Topological phases in electron glass

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Edge dynamics in topological phases@ICTS, 2019

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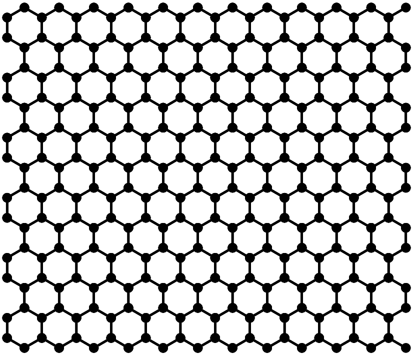
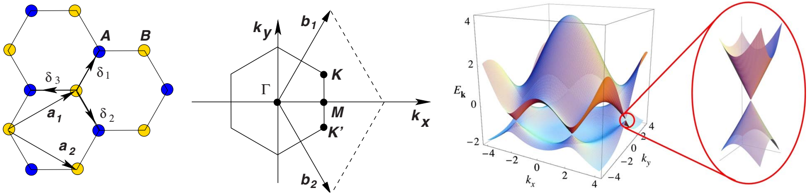
arXiv: 1606.05483, 1701.00374, 1803.01404, 1902.00507, 190X.XXXXX

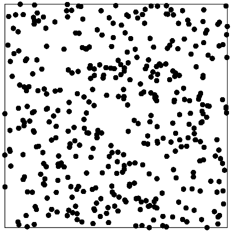
In the zone

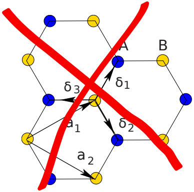


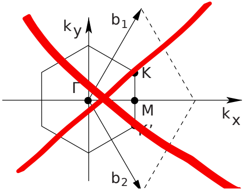


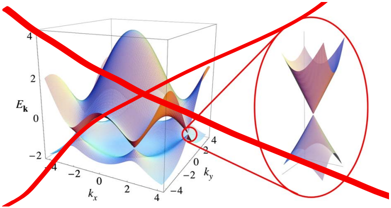
Neto. et al. RMP (2009)

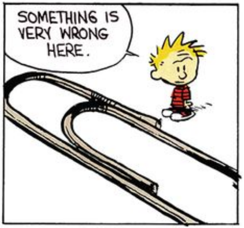
For this





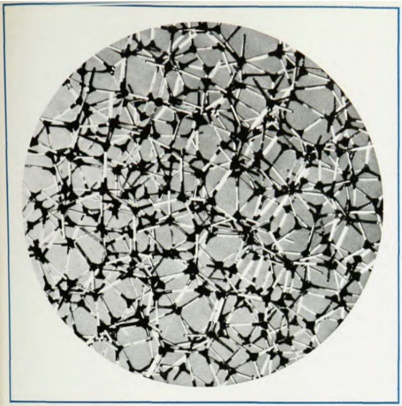






How does one think of glassy systems!

Electronic Glass



Structural model for amorphous germanium or silicon. The three-inch rods in the model represent 0.235-nanometer bonds in silicon and 0.245-nm bonds in germanium. Donald E. Polk of Harvard, who built this model, describes its details in a forthcoming issue of *J. Non-Crystalline Solids*.
Figure 1

D. E. Polk, Journal of non-crystalline solids 5 (1971) 365–376
Cohen, Physics Today 24, 5, 26 (1971)

Band gap in a glass?!

NEWS

search & discovery

Model for amorphous semiconductors predicts energy band gaps

How do amorphous semiconductors get their energy band gaps? Experi-

are not able to normalize the wave function.

atom on either side of the forbidden region.[5] We have, in other words, a

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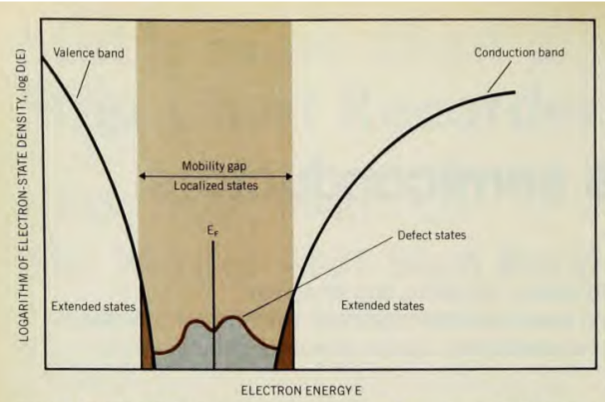
Marian S. Rothenberg, Physics Today 24, 11, 17 (1971)
Weaire and Thorpe, Phys. Rev. B. 4, (1971)

Gap in a glass



Density of electron states $D(E)$ in amorphous semiconductors. The "intrinsic" localized states were found to be restricted to energy intervals less than 0.1 eV. The other localized states in the mobility gap (shown here as a light colored band) are associated with defects and impurities (solid colored line). E_F is the Fermi level. Figure 1

Tauc, Physics Today 29, 10, 23 (1976)

The Question

Is that all? What are the electronic phases possible in such systems?

