Glassy dynamics across the melting of 2D Coulomb clusters

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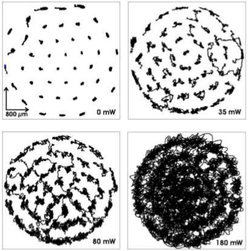
Theme:

- **Study Coulomb interacting particles in two dimensional confinements**
- **Static & Dynamic responses across ‘melting’ (thermal and quantum).**

Biswarup Ash
(Now: Weizmann)



Objective:

- **Explore spatio-temporal correlations when system size $\sim$ range of interaction**
- **Effects of ‘disorder’ (irregularity)**
- **Generic signatures of disordered dynamics in traps?**

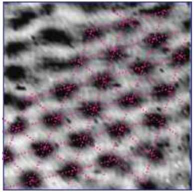
Anurag Banerjee
(IISER Kolkata)

EIOSM-ICTS, 1 **Oct,** 2018

Crystal of Coulomb particles and its melting:

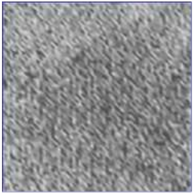
Wigner Crystal Melting
(1934)

Competetion between PE & KE



Crystal → *Liquid*

melting



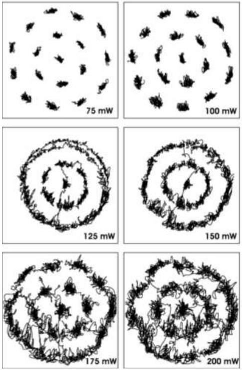
"If the electrons had no kinetic energy, they would settle in configurations which correspond to the absolute minima of the potential energy. These are close-packed lattice configurations, with energies very near to that of the body-centered lattice...." **(in 3D)**

Coulomb repulsion forces particles to stay as far as possible from each other, localizing them in regular pattern, e.g. *crystal*. Kinetic energy delocalizes them.

- KE $\sim k_B T$ (equipartition) $\Rightarrow$ Thermal / Classical melting
- KE $\sim$ Quantum (zero-point) fluctuations $\Rightarrow$ Quantum melting.

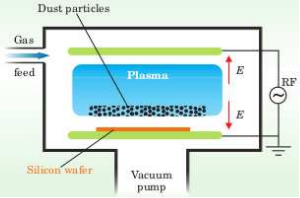
Experiments: Wigner Physics in confinements

- Realizing Wigner melting in bulk ?? Goldman *et al.* PRL (90); Yoon *et al.* PRL (99); Chen *et al.* Nat Phys.(06)
- **"Wigner molecules"** are promising !!

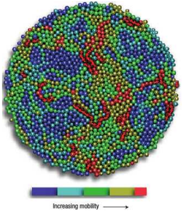


Dusty plasma
A. Melzer Group (2012)

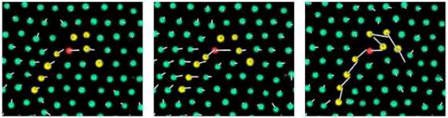
Dusty plasma setup



Dynamics across melting



air driven steel beads
(Keys *et. al.*; Nat.Phys. '07)

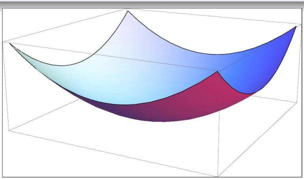


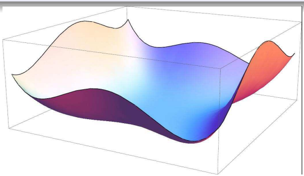
Relaxation of localized stress in 2*D* Colloid; (B. Meer, *et.al.*, PNAS'14)

Model, Method & Parameters :

Objective

- **Static & Dynamics for system size $\sim$ interaction range**
- **Effects of** *inherent* **'disorder' (irregularity) on observables**





Hamiltonian for the model system

$$\mathcal{H} = \frac{q^2}{4\pi\epsilon} \sum_{i<j}^{N} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sum_{i}^{N} V_{\rm conf}(r_i); \quad r = |\vec{r}| = \sqrt{x^2 + y^2}$$

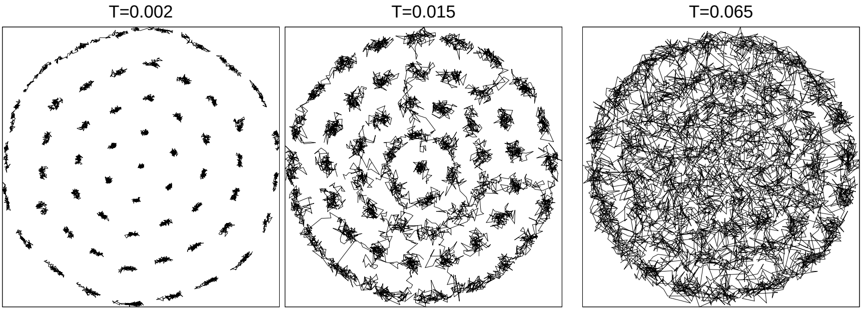
(a) Irregular: $V^{\rm Ir}_{\rm conf}(r) = a\{x^4/b + by^4 - 2\lambda x^2 y^2 + \gamma(x-y)xyr\}$, [Bohigas et.al, Phys. Rep. **223**, 43 (93)]

(b) Circular: $V^{\rm Cr}_{\rm conf}(r) = \alpha r^2$, with $\alpha = m\omega^2/2$.

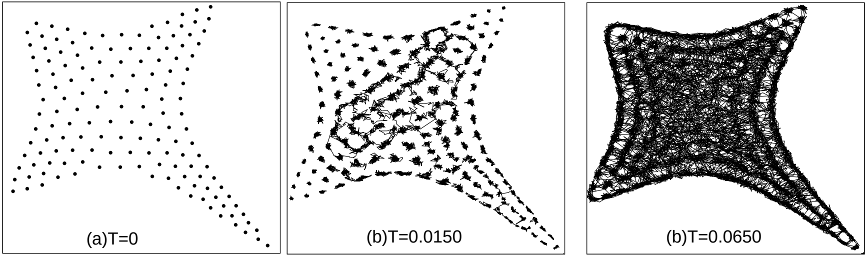
Computational Tools

Molecular dynamics (MD) and **Classical (Metropolis) Monte Carlo (MC)** with **Simulated Annealing** at finite T. **Path Integral QMC** at low T; **Variational and Diffusion QMC** at $T = 0$.

