GENERATING TREE AMPLITUDES BY INVERSE SOFT LIMIT

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Based on the following paper:
arXiv:1204.4841

DN and Congkao Wen

ICTS, INDIA Sept.2012

The study of **Scattering Amplitudes** has seen enormous progress over the last decade.

This research has revealed new structures in gauge and gravity theories.

Very powerful computational techniques have been developed that has been very useful for computing processes in LHC.

There had been a lot of progress in theories with maximal SUSY both at tree level and loop level.

Today we will concentrate only on tree level amplitudes.

PLAN OF THE TALK

- MOTIVATION
- BCFW AND INVERSE SOFT LIMIT
- CONSTRUCTING TREE AMPLITUDES BY ADDING PARTICLES
- CONSTRUCTING FORM FACTORS
- CONCLUSION

MOTIVATION

- TREE LEVEL GLUON SCATTERING AMPLITUDES IN N=4 SYM IN PLANAR LIMIT
- COLOR ORDERED SUBAMPLITUDE

$$\sum_{\text{all permutations of } \{1,2,\ldots,n\}} A(1^{h_1},\ldots,n^{h_n})\mathrm{Tr}[T^{a_1}\cdots T^{a_n}]$$

- SPINOR HELICITY

$$k^{\alpha\dot{\alpha}} = \frac{1}{2}k^{\mu}\sigma_{\mu}^{\alpha\dot{\alpha}} \longrightarrow k^{\alpha\dot{\alpha}} = \lambda^{\alpha}\widetilde{\lambda}^{\dot{\alpha}}$$

$$\langle i\, j\rangle = \epsilon_{\alpha\beta}\lambda_i^{\alpha}\lambda_j^{\beta} \qquad [i\, j] = \epsilon_{\dot{\alpha}\dot{\beta}}\widetilde{\lambda}_i^{\dot{\alpha}}\widetilde{\lambda}_j^{\dot{\beta}}$$

Soft Limit in Gauge Theories:

Soft Limit of Gauge Theories ----- Very Useful in constraining various aspects of amplitudes and consistency check

$$\lim_{j\to 0} A(1,\ldots,i,j,k,\ldots,n) = \sqrt{2}\left(-\frac{i\cdot\epsilon(j)}{s_{ij}} + \frac{k\cdot\epsilon(j)}{s_{jk}}\right) A(1,\ldots,i,\bar{j},k,\ldots,n)$$

Universal Soft Factor in Spinor Helicity:

$$\sqrt{2}\left(-\frac{i\cdot\epsilon(j)}{s_{ij}} + \frac{k\cdot\epsilon(j)}{s_{jk}}\right) = \frac{\langle ik\rangle}{\langle ij\rangle\langle jk\rangle}$$

Weinberg

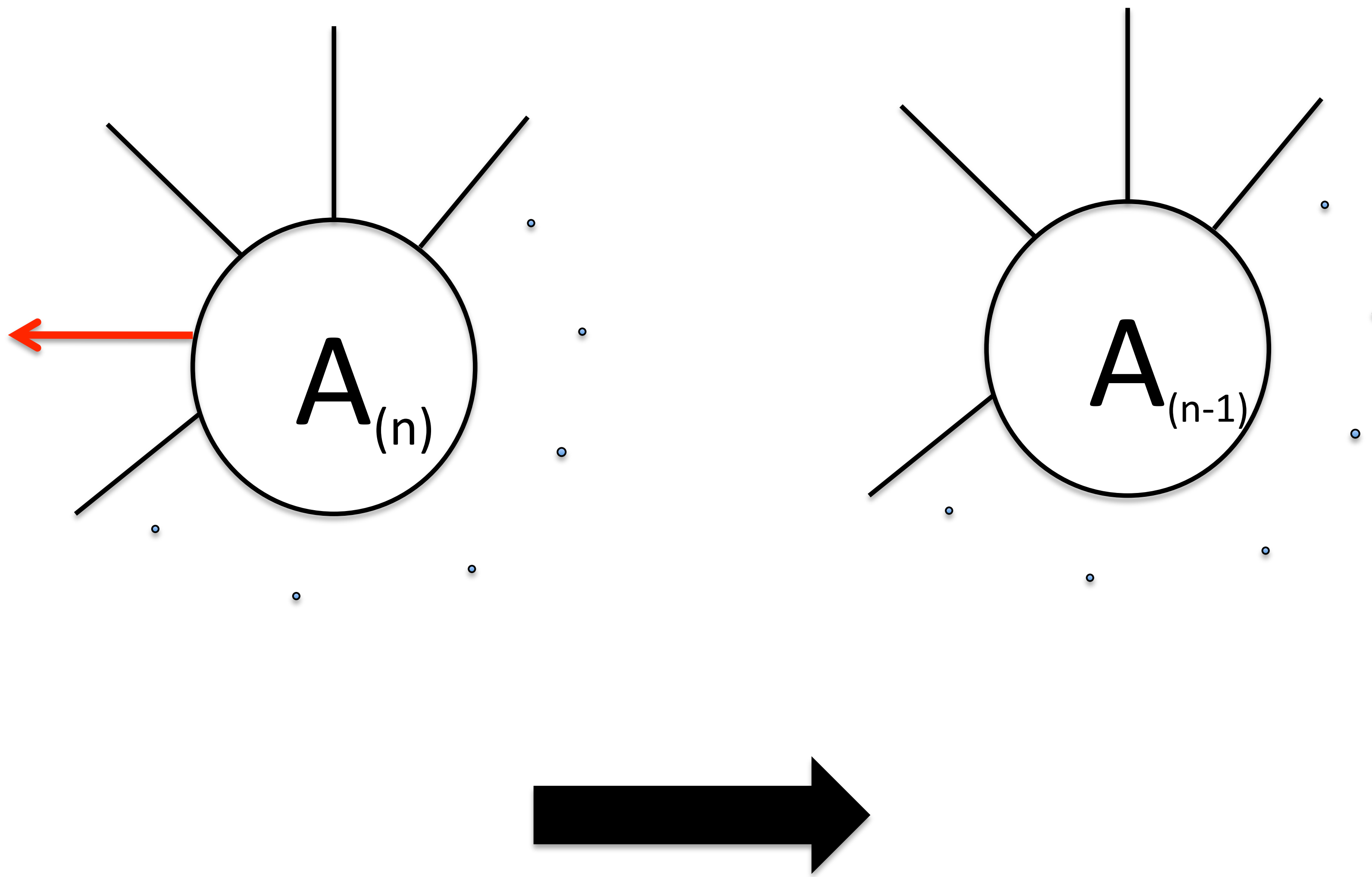
Recent idea – " INVERSE SOFT LIMIT"

