Primordial magnetogenesis and non-Gaussianities

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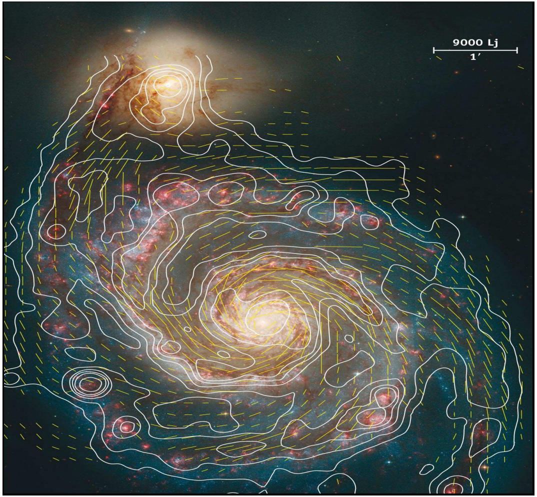
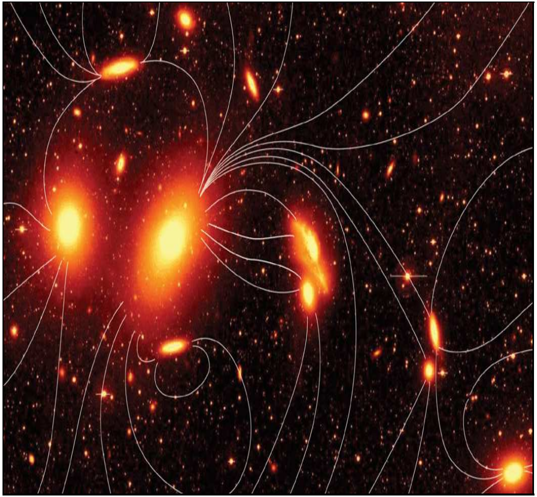
Magnetic fields in the universe



Whirlpool galaxy[1]



Virgo cluster[2]

[1] Image from http://www.mpifr-bonn.mpg.de/research/fundamental/cosmag.
[2] Image from http://science.sciencemag.org/content/311/5762/787.full.pdf+html.

Outline of the talk

1. Primordial magnetogenesis in bouncing scenarios
2. Cross-correlations of primordial magnetic fields with scalar perturbations
3. Effects of parity violation
4. Summary

The non-minimal action and equation of motion

Action:

$$S[\phi, A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,J^2(\phi)\,F_{\mu\nu}F^{\mu\nu}.$$

On quantization, the vector potential $\hat{A}_i$ can be Fourier decomposed as follows[3]:

$$\hat{A}_i\,(\eta, \boldsymbol{x}) = \sqrt{4\,\pi}\int \frac{\mathrm{d}^3\,\boldsymbol{k}}{(2\,\pi)^{3/2}}\sum_{\lambda=1}^{2}\varepsilon_{\lambda i}^{\boldsymbol{k}}\,\left[\hat{b}_{\boldsymbol{k}}^{\lambda}\,\bar{A}_k(\eta)\,\mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}_{\boldsymbol{k}}^{\lambda\dagger}\,\bar{A}_k^{*}(\eta)\,\mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right].$$

The Fourier modes $\bar{A}_k$ satisfy the differential equation

$$\bar{A}_k'' + 2\,\frac{J'}{J}\,\bar{A}_k' + k^2\,\bar{A}_k = 0.$$

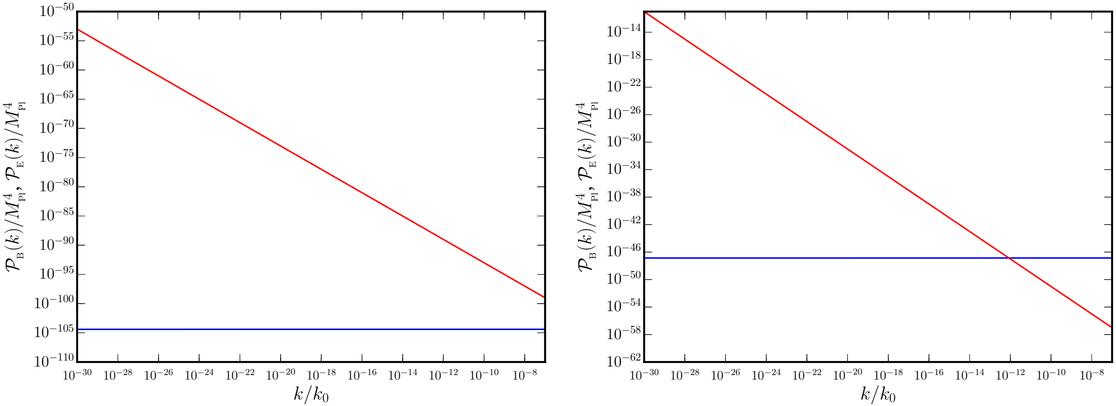
Scale factor describing the bounce:

$$a(\eta) = a_0\,\left(1 + \eta^2/\eta_0^2\right)^q = a_0\,\left(1 + k_0^2\,\eta^2\right)^q.$$

The case $q = 1$ is referred to as a matter bounce.

