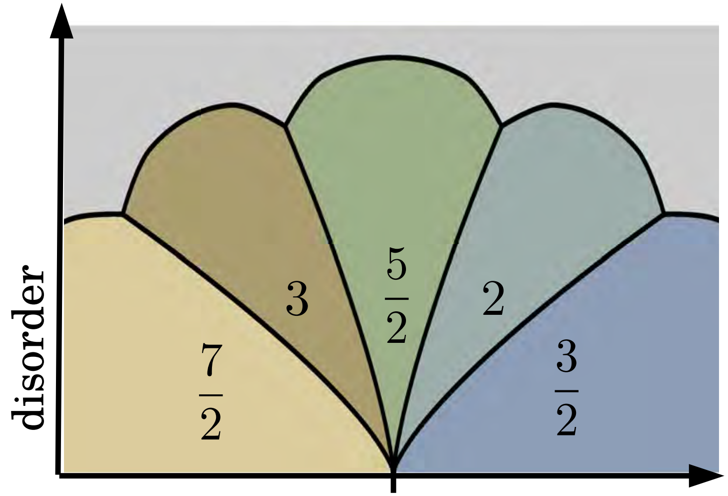
Theory of Disorder-Induced Half-Integer Thermal Hall Conductance

David F. Mross



DFM, Y. Oreg, A. Stern, G. Margalit, M. Heiblum, PRL 121, 026801 (2018)

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Physics

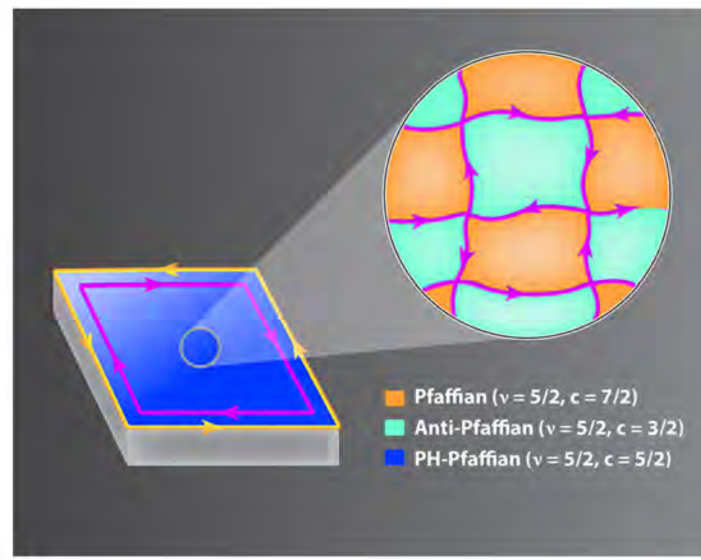
VIEWPOINT by Jason Alicea

A Hot Topic in the Quantum Hall Effect

Heat transport studies of fractional quantum Hall systems provide evidence for a new phase of matter with potential applications in fault-tolerant quantum computation.

Experiment:
M. Banerjee, M. Heiblum *et al.*,
Nature (2018)

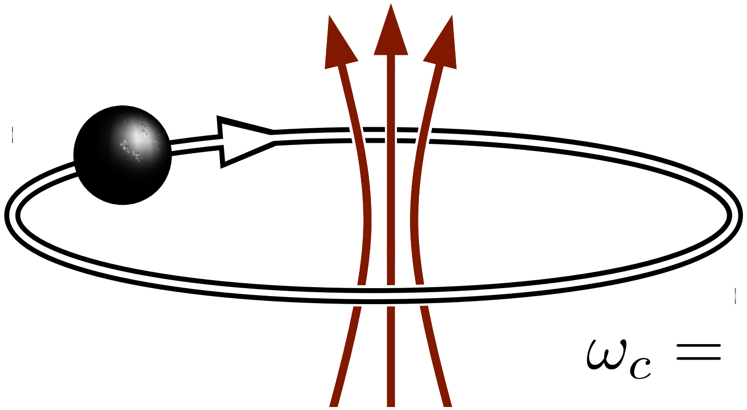
Theory:
DFM, Y. Oreg, A. Stern, G. Margalit
M. Heiblum, PRL 121, 026801 (2018)
C. Wang, A. Vishwanath,
B. Halperin, PRB 98, 045112 (2018)



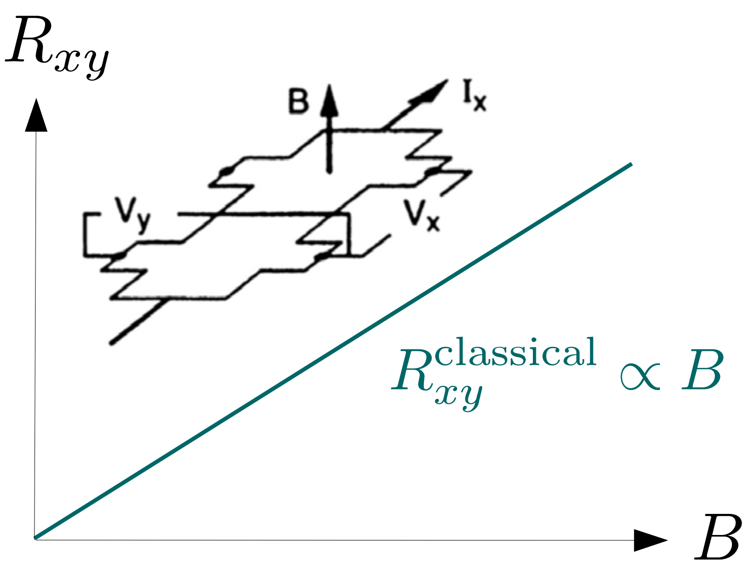
[Also related: B. Lian and J. Wang, PRB 97, 165124 (2018)]

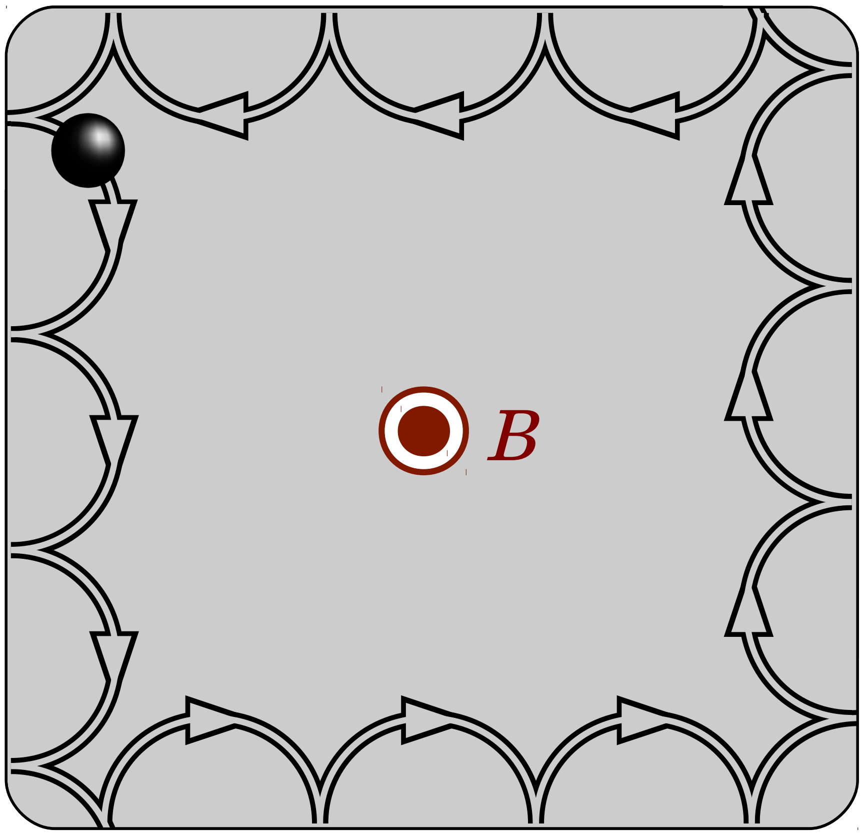
Quantum Hall effect in a nutshell

Classical: Cyclotron orbits



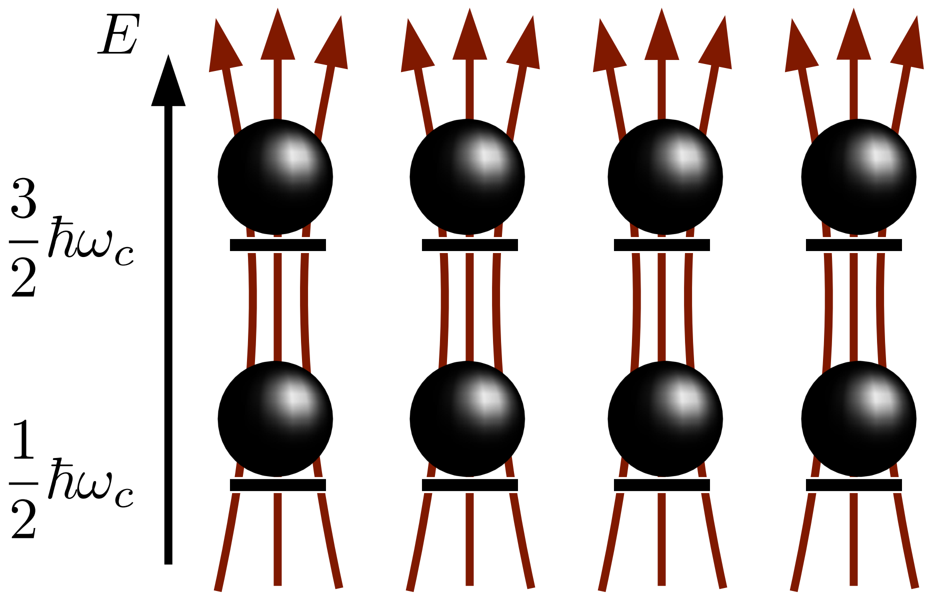
$$\omega_c = \frac{qB}{mc}$$





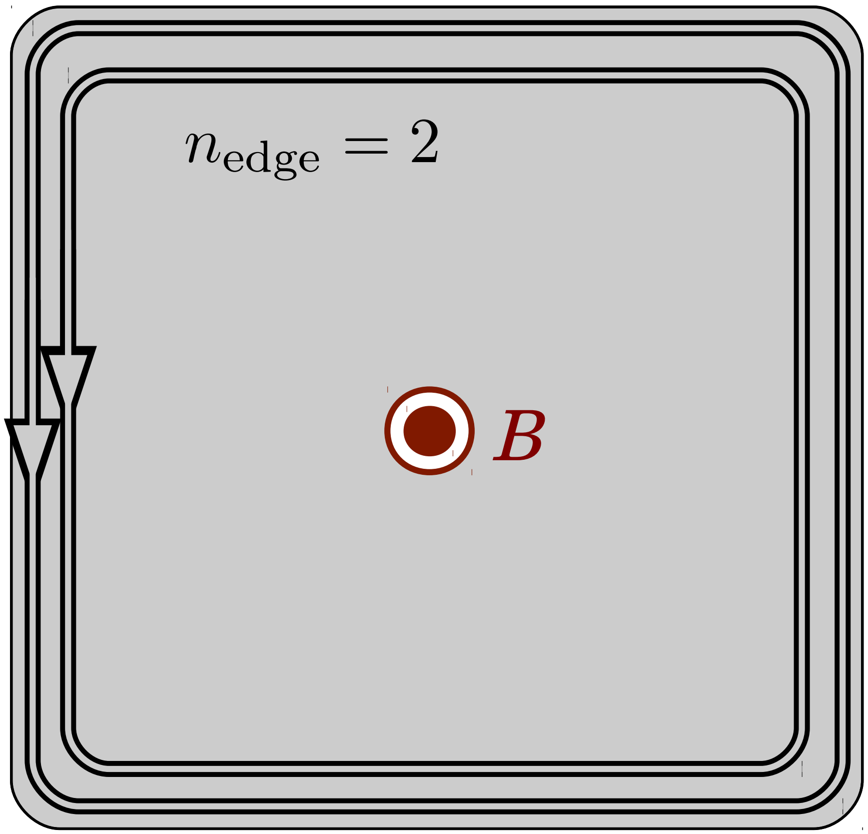
Quantum Hall effect in a nutshell

Quantum mechanical: Energy levels



N_{flux} states per energy level

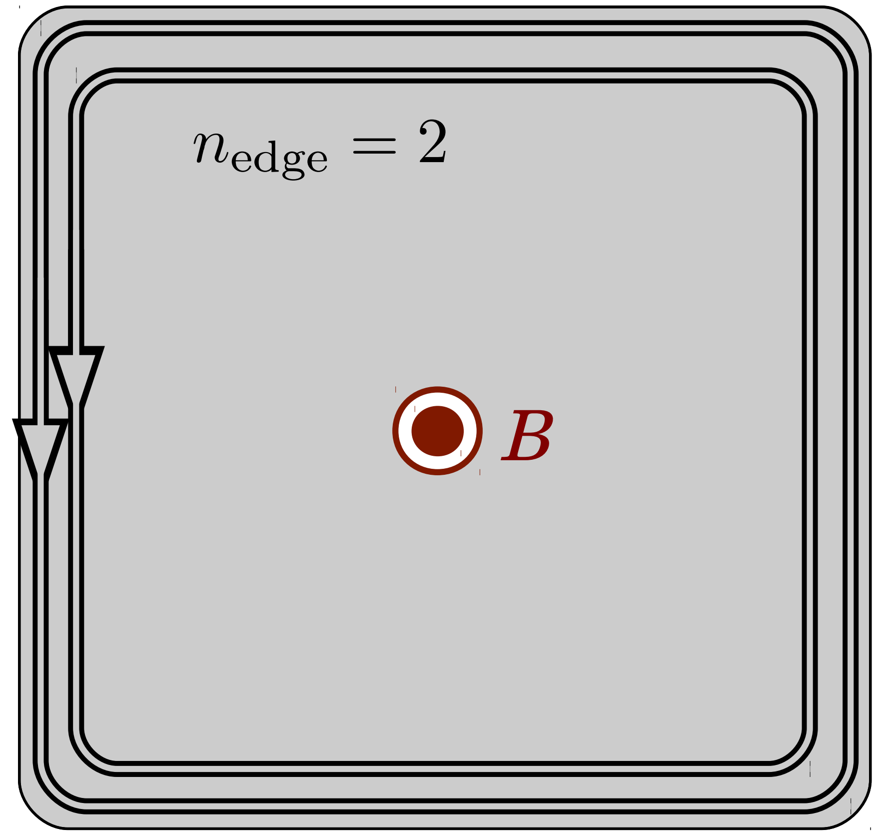
filling factor $\nu = \dfrac{N_{\text{electron}}}{N_{\text{flux}}}$



$n_{\text{edge}} = \nu$ chiral electron modes carry quantized flow of charge and energy

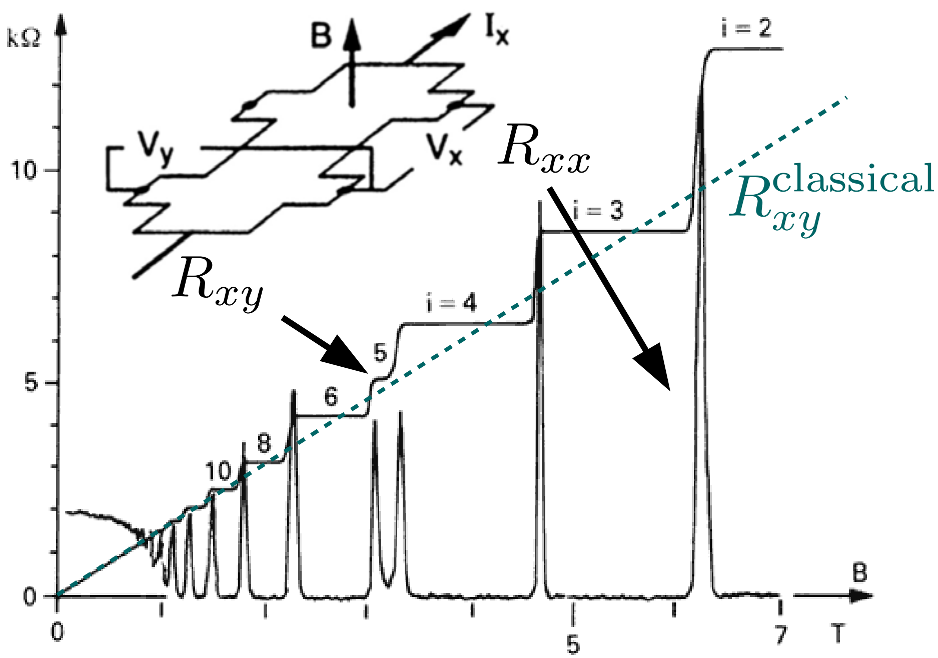
Quantum Hall effect in a nutshell

Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$



...ge $= \nu$ chiral electron modes carry
...antized flow of charge and energy

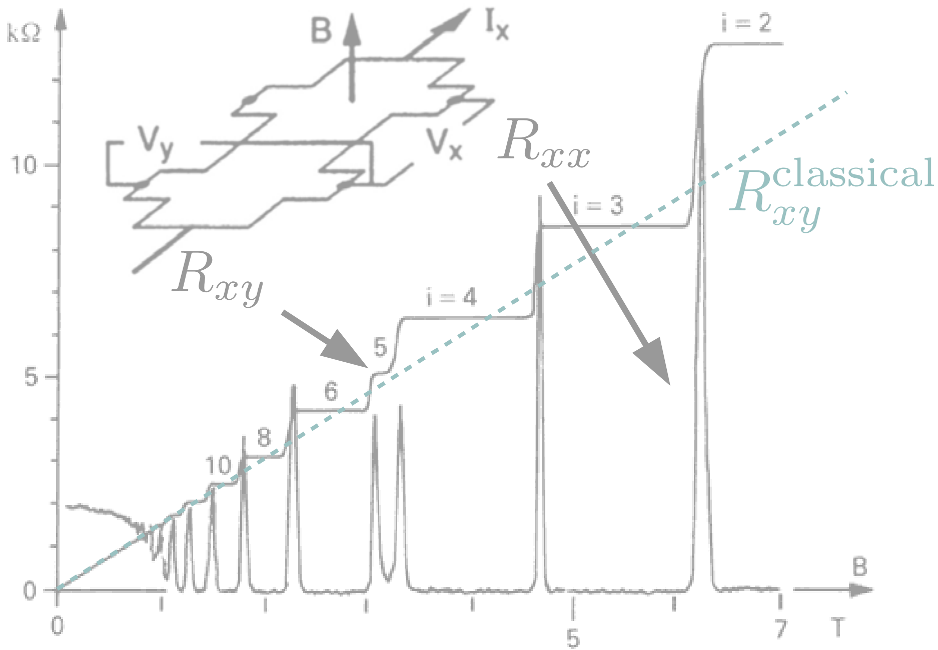
Quantum Hall effect in a nutshell

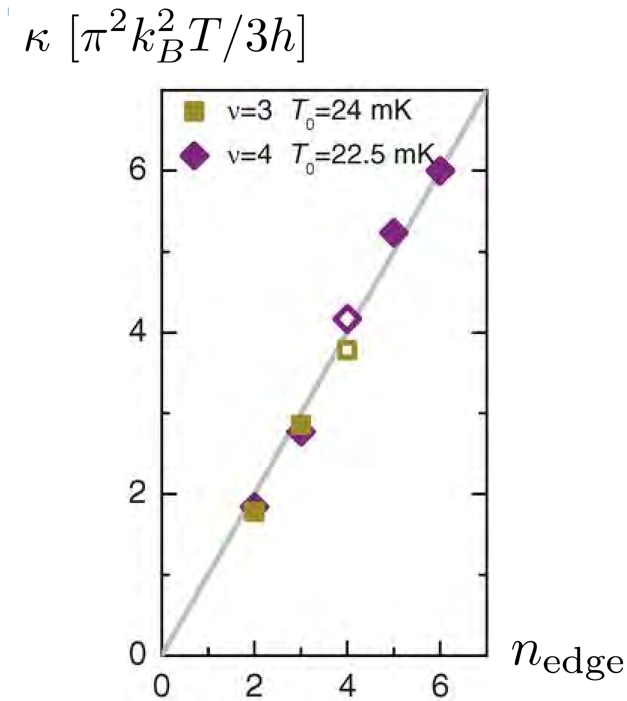


Hall conductance

$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$

Quantum Hall effect in a nutshell





Jezouin *et al.* (2013)