$\Downarrow$

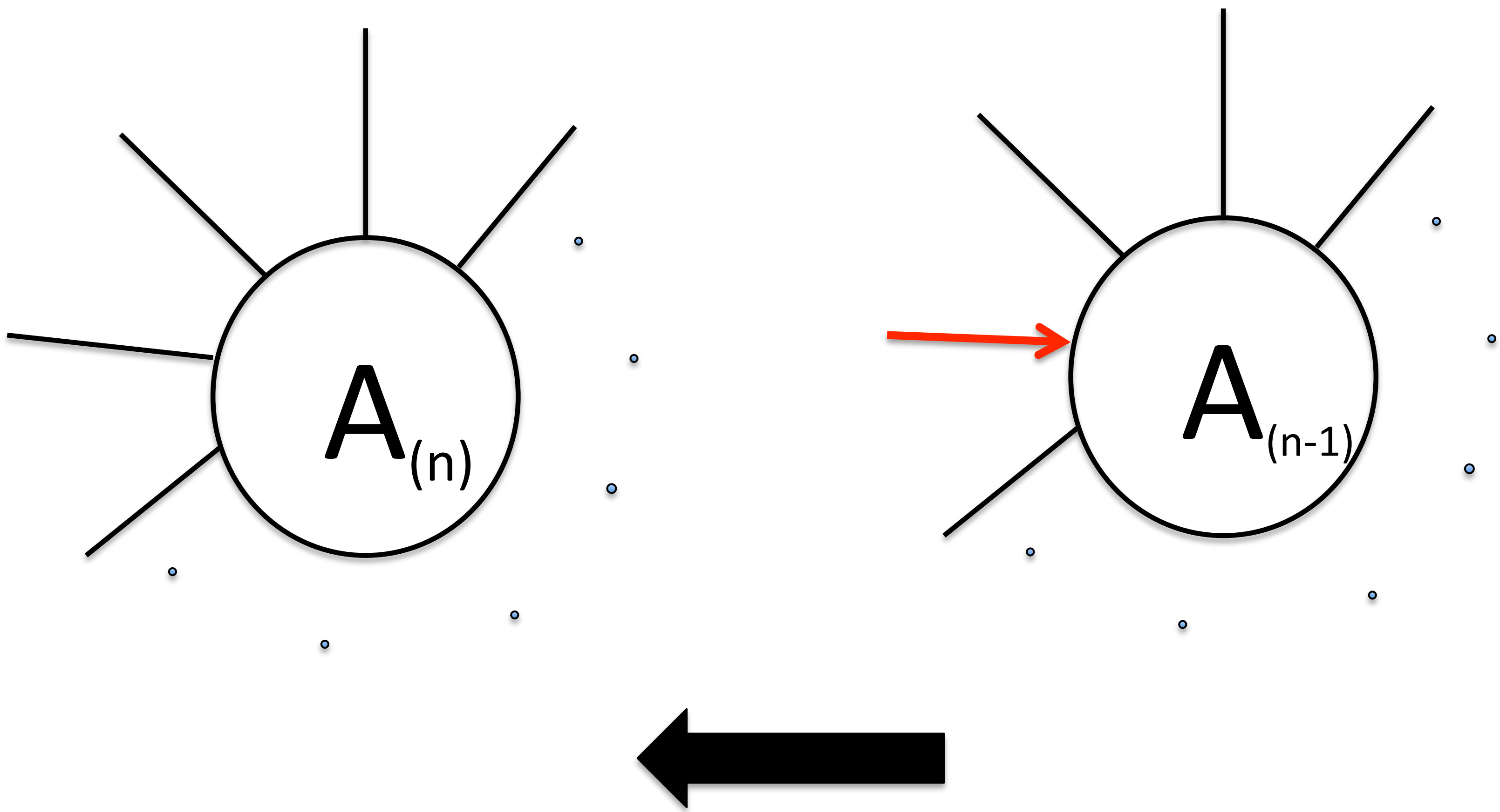
Add particle to lower point amplitudes!

- Used in the recent Grassmannian picture of amplitudes : Residues have an interpretation in terms of inverse soft limit.
- Also used to construct certain class of leading singularities.
- Some recent attempts to construct amplitudes up to NMHV.

Arkani-Hamed,Cachazo,Cheung,Kaplan;Arkani-Hamed; Bullimore; Boucher-Veroneau,Larkoski; Bourjaily,Cheung,Kaplan



A(n)
A(n-1)



Can we construct amplitudes using Soft Limit ?

INVERSE SOFT LIMIT and BCFW

BCFW Detour:

- Study analytic structure of amplitudes – external momenta undergoes complex deformation.
- Factorization channel gives amplitude as products of lower point amplitudes via recursion relation.

$$\widetilde{\lambda}_1 \to \widetilde{\lambda}_1(z) = \widetilde{\lambda}_1 + z\widetilde{\lambda}_n$$

$$\lambda_n \to \lambda_n(z) = \lambda_n - z\lambda_1$$

Britto,Cachazo,Feng,Witten

$$A(1^-, 2, \ldots, n-1, n^+) =$$

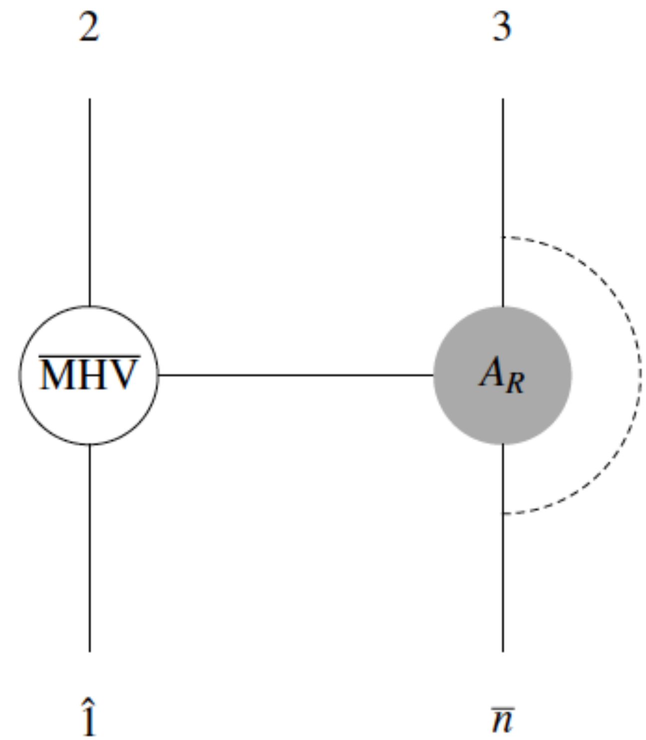
$$\boxed{\sum_{i=2}^{n-2} \sum_{h=+,-} A(\widehat{1}, 2, \ldots, i, -\widehat{K}_{1,i}^{h}) \frac{1}{K_{1,i}^2} A(+\widehat{K}_{1,i}^{-h}, i+1, \ldots, n-1, \widehat{n})}$$

$$K_{j,k} = k_j + k_{j+1} + \cdots + k_k \qquad\qquad z = \frac{K_{1,i}^2}{\langle 1|K_{1,i}|n]}$$

Now let us look at the supersymmetric version

2 Channel BCFW diagram

$\langle 1 \; n]$ shift



$$\lambda_{\hat{1}} = \lambda_1 - z\lambda_n \,;$$
$$\tilde{\lambda}_{\bar{n}} = \tilde{\lambda}_n + z\tilde{\lambda}_1 \,;$$
$$\eta_{\bar{n}} = \eta_n + z\eta_1 \,,$$

$$A(\hat{1}, 2|3, \cdots, \bar{n}) = \int d^4\eta_{\hat{P}} A_L(\hat{1}, 2, -\hat{P}) \frac{1}{s_{12}} A_R(\hat{P}, 3, \ldots, \bar{n})$$

Adding + helicity particle

$$A(\hat{1}, 2|3, \cdots, \bar{n}) = \mathcal{S}_+(n\ 1\ 2) A_R(2', 3, \ldots, n')$$

$$\mathcal{S}_+(n\ 1\ 2) = \frac{\langle n2 \rangle}{\langle n1 \rangle \langle 12 \rangle}$$