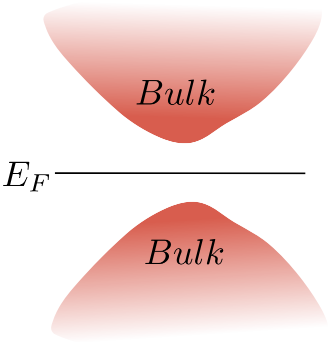
Are topological phases possible in glasses?

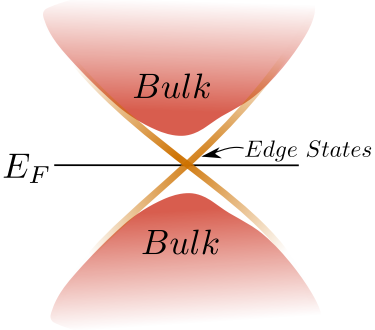
menu

- Invitation
  - Electronic glass
  - The Question
- Building a toy glass
  - Hamiltonian construction
  - Symmetries and length scales
  - Parameters and their twists
- Phenomenology and diagnostics
  - Chern system/other classes
  - Z_2 topological glass
  - Higher order generalizations
- Broad strokes, conclusion

Recap: how to cook a topological phase?

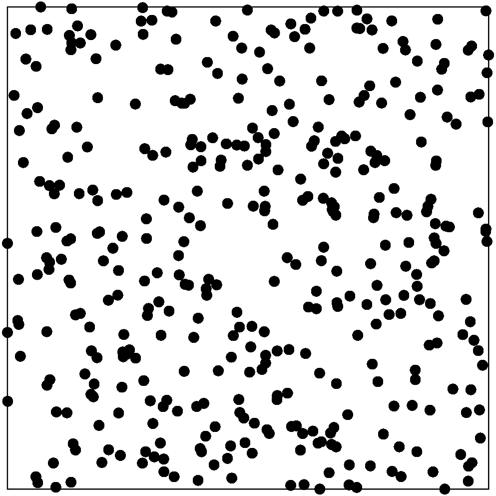
Take a Hamiltonian. Periodic system will be an insulator.
The same system with boundaries will have midgap states.
These midgap states will live on the boundary.



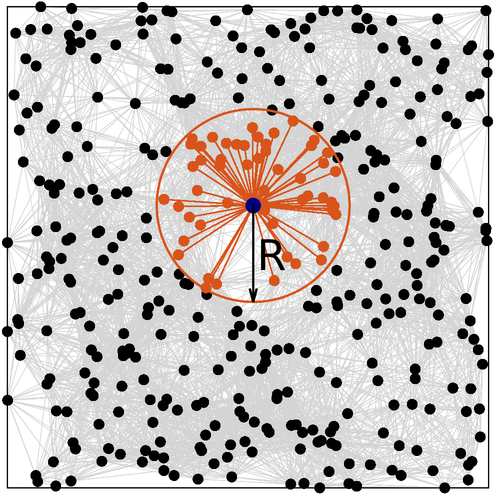


If this happens then the system can be characterized by a nontrivial topological invariant for the periodic system – involves an integral over Brillouin zone or similar things.

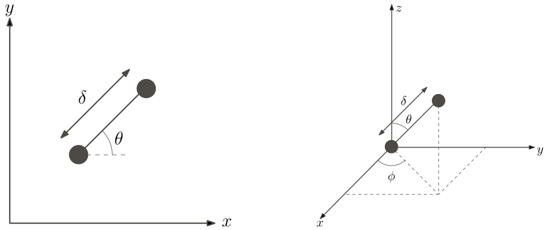
Our system



Set up a Hamiltonian



hopping between two sites



Every site has a set of "orbitals"

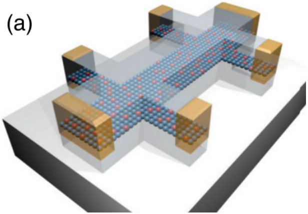
$$t_{\alpha\beta}(\delta) = t^r(\delta)t^{\theta}_{\alpha\beta}(\theta)$$

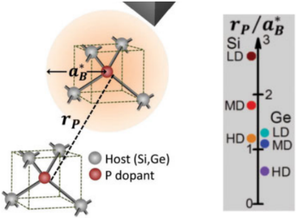
$$t^r(\delta) \sim e^{-\delta/r_o}$$

r_o is a length scale characterising strength of coupling between sites ($r_o < R$).

