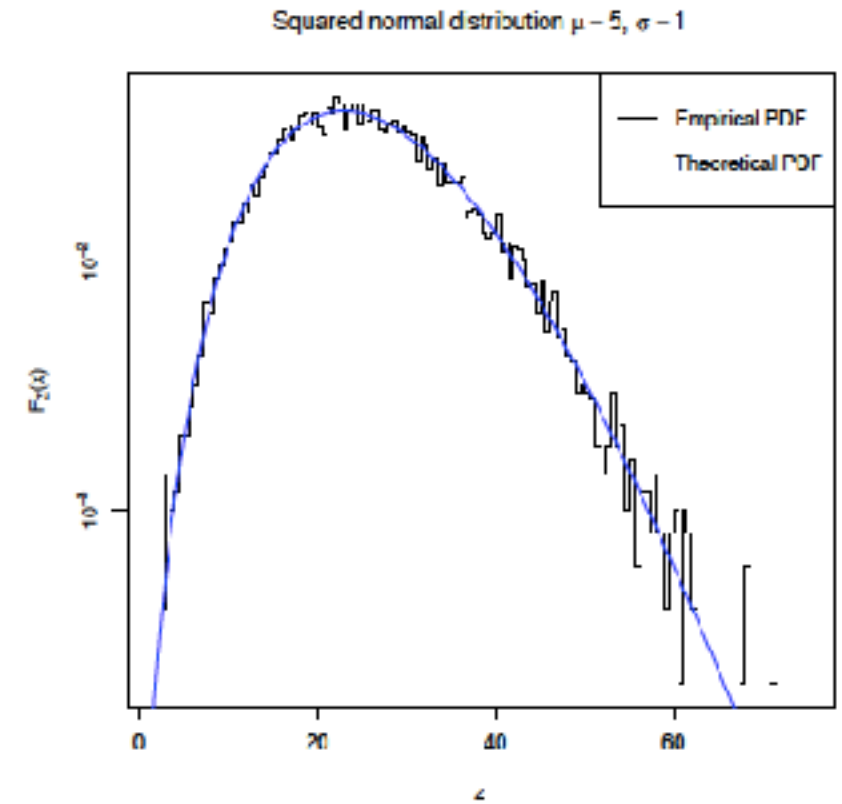
$$\begin{aligned} f_Z(z) &= \frac{dF_Z(z)}{dz} \\ &= F_X'(\sqrt{z})\,\frac{d}{dz}\sqrt{z} - F_X'(-\sqrt{z})\,\frac{d}{dz}(-\sqrt{z}) \\ &= \frac{1}{2\sqrt{z}}\left(f_X(\sqrt{z}) + f_X(-\sqrt{z})\right) \end{aligned}$$

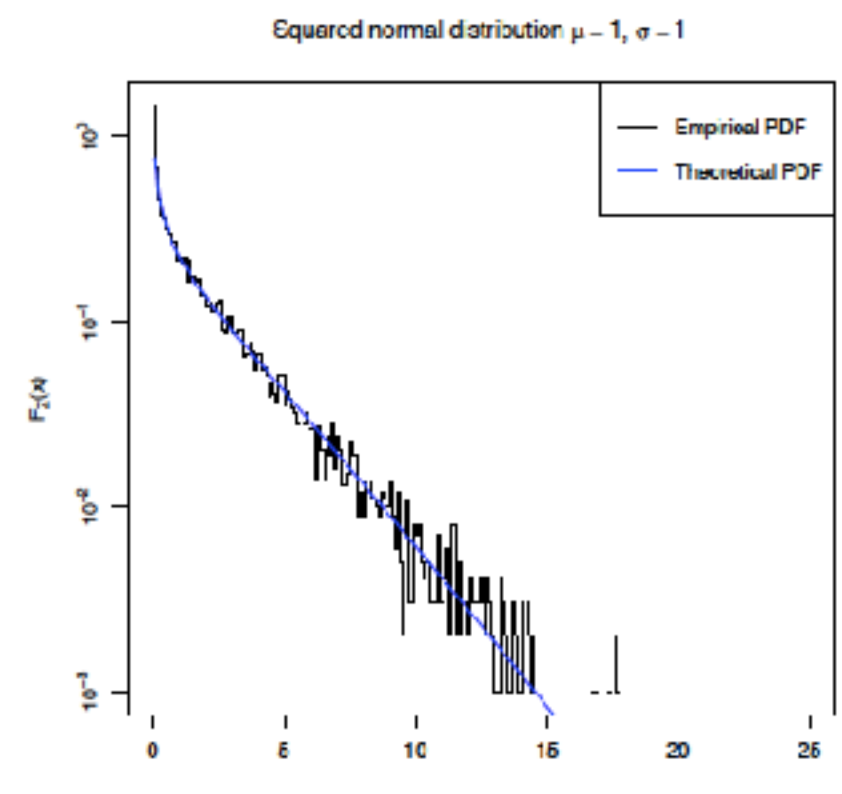
Above scheme works for any PDF $f_X(x)$. Suppose we take normal distribution with mean μ & stdev. σ :

