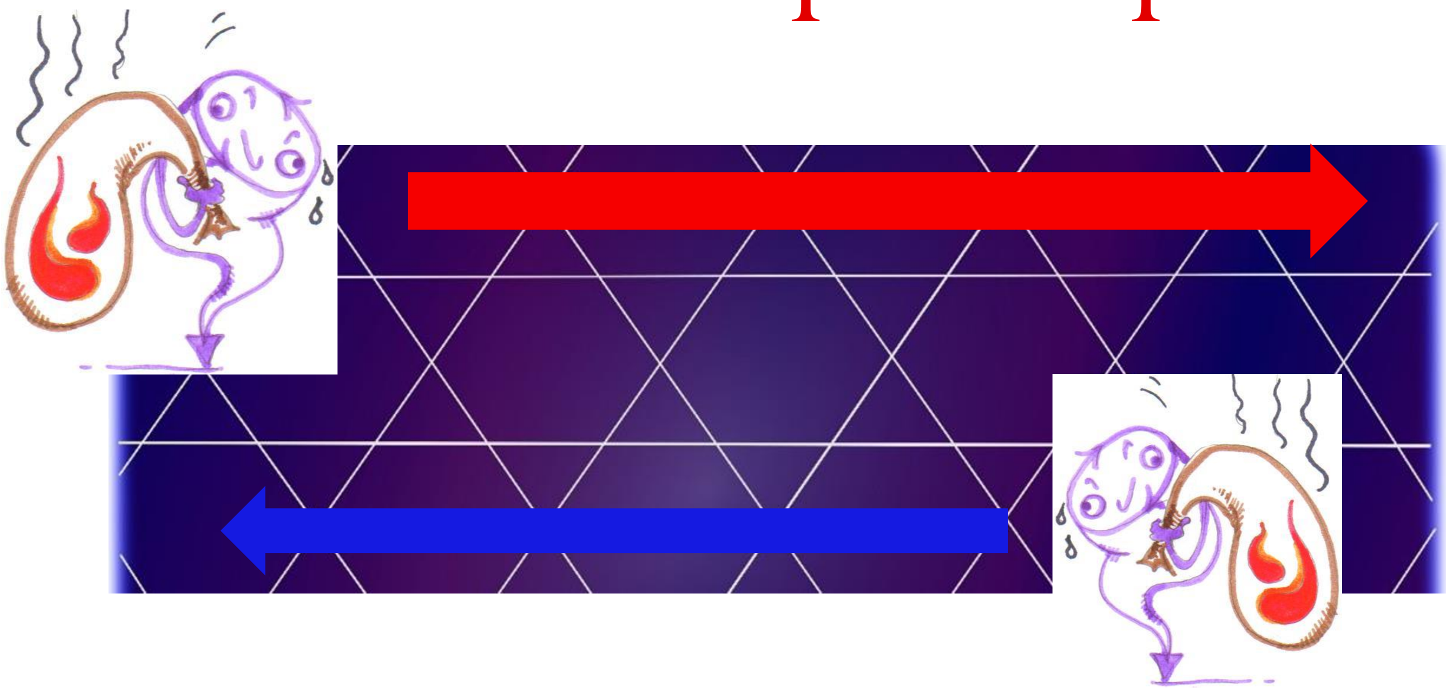
Quantum Thermal Hall Effect in a Chiral Spin Liquid



Pasha Tikhonov and E.S. (BIU)

PRB 99, 174429 (2019)

United States – Israel
Binational Science Foundation

Quantum Thermal Hall Effect in a Chiral Spin Liquid

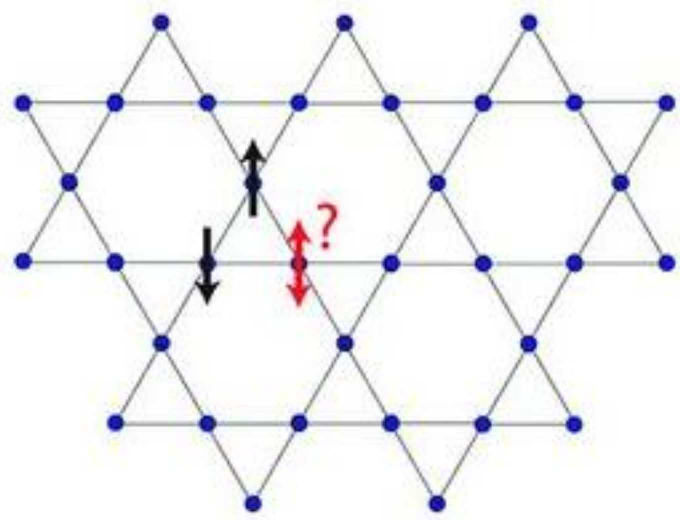
Outline

- Chiral spin liquids: charge-neutral analogues of Quantum Hall liquids
- Thermal Hall effect: experimental results
- Kagome strip model
- Quantum phases and their signature in $\kappa_{xy}\left(B\right)$

Quantum Spin Liquids

Frustration of anti-ferromagnetism:

$$H = J\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j$$



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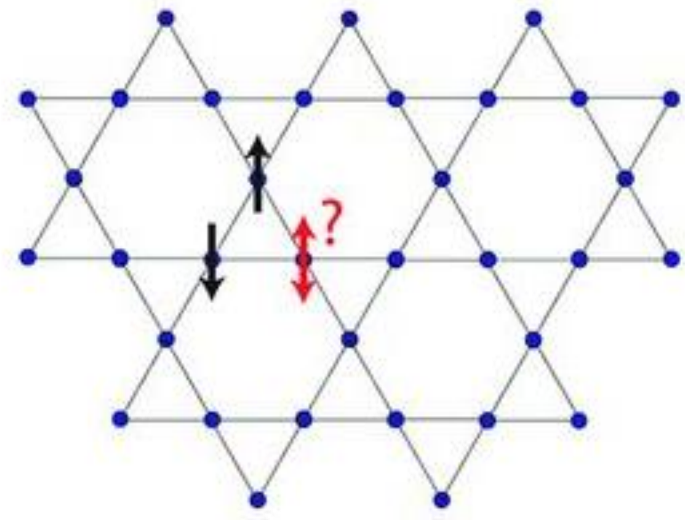
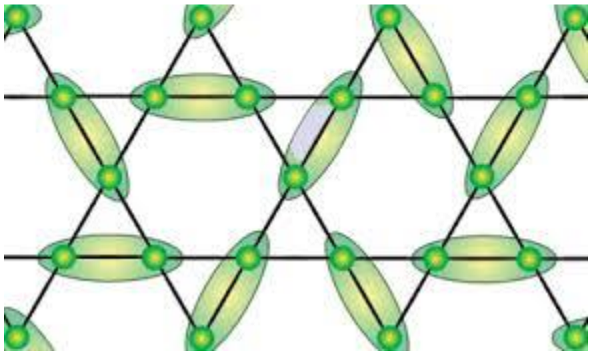


$$\left[S_i^{\alpha}, S_i^{\beta}\right] \neq 0$$



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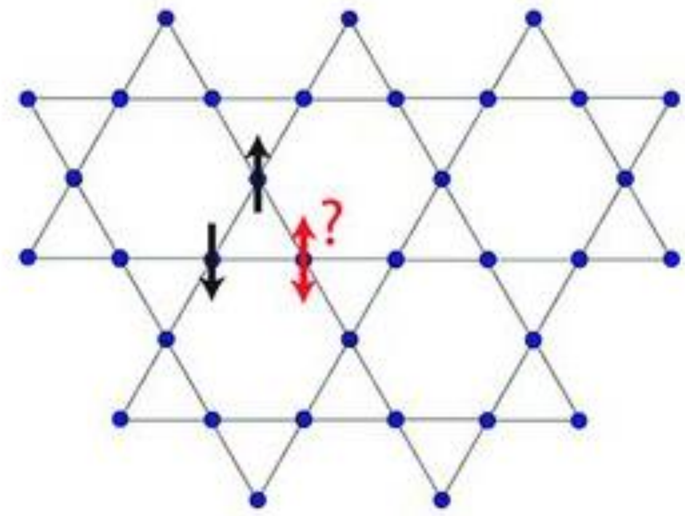

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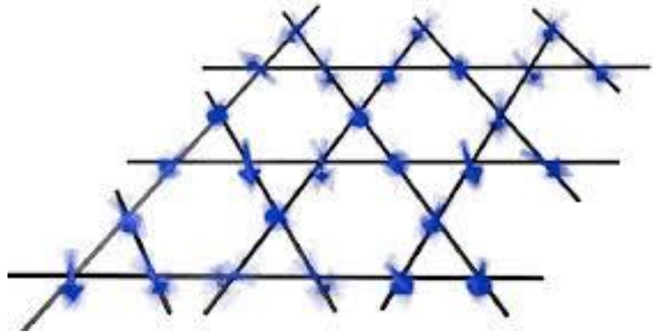

Quantum Spin Liquids

$$S^+ = \left(S^x + iS^y\right)$$

$$S^z$$

Magnetic field B_z

$$\left(\vec{S}_i \times \vec{S}_j\right)_z$$

"Spinons": $a^\dagger$

Spinons density

μ_s

Spinons current

Quantum Spin Liquids

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?

"Spinons": $a^{\dagger}$

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Chiral Spin Liquids

Kalmeyer & Laughlin (1987): analogy with a Fractional QH liquid

$$J\sum_{\langle i,j\rangle} S_i^+ S_j^- \rightarrow \sum_{\langle i,j\rangle} t_{ij} a_i^\dagger a_j$$

$$t_{ij} = -Je^{i\frac{2\pi}{\Phi_0}\int_i^j A\,dl}$$

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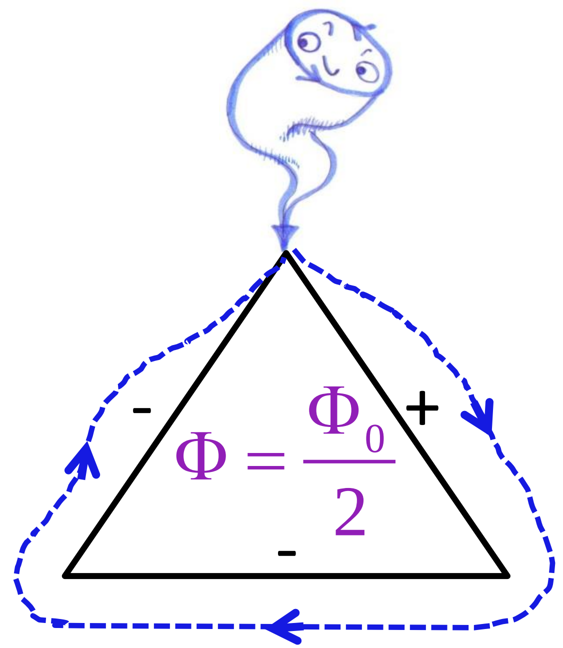