Thermal melting of Wigner molecules (WM)

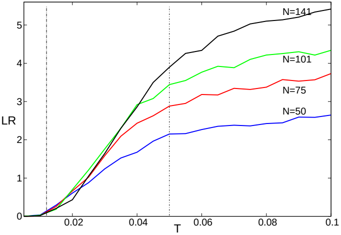


[Bedanov & Peeters, PRB (94)]

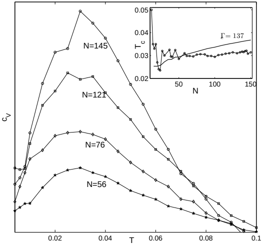


Static Correlations: EPJB **86**, 499, (2013), PRE, **96**, 042105 (2017)

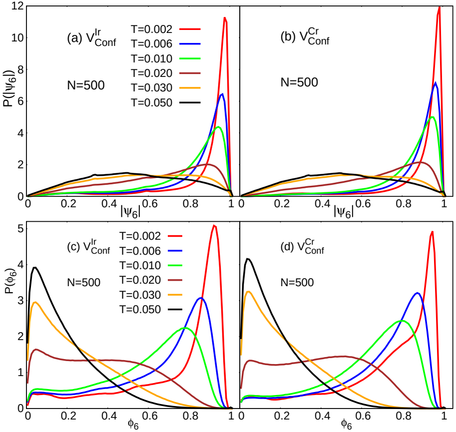
Lindemann: $\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N} a_i^{-1}\sqrt{\langle|\vec{r}_i - \vec{r}_i^{\,0}|^2\rangle}$



Specific Heat: $c_V = \frac{d\hat{E}}{dT} = T^{-2}\left[\langle E^2\rangle - \langle E\rangle^2\right]$



BOO: $\psi_6(i) = \sum_{k=1}^{N}\frac{1}{N_b}\sum_{l=1}^{N_b} e^{i6\theta_{ki}}$



$m_6(i)$: projection of $\psi_6(i)$ onto *mean local* orientation field.

$m_6(i) = \left|\psi_6^*(i)\ \frac{1}{N_b}\sum_{k=1}^{N_b}\psi_6(k)\right|$ Larsen & Grier, PRL '96

- Also studied $g(r), g_6(r)$, Generalized susceptibilities: χ_ψ, χ_ϕ

Take-home messages from **static** correlations & questions:

1. Crossover from 'solid'-like to 'liquid'-like behavior discerned by studying independent observables (unique T_x found within tolerance).
2. No apparent distinction between T_X (within errorbars) in circular & irregular confinements, dictated primarily by $\Gamma = \sqrt{\pi \bar{n}}/T_X$.
3. Qualitative responses are more-or-less independent of N (for $100 \leq N \leq 500$) though there are differences in details.

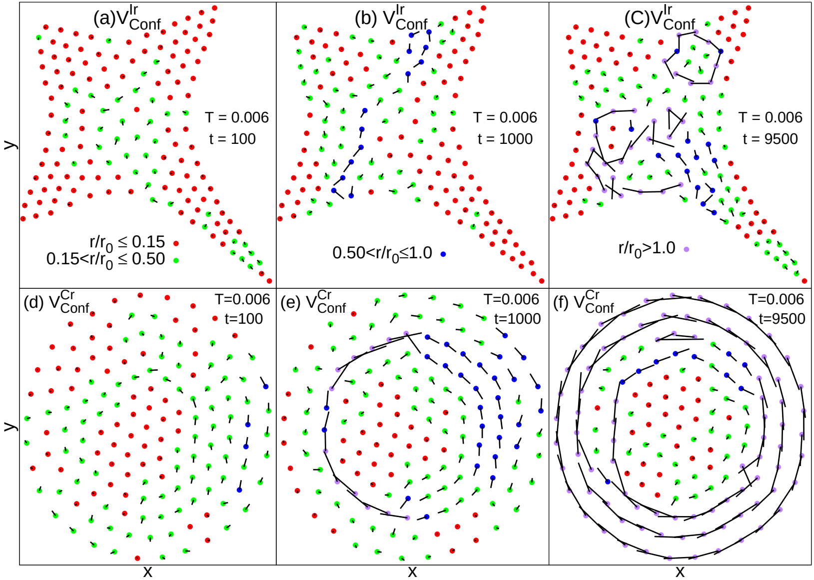
- What can dynamics tell us about the 'solid' and 'liquid' in traps?
- Can motional signatures distinguish the crossover based on the *nature* of the confinement? (e.g., circular vs. irregular)
- **Access generic signatures of disordered dynamics in traps?**

• **EPJB 86, 499, (2013);** • **EPL 114, 46001 (2016);** • **Phys. Rev. E, 96, 042105 (2017)**

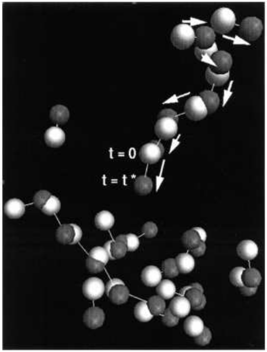
• **Preprint** *(arXiv:1805.11180), to appear in PRE*

Displacements $[\Delta\vec{r}(t) = \{\vec{r}(t) - \vec{r}(0)\}]$ in 'solid'

- Spatially correlated inhomogeneous motion at large t and at low T in irregular traps.

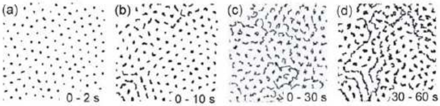


Spatially "correlated" displacements in literature...



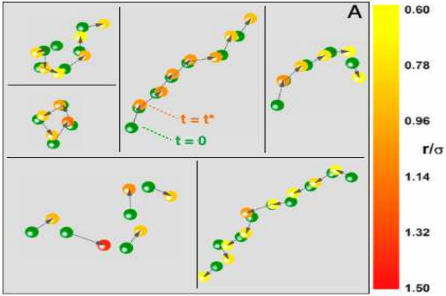
Lennard-Jones binary mixture

C.Donati *et. al.*, PRL(1998)



2D dusty plasma

C. Chan, *et. al.*, Contrib. Plasma Phys.(2009)



grain boundary in colloidal polycrystal

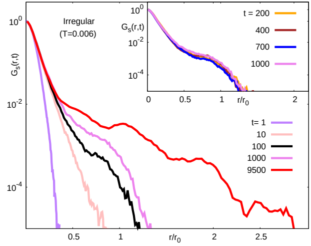
K.H.Nagamanasa, *et. al.*, PNAS (2011)

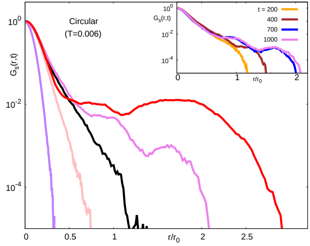
spatio-temporal density correlations:

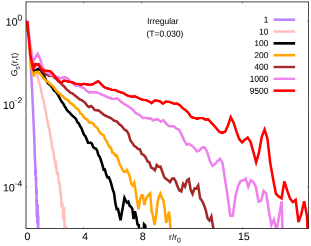
Dynamical (spatio-temporal) information best extracted from Van-Hove correlation function:

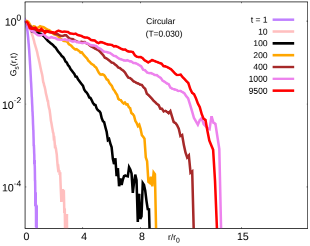
$$G(r,t) = \langle \sum_{i,j=1}^{N} \delta\left[r - |\vec{r}_i(t) - \vec{r}_j(0)|\right] \rangle$$

- Self-part $G_s(r,t)$ (when $i = j$): probability to move on an average a distance r in time t.







