Large-Spin Magnetic Impurity near a 2D Topological Edge

JETP Lett. **106**, 575 (2017); Phys. Rev. B **99**, 085407 (2019); arXiv:1903.03965

Moshe Goldstein (Tel Aviv)

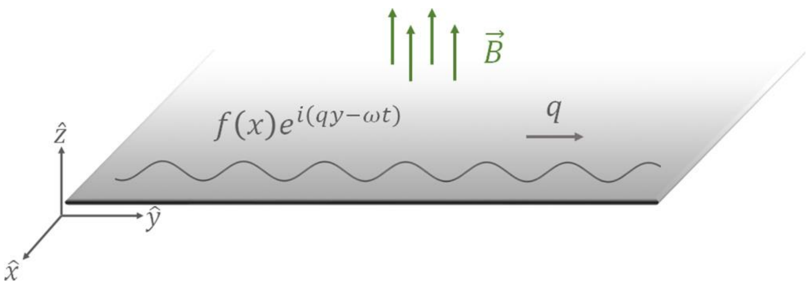
Vladislav & Pavel Kurilovich,
Igor Burmistrov (Landau Inst.),

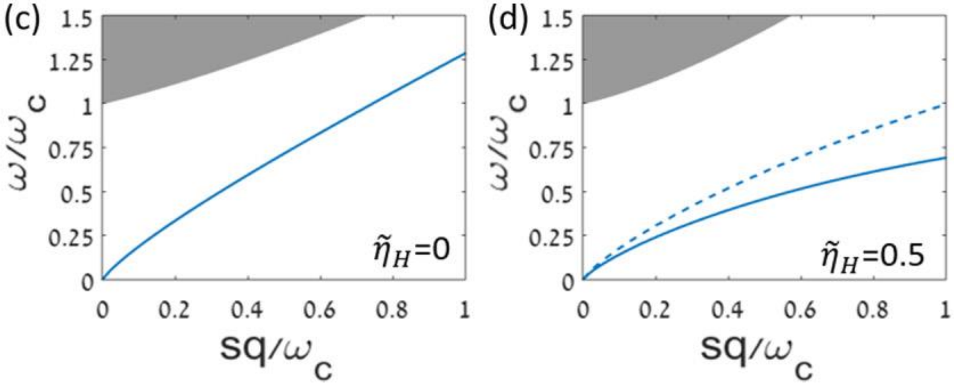
Yuval Gefen (Weizmann)

אוניברסיטת תל-אביב
TEL AVIV UNIVERSITY

Other Recent Works (I)

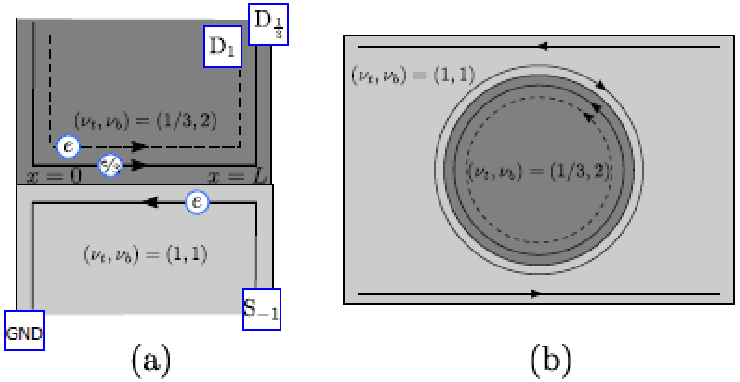
- **Hall viscosity** effects on **edge magnetoplasmons**





Cohen & MG, PRB **98**, 235103 (2018)

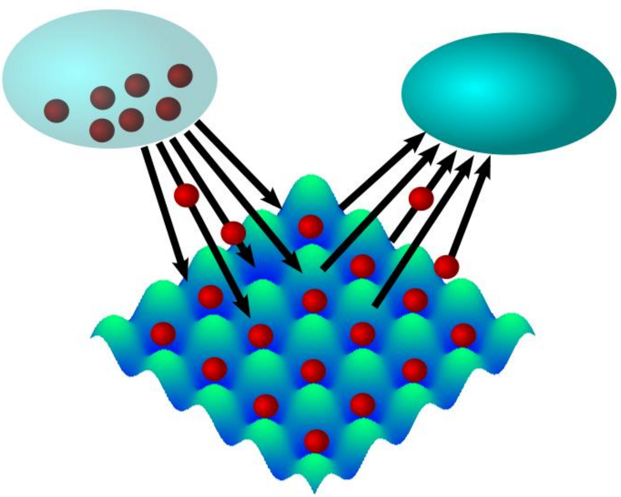
- **Superconducting correlations** out of **purely-repulsive interactions** on a **fractional edge**

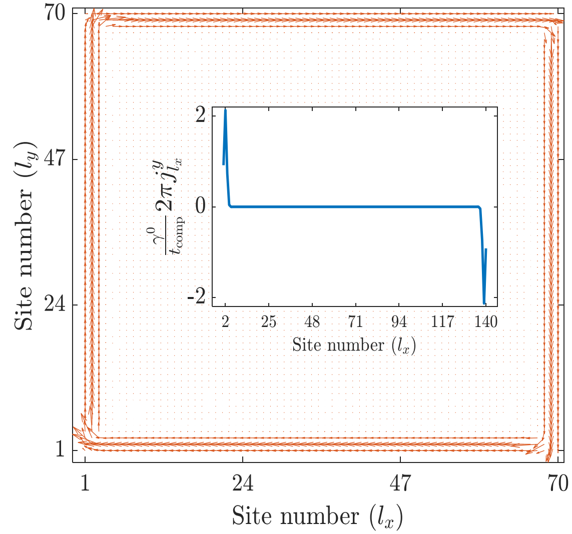


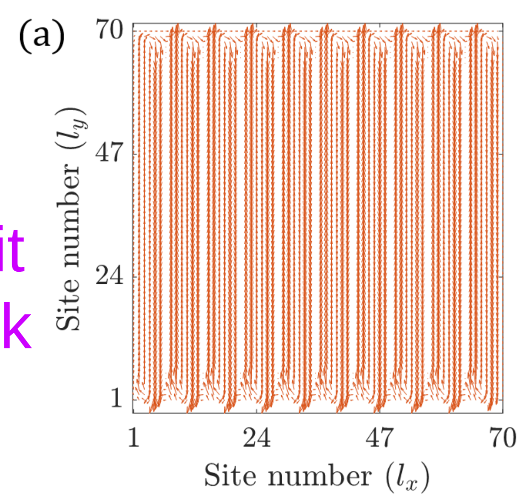
Vayrynen, MG, Gefen, PRL – to appear

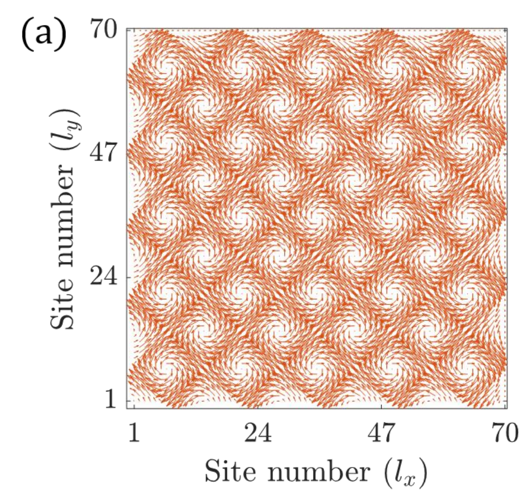
Other Recent Works (II)

- **Topology** from **driven-dissipative dynamics**









MG, arXiv:1810.12050; Shavit & MG arXiv:1903.05336; Beck & MG …

Outline

- **Transport** in **2D topological insulators**

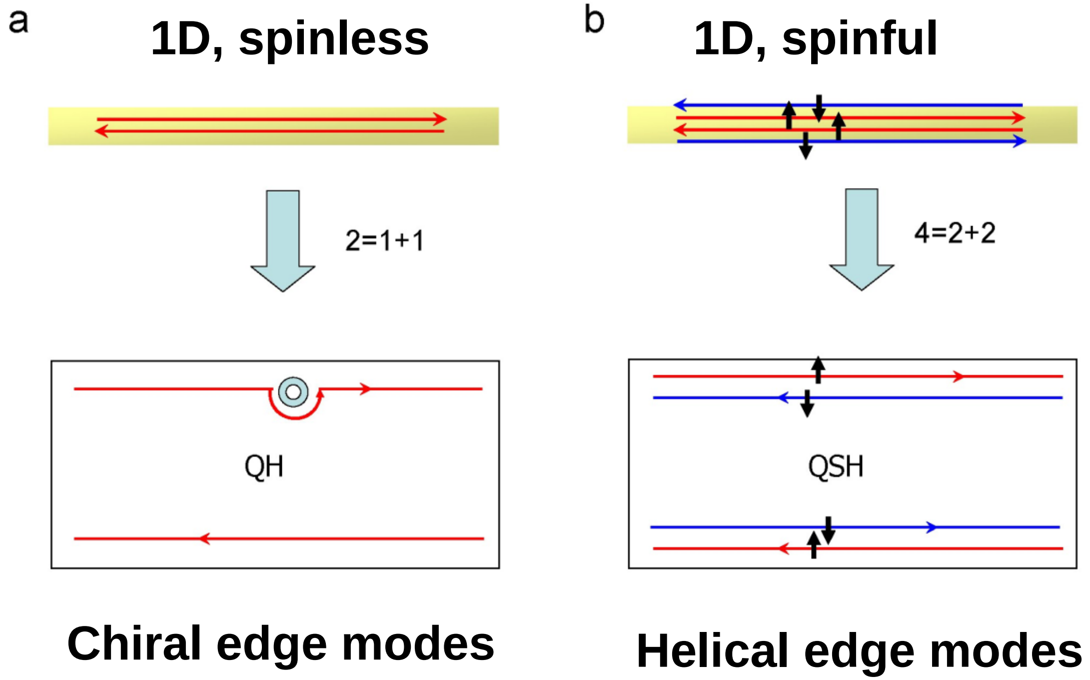
- **Large S impurity**: **low energy**

- **Large S impurity**: **finite energy transport**

- **Large S impurity**: **shot noise**

2D Topological Insulators

- "**Quantum spin Hall**": **Time reversal invariant** analogue of the **quantum Hall effect**



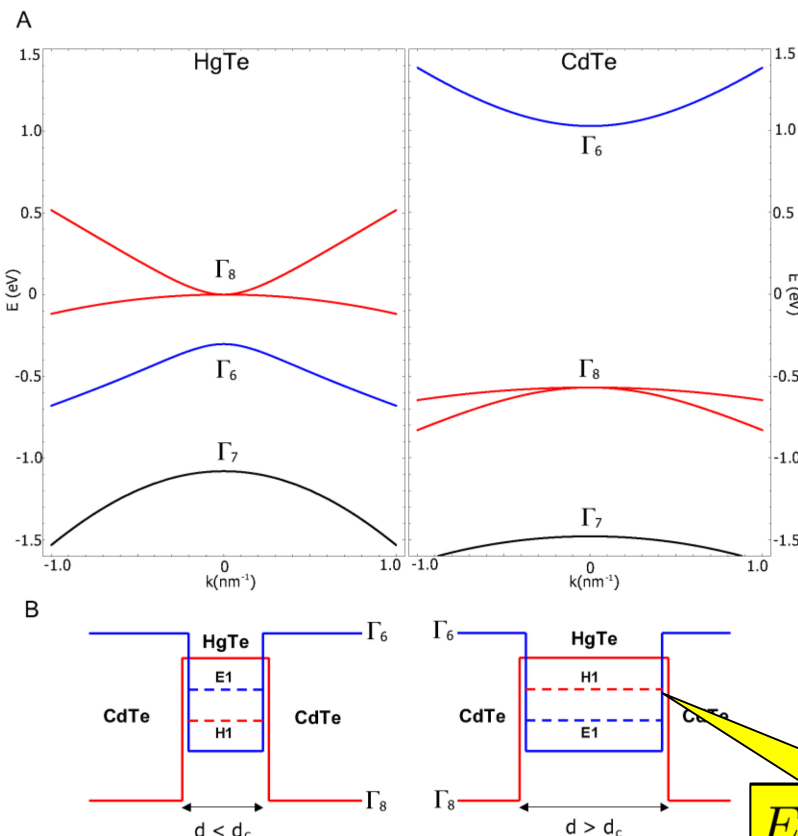
[Kane & Mele '05; Bernevig, Hughes & Zhang '06]

- **Time-reversal symmetry ► No 1-particle backscattering ► $G_0 = e^2/h$ per edge**

Proposed Systems

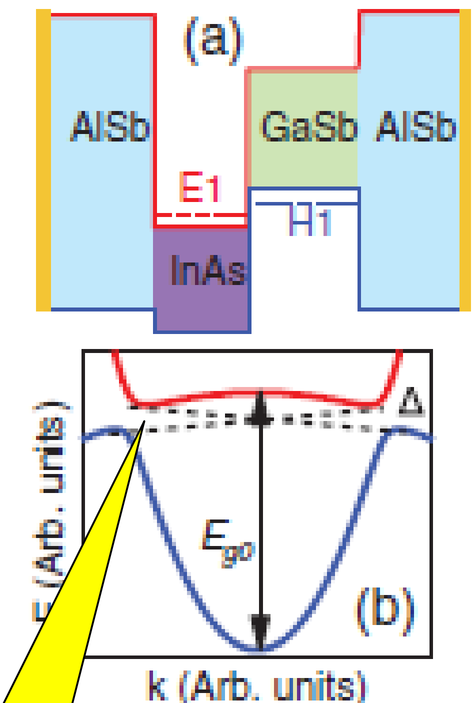
- Realizable in semiconductors with **inverted bands** due to **strong spin-orbit coupling**

Type I quantum well: HgTe



[Bernevig, Hughes & Zhang ‘06]