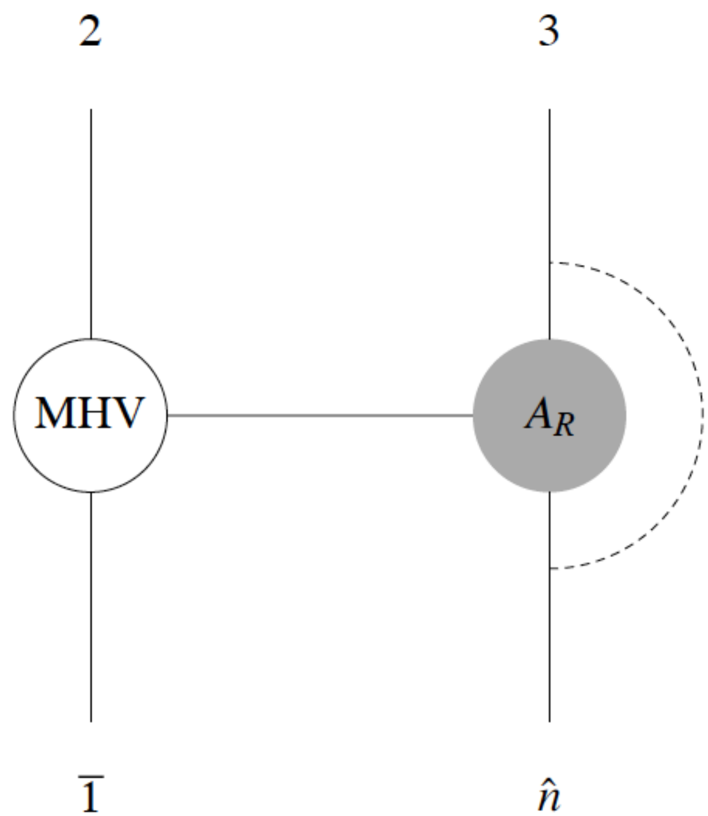
Inverse Soft Limit Shift : momenta + supercharge conserved

$$\tilde{\lambda}_2 \to \tilde{\lambda}_2 + \frac{\langle 1n \rangle}{\langle 2n \rangle} \tilde{\lambda}_1, \ \tilde{\lambda}_n \to \tilde{\lambda}_n + \frac{\langle 12 \rangle}{\langle n2 \rangle} \tilde{\lambda}_1$$

$$\eta_2 \to \eta_2 + \frac{\langle 1n \rangle}{\langle 2n \rangle} \eta_1, \ \eta_n \to \eta_n + \frac{\langle 12 \rangle}{\langle n2 \rangle} \eta_1 .$$

2 Channel BCFW diagram

$[1\ n\rangle$ shift



$$\lambda_{\bar{1}} = \tilde{\lambda}_1 - z\tilde{\lambda}_n$$

$$\lambda_{\hat{n}} = \lambda_n + z\lambda_1$$

$$\eta_{\bar{1}} = \eta_1 - z\eta_n.$$

$$A(\bar{1}, 2|3, \ldots, \hat{n}) = \mathcal{S}_{-}(n\ 1\ 2) A(2', \ldots, (n-1), n')$$

Adding - helicity particle

$$A(\bar{1}, 2|3, \ldots, \hat{n}) = \mathcal{S}_-(n\ 1\ 2) A(2', \ldots, (n-1), n')$$

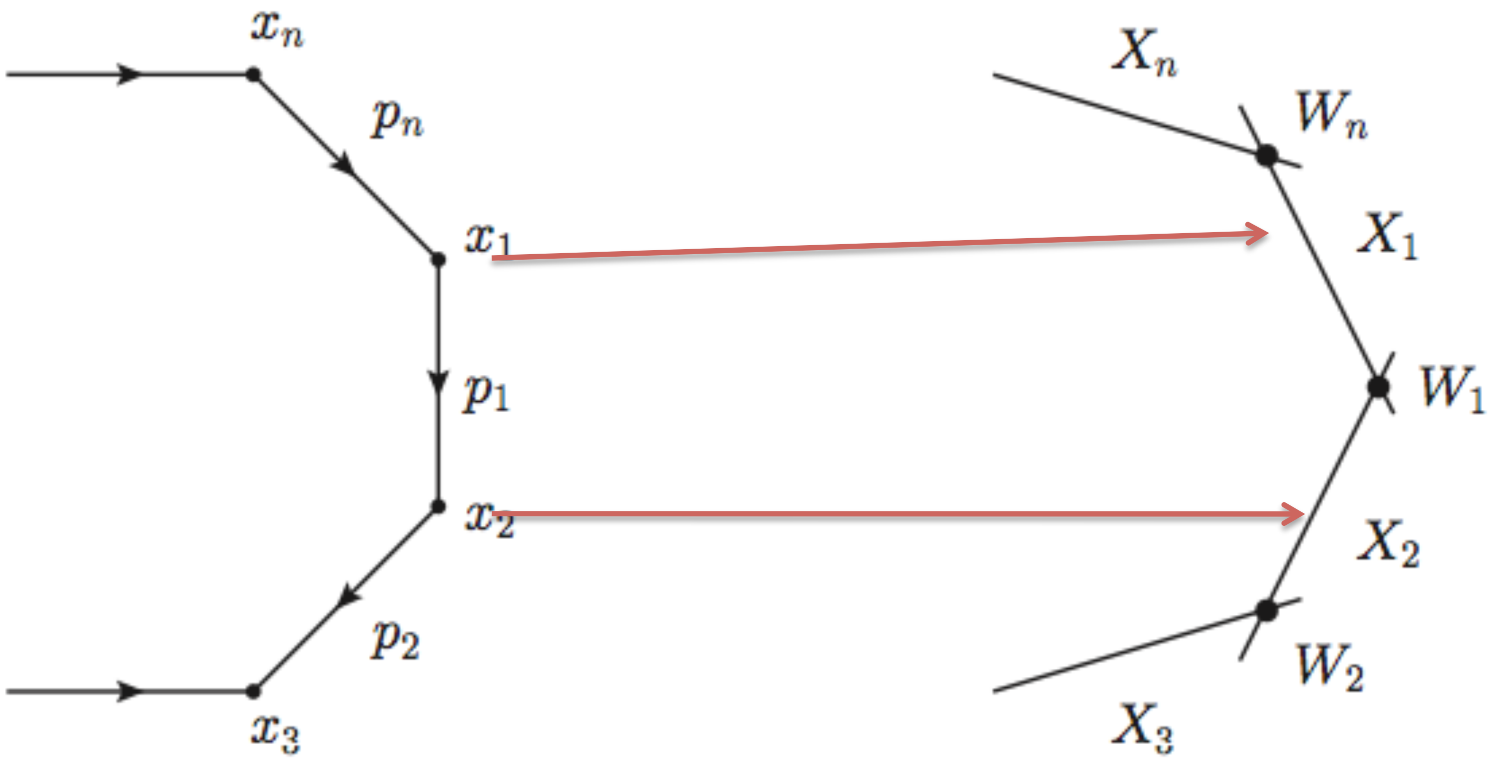
$$\mathcal{S}_-(n\ 1\ 2) = \frac{[n2]}{[n1][12]} \delta^4\left(\eta_1 + \frac{[n1]}{[2n]}\eta_2 + \frac{[12]}{[2n]}\eta_n\right)$$

- **Increases R-charge by 1 !**

Inverse Soft Limit Shift : momenta + supercharge conserved

$$\lambda_n \to \lambda_n + \frac{[21]}{[2n]}\lambda_1,\ \lambda_2 \to \lambda_2 + \frac{[1n]}{[2n]}\lambda_1$$

MOMENTUM TWISTOR :



- Manifest Dual Superconformal Symmetry,
- Region momenta x_i corresponds to lines X_i,
- Intersection defines momentum twistors

$$W_i = (\lambda_{i\alpha}, \mu_i^{\dot\alpha}, \chi_i^A)$$

$$\mu_i^{\dot\alpha} = -i x_i^{\alpha\dot\alpha}\lambda_{\alpha i} \qquad \chi_i^A = -i\theta_i^{A\alpha}\lambda_{\alpha i}$$

$$i\tilde\lambda_i = \frac{\langle i-1,i\rangle\mu_{i+1} + \langle i+1,i-1\rangle\mu_i + \langle i,i+1\rangle\mu_{i-1}}{\langle i-1,i\rangle\langle i,i+1\rangle}$$
$$i\eta_i = \frac{\langle i-1,i\rangle\chi_{i+1} + \langle i+1,i-1\rangle\chi_i + \langle i,i+1\rangle\chi_{i-1}}{\langle i-1,i\rangle\langle i,i+1\rangle}.$$

