Optimized Evolution of Multilayer Networks

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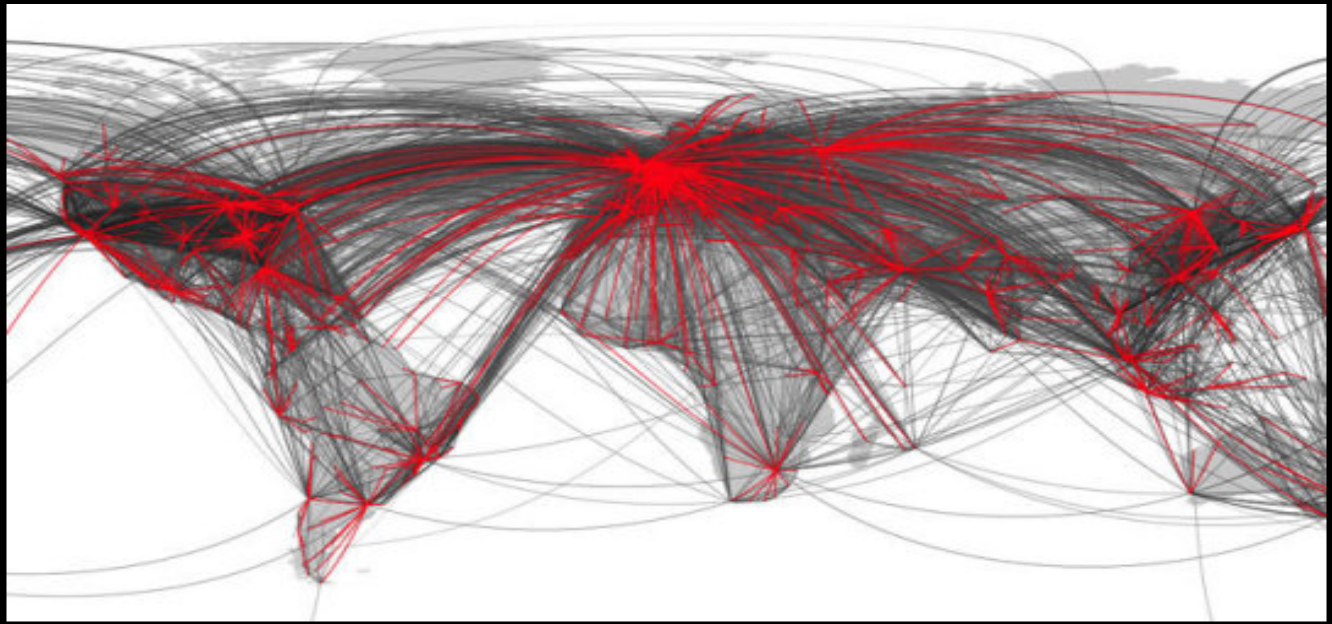
Regulating properties of a Network (epidemic spreading) by appropriate “Multiplexing”

Network Science has been tremendously successful in modeling real-world complex systems.

For example:

- Predictions of important nodes/links: applications in discovery of potential drug target, important grid in power transmission line
- Emergent time behavior of underlying complex systems; disease outbreak, social behaviors (opinion formation)

The Worldwide air transportation network



Grady, Thiemann, Brockmann. Robust classification of salient links in complex networks. Nature Communications 2012

Alison Abbott, Human Brain Project: votes for leadership change, Nature News, 04 March 2015

Real World Systems as Graphs

Biological Networks

System	Nodes	Connections
Neural network	Neurons	Axons
Cellular network	Chemicals	Reactions
Protein folding	Confirmations	Differ by one fold
Gene networks	Genes	Expression levels
Food webs	Species	Predator-prey

Social Networks

Disease network	People	Contact with infected person
Social network	People	Social relationship
Actor network	Actors	Acted in same movie
Phone call network	Phone numbers	Completed calla
Citation network	Papers	Citations
Co-authorship network	Authors	Co-authors
Terrorist network	Individual	Collaboration

Technological Networks

System	Nodes	Connections
Electrical circuit	Points	Resistances, capacitors
Internet network	Computers	Physical/wireless links
Power grid	Generators	High voltage links
Railway network	Stations	Rail line
Airport network	Airports	Flights

Online Networks

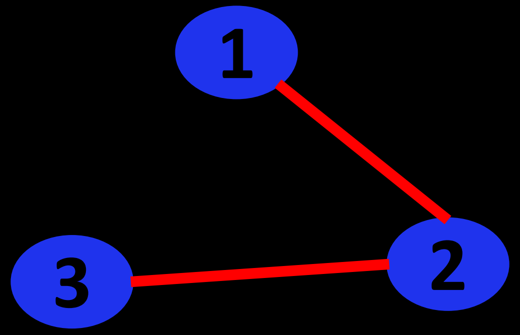
World-wide-web	Webpages	Hyperlinks
Social sites (Facebook, Twitter)	People	Friends

Others

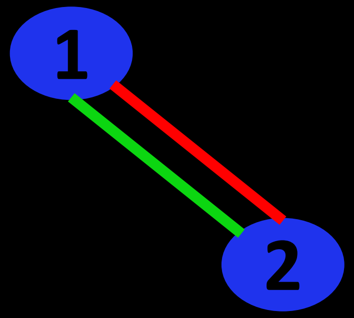
Linguistic network	Words	Synonyms
Crystal	Atoms	Bonds
Polymer	Atoms	Bonds
Percolation	Sites	Bonds

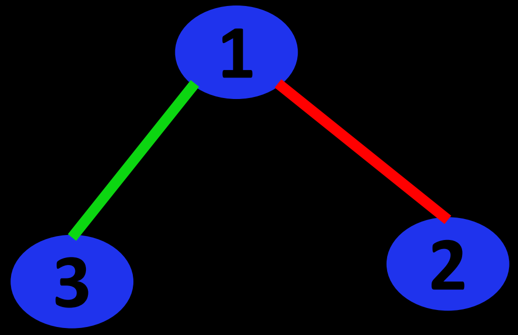
Unifying Laws Governing These Diverse Complex Systems

Network Science has so far largely focused on "one type of interactions" among individual



There may exist more than one type of interaction among the same units

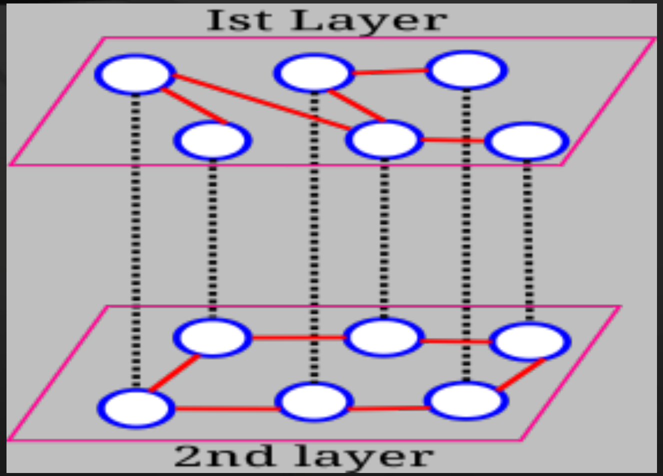




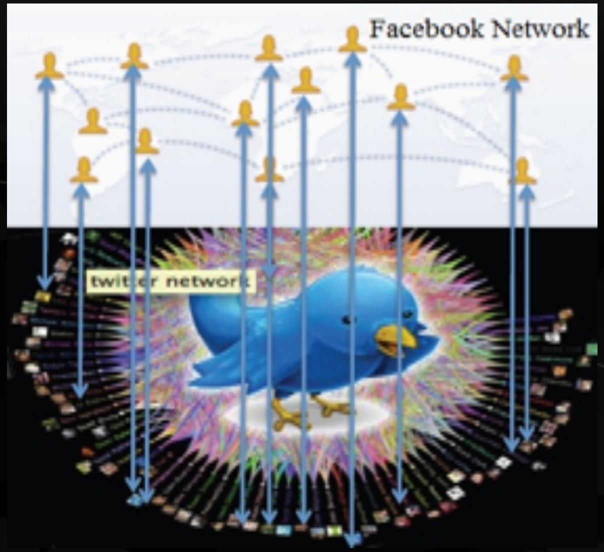
MULTIPLEX NETWORKS

Multiplex Or Multilayer Networks

A multiplex network is a set of N nodes interacting in m layers, each reflecting a distinct type of interaction among the same units