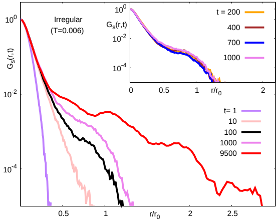


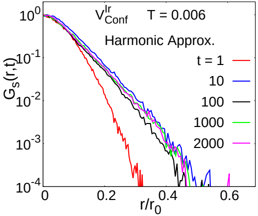
Contrast between results from MD & Harmonic approx.

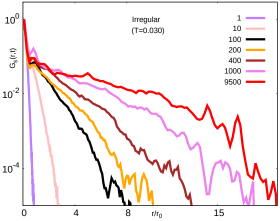
Position $\vec{r}_i(t)$ of i-th particle related to the displacement $\vec{u}_i(t)$ and its equilibrium position $\vec{r}_i^{\,0}$:

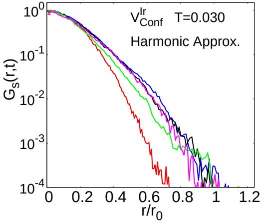
$$\vec{r}_i(t) = \vec{r}_i^{\,0} + \vec{u}_i(t); \quad V(\{\vec{r}_i\}) \simeq V(\{\vec{r}_i^{\,0}\}) + \frac{1}{2}\sum_{i,j=1}^{N}\sum_{\alpha,\beta=1}^{2} K_{i,j}^{\alpha,\beta}\, u_i^\alpha u_j^\beta \quad \text{(Harmonic approx.)}$$

$$\text{Eq. of motion:} \quad m_a \ddot{q}_a = -\sum_{b=1}^{2N} K_{a,b}\, q_b; \qquad u_i^\alpha \to q_a = q_{2(i-1)+\alpha}; \quad a = 1, 2, \cdots, 2N$$









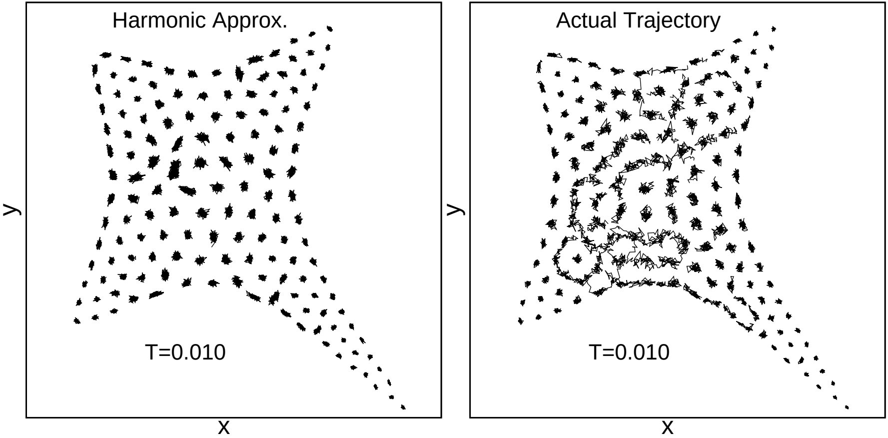
In harmonic approximation:

(a) Each DOF follows simple harmonic motion.

(b) At any T, typical amplitude for vibration derived from equipartition theorem.

Comparing trajectories from MD & Harmonic approx.

Comparing particle trajectories from full numerical simulations (molecular dynamics), with those from harmonic approximation:

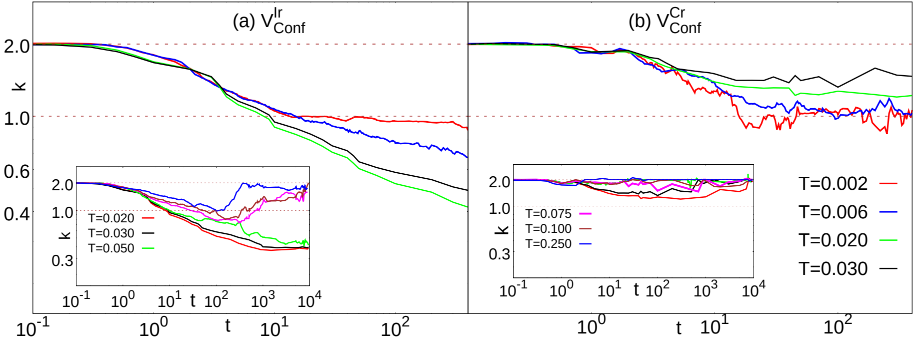


Stretched exponential decay of spatial correlation in IWM

Observation: • $G_s(r,t) \sim e^{-r^2/c}$ for small r $\forall t$. • $G_s(r,t)$ shows complex tail (large t).

Postulate: $G_s^{\rm small}(r,t) \sim e^{-r^2/c}$ for $r \le r_c$ and $G_s^{\rm large}(r,t) \sim e^{-lr^k}$ for $r > r_c$

• Optimal r_c and other parameters (**including** k) determined by minimizing **total** χ^2.



• Small t, All T: $k \simeq 2$, (Gaussian tail).

• Large t, Low T: (IWM + CWM) $k \sim 1$ (exponential tail) **[P. Chaudhuri *et al.*, PRL (2007)]**

• Large t, High T: • (CWM) $1 \ge k \le 2$, (**stretched Gaussian** tail); Expt: **[He *et al.* ACS Nano ('13)]**

• (IWM) $k < 1$, T-dependent **Stretched exponential** tail of spatial correlation!

Phenomenological model: *Dynamic Heterogeneity*

$$G_s(r,t) = \int \left[dDP(D)(4\pi Dt)^{-1} \exp[-r^2/4Dt] \right]$$ Wang et.al, Nat.Mat.2012

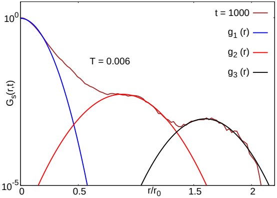
For $P(D) = \delta(D - D_0)$,

$$G_s(r,t) = (4\pi D_0 t)^{-1} \exp[-r^2/4D_0 t]$$

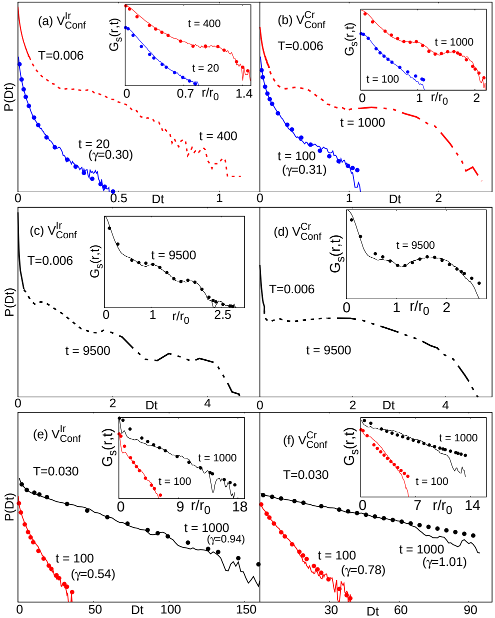
Propose:

$$G_s(r,t) = \sum_{p=1}^{N_p} \left[\int dD_p P(D_p)(4\pi D_p t)^{-1} \exp[-(r - r_p)^2/4D_p t] \right]$$