▪ Contour integral for dual formulation of all loop leading singularities in the space Grassmannnain G(k,n) using Twistors

$$\mathcal{L}_{n,k}(\mathcal{W}_a) = \frac{1}{\mathrm{vol}(\mathrm{GL}(k))} \int \frac{d^{k\times n} C_{\alpha a}}{(1\,2 \cdots k)(2\,3 \cdots k{+}1) \cdots (n\,1 \cdots k{-}1)}$$

$$\times \quad \prod_{\alpha=1}^{k} \delta^{4|4}\Big(\sum_{a=1}^{n} C_{\alpha a} \mathcal{W}_a\Big),$$

Arkani-Hamed,
BourJaily, Cachazo,Trnka

- Contour integral for dual formulation of all loop leading singularities in the space Grassmannian G(k-2,n) using Momentum Twistors

$$\mathcal{L}_{n,k} = \mathcal{A}_{\text{MHV}} \frac{1}{\text{vol}(\text{GL}(k-2))} \int \frac{d^{(k-2)\times n} D}{(1\,2 \cdots k{-}2)(2\,3 \cdots k{-}1) \cdots (n\,1 \cdots k{-}3)}$$

$$\times \prod_{\hat{\alpha}=1}^{k-2} \delta^{4|4}(D_{\hat{\alpha} a} \mathcal{Z}_a),$$

YANGIAN INVARIANTS in MOMENTUM TWISTOR :

- k- preserving

$$Y'_{n,k}(\mathcal{Z}_1, \ldots, \mathcal{Z}_{n-1}, \mathcal{Z}_n) = Y_{n-1,k}(\mathcal{Z}_1, \ldots \mathcal{Z}_{n-1})$$

- k- increasing

$$Y'_{n,k}(\ldots, \mathcal{Z}_{n-1}, \mathcal{Z}_n, \mathcal{Z}_1, \ldots) = [n-2\ n-1\ n\ 1\ 2] Y_{n-1,k-1}(\ldots, \widehat{\mathcal{Z}_{n-1}}, \widehat{\mathcal{Z}_1}, \ldots)$$

Shifts are:

$$\begin{aligned} \widehat{\mathcal{Z}_1} &= \mathcal{Z}_1 \langle 2\ n-2\ n-1\ n \rangle + \mathcal{Z}_2 \langle n-2\ n-1\ n\ 1 \rangle \\ \widehat{\mathcal{Z}_{n-1}} &= \mathcal{Z}_{n-2} \langle n-1\ n\ 1\ 2 \rangle + \mathcal{Z}_{n-1} \langle n\ 1\ 2\ n-2 \rangle, \end{aligned}$$

$$[a\ b\ c\ d\ e] = \frac{\delta^{0|4}(\eta_a \langle b\ c\ d\ e \rangle + \text{cyclic})}{\langle a\ b\ c\ d \rangle \langle b\ c\ d\ e \rangle \langle c\ d\ e\ a \rangle \langle d\ e\ a\ b \rangle \langle e\ a\ b\ c \rangle}$$

Arkani-Hamed, Bourjaily,Cachazo,Caron-Huot,Trnka

CONSTRUCTING TREE AMPLITUDES BY INVERSE SOFT LIMIT

How to construct multi-particle channel BCFW diagrams ?

To construct generic diagram $A_L(\bar{1}, 2, \cdots, m, \hat{P})\frac{1}{P^2}A_R(-\hat{P}, m+1, \cdots, n-1, \hat{n})$

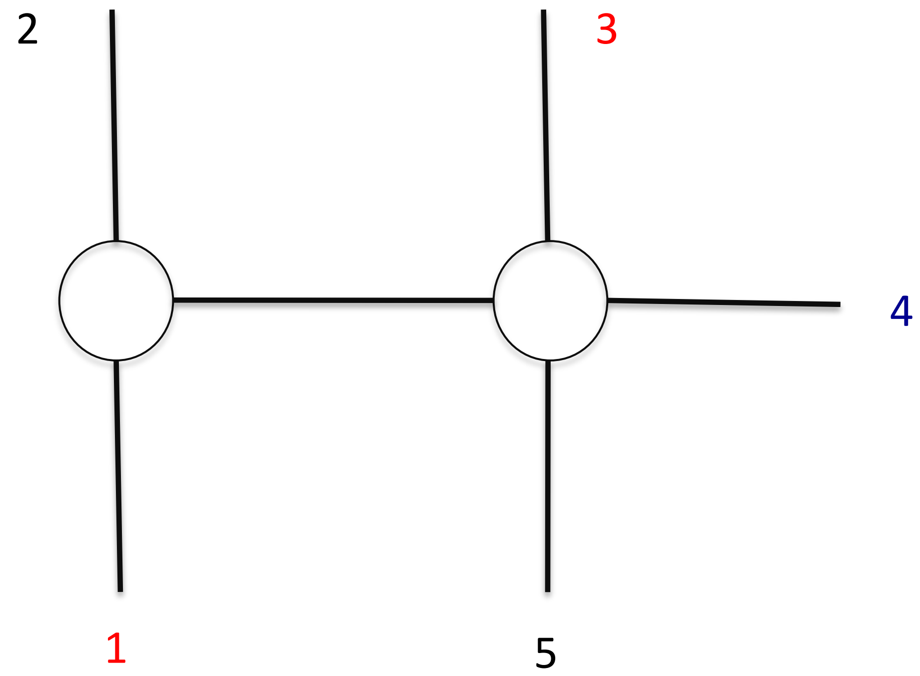
- Start with 2 channel diagram

$$A_L(\bar{1}, m, \hat{P})\frac{1}{P^2}A_R(-\hat{P}, m+1, \cdots, n-1, \hat{n}) = \mathcal{S}(n\,1\,m)A_R(m', m+1, \cdots, n-1, n').$$

  - Add particles between 1 and m

$$A_n = \sum_{i;L,R}(\prod_L \mathcal{S}'_L)(\prod_R \mathcal{S}'_R)A_{\overline{\text{MHV}}}(i', i+1, n'),$$

Now let us work out an example : $\overline{\mathrm{MHV}}$



- Start with 3 point MHV amplitude ➡ $A_{MHV}(345)$
- Add particle 1^+ ➡ $A_{MHV}(1345)$

$$\left.\begin{aligned} A_{\mathrm{MHV}}(1345) &= \frac{\langle 5\,3\rangle}{\langle 5\,1\rangle\langle 1\,3\rangle}\left(\frac{\delta^8(\lambda_{3'}\eta_{3'}+\lambda_4\eta_4+\lambda_{5'}\eta_{5'})}{\langle 3'\,4\rangle\langle 4\,5'\rangle\langle 5'\,3'\rangle}\right) \\ &= \frac{\delta^8(\lambda_1\eta_1+\lambda_3\eta_3+\lambda_4\eta_4+\lambda_5\eta_5)}{\langle 1\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}, \end{aligned}\right\} \mathcal{S}_+(513)A_{\mathrm{MHV}}(3'45')$$