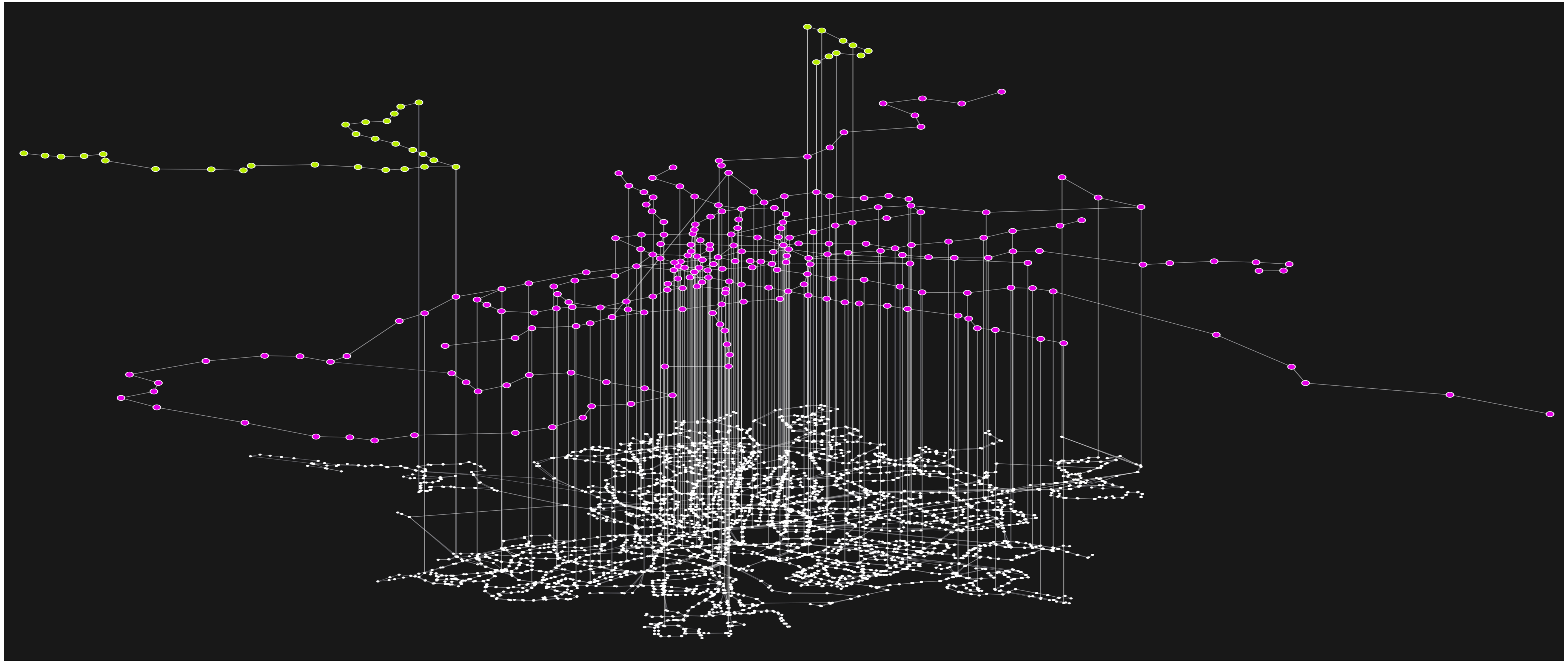


✓ In human brain, different regions can be seen connected by functional and structural neural networks [Bullmore, and Sporns, Nat. Rev. Neurosci 2010; Makovkin, Kumar, Zaikin, SJ, Ivanchenko, Phys. Rev. E 2017]



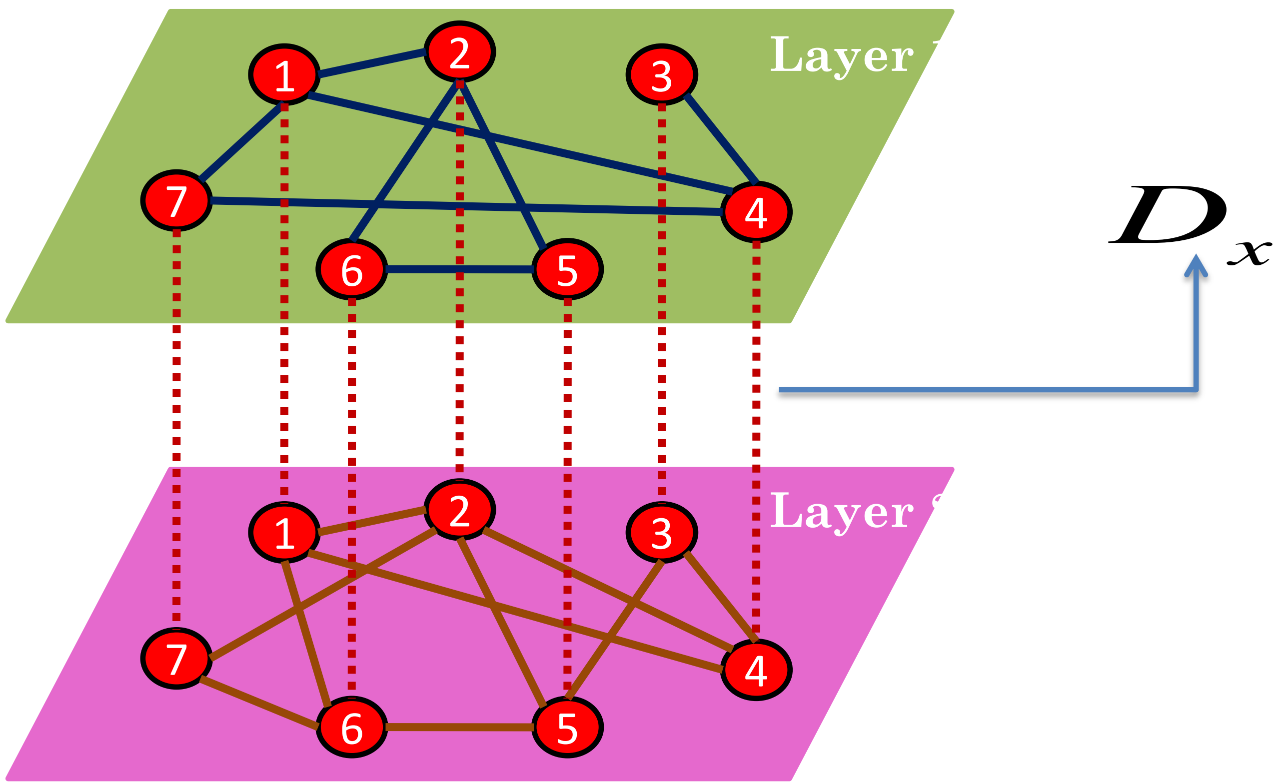
✓ Transport network: Different layers can be Air, train and bus transportation networks [Boccaletti et al. Phys. Rep. (2014)]

✓ In social networks people may be connected because of belonging to the same family, being friend or work [Camellia Sarkar, Alok Yadav and SJ, EPL (2016a)]



Madrid multilayer transportation system; tram (yellow nodes), metro (purple nodes) and buses (white nodes)

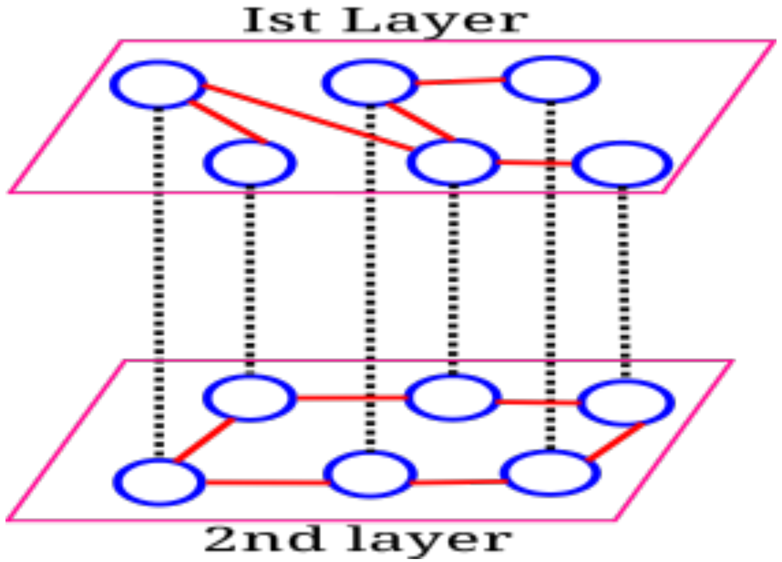
Multi-layer Networks



D_x : strength to which activities of layer 1 affects layer 2

➢Multiplex framework provides understanding to various dynamical features of underlying real-world systems which are beyond the limit of *single network incorporating only one type of coupling behavior*

Ignoring impact of multiplexity (Layer 2)
may result in wrong prediction for the behavior of a system (Layer 1)



A strike of the bus service may result in overloading the rail and air traffic routes

Multiplexing and Explosive synchronization: Kachhvah and SJ, NJP 2019

We can control properties of the entire system (multilayer network) by tuning structural properties of only one *accessible* layer