Type II quantum well: InAs/GaSb



$E_G \sim 10\text{meV}$

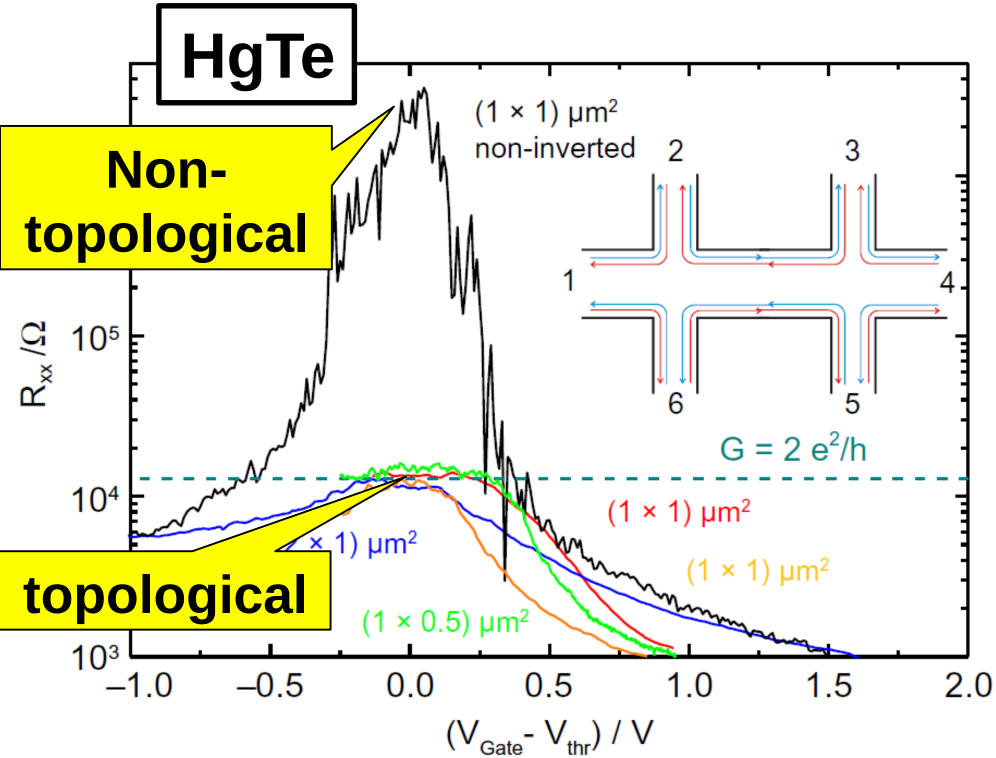
[Liu, Hughes, Qi, Wang, & Zhang ‘08]

- **2D materials**: Bismuth bilayers, **TMD**s

Transport Experiments (I)

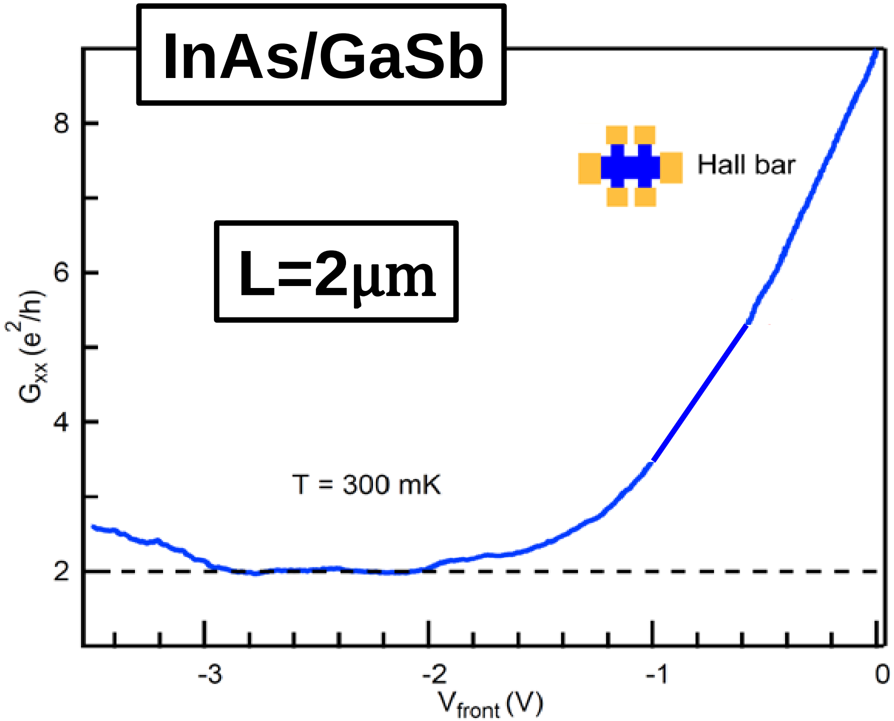
- Semiconductor **quantum wells**:

Type I quantum well



[Würzburg: König *et al.* ’07; Novosibirsk; Stanford; Harvard]

Type II quantum well

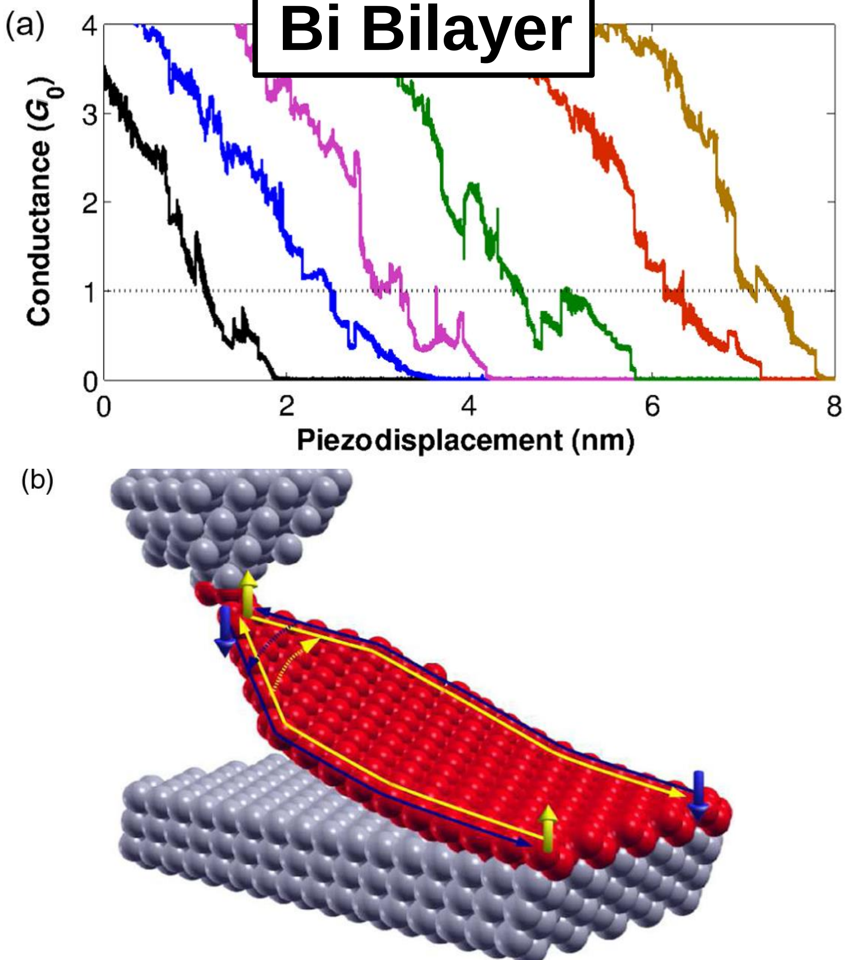


[Rice: Du *et al.* ’13; NTT; ETH]

Transport Experiments (II)

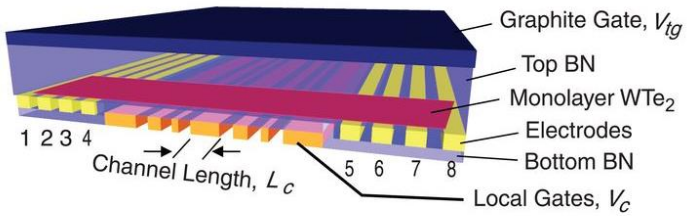
- **2D materials**:

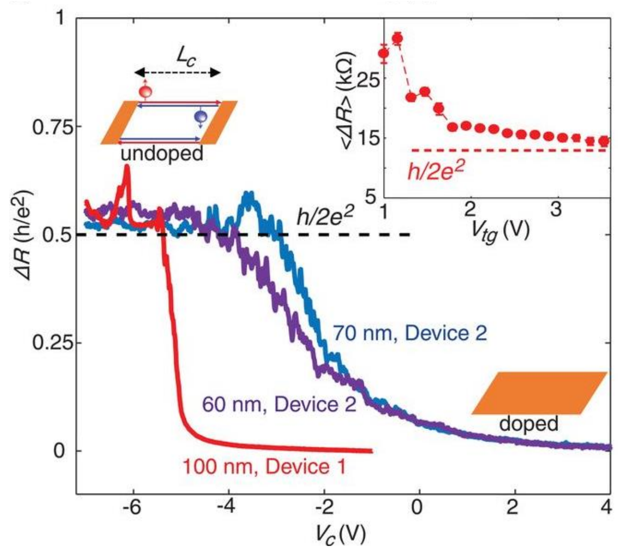
Bi Bilayer



[Alicante ‘13]

1T’-WTe$_2$



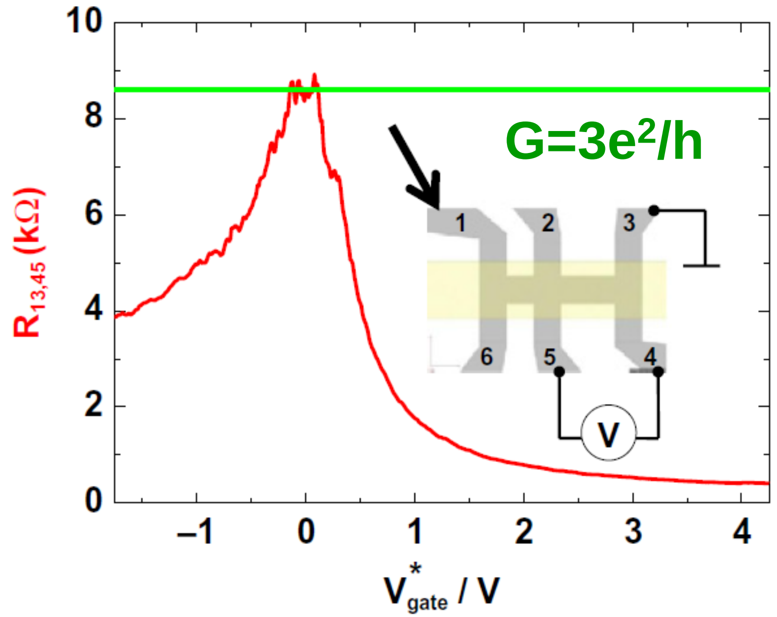


[MIT ‘18;

Seattle ‘17, Nanjing ‘17]

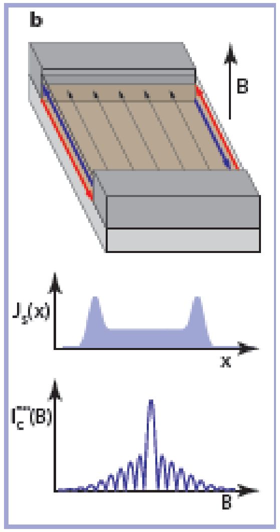
Evidence for Edges (I)

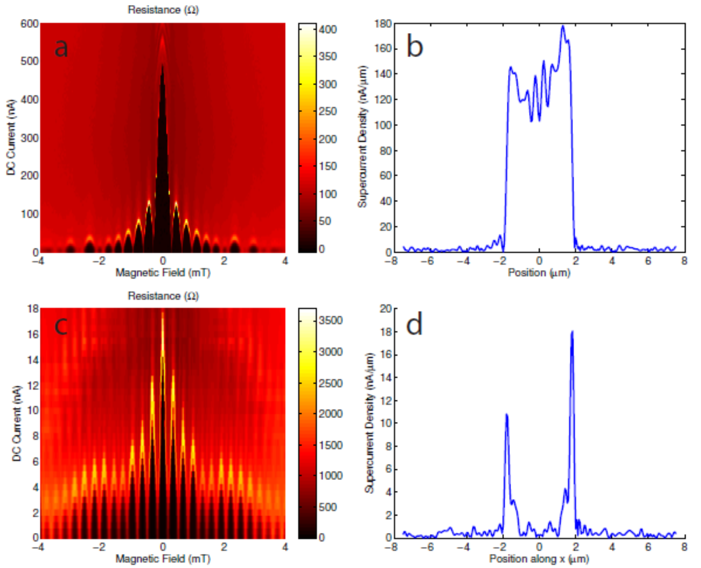
- **No** dependence on sample **width**
- **Nonlocal** transport: **Landauer-Büttiker**



[Würzburg]