$$\begin{aligned} f_X(x) &= \frac{1}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\,\sigma^2}}, \\ F_X(x) &= \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right] \end{aligned}$$

For specific case of normal distribution we obtain:

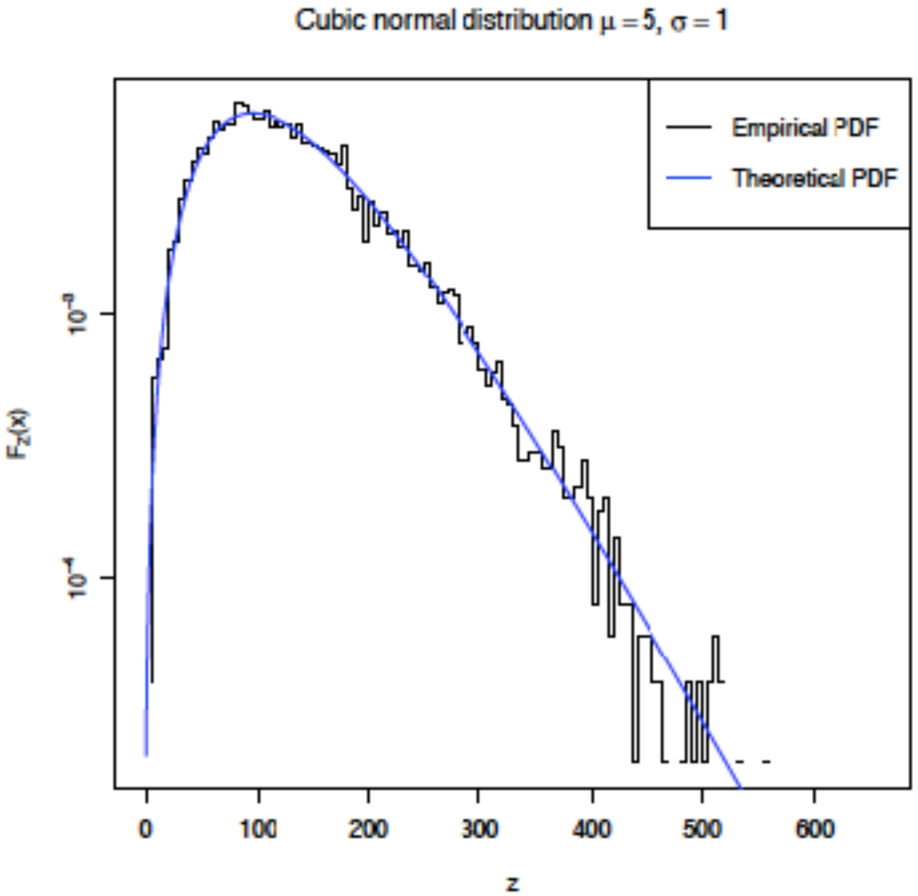
$$f_Z(z) = \frac{1}{2\sqrt{2\,\pi\,\sigma^2\,z}}\left(e^{-\frac{(\sqrt{z}-\mu)^2}{2\,\sigma^2}} + e^{-\frac{(\sqrt{z}+\mu)^2}{2\,\sigma^2}}\right)$$