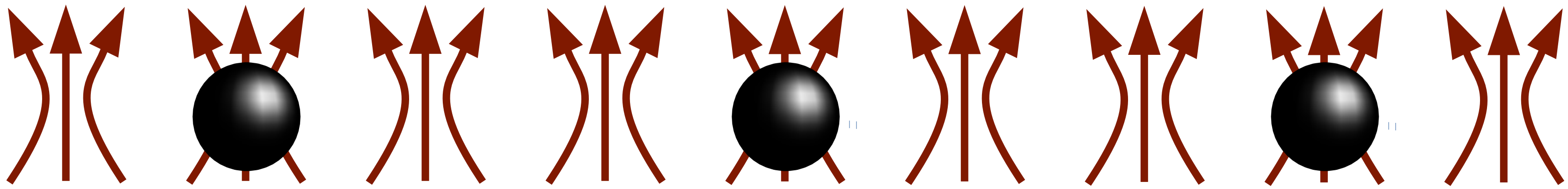
Hall conductance

$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$

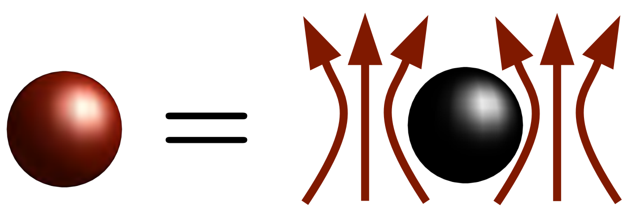
Thermal Hall conductance

$$\kappa_{xy} = n_{\text{edge}} \kappa_0$$

Fractional quantum Hall effect

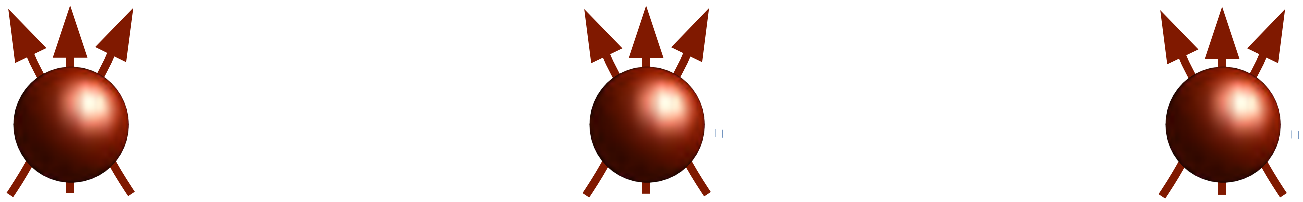


Fewer electrons than flux quanta: many possible states

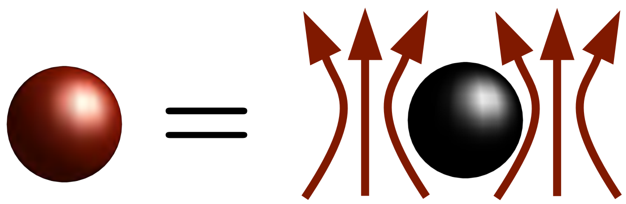


Composite fermions

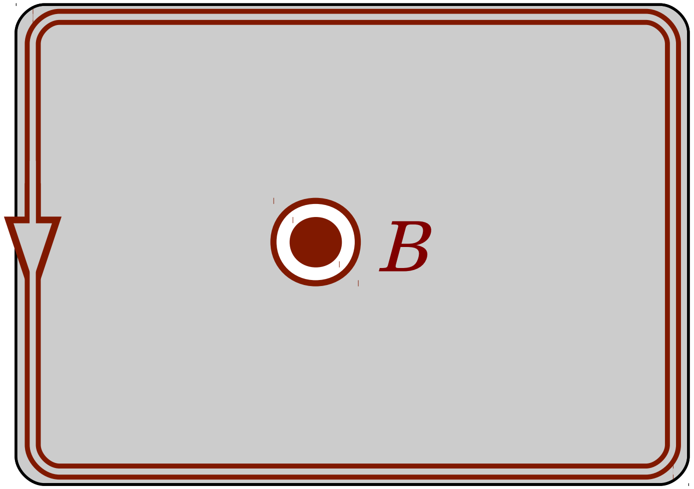
Fractional quantum Hall effect



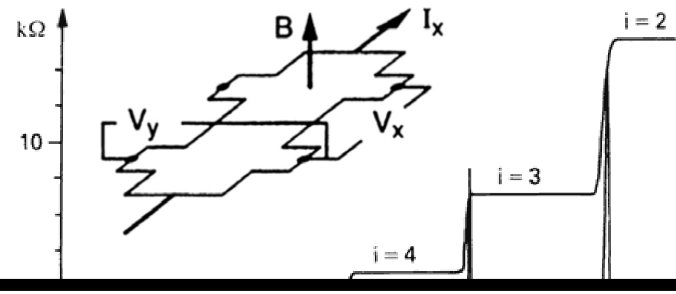
One composite fermions per flux quantum

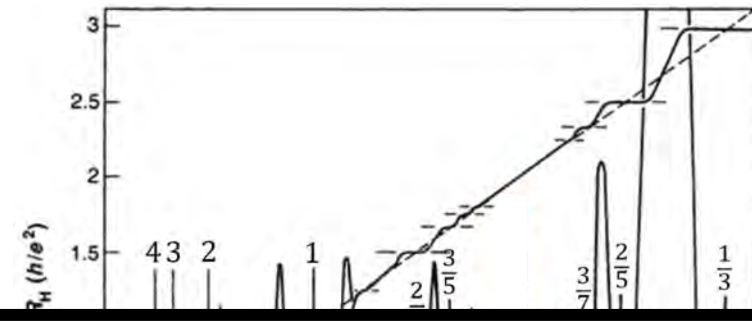


Composite fermions



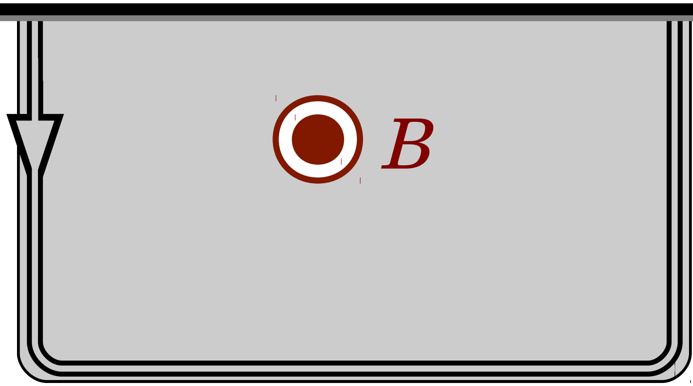
Fractional quantum Hall effect

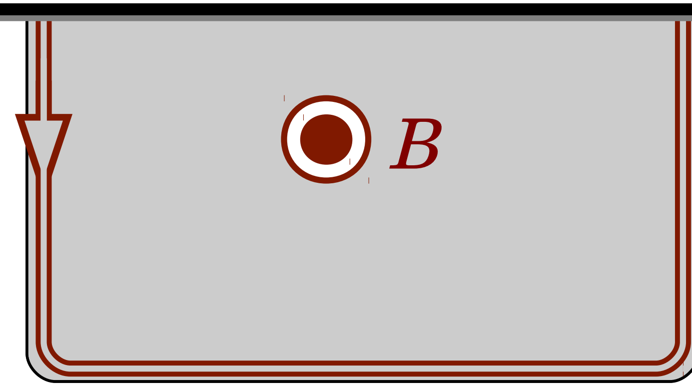




Any charge carrying edge state, fractional or integer, carries an integer thermal conductance κ_0

Theory: Kane and Fisher (1997)
Experiment: Banerjee *et al.* (2017)



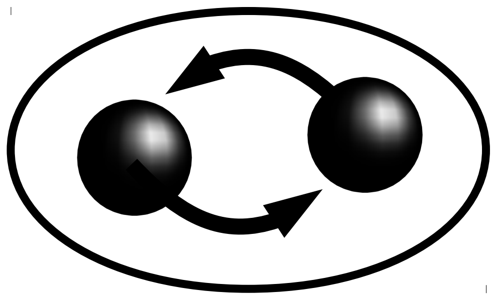


$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$
$$\kappa_{xy} = n_{\text{edge}} \kappa_0$$

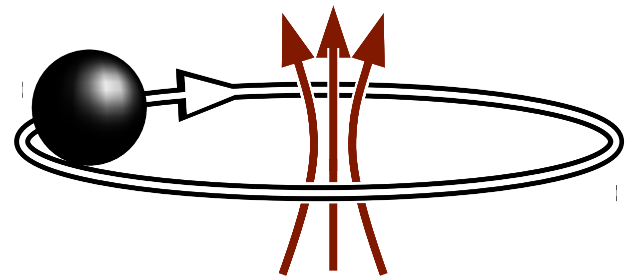
$$\sigma_{xy} = \frac{1}{3} \frac{e^2}{h}$$
$$\kappa_{xy} = \kappa_0$$

Topological superconductors

Pairing of spinless electrons

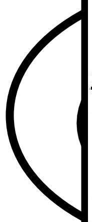


Electrons in magnetic field



Topological superconductors

Pairing



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D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
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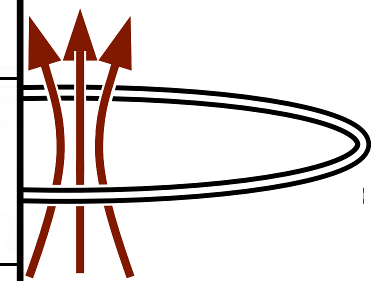
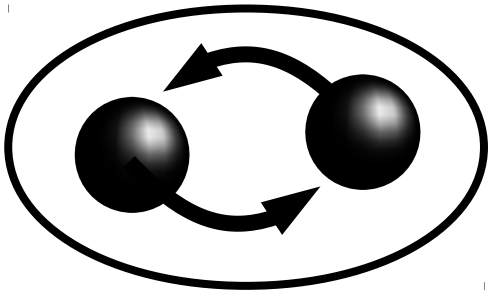
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

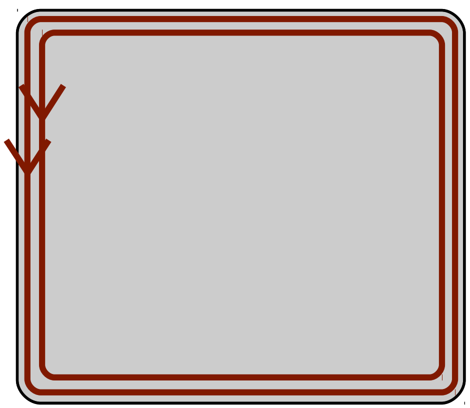
magnetic field



Topological superconductors

Pairing of spinless electrons

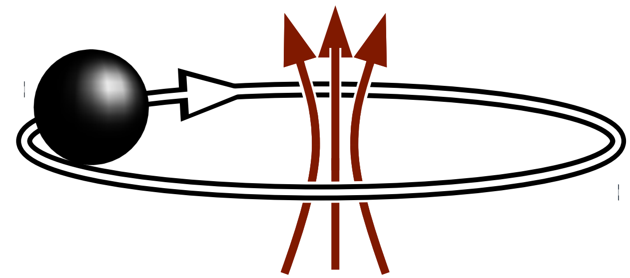


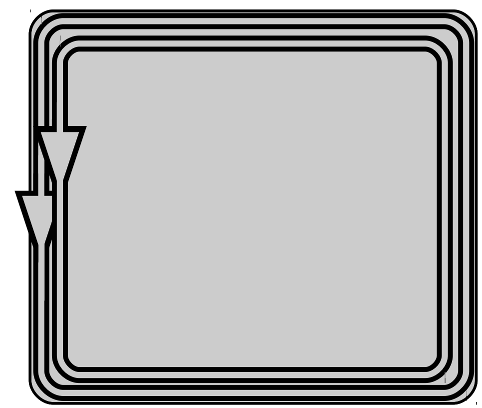


n_{Majorana} chiral Majoranas

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2}\kappa_0$$

Electrons in magnetic field



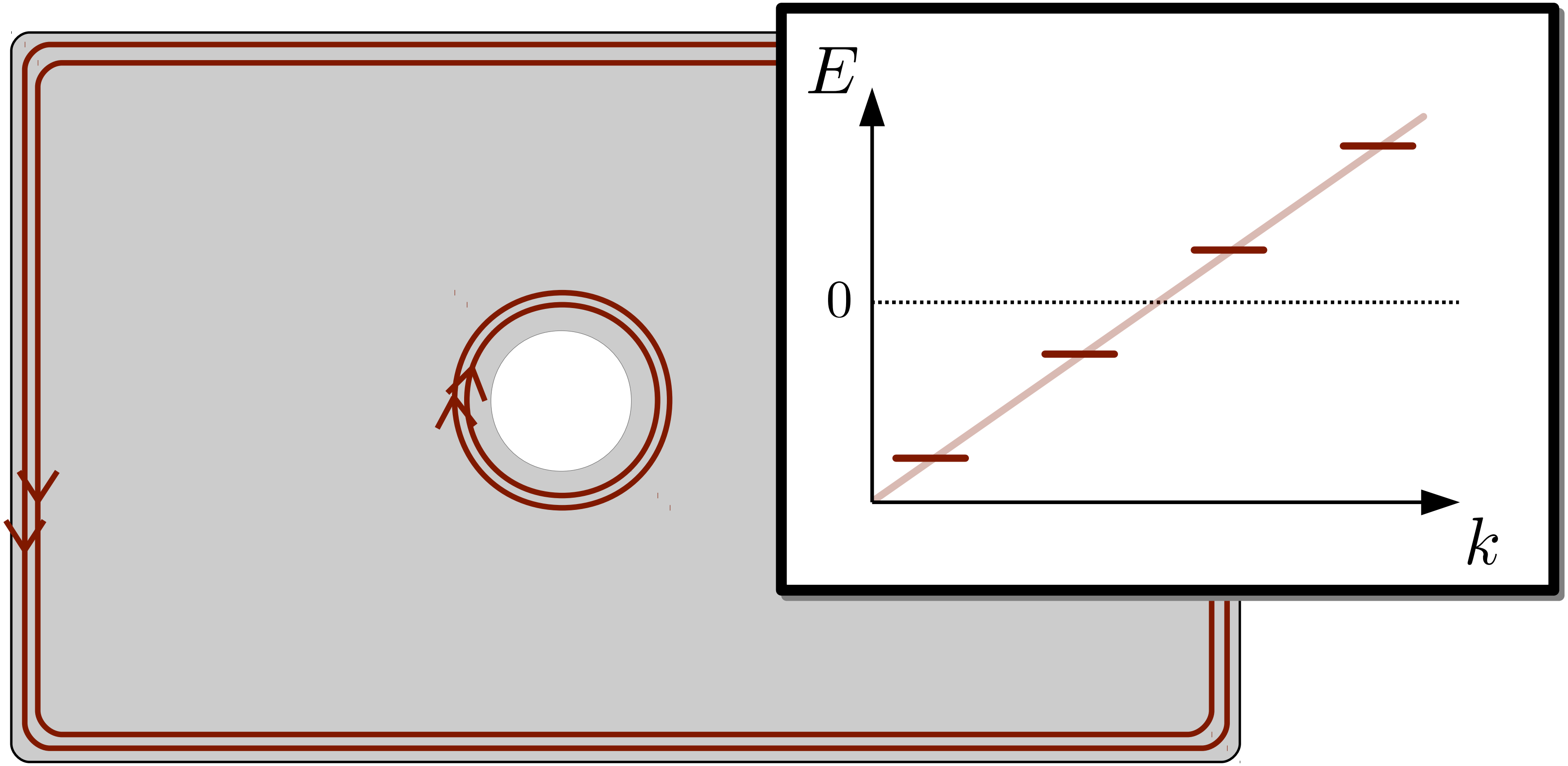


n_{edge} chiral electrons

$$\sigma_{xy} = n_{\text{edge}}\frac{e^2}{h}$$

$$\kappa_{xy} = n_{\text{edge}}\kappa_0$$

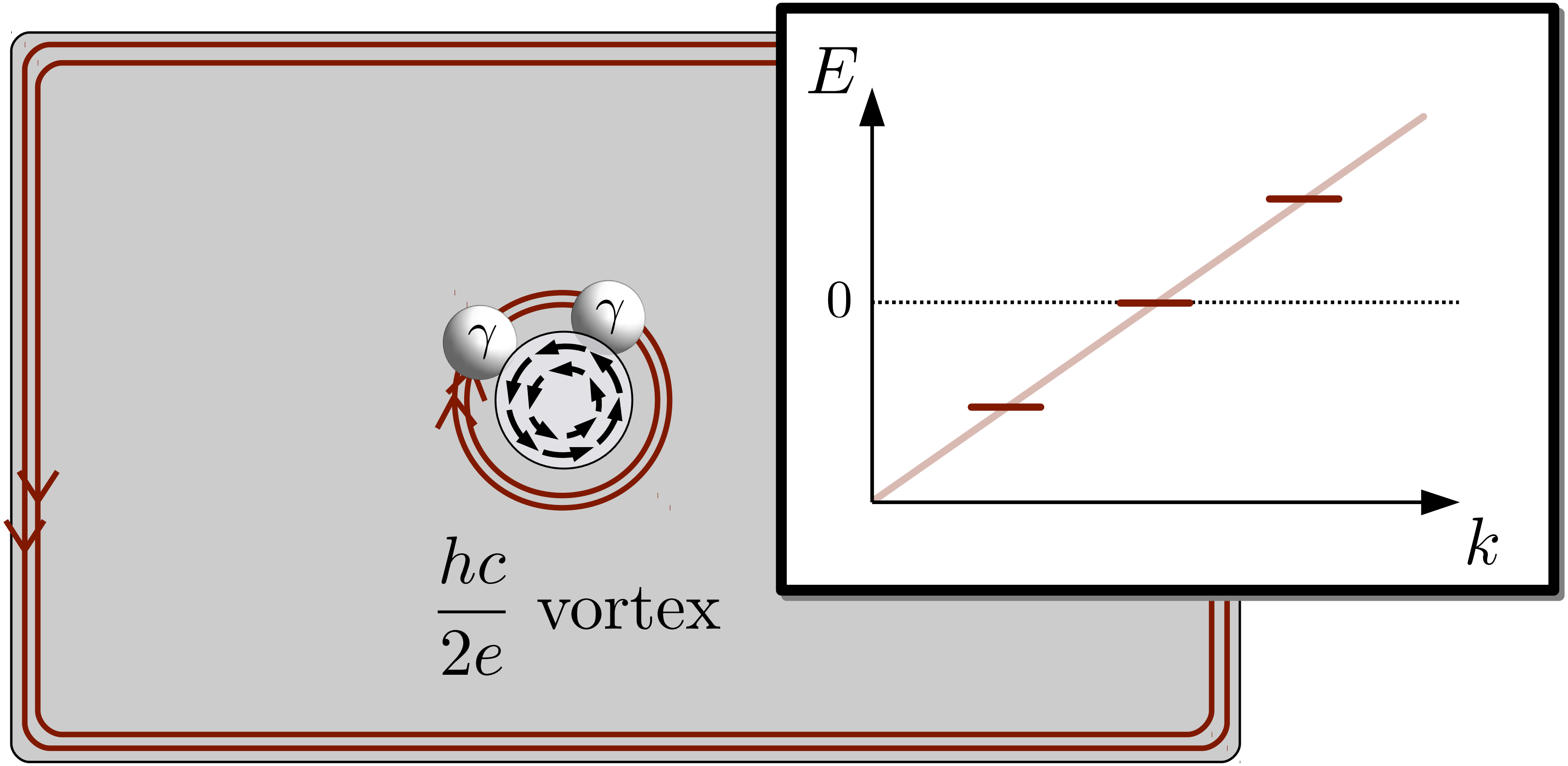
Topological superconductors



n_{Majorana} chiral Majoranas propagating at the edge (absolutely stable)

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2}\kappa_0$$

Topological superconductors

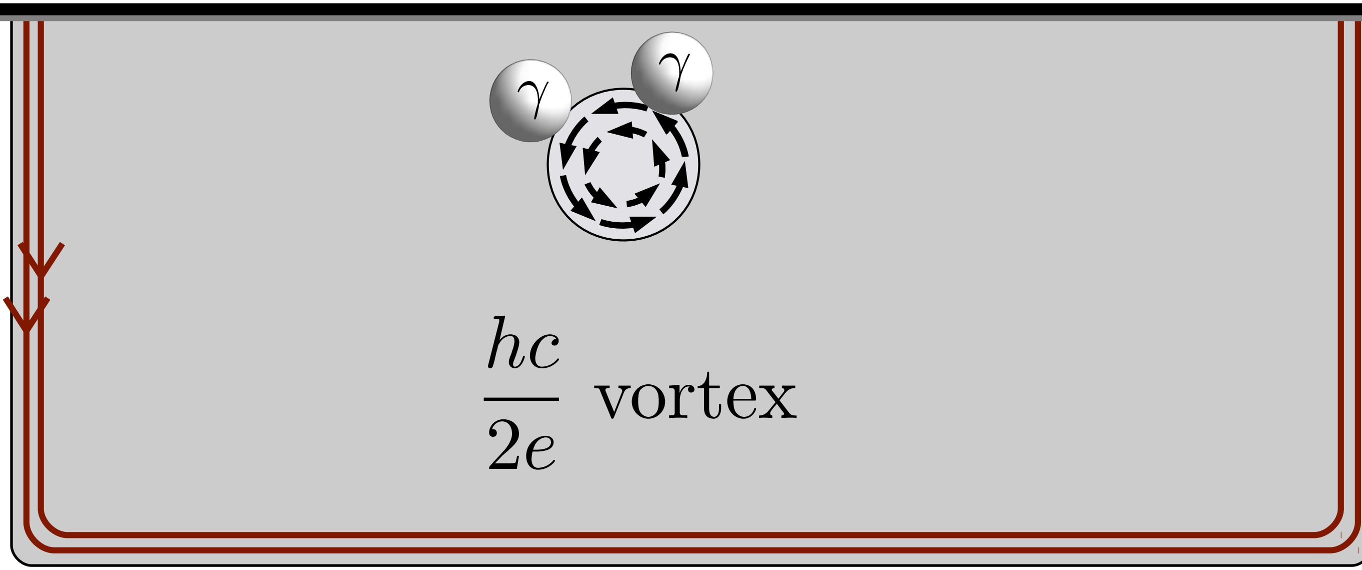


n_{Majorana} chiral Majoranas
propagating at the edge
(absolutely stable)

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2}\kappa_0$$

Topological superconductors

Half-odd integer κ_{xy} $\rightarrow$ Majorana zero modes

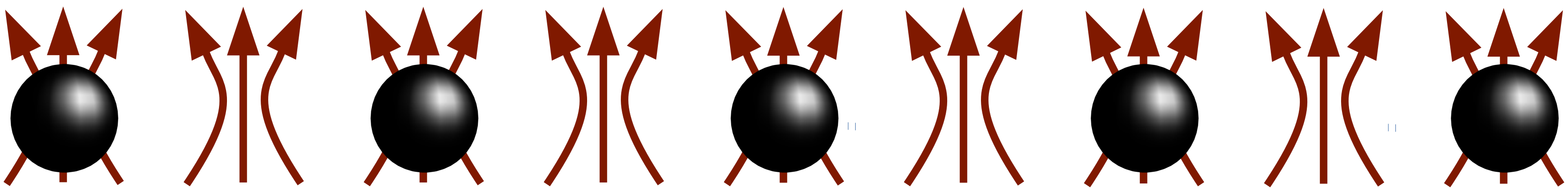


n_{Majorana} chiral Majoranas propagating at the edge (absolutely stable)

n_{Majorana} Majorana zero modes localized at a vortex (stable mod 2)

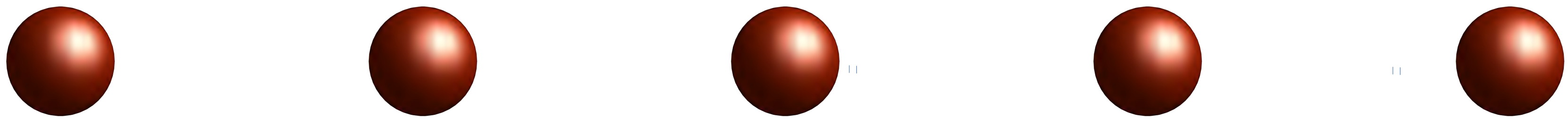
$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2} \kappa_0$$