- **Josephson junctions**: **Fraunhofer patterns**

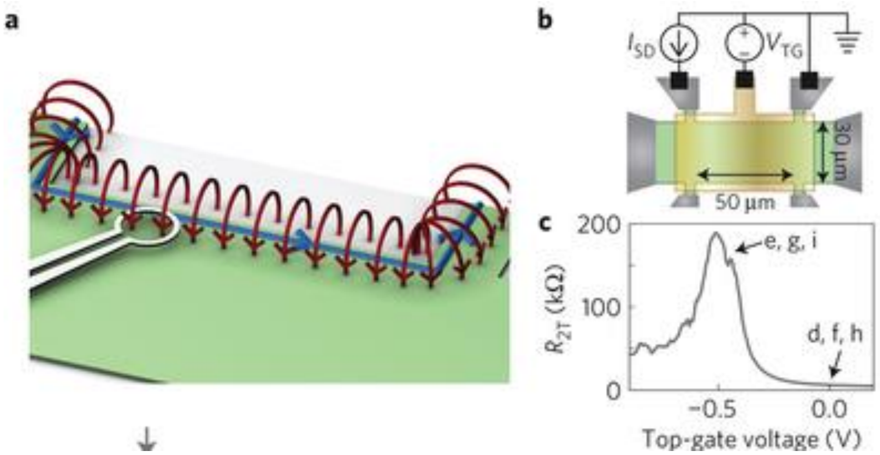


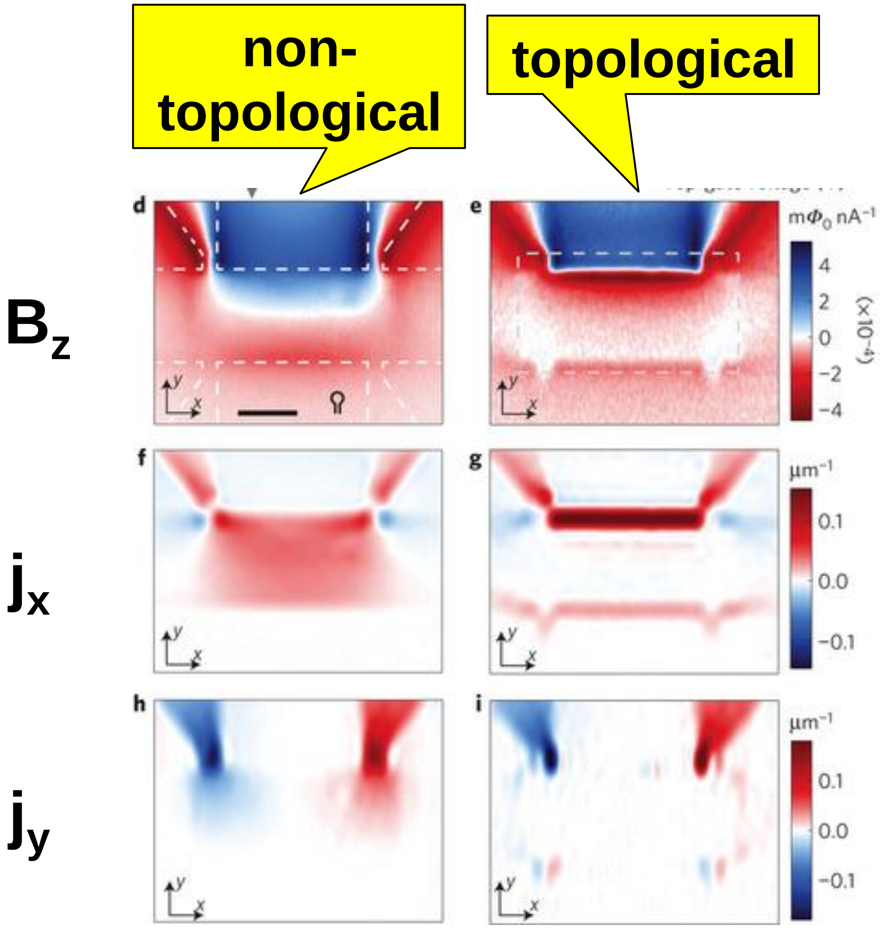


[Harvard]

Evidence for Edges (II)

- **Direct** measurement:
  Scanning SQUID

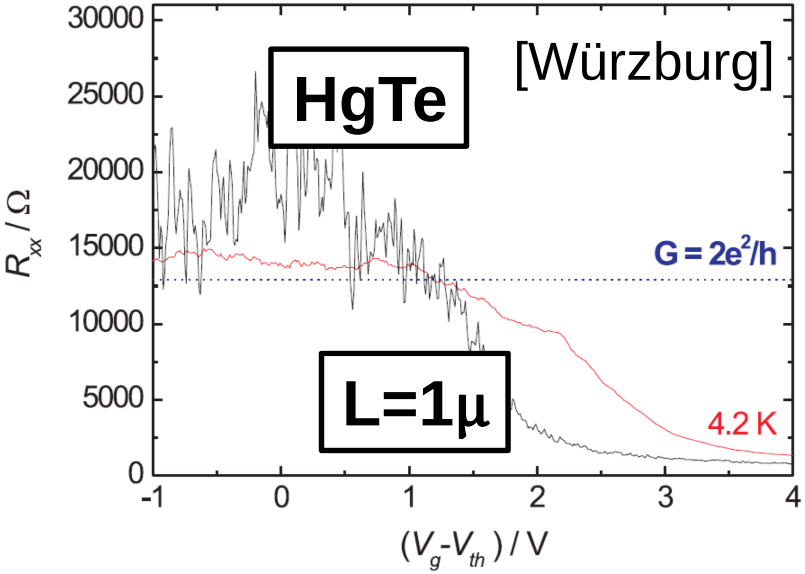




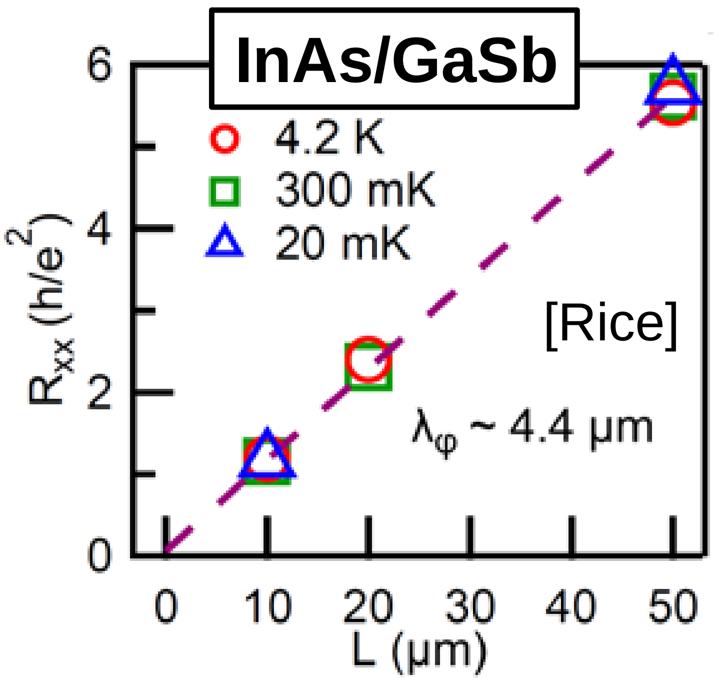
[Moler group]

But …

- **Short** samples (**L=0.5-2μm**):
  - **Strong low T fluctuations**



- **Long** samples (**L>10μm**):
  - **Resistive** behavior
  - **Weak T** and **gate dependence**



Point Defects?

- **Elastic** backscattering ▶ **forbidden** $\cancel{\psi_L^\dagger(x_0)\psi_R(x_0)}$
- **Inelastic** backscattering ▶ **strong T** dependence
  - **Luttinger liquid** effects **typically insignificant**

$$\psi_L^\dagger(x_0)\partial_x\psi_L^\dagger(x_0)\psi_R(x_0)\partial_x\psi_R(x_0) \Rightarrow \Delta G \sim (T/E_G)^6$$

$\sim T^2$

$$\psi_L^\dagger(x_0)\psi_R^\dagger(x_0)\psi_R(x_0)\partial_x\psi_R(x_0) \Rightarrow \Delta G \sim (T/E_G)^4$$

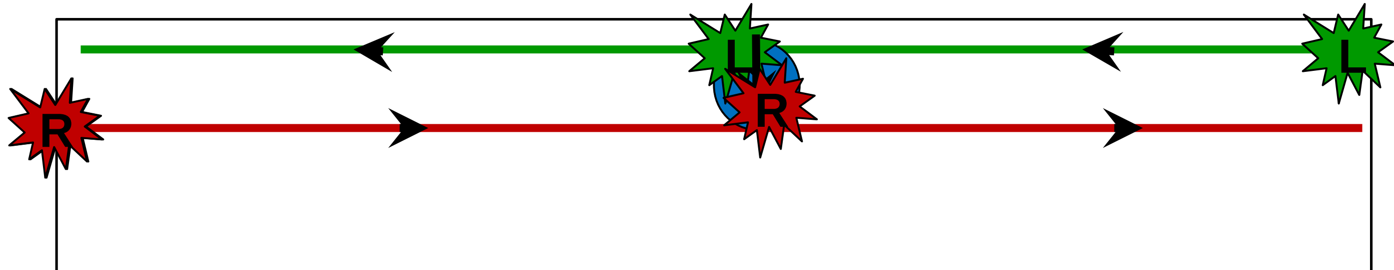
[Kane & Mele '05]

[Schmidt *et al.* '12; Lezmy *et al.* '12]

- **Magnetic impurities** (**1 level Anderson**) ▶ **cannot** backscatter

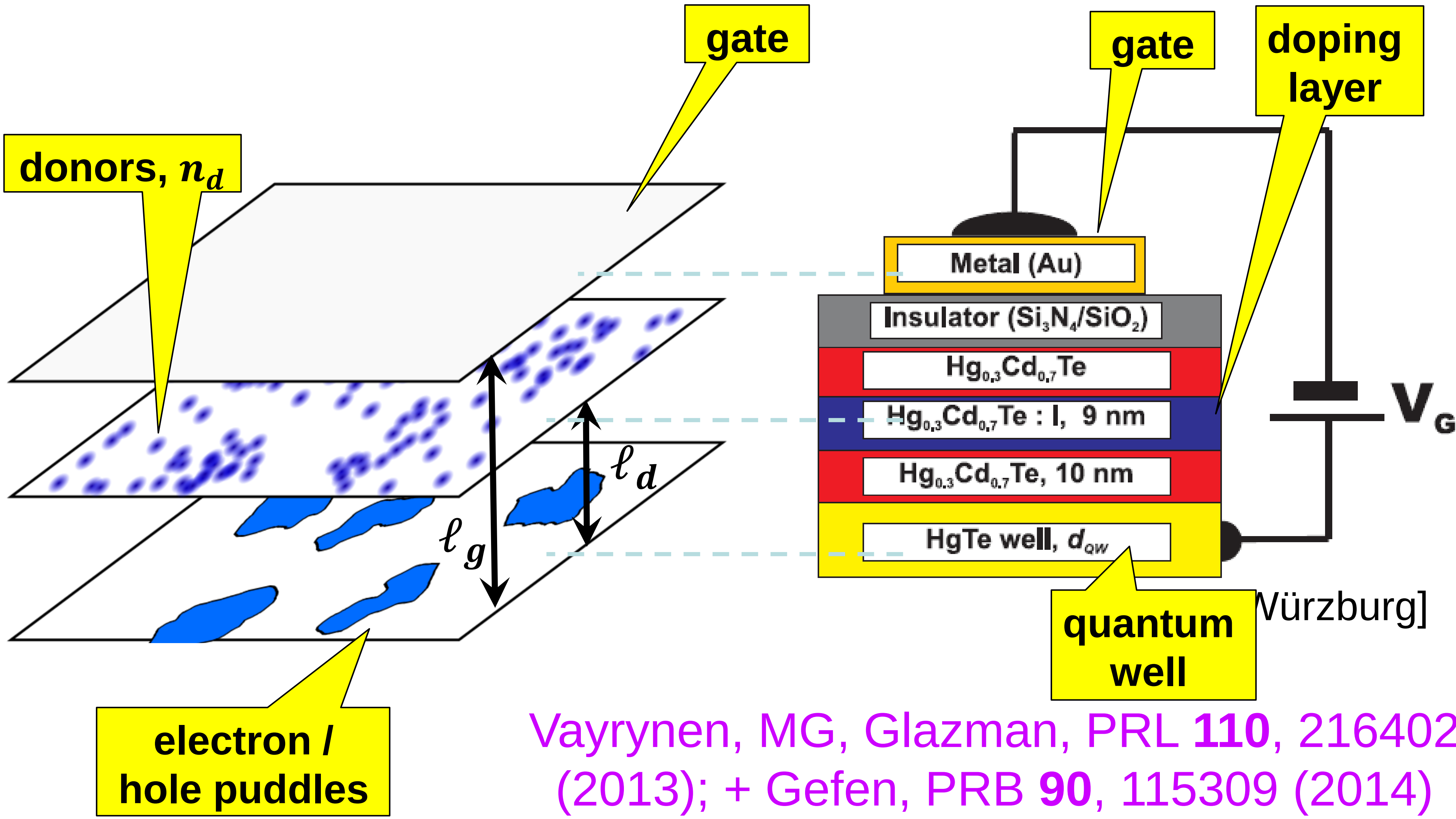
$$\frac{1}{2}J_{xy}S_+\psi_L^\dagger(x_0)\psi_R(x_0) + \text{h.c.}$$

[Maciejko *et al.* '09; Tanaka, *et al.* '11; **many impurities**: Altshuler, Aleiner, Yudson '13]