$$P(D_p) \rightarrow \langle r_p^2(t) \rangle = 4D_p t$$



unusually wide $P(D)$ sets in subtle dynamics



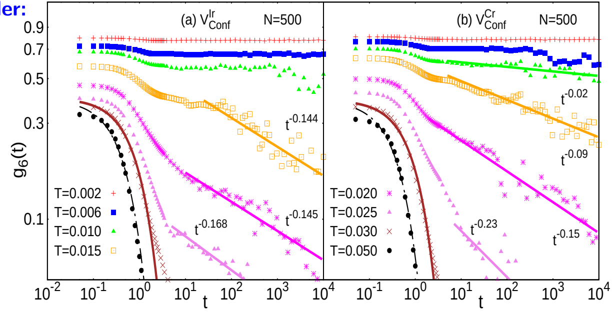
Results: Dynamical Correlations

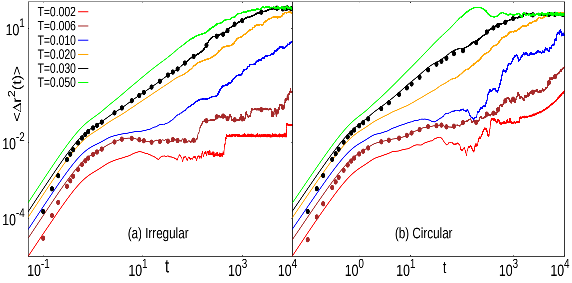
- **Temporal Bond Orientational Order:**

$$g_6(t) = \langle \psi_6^*(t)\psi_6(0) \rangle$$

(Zahn & Maret, PRL, 2000)

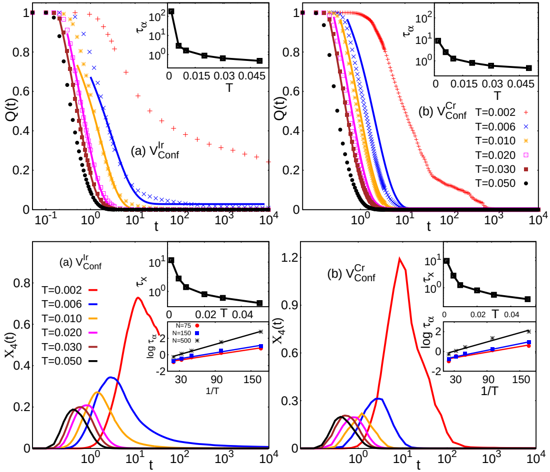
- $g_6(t) \sim 1$ in solid phase.
- $g_6(t)$ shows algebraic decay in the 'hexatic phase'.
- $g_6(t)$ decays exponentially in liquid phase.





- **Mean Square Displacement:**
  - $\langle \Delta r^2(t) \rangle = \int r^2 G_s(r,t) d^2 r$
  - 'Fickian yet non-Gaussian' dynamics.

Time scales: $\alpha-$ relaxation time from overlap Function



- $Q(t) = \dfrac{1}{N}\sum_{i=1}^{N} W(|\vec{r}_i(t) - \vec{r}_i(0)|)$

  where $W(r_i) = 1$ if $r_i < r_{\rm cut}$, & $W(r_i) = 0$ if $r_i > r_{\rm cut}$ (satisfied once, only on first passage).

  [Kob et al. ('12); Karmakar et al. ('14)]

- $\alpha-$ relaxation time (τ_α) from $Q(t)$: $Q(\tau_\alpha) = e^{-1}$

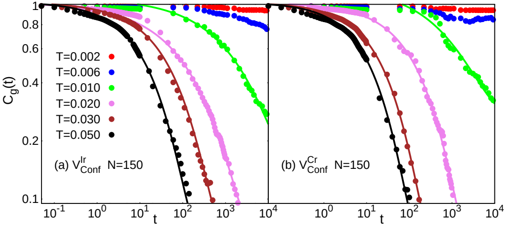
- $\chi_4(t) = \dfrac{1}{N}[\langle Q^2(t)\rangle - \langle Q(t)\rangle^2]$

  [Karmakar et.al. PNAS, ('08)]

- $\chi_4(t)$ measures extent of dynamic heterogeneity (spatial correlations in particles' dynamics).

- $\tau_x(T)$ is the time-scale when dynamic heterogeneity is maximum at the given T.

Dynamical Correlations



- When particles' cages rearrange, the system relaxes & particles diffuse.
  ⇒ corresponding structural change characterized by a cage correlation (CC) function.

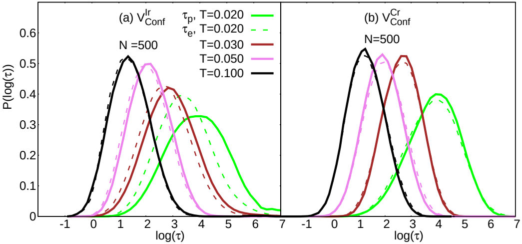
Cage correlation function:

$$C_g(t) = \frac{\langle \mathbf{L^{(i)}(t)\cdot L^{(i)}(0)} \rangle}{\langle \mathbf{L^{(i)2}(0)} \rangle}$$ [Rabani et.al. PRL'99]

- $C_g(t) \sim \exp[-(t/\tau_g)^c]$; $c \sim 0.5$ for irregular, and $c \sim 0.6$ for circular traps.

- Persistence time (τ_p, solid line): a particle displaced beyond a cut-off for the first time.

- Exchange time (τ_e, dotted line): time required for subsequent passage by cut-off distance. [Hedges *et. al.*, J. Chem. Phys.(2007)]



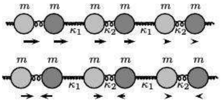
- Two distributions decouple for Irregular confinement but signature of decoupling is weaker for Circular confinement.

Normal Mode Analysis: Preprint *(arXiv:1805.11180)*, to appear in PRE

Goal:

- **Develop deeper insight into the dynamics of Coulomb particles in traps.**

- Addresses dynamic responses: how each particle proposes to move in a given configuration (remember phonon in crystals!)



- **Normal Modes:** Construct $2N \times 2N$ Hessian matrix

$$A = \left(\frac{\partial^2 H}{\partial r_{i\alpha} \partial r_{j\beta}} \right)$$

with INSTANTANEOUS configuration of N particles $\{(r_{1x}, r_{1y}), \cdots (r_{Nx}, r_{Ny}))\}$

$\Rightarrow$ results into Instantaneous Normal Modes (INM).

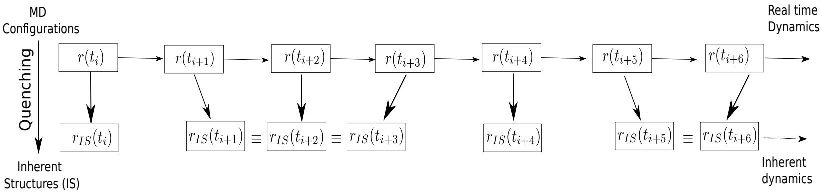
Eigenvalues of $A \rightarrow$ square of eigen frequencies ω_n $(n = 1, 2, 3, \cdots, 2N)$

Eigenvector $e_n \rightarrow$ oscillation pattern of the particles in mode number n.