Half-filled Landau level

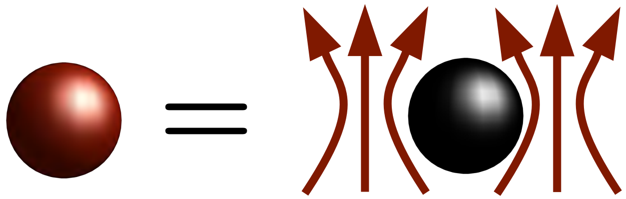


Electrons at $\nu = \frac{1}{2}$

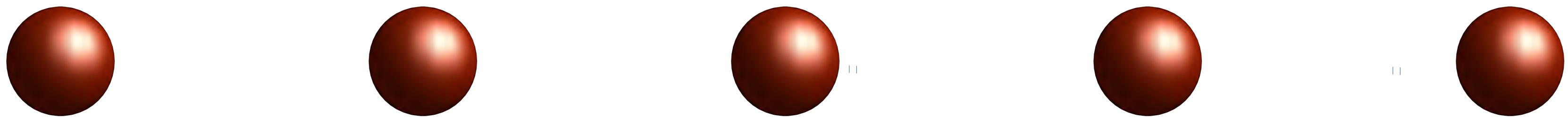
Half-filled Landau level



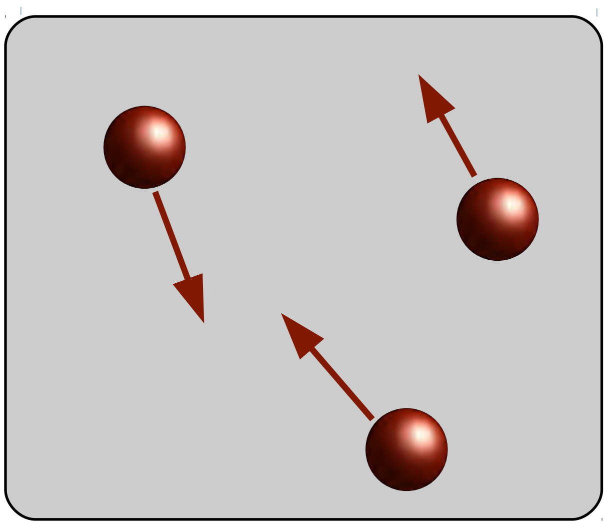
Composite fermions in zero flux



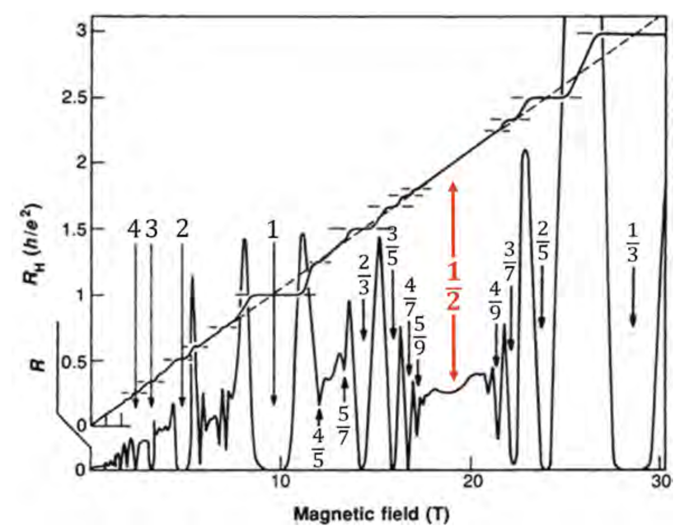
Half-filled Landau level



Composite fermions in zero flux

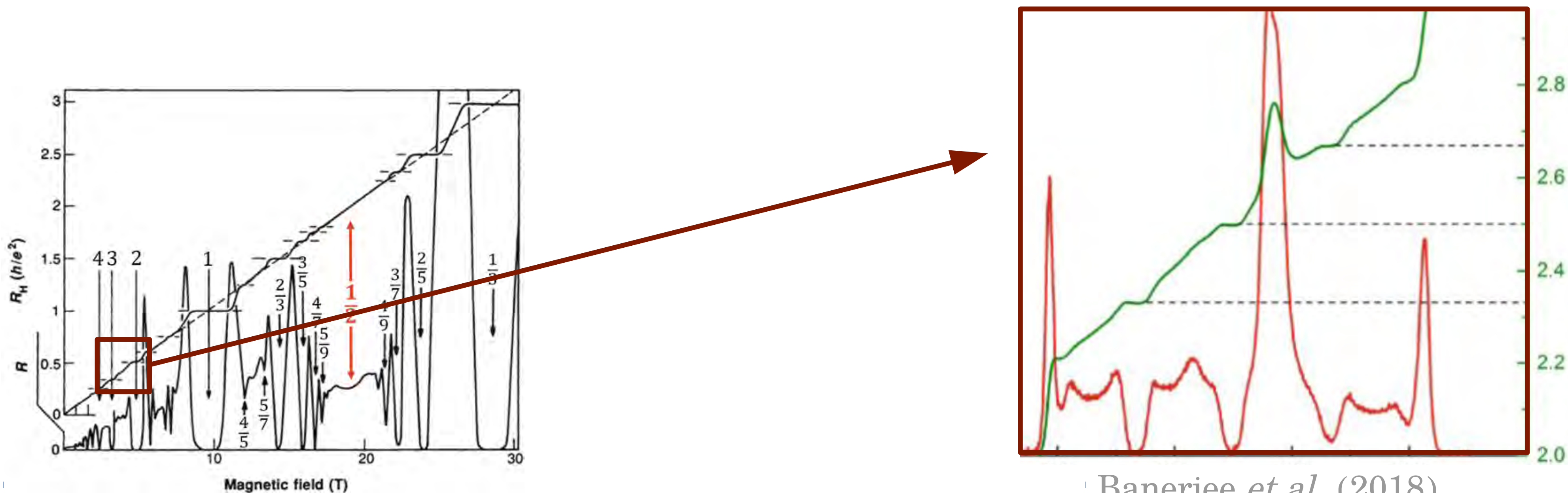


Halperin, Lee, and Read (1993)

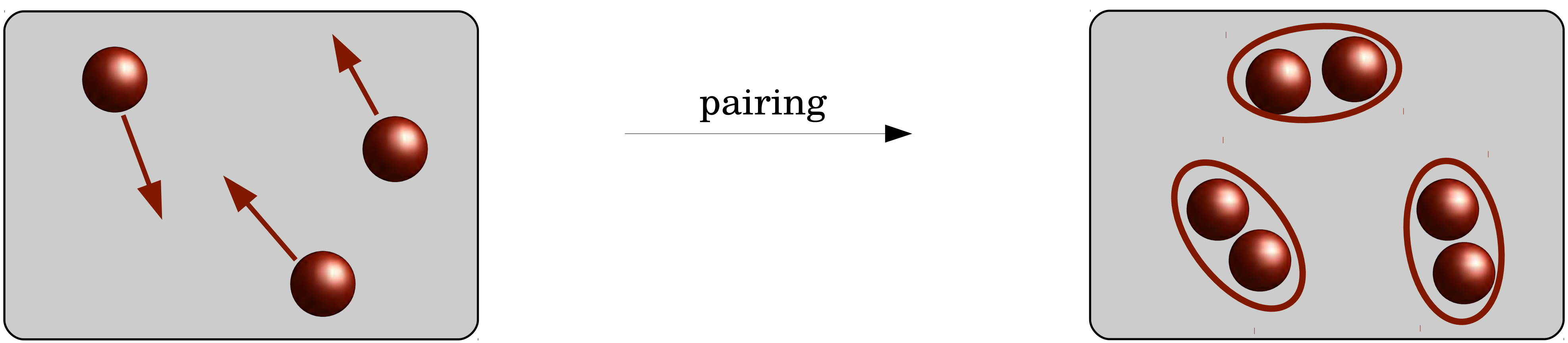


Composite fermions in zero effective magnetic field move in straight lines ⟶ form Fermi surface ⟶ metallic state at $\nu = \frac{1}{2}$

Half-filled Landau level



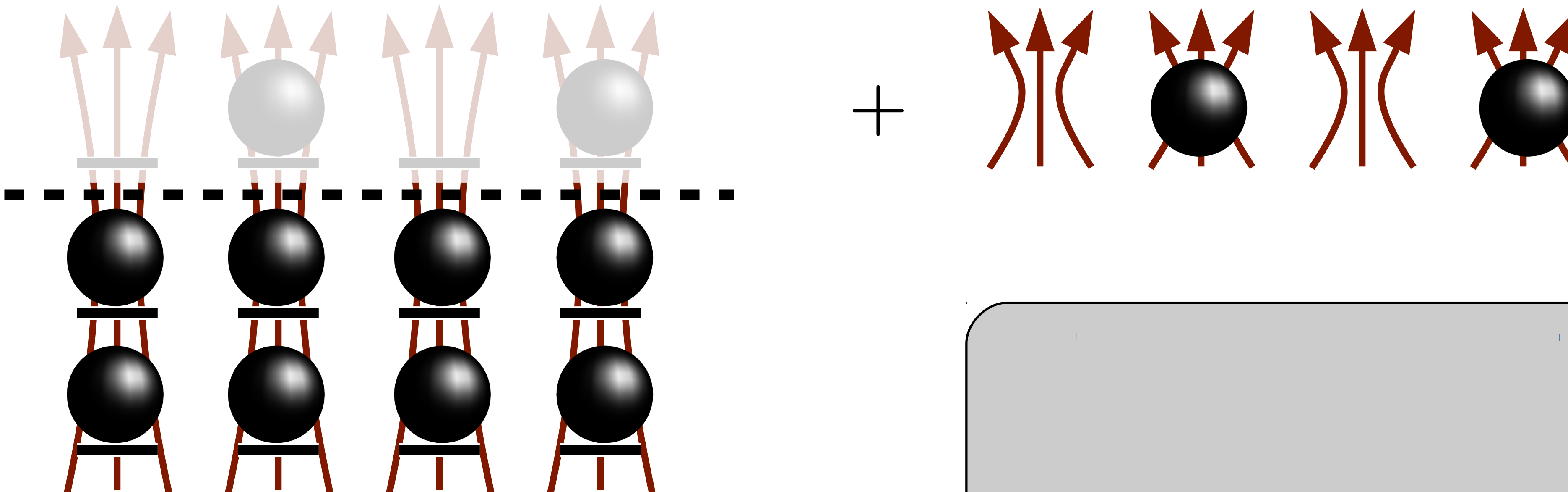
Banerjee *et al.* (2018)



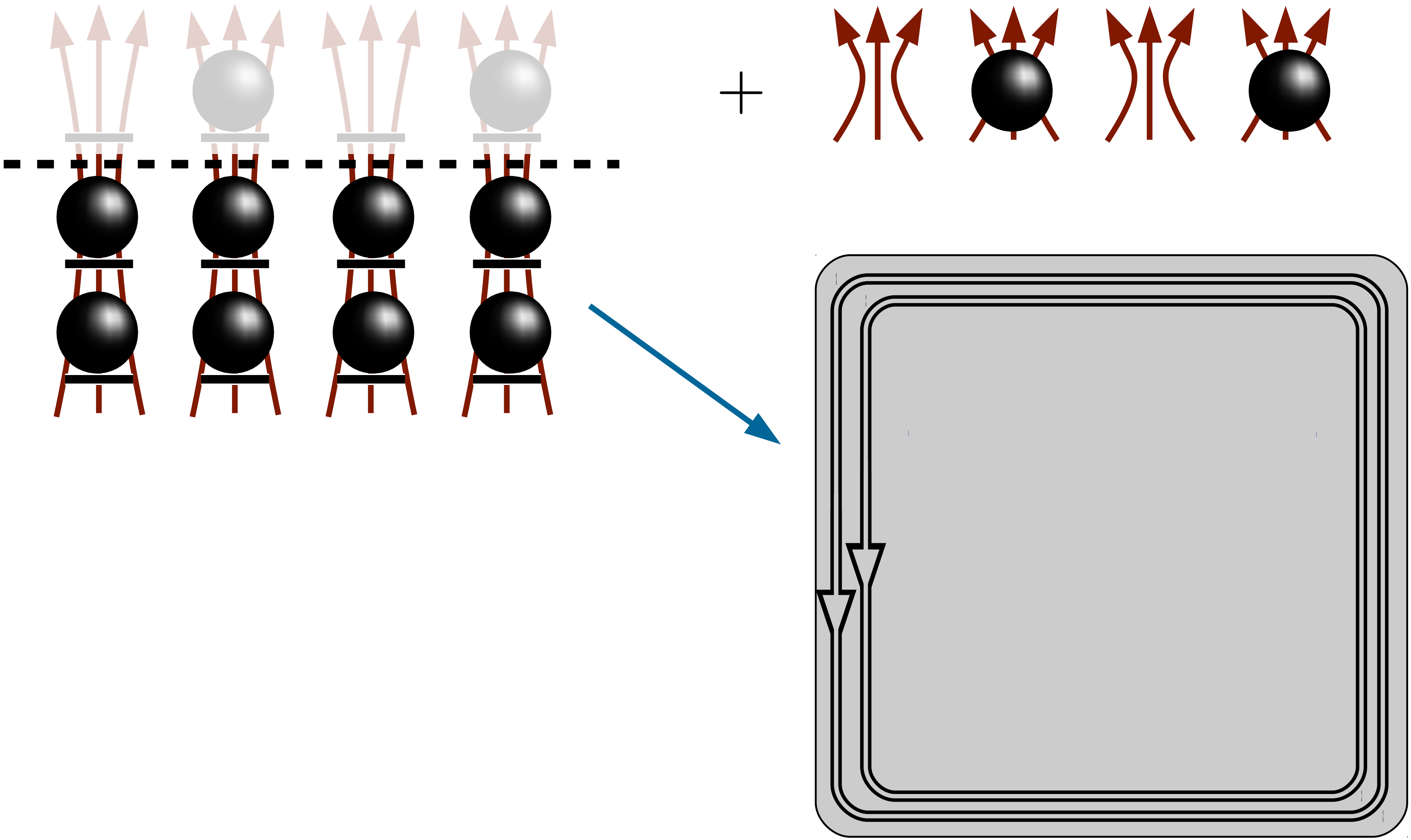
- Compressible state
- Hall conductance not quantized

- Incompressible state
- Quantized Hall conductance

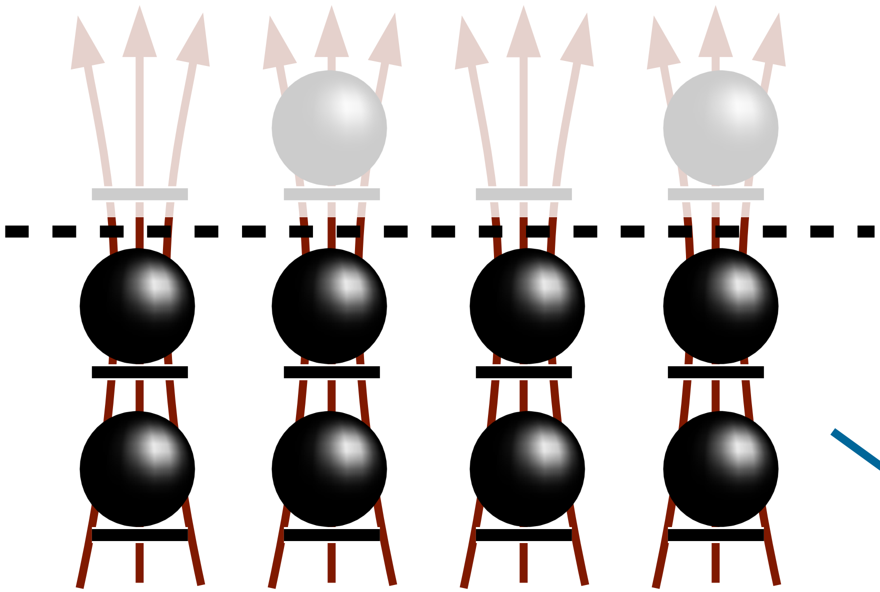
Electrons at $\nu=5/2$



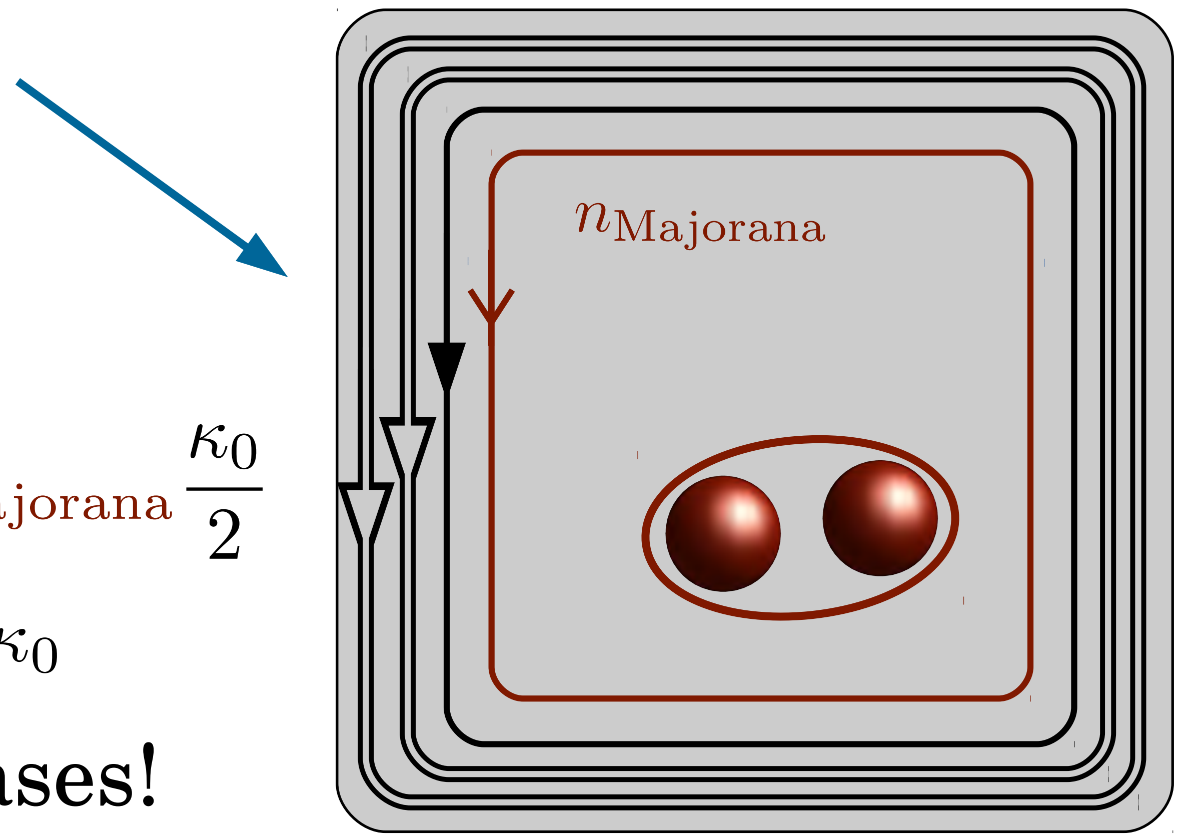
Electrons at ν=5/2



Electrons at ν=5/2



$+$



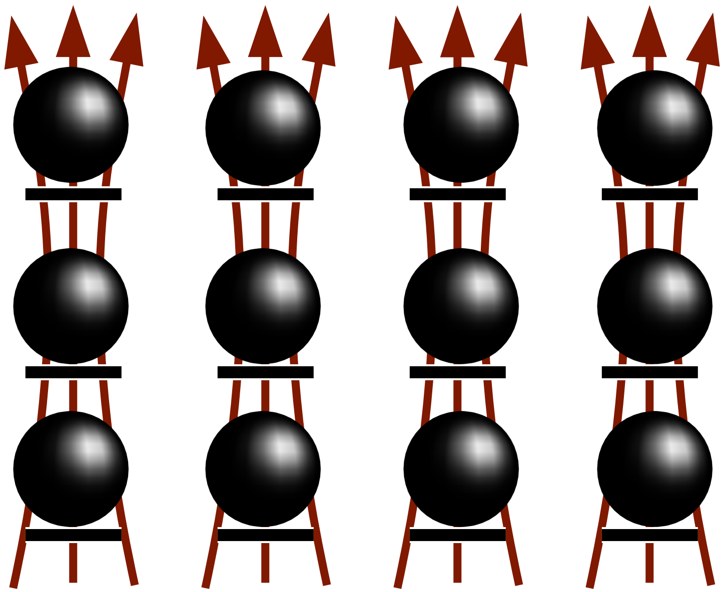
$$\sigma_{xy} = 2 * 1 + \frac{1}{2} = \frac{5}{2}$$

$$\kappa_{xy} = 2 * \kappa_0 + \kappa_0 + n_{\text{Majorana}} \frac{\kappa_0}{2}$$

$$= \left(3 + \frac{n_{\text{Majorana}}}{2}\right) \kappa_0$$

Many possible phases!

Particle-hole symmetry at ν=5/2



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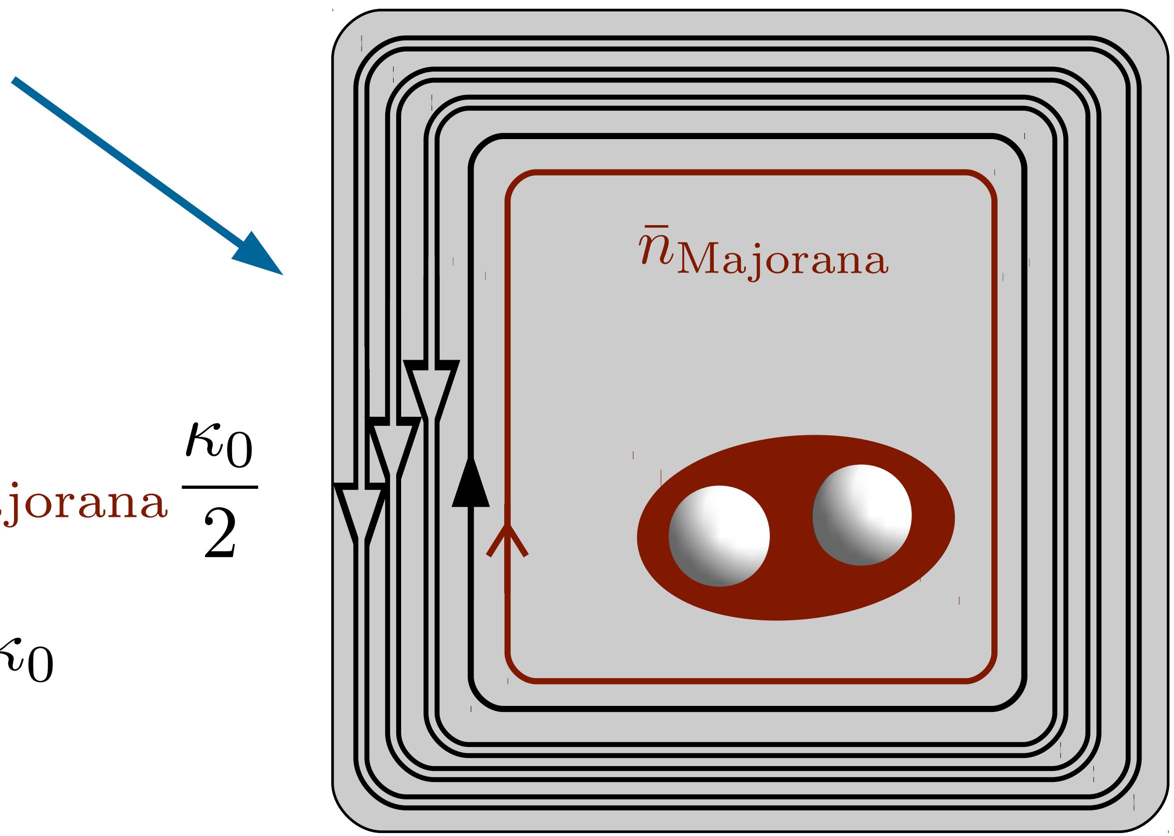


$$\sigma_{xy} = 3 * 1 - \frac{1}{2} = \frac{5}{2}$$

$$\kappa_{xy} = 3 * \kappa_0 - \kappa_0 - \bar{n}_{\text{Majorana}} \frac{\kappa_0}{2}$$

$$= \left(2 - \frac{\bar{n}_{\text{Majorana}}}{2}\right) \kappa_0$$

Particle-hole symmetry at ν=5/2

$$\left(3 + \frac{n_{\text{Majorana}}}{2}\right)\kappa_0 = \left(2 - \frac{\bar{n}_{\text{Majorana}}}{2}\right)\kappa_0$$

Particle-hole transformation:

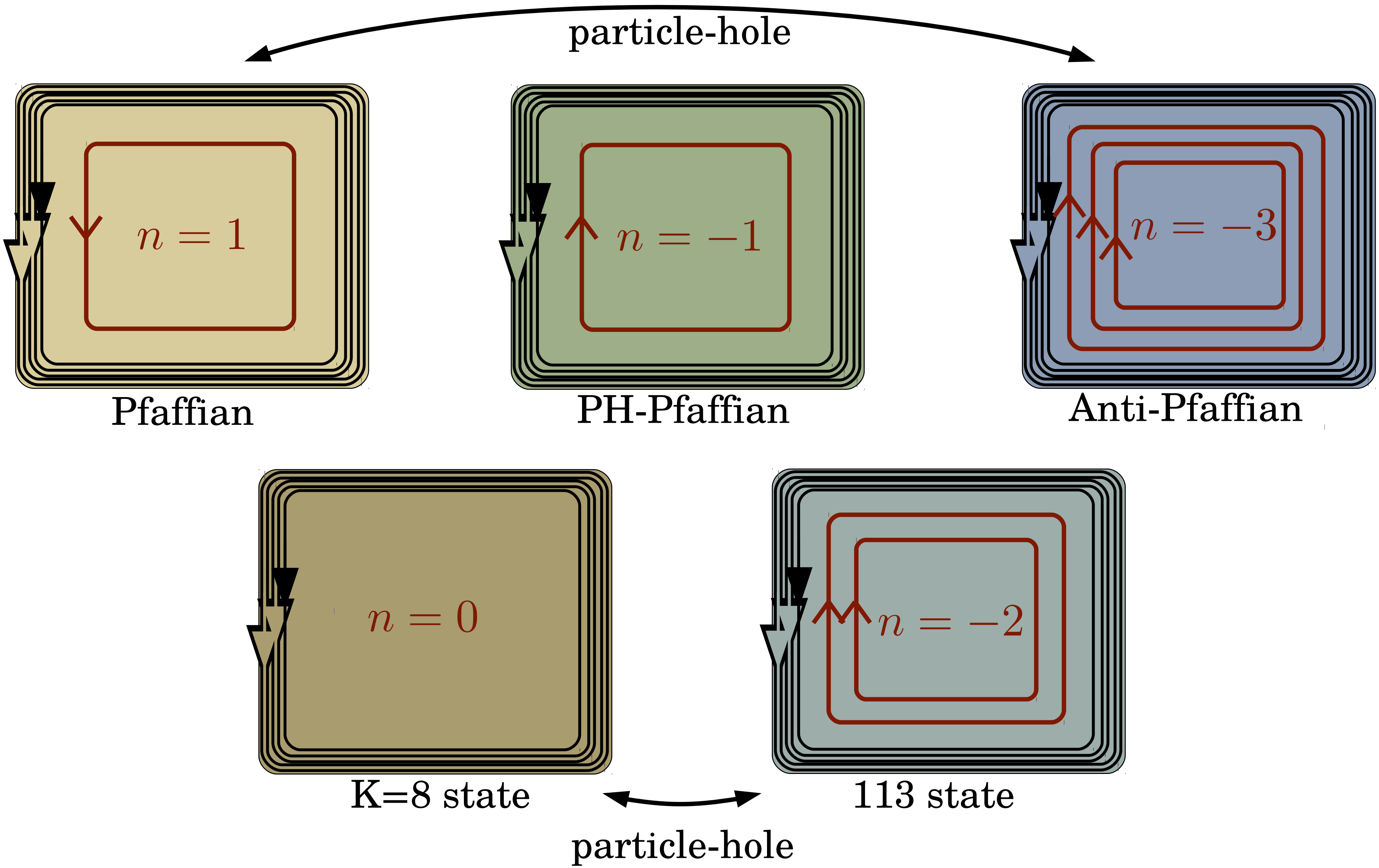
$$P_{\text{electron}} : n_{\text{Majorana}} \to -2 - n_{\text{Majorana}}$$
$$\Psi_{\text{Dirac CF}} \to i\sigma^y \Psi_{\text{Dirac CF}}$$

- acts as time reversal on composite fermions
- may or may not be present

$$P_{\text{composite fermion}} : n_{\text{Majorana}} \to n_{\text{Majorana}}$$

- always present in any composite fermion superconductor

Electrons at ν=5/2



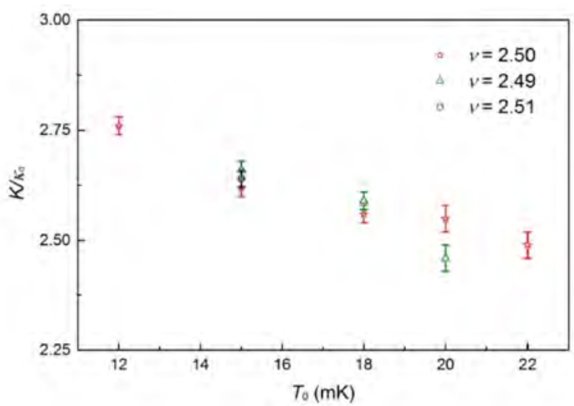
Input from experiment

ARTICLE

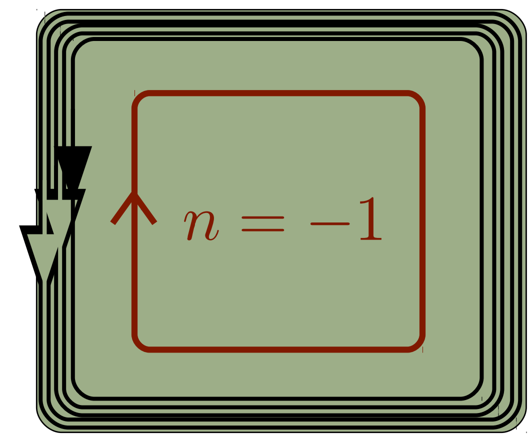
https://doi.org/10.1038/s41586-018-0184-1

Observation of half-integer thermal Hall conductance

Mitali Banerjee[1], Moty Heiblum[1*], Vladimir Umansky[1], Dima E. Feldman[2], Yuval Oreg[1] & Ady Stern[1]



$$K/\kappa_0 = 3 + \frac{n_{\text{Majorana}}}{2}$$



PH-Pfaffian

PRL 117, 096802 (2016) PHYSICAL REVIEW LETTERS week ending 26 AUGUST 2016

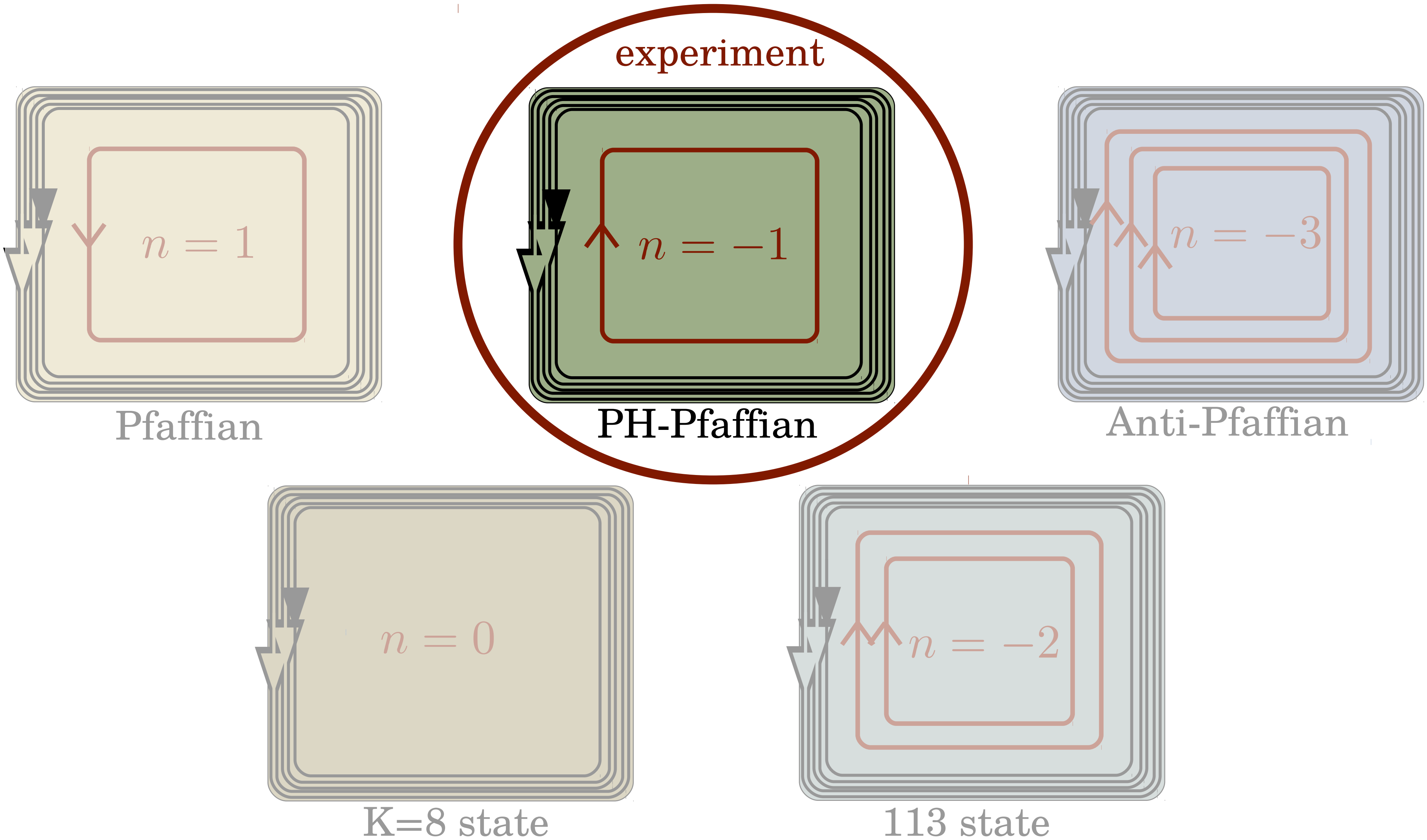
Stabilization of the Particle-Hole Pfaffian Order by Landau-Level Mixing and Impurities That Break Particle-Hole Symmetry

P. T. Zucker and D. E. Feldman
Department of Physics, Brown University, Providence, Rhode Island 02912, USA

(Received 30 March 2016; published 22 August 2016)

Numerical results suggest that the quantum Hall effect at $\nu = 5/2$ is described by the Pfaffian or anti-Pfaffian state in the absence of disorder and Landau-level mixing. Those states are incompatible with the observed transport properties of GaAs heterostructures, where disorder and Landau-level mixing are strong. We show that the recent proposal of a particle-hole (PH)-Pfaffian topological order by Son is consistent with all experiments. The absence of particle-hole symmetry at $\nu = 5/2$ is not an obstacle to the

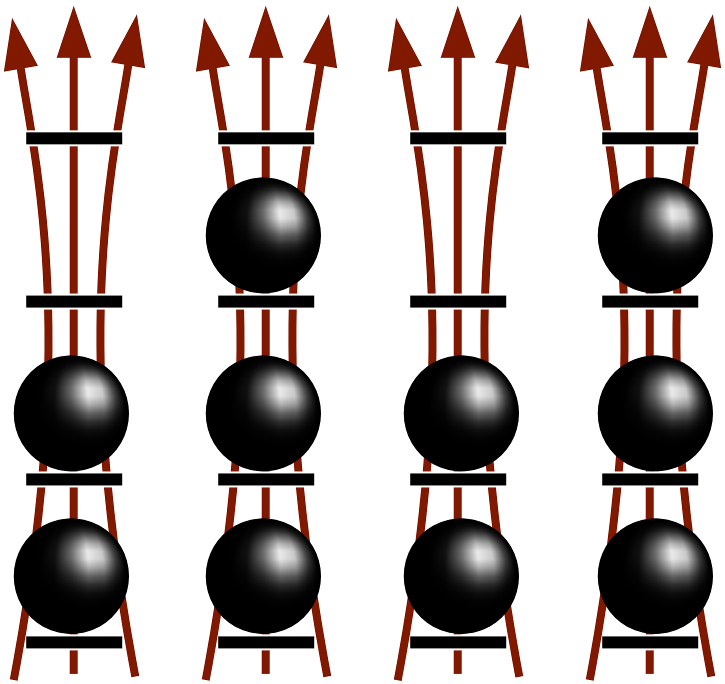
Electrons at ν=5/2



Input from theory

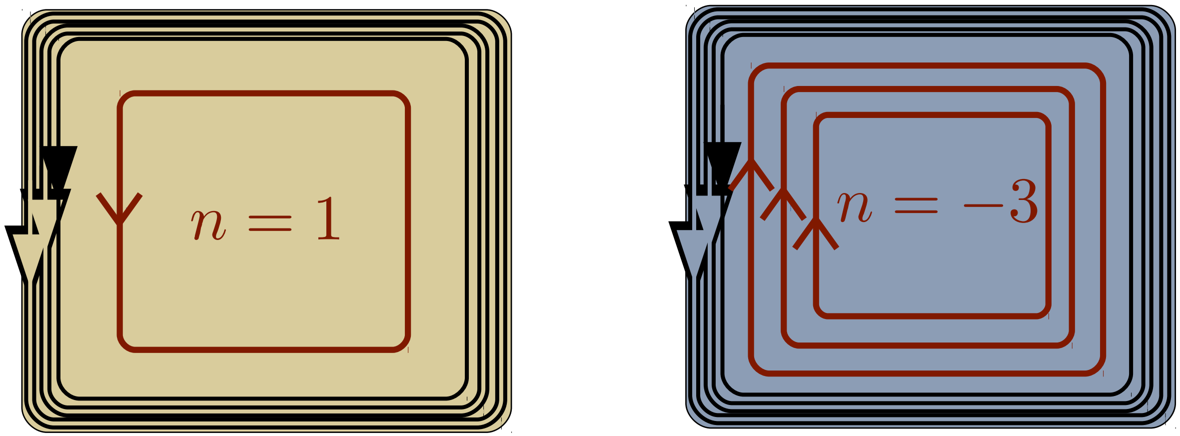
1. Exact Diagonalization and DMRG

Morf (1998), Rezayi, Haldane (2000), Peterson, Jolicoeur, Das Sarma (2008)
Feiguin, Rezayi, Nayak, Das Sarma (2008), Feiguin *et al.* (2009)
Storni, Morf, Das Sarma (2010), Wójs, Tőke, Jain (2010), Rezayi, Simon (2011)
Papić, Haldane, Rezayi (2012), Pakrouski *et al.* (2015)
Zaletel, Mong, Pollmann, Rezayi (2015), Rezayi (2017).



Without particle-hole symmetry:

Pfaffian or Anti-Pfaffian



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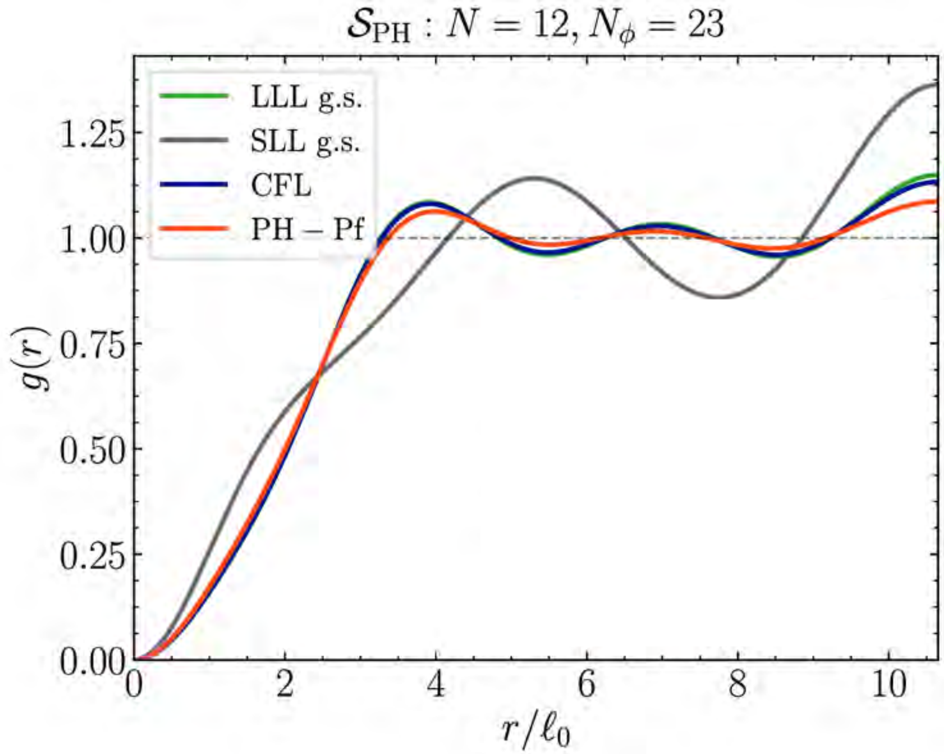


$N = 12$ particles

$|\langle \Psi_{\text{CFL}} | \Psi_{\text{PH-Pfaffian}} \rangle|^2 \approx 0.91$

$|\langle \Psi_{\text{CFL}} | \Psi_{\text{MR-Pfaffian}} \rangle|^2 \approx 0.38$

Mishmash, **DFM**, Alicea, Motrunich (2018)

2. Trial wavefunctions for PH-Pfaffian

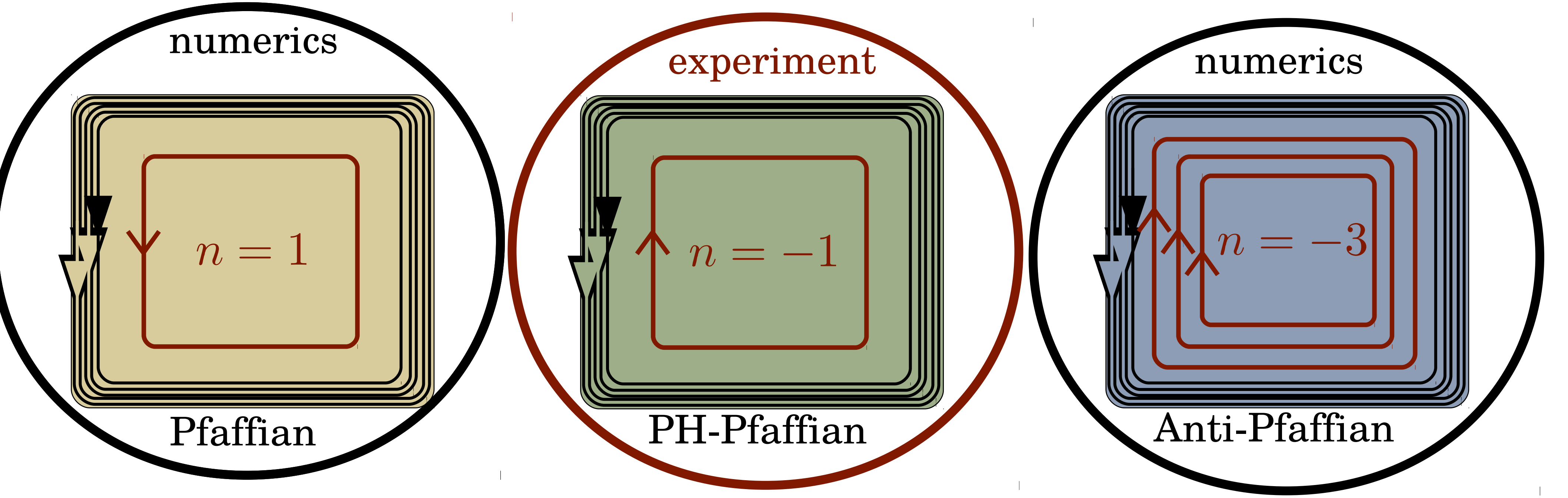
$$\Psi_{\text{PH-Pfaffian}} = \mathcal{P}_{\text{LLL}} \text{Pf}\left[\frac{1}{z_i^* - z_j^*}\right] \prod_{i<j} (z_i - z_j)^2$$

Zucker, Feldmann (2016)

- almost zero overlap with Coulomb in 2^{nd} LL
- high overlap with gapless Composite Fermi Liquid

Balram, Barkeshli, Rudner (2018)
Mishmash, **DFM**, Alicea, Motrunich (2018)

Electrons at ν=5/2



Possible resolutions:

‘numerics are wrong’

- Incorrect Hamiltonian
- Finite size not representative

‘experiment is wrong’

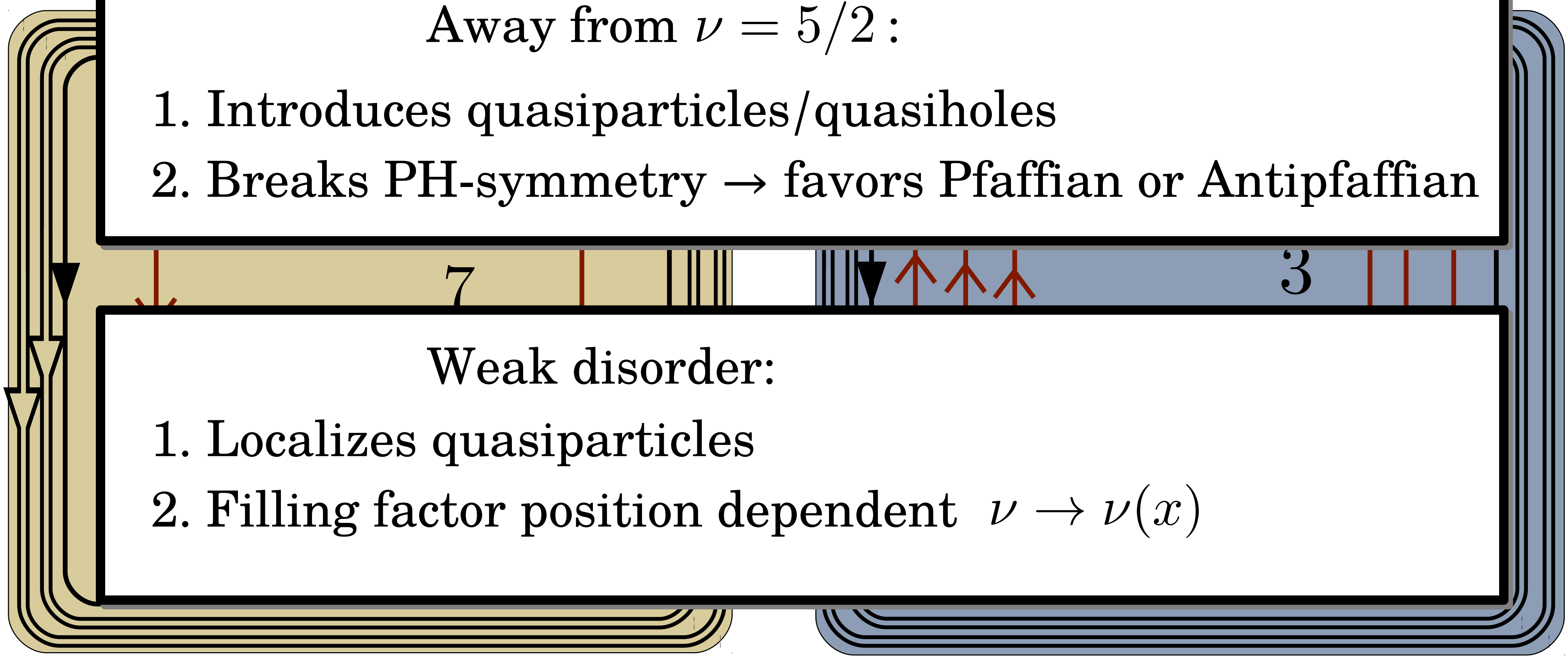
- Alternative interpretation possible?