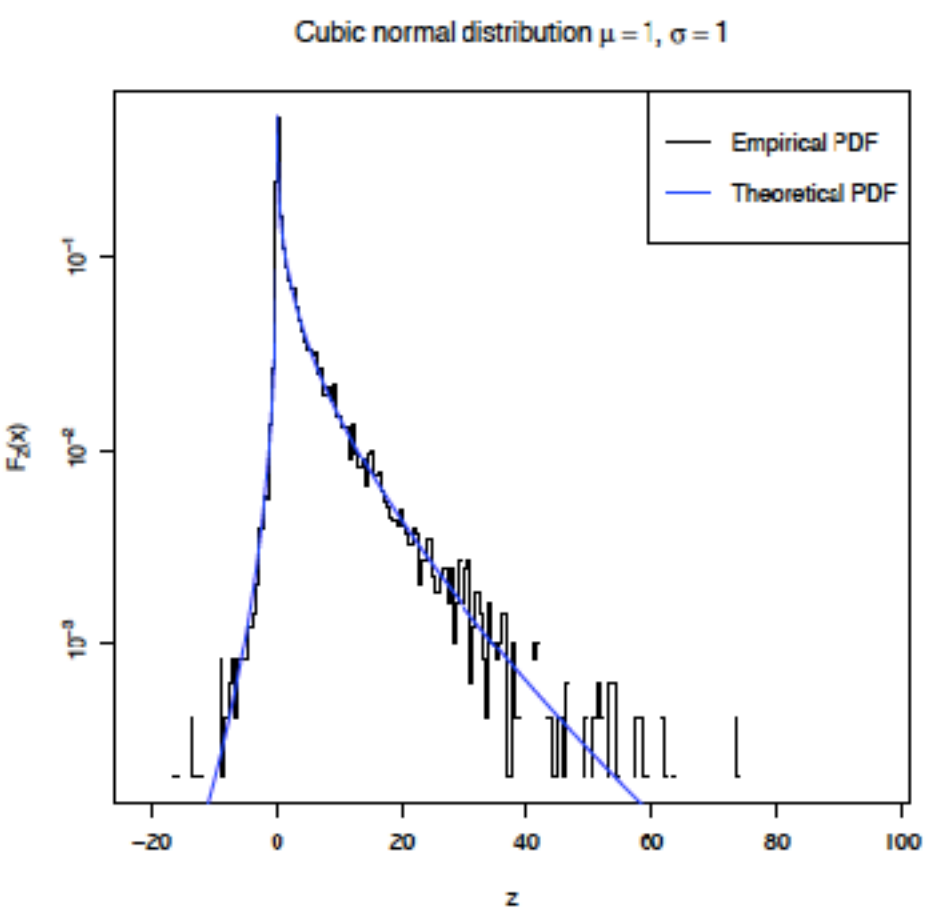




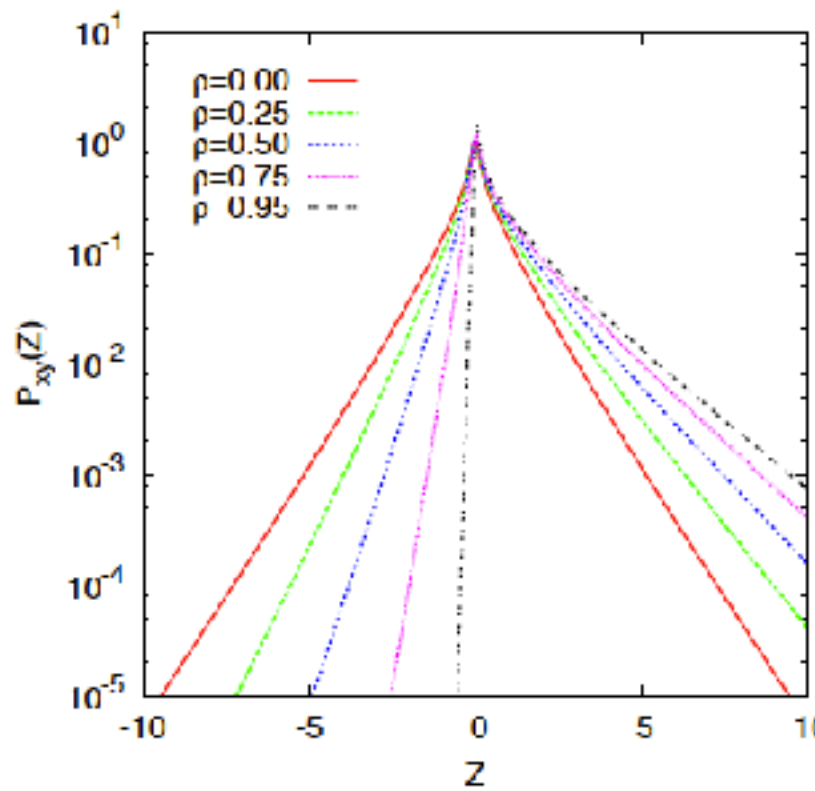
Similar calculation also works for Z = X^3

