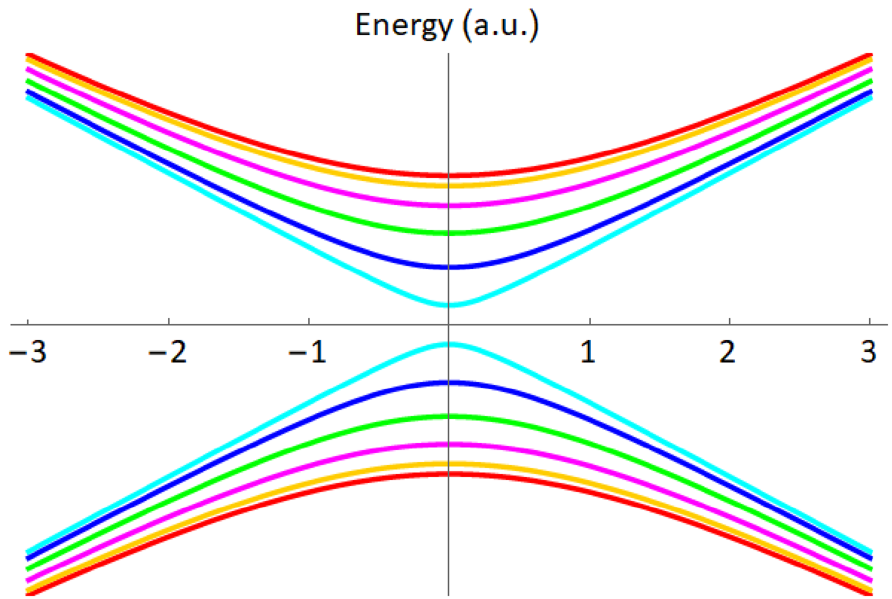
Universal quantum noise in adiabatic pumping



Yaroslav Herasymenko, **Kyrylo Snizhko**, Yuval Gefen

PRL **120**, 226802 (2018)

Universiteit Leiden

ICTS, Bengaluru
13.06.2019

מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

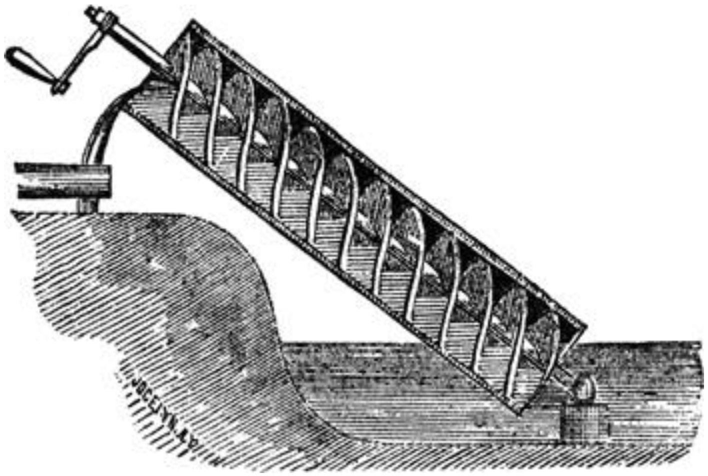
Outline

I. Adiabatic pumps

II. Essentials about parafermionic zero modes

III. Adiabatic pumping with parafermions

Archimedes' screw



Clore Garden of Science
at the Weizmann Institute

http://www.museumsinisrael.gov.il/en/items/Pages/ItemCard.aspx?IdItem=ICMS-SCG-47

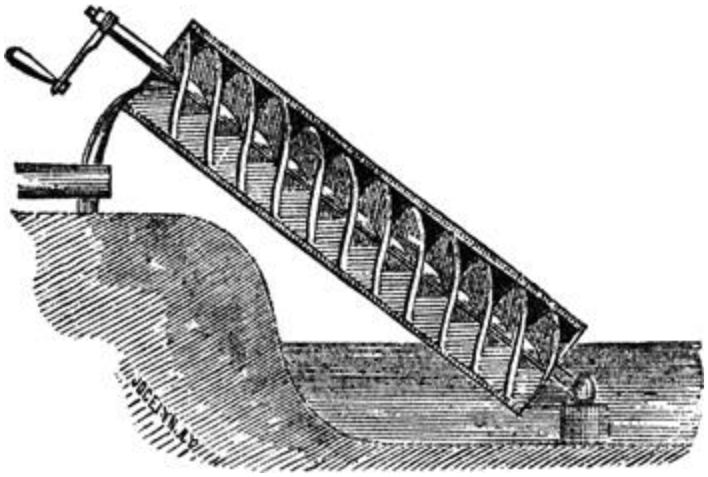
Archimedes’ screw

Koffler accelerator
at the Weizmann Institute

https://wis-wander.weizmann.ac.il/gallery/campus-buildings

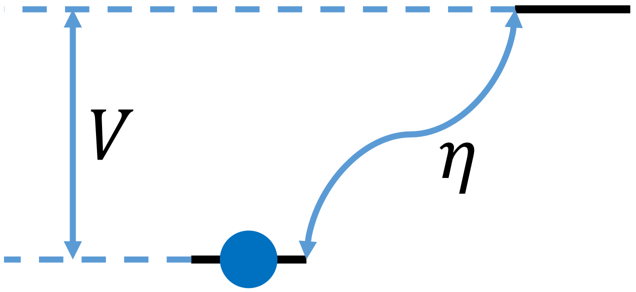
http://www.museumsinisrael.gov.il/en/items/Pages/ItemCard.aspx?IdItem=ICMS-SCG-47

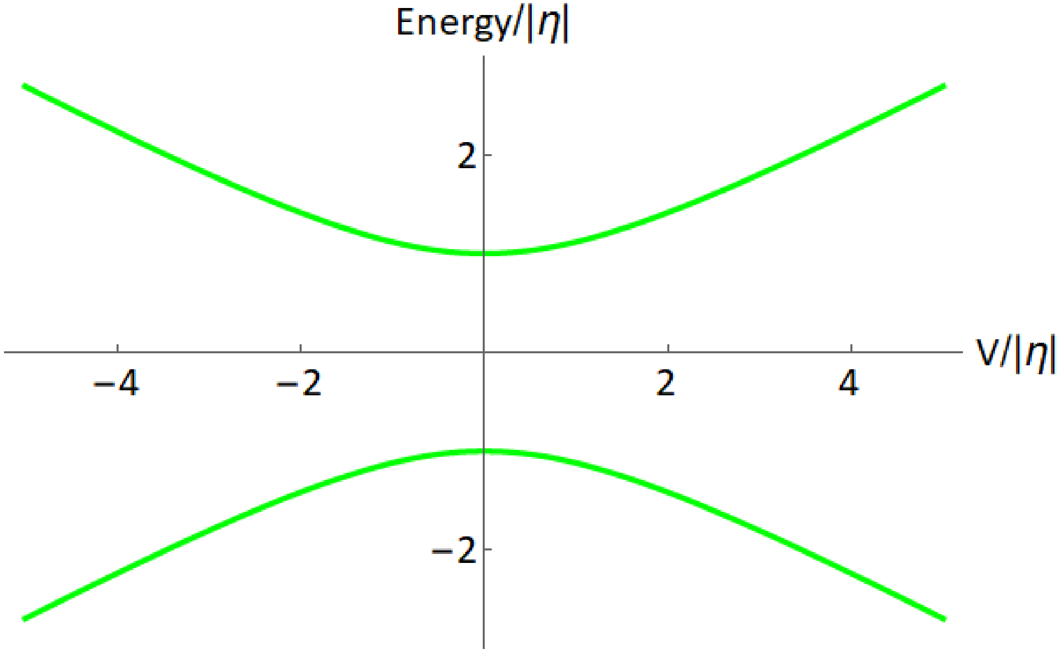
Archimedes' screw



An exhibit
at Visvesvaraya Industrial &
Technological Museum
in Bengaluru

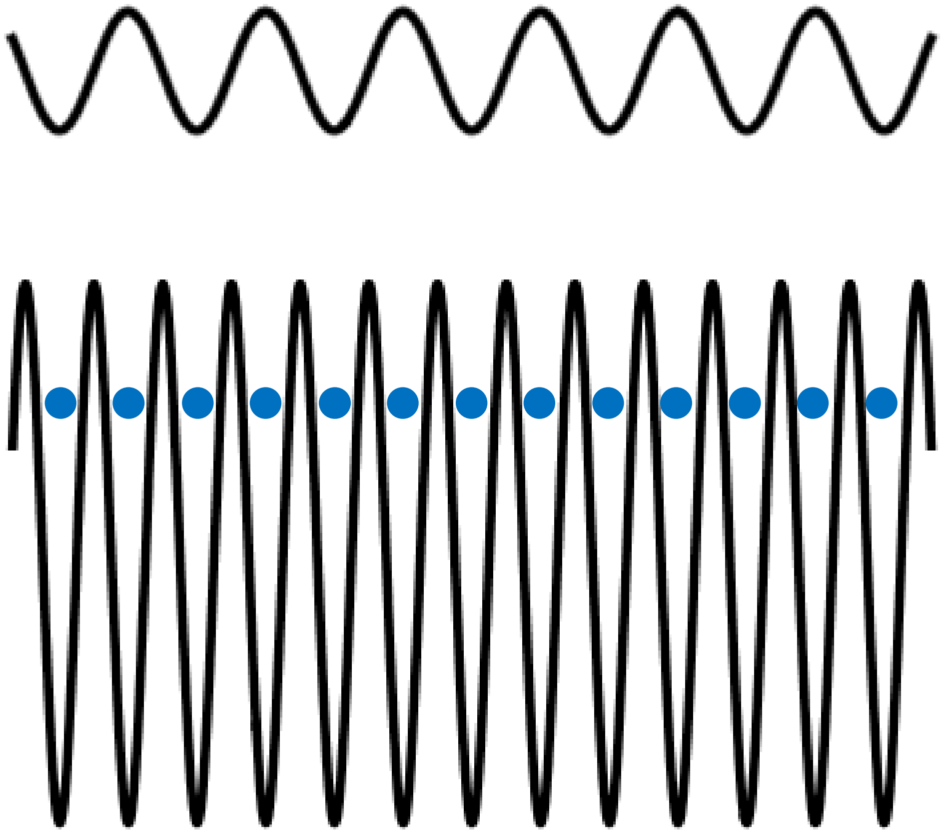
Adiabatic theorem





$$H = \begin{pmatrix} V/2 & \eta^* \\ \eta & -V/2 \end{pmatrix}$$

Thouless pump

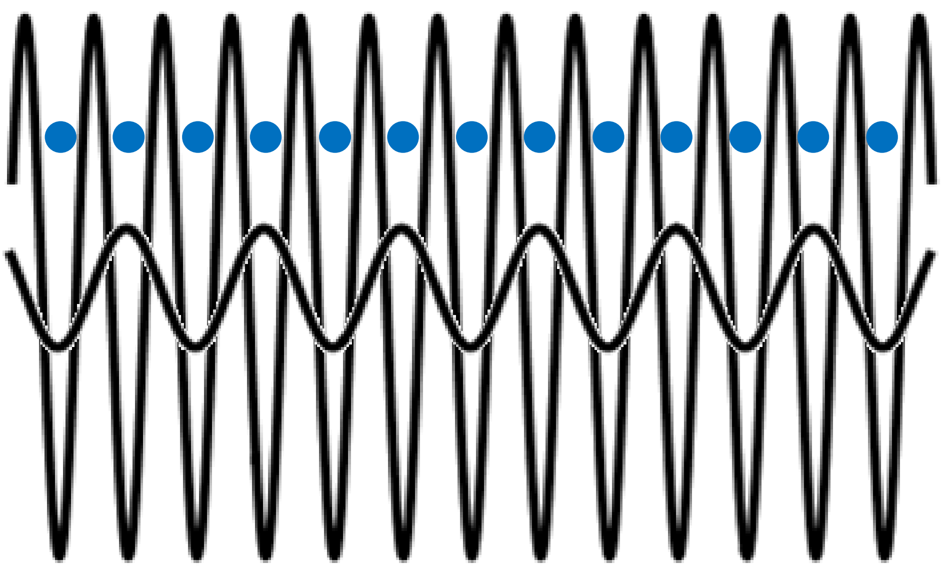


D. Thouless, PRB'83

Thouless pump

- Commensurate potentials
- Fully occupied bands
- One potential adiabatically shifted by the common period