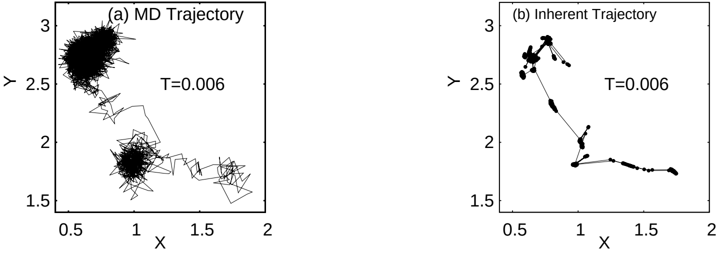
• **Preprint** ***(arXiv:1805.11180)*****, to appear in PRE** •

Quenched Normal Mode (QNM) Analysis

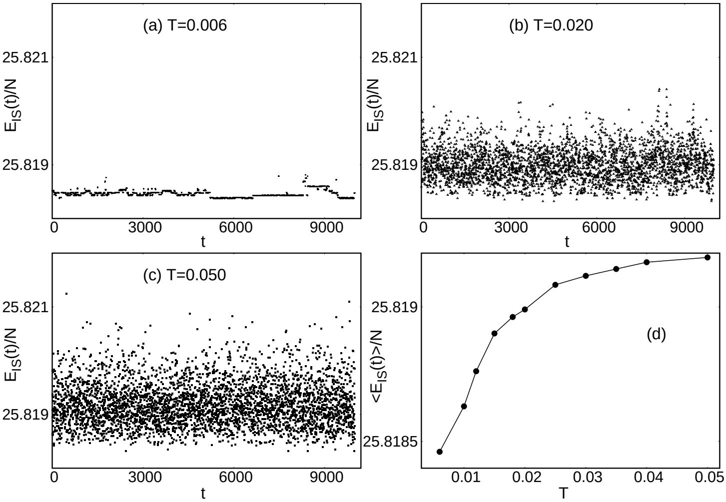
- Procedure to obtain the inherent structures from instantaneous configurations



- Effect of quenching on the actual dynamics of the particles $\rightarrow$ get rid of small amplitude vibrations.



Energies of Quenched Normal Modes

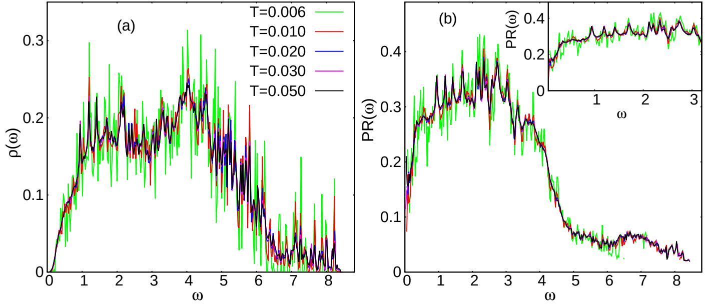


- Different eqlbm configs 'quench' to same IS at low T. System stays in the basin of that IS, only few distinct IS explored.
- System makes frequent transitions between the basins of distinct *IS*s beyond melting ($T > T_X$), $\langle E_{\rm IS} \rangle / N$ saturates.

Analysis of Quenched Normal Modes (QNM)

QNM: Energy minimized configurations starting from equilibrium configurations at a T.

Density of states (DOS): Distribution of normal mode freq. with normalization $\int d\omega \rho(\omega) = 1$.



Participation ratio $\mathrm{PR}(\omega_m) = \left[N \sum_{i=1}^{N} \left(\vec{e}_i(m) \cdot \vec{e}_i(m)\right)^2\right]^{-1}$

- $\mathbf{PR}(\omega_m) < 0.05$: small (very low and high ω_m) $\Rightarrow$ Localized modes.
- $\mathbf{PR}(\omega_m) > 0.35$: Large (intermediate ω_m) $\Rightarrow$ Delocalized modes.

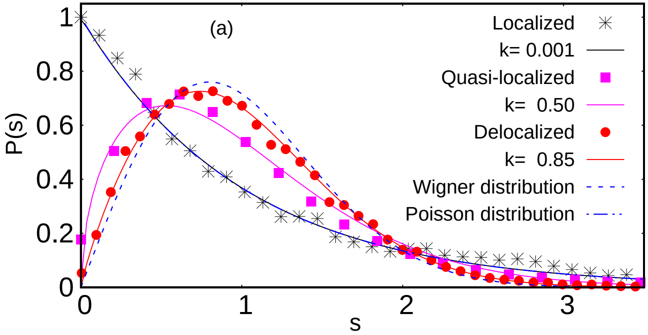
quasi-localized modes for $0.05 < \mathbf{PR} < 0.35$?

Analysis of QNM: Spectral Statistics

Localized, **quasi-localized** and **delocalized** modes. [Shastry, Deo & Franz, PRE (2001)]

Distribution of spacings s between the eigenvalues (λ) of the Hessian matrix.

$\boldsymbol{s_i = (\lambda_{i+1} - \lambda_i)/\Delta}$; Δ is the mean-level spacing.



For modes with

Brody function: $p_k(s) = (k+1)bs^k \mathrm{e}^{-bs^{k+1}}$

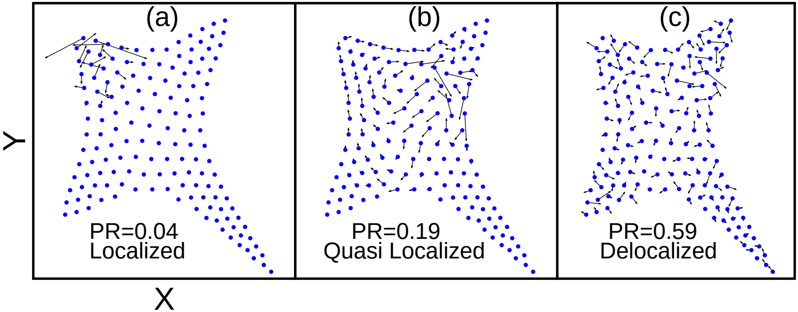
- **PR**< 0.05 Poisson distribution $\Rightarrow$ Localized modes.
- **PR**> 0.35 Wigner distribution $\Rightarrow$ Delocalized modes.

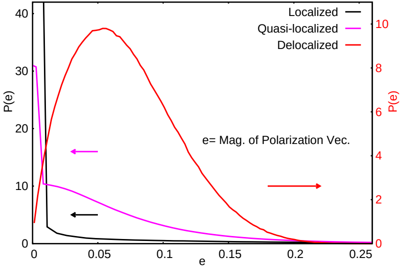
Here, $b = \left[\Gamma\left(\frac{k+2}{k+1}\right)\right]^{k+1}$

- $0.05 <$**PR**$< 0.35 \Rightarrow P(s)$ intermediate between Poisson and Wigner distribution $\Rightarrow$ Quasi-localized modes.

Analysis of QNM: Distribution of polarization vector

Representative modes: Localized, quasi-localized and **delocalized**.