Tuning Bohr's radius

not just on paper

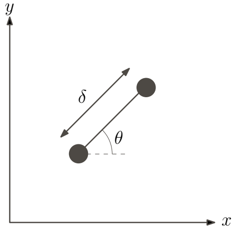


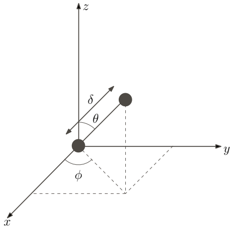


δ doped amorphous systems @IISc, Bangalore

S. Shamim et al. PRL 112, 236602 (2014)

chemistry of hopping between two sites





$$t_{\alpha\beta}(\delta) = t^r(\delta) t^\theta_{\alpha\beta}(\theta) \qquad t^r(\delta) \sim e^{-\delta/r_o}$$

$$\mathcal{H} = \sum_{I\alpha} \sum_{J \neq I \beta} t_{\alpha\beta}(\boldsymbol{r}_{IJ}) c^\dagger_{I,\alpha} c_{J,\beta} + \sum_{I\alpha,\beta} \epsilon_{\alpha\beta}(\boldsymbol{r}_{IJ}) c^\dagger_{I,\alpha} c_{I,\beta}.$$

$$t^\theta_{\alpha\beta}(\theta) = ?$$

$$\epsilon_{\alpha,\beta} = ?$$

need to put in the symmetries and appropriate physics in

The other periodic table

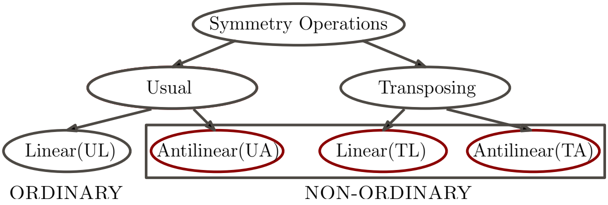
Based on three symmetries – "time reversal, charge conjugation and sublattice" and spatial dimension

Class	(T,C,S)	d=0	d=1	d=2	d=3	d=4	d=5	d=6	d=7
A	(0,0,0)	Z	0	Z	0	Z	0	Z	0
AIII	(0,0,1)	0	Z	0	Z	0	Z	0	Z
AI	(+1,0,0)	Z	0	0	0	2Z	0	Z_2	Z_2
BDI	(+1,+1,1)	Z_2	Z	0	0	0	2Z	0	Z_2
D	(0,+1,0)	Z_2	Z_2	Z	0	0	0	2Z	0
DIII	(-1,+1,1)	0	Z_2	Z_2	Z	0	0	0	2Z
AII	(-1,0,0)	2Z	0	Z_2	Z_2	Z	0	0	0
CII	(-1,-1,1)	0	2Z	0	Z_2	Z_2	Z	0	0
C	(0,-1,0)	0	0	2Z	0	Z_2	Z_2	Z	0
CI	(+1,-1,1)	0	0	0	2Z	0	Z_2	Z_2	Z

Kitaev(2009); Ryu, Ludwig, Furusaki, Schnyder(2008); Moore, Ryu, Ludwig(2012)

Brief recap: symmetries

Usual symmetries:	$\mathcal{U}\psi\mathcal{U}^{-1} \sim \psi$
Transposing symmetries:	$\mathcal{U}\psi\mathcal{U}^{-1} \sim \psi^\dagger$
Linear symmetries:	$\mathcal{U}i\mathcal{U}^{-1} \sim i$
Antilinear symmetries:	$\mathcal{U}i\mathcal{U}^{-1} \sim -i$



Usual antilinear: Time Reversal ($\mathcal{T}$)
Transposing linear: Charge Conjugation ($\mathcal{C}$)
Transposing antilinear: Sublattice ($\mathcal{S}$)

chemistry of hopping between two sites

Usual:

$$\mathcal{U}\mathcal{H}\mathcal{U}^{-1} = \mathcal{H}$$

Transposing:

$$:\mathcal{U}\mathcal{H}\mathcal{U}^{-1} := \mathcal{H}$$

Example: For $\mathcal{C}^2 = -1$

$$\mathcal{C} c^{\dagger}_{I,\alpha} \mathcal{C}^{-1} = \alpha c_{I,\bar{\alpha}}$$
$$\mathcal{C} c_{I,\alpha} \mathcal{C}^{-1} = \alpha c^{\dagger}_{I,\bar{\alpha}}.$$

$$-t_{\alpha\beta}(\boldsymbol{r}_{IJ})\alpha\beta = t_{\bar{\beta}\bar{\alpha}}(\boldsymbol{r}_{JI}).$$

Structures like this constraint the forms of t_{IJ} in all classes.

AA et al. 1606.05483

chemistry of hopping between two sites

In two dimensions five classes are topological.