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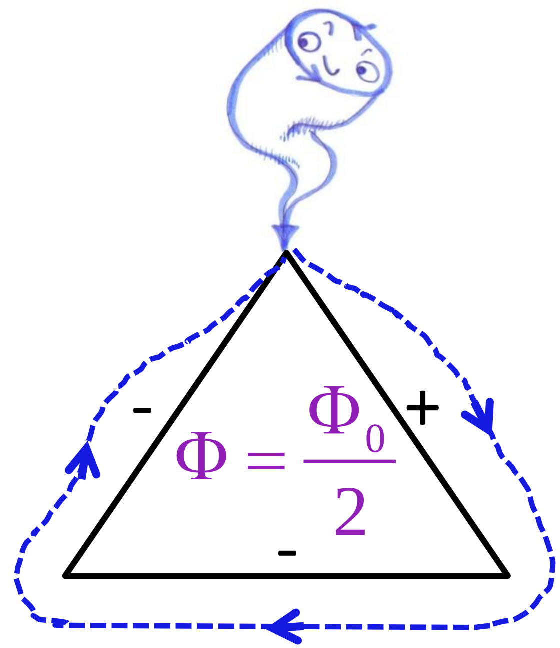
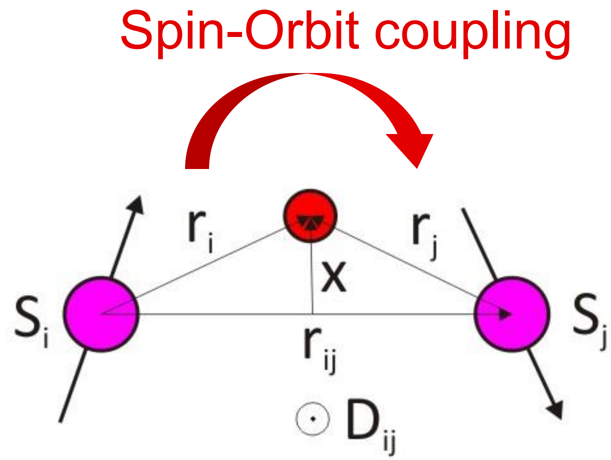


$$\Phi \overset{?}{\neq} \frac{\Phi_0}{2}$$

Chiral Spin Liquids

Dzyaloshinskii-Moriya interaction:

$$\underbrace{\vec{D}_{ij}}_{\vec{A}_s} \cdot \underbrace{\left(\vec{S}_i \times \vec{S}_j\right)}_{\vec{J}_s}$$



Chiral Spin Liquids

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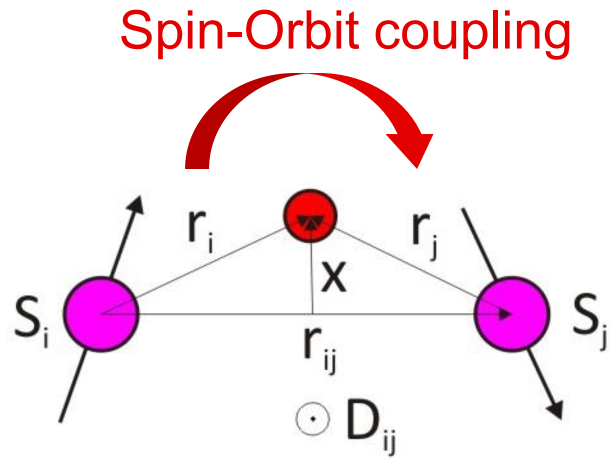
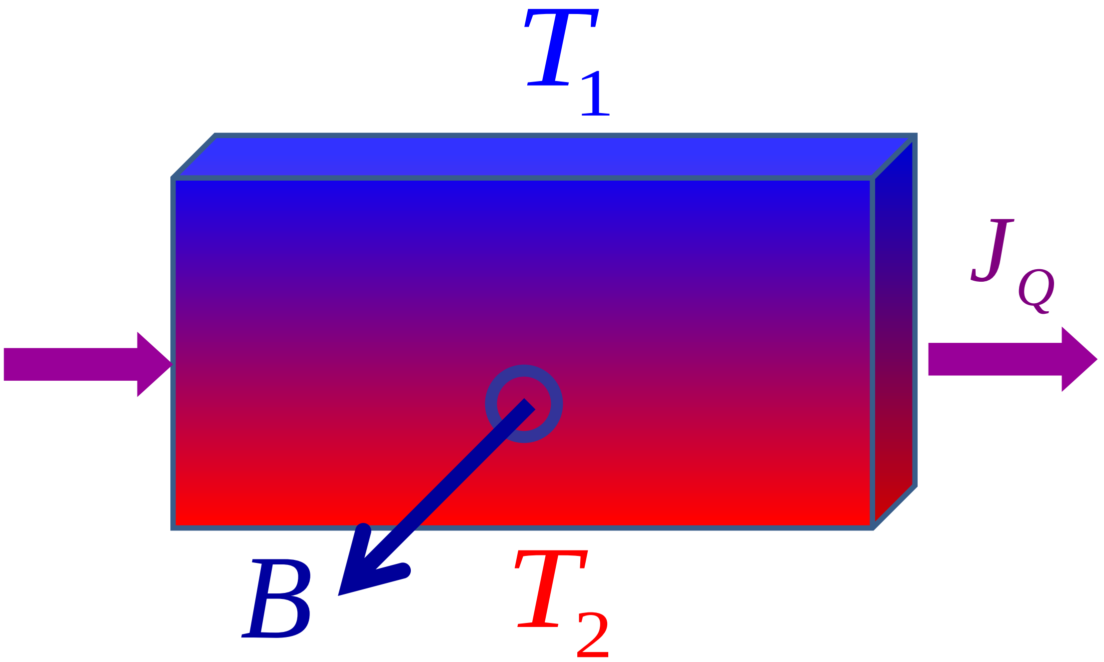


Wen, Wilczek & Zee; Baskaran (1989): chiral order parameter

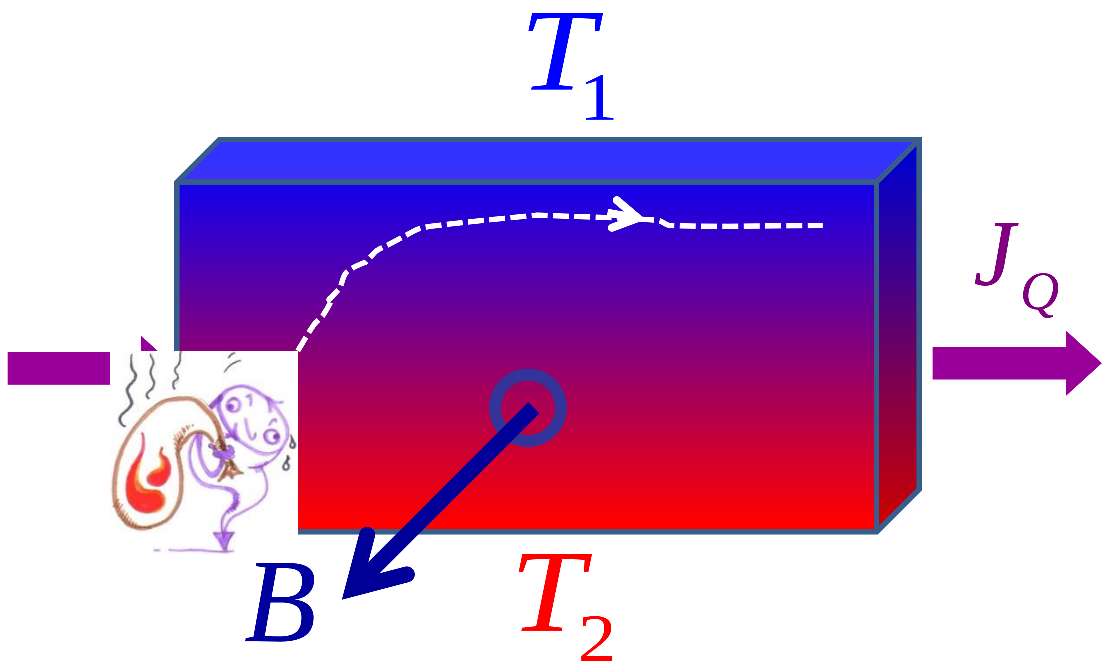
$$\boxed{O_\chi = \vec{S}_i \cdot \left(\vec{S}_j \times \vec{S}_k\right)}$$

Thermal Hall Effect



$$\kappa_{xy} \equiv \frac{J_Q}{\nabla_y T}$$

Thermal Hall Effect



$$\kappa_{xy} \equiv \frac{J_Q}{\nabla_y T}$$

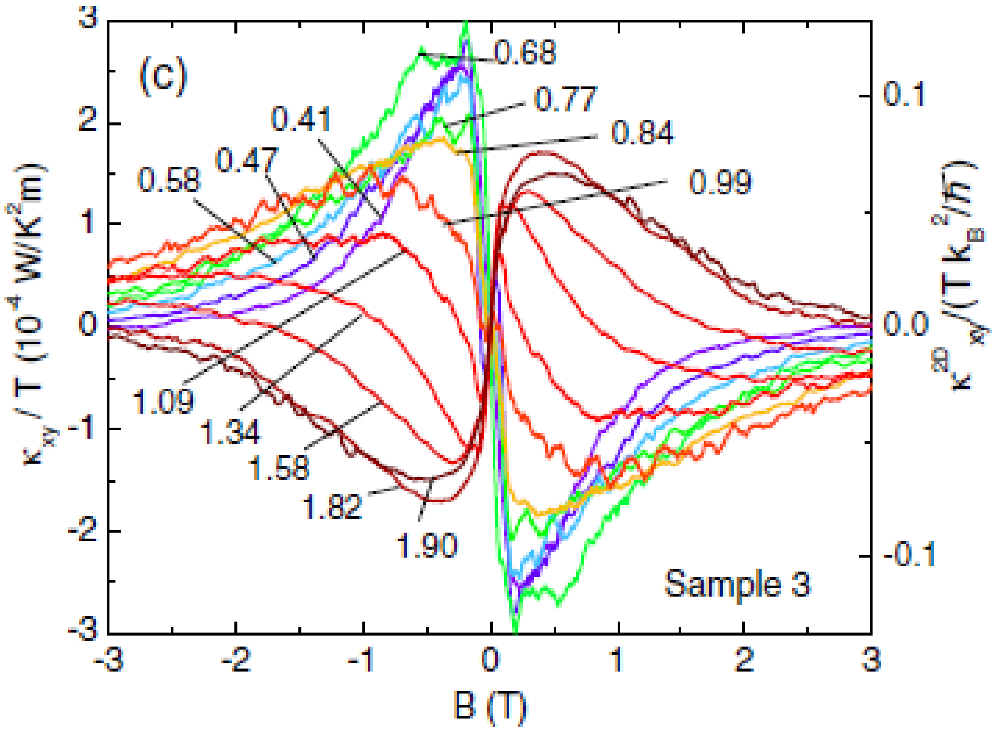
Thermal Hall Effect

PRL 115, 106603 (2015) PHYSICAL REVIEW LETTERS week ending 4 SEPTEMBER 2015

Thermal Hall Effect of Spin Excitations in a Kagome Magnet

Max Hirschberger,[1] Robin Chisnell,[2,*] Young S. Lee,[2,3] and N. P. Ong[1]