Simon (2018), Feldman (2018),
Ma, Feldman (2018)

Can both be right?

Electrons at ν=5/2

Numerics: In **clean** system, Pfaffian or Antipfaffian



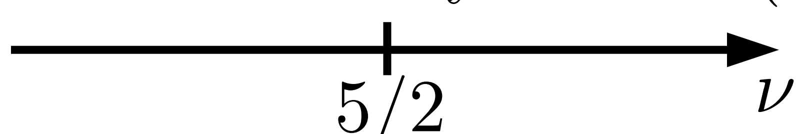
Away from $\nu = 5/2$:

1. Introduces quasiparticles/quasiholes
2. Breaks PH-symmetry → favors Pfaffian or Antipfaffian

Weak disorder:

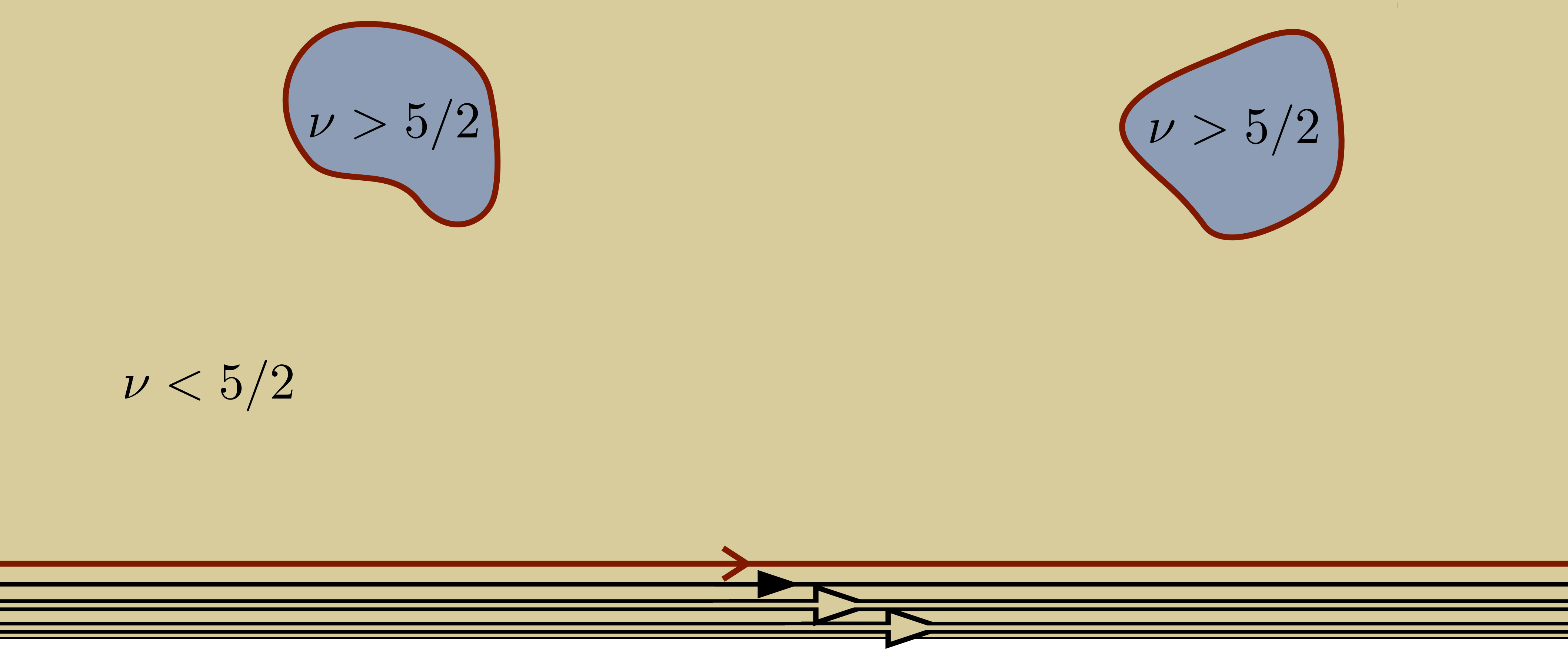
1. Localizes quasiparticles
2. Filling factor position dependent $\nu \to \nu(x)$

degenerate when PH-symmetric ($\nu = 5/2$)