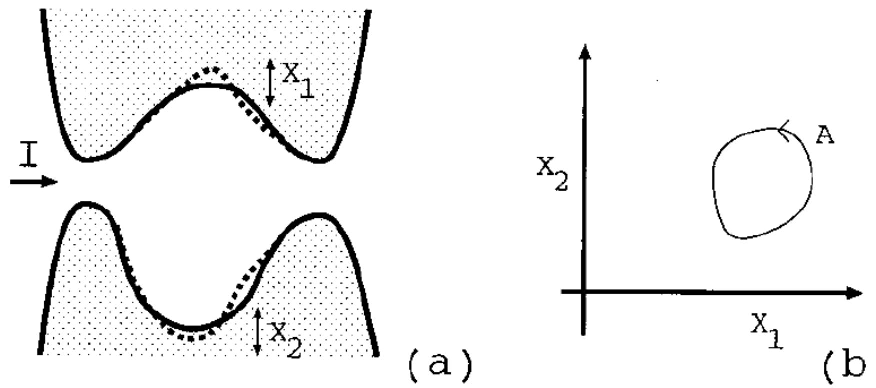
Integer number of electrons transferred



Robust against small changes in the potentials

D. Thouless, PRB'83

Pumping by changing scattering matrices



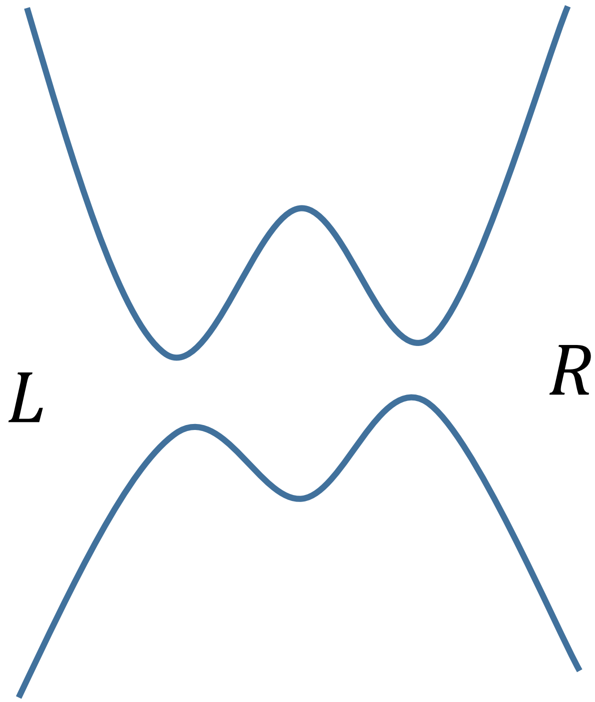
$$Q(m,\tau)=\frac{e}{\pi}\int_A dX_1 dX_2 \sum_{\beta} \sum_{\alpha\in m} \mathrm{Im}\frac{\partial S^*_{\alpha\beta}}{\partial X_1}\frac{\partial S_{\alpha\beta}}{\partial X_2}.$$

Transferred charge not quantized. But electrons are!

=> Noisy pumping

P. Brouwer, PRB’98

An example of noisy pumping



$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ S = & \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{array}{c} L \\ R \end{array} \end{array}$$

$$r = |r|e^{i\theta(\tau)}$$
$$r' = |r'|e^{-i\theta(\tau)} = |r|e^{-i\theta(\tau)}$$
$$\theta(\tau + \tau_0) - \theta(\tau) = 2\pi$$

$$P_N(Q) = \frac{N!}{Q!\,(N-Q)!}(|r|^2)^Q(1-|r|^2)^{N-Q}$$

Sensitive to $|r|$!

A. Andreev, A. Kamenev, PRL'00

Two types of adiabatic pumps

- Pump integer number of quanta each cycle. Noiseless.

  - Robust. (Topological invariants).

    Thouless PRB’83, Fu&Kane PRB’06, …

  - Fine-tuned.

    Andreev&Kamenev PRL’00, Avron et al. PRL’01, …

- Produce noise. Sensitive to perturbations.

  Andreev&Kamenev PRL’00, Makhlin&Mirlin PRL’01, Moskalets&Büttiker PRB’02, …

Two types of adiabatic pumps

- Pump integer number of quanta each cycle. Noiseless.
    - Robust. (Topological invariants).
    - Fine-tuned.

- Produce noise. Sensitive to perturbations.

- Produce noise but are robust. Do they exist?

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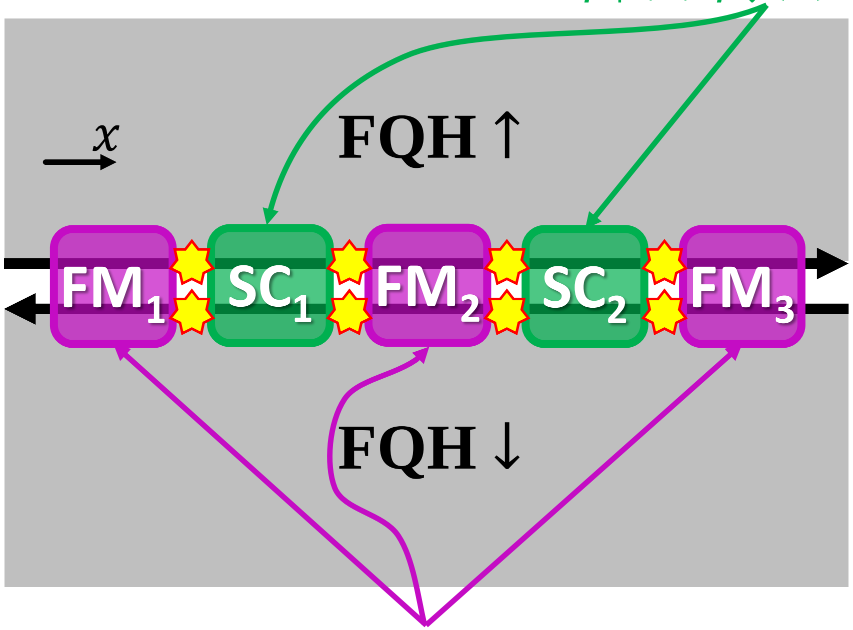
III. Adiabatic pumping with parafermions

Parafermions (PFs) at $\nu = 1/(2k+1)$ FQHE edge

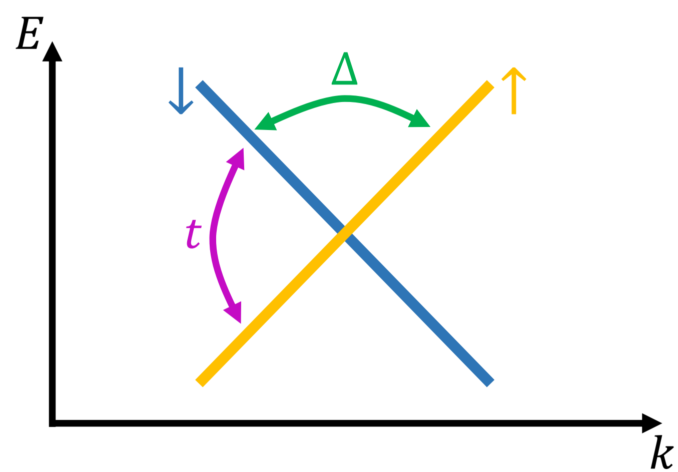
Lindner, Berg, Refael, Stern, PRX (2012);

Clarke, Alicea, Shtengel, Nature Commun. (2013)

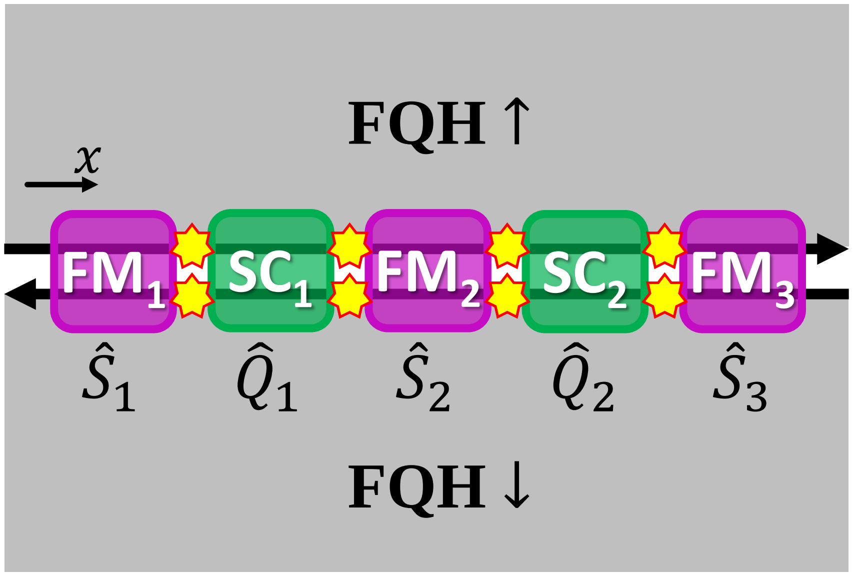
$\Delta\psi_\uparrow(x)\psi_\downarrow(x) + \mathrm{h.c.}$