Cu(1,3-bdc)



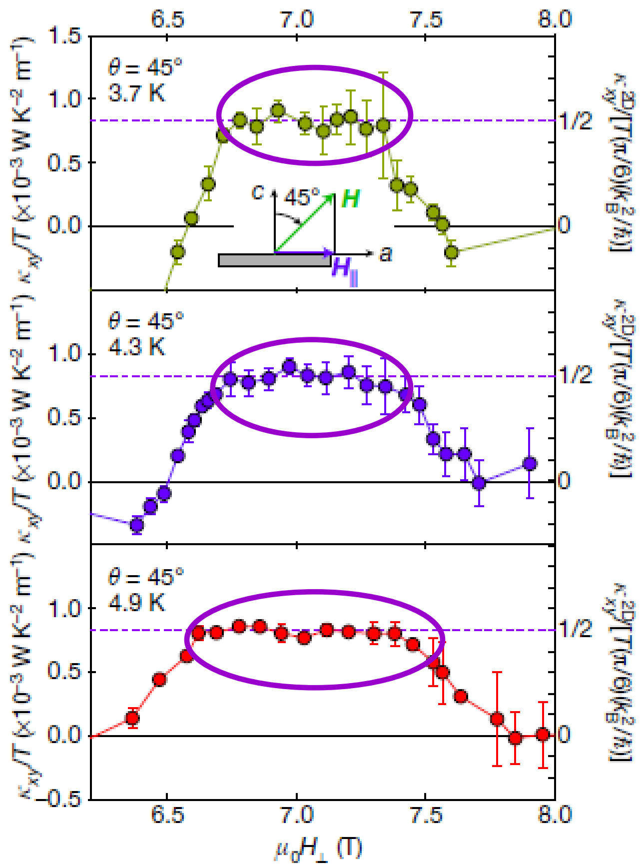
LETTER

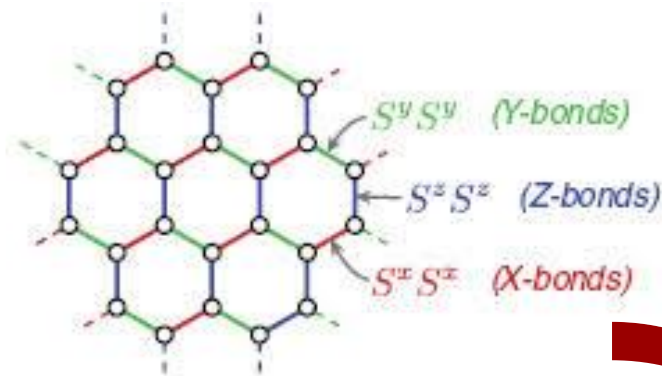
https://doi.org/10.1038/s41586-018-0274-0

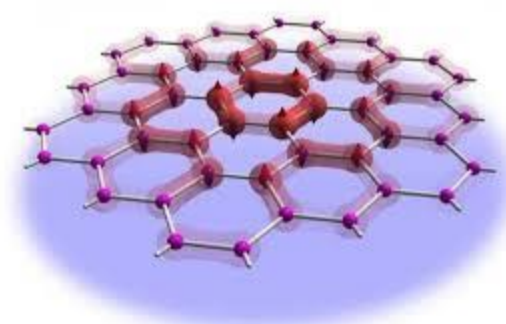
Majorana quantization and half-integer thermal quantum Hall effect in a $\boxed{\text{Kitaev spin liquid}}$

$\alpha - RuCl_3$

Y. Kasahara[1], T. Ohnishi[1], Y. Mizukami[2], O. Tanaka[2], Sixiao Ma[1], K. Sugii[3], N. Kurita[4], H. Tanaka[4], J. Nasu[4], Y. Motome[5], T. Shibauchi[2] & Y. Matsuda[1]*





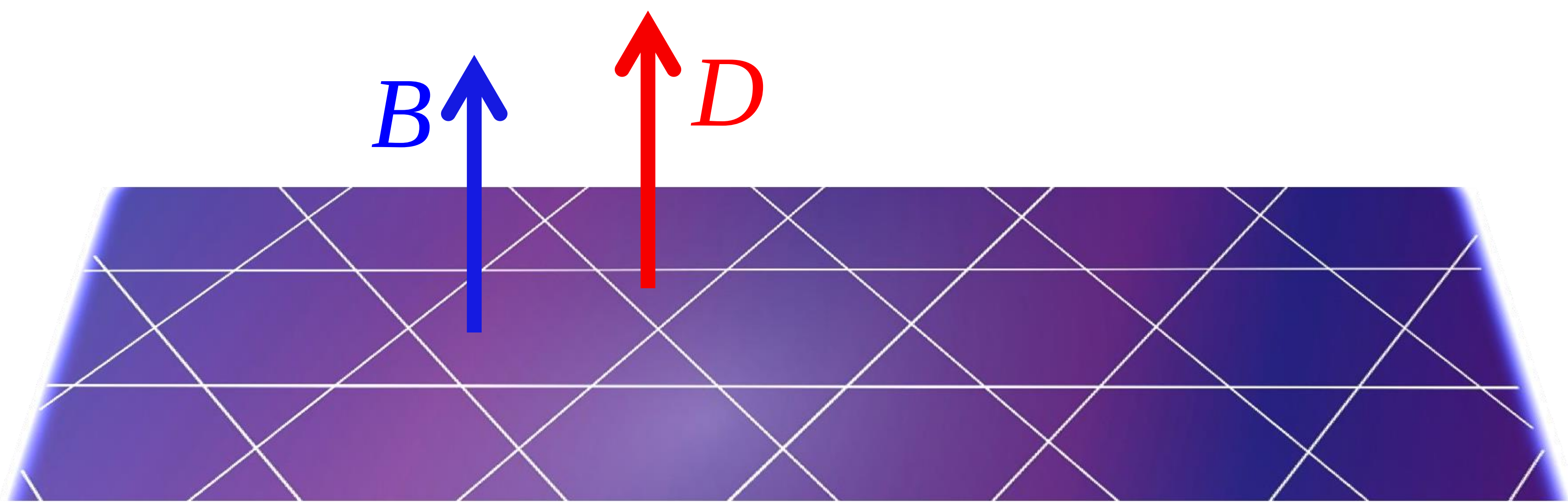


Coupled spin-chains model

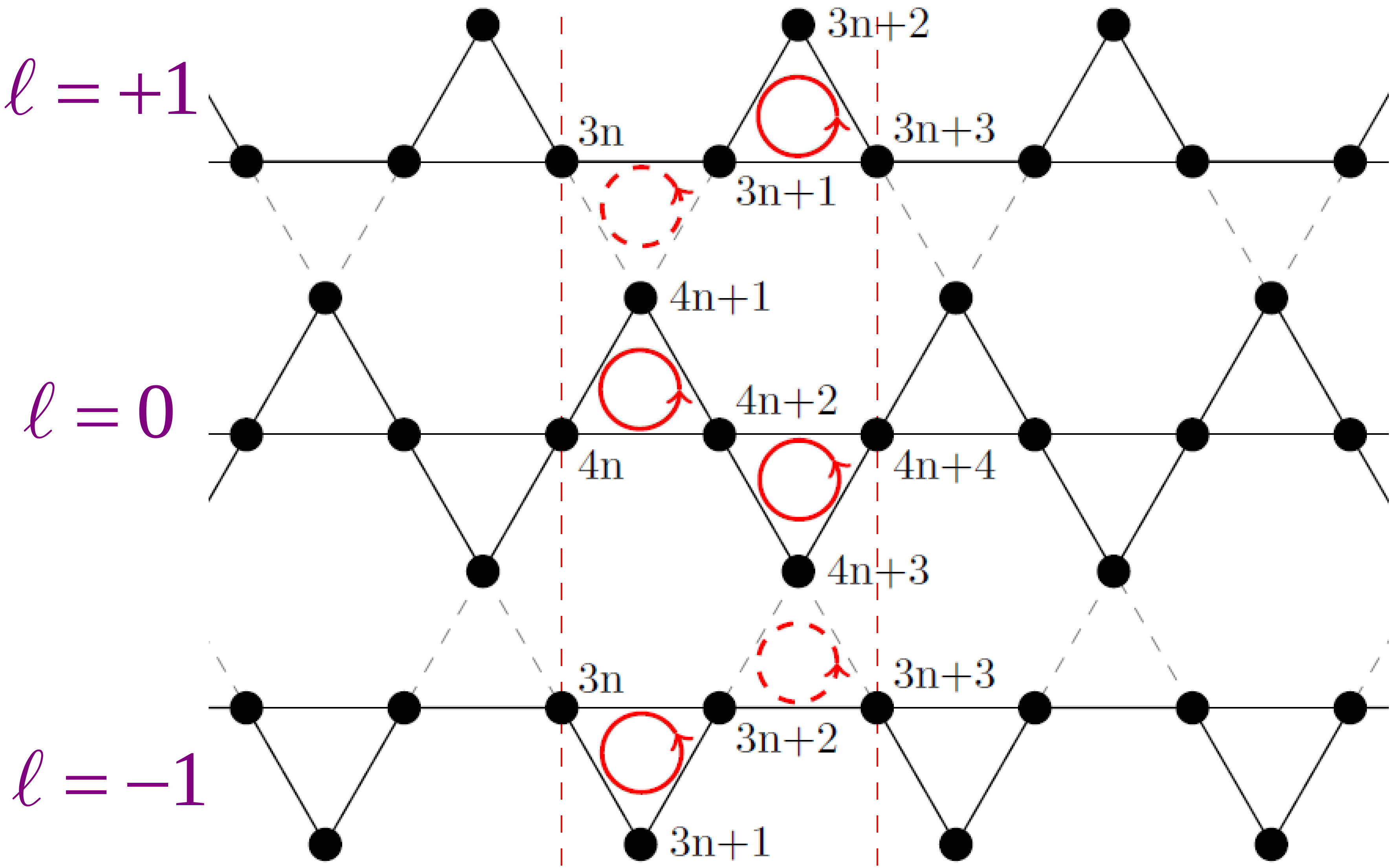
Gorohovsky, Pereira & Sela; Meng, Neupert, Greiter & Tomale (2015):
a CSL-phase can be stable ($O_\chi = \vec{S}_i \cdot \left(\vec{S}_j \times \vec{S}_k \right)$ relevant)