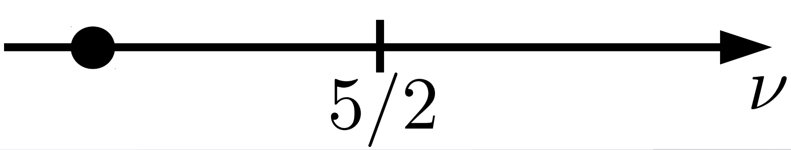


Electrons at ν=5/2

With disorder: Regions of Pfaffian and Antipfaffian

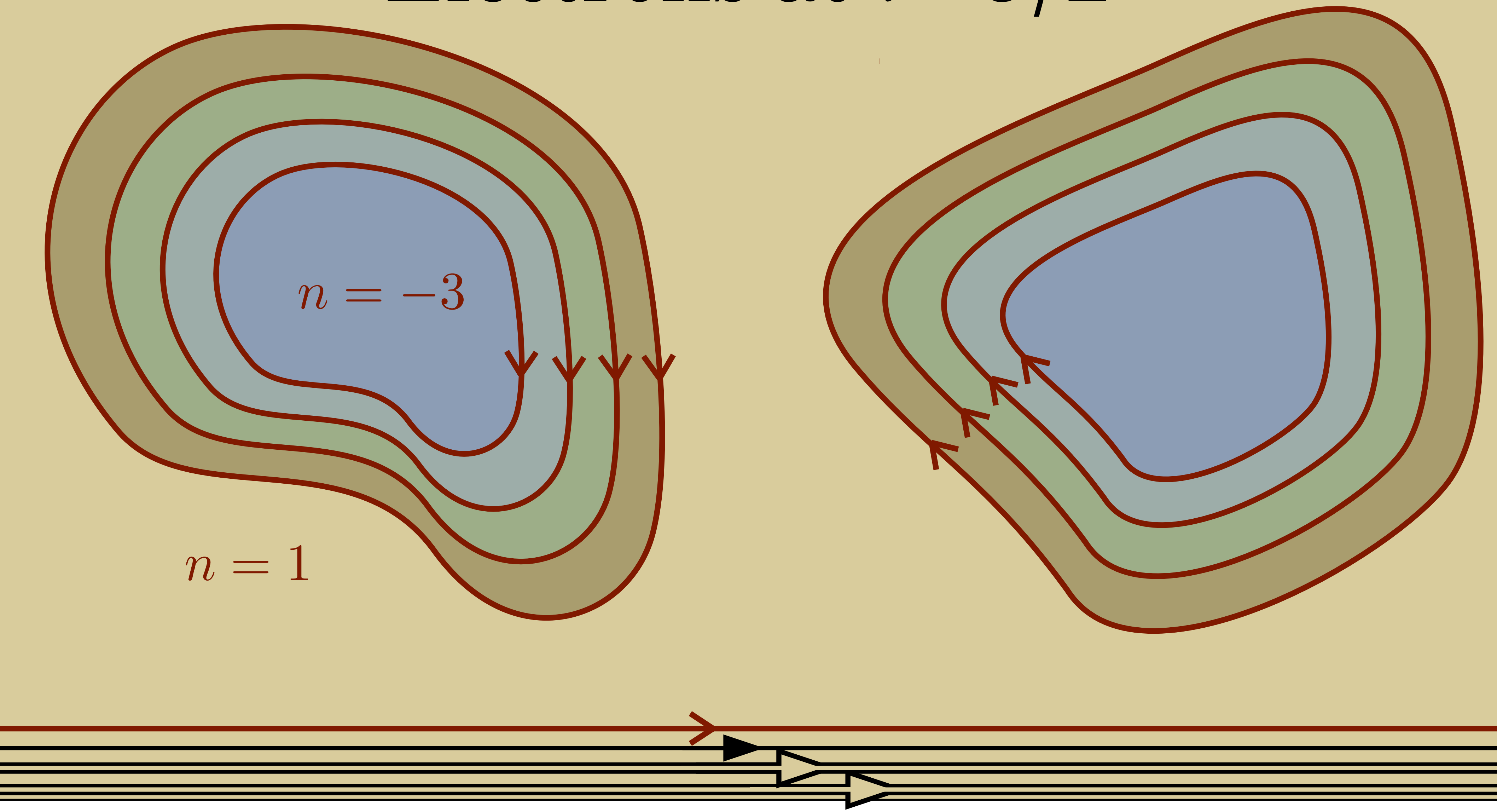


$\kappa_{xy} = \frac{7}{2}$

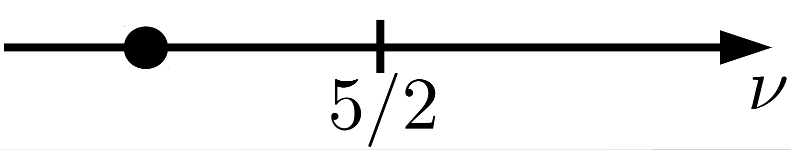


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

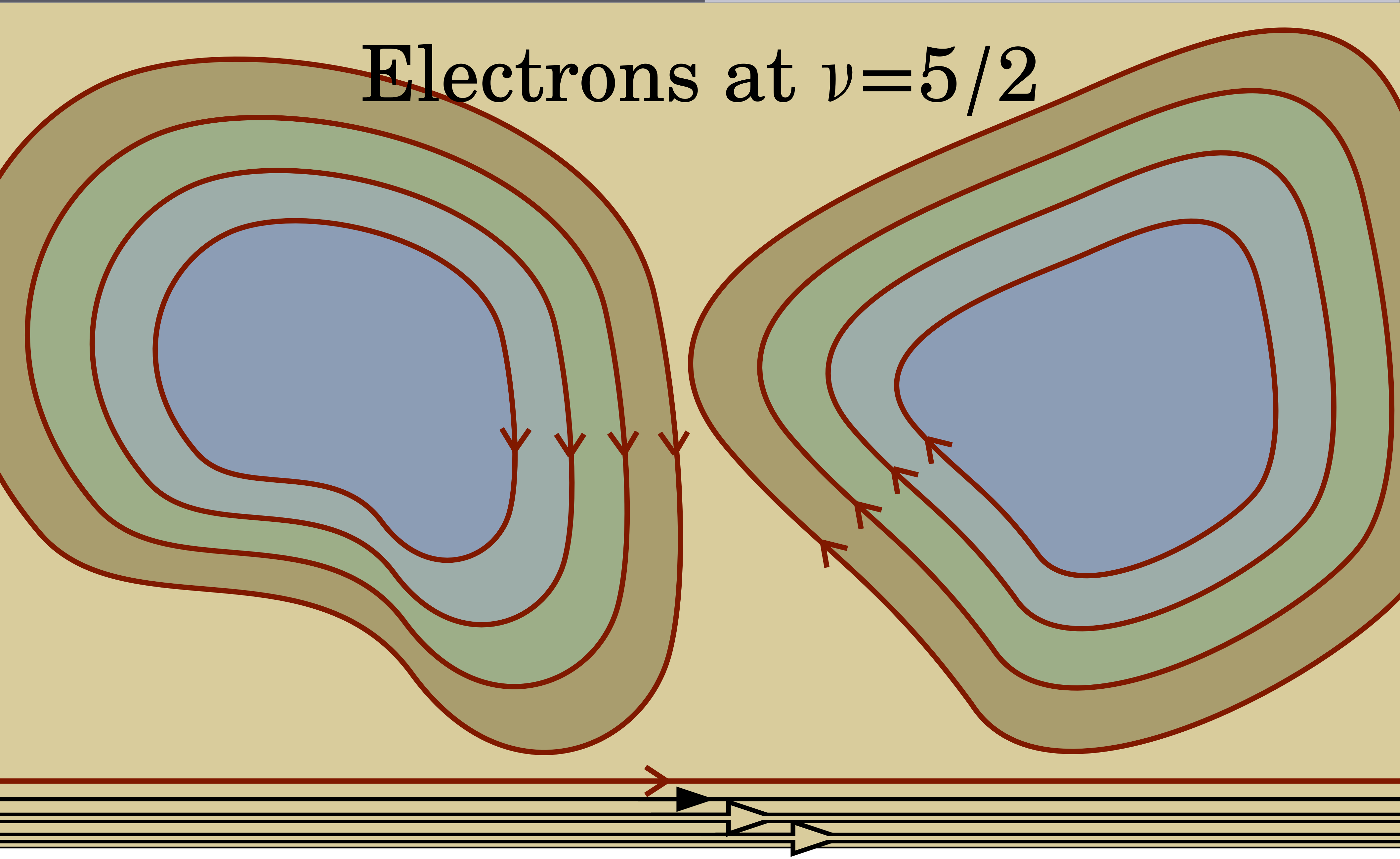


$\kappa_{xy} = \frac{7}{2}$

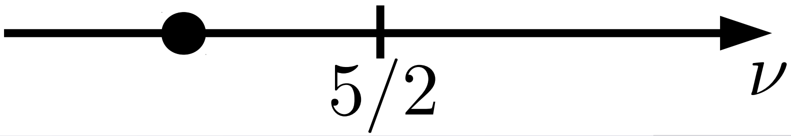


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

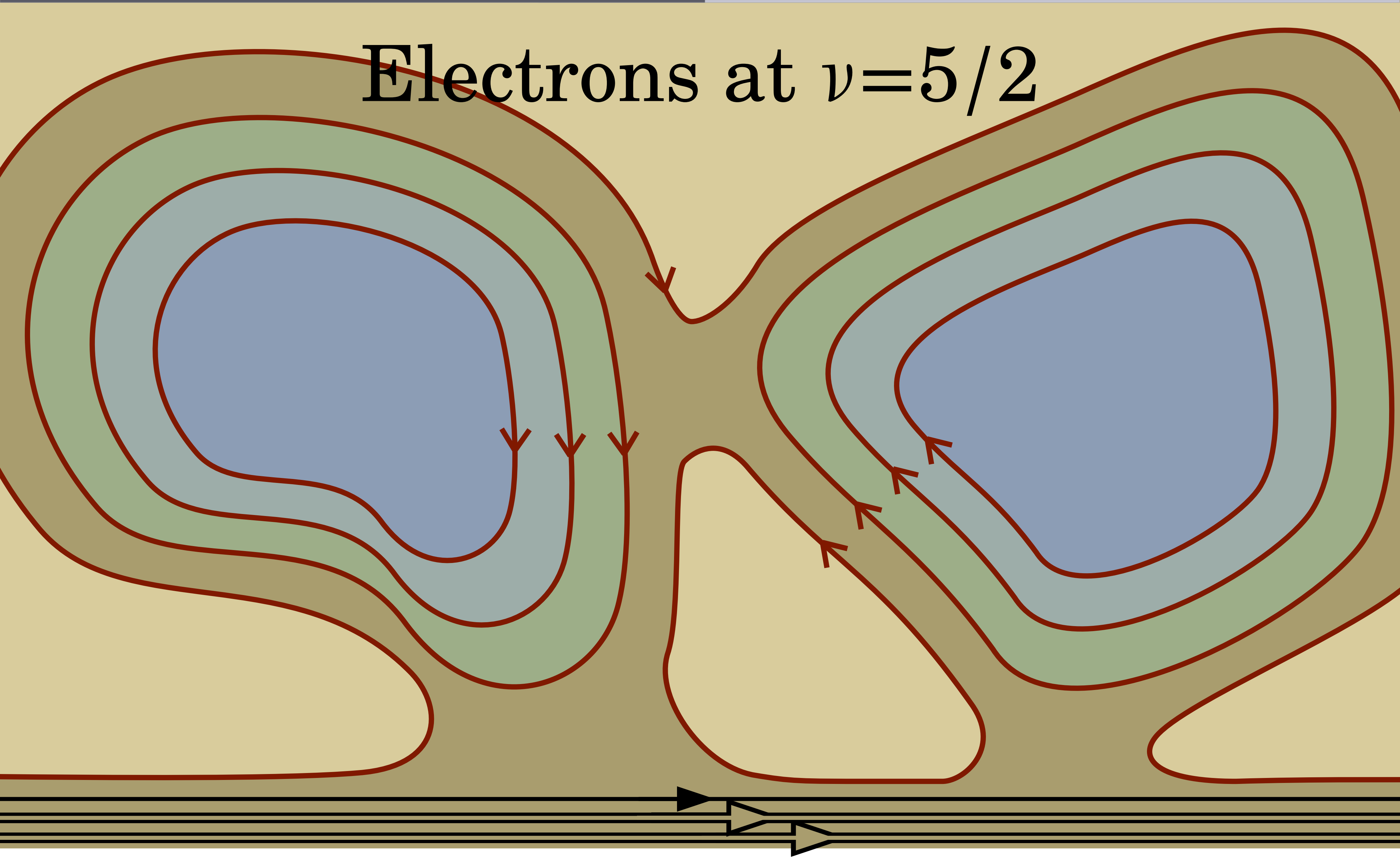


$$\kappa_{xy} = \frac{7}{2}$$

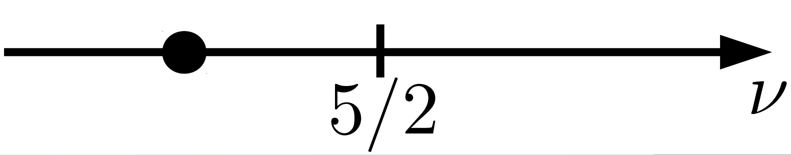


$$\sigma_{xy} = \frac{5}{2}$$

Electrons at ν=5/2

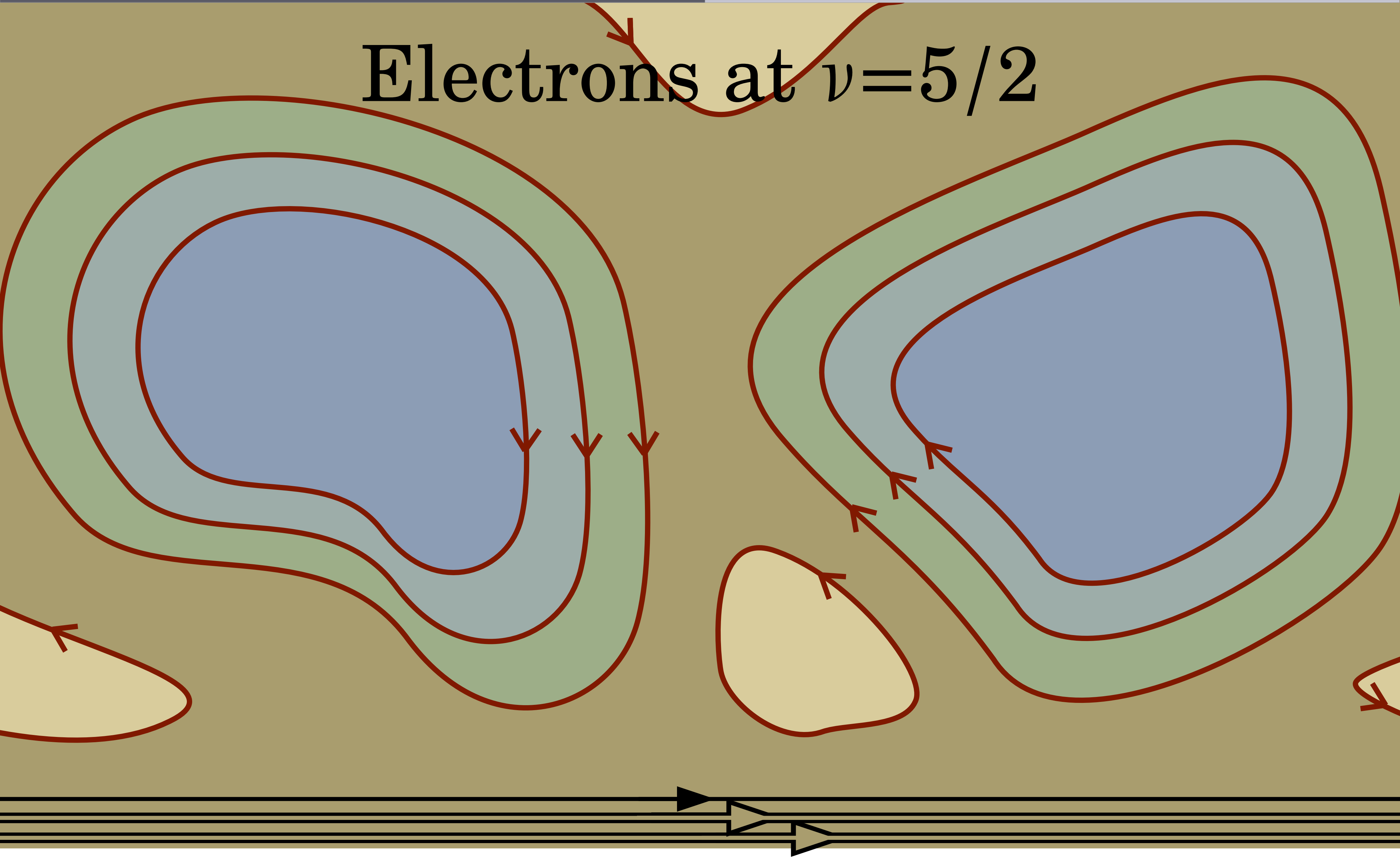


κ_{xy}
not quantized

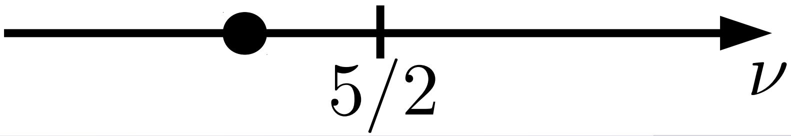


$$\sigma_{xy} = \frac{5}{2}$$

Electrons at $\nu=5/2$

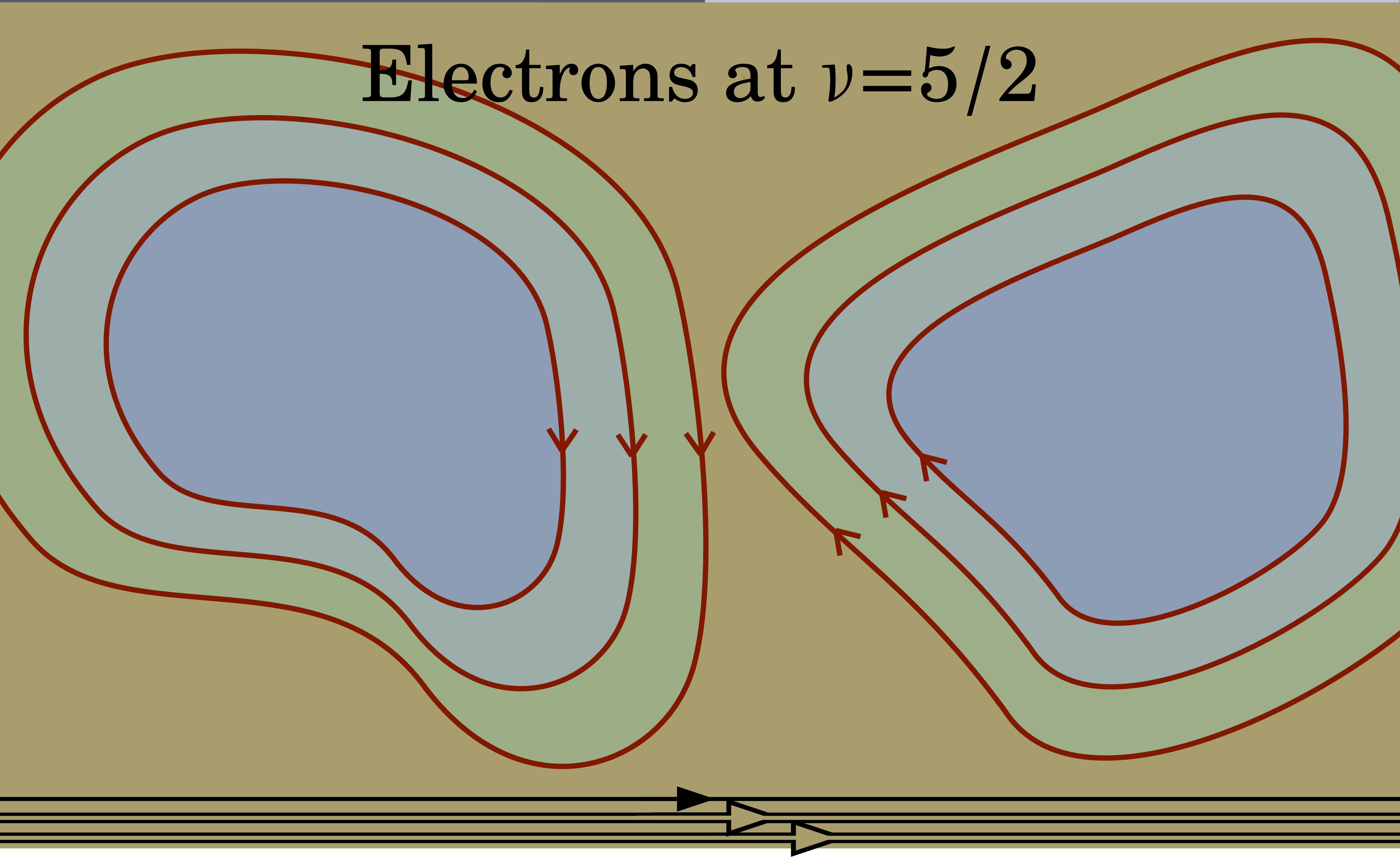


$\kappa_{xy} = 3$

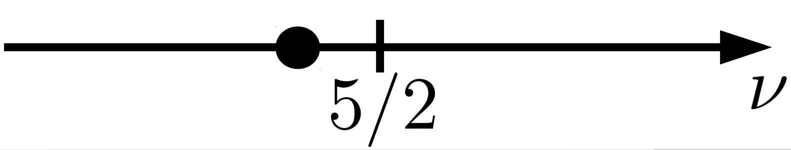


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

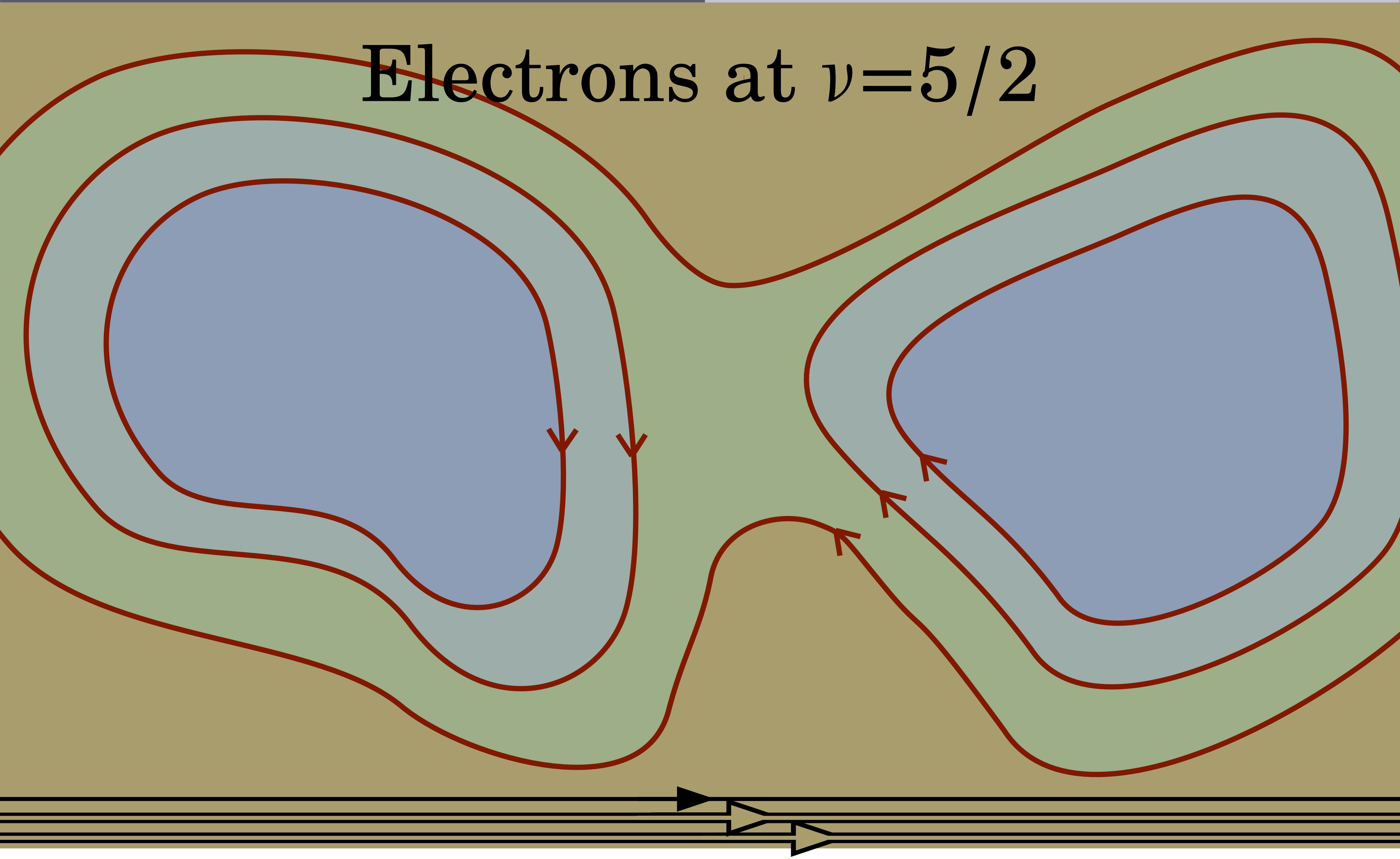


$\kappa_{xy} = 3$

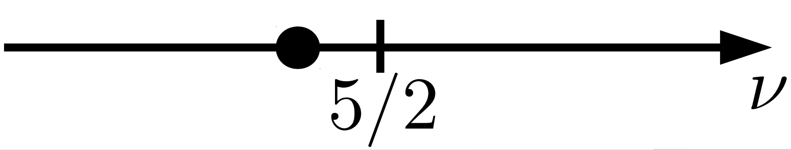


$\sigma_{xy} = \frac{5}{2}$

Electrons at $\nu=5/2$

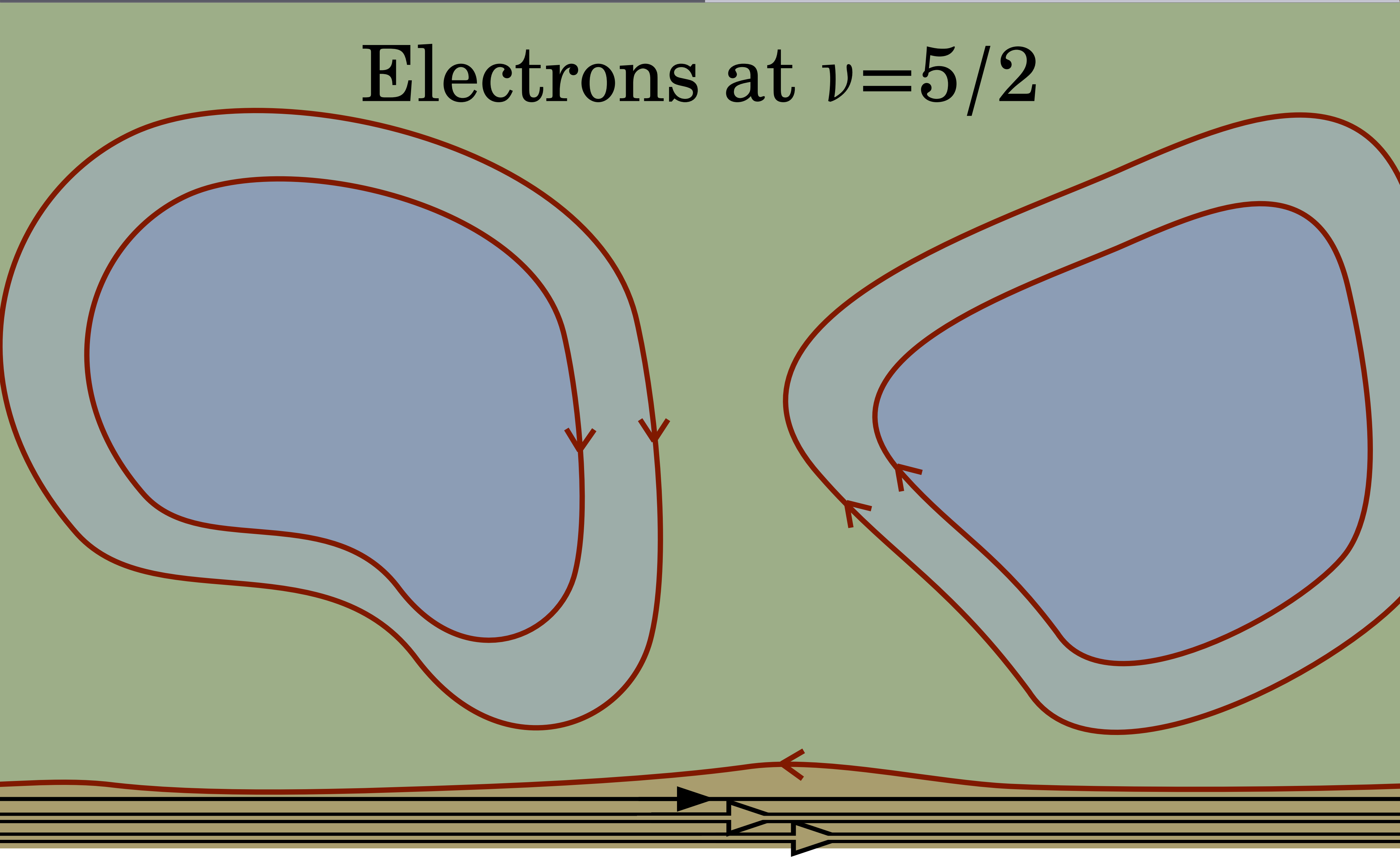


$\kappa_{xy} = 3$

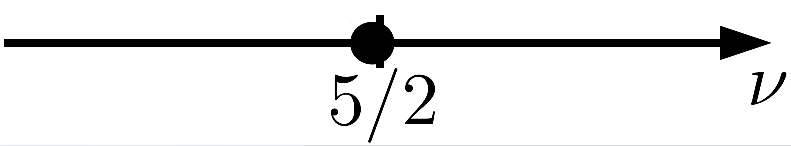


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

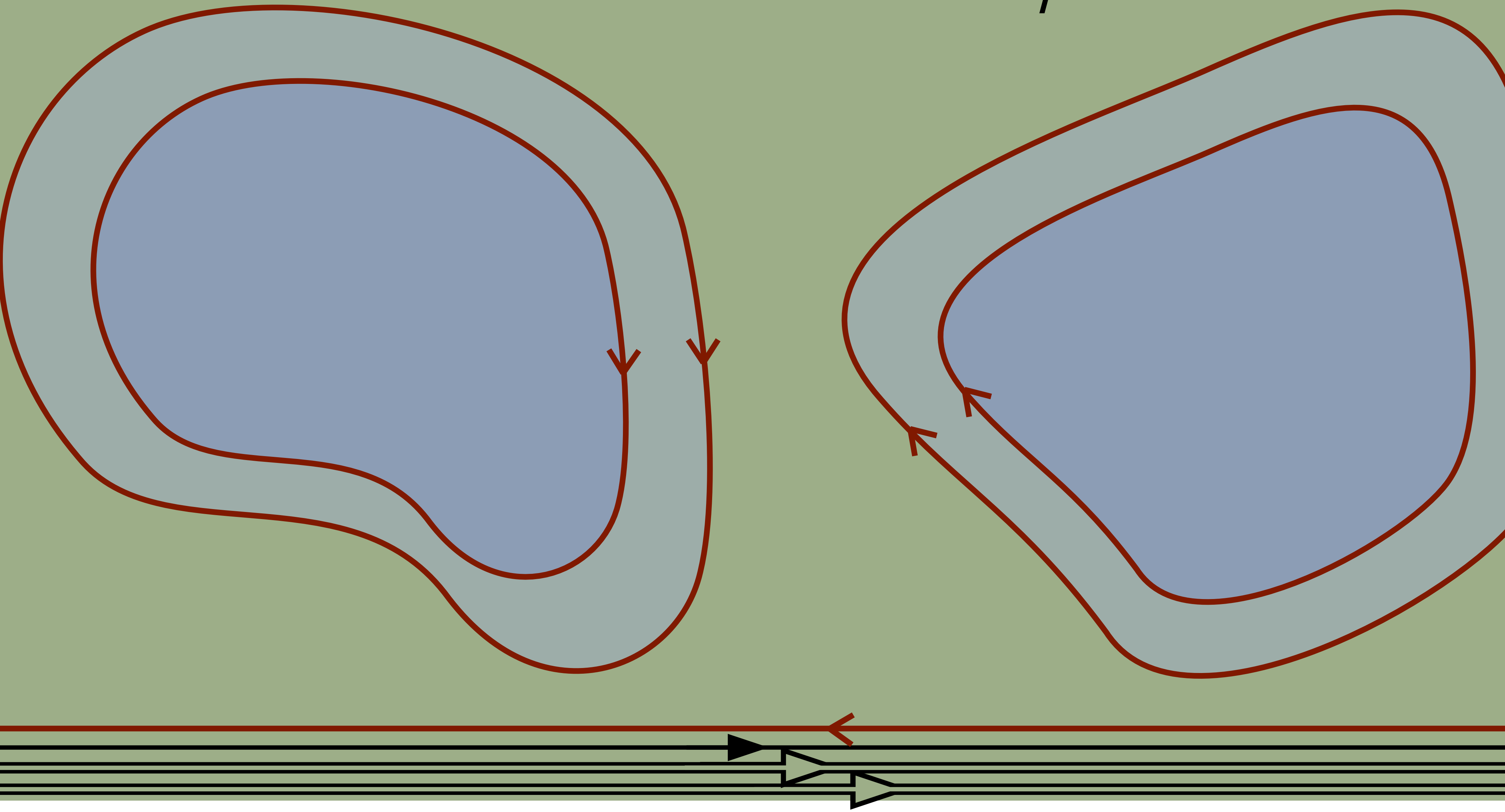


$\kappa_{xy} = \frac{5}{2}$

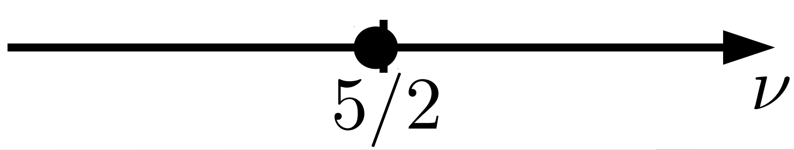


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

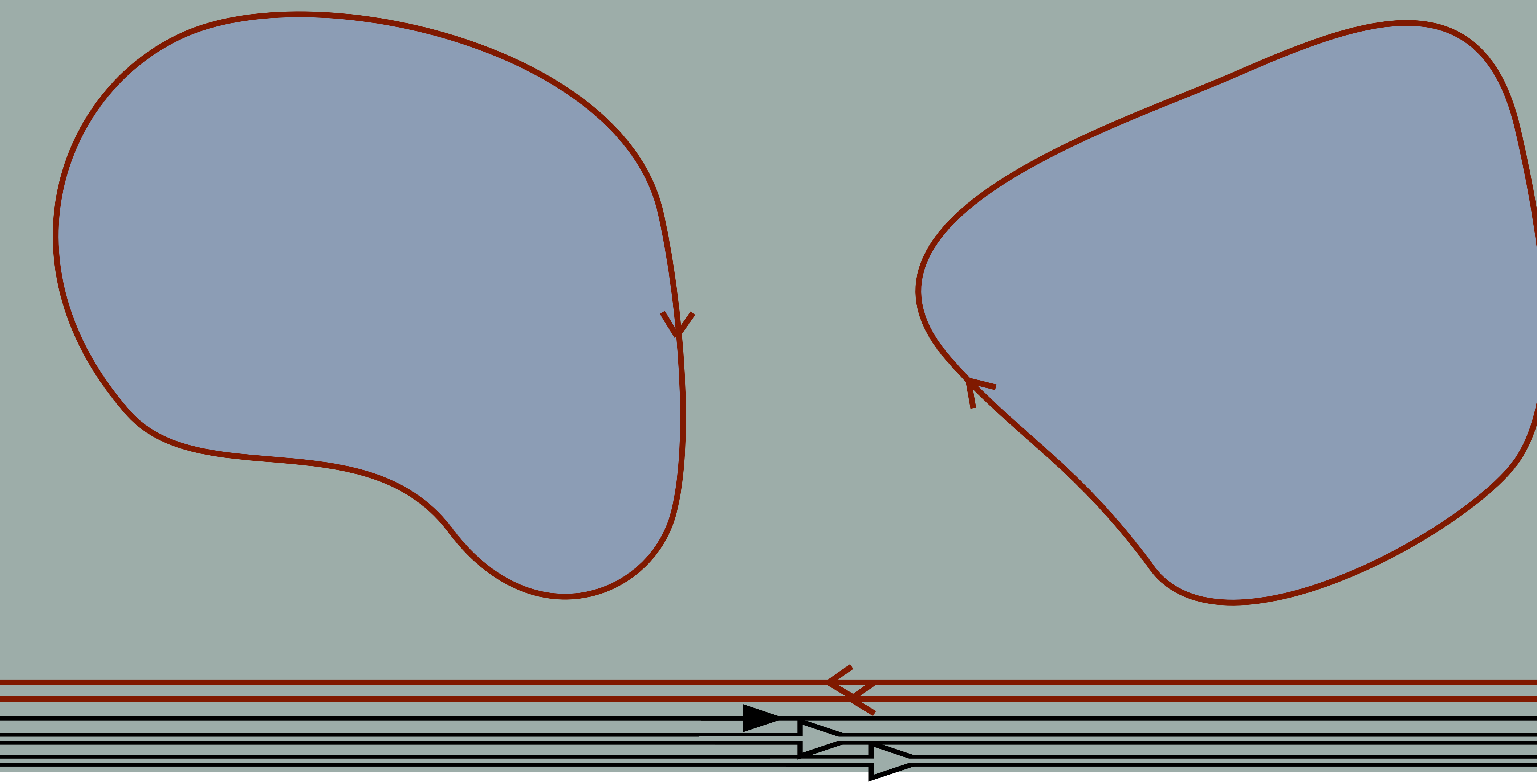


$$\kappa_{xy} = \frac{5}{2}$$

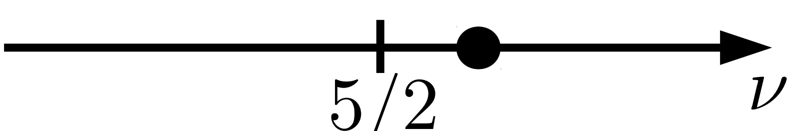


$$\sigma_{xy} = \frac{5}{2}$$

Electrons at ν=5/2

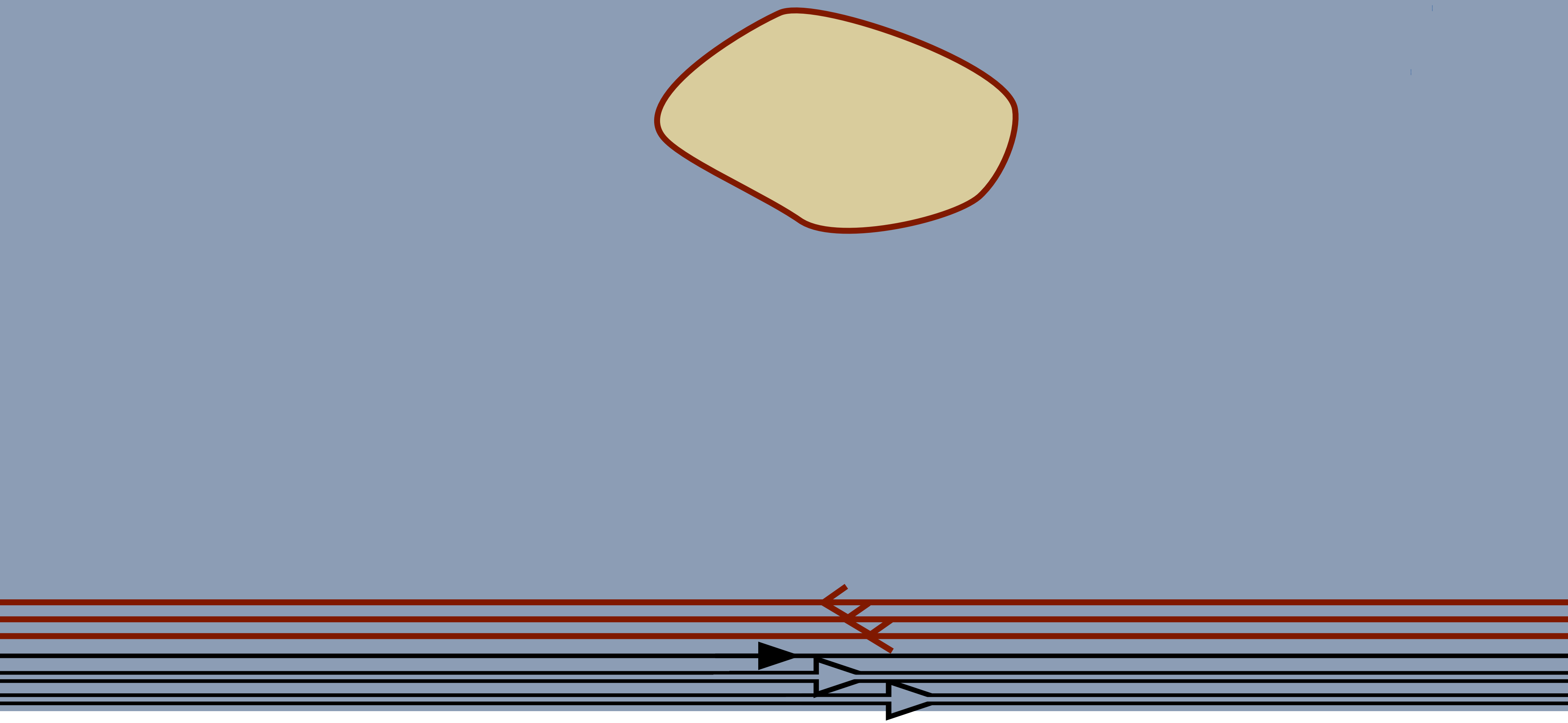


$\kappa_{xy} = 2$

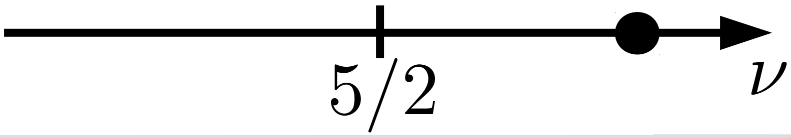


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

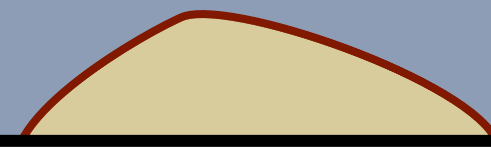


$$\kappa_{xy} = \frac{3}{2}$$



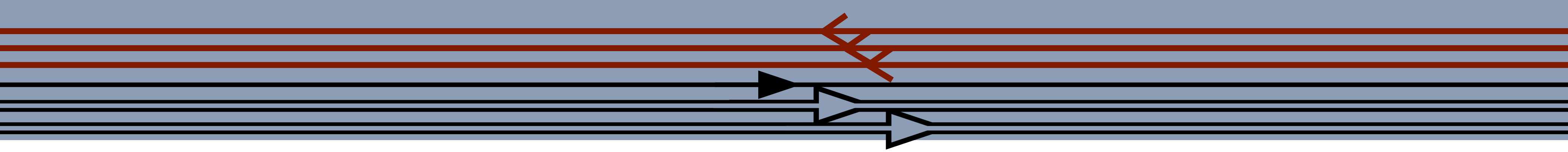
$$\sigma_{xy} = \frac{5}{2}$$

Electrons at ν=5/2

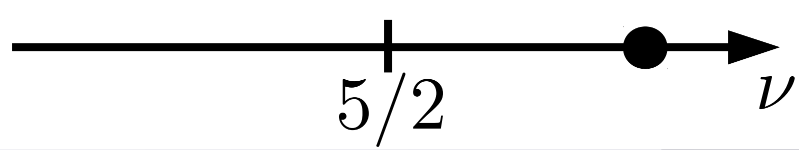


tuning the filling factor **within the** $\boldsymbol{\sigma_{xy} = 5/2}$ **plateau**,

plateaus with $\boldsymbol{\kappa_{xy} = \frac{7}{2}, 3, \frac{5}{2}, 2, \frac{3}{2}}$