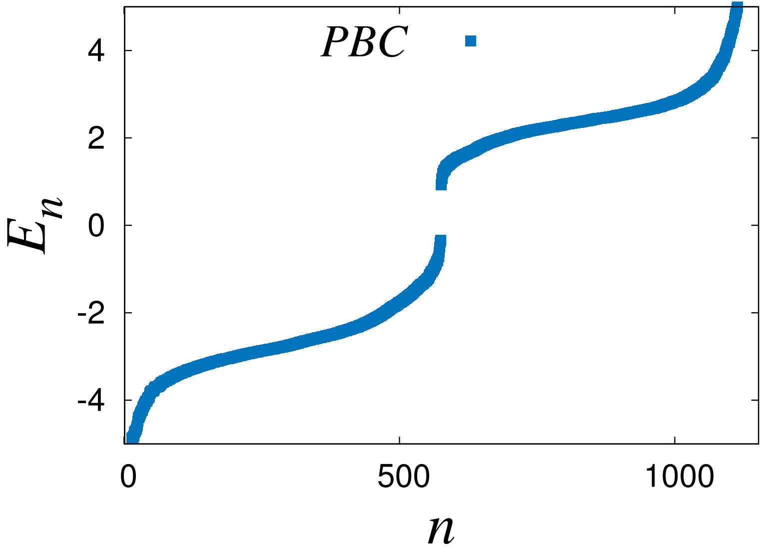
Class (par)	$\epsilon_{\alpha\beta}$	$t_{\alpha\beta}(\hat{\boldsymbol{r}})$
A (λ, M, t_2)	$\begin{pmatrix} 2+M & (1-i)\lambda \\ (1+i)\lambda & -(2+M) \end{pmatrix}$	$\begin{pmatrix} \frac{-1+t_2}{2} & \frac{-ie^{-i\theta}+\lambda(\sin^2\theta(1+i)-1)}{2} \\ \frac{-ie^{i\theta}+\lambda(\sin^2\theta(1-i)-1)}{2} & \frac{1+t_2}{2} \end{pmatrix}$
AII (λ, M, t_2, g)	$\begin{pmatrix} 2+M+2t_2 & -i2\lambda & 0 & 0 \\ i2\lambda & -(2+M)+2t_2 & 0 & 0 \\ 0 & 0 & 2+M+2t_2 & i2\lambda \\ 0 & 0 & -i2\lambda & -(2+M)+2t_2 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2}-\frac{t_2}{2} & -\frac{i}{2}e^{-i\theta}+\frac{i\lambda}{2} & 0 & -\frac{ig}{2}e^{-i\theta} \\ -\frac{i}{2}e^{i\theta}-\frac{i\lambda}{2} & \frac{1}{2}-\frac{t_2}{2} & -\frac{ig}{2}e^{-i\theta} & 0 \\ 0 & -\frac{ig}{2}e^{i\theta} & -\frac{1}{2}-\frac{t_2}{2} & \frac{i}{2}e^{i\theta}-\frac{i\lambda}{2} \\ -\frac{ig}{2}e^{i\theta} & 0 & \frac{i}{2}e^{-i\theta}+\frac{i\lambda}{2} & \frac{1}{2}-\frac{t_2}{2} \end{pmatrix}$
D (μ, Δ)	$\begin{pmatrix} 2-\mu & 0 \\ 0 & -(2-\mu) \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & \Delta e^{i\theta} \\ -\Delta e^{-i\theta} & \frac{1}{2} \end{pmatrix}$
DIII (M, g)	AII$(\lambda = 0, t_2 = 0)$	AII$(\lambda = 0, t_2 = 0)$
C (M)	$\begin{pmatrix} 2+M & 0 \\ 0 & -(2+M) \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}e^{-i2\theta} \\ -\frac{1}{2}e^{i2\theta} & \frac{1}{2} \end{pmatrix}$

Notice the terms which build in real space "*twists*".

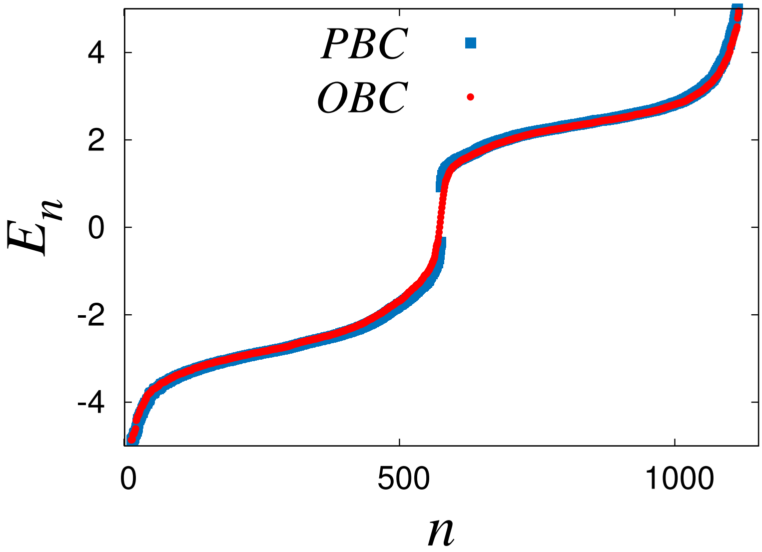
Note a mass term M which will take the system to "gapped" trivial phase. Generic to systems we will consider.