- Add 2^- particle ➡ $A_{\mathrm{NMHV}}(12345)$

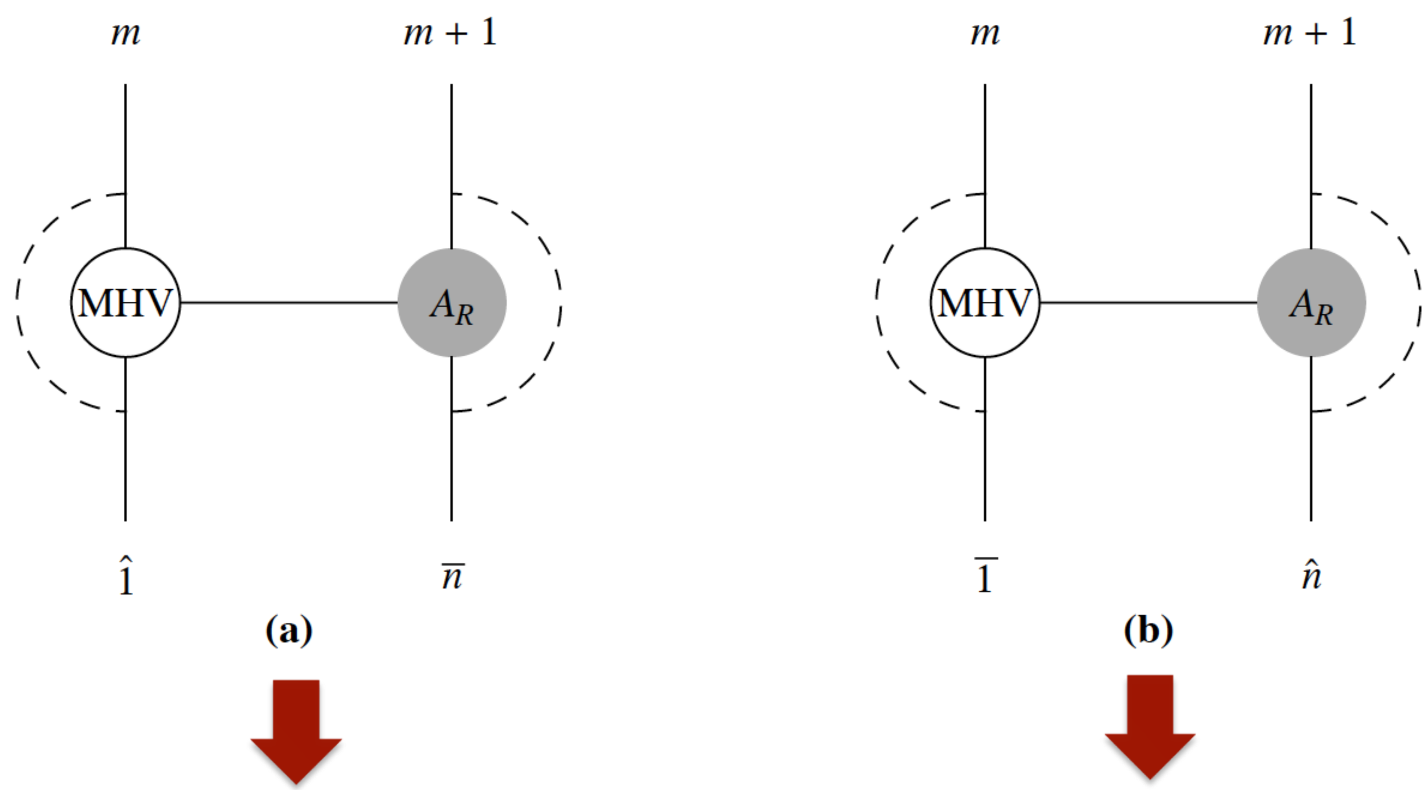
$$\begin{aligned} A_{\overline{\mathrm{MHV}}}(12345) &= \mathcal{S}_-(123)A_{\mathrm{MHV}}(1'3'45) \\ &= \frac{\delta^4(\eta_1[23]+\eta_2[31]+\eta_3[12])\,\delta^8(\sum_{i=1}^5\lambda_i\eta_i)}{[1\,2][2\,3][3\,4][4\,5][5\,1]\langle 4\,5\rangle^4} \end{aligned}$$

General Anti-MHV Amplitude:

$$A_{\overline{\mathrm{MHV}}}(1,2,\cdots,n) = \frac{\delta^8(\sum\lambda_i\eta_i)\prod_{i=2}^{n-3}\delta^4(\eta_1[i\,i+1]+\eta_i[i+1\,1]+\eta_{i+1}[1\,i])}{\langle n-1\,n\rangle^4\prod_{i=1}^{n}[i\,i+1]\prod_{i=3}^{n-3}[1\,i]^4}$$

Construct MHV Amplitude:

- Two possible diagrams for 2 types of shifts $A_{MHV}(1\ldots m) X A_R(m+1\ldots\ldots n)$



$$\hat{\mathbb{A}}^{(m)}_{\mathrm{MHV}} \equiv \{1^+, 2^-, 3^+, \cdots, (m-1)^+\} \quad \bar{\mathbb{A}}^{(m)}_{\mathrm{MHV}} \equiv \{1^-, 2^+, \cdots, (m-1)^+\}$$

How to relate Inverse Soft with BCFW in Momentum Twistor ?

$$M_{n,k}(1,\cdots,n) = M_{n-1,k}(2,\cdots,n) + \sum_{n_R,k_R;j} [j{+}1\ j\ 2\ 1\ n] M_{n_L,k_L}(\hat{1}_{j+1},\cdots,j,I_{j+1}) M_{n_R,k_R}(I_{j+1},j{+}1,\cdots,n)$$

$n_L + n_R = n+2, k_L + k_R = k-1$ & $\hat{1}_{j+1} = (12)\bigcap(j\,j{+}1\,n), I_{j+1} = (j\,j{+}1)\bigcap(n12)$

MHV: $M_{n_L,k_L}(\hat{1}_{j+1},\cdots,j,I_{j+1}) = 1$

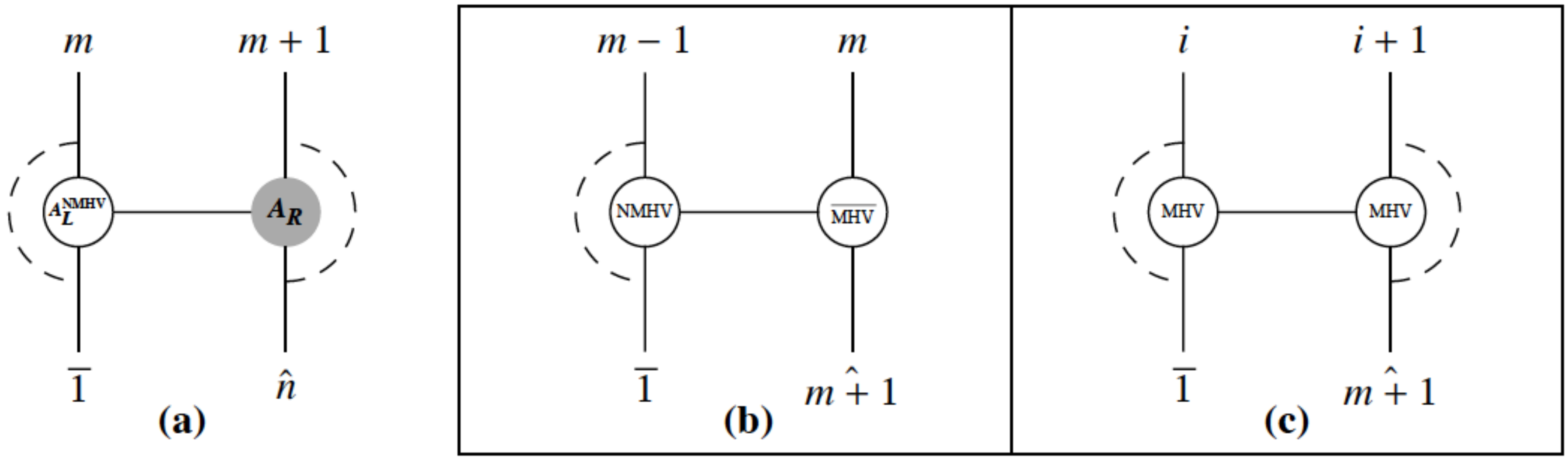
1^+

$$M_{n,k}(1,\cdots,n) = M_{n-1,k}(2,\cdots,n) + \sum_{n_R,k_R;j} [j{+}1\ j\ 2\ 1\ n] M_{n_R,k_R}(I_{j+1},j{+}1,\cdots,n)$$

$\{1^+,2^-,3^+,\cdots,(j{-}1)^+\}$

Connection not clear beyond MHV !!