We focus on:

Eigenvector localization of complex networks

Localization properties of eigenvectors have diverse applications ranging from detection of influential nodes to **disease-spreading phenomena in underlying networks**

Principle Eigenvector (PEV)

- PEV is useful for getting insight into the propagation or localization of perturbation in the underlying systems (Golstev et. al. PRL 2012)

- Provides insight to disease spreading, for instance, using SIS model (Satorras and Castellano, Sci. Rep. 2016)

- PEV localization provides insight into the propagation of perturbation in mutualistic ecological networks (Suweis et al., Nat. Commun. 2015)
- In the analysis the existence of rare-regions in brain networks study (Moretti and Munoz, Nat. Commun. 2013)
- To enhance the efficiency of Google matrix (Ermann, Frahm and Shepelyansky, Rev. Mod. Phys. 2015)

Susceptible Infected Disease Spreading model

$I_k(t)$: probability that a node k becomes infected at time t
$1 - I_k(t)$: probability of being susceptible (not infected)

$$\frac{dS_k}{dt} = -\beta S_k \sum_j G_{kj} I_j = -\beta S_k \sum_j G_{kj}(1 - S_j),$$
$$\frac{dI_k}{dt} = \beta S_k \sum_j G_{kj} I_j = \beta(1 - I_k) \sum_j G_{kj} I_j,$$

Solution can be written in terms of eigenvalue and PEV of the connectivity matrix:

$$\vec{I}(t) \sim e^{\beta\lambda_k t}\vec{u}_1$$

The Eigenvector Centrality of Nodes of PEV: Controls Distribution of Probabilities of getting Infected

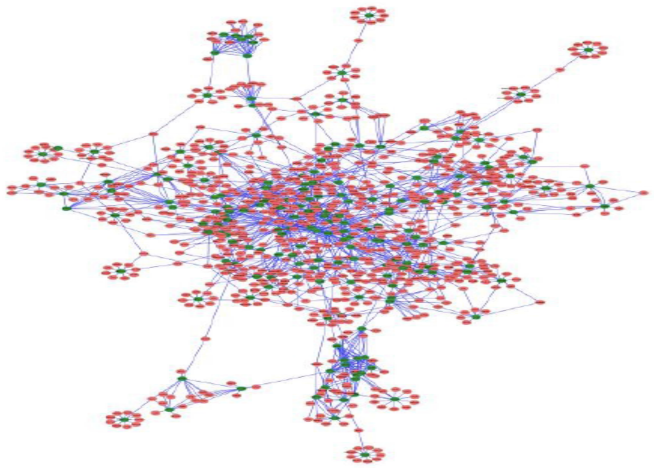
Aguirre, Papo and Buldú, Nature Physics 2013

PEV localization and Linear dynamics

- The PEV of an adjacency matrix encodes steady state vector of many linear dynamical systems
- Localized PEV indicates a network structure which have a few nodes contributing more and rest of the nodes having a tiny contribution in the dynamical process

PEV Localization: Disease is Localized on a Smaller Section of the Graph

Adjacency matrix of single layer networks



$$\begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & 0 & 1 & 1 & 0 & \dots \\ \dots & 1 & 0 & 1 & 1 & \dots \\ \dots & 1 & 0 & 1 & 0 & \dots \\ \dots & 0 & 1 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Eigenvalues of the underlying Adjacency matrix are called the spectra of network

$$\{\lambda_i\}, \; i = 1, \dots, N$$

Corresponding eigenvectors are: $X_1, X_2, \cdots, X_N$

Identification in complex diseases. Aparna Rai, Pramod Shinde, and **SJ**. Applied Network Science (2018)

Localization of an eigenvector:

- Few components of the vector take very high values
- Rest of the components take very small values

$$\left(1, 0, 0, 0, 0, \ldots\ldots\ldots\ldots\ldots\ldots, 0, 0\right)$$

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0, \ldots\ldots, 0, 0\right)$$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_k, \varepsilon_l\right)$$

$$\sum_{j=1}^{N} (x_i^j)^2 = 1$$

Normalization

Measure of Eigenvector Localization:

$$Y_{X_k} = \sum_{i=1}^{N} x_i^4, \qquad (1)$$

For a connected network

1/N < IPR < 1

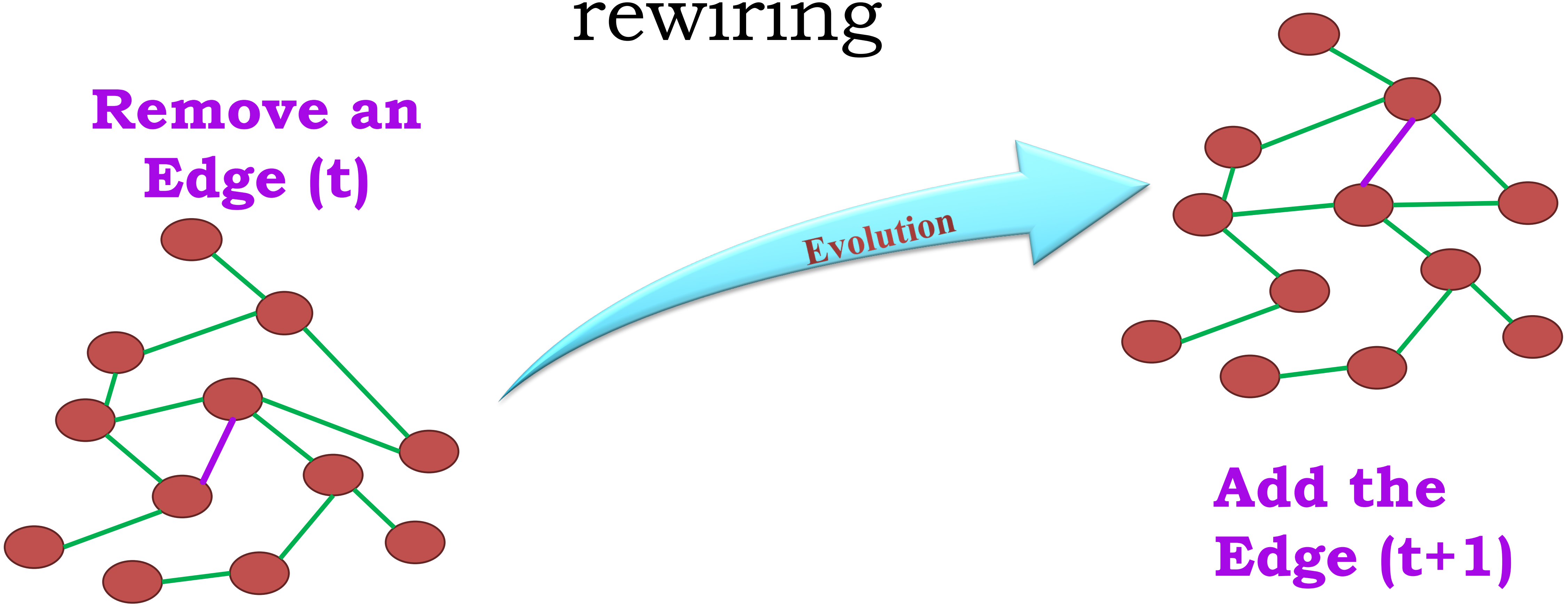
For given size (number of nodes *N*) and building cost (number of connections in the network *NC*), what network structure will correspond to the most localized PEV ??

Theoretical Framework:

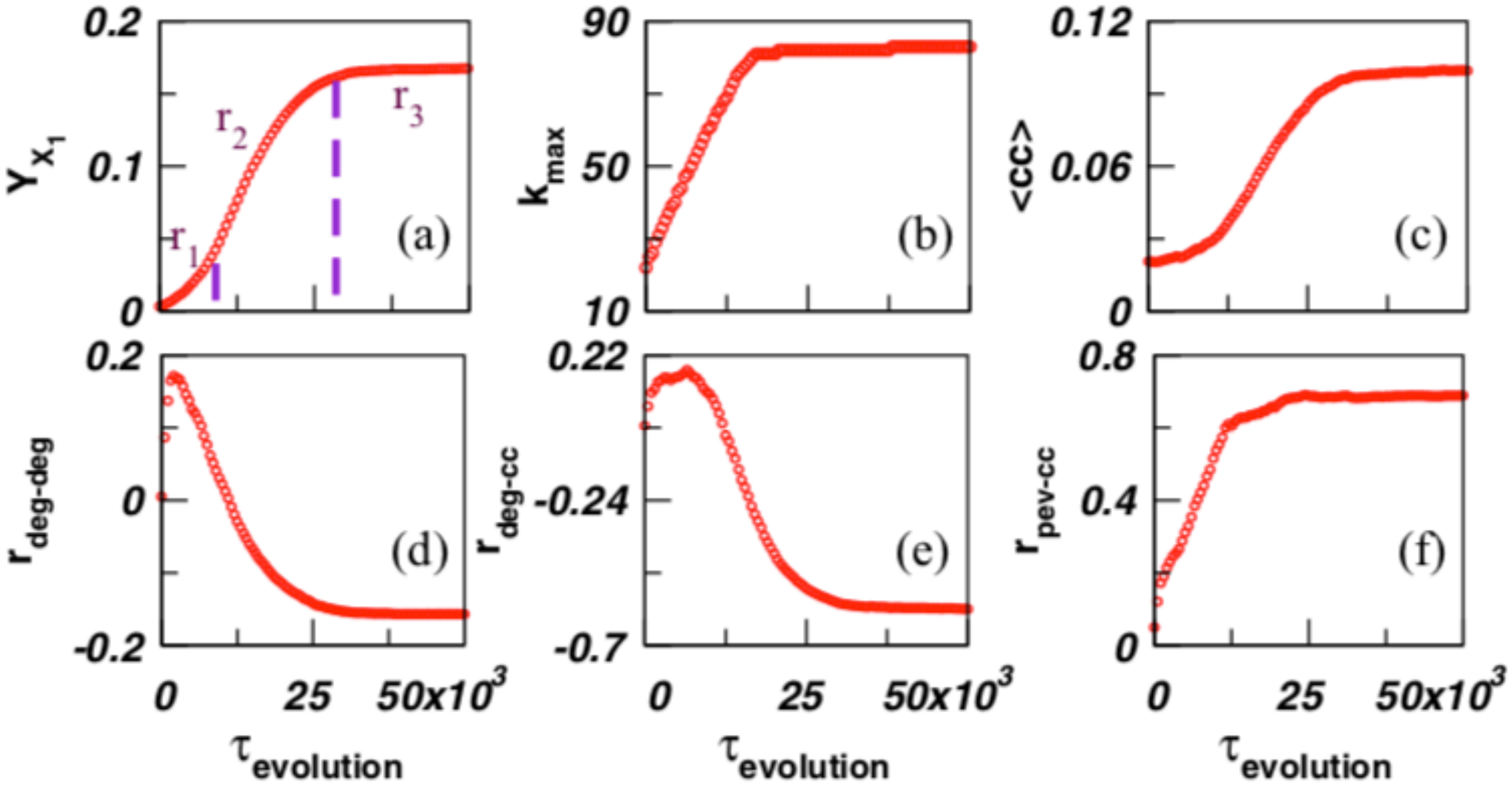
- For give N (size of the system) and NC (building cost), if we can enumerate all the possible configurations, the network corresponding to the highest IPR value will be our desired one
- Enumerating all the network configurations for a given N and NC is computationally exhaustive
- We formulate this problem through an optimization technique

Our optimization aims at maximizing IPR

Evolution of a layer (network) by Edge rewiring

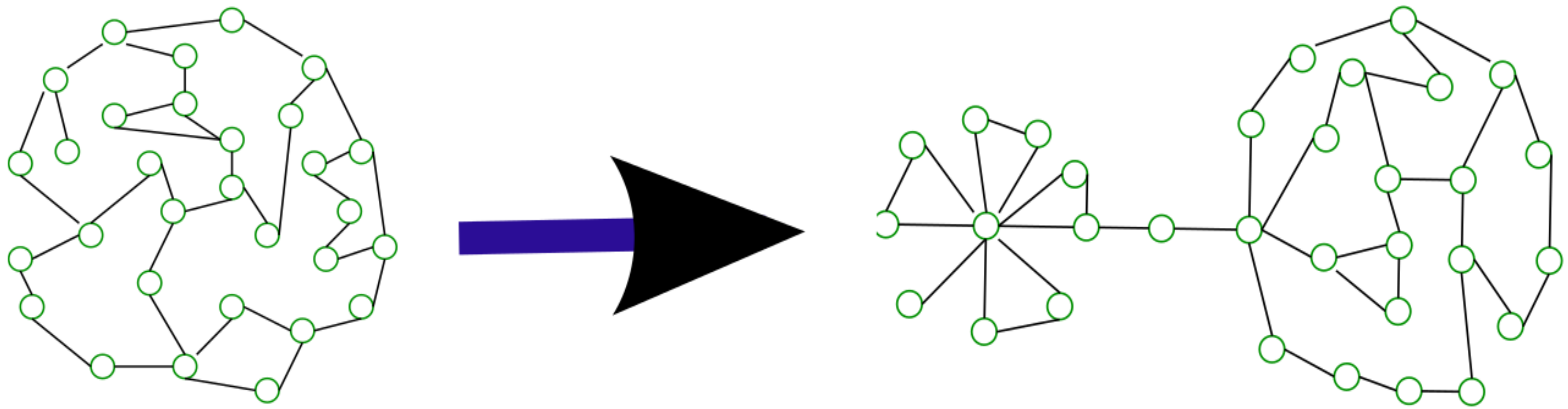


$$IPR(t+1) > IPR(t)$$



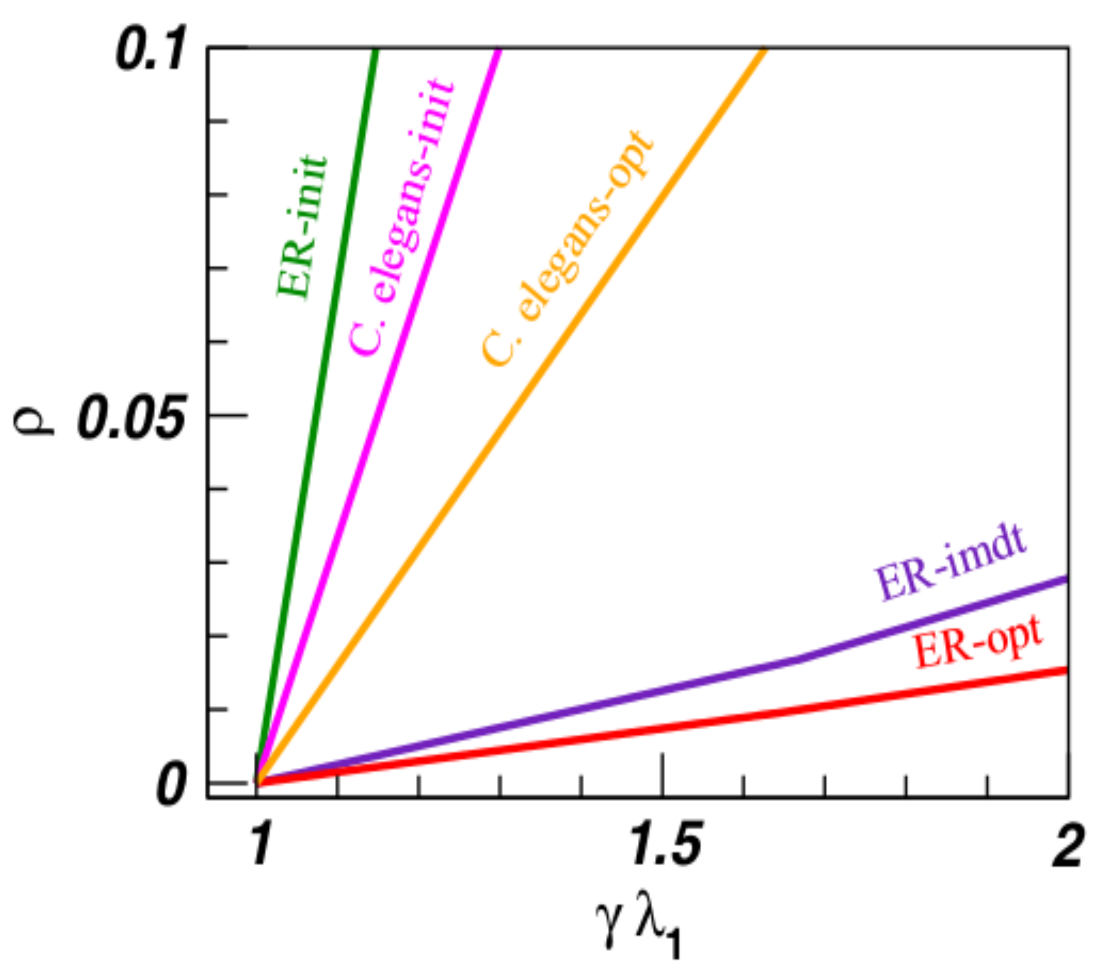
Final network possess various special network architecture features: high degree, cluster coefficient, negative degree-degree correlations

Evolution of Random Networks using IPR as fitness function:



The optimized structure is robust against change in the initial network structure