Our study

- tractable minimal model: a Kagome strip
- tunable breaking of T and P symmetries

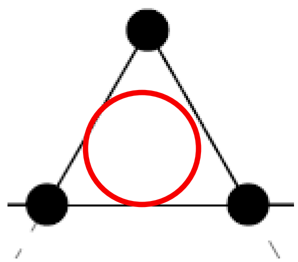


Coupled spin-chains model

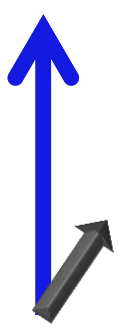


Coupled spin-chains model

$$= \frac{1}{2} J_{xy} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) + J_z S_i^z S_j^z$$

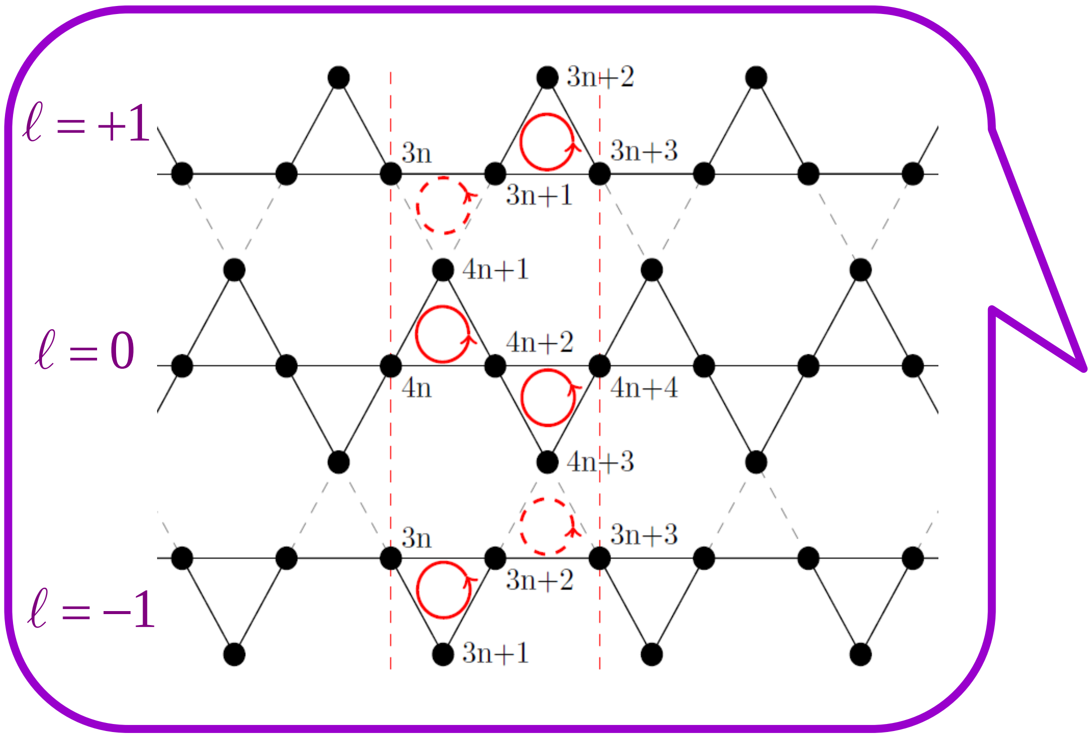


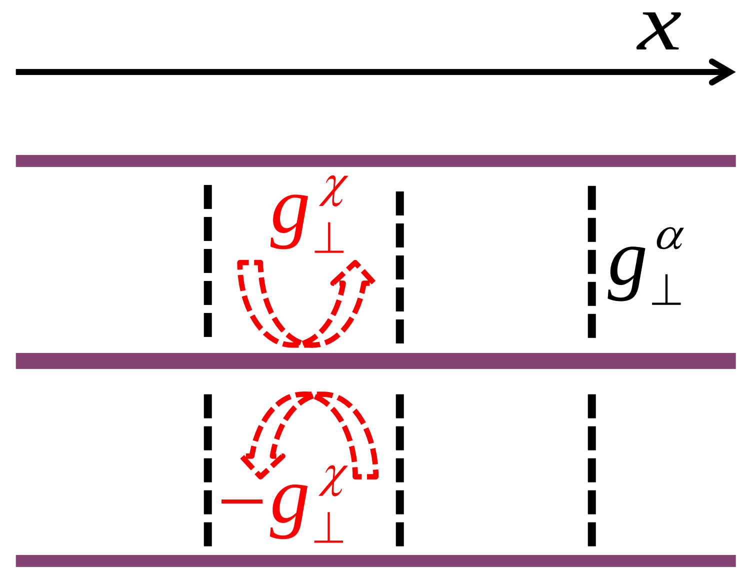
$$= D \sum_{\langle i, j \rangle \in \Delta} \left(\vec{S}_i \times \vec{S}_j \right)_z$$



$$= -B S_i^z$$

Coupled spin-chains model





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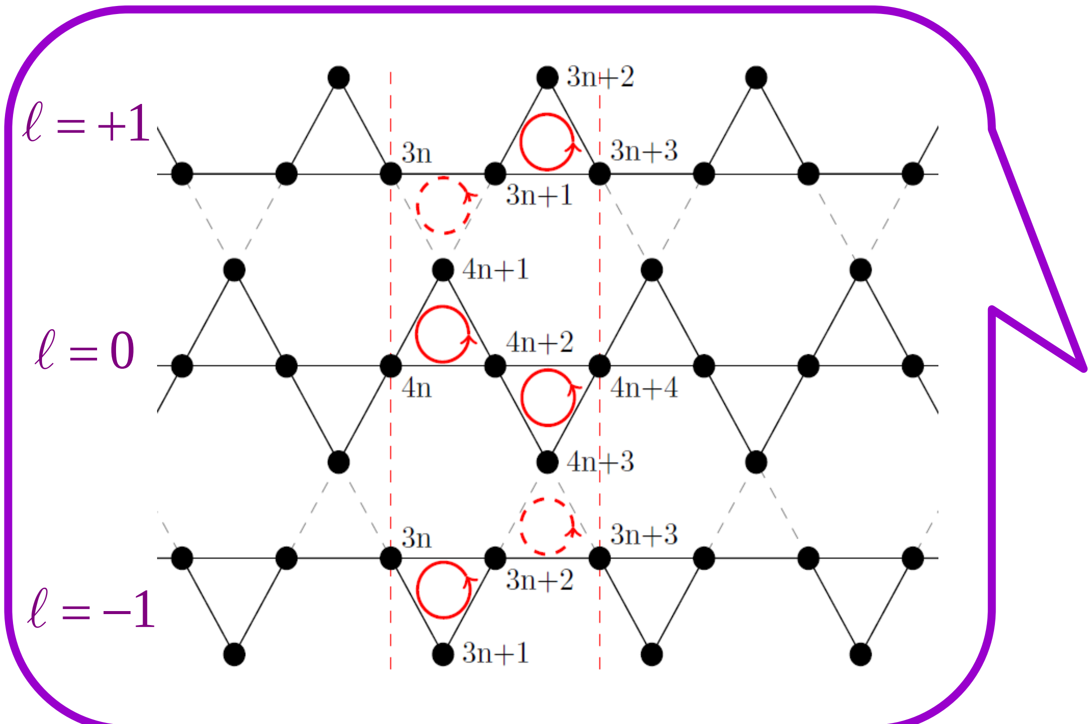
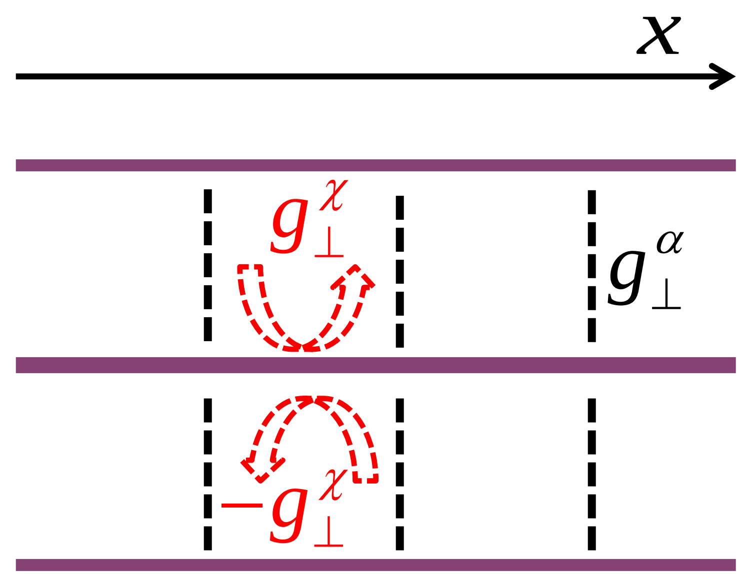




Bosonization of spin operators

$$\left[\phi_\ell(x), \partial_x\theta_{\ell'}(x')\right] = i\pi\delta_{\ell\ell'}\delta(x - x')$$

$$\begin{cases} S_\ell^\pm(x) = \dfrac{e^{\mp i\theta_\ell}}{\sqrt{2\pi a}}\left((-)^x + \cos 2\phi_\ell\right) \\ S_\ell^z(x) = -\dfrac{\partial_x\phi_\ell}{\pi} + \dfrac{(-)^x}{2\pi a}\cos 2\phi_\ell \end{cases}$$