AA et al. 1701.00374

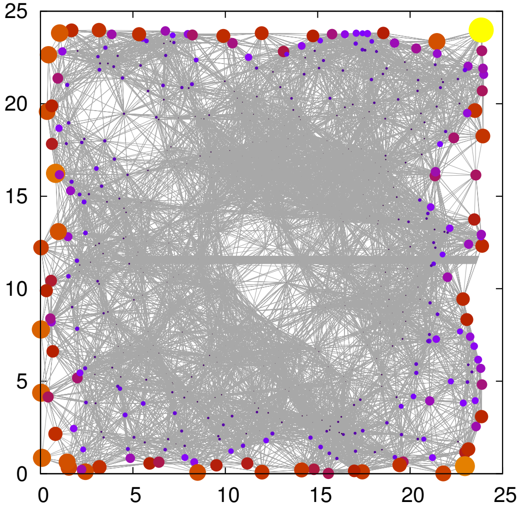
Diagonalize, with and without PBC



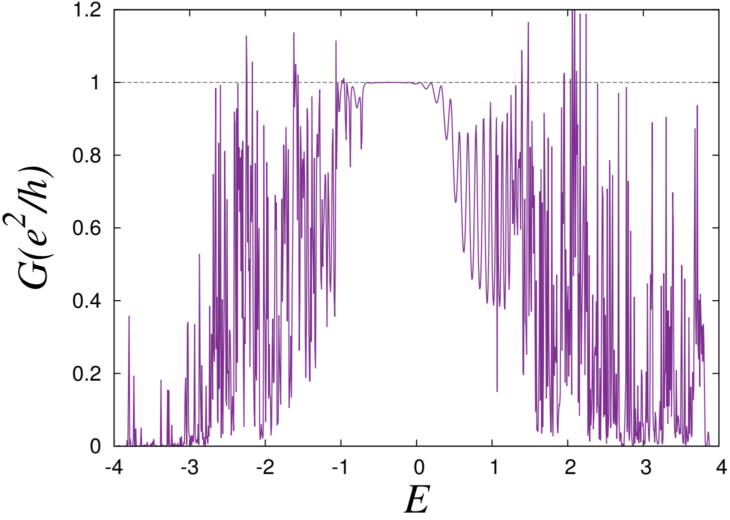
Diagonalize, with and without PBC



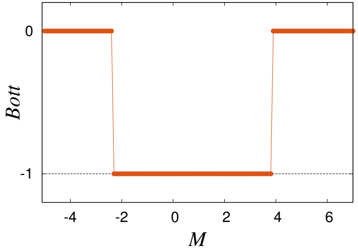
The edge

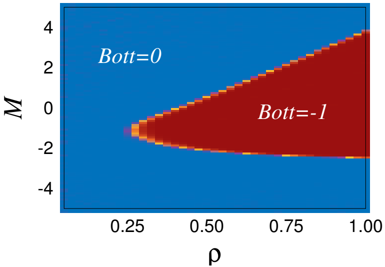


Conductance



Topological invariant – Bott Index

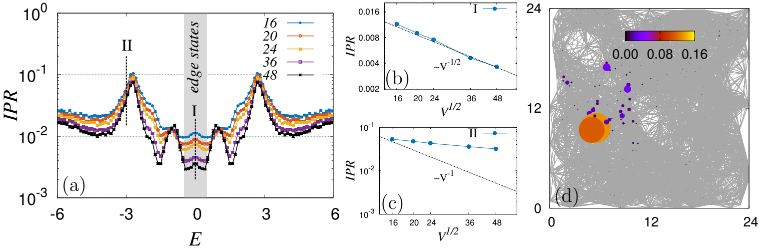




Hence Proved.

Hastings and Loring, EPL (2010)

Edges

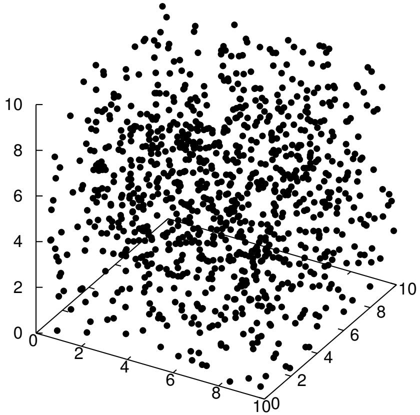


"edges" are the most delocalized states! Bulk is localized!