Extended Defects: Puddles

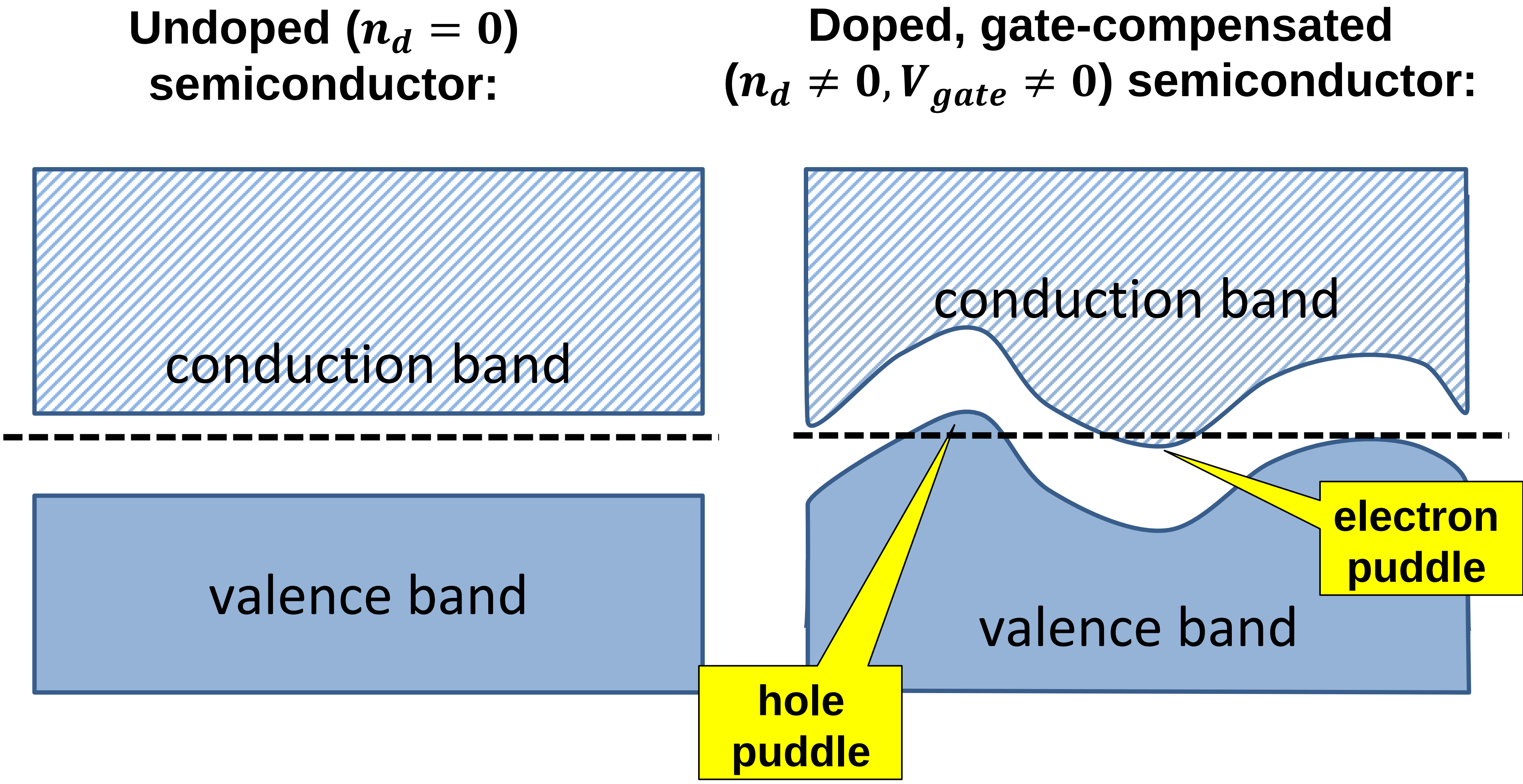
- **HgTe heterostructure** (similarly for **InAs/GaSb**):



Vayrynen, MG, Glazman, PRL **110**, 216402 (2013); + Gefen, PRB **90**, 115309 (2014)

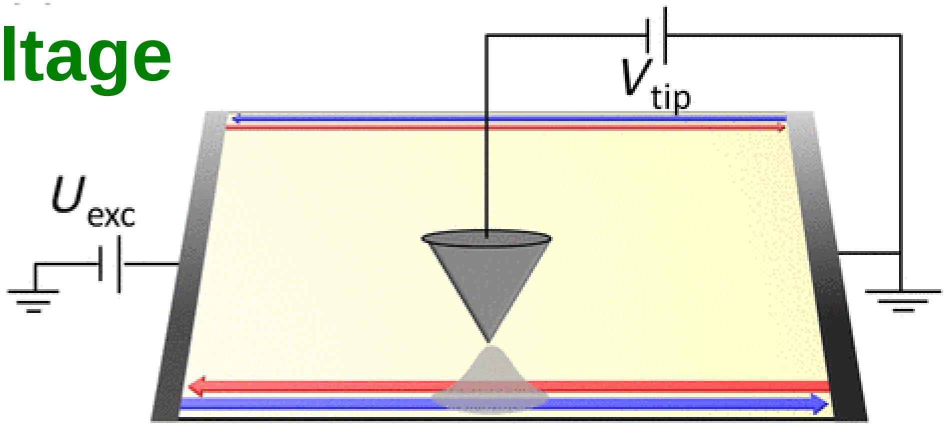
Puddles in Semiconductors

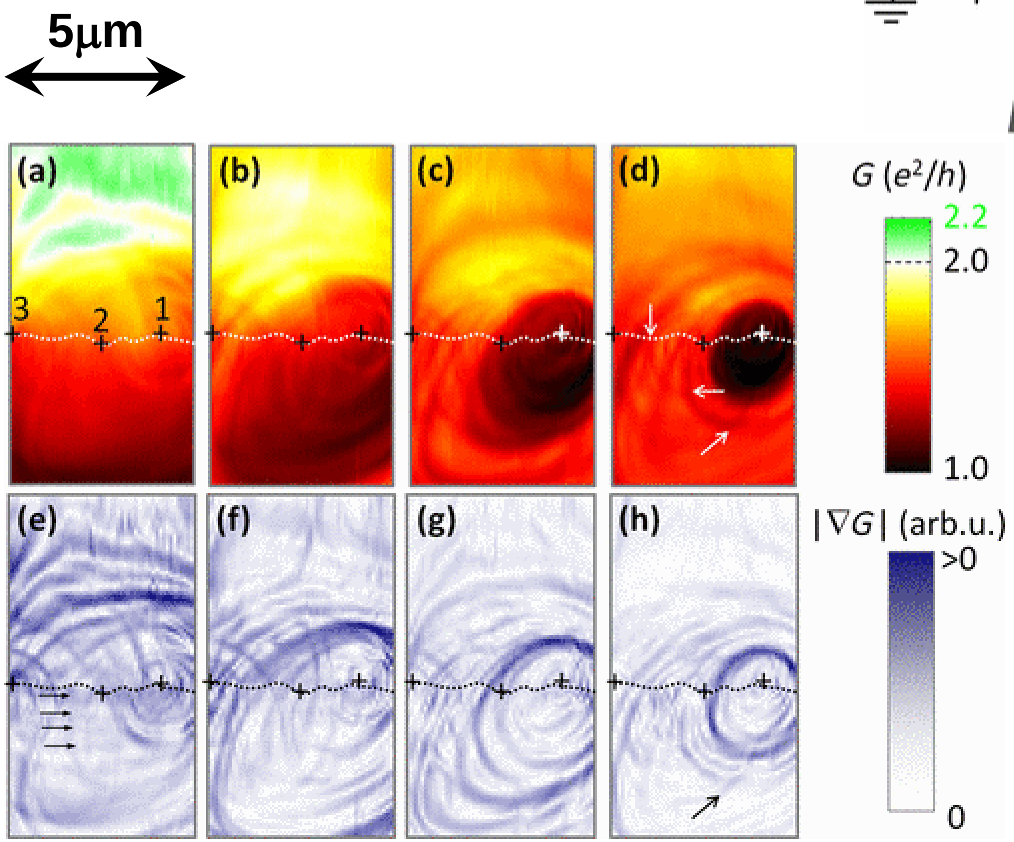
- **Potential fluctuations** + **narrow-band** semiconductor



Evidence for Puddles

- Applying a **local** **gate voltage**

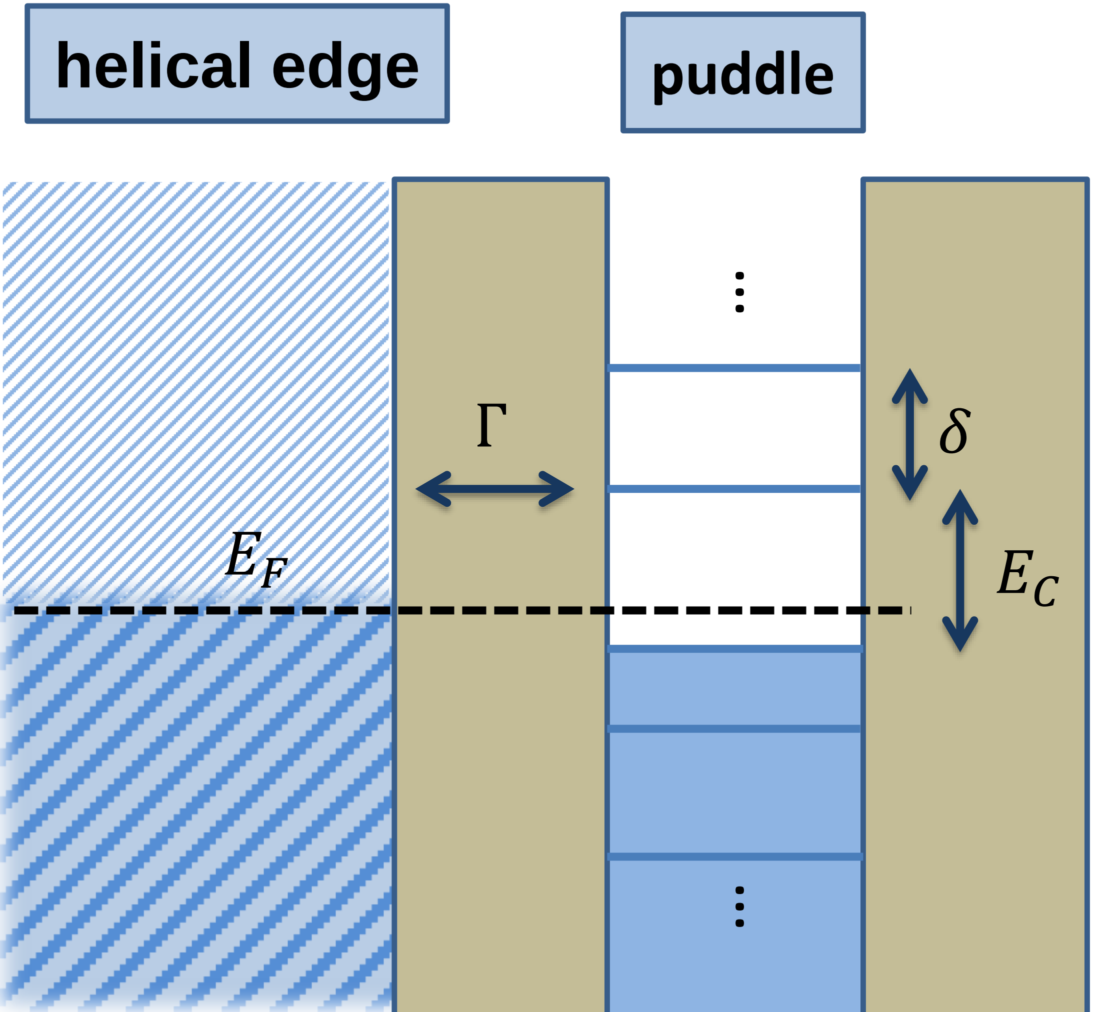


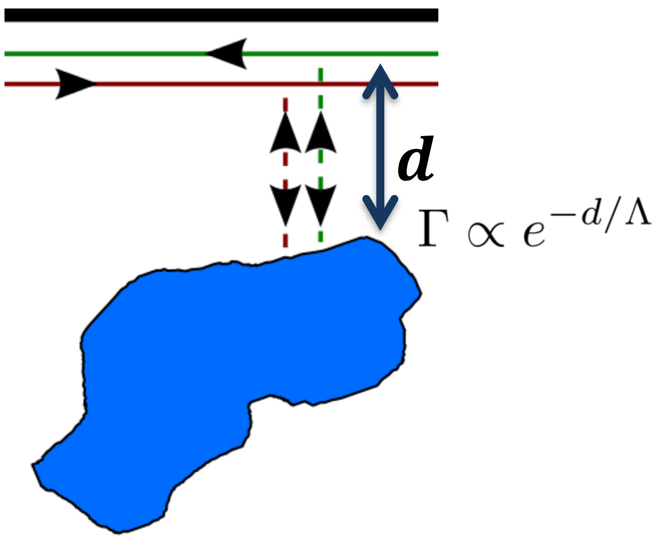


[Goldhaber-Gordon group]