SIS Model On Optimized Networks



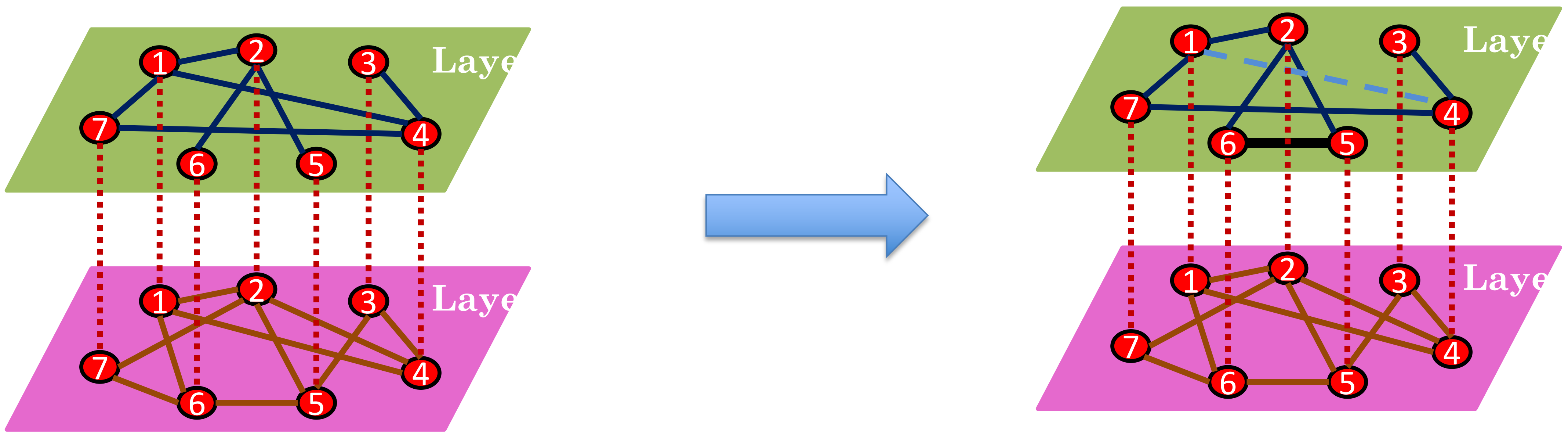
- Critical epidemic threshold $\gamma_c = \frac{1}{\lambda_1}$ and γ is the epidemic threshold
- ρ is the fraction of infected nodes at steady state

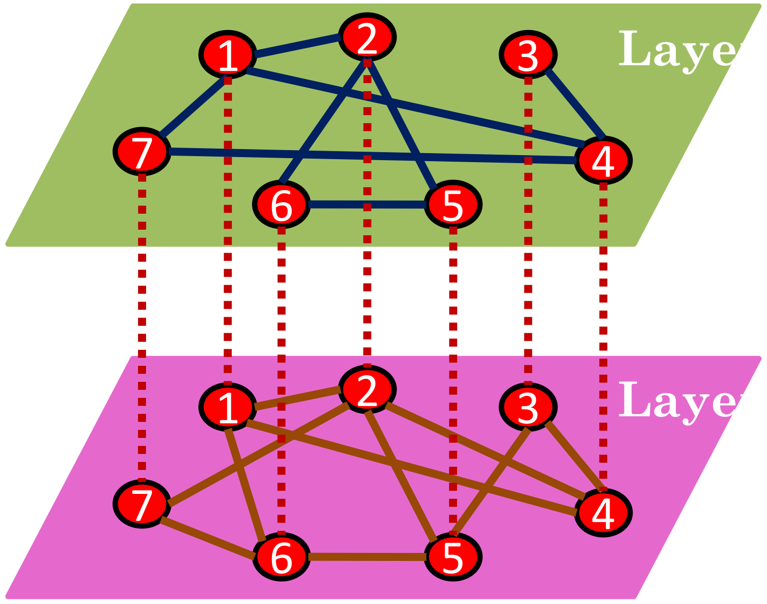
FIG. 8. Spreading process of SIS model on the initial ER random network ($\lambda_1^{init} \approx 11.34$, $IPR_{init} \approx 0.0007$), intermediate ($\lambda_1^{imdt} \approx 11.14$, $IPR_{imdt} \approx 0.21$) and optimized networks ($\lambda_1^{opt} \approx 10.77$, $IPR_{opt} \approx 0.22$) and the C.elegans neural networks ($\lambda_1^{init} \approx 24.36$, $IPR_{init} \approx 0.019$ and $\lambda_1^{opt} \approx 17.18$, $IPR_{opt} \approx 0.1$) have been depicted. ER network has $N = 2000$ nodes with $\langle k \rangle = 10$.

Pradhan, Yadav, Dwivedi, SJ, *Optimized evolution of networks for principal eigenvector localization*
Phys. Rev. E 2017

Aim: To control a desired property of systems represented by Multiplex networks

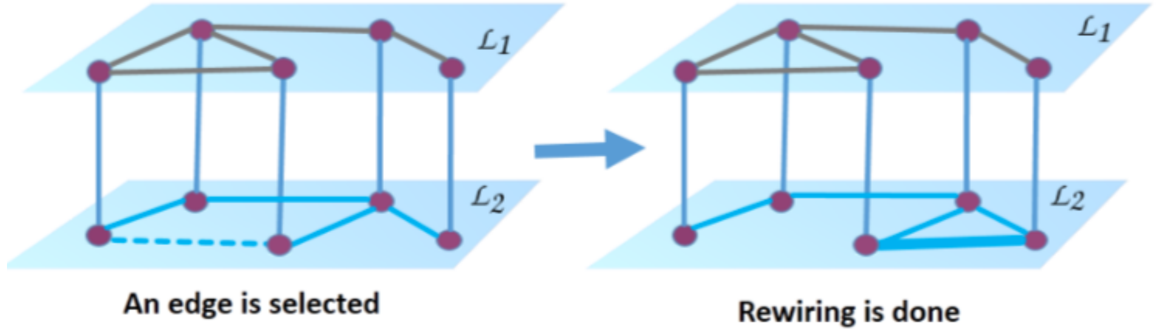
Restriction: only one layer can be altered or in our control