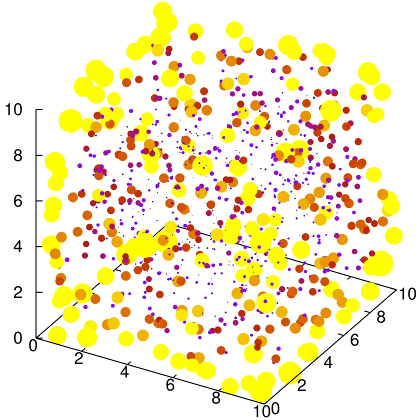
(All classes in 2D show such phases!)

AA et al. 1701.00374

The electron glass



The electron glass



When the glass is half-filled

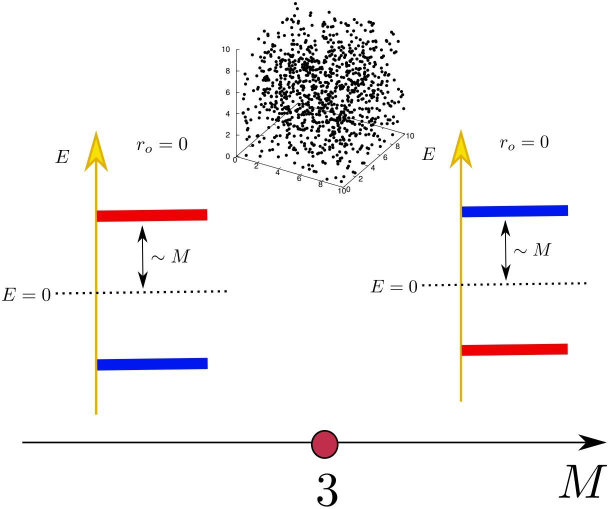
What are the phases and phase transitions possible?

tune r_o (Bohr's radius or density) and onsite energy M.

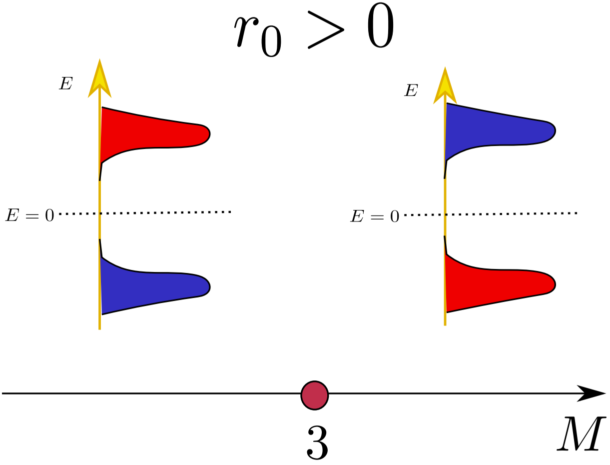
2 orbitals per site. Spinful electrons.

r_o is zero

All sites are decoupled. Two orbitals gapped via a parameter M – just cross each other.