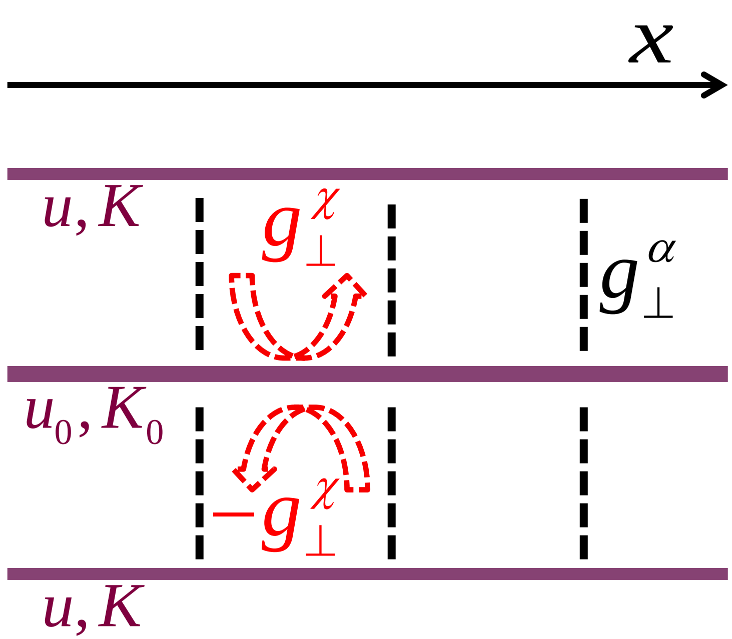
Coupled spin-chains model

$$H = \sum_{\ell=-1,0,1} H_\ell + H_\perp + H_\perp^\chi$$

$$H_\ell \simeq \frac{u_\ell}{2\pi}\int dx\left(K_\ell(\partial_x\theta_\ell)^2 + \frac{1}{K_\ell}(\partial_x\phi_\ell)^2\right)$$
$$+ g_\ell \int dx \cos 4\phi_\ell$$
$$+ \frac{1}{\pi}\int dx\{B\partial_x\phi_\ell + D2\ell\partial_x\theta_\ell\}$$

$$H_\perp = g_\perp \int dx(\partial_x\phi_1 + \partial_x\phi_{-1})\partial_x\phi_0$$

$$H_\perp^\chi \simeq g_{\perp,0}^\chi \int dx(\partial_x\theta_1 - \partial_x\theta_{-1})\partial_x\phi_0$$
$$+ \sum_{\ell=1,-1,\sigma=\pm} g_{\perp,\sigma}^\chi \int dx \cos\left(2(\phi_0+\phi_\ell) + \sigma(\theta_0-\theta_\ell) + \delta_{\sigma;\ell}\right)$$



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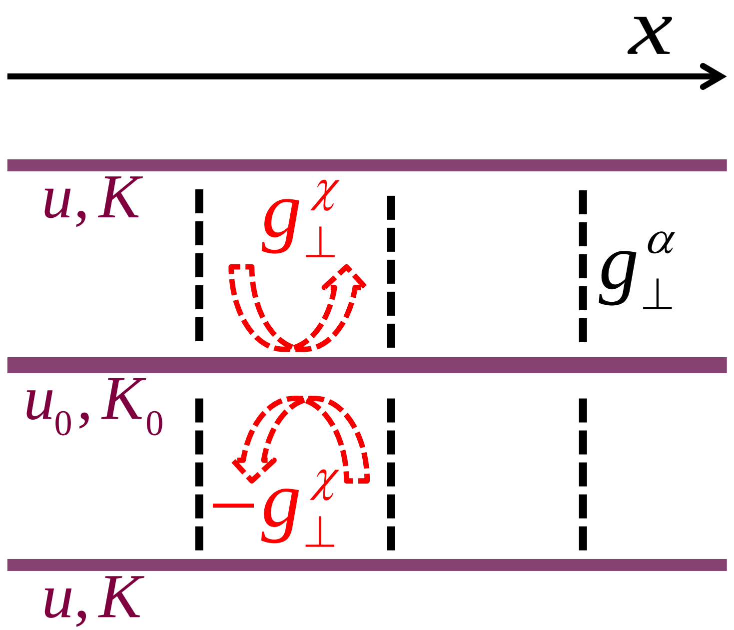

$$H_\ell\simeq\frac{u_\ell}{2\pi}\int dx\left(K_\ell\left(\partial_x\theta_\ell\right)^2+\frac{1}{K_\ell}\left(\partial_x\phi_\ell\right)^2\right)$$
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$$H_\perp=g_\perp\int dx\left(\partial_x\phi_1+\partial_x\phi_{-1}\right)\partial_x\phi_0$$

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$$+\sum_{\ell=1,-1,\sigma=\pm}g_{\perp,\sigma}^\chi\int dx\,\underbrace{\cos\left(2\left(\phi_0+\phi_\ell\right)+\sigma\left(\theta_0-\theta_\ell\right)+\delta_{\sigma;\ell}\right)}_{O_\chi=\vec{S}_i\cdot\left(\vec{S}_j\times\vec{S}_k\right)}$$

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$$\boxed{+\frac{1}{\pi}\int dx\{B\partial_x\phi_\ell + D2\ell\partial_x\theta_\ell\}}$$

$$\begin{cases}\partial_x\phi_\ell \to \partial_x\phi_\ell + k_\ell^B\\ \partial_x\theta_\ell \to \partial_x\theta_\ell + k_\ell^D\end{cases}$$

$$H_\perp = g_\perp\int dx(\partial_x\phi_1 + \partial_x\phi_{-1})\partial_x\phi_0$$

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$$\boxed{+\frac{1}{\pi}\int dx\left\{B\partial_x\phi_\ell + D2\ell\partial_x\theta_\ell\right\}}$$

$$\begin{cases} \partial_x\phi_\ell \to \partial_x\phi_\ell + k_\ell^B \\ \partial_x\theta_\ell \to \partial_x\theta_\ell + k_\ell^D \end{cases}$$

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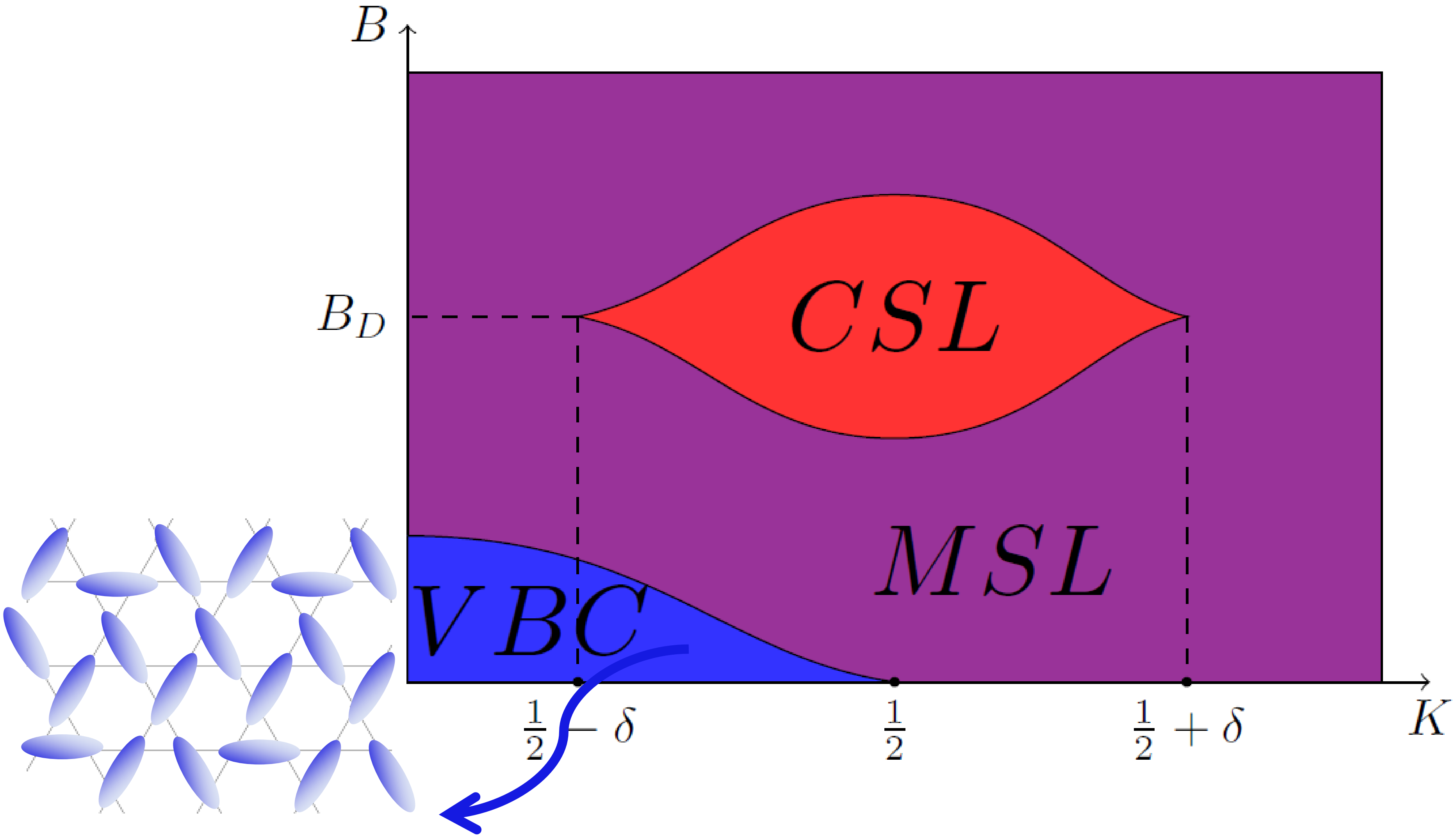
$$H_\perp^\chi \simeq g_{\perp,0}^\chi \int dx\left(\partial_x\theta_1 - \partial_x\theta_{-1}\right)\partial_x\phi_0$$

$$+\sum_{\ell=1,-1,\sigma=\pm} g_{\perp,\sigma}^\chi \int dx \cos\left(2(\phi_0+\phi_\ell) + \sigma(\theta_0-\theta_\ell) + \delta_{\sigma;\ell} - (4k_B + \sigma k_D)x\right)$$