$t\psi_\uparrow^\dagger(x)\psi_\downarrow(x) + \mathrm{h.c.}$



FQH-based PFs: physics



- SC domains:

host fractional charge ($e = 1$)

$$\hat{Q}_j \bmod 2 = 0, \nu, \dots, 2 - \nu$$

- FM domains:

host fractional spin ($\hbar/2 = 1$)

$$\hat{S}_j \bmod 2 = 0, \nu, \dots, 2 - \nu$$

$$[\hat{Q}_j, \hat{S}_l] \neq 0 \quad \Longrightarrow$$

- Ground state subspace basis:

$$|Q_1 \bmod 2, Q_2 \bmod 2\rangle$$

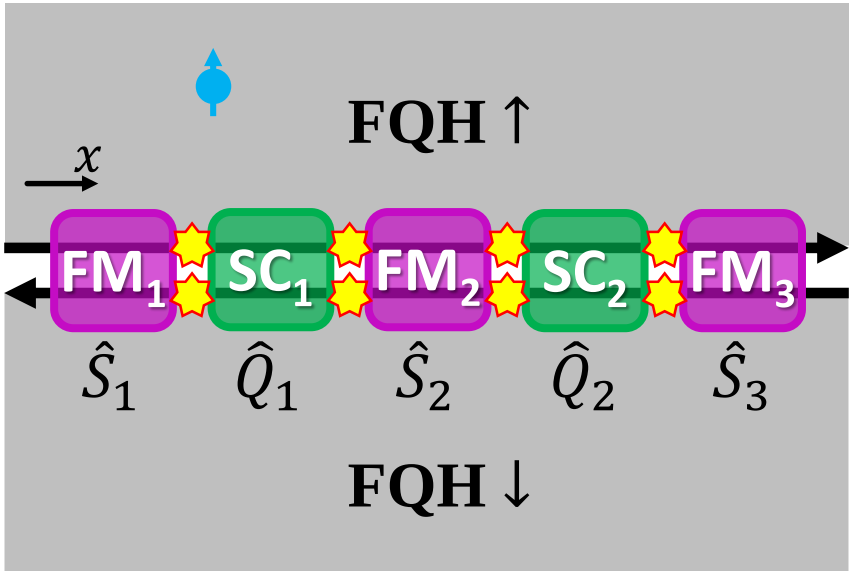
or

$$|Q_1 + Q_2 \bmod 2, Q_1 \bmod 2\rangle$$

or

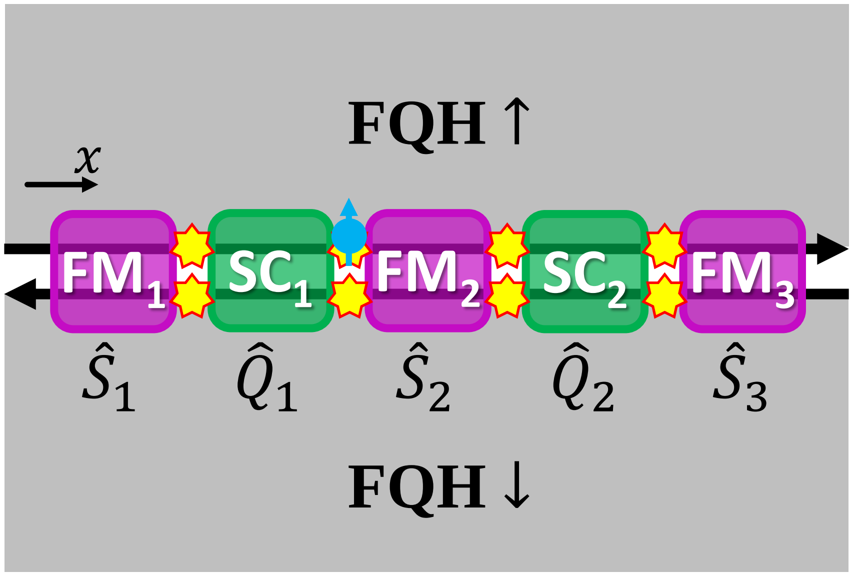
$$|Q_1 + Q_2 \bmod 2, S_2 \bmod 2\rangle$$

FQH-based PFs: physics



Quasiparticles enter domains at domain walls only

FQH-based PFs: physics



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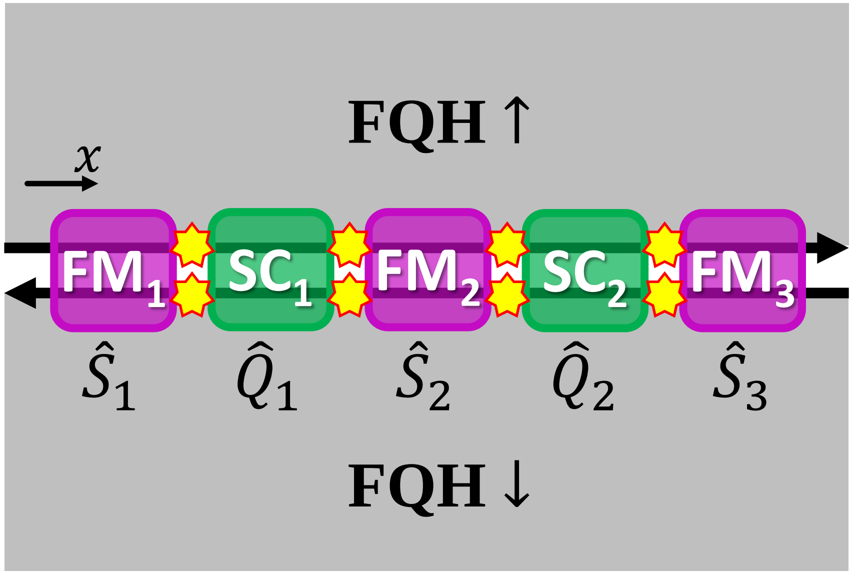
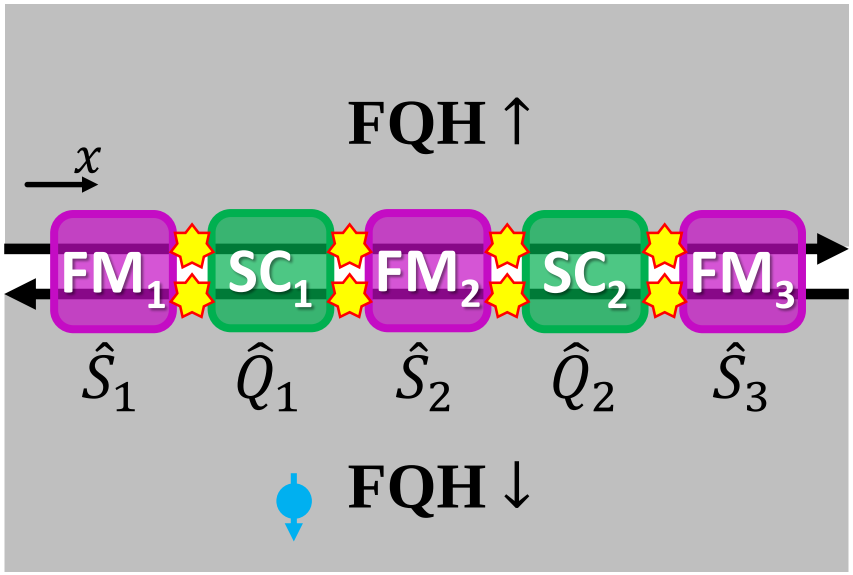
FQH-based PFs: physics



Quasiparticles enter domains at domain walls only

$$Q_1 \to Q_1 + \nu$$
$$S_2 \to S_2 + \nu$$

FQH-based PFs: physics



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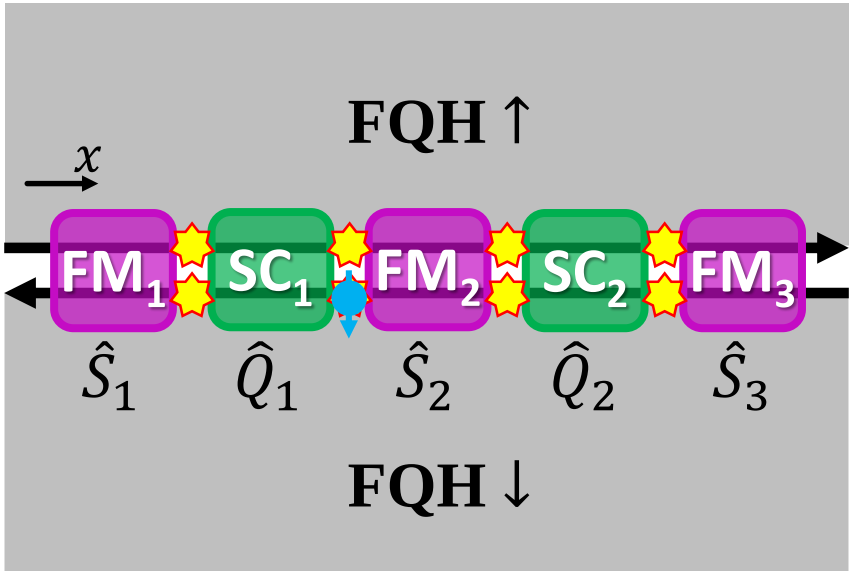
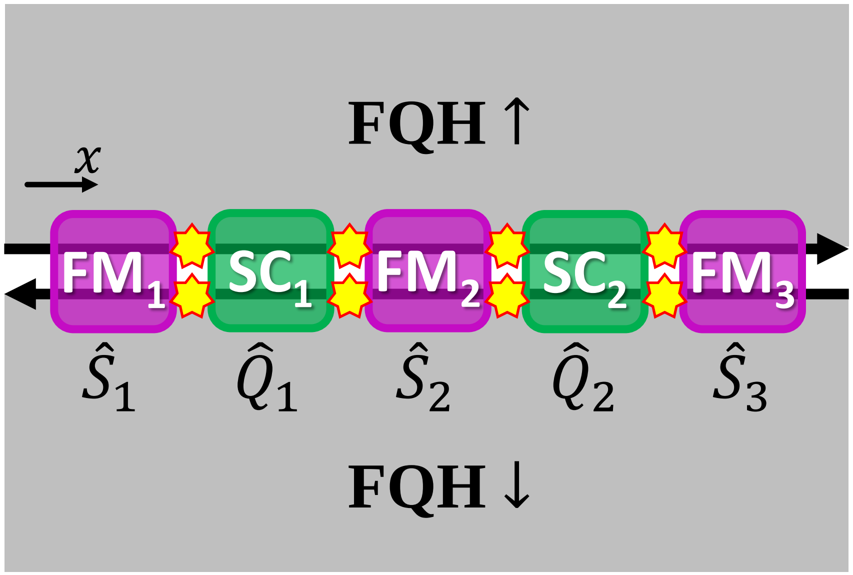
FQH-based PFs: physics



Quasiparticles enter domains at domain walls only

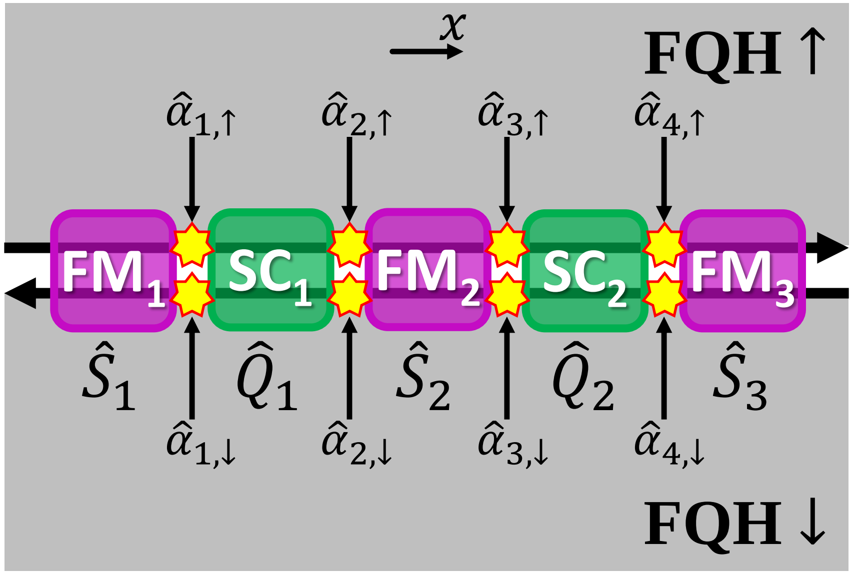
FQH-based PFs: physics



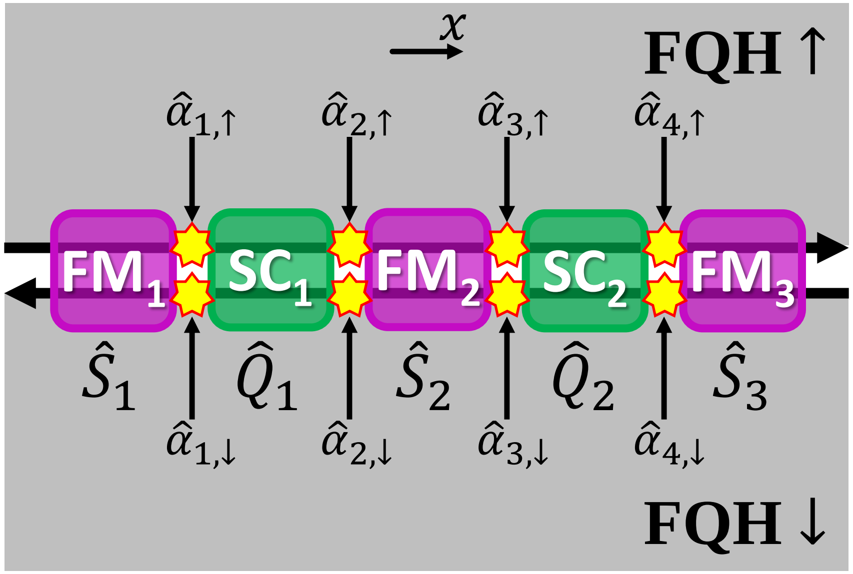
Quasiparticles enter domains at domain walls only