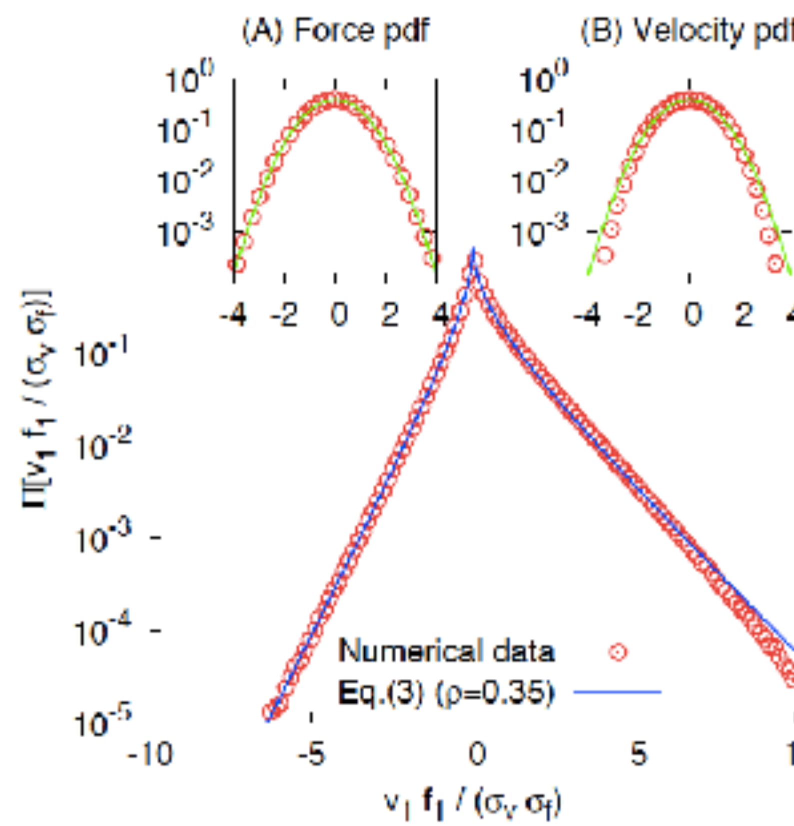
$$\begin{aligned} F_Z(z) &= \mathbb{P}(Z \le z) \\ &= \mathbb{P}(X^3 \le z) \\ &= \mathbb{P}(X \le \mathrm{sgn}(z)\,|z|^{\frac{1}{3}}) \\ &= F_X(\mathrm{sgn}(z)\,|z|^{\frac{1}{3}}). \end{aligned}$$





1) Power (in any form) always product of random variables.
2) These PDFs are system or model independent.
3) Apply to any Z that is product of random variables, many applications.
4) There's more to life beyond a Gaussian :).





MMB & CP Connaughton *PRE* **77**, 036318 (2008)
MMB et al *PRE* **79**, 016309 (2009)

Am I doing it because I can or are these PDFs useful for anything at all?

Character of Atmospheric flows

- Long-standing debate: Are atmospheric flows 2D or 3D? Both characteristics observed.
- Also, does atmospheric turbulence have a correlation length (integral scale)?

Known empirical evidence:

- Atmospheric flows are broadband forced, no fixed forcing length.
- But correlation length cutoff has to exist: Earth is finite, after all.