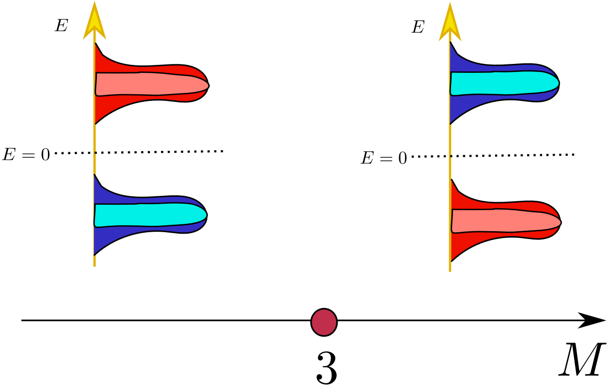
slight hopping



expectation: two bands of localized states

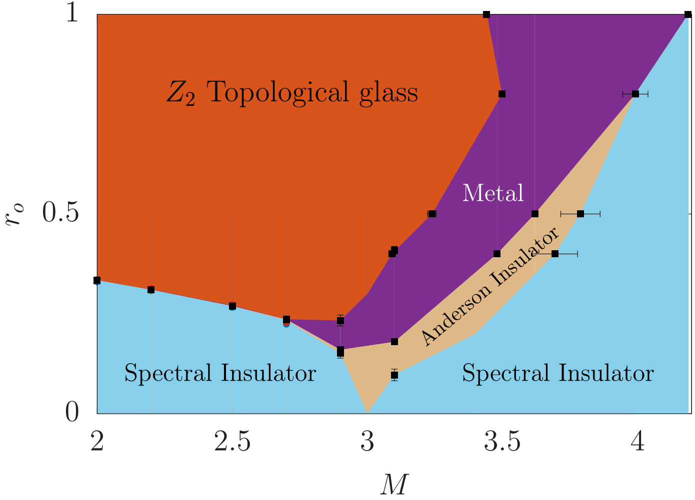
increase more

$$r_0 > 0$$



expectation: delocalized states with Lifshitz tails

this topological glass is half-filled

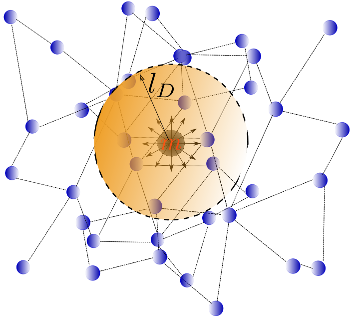


Witten effect

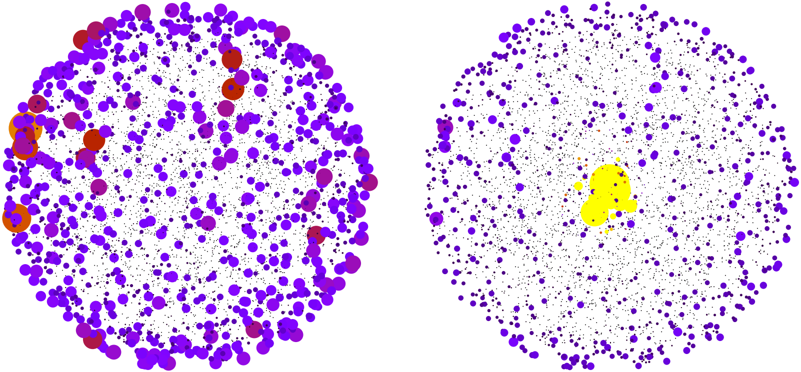
We are describing a insulator.

Effective electromagnetic action contains a $\pi(0)E.B$ term.

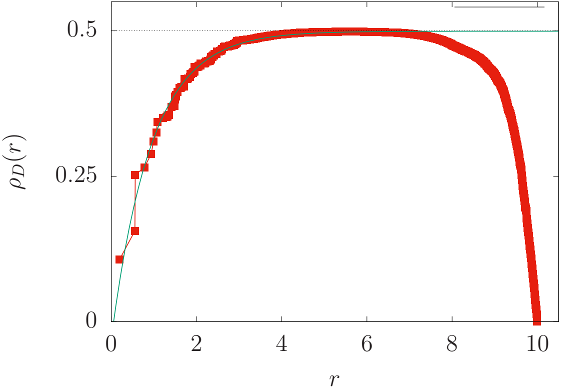
Put a magnetic monopole inside the glass!



Formation of dyon

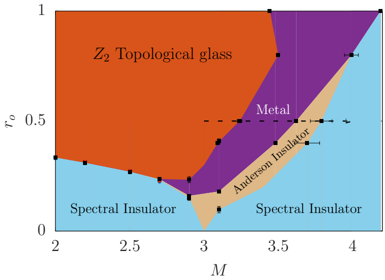


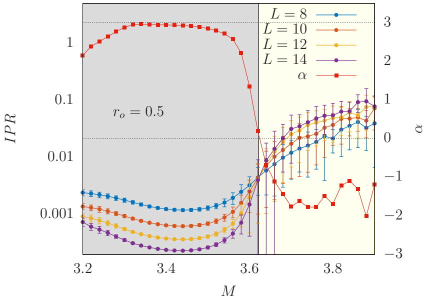
Witten charge



0.5 charge! Z_2 topological glass!

Metal and Lifshitz dominated localized phase



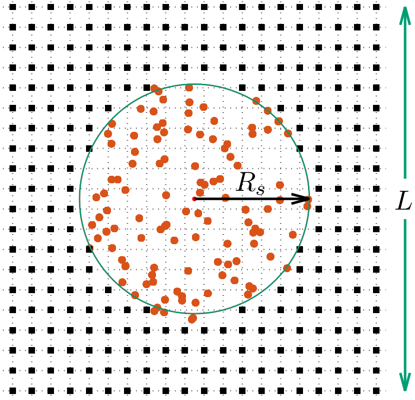


states near the Fermi energy scale as $\sim 1/L^{\alpha}$

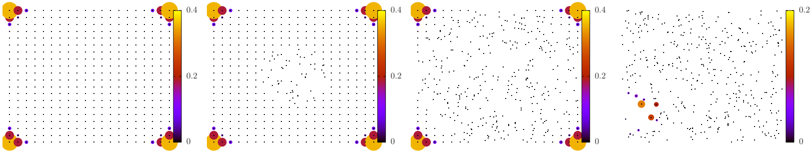
Higher order phases

crystalline symmetries are crucial to realise these phases. Can one realise them in amorphous systems?

Scramble the system!



Higher order phases



The state remain stable right upto the boundary!

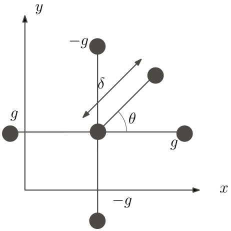
What one needs is the symmetry to be respected at the "boundary".

Why is the phase stable at all?

Symmetry remains in a statistical sense.