$$Q_1 \to Q_1 + \nu$$
$$S_2 \to S_2 - \nu$$

FQH-based PFs



FQH-based PFs: properties



$$2\nu^{-1} = d$$

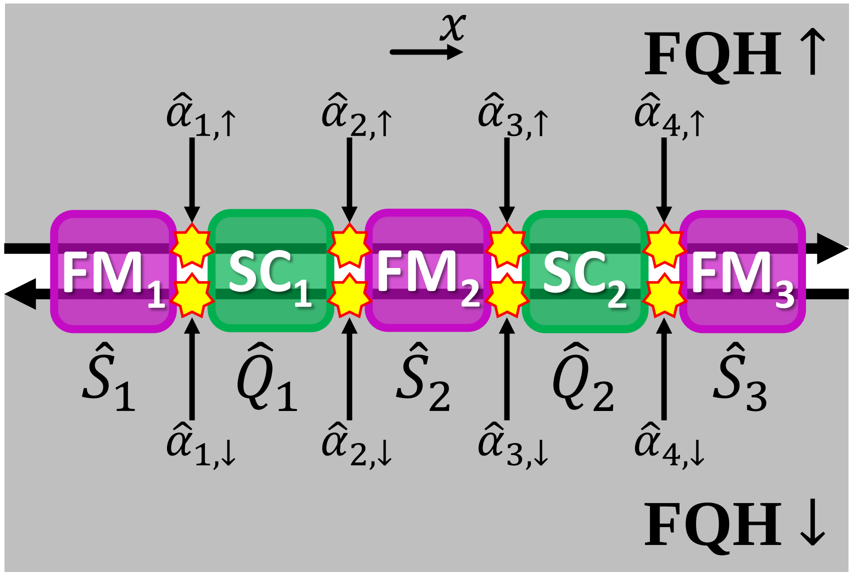
$$\hat{\alpha}_{j,s}\hat{\alpha}_{l,s} = e^{\pm i\pi\nu}\hat{\alpha}_{l,s}\hat{\alpha}_{j,s} \qquad j \neq l$$

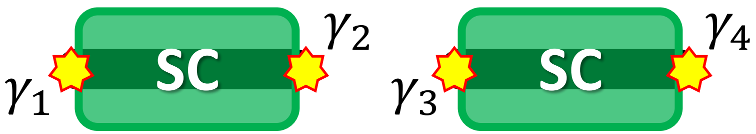
$$\hat{\alpha}_{j,s}^{d} = 1, \hat{\alpha}_{j,s}^{\dagger} = \hat{\alpha}_{j,s}^{d-1} \qquad j = l$$

$$\hat{\alpha}_{2,s}\hat{\alpha}_{1,s}^{\dagger} = e^{i\pi s\left(\hat{Q}_1+\frac{\nu}{2}\right)}$$

$$\hat{\alpha}_{3,s}\hat{\alpha}_{2,s}^{\dagger} = e^{i\pi\left(\hat{S}_2+s\frac{\nu}{2}\right)}$$

FQH-based PFs: properties





$\hat{\alpha}_{j,s}\hat{\alpha}_{l,s} = e^{\pm i\pi\nu}\hat{\alpha}_{l,s}\hat{\alpha}_{j,s}$	$j \neq l$	$\gamma_j\gamma_l = -\gamma_l\gamma_j$
$\hat{\alpha}_{j,s}^{d} = 1, \hat{\alpha}_{j,s}^{\dagger} = \hat{\alpha}_{j,s}^{d-1}$	$j = l$	$\gamma_j^2 = 1, \gamma_j^{\dagger} = \gamma_j$
$\hat{\alpha}_{2,s}\hat{\alpha}_{1,s}^{\dagger} = e^{i\pi s\left(\hat{Q}_1+\frac{\nu}{2}\right)}$		$i\gamma_2\gamma_1$
$\hat{\alpha}_{3,s}\hat{\alpha}_{2,s}^{\dagger} = e^{i\pi\left(\hat{S}_2+s\frac{\nu}{2}\right)}$		$i\gamma_3\gamma_2$

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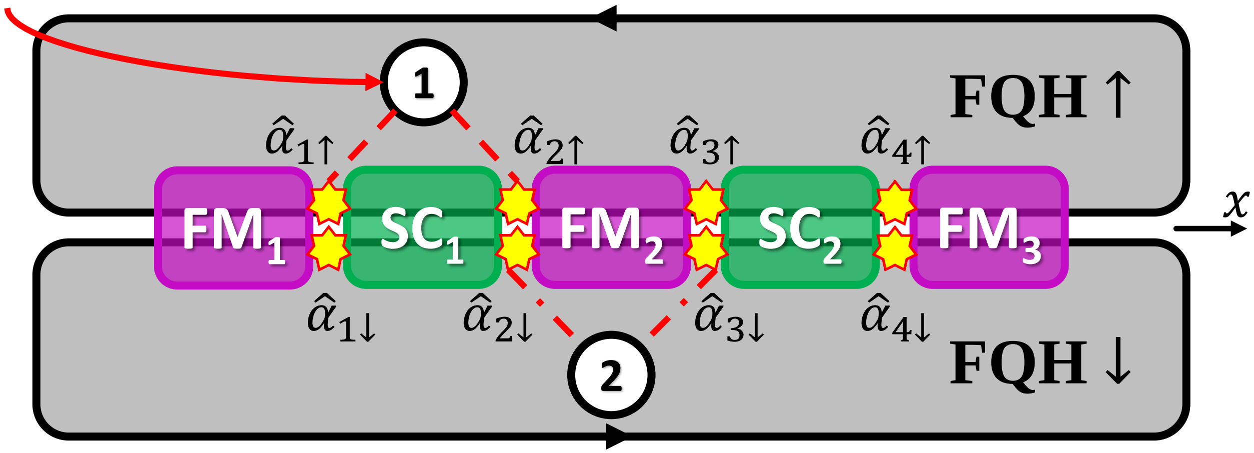
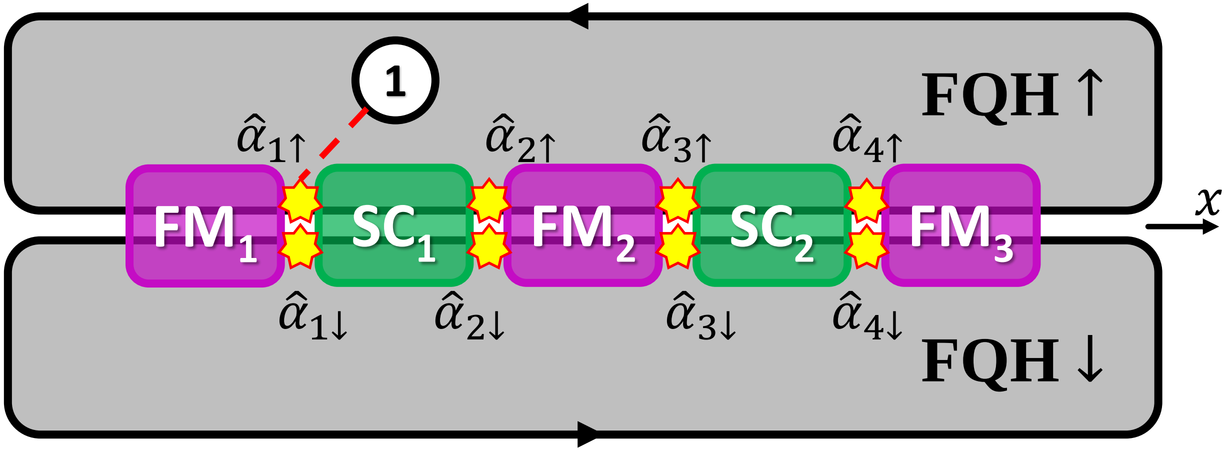
The pump

Quantum anti-dot (QAD)

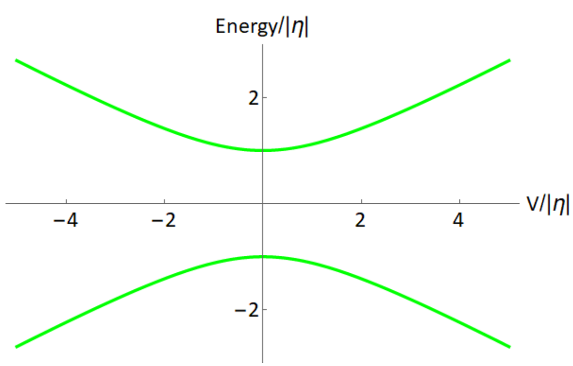


The pump: elementary ingredients

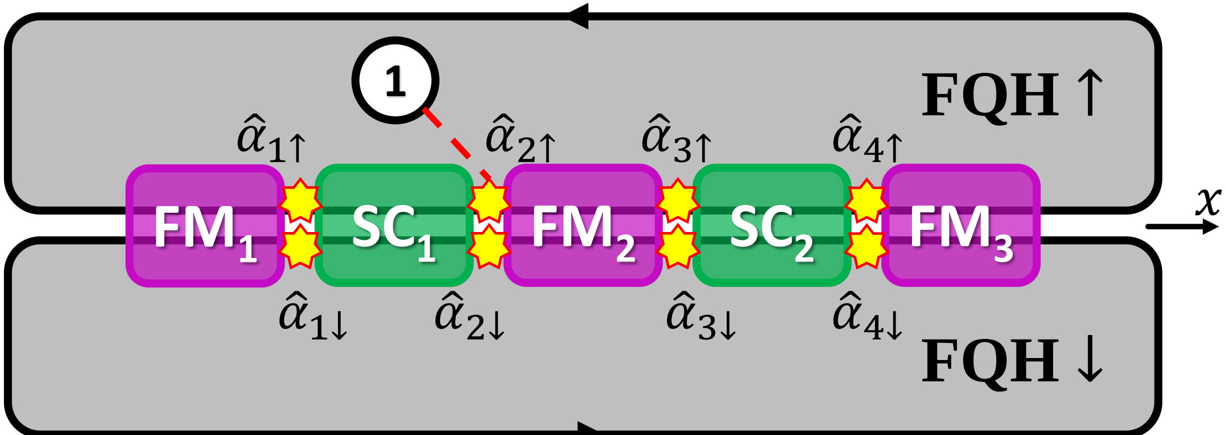


$$H = \begin{pmatrix} \nu V/2 & \eta_1^* \\ \eta_1 & -\nu V/2 \end{pmatrix}$$

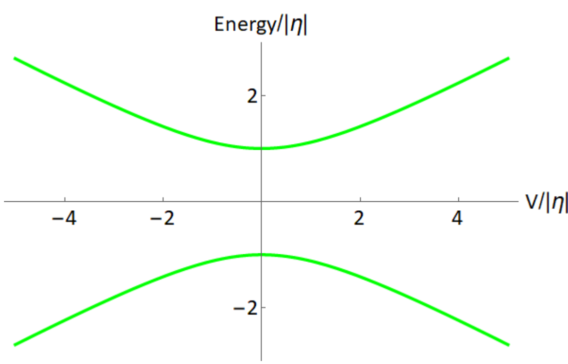
$$|\nu\rangle_{\mathrm{QAD}} |Q_1 \bmod 2\rangle$$
$$|0\rangle_{\mathrm{QAD}} |Q_1 + \nu \bmod 2\rangle$$