Phase Diagram



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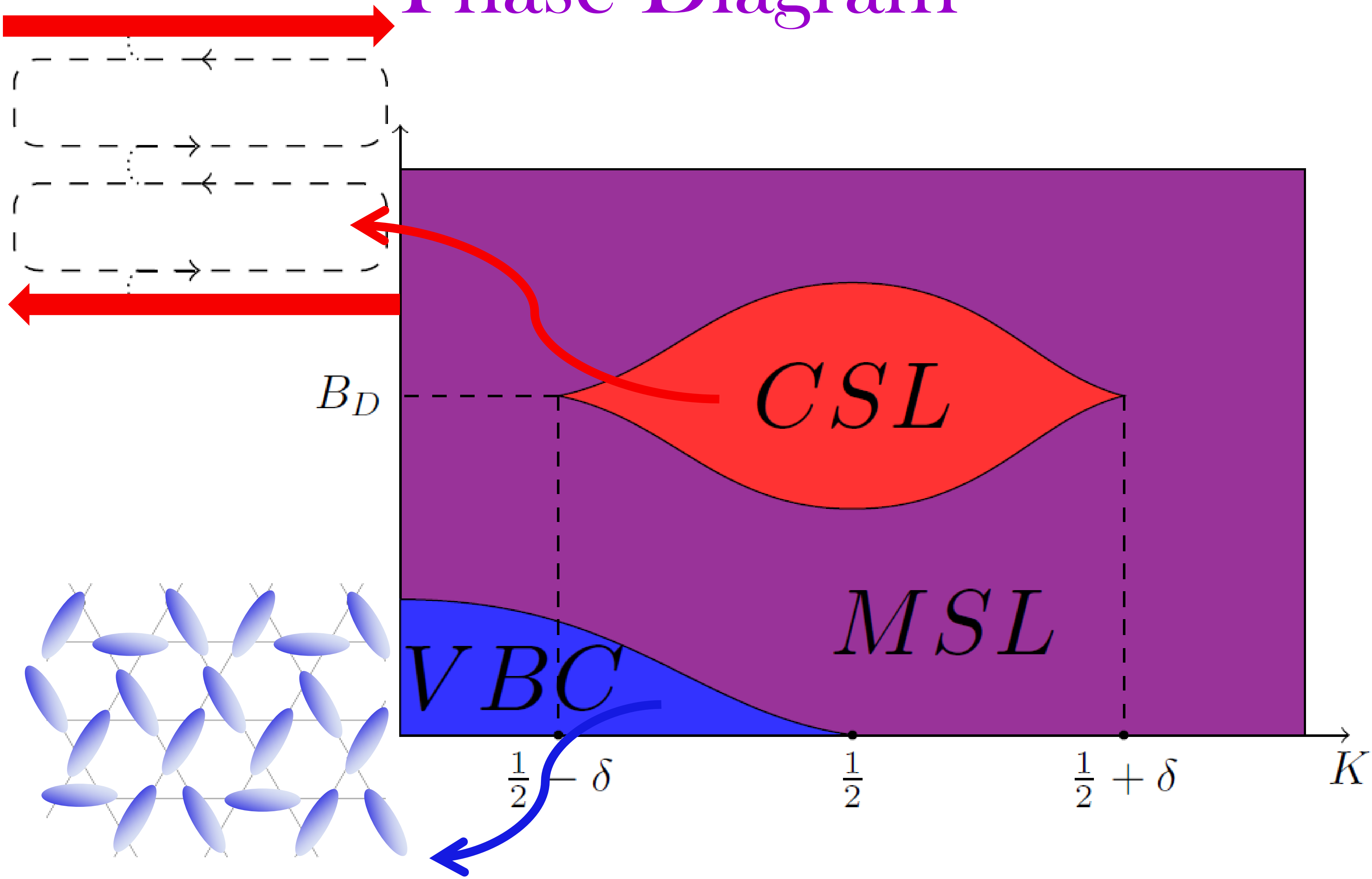

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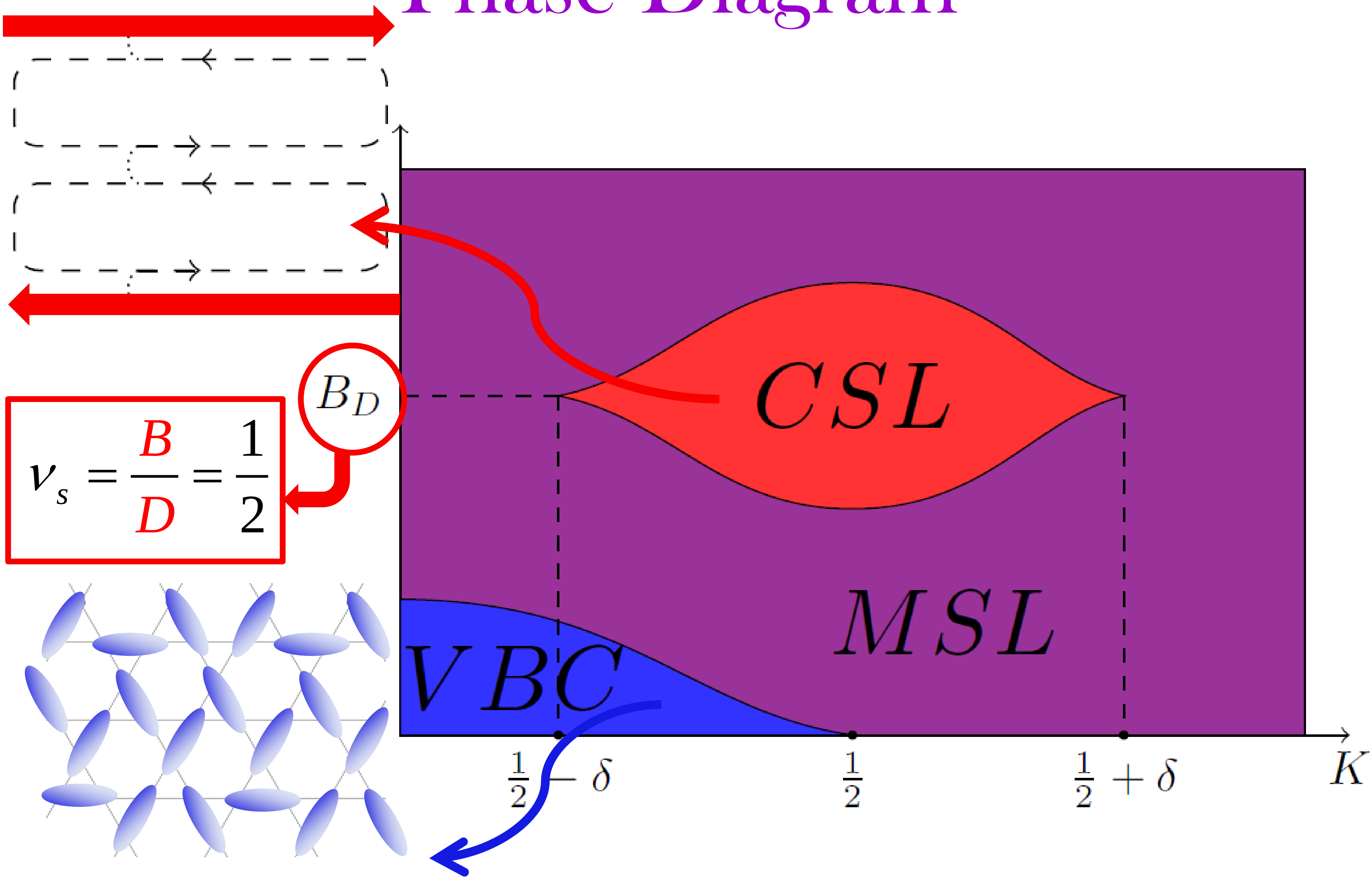
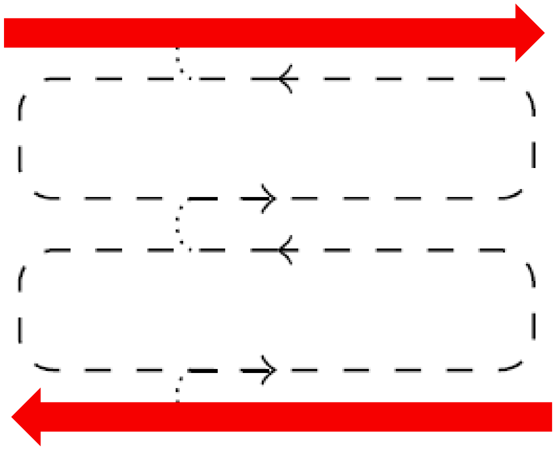


Chiral Spin Liquid phase



$$O_\chi \sim g_\perp^\chi \cos\left(2\left(\phi_0 + \phi_\ell\right) \pm \left(\theta_0 - \theta_\ell\right)\right)$$

RG equations:

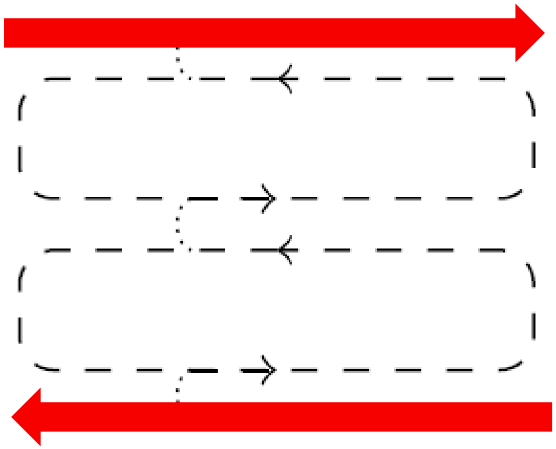
$$\frac{dK}{d\ell} \approx \left(1 - 4K^2\right)\left(\tilde{g}_\perp^\chi\right)^2$$

$$\frac{d\tilde{g}_\perp^\chi}{d\ell} \approx \left(2 - \frac{1}{2K_-} - 2K_+\right)\tilde{g}_\perp^\chi$$

$$K_+ - K_- \sim \frac{g_\perp}{u}$$

$$\boxed{\nu_s = \frac{B}{D} = \frac{1}{2}}$$

Chiral Spin Liquid phase



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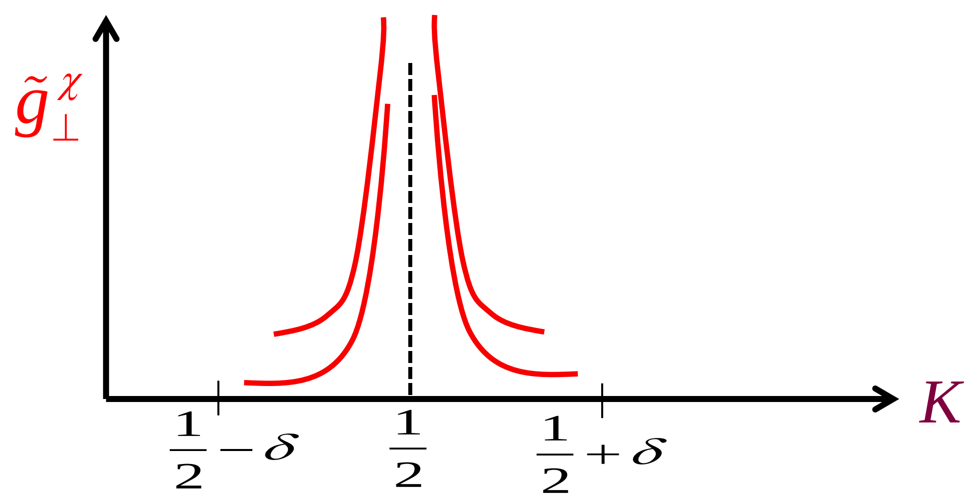
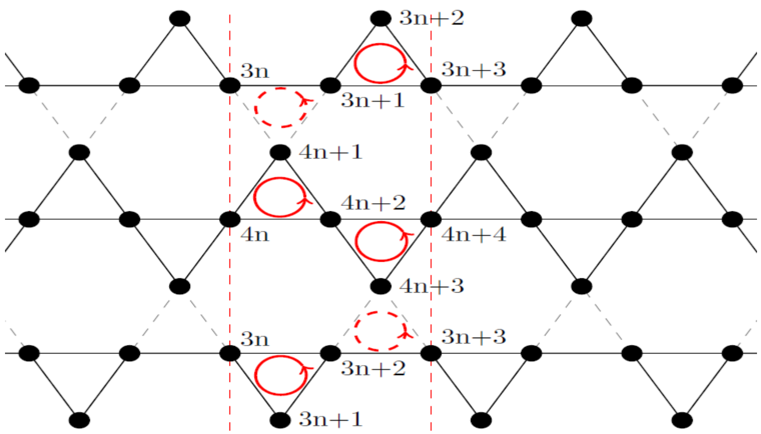