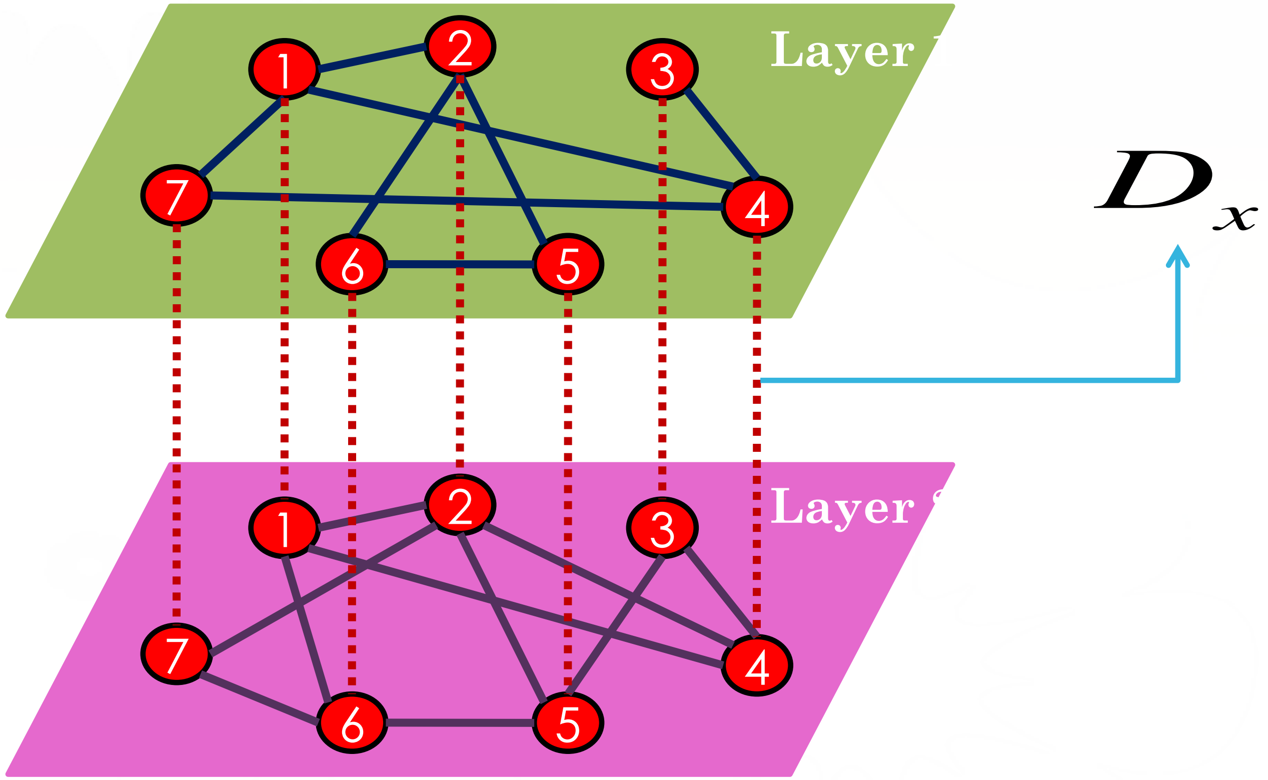


$$\left(\begin{array}{c|c} A_1 & I \\ \hline I & A_2 \end{array}\right)$$

Optimized evolution of multiplex network by *one layer* rewiring



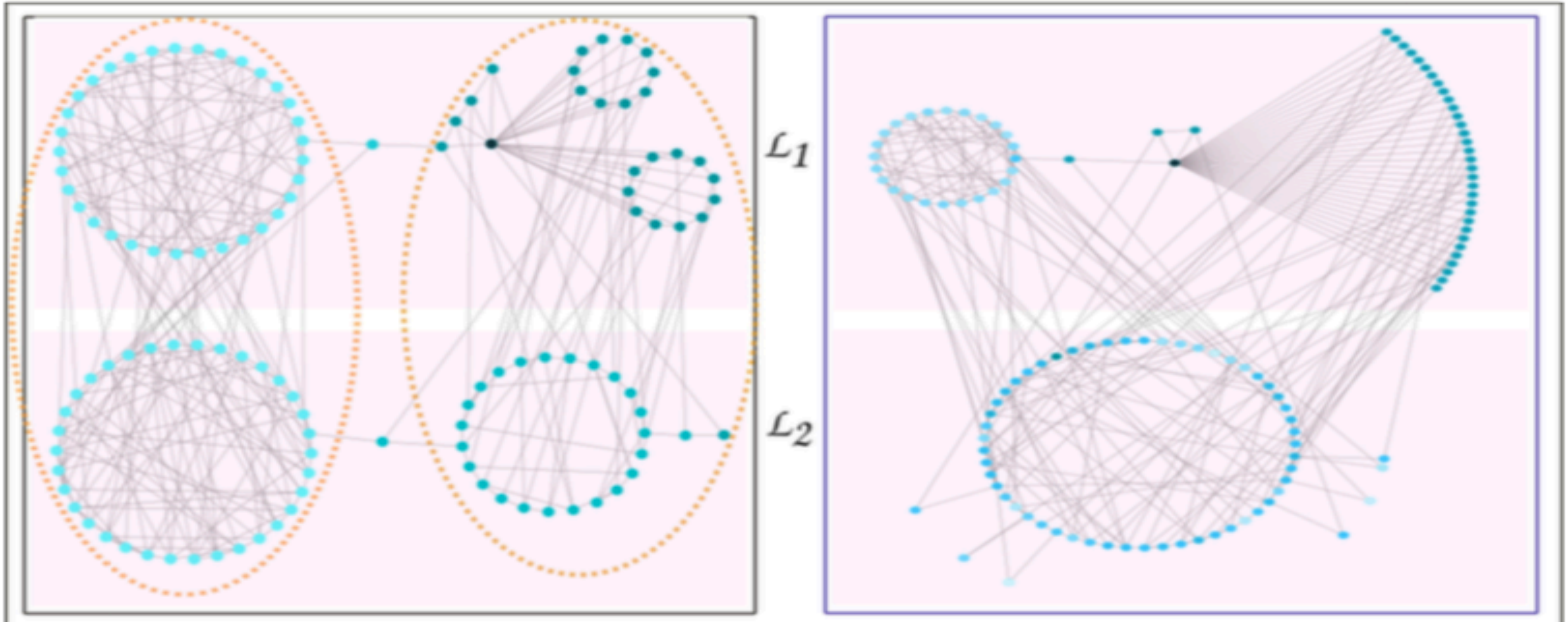
Adjacency matrix of Multi-layer networks



$$\left(\begin{array}{c|c} A_1 & D_x I \\ \hline D_x I & E_y A_2 \end{array}\right)$$

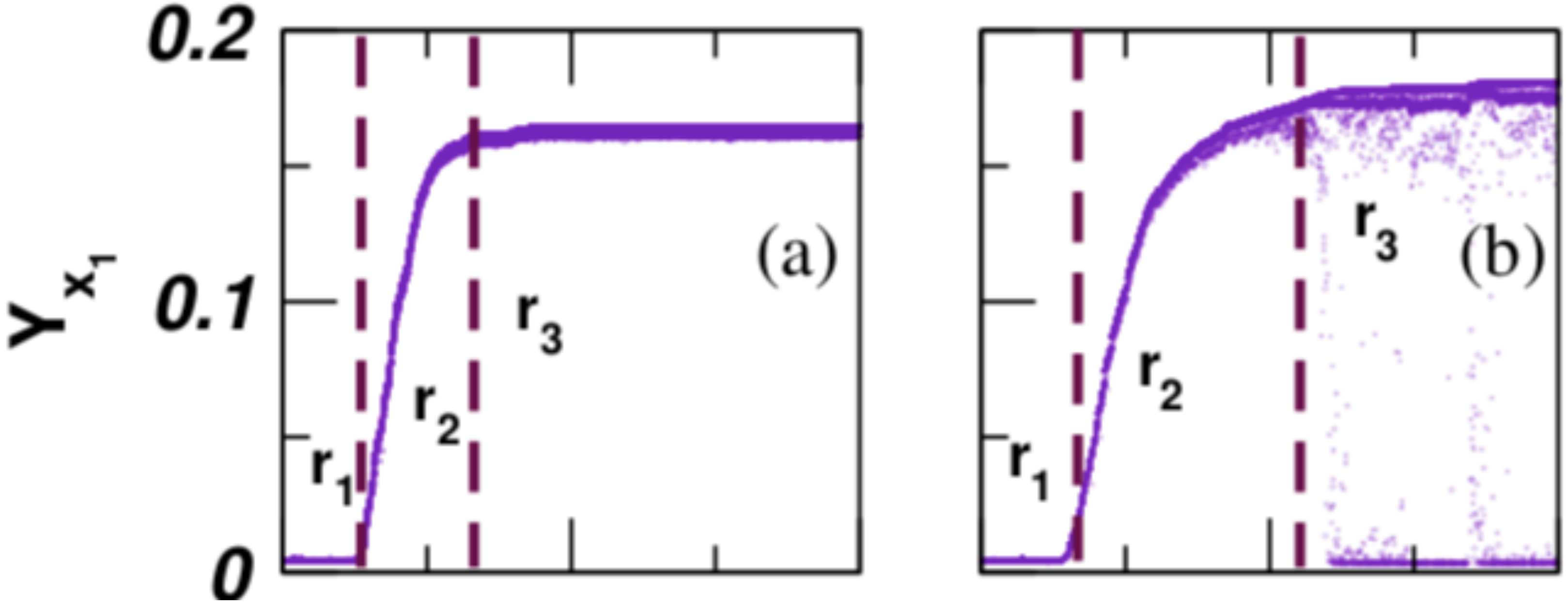
D_x : strength to which activities of layer 1 affects layer 2

E_y : relative coupling strength of the layers



(a) Both-layer rewiring (b) Single-layer rewiring

Optimized evolution of multiplex network



Left: Single layer rewiring   Right: Both layers rewiring

Single layer rewiring leads to almost the same value of IPR as the both layer rewiring

Importance of Optimization

- Optimization of complex networks is behind the success of technological as well as natural adaptive processes
- The brain learns by rewiring its synaptic connections. Deep learning machines changes internal structures of neural network to optimize its logical outputs
- The difficulty lies in the fact that optimization complexity increases exponentially by the size of the system
- We show that localization behavior of a whole multiplex network can be optimized by only rewiring a single network layer

Optimization complexity can be drastically reduced

Conclusion:

- ✓ We develop a learning framework to explore localization of eigenvector through an optimization method
- ✓ Localized networks possess several structual properties, incorporating only one of them does not lead to localization
- ✓ Localization of entire system represented by multiplex network by single layer rewiring

- ➢ Localization of principal eigenvector localization
  - Localization of disease, computer virus on smaller section
  - Faster spread of information: e.g. awareness of vaccination

Pradhan, Yadav, Dwivedi and SJ, Phys. Rev. E (2017)