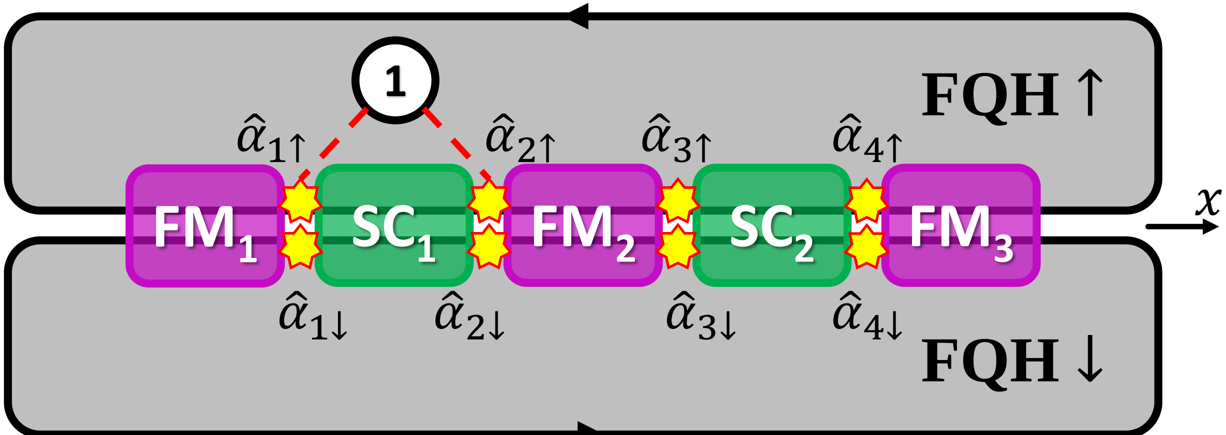
The pump: elementary ingredients



$$H = \begin{pmatrix} \nu V/2 & \eta_2^* e^{i\pi Q_1} \\ \eta_2 e^{-i\pi Q_1} & -\nu V/2 \end{pmatrix} \qquad \begin{matrix} |\nu\rangle_{\mathrm{QAD}} |Q_1 \bmod 2\rangle \\ |0\rangle_{\mathrm{QAD}} |Q_1 + \nu \bmod 2\rangle \end{matrix}$$

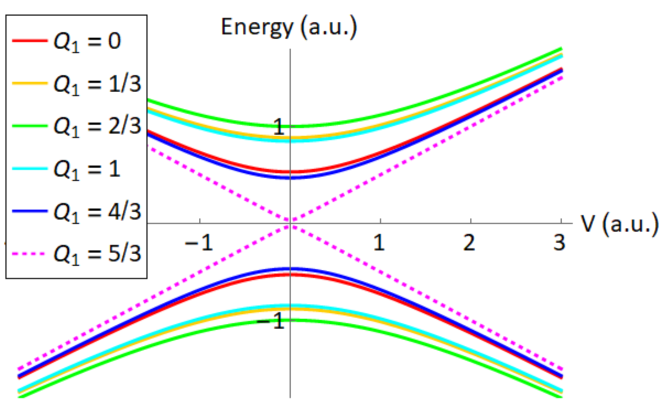


The pump: elementary ingredients



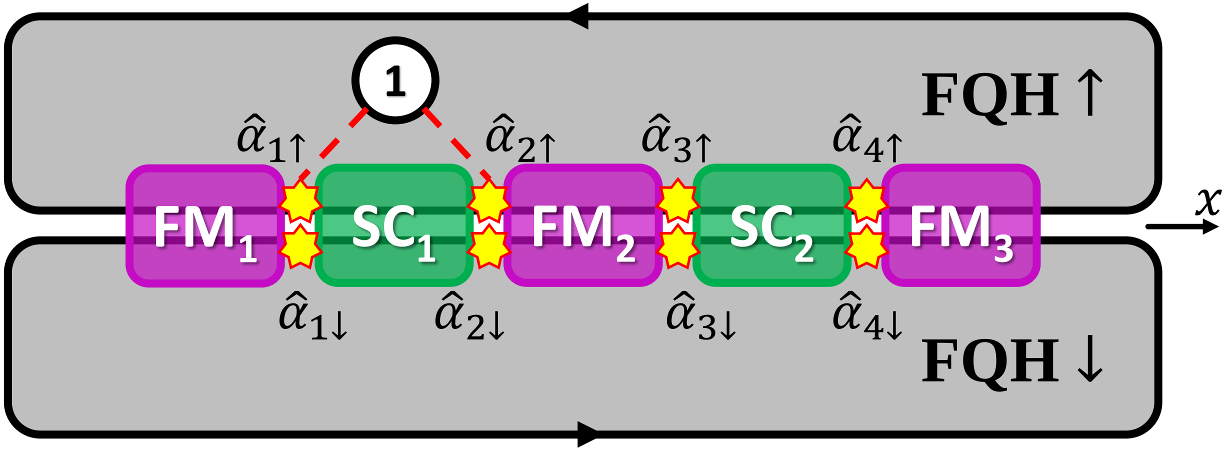
$$H = \begin{pmatrix} \nu V/2 & \eta^*_{Q_1} \\ \eta_{Q_1} & -\nu V/2 \end{pmatrix}$$

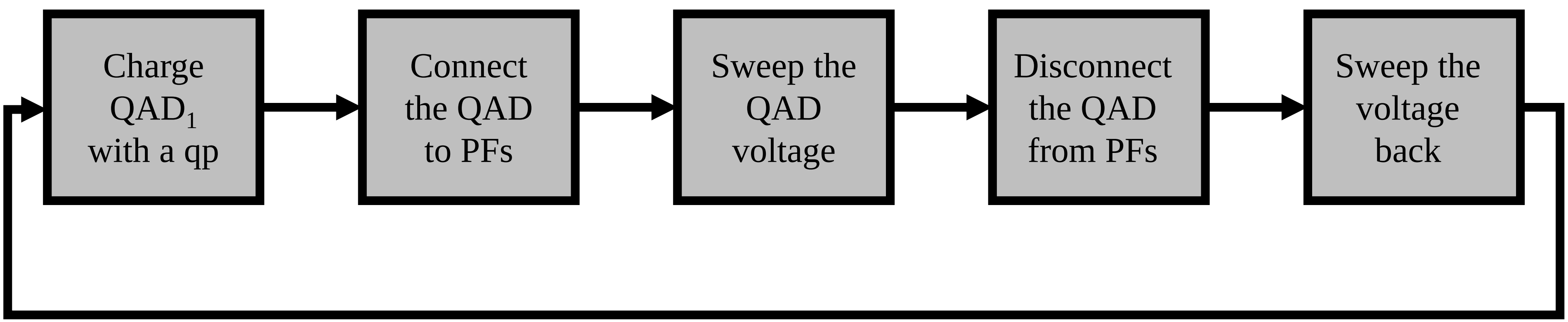
$$|\nu\rangle_{\mathrm{QAD}} |Q_1 \bmod 2\rangle$$
$$|0\rangle_{\mathrm{QAD}} |Q_1 + \nu \bmod 2\rangle$$



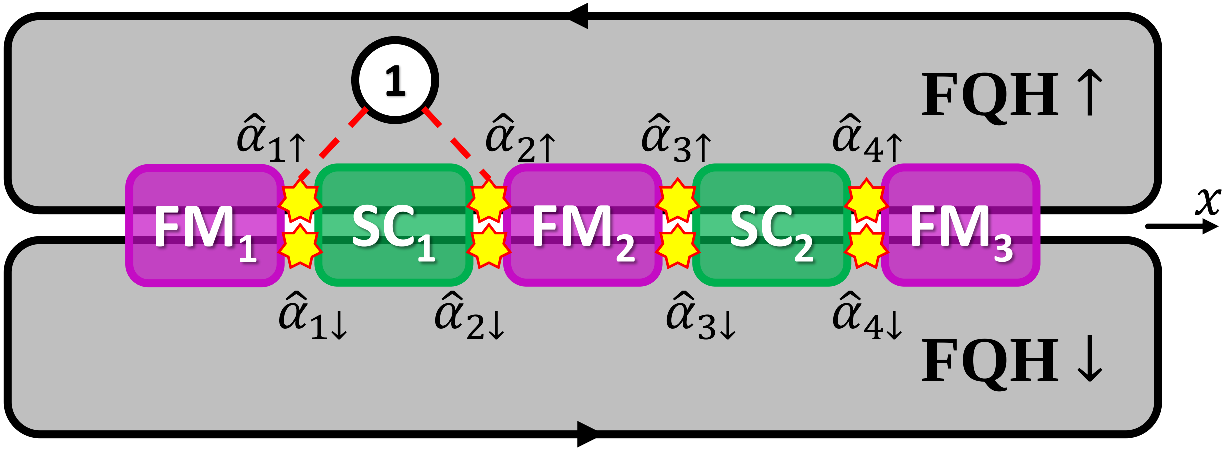
$$\eta_{Q_1} = \eta_1 + \eta_2 e^{-i\pi Q_1}$$
$$P(\text{no transfer of qp}) = e^{-|\eta_{Q_1}|^2/\lambda}$$
$$V(t) \propto \lambda t$$