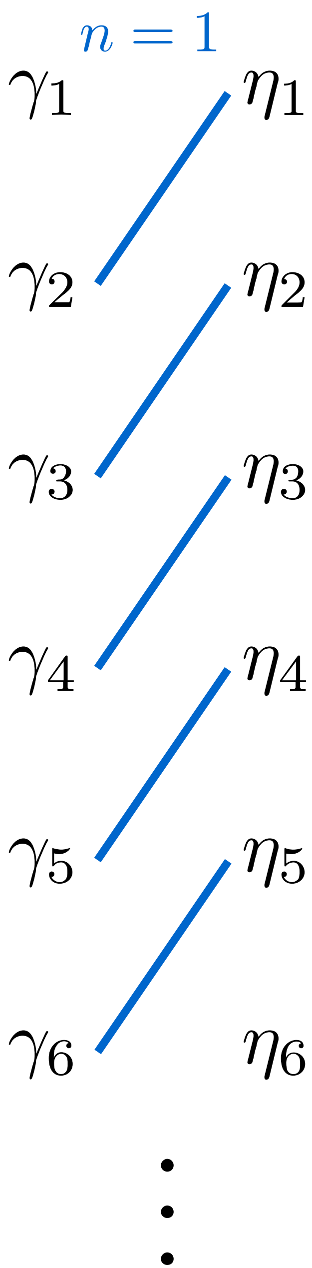


$\kappa_{xy} = \frac{3}{2}$



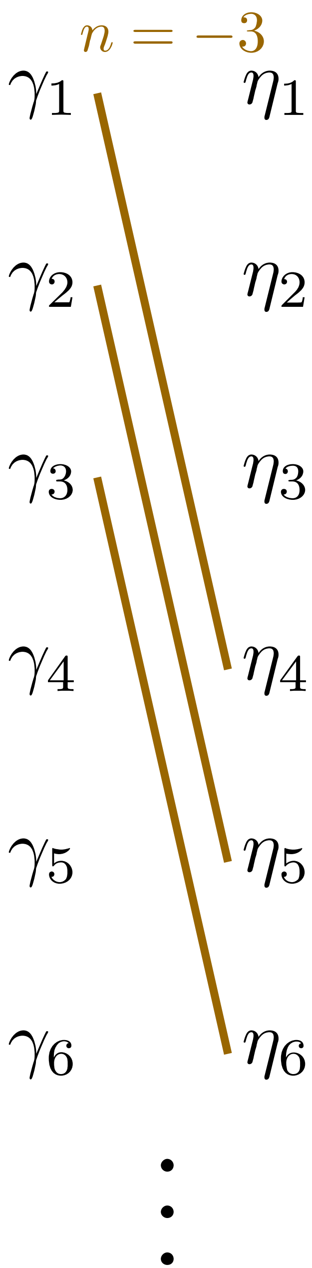
$\sigma_{xy} = \frac{5}{2}$

Toy model in one dimension



Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

Toy model in one dimension



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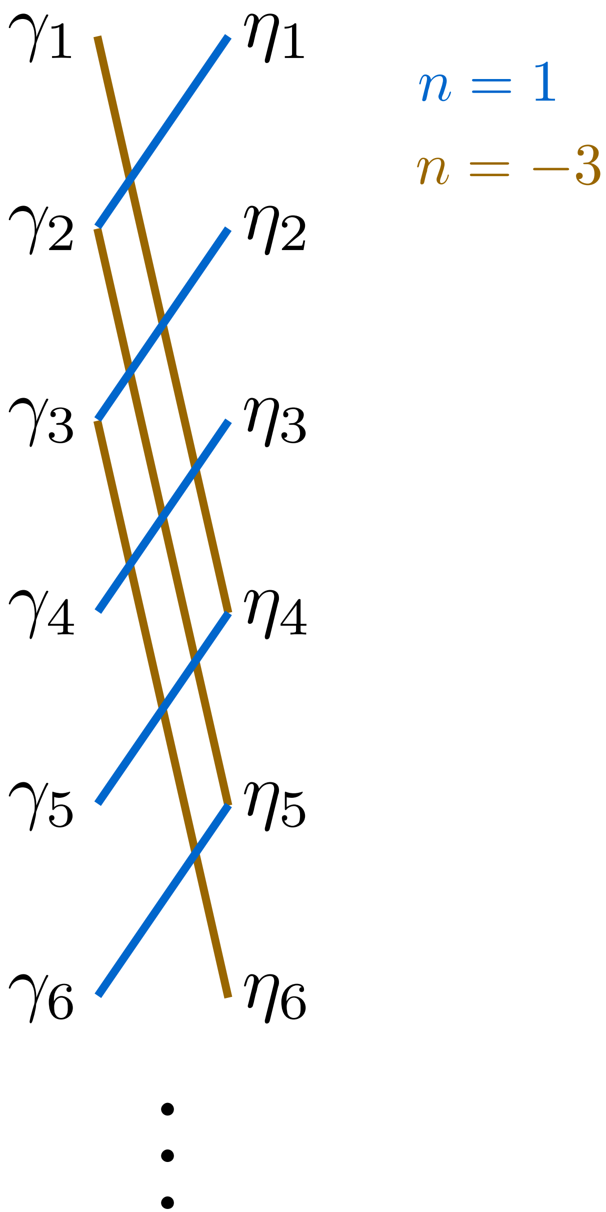
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

Toy model in one dimension



	Symmetry			d		
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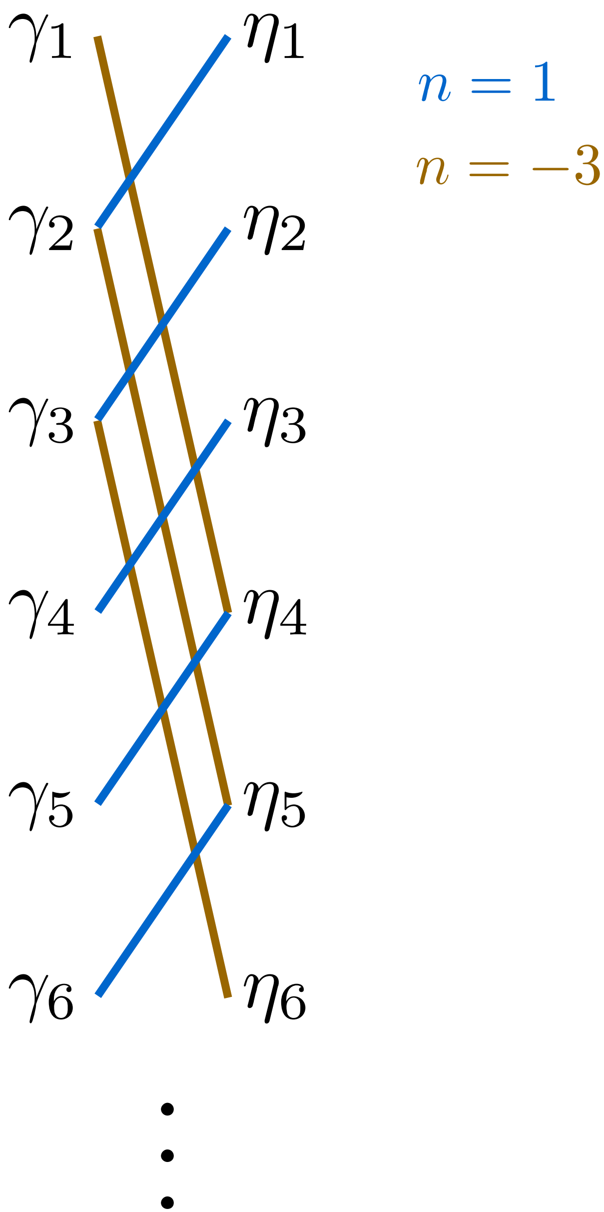
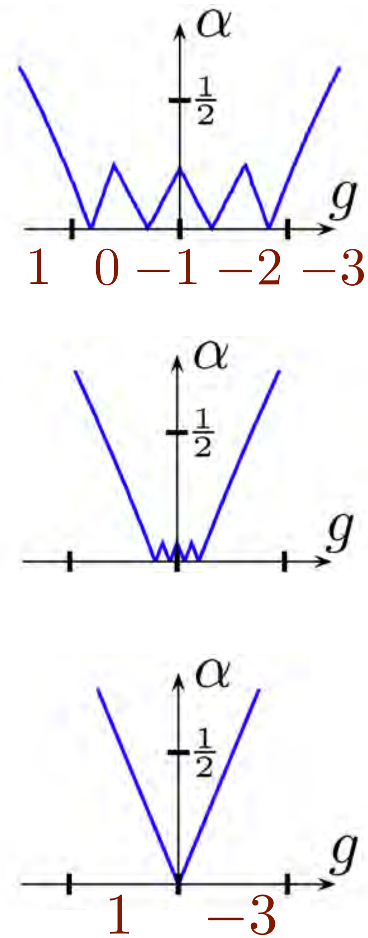
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	−1	0	0	$\mathbb{Z}$	0
CI	1	−1	1	0	0	$\mathbb{Z}$

Toy model in one dimension

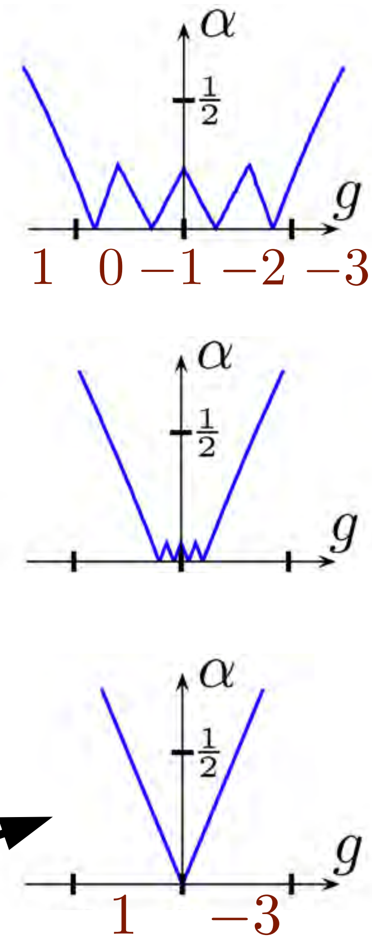




One-dimensional superconductor (BDI)

Motrunich, Damle, Huse (2001), Rieder, Brouwer, Adagideli (2013)

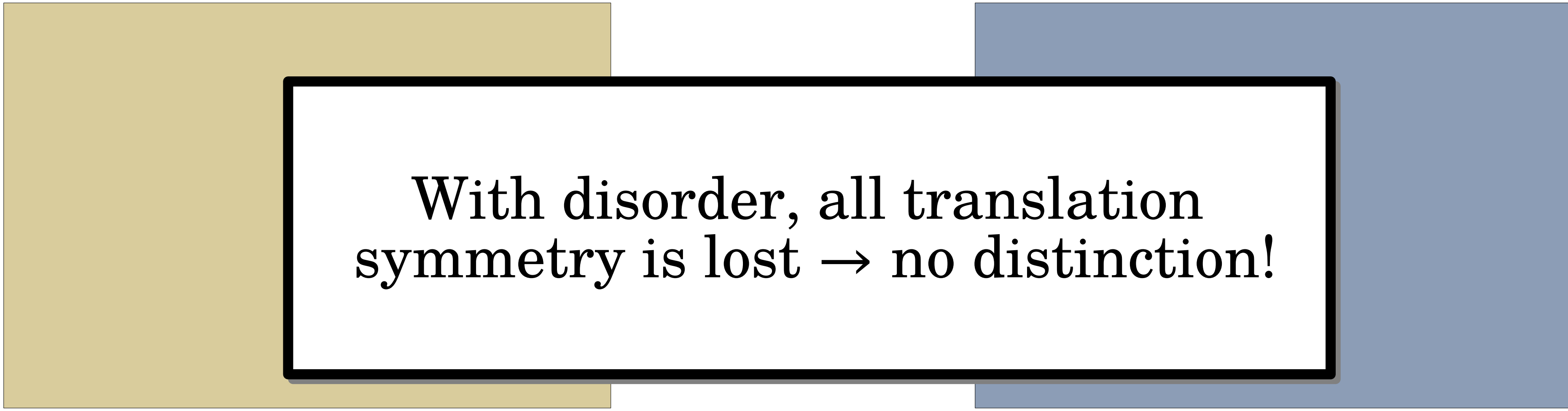
Toy model in one dimension



$$H = \sum_{i=1}^{4} \xi_i^T \left[\tau_x(-i\partial_x) + m\tau_y\right] \xi_i$$

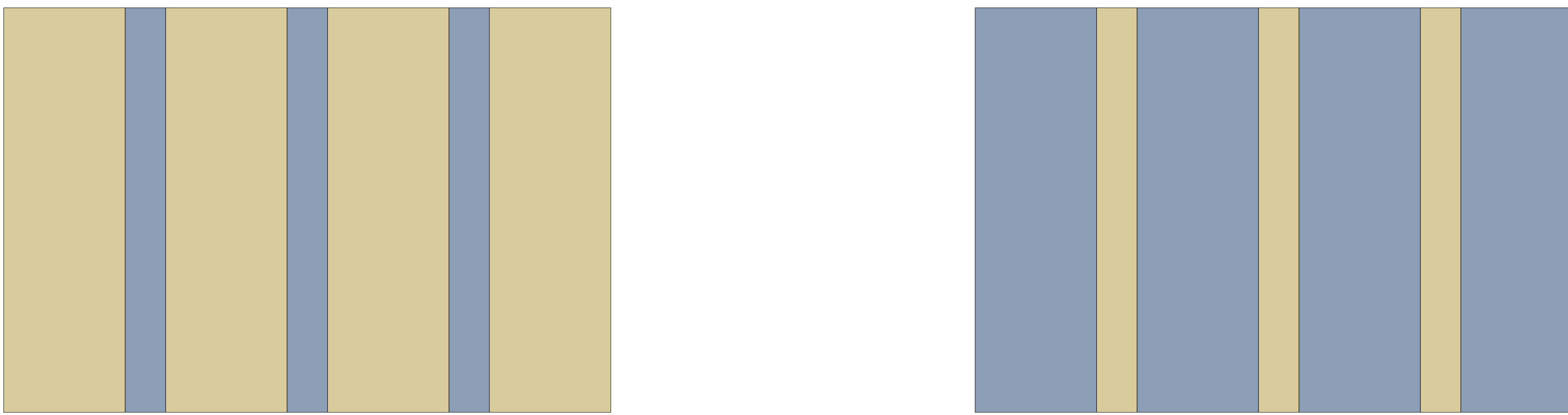
Continuous phase transition in clean system

Two dimensions

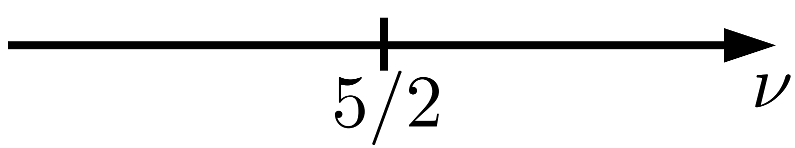


With disorder, all translation symmetry is lost → no distinction!

Continuous translation symmetry

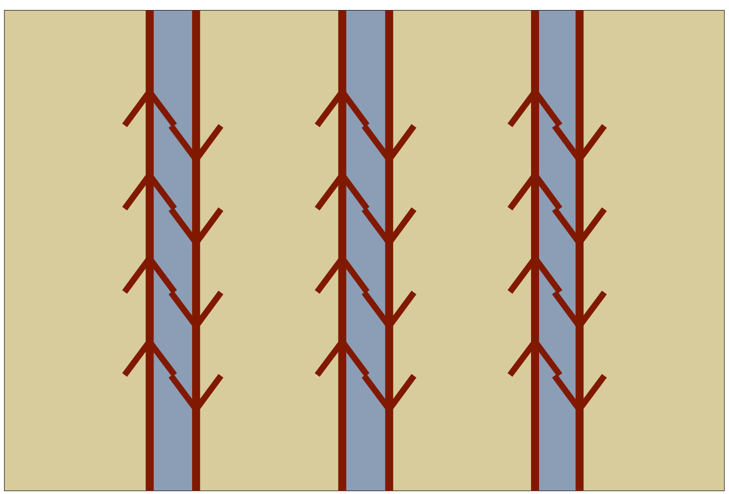


Discrete translation symmetry

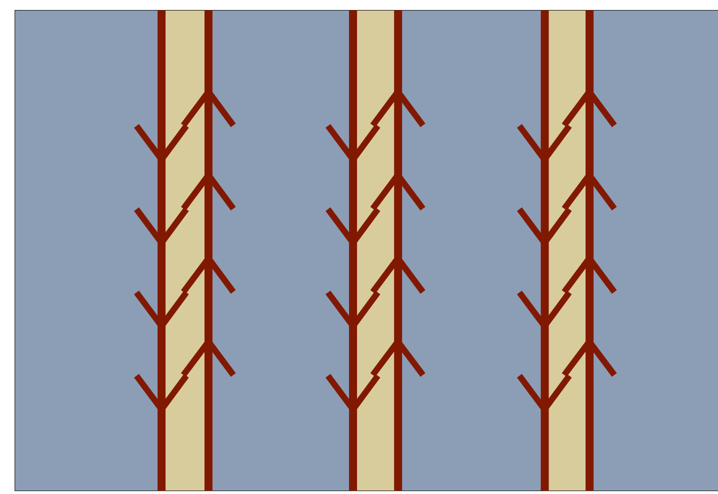


Two dimensions

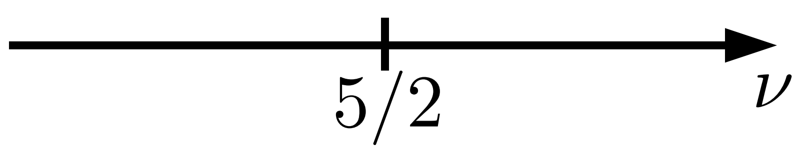
$$H = \sum_{i=1}^{4} \xi_i^T \underbrace{\left[\tau_z(-i\partial_z)\right.}_{\text{motion along domain walls}} + \underbrace{\left.\tau_x(-i\partial_x) + m\tau_y\right]}_{\text{tunneling across domains}} \xi_i$$



2nd order transition



Discrete translation symmetry



A useful analogy

Integer quantum Hall

- Class A for electrons

Electrons at $\nu=5/2$

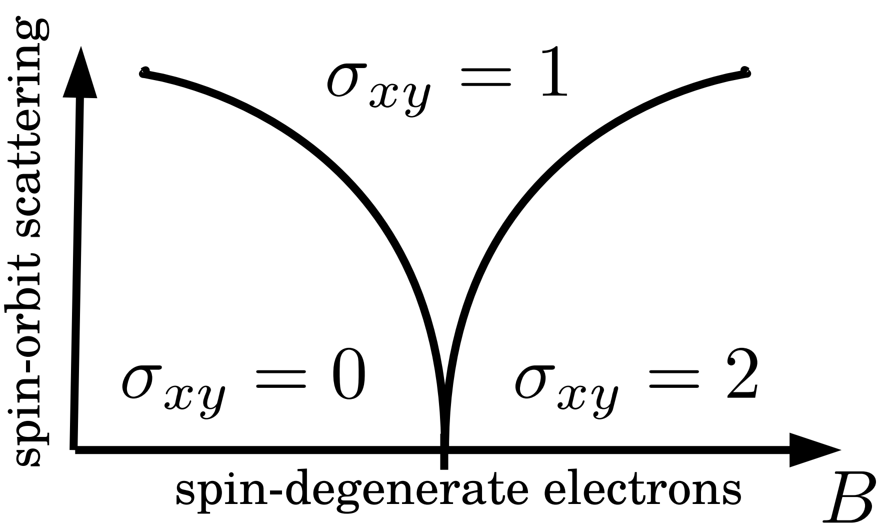
- Class D for comp. fermions

Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$

A useful analogy

Integer quantum Hall

- Class A for electrons
- Integer classification ($n =$ # of edge electrons)
- Generic transition: $\Delta n = 1$



Electrons at ν=5/2

- Class D for comp. fermions

Electrons with full spin rotation symmetry: $\Delta n = 2$

With spin-orbit scattering: Two transitions with $\Delta n = 1$

Lee and Chalker (1994)