Construct NMHV Amplitude:



$$\begin{aligned}\bar{\mathbb{A}}^{(m)}_{\mathrm{NMHV}} &= \{1^-, (m-1)^+, \bar{\mathbb{A}}^{(m-1)}_{\mathrm{NMHV}}(1^-)\} \\ &+ \sum_{i=2}^{m-2} \{1^-, i^+, \mathcal{R}^{i-1}\left[\hat{\mathbb{A}}^{(m-i+1)}_{\mathrm{MHV}}(\cancel{1^+})\right], \bar{A}^{(i)}_{\mathrm{MHV}}(\cancel{1^-})\}\end{aligned}$$

Solution NMHV

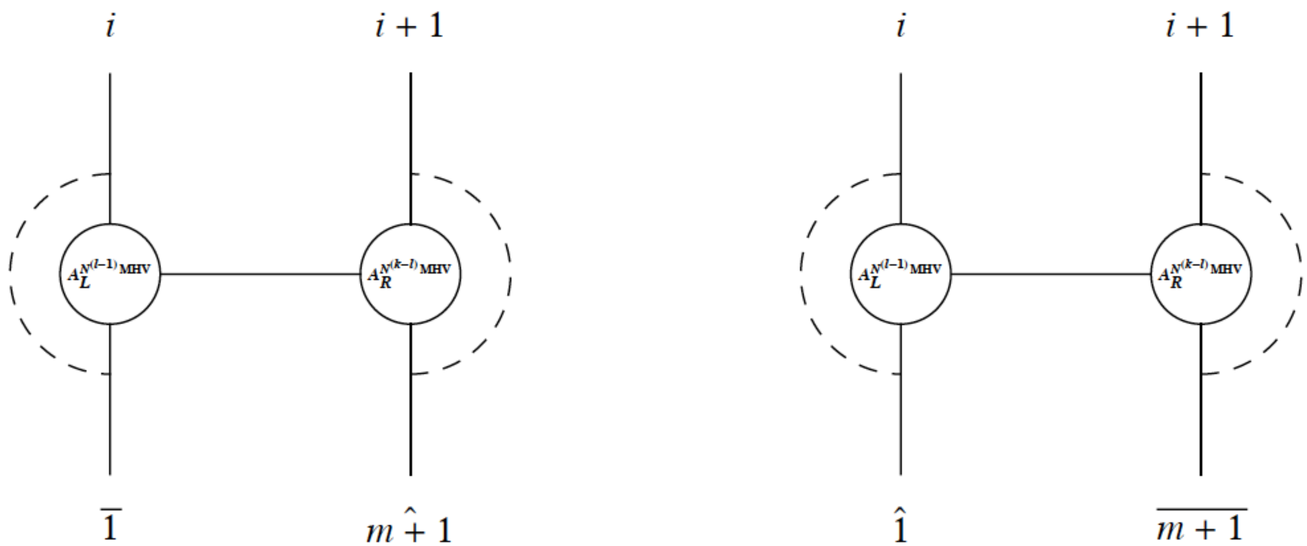
$$
\begin{aligned}
\bar{\mathbb{A}}^{(m)}_{\mathrm{NMHV}} &= \sum_{i=4}^{m-1}\sum_{j=2}^{i-2}\{1^-,(m-1)^+,\cdots,i^+,j^+,(j+1)^-,(j+2)^+,\cdots,(i-1)^+\} \\
&\quad + \sum_{i=2}^{m-2}\{1^-,i^+,(i+1)^-,(i+2)^+,\cdots,(m-1)^+,2^+,\cdots,(i-1)^+\} \\
&= \sum_{i=4}^{m}\sum_{j=2}^{i-2}\{1^-,(m-1)^+,\cdots,i^+,j^+,(j+1)^-,R^+\},
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathbb{A}}^{(m)}_{\text{NMHV}} = &\{1^+, 2^-, \mathcal{R}[\bar{\mathbb{A}}^{(m-1)}_{\text{NMHV}}(\not{1}^-)]\} \\
&+ \sum_{i=3}^{m-1} \{1^+, i^-, \mathcal{R}^{i-1}[\bar{\mathbb{A}}^{(m-i+1)}_{\text{MHV}}(\not{1}^-)], \hat{\mathbb{A}}^{(i)}_{\text{MHV}}(\not{1}^+)\}
\end{aligned}
$$

Solution NMHV

$$
\begin{aligned}
\hat{\mathbb{A}}^{(m)}_{\text{NMHV}} = &\sum_{i=3}^{m-1} \{1^+, i^-, (i+1)^+, \cdots, (m-1)^+, 2^-, R_1^+\} \\
&+ \sum_{i=4}^{m-1} \sum_{j=3}^{i-1} \{1^+, 2^-, (m-1)^+, \cdots, (i+1)^+, j^+, (j+1)^-, R_2^+\}
\end{aligned}
$$

Construct N^kMHV Amplitude:



(a) **(b)**

$$\bar{\mathbb{A}}^{(m)}_{\mathrm{N}^k\mathrm{MHV}} = \{1^-, (m-1)^+, \bar{\mathbb{A}}^{(m-1)}_{\mathrm{N}^k\mathrm{MHV}}(\cancel{1^-})\}$$
$$+ \sum_{l=1}^{k} \sum_{\substack{i=l+2 \\ (2 \text{ for } l=1)}}^{m-k+l-2} \{1^-, i^+, \mathcal{R}^{i-1}\left[\hat{\mathbb{A}}^{(m-i+1)}_{\mathrm{N}^{k-l}\mathrm{MHV}}(\cancel{1^+})\right], \bar{\mathbb{A}}^{(i)}_{\mathrm{N}^{l-1}\mathrm{MHV}}(\cancel{1^-})\}$$

$$\hat{\mathbb{A}}^{(m)}_{\mathrm{N}^k\mathrm{MHV}} = \{1^+, 2^-, \mathcal{R}\left[\bar{\mathbb{A}}^{(m-1)}_{\mathrm{N}^k\mathrm{MHV}}(\cancel{1^-})\right]\} + \sum_{l=1}^{k} \sum_{i=l+2}^{\substack{(m-k+l-1\,\text{for}\,l=k)\\ m-k+l-2}} \{1^+, i^-, \mathcal{R}^{i-1}\left[\bar{\mathbb{A}}^{(m-i+1)}_{\mathrm{N}^{k-l}\mathrm{MHV}}(\cancel{1^-})\right], \hat{\mathbb{A}}^{(i)}_{\mathrm{N}^{l-1}\mathrm{MHV}}(\cancel{1^+})\}$$

$$\bar{\mathbb{A}}^{(m)}_{\mathrm{N}^k\mathrm{MHV}} = \mathcal{P}\left[\hat{\mathbb{A}}^{(m)}_{\mathrm{N}^{m-k-3}\mathrm{MHV}}\right]$$