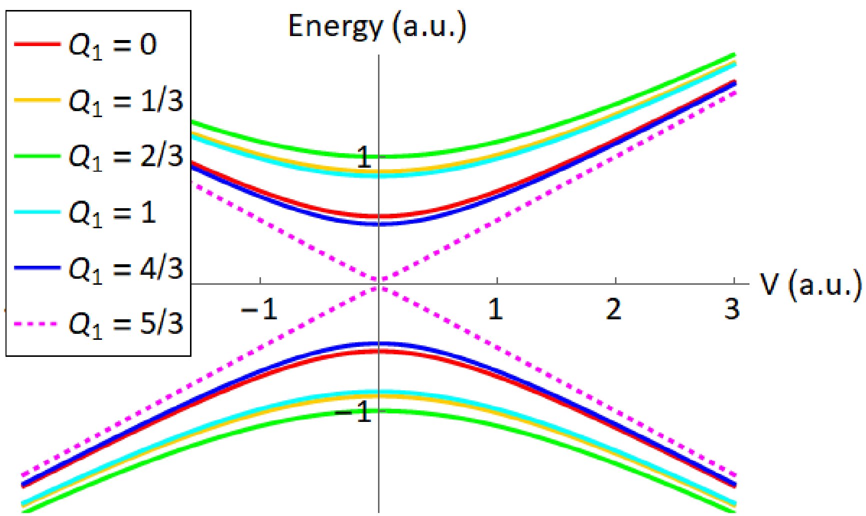
Elementary pumping step

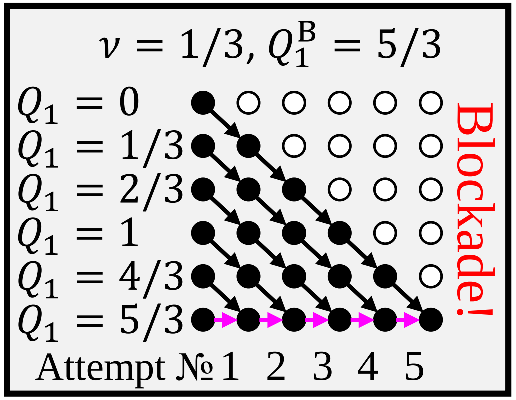




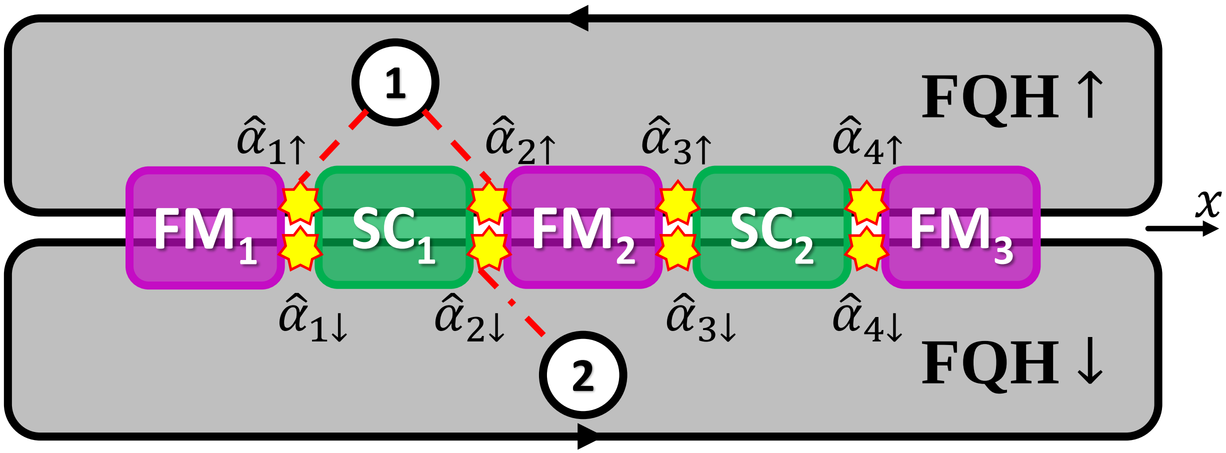
Topological pumping blockade

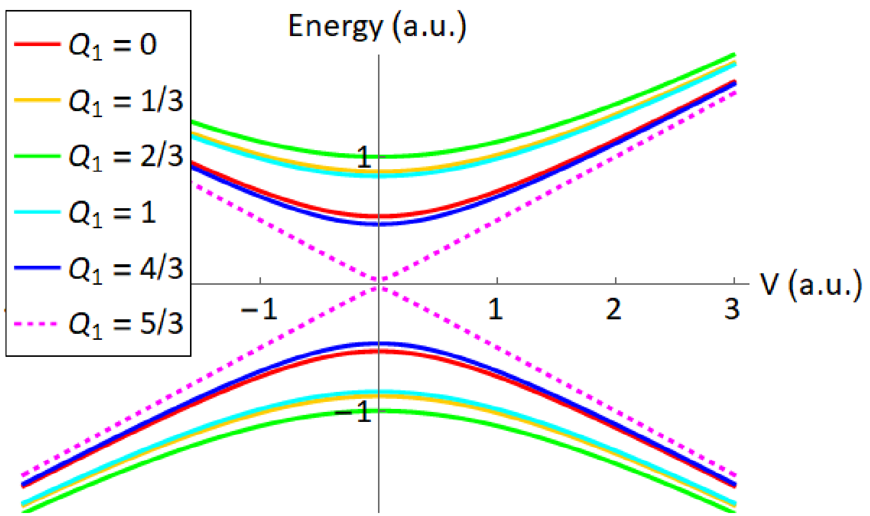


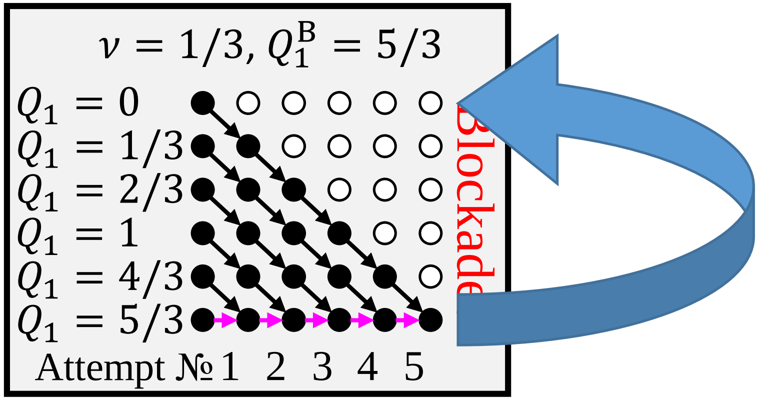




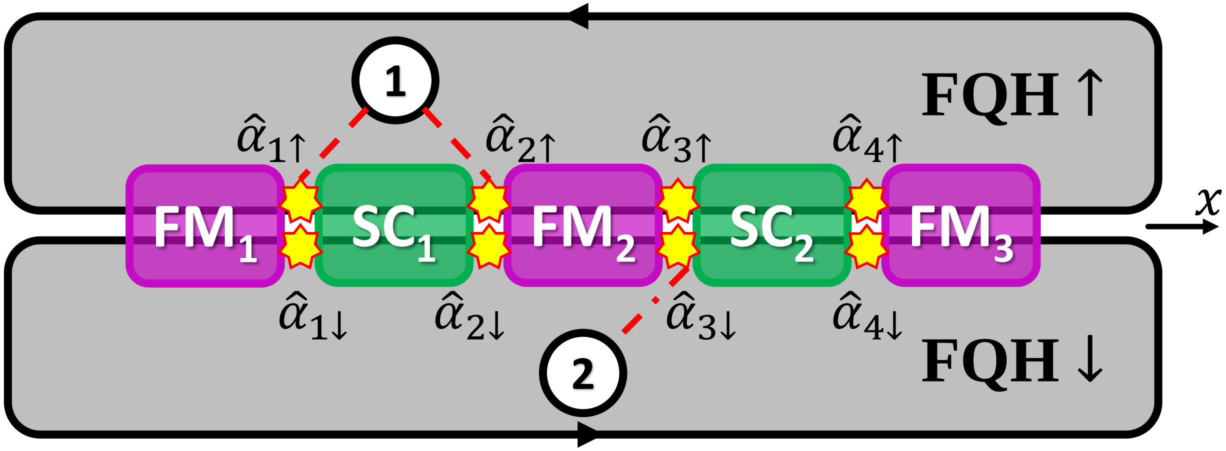
Blockade lifting

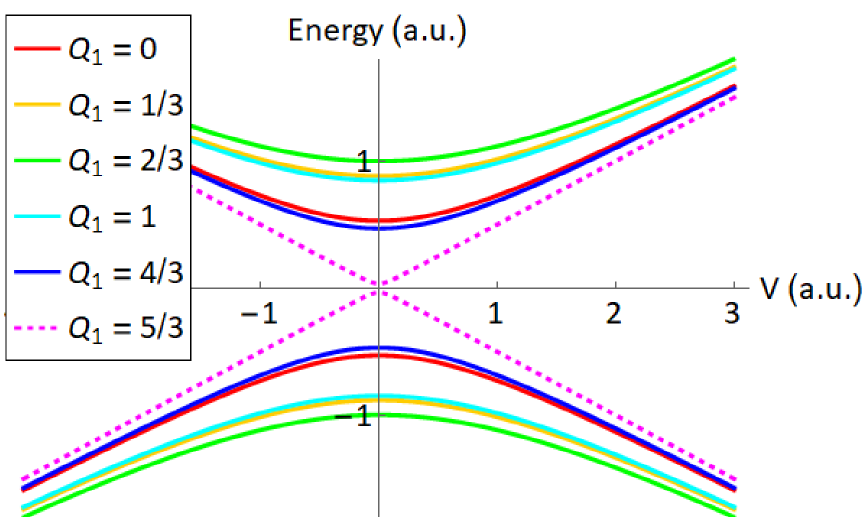


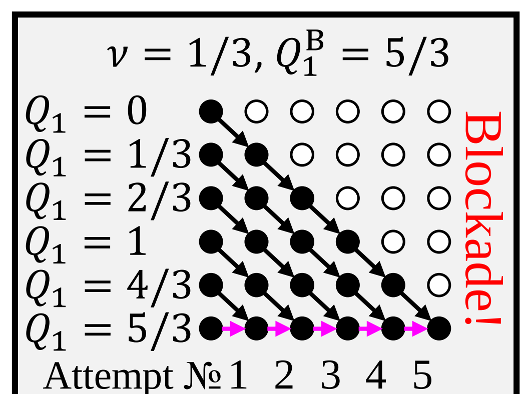




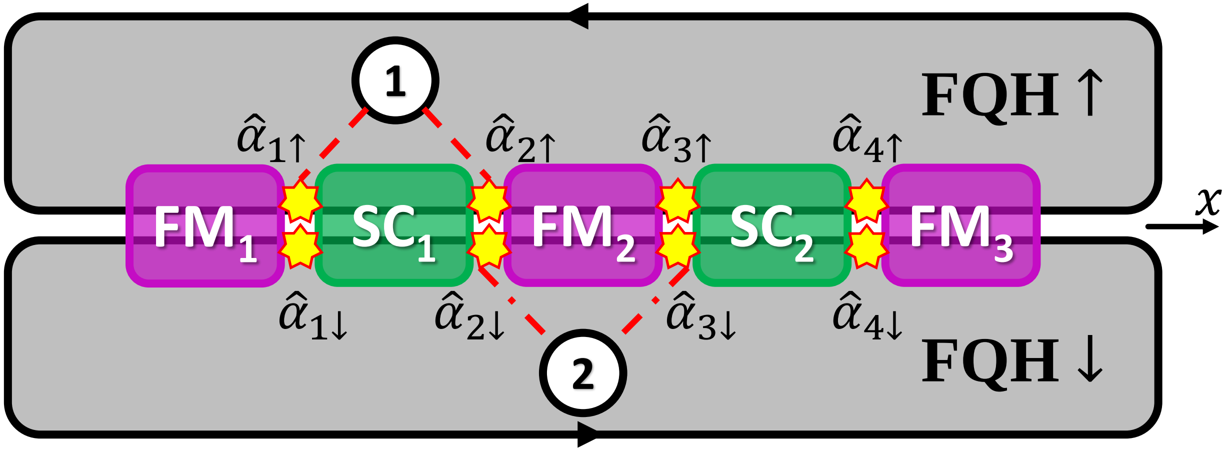
...or not lifting

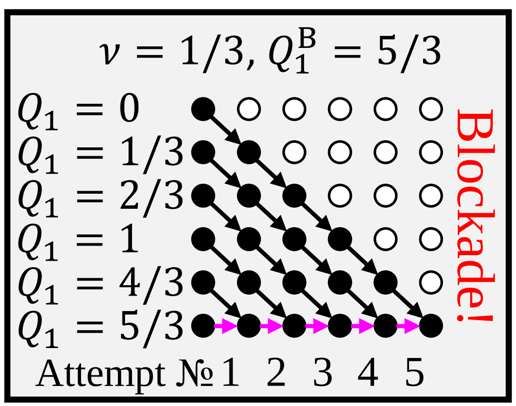


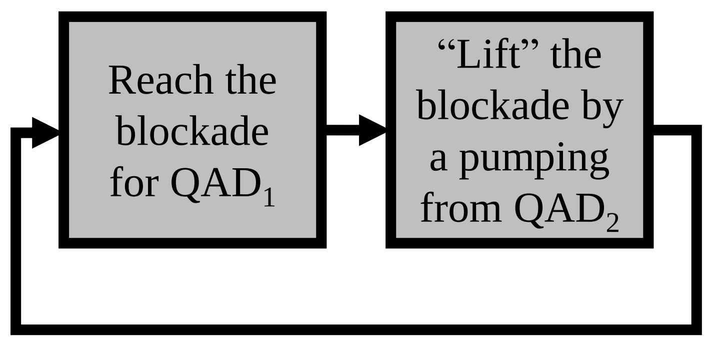




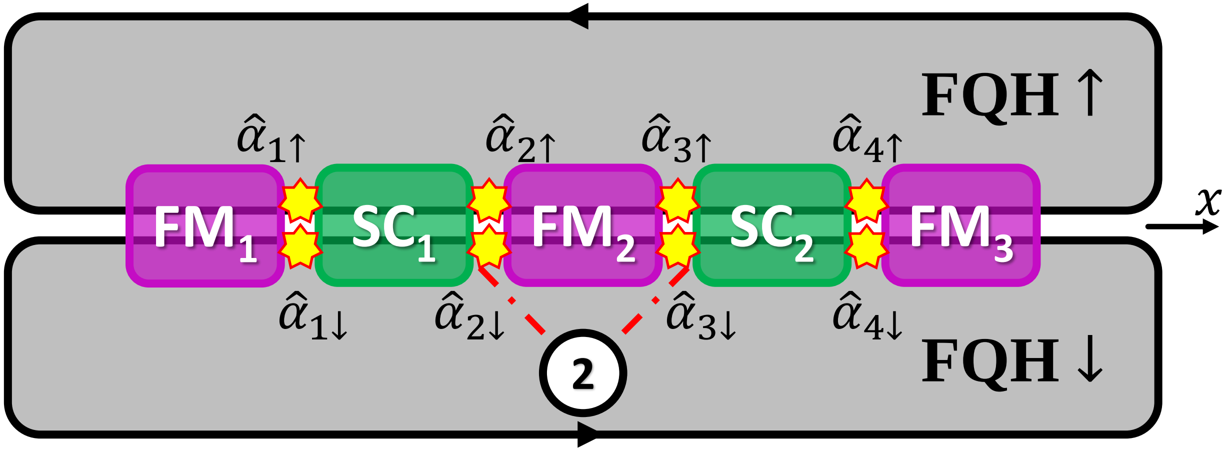
The origin of noise

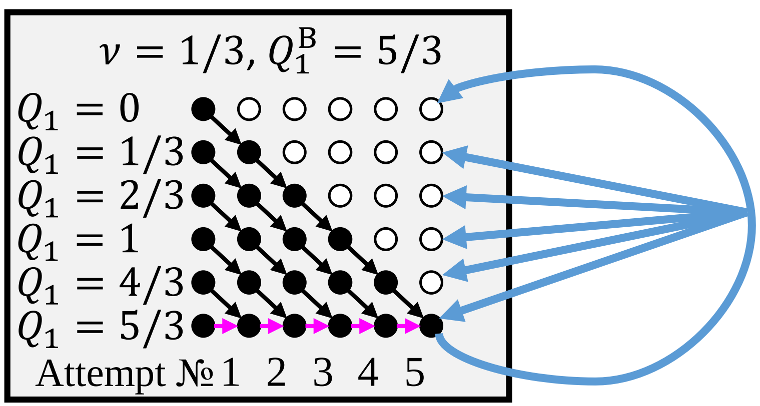






Lifting the blockade - in detail



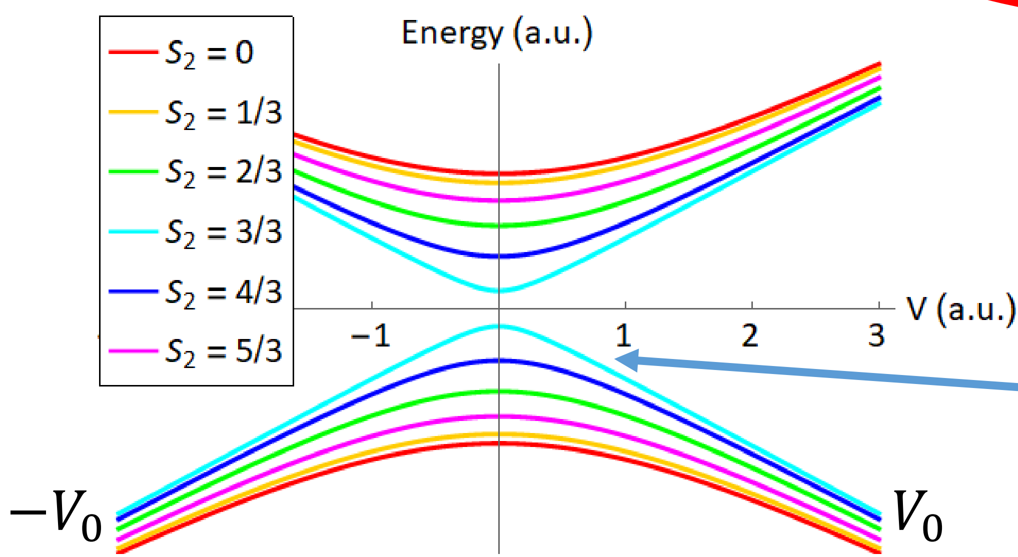


with probability $\left|A_{Q_1}\right|^2$ of going to $|Q_1 \bmod 2\rangle$

Lifting the blockade: what are the phases?

$$|A_{Q_1}|^2 = \frac{1}{d^2}\sum_{S_2,\,S_2'} e^{i\pi d(Q_1^{\mathrm{B}}-Q_1)(S_2-S_2')/2+i(\theta_{S_2}-\theta_{S_2'})}$$