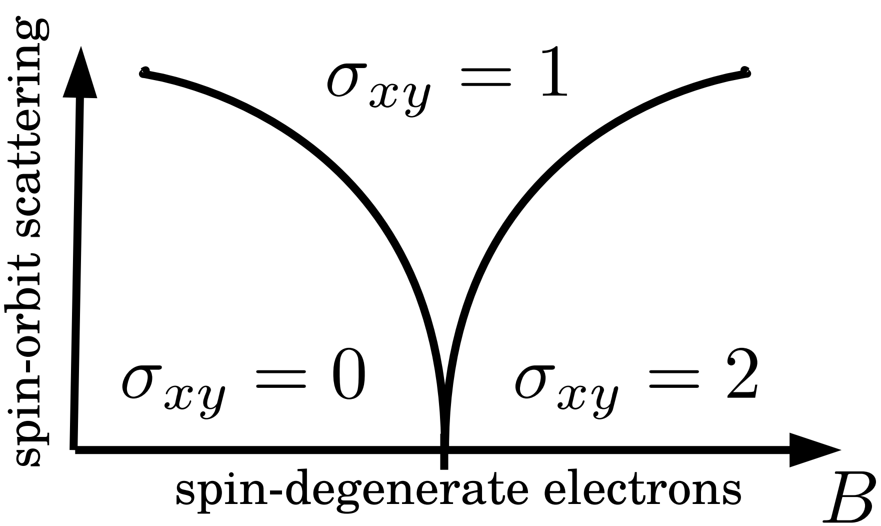
A useful analogy

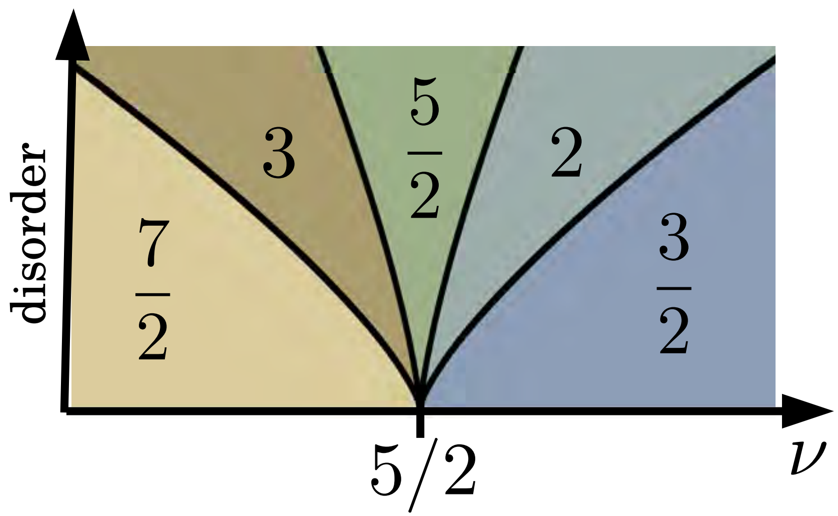
<u>Integer quantum Hall</u>

- Class A for electrons
- Integer classification
  ($n =$ # of edge electrons)
- Generic transition: $\Delta n = 1$

<u>Electrons at ν=5/2</u>

- Class D for comp. fermions
- Integer classification
  ($n =$ # of edge Majoranas)
- Generic transition: $\Delta n = 1$



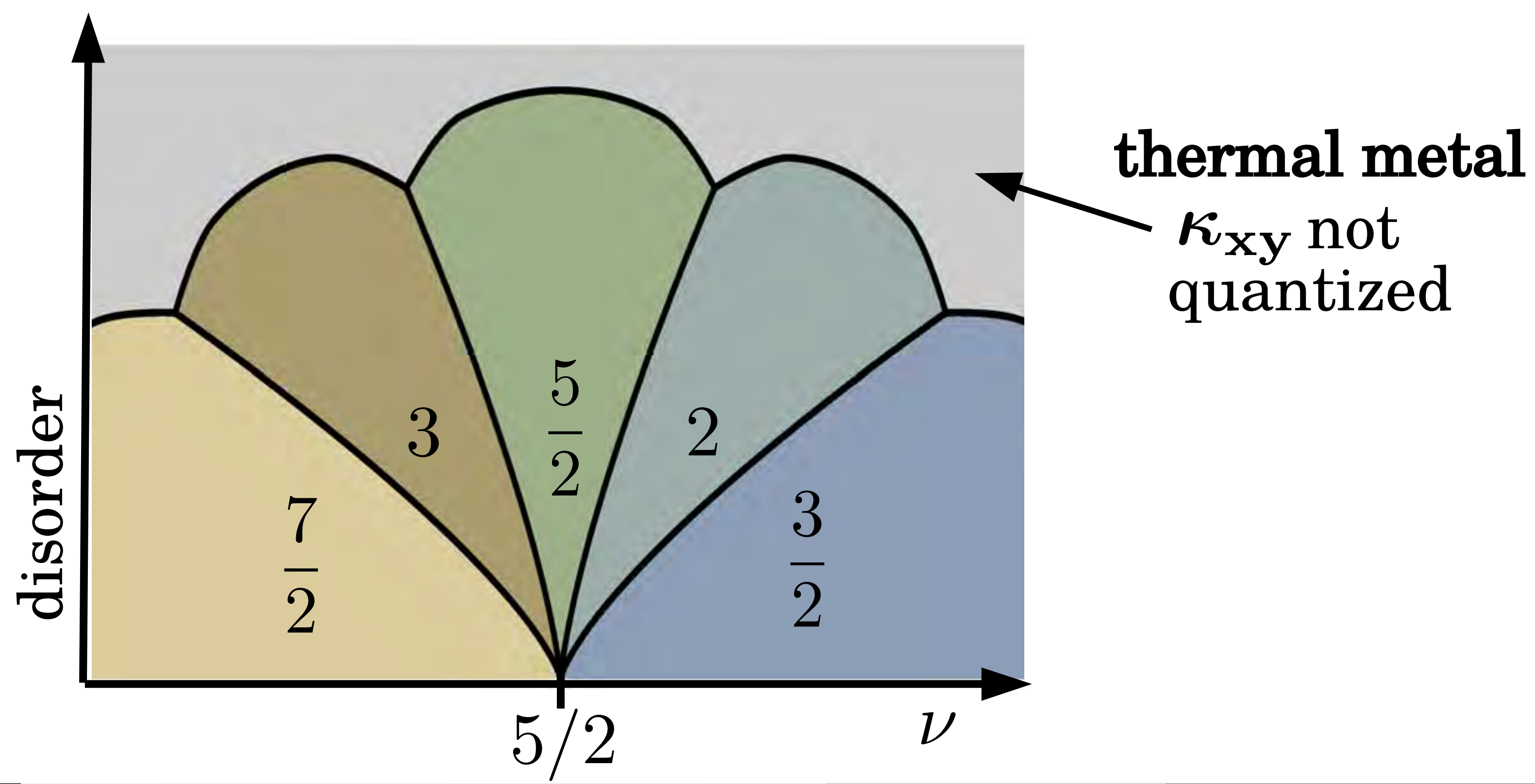


Strong disorder

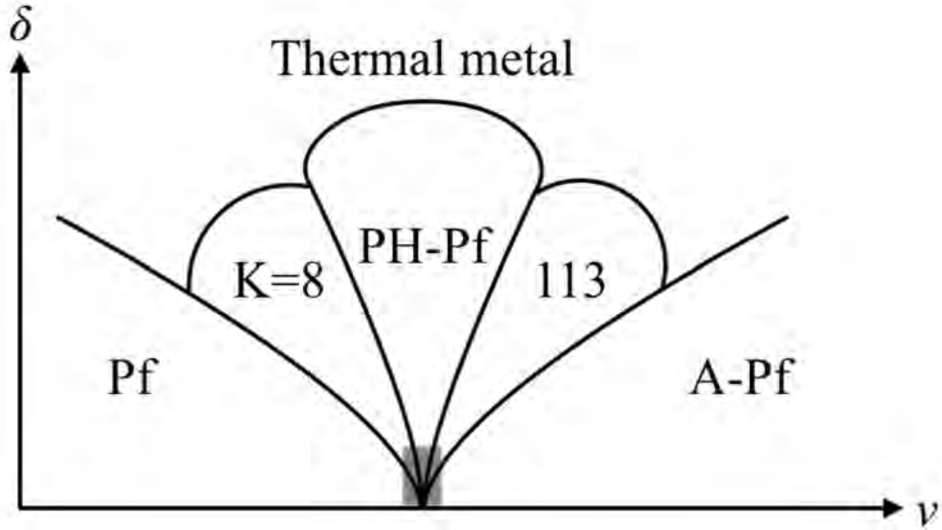
- a localized phase (well defined n_{Majorana}) not guaranteed

Cho and Fisher (1997), Senthil and Fisher (2000), Bocquet, Serban and Zirnbauer (2000)
Read and Ludwig (2000), Chalker *et al.* (2001)

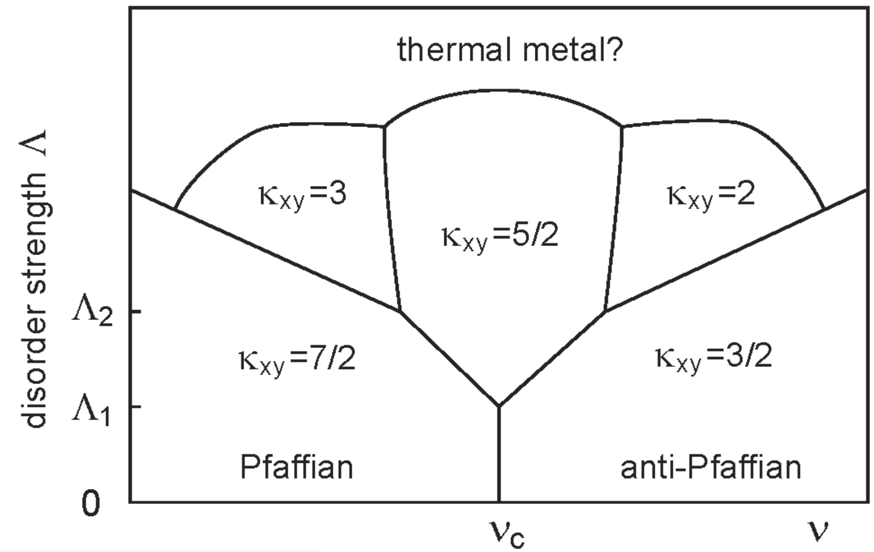
- depends on details of the disorder potential



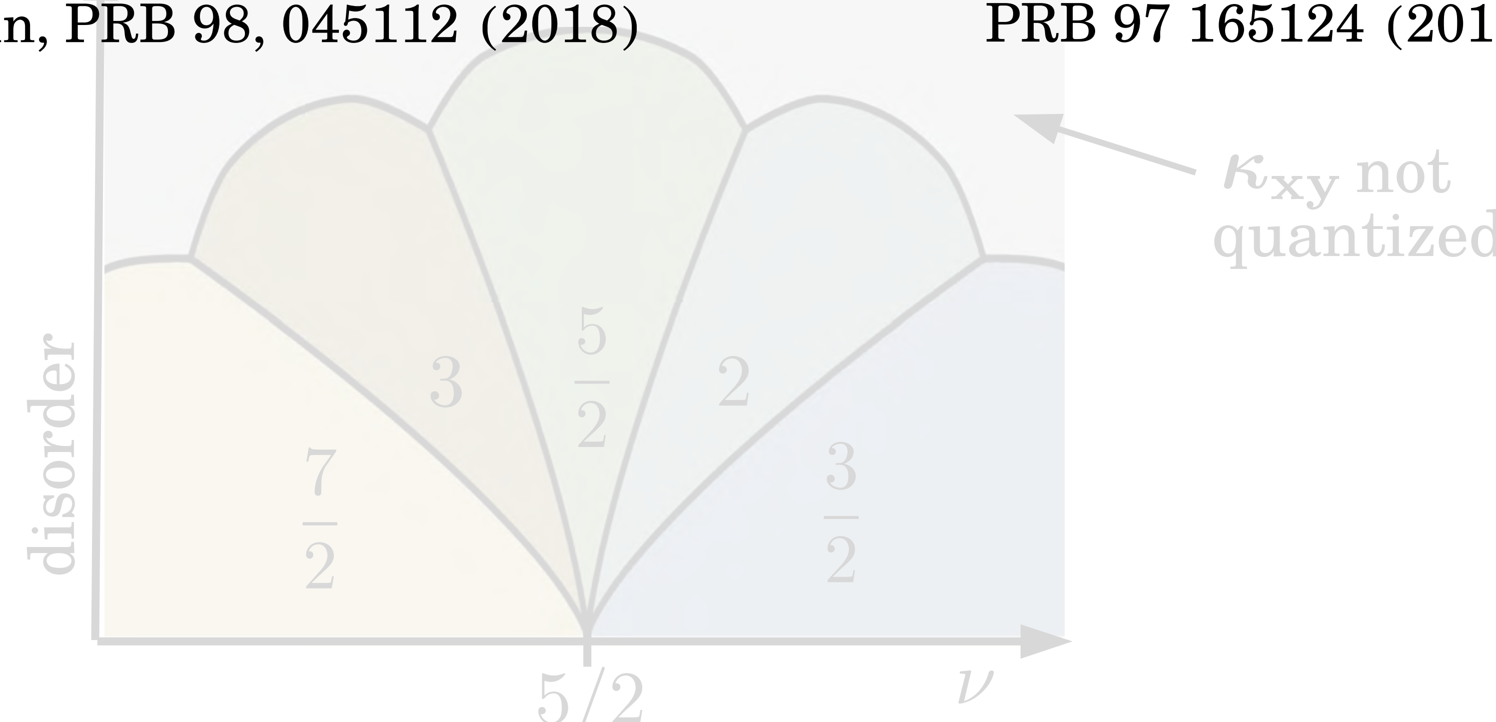
Related work



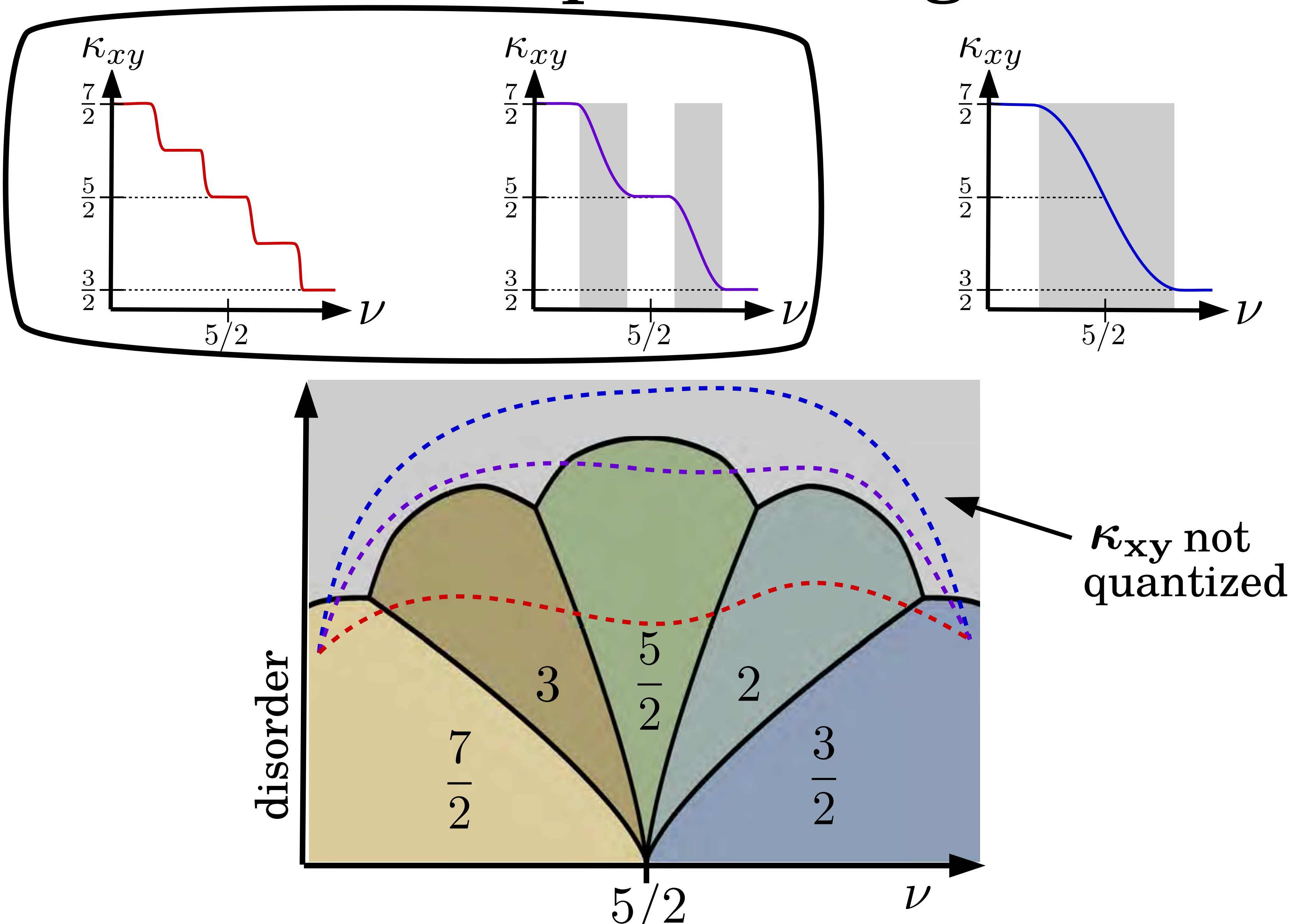
C. Wang, A. Vishwanath,
B. Halperin, PRB 98, 045112 (2018)



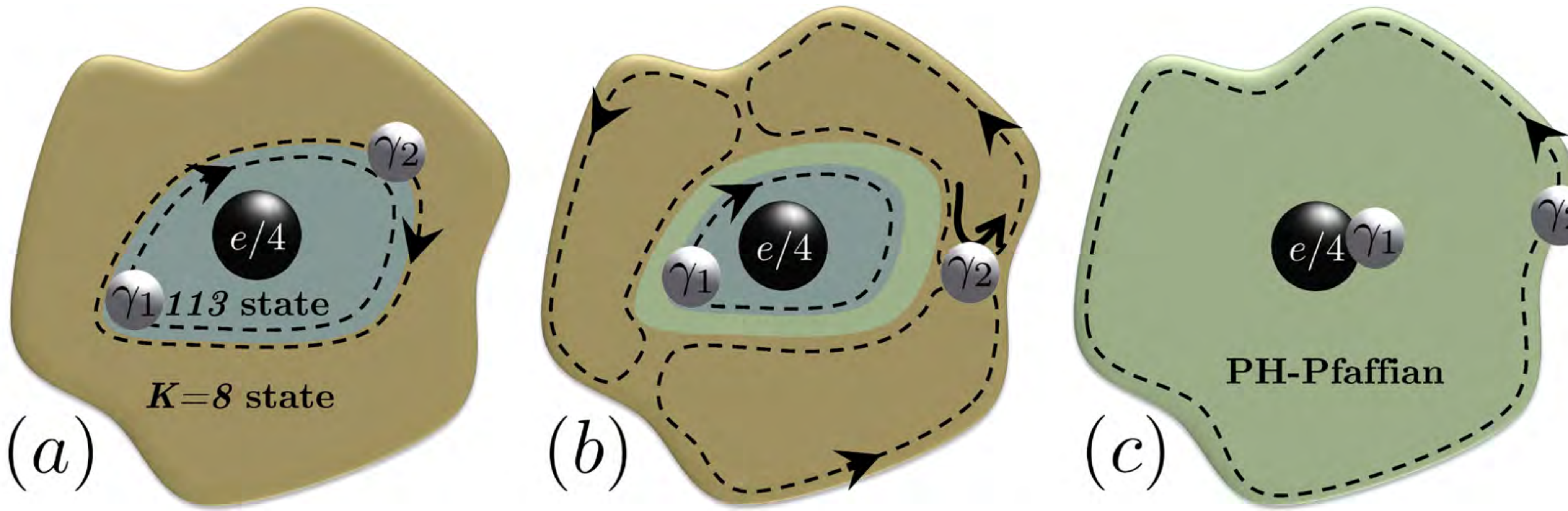
B. Lian and J. Wang
PRB 97 165124 (2018)



General phase diagram



From Abelian to non-Abelian



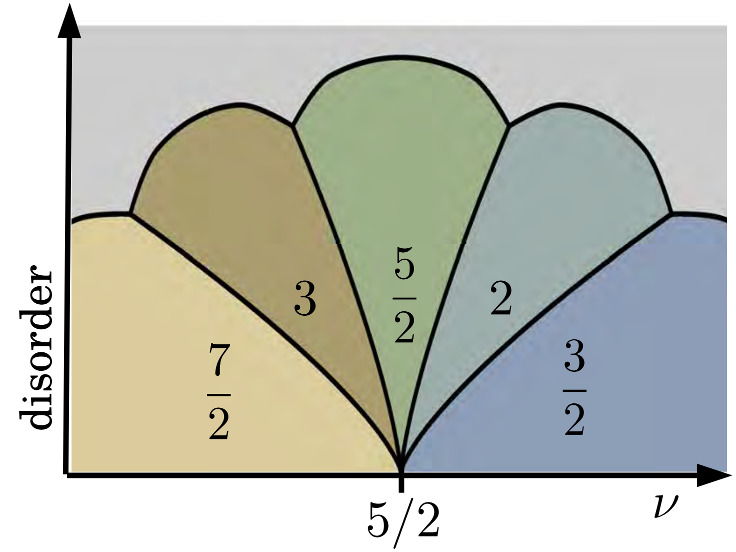
(a) No isolated Majorana modes in Abelian phase

(b) Transfer of Majorana mode at transition

(c) Isolated Majorana mode, i.e., non-Abelian phase

Conclusions / Outlook

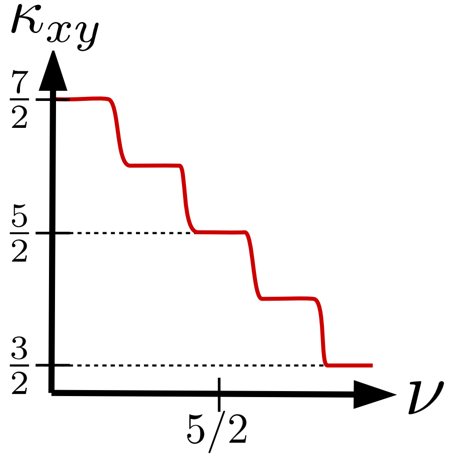
Weak disorder can resolve discrepancy between numerics and experiment.



- Are edge modes fully equilibrated?

Simon (2018), Feldman (2018), Ma, Feldman (2018)

- Microscopic treatment of disorder.



Predict additional plateaus in thermal Hall conductance

- Are different plateaus accessible?
- What about thermal metal?

- Is PH-Pfaffian possible in a clean PH-symmetric system and what is its wave function?

Milovanović (2017), Antonić, Vučičević, and Milovanović (2018)

- What interactions realize PH-Pfaffian in a clean system?