Distribution of polarization vectors (magmitude)

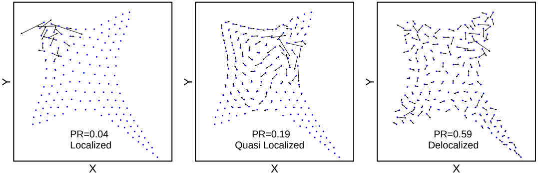
- Localized modes $\Rightarrow$ Sharp peak at $e \sim 0$.
- Delocalized modes $\Rightarrow$ Broad distribution.
- Quasi-localized modes $\Rightarrow$ Peak at zero

  $+$ Long tail.

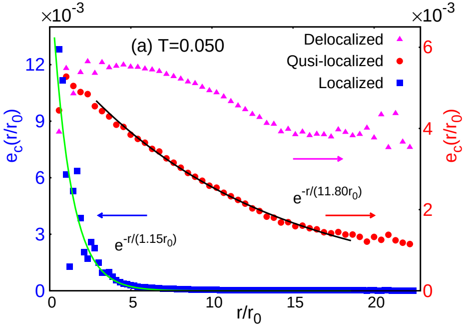
Analysis of QNM: Length scale of quasi-localized modes?

Localized, **quasi-localized** and **delocalized**

Representative modes:



- Magnitude of polarization vector $n(\vec{r}_i) = |\vec{e}(\vec{r}_i)|$; **measure correlation** $e_c(\vec{r}) = \langle n(\vec{r}_i)n(\vec{r}_i + \vec{r})\rangle$



- Delocalized modes: Weak r dependence.
- Localized modes: Sharp fall. $e_c \rightarrow 0$ by $r \sim r_0$.
- **Quasi-localized** modes: $e_c(r) \propto \exp[-r/\xi_{\rm ql}]$; $\xi_{\rm ql} \sim 12r_0$.

Cluster Analysis:

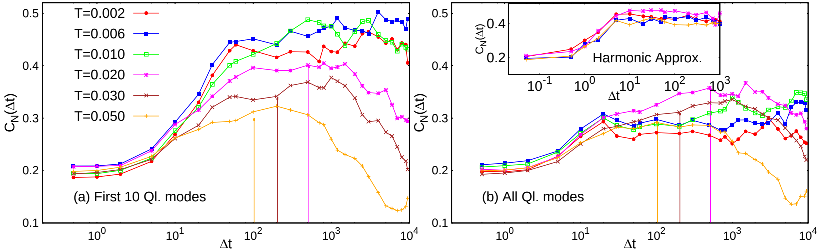
- Particles with significant contribution in quasi-localized modes tend to form clusters.
  $\rightarrow$ what is the typical cluster size? $\sim 10r_0$
  consistent with $\xi_{\rm ql}$!

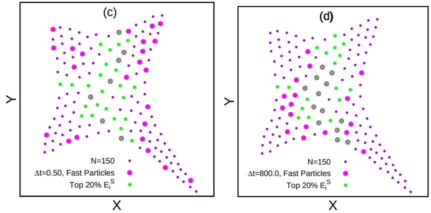
  $\rightarrow$ T-dependence of cluster sizes? No!

Identify 'fast' particles at long time from quasi-loc modes

$C_d(i, \Delta t) = 1$ only if i among top 20% particles with largest displacement during time $(t_0, t_0 + \Delta t)$;
$C_e(i, t_0) = 1$ if i among top 20% of highest E_i; Here, $E_i = \frac{1}{N_e} \sum_{m=1}^{N_e} |e_i(m)|^2$ with QNMs at t_0; $N_e \ll 2N$ (typically).

Define correlation: $\boldsymbol{C_N(\Delta t) = \sum_{i=1}^{N} \langle C_d(i, \Delta t) C_e(i, t_0) \rangle}$





- Only few QL modes identify 'fast' particles.
- Harmonic approx. good for small t, but, fails to characterize structural relaxation, which requires jumps over barrier beyond HA.
- $C_N(\Delta t)$ attains maximum for $\Delta t \sim \tau_\alpha$.
- Explains structural origin of heterogeneous and slow dynamics in supercooled liquids.
  [Widmer-Cooper *et al.* Nat. Phys. ('08); JCP ('09)]

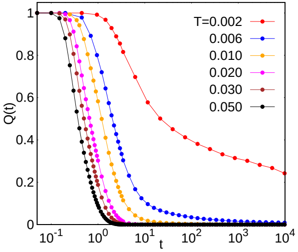
Overlap function and structural relaxation time

$Q(t) = \frac{1}{N}\sum_{i=1}^{N} W(|\vec{r}_i(t) - \vec{r}_i(0)|)$; where $W(r_i) = 1$ if $r_i < r_{\rm cut}$, & $W(r_i) = 0$ if $r_i > r_{\rm cut}$

[Karmakar et al. ('14)]

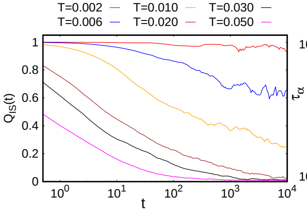
From MD configurations: [Ash, Chakrabarti & AG, PRE, **96**, 42105]

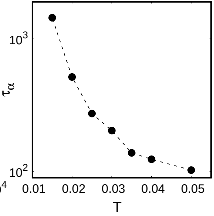
- Small time decay due to vibrational motion, while long time decay corresponds to structural relaxation.
- The two step decay makes the extraction of structural relaxation time (τ_α) difficult, even erroneous!



Inherent structure space:

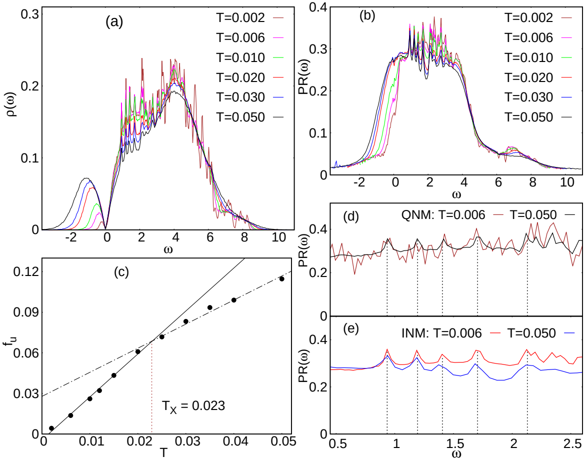
- Structural relaxation through transition from one inherent structure to another.
- Estimate (τ_α) from the area under the $Q_{IS}(t)$ vs. t curve for all T (provided $Q_{IS}(t) \to 0$ at long time for that T).
- τ_α increases rapidly as T is decreased.





Analysis of Instantaneous Normal Modes (INM)

Hessian matrix evealuated from *instantaneous* equilibrium configurations !!