$$\theta_{S_2} = -\pi - i\arg(\eta_{S_2}) + \frac{(\nu V_0)^2}{2\lambda} + \frac{|\eta_{S_2}|^2}{\lambda}\left(1+\frac{\ln(\nu V_0)^2}{|\eta_{S_2}|^2}\right)$$



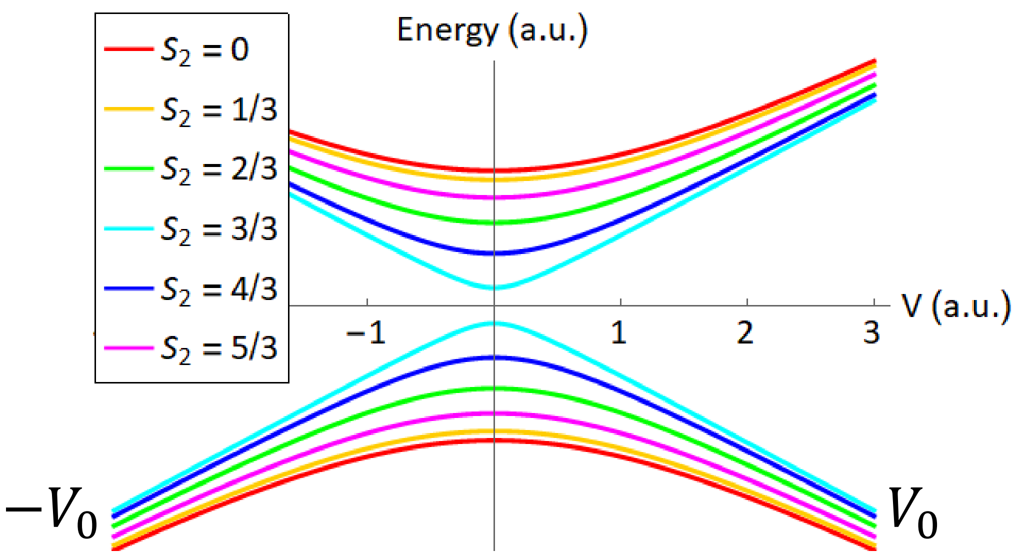
- Is the dynamic phase
- Diverges as $\lambda \to 0$

Small parameter changes lead to large phase changes

$$V_0 \to V_0 + \delta V_0$$

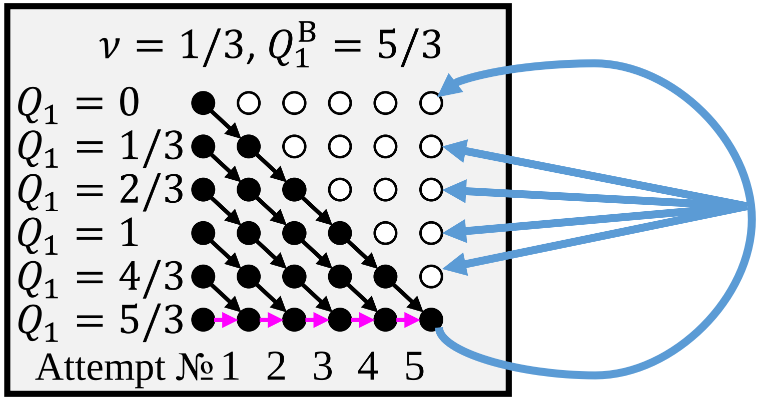
$$\Rightarrow \theta_{S_2} \to \theta_{S_2} + \delta\theta_{S_2}$$

$$\delta\theta_{S_2} = \frac{(\nu V_0)^2}{\lambda}\frac{\delta \mathrm{V}_0}{\mathrm{V}_0} + 2\frac{\left|\eta_{S_2}\right|^2}{\lambda}\frac{\delta \mathrm{V}_0}{\mathrm{V}_0}$$



- Diverges as $\lambda \to 0$

Lifting the blockade: what are the probabilities?

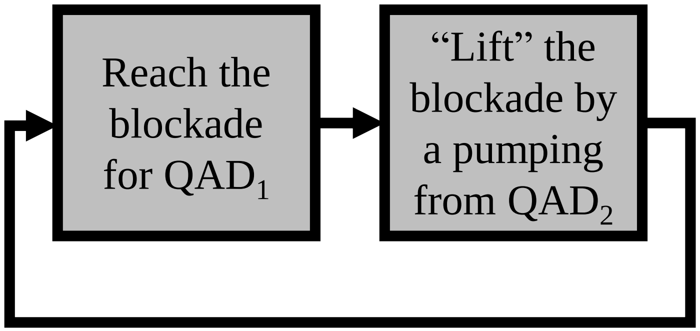


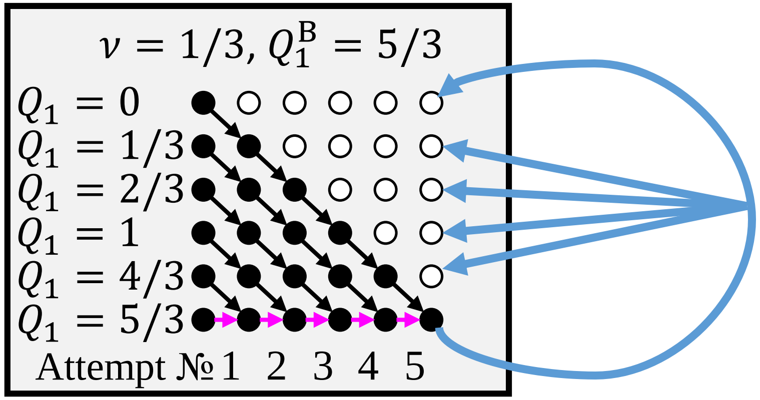
$$\left\langle \left|A_{Q_1}\right|^2 \right\rangle = \frac{1}{d^2} \sum_{S_2, S_2'} e^{i\pi d (Q_1^{\mathrm{B}} - Q_1)(S_2 - S_2')/2} \left\langle e^{i(\theta_{S_2} - \theta_{S_2'})} \right\rangle = \frac{1}{d}$$

Equal probability of starting the next cycle in any state of Q_1

$\Rightarrow$ Universal counting statistics!