Modeling a Single Puddle

- A **puddle** is a **quantum dot**

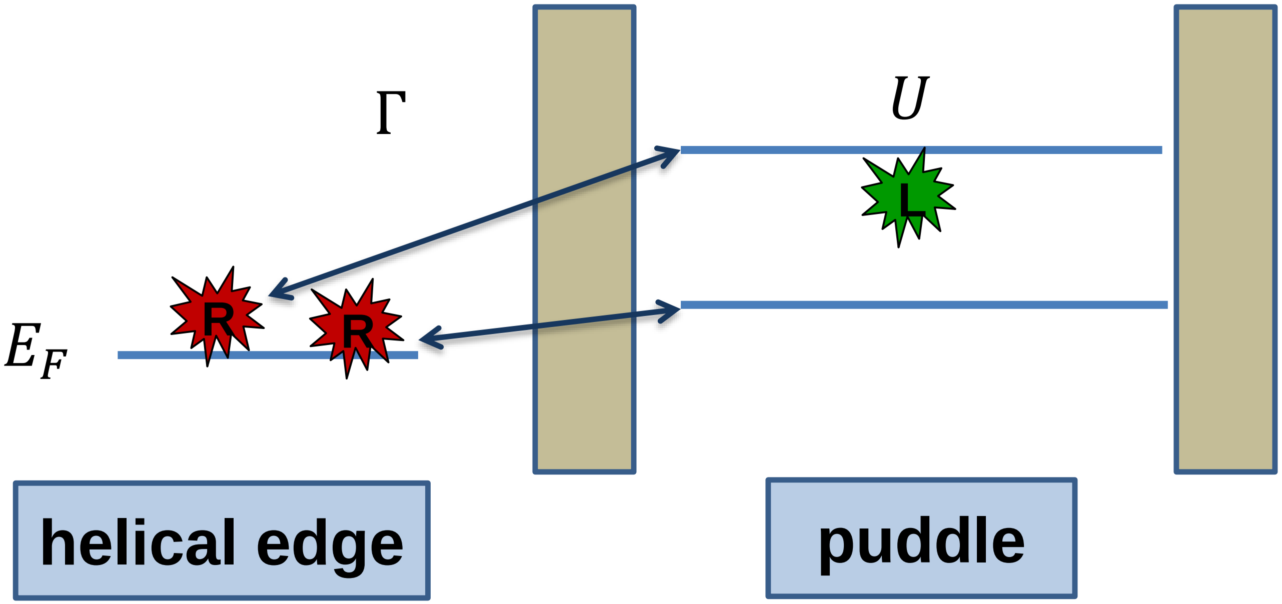




$$\Gamma \ll \delta \lesssim E_C \ll E_G$$

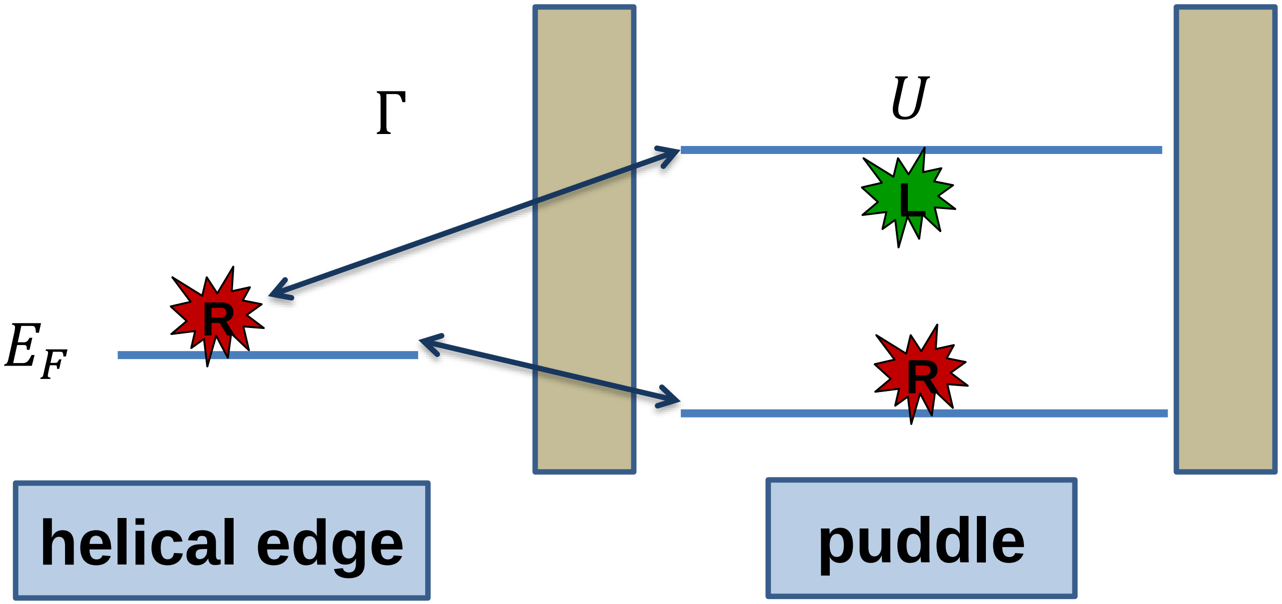
Single Puddle Backscattering

- **Elastic** backscattering: **forbidden**
  - **tunneling**: can be chosen to **conserve R/L**
- **Inelastic** backscattering:
  - **charging**: **conserves R/L**
  - "**non-universal interaction**" (order δ/g) **!**
- **Coulomb blockade**: **Even** vs. **odd valley**



Single Puddle Backscattering

- **Elastic** backscattering: **forbidden**
  - **tunneling**: can be chosen to **conserve R/L**
- **Inelastic** backscattering
  - **charging**: **conserves R/L**
  - "**non-universal interaction**" (order δ/g) **!**
- **Coulomb blockade**: **Even** vs. **odd valley**



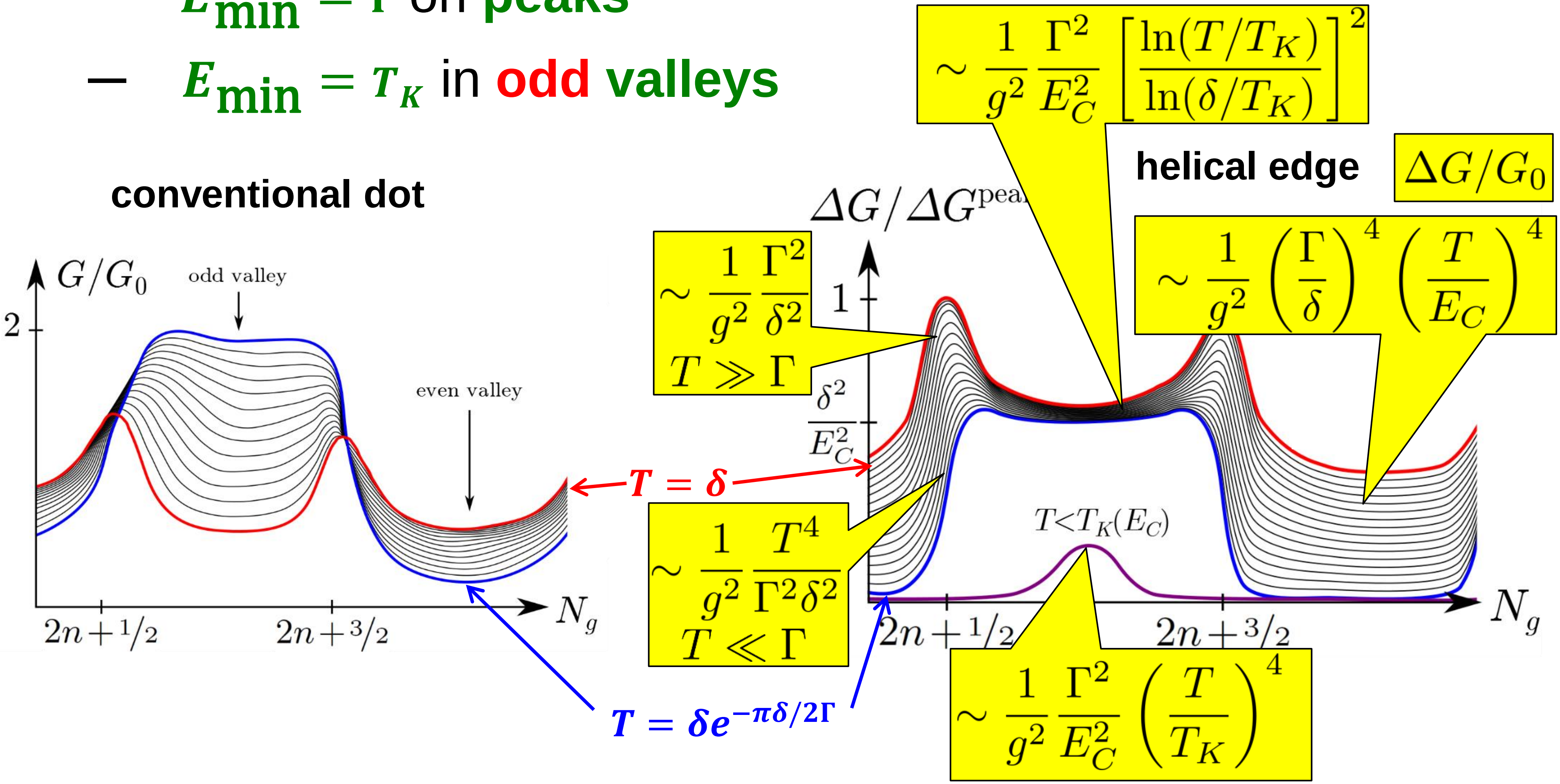
$$H_{\text{eff}} = J\vec{S}^{\text{dot}} \cdot \vec{s}^{\text{edge}} + \sum_{i,j} \Delta J_{ij} S_i^{\text{dot}} s_j^{\text{edge}}$$

$\begin{aligned} J_{xx} = \; & 2 \sum_{n,m<1} \frac{t_m t_n \mathrm{Re}(U_{nL1L;mR1R} + U_{nR1L;mL1R})}{(\varepsilon_m - E_-)(\varepsilon_n - E_-)} \\ & +2 \sum_{n,m>1} \frac{t_n t_m \mathrm{Re}(U_{1LmR;1RnL} + U_{1RmR;1LnL})}{(\varepsilon_m + E_+)(\varepsilon_n + E_+)} \\ & +4 \sum_{\mu} \sum_{n,m<1} \frac{t_1 t_n \mathrm{Re} U_{nLm\mu;1Lm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n - E_-)} \\ & +4 \sum_{\mu} \sum_{n>1} \sum_{m<1} \frac{t_1 t_n \mathrm{Re} U_{nLm\mu;1Lm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n + E_+)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{nL1L;mR1R} + U_{nR1L;mL1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_n - E_-)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{nL1L;mR1R} + U_{nR1L;mL1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_m + E_+)} \end{aligned}$	$\begin{aligned} J_{yy} = \; & 2 \sum_{n,m<1} \frac{t_m t_n \mathrm{Re}(U_{nR1L;mL1R} - U_{nL1L;mR1R})}{(\varepsilon_m - E_-)(\varepsilon_n - E_-)} \\ & +2 \sum_{n,m>1} \frac{t_n t_m \mathrm{Re}(U_{1LmR;1RnL} - U_{1RmR;1LnL})}{(\varepsilon_m + E_+)(\varepsilon_n + E_+)} \\ & +4 \sum_{\mu} \sum_{n,m<1} \frac{t_1 t_n \mathrm{Re} U_{nLm\mu;1Lm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n - E_-)} \\ & +4 \sum_{\mu} \sum_{n>1} \sum_{m<1} \frac{t_n t_1 \mathrm{Re} U_{nLm\mu;1Lm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n + E_+)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{nR1L;mL1R} - U_{nL1L;mR1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_n - E_-)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{nR1L;mL1R} - U_{nL1L;mR1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_m + E_+)} \end{aligned}$
$\begin{aligned} J_{zz} = \; & 2 \sum_{m \neq n<1} \sum_{n<1} \frac{t_m t_n \mathrm{Re}(U_{nL1L;mL1L} - U_{nR1L;mR1L})}{(\varepsilon_m - E_-)(\varepsilon_n - E_-)} \\ & +2 \sum_{m \neq n>1} \sum_{n>1} \frac{t_n t_m \mathrm{Re}(U_{mL1L;nL1L} - U_{mR1L;nR1L})}{(\varepsilon_m + E_+)(\varepsilon_n + E_+)} \\ & -4 \sum_{\mu} \sum_{n,m<1} \frac{t_1 t_n \mathrm{Re} U_{nRm\mu;m\mu1R}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n - E_-)} \\ & -4 \sum_{\mu} \sum_{n>1} \sum_{m<1} \frac{t_1 t_n \mathrm{Re} U_{nRm\mu;m\mu1R}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n + E_+)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{nL1L;mL1L} - U_{nL1R;mL1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_n - E_-)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{nL1L;mL1L} - U_{nL1R;mL1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_m + E_+)} \end{aligned}$	$\begin{aligned} J_{yx} = \; & -4 \sum_{n,m<1} \frac{t_m t_n \mathrm{Im} U_{nL1L;mR1R}}{(\varepsilon_m - E_-)(\varepsilon_n - E_-)} \\ & -4 \sum_{n,m>1} \frac{t_m t_n \mathrm{Im} U_{nL1L;mR1R}}{(\varepsilon_m + E_+)(\varepsilon_n + E_+)} \\ & +8 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Im} U_{nL1L;mR1R}}{(\varepsilon_n - \varepsilon_m)(\varepsilon_n - E_-)} \\ & +8 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Im} U_{nL1L;mR1R}}{(\varepsilon_n - \varepsilon_m)(\varepsilon_m + E_+)} \end{aligned}$
$\begin{aligned} J_{zx} = \; & 4 \sum_{n,m<1} \frac{t_n t_m \mathrm{Re}(U_{nL1L;mR1L} + U_{nL1L;mL1R})}{(\varepsilon_m - E_-)(\varepsilon_n - E_-)} \\ & +4 \sum_{n,m>1} \frac{t_n t_m \mathrm{Re}(U_{nL1L;mR1L} + U_{nL1L;mL1R})}{(\varepsilon_m + E_+)(\varepsilon_n + E_+)} \\ & -4 \sum_{\mu} \sum_{n,m<1} \frac{t_1 t_n \mathrm{Re} U_{nLm\mu;1Rm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n - E_-)} \\ & -4 \sum_{\mu} \sum_{n>1} \sum_{m<1} \frac{t_1 t_n \mathrm{Re} U_{nLm\mu;1Rm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n + E_+)} \\ & +4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{mR1R;nL1R} + U_{mL1R;nR1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_n - E_-)} \\ & +4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Re}(U_{mR1R;nL1R} + U_{mL1R;nR1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_m + E_+)} \end{aligned}$	$\begin{aligned} J_{zy} = \; & -4 \sum_{n,m<1} \frac{t_m t_n \mathrm{Im}(U_{nL1L;mR1L} + U_{nL1L;mL1R})}{(\varepsilon_m - E_-)(\varepsilon_n - E_-)} \\ & -4 \sum_{n,m>1} \frac{t_m t_n \mathrm{Im}(U_{nL1L;mR1L} + U_{nL1L;mL1R})}{(\varepsilon_m + E_+)(\varepsilon_n + E_+)} \\ & +4 \sum_{\mu} \sum_{n,m<1} \frac{t_1 t_n \mathrm{Im} U_{nLm\mu;1Rm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n - E_-)} \\ & +4 \sum_{\mu} \sum_{n>1} \sum_{m<1} \frac{t_1 t_n \mathrm{Im} U_{nLm\mu;1Rm\mu}}{(\varepsilon_n - \varepsilon_1)(\varepsilon_n + E_+)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Im}(U_{mR1R;nL1R} + U_{nR1R;mL1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_n - E_-)} \\ & -4 \sum_{n<1} \sum_{m>1} \frac{t_m t_n \mathrm{Im}(U_{mR1R;nL1R} + U_{nR1R;mL1R})}{(\varepsilon_n - \varepsilon_m)(\varepsilon_m + E_+)} \end{aligned}$

TABLE II. Components of the exchange tensor $\mathbf{J}$. The components are obtained by matching terms in Eqs. (4.7) and (4.8).