Thermal Hall Effect

Definition of heat current: $\partial_x J_h = -\partial_t h \qquad \left(H = \int dx h(x)\right)$



$\longrightarrow \quad \left\langle J_{h;+} \right\rangle$

$\longleftarrow \quad \left\langle J_{h;-} \right\rangle$

Thermal Hall Effect

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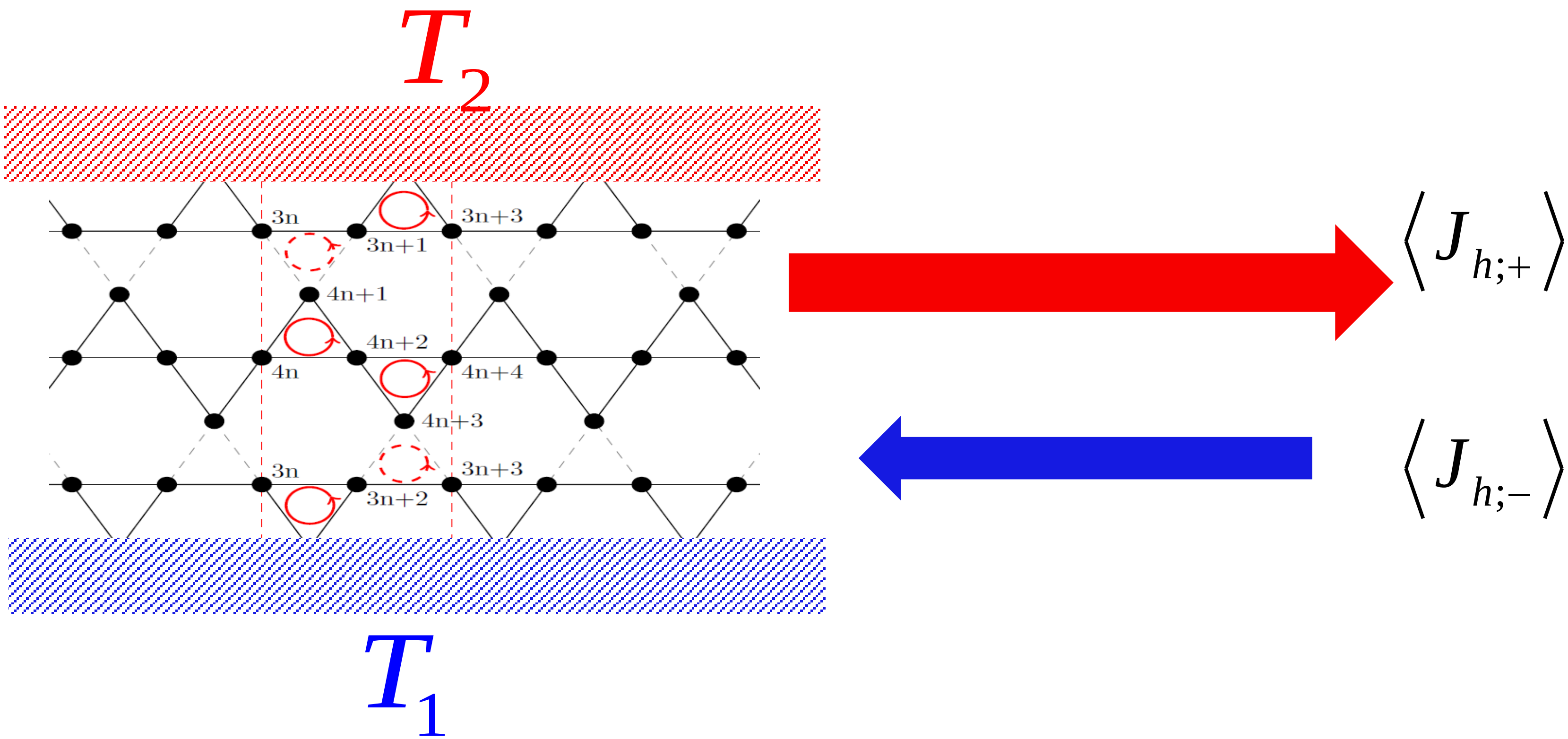
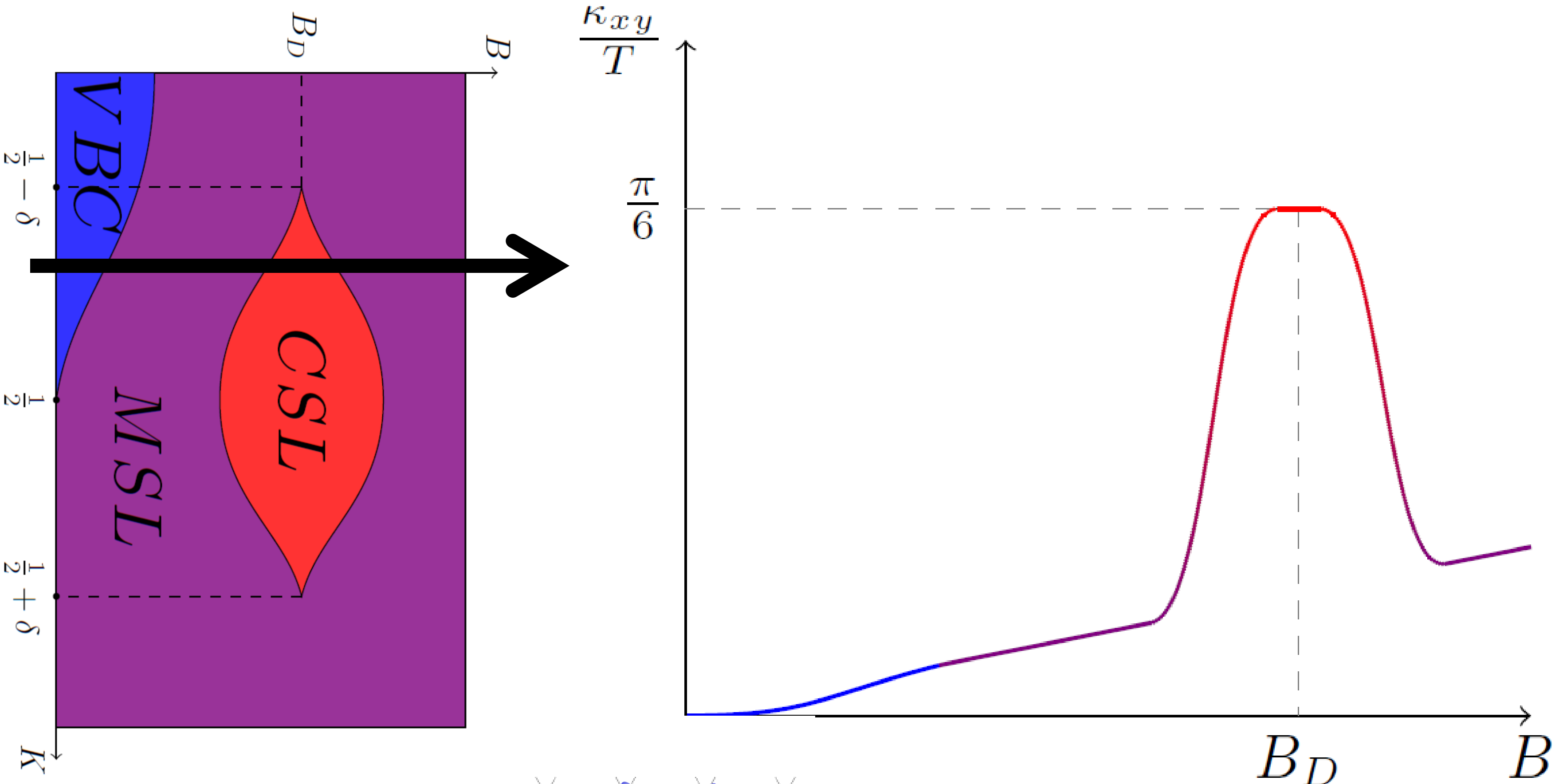
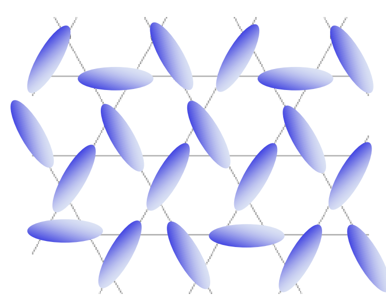
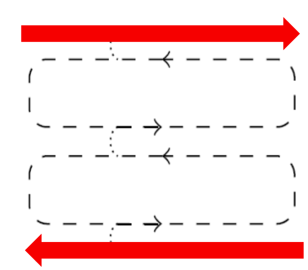