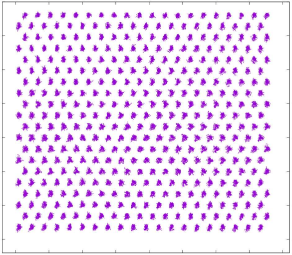
- Unstable mode (negative $\lambda \rightarrow$ imaginary ω) $\Rightarrow$ configurational transitions over potential hills.
- Some stable ($\omega > 0$) modes are robust, features peak in $\rho(\omega)$. [insensitive to T or N !!].
- Robust mode occur at similar ω in INM & QNM for different Ns!
- Identify T_X from T-dependence of fraction of unstable modes !!

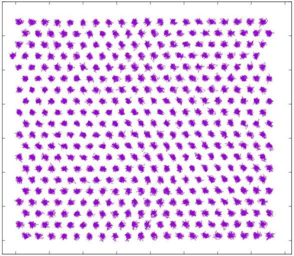
Outlook and Future:

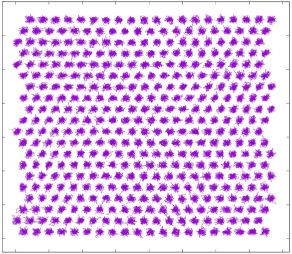
1. **Bulk melting in disordered environment?**
2. **Quantum melting – glassiness – defects?**

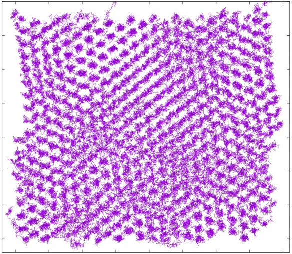
Melting of **bulk** Wigner X'tal: Nishchhal Verma & Swarnadeep Seth

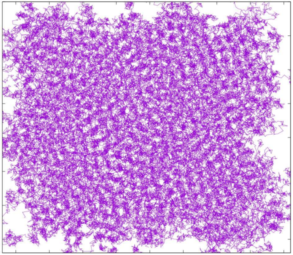
$\Gamma = 308$ $\Gamma = 180$ $\Gamma = 140$ Here: $\left(\Gamma = \sqrt{\pi \bar{n}}/T\right)$

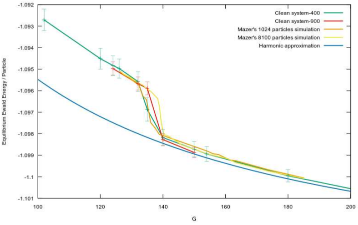










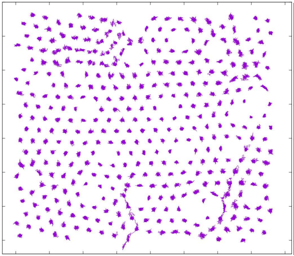


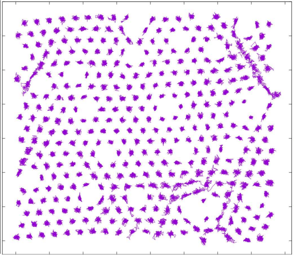
$\Gamma = 135$ $\Gamma = 124$

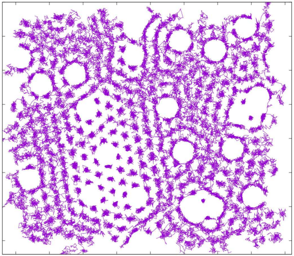
Clean System

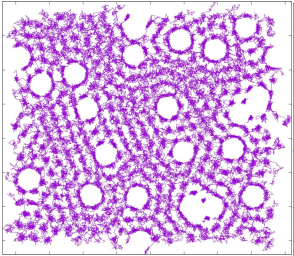
Bulk Wigner melting: Nishchhal Verma & Swarnadeep Seth

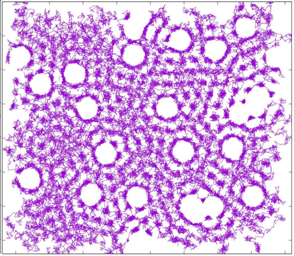
$\Gamma = 308$ $\Gamma = 180$ $\Gamma = 140$

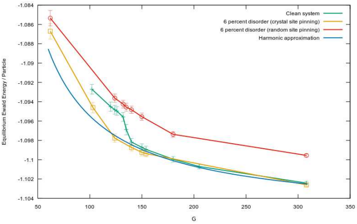










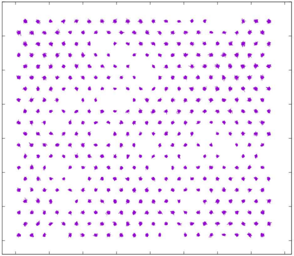


$\Gamma = 135$ $\Gamma = 124$

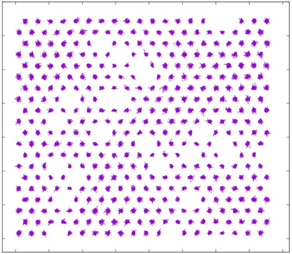
6% "random" pinning

Bulk Wigner melting: Nishchhal Verma & Swarnadeep Seth

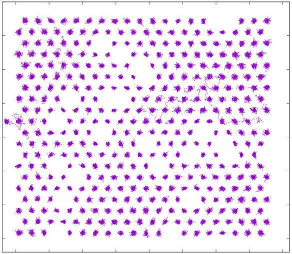
$\Gamma = 308$



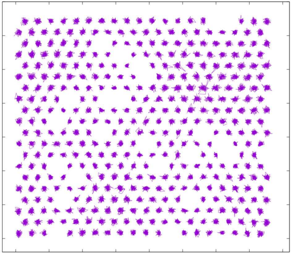
$\Gamma = 154$



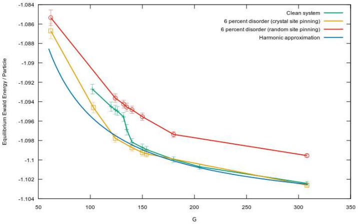
- Nature of melting in the presence of ‘disorder’ depends on the location of pinning impurities!



$\Gamma = 135$



$\Gamma = 124$



6% **“Crystal” pinning**

Qtm. melting in traps: [D. Bhattacharya *et. al.* EPJB, **89**, 60 (2016)]

- **Hamiltonian (for Harmonic trap):**

$$H = \sum_{i=1}^{N}\left[-\frac{n^2}{2}\nabla_i^2 + r_i^2\right] + \sum_{i<j}^{N}\frac{1}{r_{ij}}$$

$n = \sqrt{2}l_0^2/r_0^2$, $l_0^2 = \hbar/m\omega_0$
$(E_0 = e^2/\epsilon r_0 = m\omega_0^2/2)$ and $r_s = 1/n^2$.

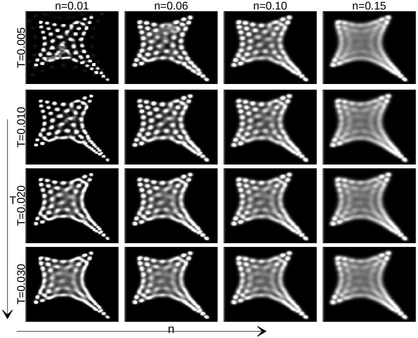
$n = 0 \Rightarrow$ classical, increase of n induces quantum fluctuations.