Universal noise in adiabatic pumping





Equal probability of starting the next cycle in any state of Q_1

$\Rightarrow$ Universal counting statistics!

In particular,

$$\langle I \rangle = \frac{d-1}{2d}\frac{e^*}{\tau} \qquad S = \frac{d+1}{6}e^* I$$

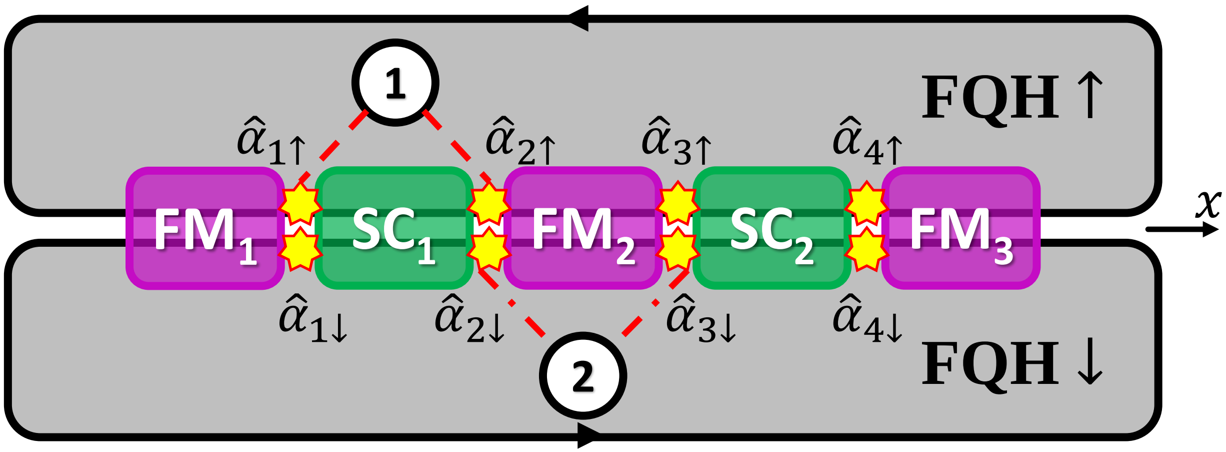
Outline

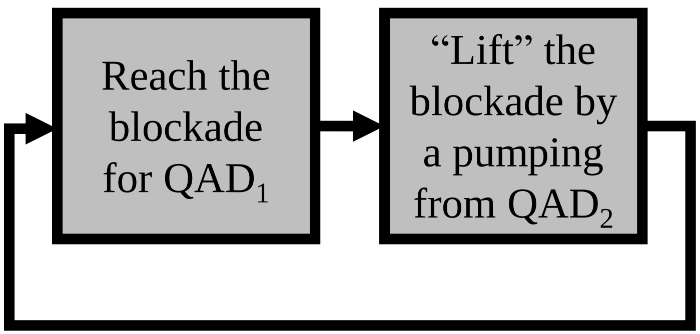
I. Adiabatic pumps

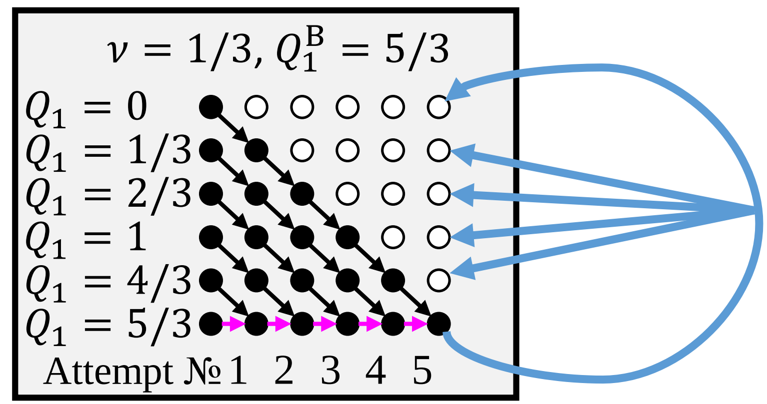
II. Essentials about parafermionic zero modes

III. Adiabatic pumping with parafermions

Conclusions







Equal probability of starting the next cycle in any state of Q_1

⇒ Noiseful; universal counting statistics!

Thank you for attention!

Outline

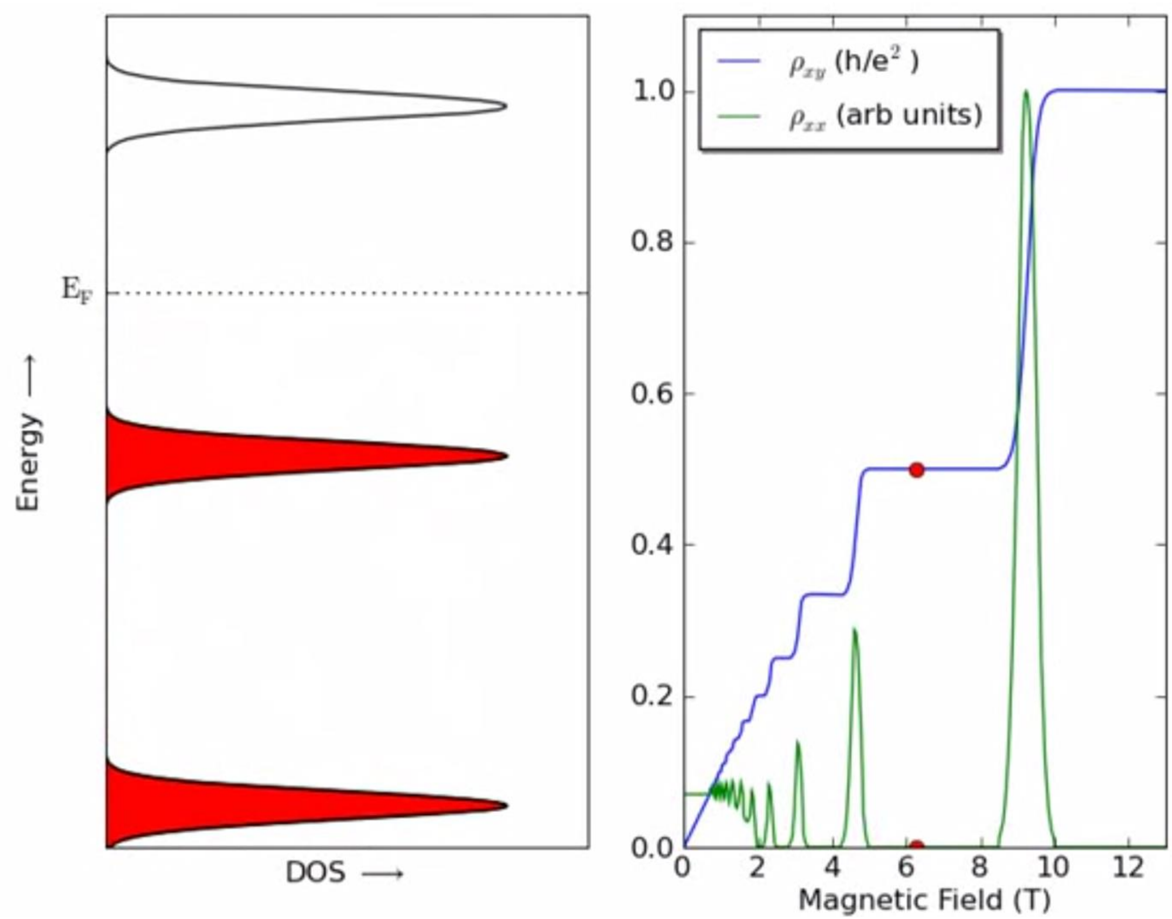
I. Adiabatic pumps

II. Essentials about parafermionic zero modes

III. Adiabatic pumping with parafermions

IV. What about the fluctuations and the fine-tuning?

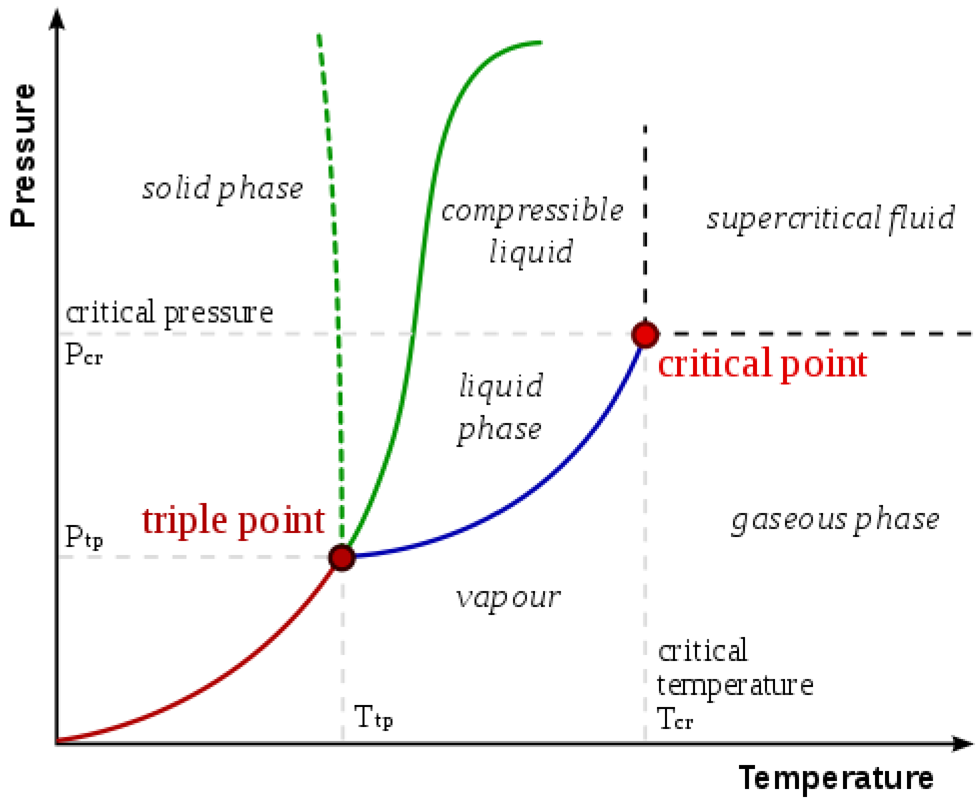
Fluctuations:
it’s like the quantum Hall effect



https://commons.wikimedia.org/wiki/File:QuantumHallEffectExplanationWithLandauLevels.ogv

Disorder needed to observe the universality (quantization)

Fine-tuning:
it's like phase transitions



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To observe universal critical exponents fine-tune to a critical point