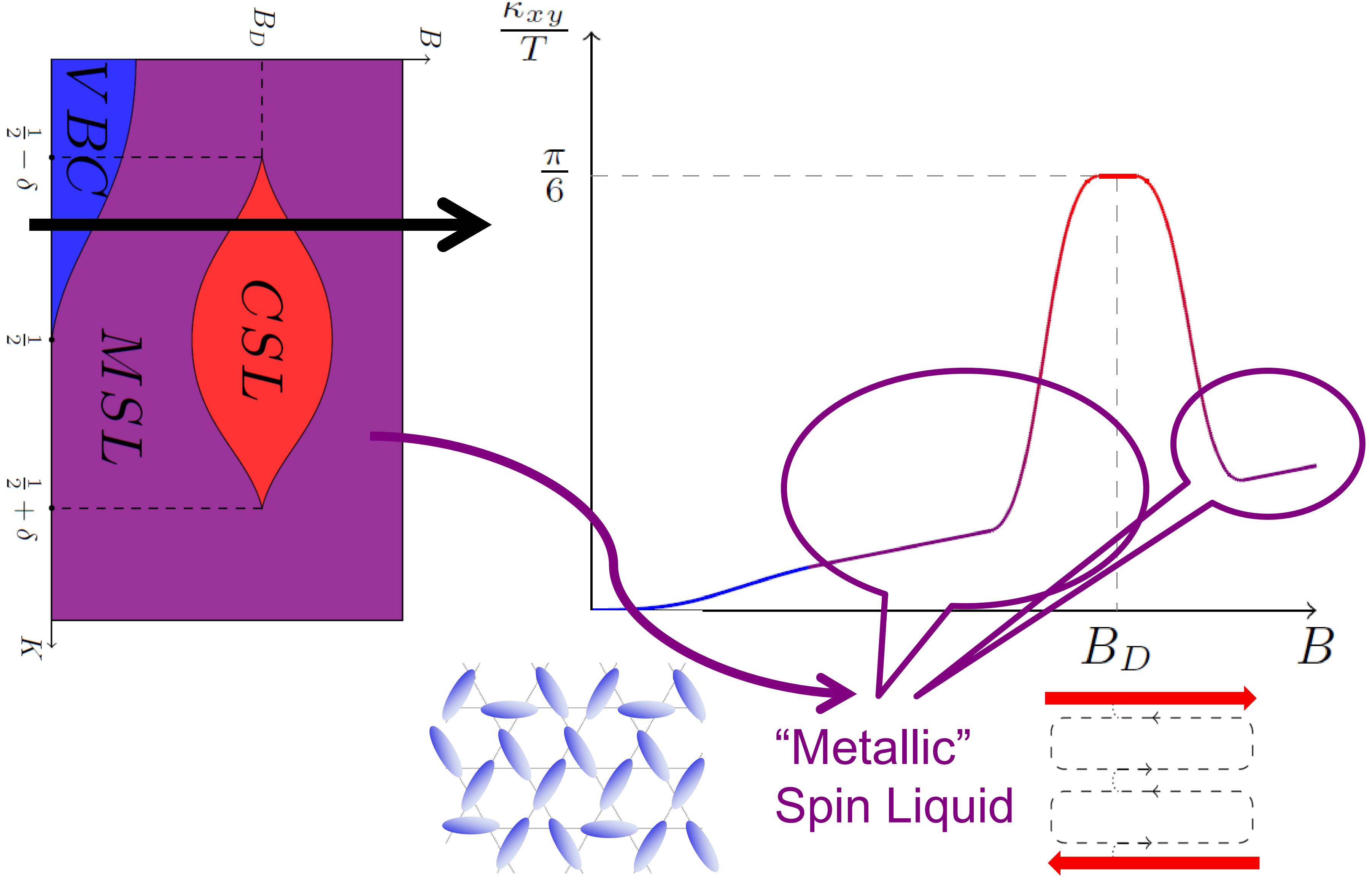
Thermal Hall Effect



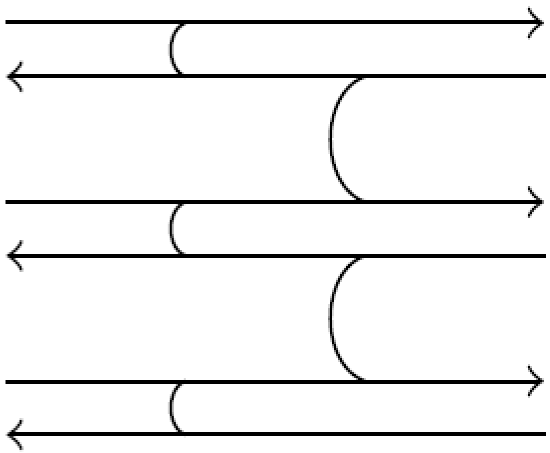


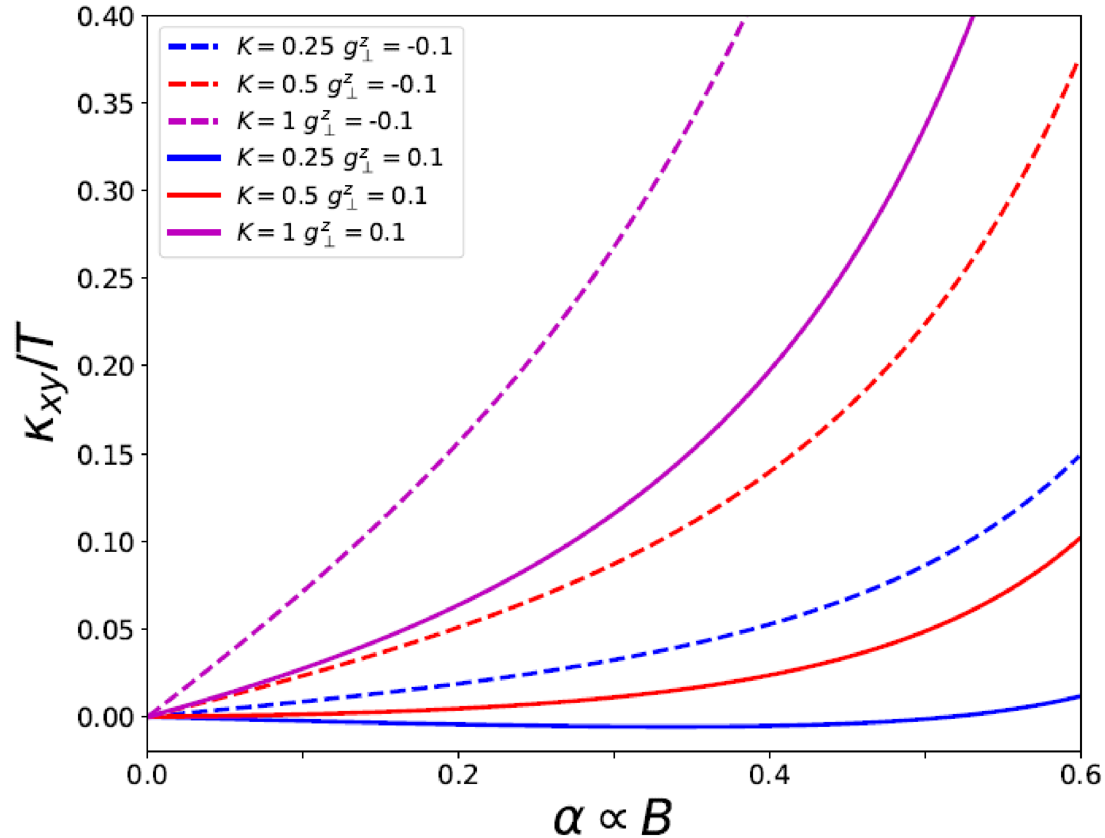


Thermal Hall Effect



Metallic Spin Liquid phase

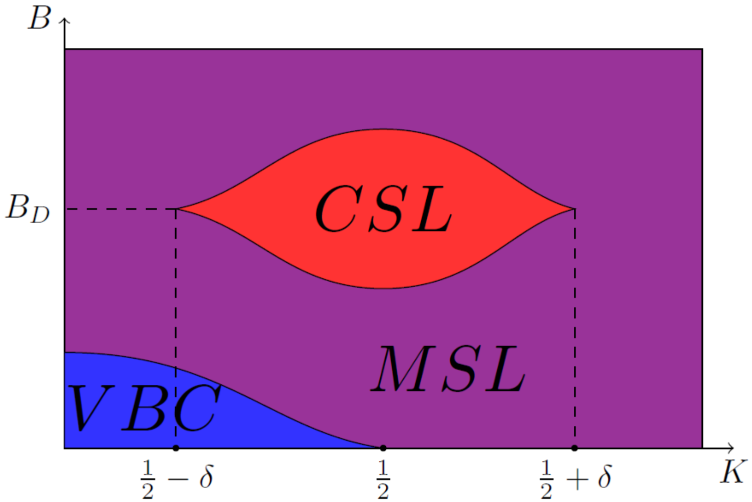


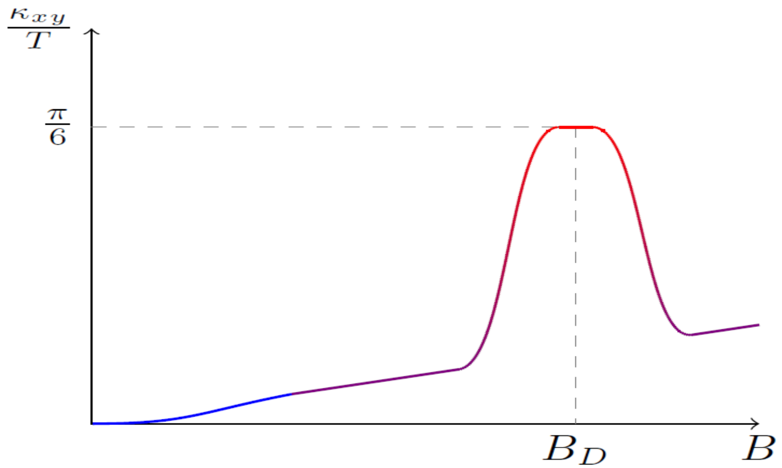


$$\kappa_{xy} = f\left(K, K_0, g_\perp, g_\perp^\chi, D, B\right)T$$

Summary

- Phase diagram of coupled Spin-1/2 chains on a Kagome strip + magnetic field B + DM interaction:





- Valence bond crystal (VBC) – a spinon insulator

- Metallic spin liquid (MSL) – coupled LL with chiral terms, non-universal κ_{xy}

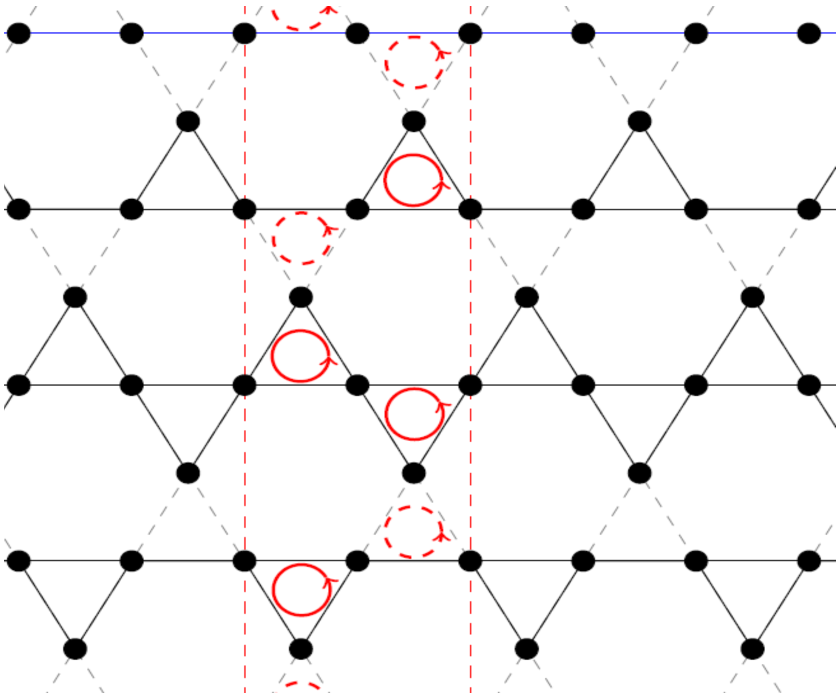
- Chiral spin liquid (CSL) – analogue of a Bosonic QH state

$$\boxed{\nu_s = \frac{B}{D} = \frac{1}{2}}$$

$$\boxed{\kappa_{xy} = \frac{\pi k_B^2}{6\hbar} T}$$

Outlook

? Extension to 2D



? Disorder vs. topological chiral terms $\cos\left(2\phi \pm \theta\right)$

? Relation to experimental data