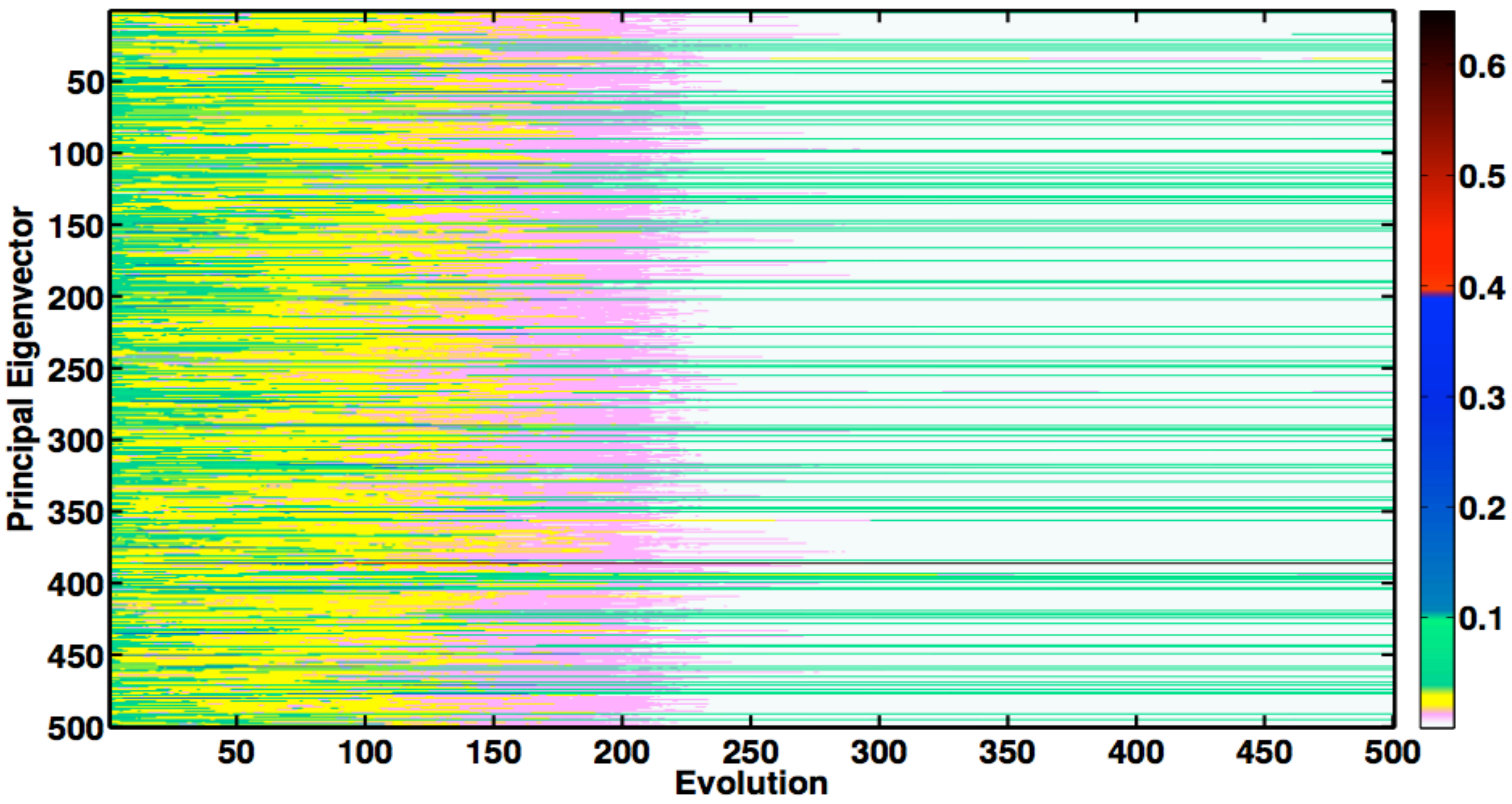
- ➢ Controlling PEV Localization of one layer: controlling propagation of perturbation in a network by appropriate multiplexing

SJ and Pradhan, Phys. Rev. E (2018)

Thank you !!

Acknowledgements: DST, CSIR and BRNS

Principal Eigenvectors during evolution

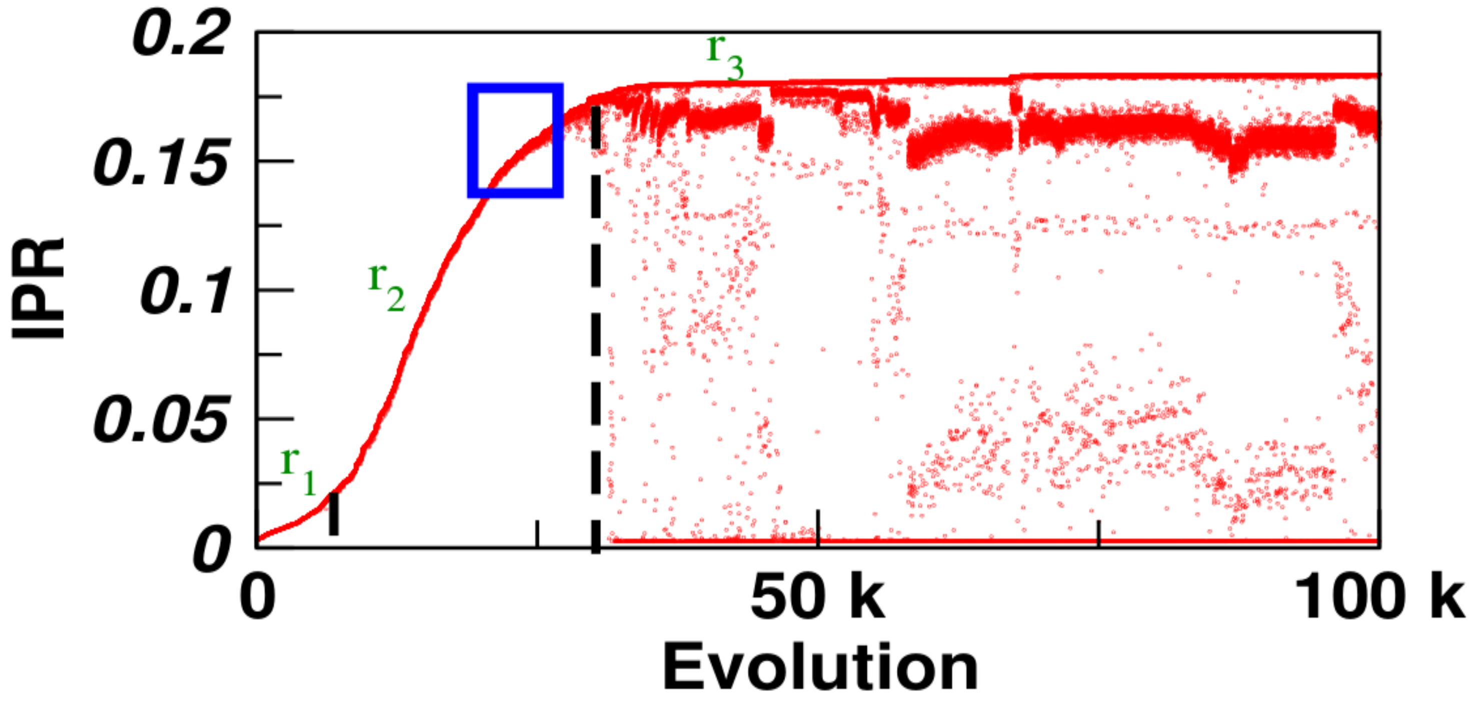


- Black line indicates the hub node weight

- During the evolution, formation of the hub node happens much before the IPR value gets saturated