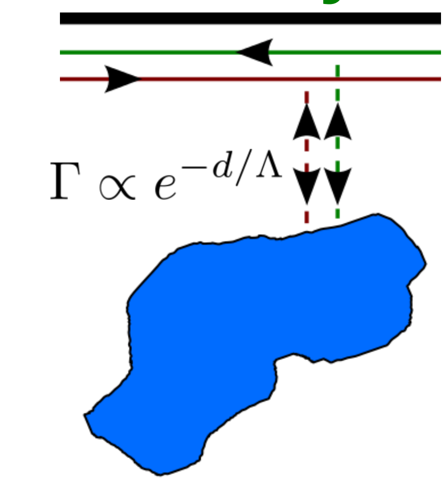
Single Puddle Conductance

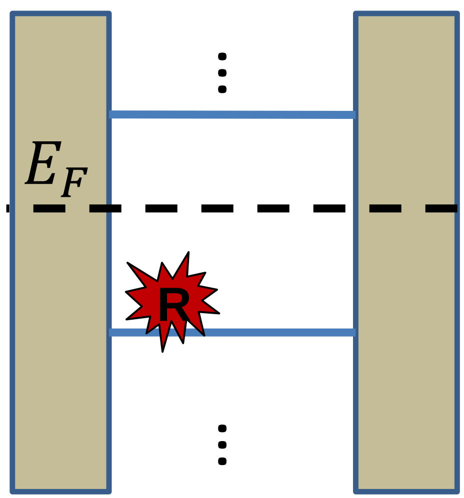
- **Low-T** behavior $\sim (T/E_{\min})^4$ **only** for $T \ll E_{\min} \ll E_G$
  - $E_{\min} = \delta$ in **even valleys**
  - $E_{\min} = \Gamma$ on **peaks**
  - $E_{\min} = T_K$ in **odd valleys**



Long Edge

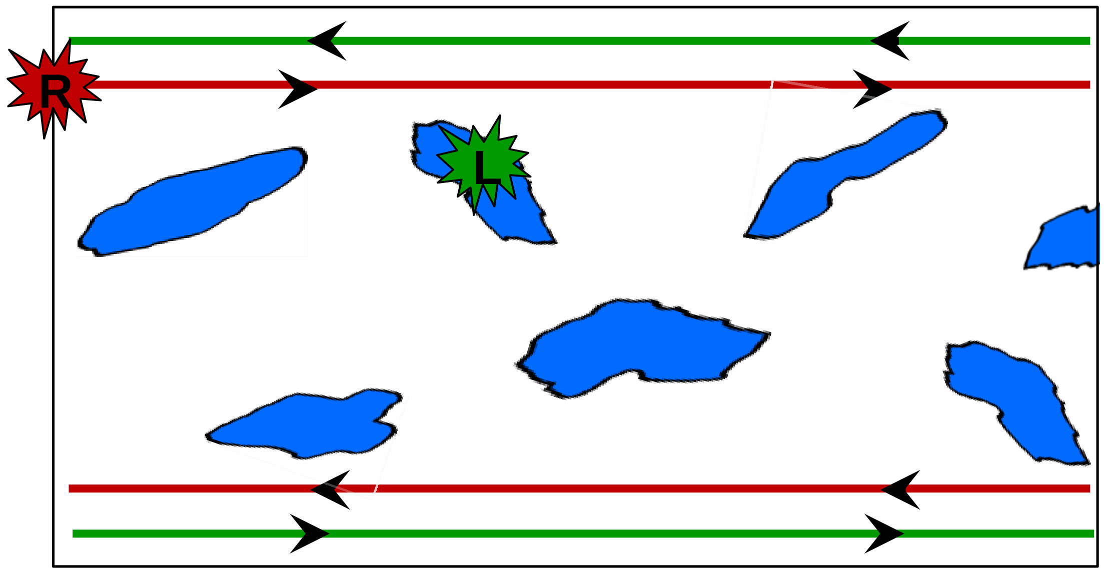
- **Sum** over puddles:
- **Sum** over **gate voltage dominated** by **odd valleys**
- **Sum** over Γ:
  - **nearby ? ▶ $T \ll T_K$ ▶** $\sim \frac{1}{g^2}\frac{\Gamma^2}{E_C^2}\left(\frac{T}{T_K}\right)^4$
    weak backscattering
  - **far away ? ▶ weak backscattering**
    $$\sim \frac{1}{g^2}\frac{\Gamma^2}{E_C^2}\left[\frac{\ln(T/T_K)}{\ln(\delta/T_K)}\right]^2$$
  - **in between ! ▶ $T_K \sim T$**: $\Gamma \sim \delta \ln(\delta/T)$





- **Resistivity**: $\rho_{\text{long}} \sim \frac{n_p \Lambda}{G_0}\frac{1}{g^2}\frac{1}{\ln^2(\delta/T)}$

Charge Puddles



- **Backscattering** still **has to be inelastic !**
- **Long dwell time in puddles ► weak T dependence**
- **Quantum dot** like: **spin ½** with **weak anisotropy**

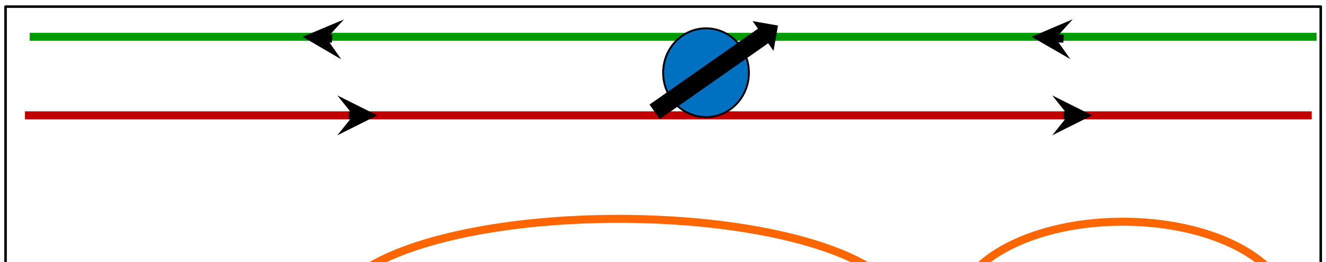
Vayrynen, MG, Glazman, PRL **110**, 216402 (2013);
+ Gefen, PRB **90**, 115309 (2014)

Outline

- **Transport** in **2D topological insulators**
- **Large S impurity**: **low energy**
- **Large S impurity**: **finite energy transport**
- **Large S impurity**: **shot noise**

Large Spin Magnetic Impurity

- **Naturally present**
  - example: **Mn** (S=5/2)



$$H_{\text{imp}} = J_{ij} S_i s_j (x=0) + D_{ij} S_i S_j$$

- **Isotropic bulk exchange ▶ fully anisotropic edge exchange**

$$J_{ij}/v_F \sim 10^{-3}$$

Kimme, Rosenow, Brataas, PRB 2016;
Kurilovich[2] & Burmistrov, PRB 2016

- Local **anisotropic self-exchange**
  - **Very relevant** under **RG**!

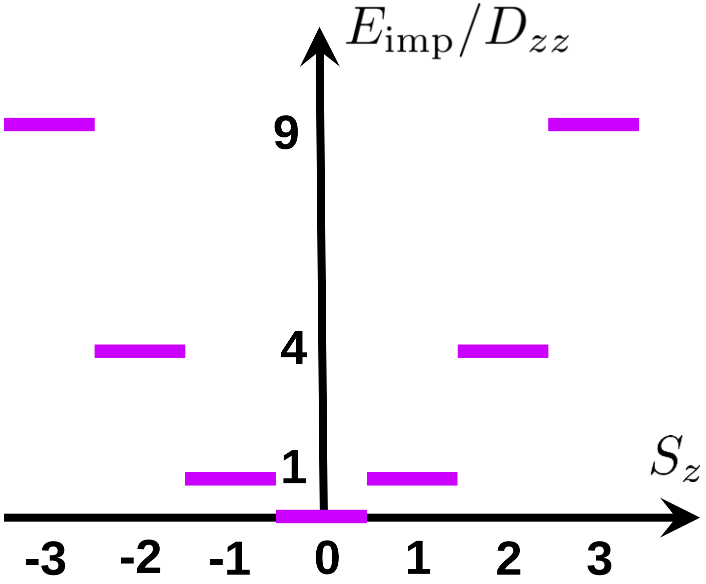
$$D_{ij} \sim \left(\frac{J_{mn}}{v_F}\right)^2 \left(\frac{v_F}{E_G a_{\text{imp}}}\right)^3 E_G$$

$$\sim 0.1\text{K}$$

$$D_{ij} S_i S_j \to D_{zz} S_z^2 + D_{xx} S_x^2, \quad |D_{zz}| > |D_{xx}|$$

Low Energies I

- $D_{zz} > 0$, S **integer**
  – effectively **spin 0**

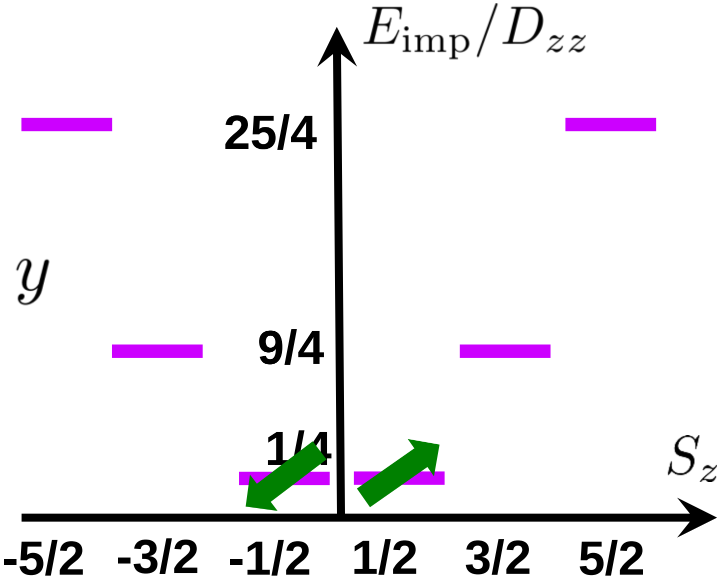


- Spin **self-screened** ► **residual backscattering**

$$V_{RR\to RL} \sim \frac{J^2 S(S+1)}{D_{zz}} \qquad \frac{\Delta G}{G_0} \sim |V_{RR\to RL}|^2 \frac{[\max(T,V)]^4 a_{\text{imp}}^4 k_F^2}{v_F^4}$$

Low Energies II

- $D_{zz} > 0$, S **half-integer**
  - effectively **spin 1/2**



$$J_{ij} \to (S+1/2)J_{ij}, \quad i = x, y$$

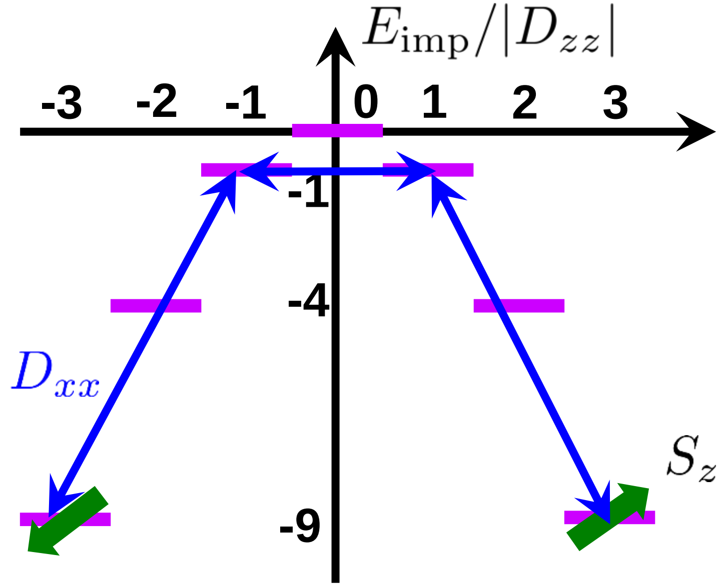
- Spin **Kondo-screened** ► **residual backscattering**

$$\frac{\Delta G}{G_0} \sim \left[\frac{\max(T,V)}{T_K^{(1/2)}}\right]^4$$

Low Energies III

- $D_{zz} < 0$, S **integer**
  - effectively **spin 1/2**
  - very small effective **“magnetic” field**

$$\delta_S \sim \frac{|D_{xx}|^S}{|D_{zz}|^{S-1}}$$



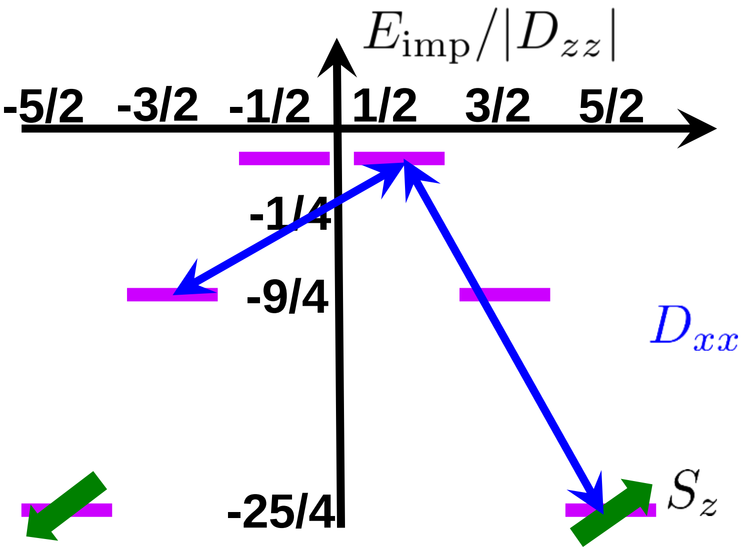
- Spin **self-screened** below δ_S ► **residual backscattering**

$$V_{RR\to RL} \sim \frac{J^2 S(S+1)}{\delta_S} \qquad \frac{\Delta G}{G_0} \sim |V_{RR\to RL}|^2 \frac{[\max(T,V)]^4 a_{\mathrm{imp}}^4 k_F^2}{v_F^4}$$

Low Energies IV

- $D_{zz} < 0$, S **half-integer**
  - effectively **spin 1/2**

$$J_{ij} \to J_{ij} \left| \frac{D_{xx}}{D_{zz}} \right|^{S-1/2}, \quad i = x, y$$



  - **very small** $T_K^{(1/2)}$!