Our analysis:

- Wind & UAV analysis: atmospheric flow is 3D at least within shear boundary layer.
- 2D character not observed over length scales smaller than boundary layer height ~ 1km.
- 2D behavior exists (e.g. growth of typhoon vortices), but only over larger scales.

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Thank You

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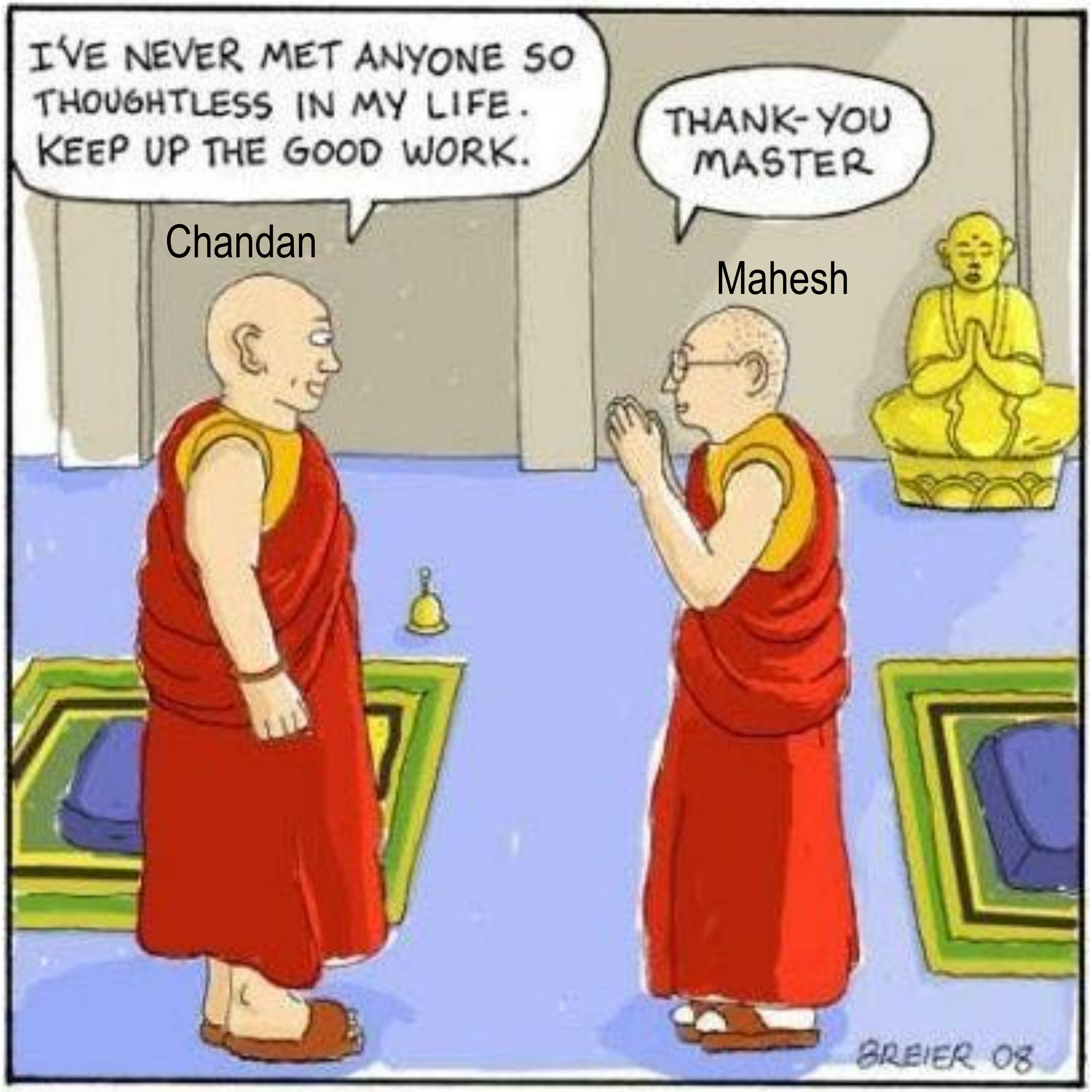

~~Buddhist~~ Compliment
ICTS