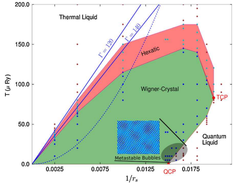
- Included: **Zero-point motion / quantum dynamics.**
- Quantum statistics: Boltzmannons **(PIMC)**
- Thermal fluctuations $\rightarrow$ tortuous path of melting.
- Quantum fluctuations $\rightarrow$diffusion around equilibrium position.

A study similar to that on bulk systems.

B. Clark, M. Casula, and D. Ceperley PRL (2009)

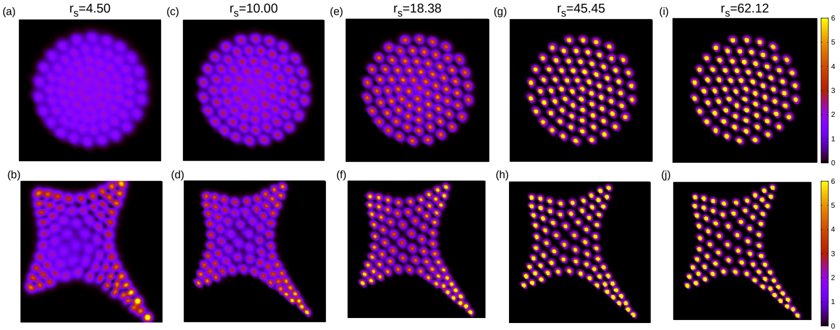
[with Dyuti Bhattacharya, A. Filinov & M. Bonitz]





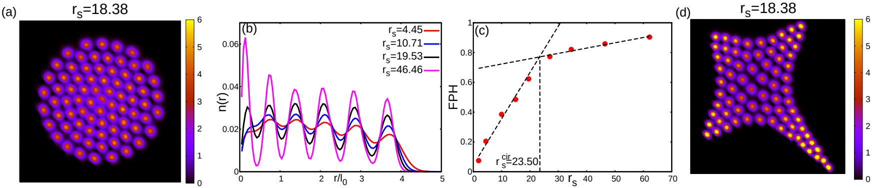
Quantum melting (**Fermions**): Anurag Banerjee & A. D. Güçlü

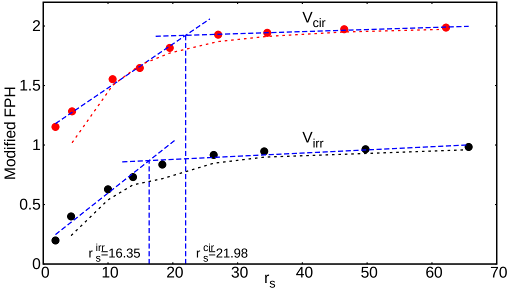
- Study irregular Wigner molecule with Spin-$\frac{1}{2}$ Fermions (VMC + DMC)



- Above density profiles show electrons localize **gradually** as (r_s) increases.
- Ours is a true $T = 0$ calculation.

Signature of melting: Fractional peak height (FPH)





- **Fractional Peak Height**

  measures localization of electrons on concentric rings in circular traps

  changes $0 \to 1 \implies$ quantum melting in traps.

- **Modified FPH**

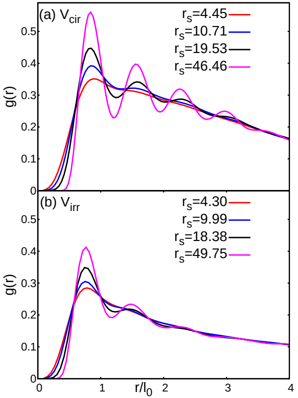
  generalizes to include irregular traps, where its contribution from each particle is the FPH w.r.t all its neighbors on an average.

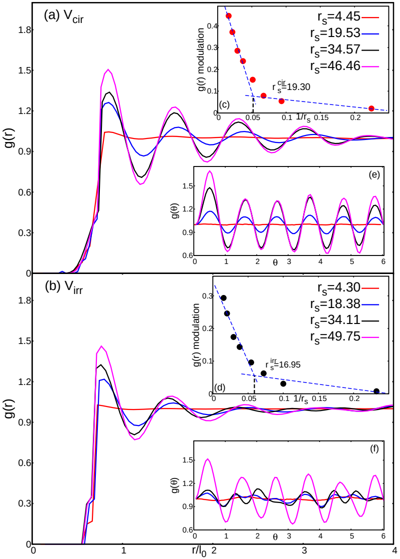
- Harmonic approx. (dotted lines) works good!

Signature of melting: Pair-distribution function $g(|\vec{r}|)$

- $g(r)$ vanishes outside the confinement.
- Weaker modulation of $g(r)$ in $V_{\rm Irr}(r)$ at low T

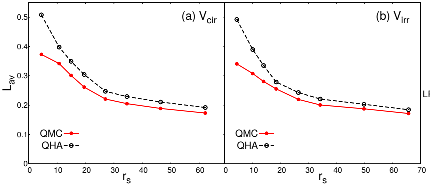
  $\Longrightarrow$ due to amorphous solid.
- Presented on right after removing smooth part.
- The modulation of $g(r)$ decreases by lowering r_s.

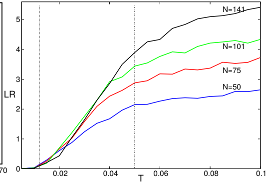




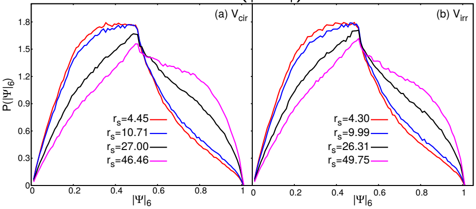
‘melting’ obscure due to quantum fluctuations

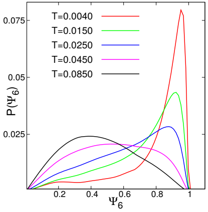
(1) Lindemann Ratio ($\mathcal{L}_{\mathcal{R}}$):





(2) Bond-orientational order, $P(|\Psi_6|)$:





- **Traditional markers, e.g. Lindemann Ratio ($\mathcal{L}_{\mathcal{R}}$) or Bond Prientational order ($|\Psi_6|$) show a very smooth evolotion (lacking any sign of threshold!) without any obvious signature of solidity, unlike in** *classical melting*,

Acknowledging collaborators

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(IISc Bangalore)

<u>Also:</u> A. V. Fillinov & M. Bonitz (Kiel U. Germany); A. D. Güçlü (Izmir Tech.)

Conclusions

• Comprehension of classical *Coulomb-glass* •

1. Intriguing motional signatures across ‘solid’ to ‘liquid’ crossover in confinements with long-range (Coulomb) interacting particles!
2. Multiple time-scales for relaxation identified.
3. Complex motion yields slow relaxations, akin to supercooled liquids. Glassiness clarified through Normal mode analysis.
4. Quasi-localized modes appraised via participation ratio and spectral analysis. Length-scale associated with quasi-localized mode extracted by cluster analysis.
5. **Role of low lying quasi-localized modes elucidated in dictating the long-time dynamic heterogeneity**.
6. Outlook:
   - **Bulk melting in disordered environment?**
   - **Quantum glassiness? Role of defects?**