- Spin **Kondo-screened** ► **local backscattering**

$$\frac{\Delta G}{G_0} \sim \left[\frac{\max(T, V)}{T_K^{(1/2)}} \right]^4$$

Outline

- Transport in 2D topological insulators

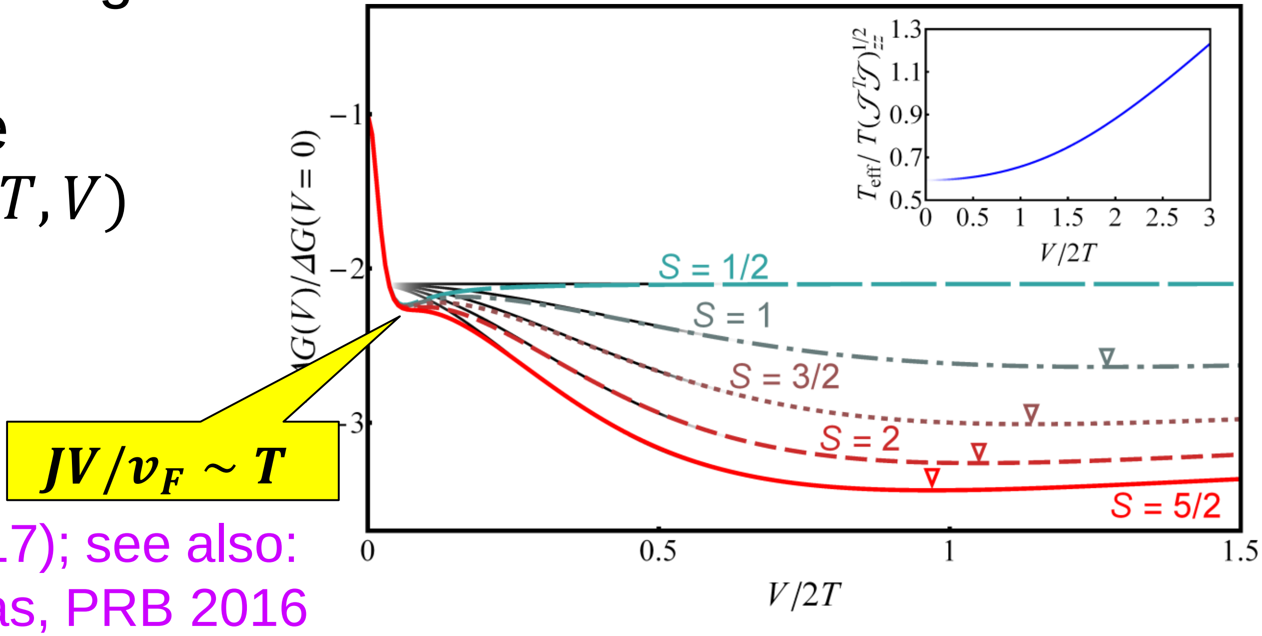
- Large S impurity: low energy

- Large S impurity: finite energy transport

- Large S impurity: shot noise

High Energies

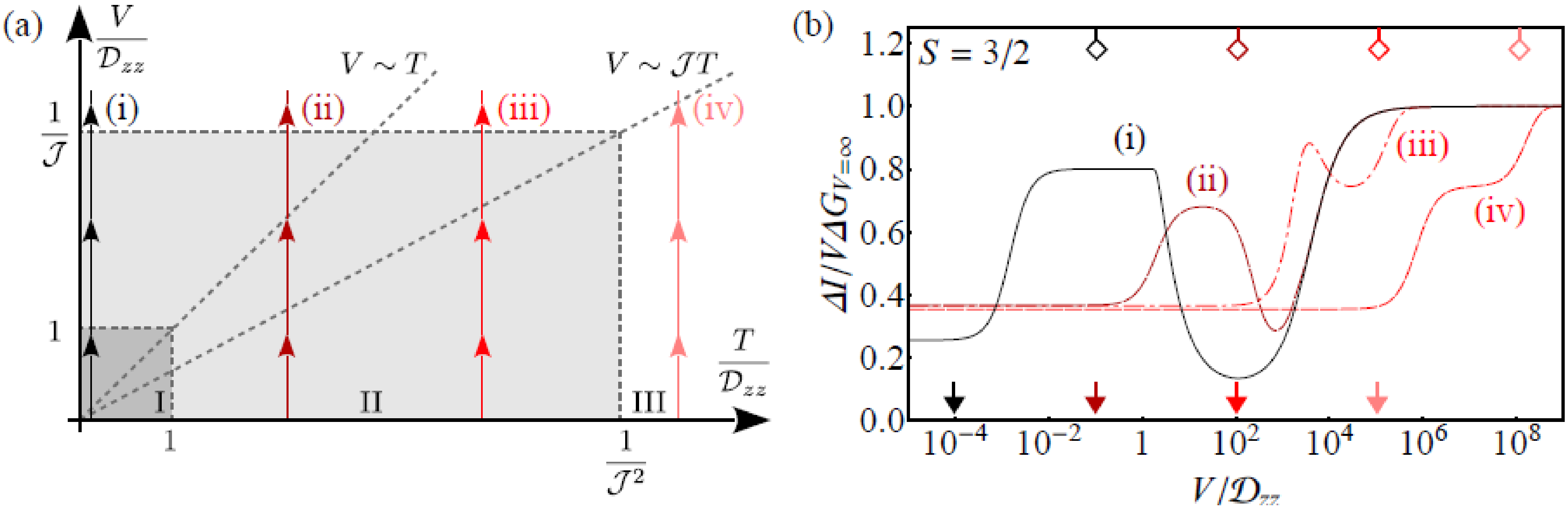
- **Negligible local anisotropy**
  - master equation
  - analytic solution for **spin 1/2 or linear response**
- Several competing scales
  - **T, V**
  - edge-induced magnetic field $\sim JV/v_F$
  - relaxation rate $\sim (J/v_F)^2 \max(T,V)$



JETP Lett. **106**, 575 (2017); see also: Kimme, Rosenow, Brataas, PRB 2016

Intermediate Energies

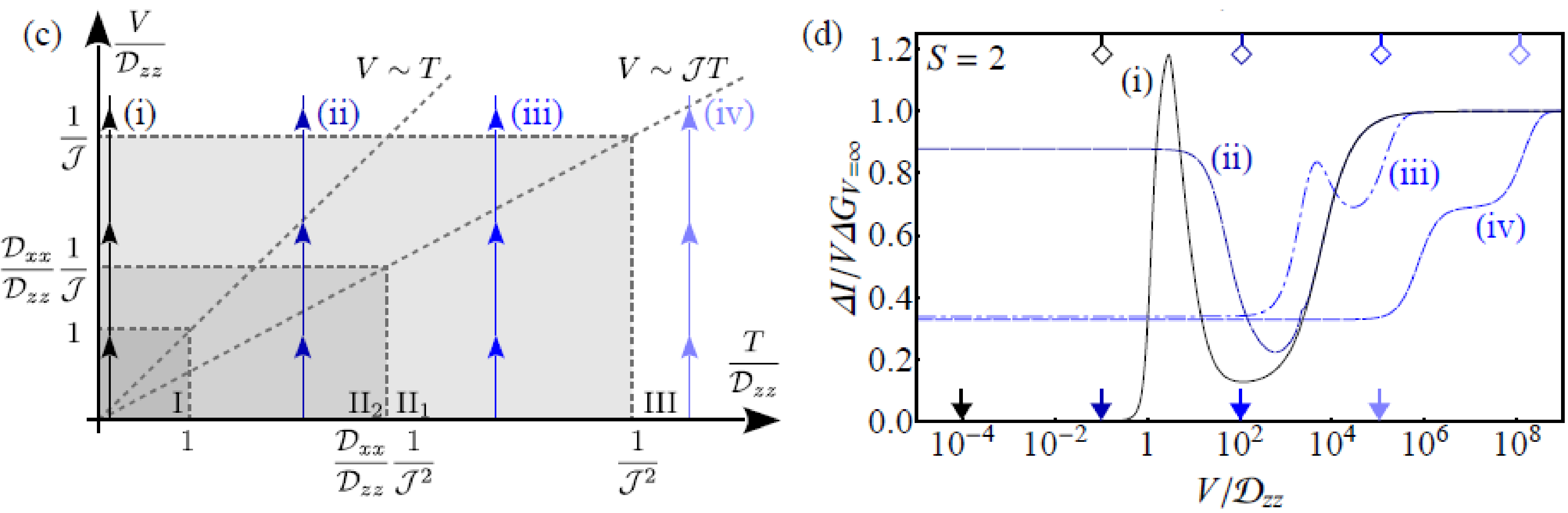
- **Half-integer** S
  - Additional scale: D_{zz} ► **nonmonotonic** behavior



Phys. Rev. B **99**, 085407 (2019)

Intermediate Energies

- **Integer** S
  - Additional scales: D_{zz}, $\delta_{|S_z|} \sim |D_{xx}|^{|S_z|}/|D_{zz}|^{|S_z|-1}$ ► **nonmonotonic** behavior



Phys. Rev. B **99**, 085407 (2019)

Outline

- **Transport** in **2D topological insulators**
- **Large S impurity**: **low energy**
- **Large S impurity**: **finite energy transport**
- **Large S impurity**: **shot noise**

Shot Noise

- Backscattering Fano factor: $\mathrm{F} \equiv \langle \delta I_{bs}^2 \rangle / 2e \langle I_{bs} \rangle$ (large V)
- S=1/2

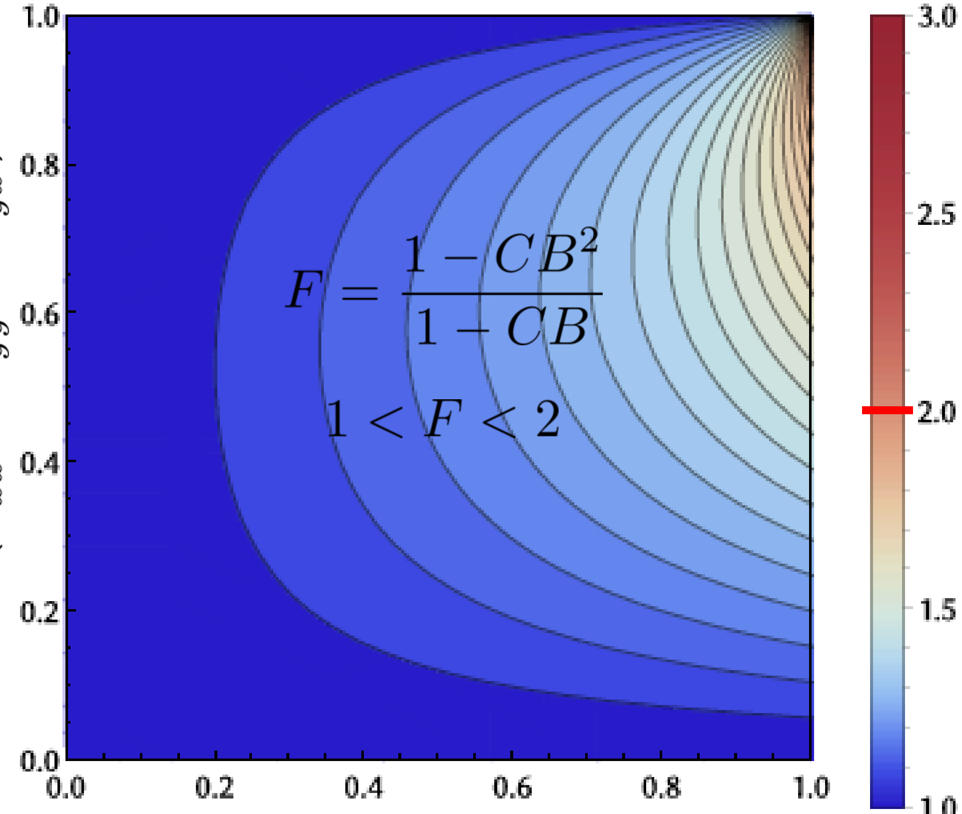
$$H_{\rm imp} = J_{ij} S_i s_j (x=0)$$

2 particle backscattering
$(J_{xx} - J_{yy} + iJ_{yx}) S_+ s_+ + \cdots$

$$J_{ij} = \begin{pmatrix} J_{xx} & 0 & 0 \\ J_{yx} & J_{yy} & 0 \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}$$

1 particle backscattering

$$B = \left(\frac{2 J_{xx} J_{yy}}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2} \right)^2$$



$$C = \frac{J_{xx}^2 + J_{yy}^2 + J_{yx}^2}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2 + J_{zx}^2 + J_{zy}^2}$$

arXiv:1903.03965 [**weak** anisotropy: Vayrynen & Glazman '17]