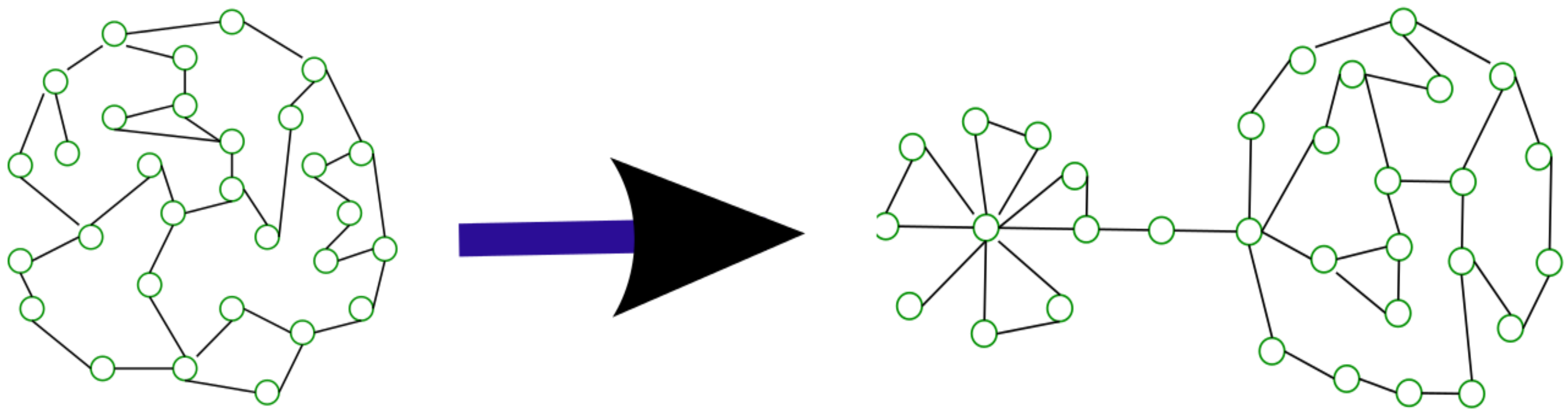
Critical Regime

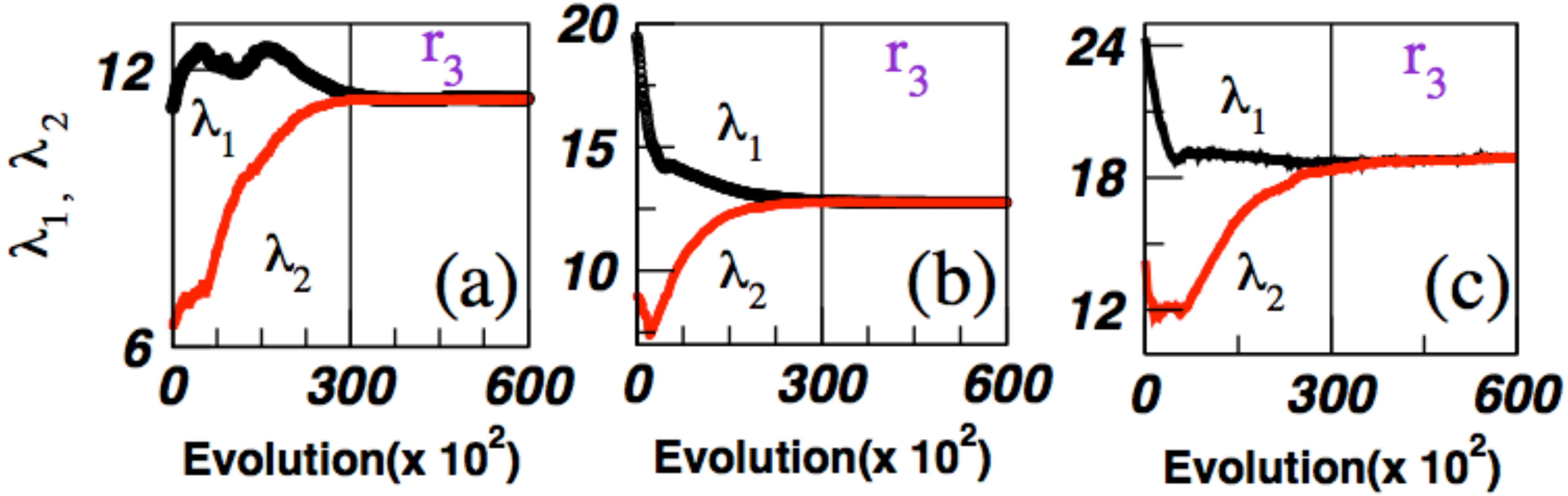
- As IPR gets saturated, there exists few edge rewiring that leads to a complete delocalization of PEV



- r3 region is very sensitive to edge rewiring

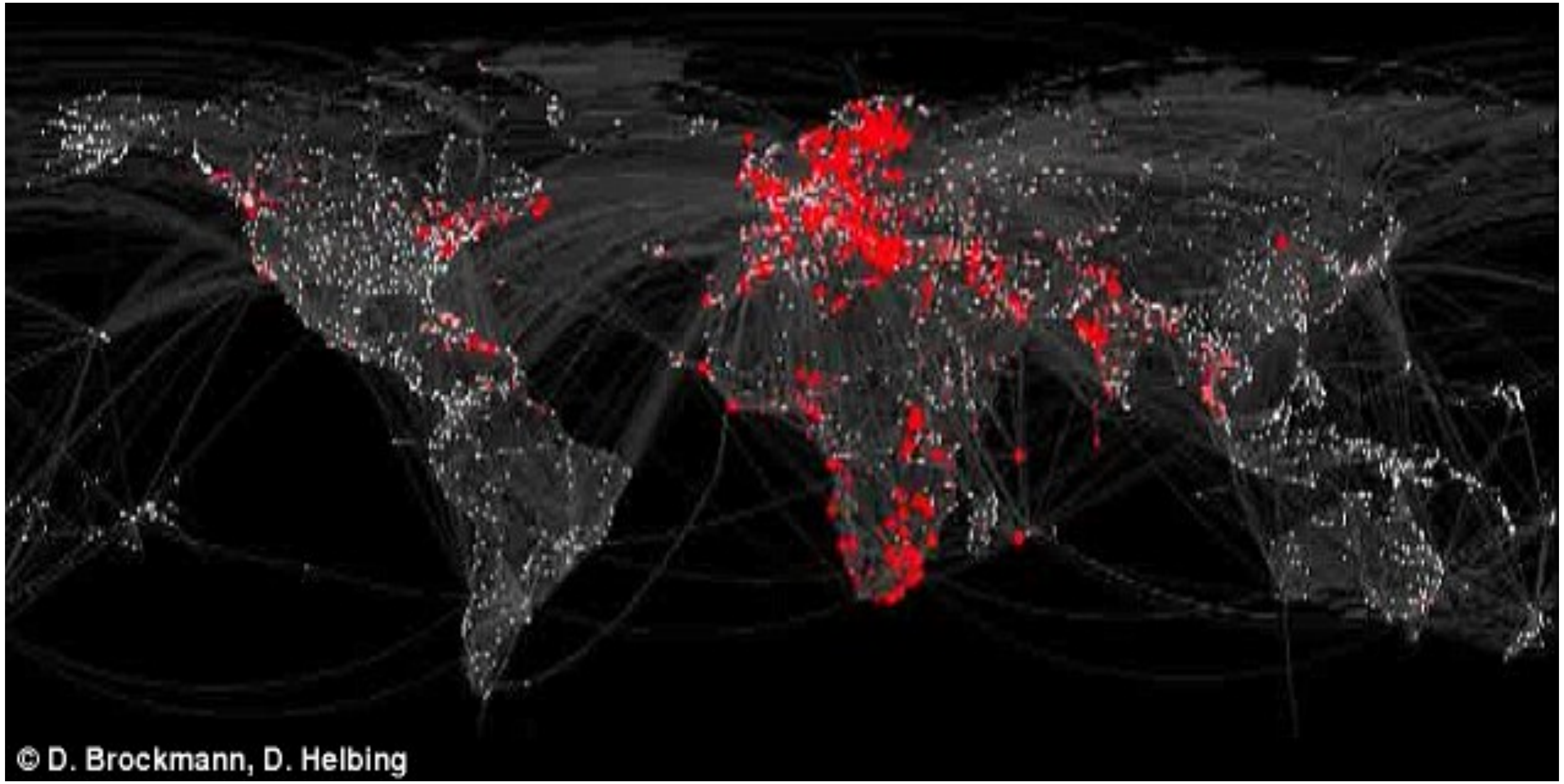
- In r2 region, IPR is close to r3, but this region is robust against edge removal/rewiring





λ_1, λ_2
12
6
λ_1
λ_2
r_3
(a)
0
300
600
Evolution(x 10^2)
20
15
10
λ_1
λ_2
r_3
(b)
0
300
600
Evolution(x 10^2)
24
18
12
λ_1
λ_2
r_3
(c)
0
300
600
Evolution(x 10^2)

How London Heathrow can spread a pandemic



http://www.dailymail.co.uk/sciencetech/article-2523302/Terrifying-video-reveals-London-Heathrow-spread-pandemic-DAYS.html

Various Real World Multiplex Networks

Network	l	N	$\langle k \rangle$	$Y_{x_1^{\mathcal{M}}}$	k_{max}	$\langle CC \rangle$	$r_{deg-deg}$	λ_1	λ_2
Moscow Athl.	3	124423	4.01	0.03	4840	0.07	-0.12	75.22	71.5
NYClimate	3	148936	5.39	0.07	9742	0.08	-0.10	118.5	99.2
MLKing2013	3	318962	2.51	0.08	8689	0.02	-0.11	93.2	85.5
Cannes2013	3	573353	3.98	0.2	8675	0.05	-0.09	94.26	86.9
Higgs mux	2	886744	31.09	0.003	51387	0.09	-0.09	653.5	436.7
ObamaIsrael	3	2258678	3.55	0.15	21650	0.07	-0.04	151.77	139.9
Drosophila	4	10255	7.62	0.008	175	0.09	0.07	46.96	31.0
Homo	4	34363	10.22	0.09	9570	0.16	-0.05	118.76	67.2

IPR of the corresponding random networks ~ N/3 ~ 0.00001

- ➢IPR of these real world MNs are much higher than the corresponding random networks
- ➢Largest two eigenvalues are not that close or well separated