A quantum spin Hall insulator produces a higher order phase with a term of the kind $\sim g(\cos k_x - \cos k_y)$

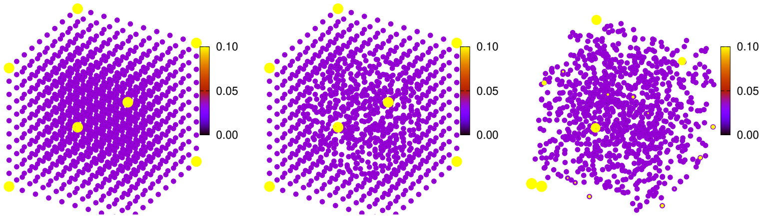
How to put this in the local “chemistry"?



$\sim g(\cos(2\theta) + i\alpha \sin(2\theta))$

In fact for (a fine tuned) $\alpha = 0$ HOTI gets stabilized for fully amorphous system! (why?)

Similar story in three dimensions



How to characterise these (HOTI) phases?

All through, we are trying to characterize insulators!

In amorphous cases – how do we do this with no reference to Brillouin zones?

Electromagnetic "response" of (localized?) many-body wavefunctions.

Polarization

Lets visit a one-dimensional system

A constant electric field will imply a increasing real space potential ($\propto x$) – in a periodic system?

then instead thread a magnetic flux!

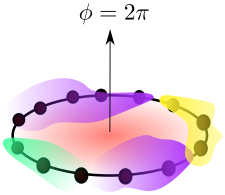
$$\sim t_{ij} c_i^\dagger c_j \rightarrow t_{ij} e^{i\left(A(x_i) - A(x_j)\right)/L} c_i^\dagger c_j$$

A operator which implements constant flux $\phi = 2\pi$ through this –

$$O = e^{i\frac{2\pi}{L}\sum_i n_i x_i}$$

Polarization

Take a wavefunction $|\Psi\rangle$



Calculate

$$\langle\Psi|O|\Psi\rangle = \langle\Psi|e^{i\frac{2\pi}{L}\sum_i n_i x_i}|\Psi\rangle = e^{iP}$$

Compactified "Polarization"!

Becomes winding number in periodic systems!

In two dimensions these two polarization operators do not commute and gives the Bott index!

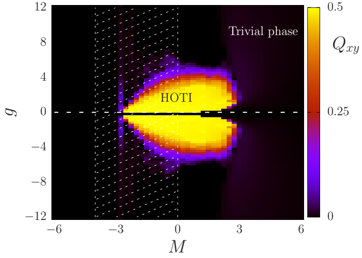
Vanderbilt, Resta, Oskhikawa, Loring and Hastings(2010), Toniolo (2017).

Quadrupolar and Octopolar moments

$$\langle\Psi|O|\Psi\rangle = \langle\Psi|e^{i\frac{2\pi}{L^2}\sum_i n_i x_i y_i}|\Psi\rangle = e^{iQ_{xy}}$$

$$\langle\Psi|O|\Psi\rangle = \langle\Psi|e^{i\frac{2\pi}{L^3}\sum_i n_i x_i y_i z_i}|\Psi\rangle = e^{iQ_{xyz}}$$

Similar quantities seem to characterize amorphous higher order phases!
(appropriately accounting for atomic limit)

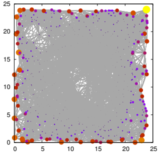


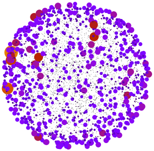
Vanderbilt, Resta, Oskhikawa, Hastings, Gill Young Cho (2019), Watanabe (2019)

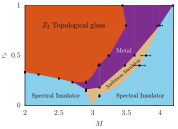
Summary: looking through a topological glass

- Symmetries, parameters and local twists. Symmetry classification.

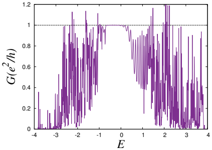
- Forming appropriate Hamiltonians are rich topological phases.

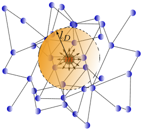


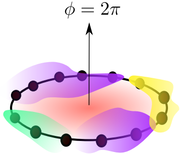




- Diagnostics – conductances, witten effect, higher-order electromagnetic moments to characterize these.







Thoughts, yet to crystallize

- Generic phases and phase transitions in such systems? Scaling theory?
- Response functions. Specially for higher order systems.
- Role of interactions? Strong disorder renormalization group?

References: arXiv: 1606.05483, 1701.00374, 1902.00507, 190X.XX
Early part in – Thesis: Excursions in ill condensed quantum matter.
Related works: Fractals – 1803.01404, Percolating clusters – 1612.01847

Mixed Topology (coexistence of $d-1$ and $d-2$ states), Continuum versions (nXX.XXXX)

So do we have all the topological phases of electron glasses?

..there are interesting things here..

Thank you