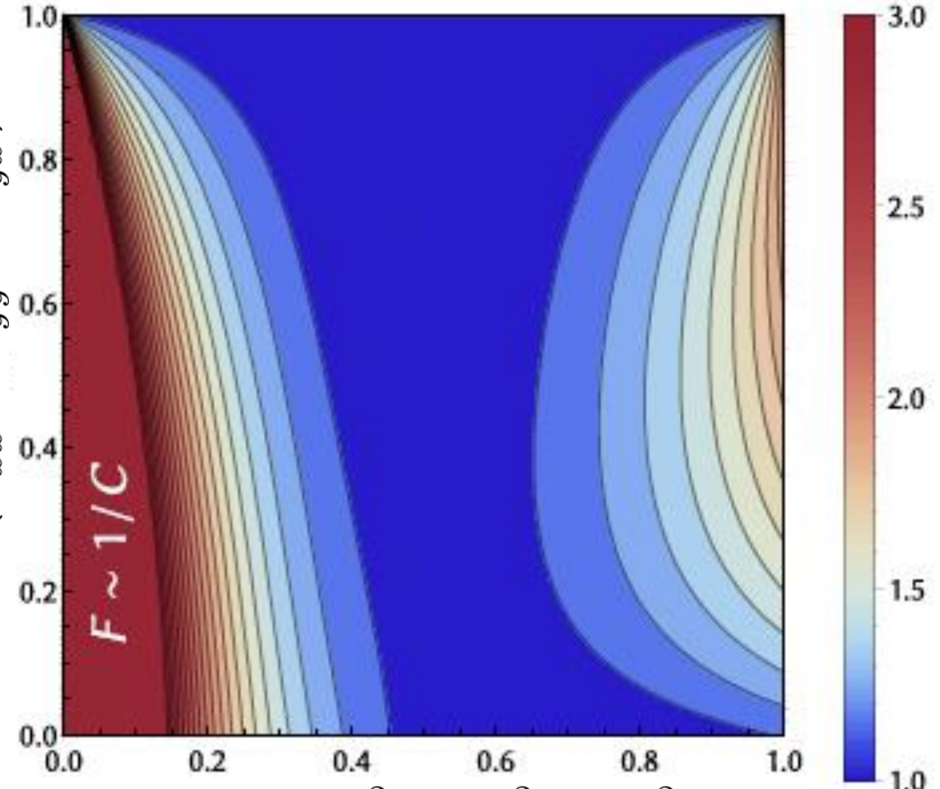
Shot Noise, S>1/2

- Backscattering Fano factor: $\mathrm{F} \equiv \langle \delta I_{bs}^2 \rangle / 2e \langle I_{bs} \rangle$ (large V)
- S=1

$$H_{\text{imp}} = J_{ij} S_i s_j (x=0)$$

$$J_{ij} = \begin{pmatrix} J_{xx} & 0 & 0 \\ J_{yx} & J_{yy} & 0 \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}$$

$$B = \left(\frac{2 J_{xx} J_{yy}}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2} \right)^2$$



$$C = \frac{J_{xx}^2 + J_{yy}^2 + J_{yx}^2}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2 + J_{zx}^2 + J_{zy}^2}$$

- $\boldsymbol{F \gg 2}$

  electron bunching??

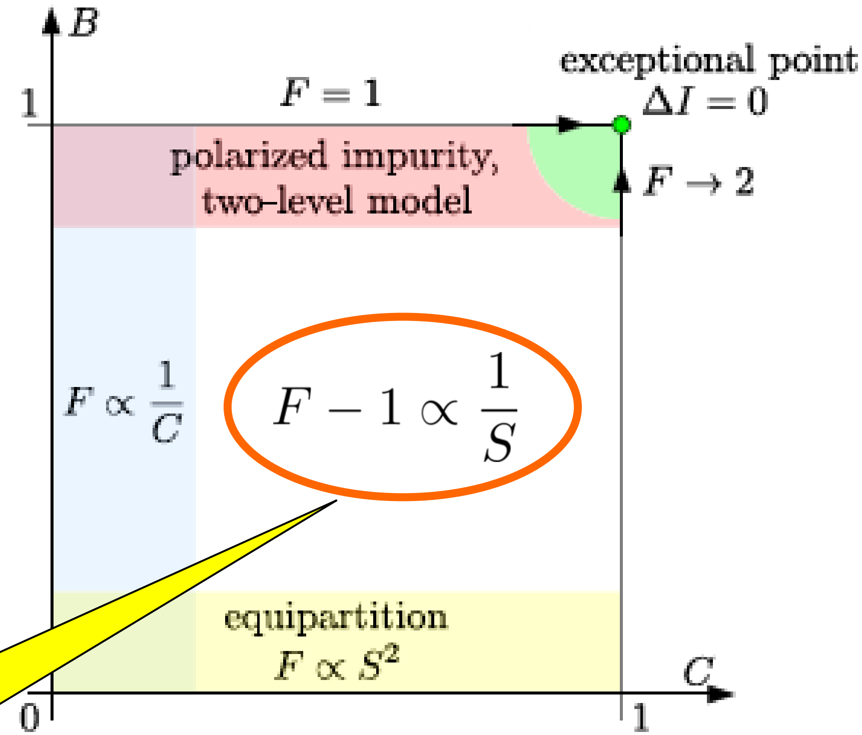
arXiv:1903.03965

Tractable limits

$$H_{\text{imp}} = J_{ij} S_i s_j (x=0)$$

$$J_{ij} = \begin{pmatrix} J_{xx} & 0 & 0 \\ J_{yx} & J_{yy} & 0 \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}$$

$$B = \left(\frac{2 J_{xx} J_{yy}}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2} \right)^2$$



$$C = \frac{J_{xx}^2 + J_{yy}^2 + J_{yx}^2}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2 + J_{zx}^2 + J_{zy}^2}$$

Typically: $S \to \infty$ ▶
1 particle backscattering

Tractable limits

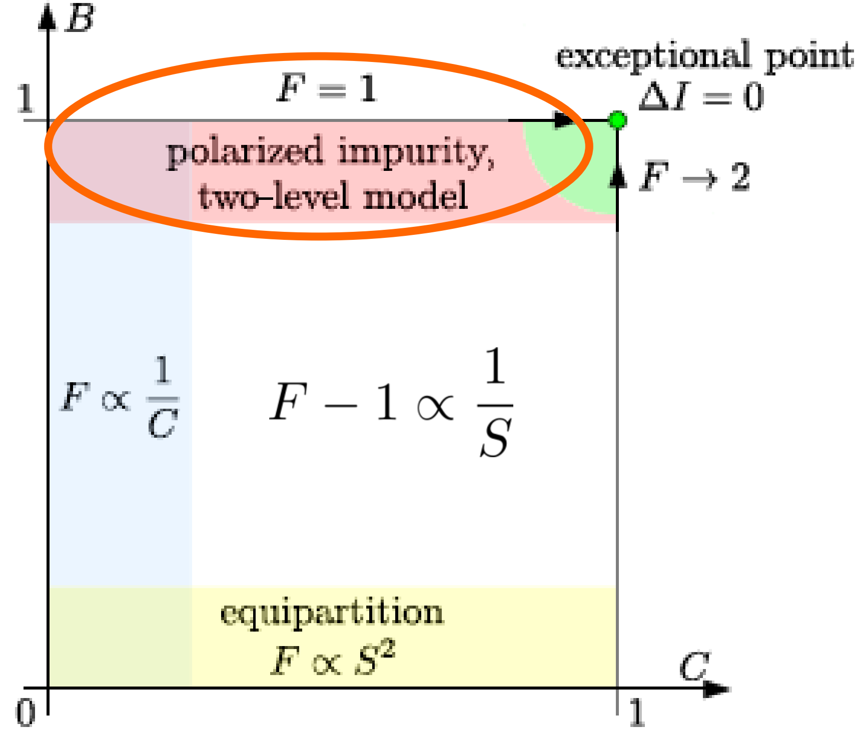
$$H_{\rm imp} = J_{ij} S_i s_j(x=0)$$

$$J_{ij} = \begin{pmatrix} J_{xx} & 0 & 0 \\ J_{yx} & J_{yy} & 0 \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}$$

$$B = \left(\frac{2J_{xx}J_{yy}}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2} \right)^2$$

- **Gibbs** distribution:

$$P(S_z) \propto \theta^{S_z} \qquad \theta = \frac{1+\sqrt{B}}{1-\sqrt{B}} \geq 1$$

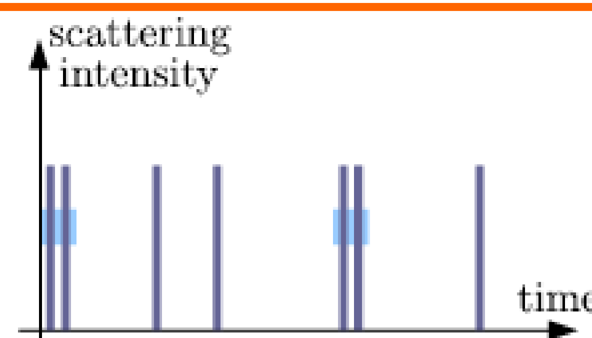


$$C = \frac{J_{xx}^2 + J_{yy}^2 + J_{yx}^2}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2 + J_{zx}^2 + J_{zy}^2}$$

- **B=1** ▶ **F=1**
  - $\theta \to \infty$ ▶ **polarized** impurity
  - $J_{xx} - J_{yy} = J_{yx} = 0$ ▶ 1 particle backscattering

- **1-B<<1**:

$$F \sim 1 + \frac{1-B}{1-C} \frac{(2S_1 - C(3S-1))^2}{2CS^3}$$



Tractable limits

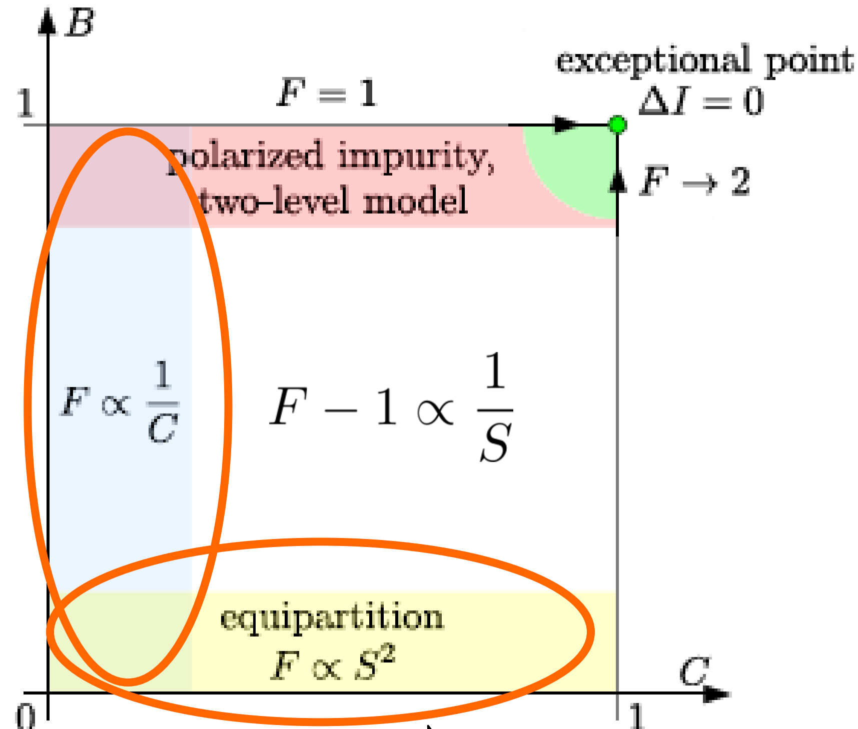
$$H_{\text{imp}} = J_{ij} S_i s_j (x=0)$$

$$J_{ij} = \begin{pmatrix} J_{xx} & 0 & 0 \\ J_{yx} & J_{yy} & 0 \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}$$

- **Gibbs** distribution:

$$P(S_z) \propto \theta^{S_z} \qquad \theta = \frac{1+\sqrt{B}}{1-\sqrt{B}} \geq 1$$

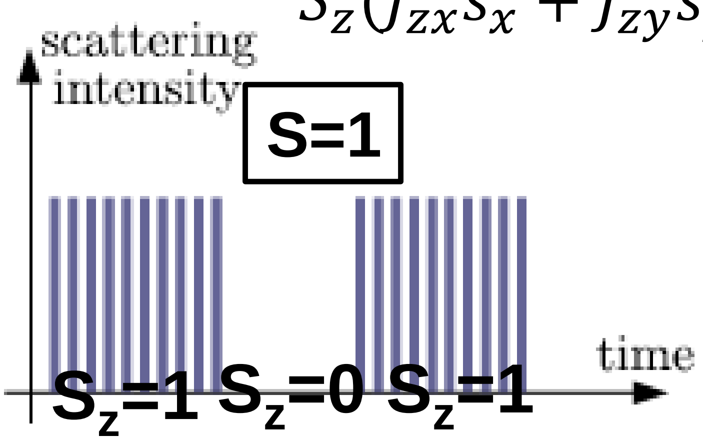
$$B = \left(\frac{2 J_{xx} J_{yy}}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2} \right)^2$$



$$C = \frac{J_{xx}^2 + J_{yy}^2 + J_{yx}^2}{J_{xx}^2 + J_{yy}^2 + J_{yx}^2 + J_{zx}^2 + J_{zy}^2}$$

- **C<<1 ▶ J_{zx}, J_{zy} dominant**

$$S_z (J_{zx} s_x + J_{zy} s_y)$$



$$F \sim 1 + \frac{(2S+1)(2S+3)}{45} \frac{(3C-2)^2}{C}$$

Conclusions

- **Large S** ► **strong backscattering**
- **Low T, V**:
  - Impurity **always screened**
  - **scale can** be **anomalously low** ($D_{zz} < 0$, especially half-integer S)
- **Higher T,V**:
  - **Multiple** competing scales
  - Complex T,V dependence ► **impurity parameters**
- **Shot noise**: $\boldsymbol{F \gg 2}$ – **electron bunching**

JETP Lett. **106**, 575 (2017); Phys. Rev. B **99**, 085407 (2019); arXiv:1903.03965