ISL shifts & BCFW deformations: equivalence

$$A_L(\bar{1},2,\cdots,i,\hat{P})\frac{1}{P^2}A_R(-\hat{P},i+1,\cdots,\hat{n}).$$

$$A_n=\sum_{i;L,R}(\prod_L \mathcal{S}'_L)(\prod_R \mathcal{S}'_R)A_{\overline{\text{MHV}}}(i',i+1,n')$$

$$A_L(\bar{1},2,\cdots,i,\hat{P})\frac{1}{P^2}A_R(-\hat{P},(i+1),\cdots,\hat{n})=\sum(\prod\mathcal{S}')A_R(i_s,(i+1),\cdots,(n-1),n_s)$$

$$i_s=\hat{P},\quad 1_s=\bar{1},\quad n_s=\hat{n}$$

$$i_s=(1+2+\cdots+i)+\frac{s_{12\cdots i}}{\langle 1|2+\cdots+i|n]}\lambda_1\tilde{\lambda}_n=\hat{P}$$

$$n_s=n+\frac{s_{12\cdots i}}{\langle 1|2+\cdots+i|n]}\lambda_1\tilde{\lambda}_n=\hat{n},$$

Amplitudes from ISL.

$$A_L(\bar{1},2,\cdots,i,\hat{P})\frac{1}{P^2}A_R(-\hat{P},i+1,\cdots,\hat{n})$$

$$A_n = \sum_{i;L,R}\left(\prod_L \mathcal{S}'_L\right)\left(\prod_R \mathcal{S}'_R\right)A_{\overline{\mathrm{MHV}}}(i',i+1,n')$$

Add between i+1 and n. to get A_R use $\hat{\mathbb{A}}^{(n-i)}$

$k \rightarrow n-k+1$

Add between 1 and i. to get A_L use $\bar{\mathbb{A}}^{(i)}$

Start with this- By construction 1^{st} Particle 1^- between i and n

CONSTRUCTING FORM FACTORS

Matrix elements of a gauge invariant, composite operator between the vacuum and some external scattering states

$$F(q;1,2,\cdots,n) = \langle 1,2,\cdots,n|\mathcal{O}(q)|0\rangle$$

$$\sum_{i=1}^{n} p_i = q \qquad\qquad q^2 \neq 0.$$

N=4 SYM : Supersymmetric form factors

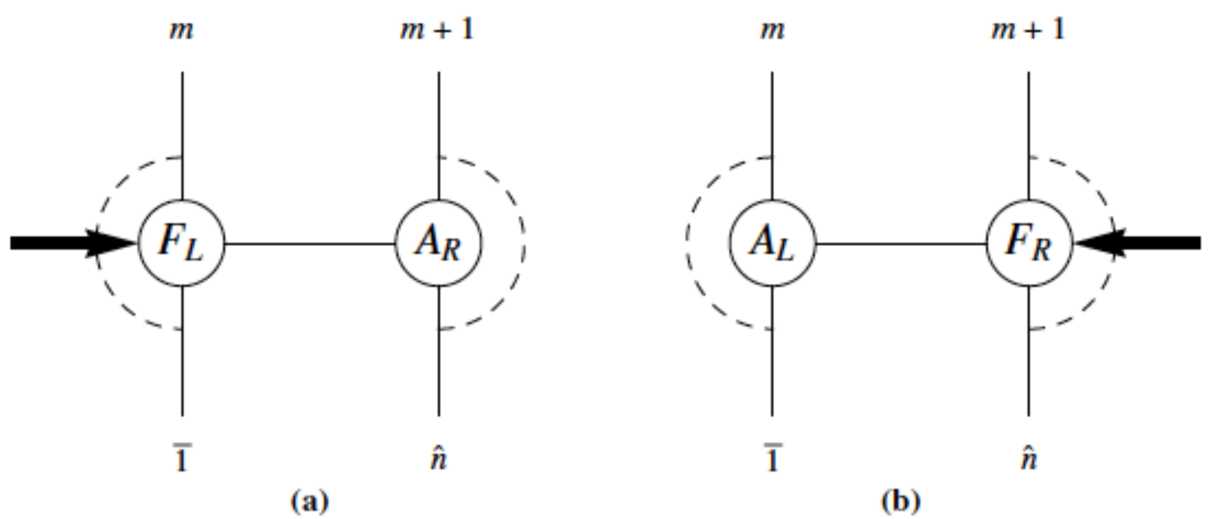
$$F(q,\gamma^+;1,2,\cdots,n) = \langle 1,2,\cdots,n|\mathcal{T}(q,\gamma^+)|0\rangle$$

Chiral part of Stress Tensor: $\mathcal{T}(x,\theta^+) = \mathcal{T}(x,\theta^+,\theta^- = 0)$

Brandhuber, Gurdogan,Mooney,Travaglini,Yang; Brandhuber,Travaglini,Yang; Maldacena,Zhibodev; Bork,Kazakov,Vartanov; Sen;Sterman;Catani,Trentadue; Magnea,Sterman.

Supersymmetric BCFW for Form Factors:

$$F(q,\gamma^+;1,2,\cdots,n) = \sum_i \Big[\int d^4\eta F(q,\gamma^+;\bar{1},2,\cdots,m,\hat{P})A(-\hat{P},(m+1),\cdots,\hat{n}) + \int d^4\eta A(\bar{1},2,\cdots,m,\hat{P})F(q,\gamma^+;-\hat{P},(m+1),\cdots,\hat{n})\Big]$$



MHV Example of super Form Factor

$$F_{\mathrm{MHV}}(q,\gamma^+;1,2,\cdots,n)=\frac{\delta^4(\sum_{i=1}^n\lambda_i\tilde{\lambda}_i-q)\delta^4(\sum_{i=1}^n\lambda_i\eta_i^-)\delta^4(\sum_{i=1}^n\lambda_i\eta_i^+-\gamma^+)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- Analogous to Parke-Taylor Formula
- Conservation Delta Functions different
- Many Properties same as amplitudes

Form Factor : Same soft limit by taking external particle soft !

ISL for Form Factors :

$$
\begin{aligned}
F(q,\gamma^+;1,2,\cdots,n) = \sum_{m;L,R} \Bigg[& \left(\prod_R \mathcal{S}'_R\right) F(q,\gamma^+;1',2,\cdots,m,(m+1)') \\
& + \left(\prod_L \mathcal{S}'_L\right) F(q,\gamma^+;m',(m+1),\cdots,n-1,n') \Bigg]
\end{aligned}
$$

- Add particles from both sides of BCFW diagram
- Use recursion relation for [1 n> and <1 n] shifts
- Eg: If adding particle "i" from left, need to add from right another particle "n+1-i"

ISL for Form Factors : Self Dual Field Strength

$$F_{\mathrm{MNMHV}}(q;1,\cdots,n) = \langle 1\cdots n|\mathrm{Tr}(F_{\mathrm{SD}}^2)|0\rangle|_{\mathrm{MNMHV}}$$

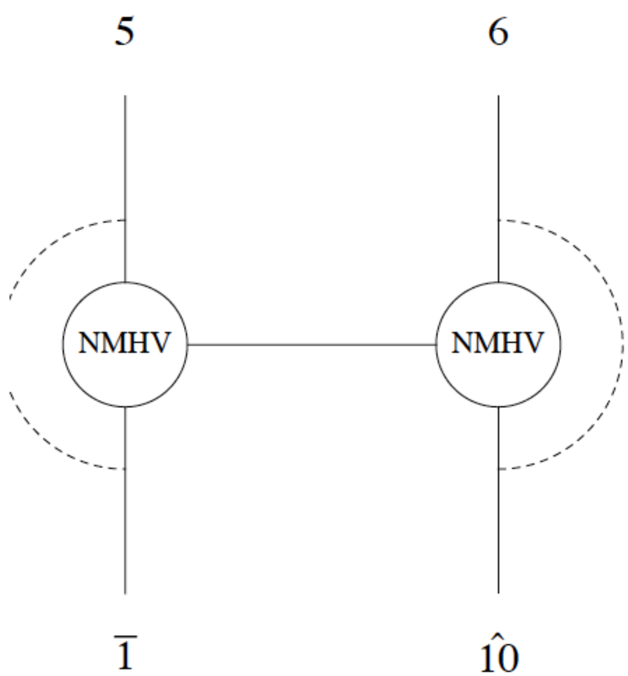
$$F_{\mathrm{MNMHV}}(q;1,\cdots,n) = \delta^8\left(\sum_{i=1}^{n}\lambda_i\eta_i\right)\delta^4\left(\sum_{i=1}^{n}\lambda_i\tilde{\lambda}_i - q\right)\frac{q^4}{[12]\cdots[n1]}\eta_1^4\cdots\eta_n^4$$

- Only 2 particle channel BCFW diagram

$$F_{\mathrm{MNMHV}}(q;1,2,\cdots,n) = \mathcal{S}_-(n\ 1\ 2)F_{\mathrm{MNMHV}}(q;2',3,\cdots,n')$$

CONCLUSION

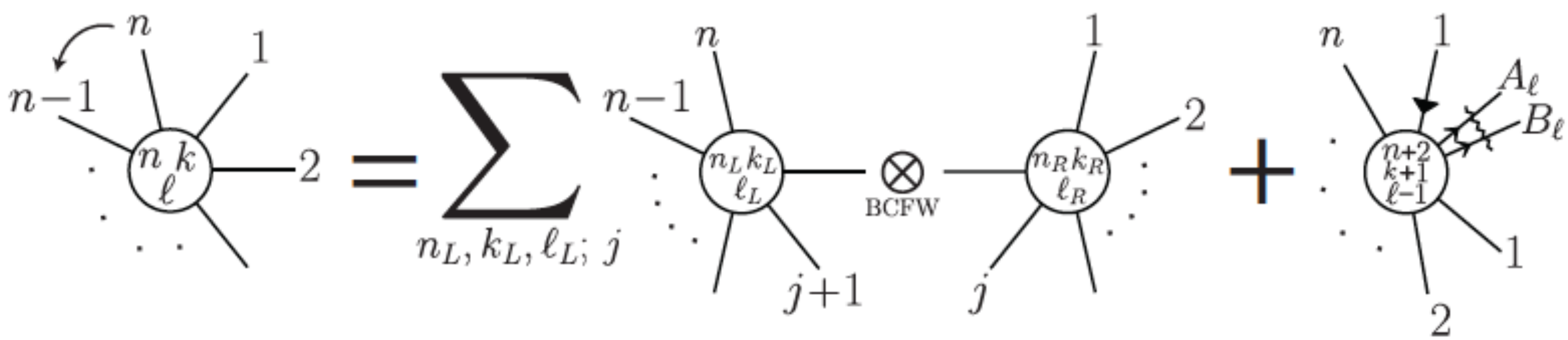
- Loop Integrands
- Gravity Theory
- Can we construct amplitudes solely from soft limits for less Supersymmetry
- Underlying geometric picture?



$A_{\text{NMHV}}(\bar{1},2,3,4,5,\hat{P}) \times A_{\text{NMHV}}(-\hat{P},6,7,8,9,\hat{10})$

$$A_{\overline{\text{MHV}}}(5,6,10)$$

$$\{1^-,4^+,2^+,3^-\}_L\,\{9^-,8^+,7^-\}_R \equiv [9\ 8\ \mathbf{7}\ 6\ 5][1\ 10\ \mathbf{9}\ 6\ 5][1\ 2\ \mathbf{3}\ 4\ 5]$$
$$\{1^-,2^+,3^-,4^+\}_L\,\{9^-,8^+,7^-\}_R \equiv [9\ 8\ \mathbf{7}\ 6\ 5][1\ 10\ \mathbf{9}\ 6\ 5][1\ 2\ \mathbf{3}\ 5\ \hat{\hat{6}}]$$
$$\{1^-,3^+,4^-,2^+\}_L\,\{9^-,8^+,7^-\}_R \equiv [9\ 8\ \mathbf{7}\ 6\ 5][1\ 10\ \mathbf{9}\ 6\ 5][1\ 3\ \mathbf{4}\ 5\ \hat{\hat{6}}]$$
$$\{1^-,4^+,2^+,3^-\}_L\,\{9^-,8^+,7^-\}_R \equiv [1\ 10\ \mathbf{9}\ 8\ 7][1\ \hat{10}\ \hat{\mathbf{8}}\ 6\ 5][1\ 2\ \mathbf{3}\ 4\ 5]$$
$$\{1^-,2^+,3^-,4^+\}_L\,\{8^-,7^+,9^-\}_R \equiv [1\ 10\ \mathbf{9}\ 8\ 7][1\ \hat{10}\ \hat{\mathbf{8}}\ 6\ 5][1\ 2\ \mathbf{3}\ 5\ \hat{\hat{6}}]$$
$$\{1^-,3^+,4^-,2^+\}_L\,\{7^-,9^-,8^+\}_R \equiv [1\ 10\ \mathbf{9}\ 8\ 7][1\ \hat{10}\ \hat{\mathbf{8}}\ 6\ 5][1\ 3\ \mathbf{4}\ 5\ \hat{\hat{6}}]$$
$$\{1^-,4^+,2^+,3^-\}_L\,\{9^-,8^+,7^-\}_R \equiv [1\ 10\ \mathbf{9}\ 7\ 6][1\ \hat{10}\ \hat{\mathbf{7}}\ 6\ 5][1\ 2\ \mathbf{3}\ 4\ 5]$$
$$\{1^-,2^+,3^-,4^+\}_L\,\{8^-,7^+,9^-\}_R \equiv [1\ 10\ \mathbf{9}\ 7\ 6][1\ \hat{10}\ \hat{\mathbf{7}}\ 6\ 5][1\ 2\ \mathbf{3}\ 5\ \hat{\hat{6}}]$$
$$\{1^-,3^+,4^-,2^+\}_L\,\{7^-,9^-,8^+\}_R \equiv [1\ 10\ \mathbf{9}\ 7\ 6][1\ \hat{10}\ \hat{\mathbf{7}}\ 6\ 5][1\ 3\ \mathbf{4}\ 5\ \hat{\hat{6}}]$$



$$
\begin{aligned}
M_{n,k,\ell}(1,\ldots,n) = \quad & M_{n-1,k,\ell}(1,\ldots,n-1) \\
& + \sum_{n_L,k_L,\ell_L;j} [j\ j{+}1\ n{-}1\ n\ 1]\ M^R_{n_R,k_R,\ell_R}(1,\ldots,j,I_j) \times M^L_{n_L,k_L,\ell_L}(I_j,j{+}1,\ldots,\widehat{n}_j) \\
& + \int_{\mathrm{GL}(2)} [AB\ n{-}1\ n\ 1] \times M_{n+2,k+1,\ell-1}(1,\ldots,\widehat{n}_{AB},\widehat{A},B).
\end{aligned}
$$

$$n_L + n_R = n+2,\ k_L + k_R = k-1,\ \ell_L + \ell_R = \ell$$

$$
\begin{aligned}
\widehat{n}_j &= (n{-}1\ n)\bigcap(j\ j{+}1\ 1), & I_j &= (j\ j{+}1)\bigcap(n{-}1\ n\ 1) \\
\widehat{n}_{AB} &= (n{-}1\ n)\bigcap(AB\ 1), & \widehat{A} &= (AB)\bigcap(n{-}1\ n\ 1).
\end{aligned}
$$