[3] J. Martin and J. Yokoyama, JCAP **0801**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110-120 (2010); K. Subramanian, Rept. Prog. Phys. **79**, 076901 (2016).

The electromagnetic power spectra before and after the bounce



Left: Power spectra of the magnetic field (in blue) and the electric field (in red), before the bounce.
Right: Power spectra of the magnetic field (in blue) and the electric field (in red), after the bounce[4].

We have considered the coupling function $J(\eta) = J_0\, a^{\bar{n}}(\eta)$.

Choosing $\bar{n} = 3/2$ leads to scale invariant magnetic power spectrum, *i.e.* $n_{\rm B} = 0$.

[4] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016).

Cross-correlation of magnetic fields and scalar perturbations

The three-point function can be defined as[5]

$$\left\langle \frac{\hat{\delta\phi}_{\boldsymbol{k_1}}(\eta_{\rm e})}{M_{_{\rm Pl}}} \hat{B}^i_{\boldsymbol{k_2}}(\eta_{\rm e})\, \hat{B}_{i\,\boldsymbol{k_3}}(\eta_{\rm e}) \right\rangle \equiv (2\,\pi)^{-3/2}\, G_{\delta\phi BB}\,(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})\;\delta^{(3)}\,(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3})\,,$$

where

$$\begin{aligned}G_{\delta\phi BB}\,(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) \;=\; & \frac{8\,\pi}{M_{_{\rm Pl}}\,a^2(\eta_{\rm e})}\, f_{k_1}(\eta_{\rm e})\,\bar{A}_{k_2}(\eta_{\rm e})\,\bar{A}_{k_3}(\eta_{\rm e}) \biggl\{2\,(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)\;\mathcal{G}^1_{\delta\phi BB}\,(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) \\ & -\left[\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2\,k_3}+k_2\,k_3\right]\,\mathcal{G}^2_{\delta\phi BB}\,(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})\biggr\} \\ & + \text{complex conjugate.}\end{aligned}$$

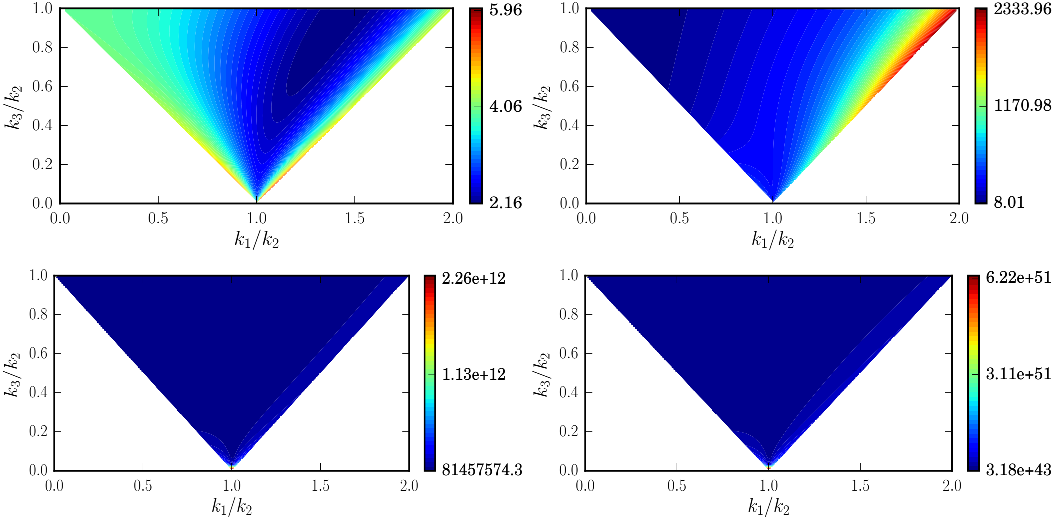
The integrals to be evaluated are of the following two types:

$$\mathcal{G}^1_{\delta\phi BB}\,(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) = i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; J\,\frac{{\rm d}J}{{\rm d}\phi}\; f^{*}_{k_1}(\eta)\,\bar{A}'^{*}_{k_2}(\eta)\,\bar{A}'^{*}_{k_3}(\eta),$$

$$\mathcal{G}^2_{\delta\phi BB}\,(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) = i\,k_2\,k_3\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; J\,\frac{{\rm d}J}{{\rm d}\phi}\; f^{*}_{k_1}(\eta)\,\bar{A}^{*}_{k_2}(\eta)\,\bar{A}^{*}_{k_3}(\eta).$$

[5] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, arXiv:1807.05530 [astro-ph.CO], to appear in JCAP.

The non-Gaussianity parameter $b_{\rm NL}$



Top: Density plots of $b_{\rm NL}$ in de Sitter inflation for $n_{\rm B} = 2$ (left) and $n_{\rm B} = 0$ (right).
Bottom: Density plots of $b_{\rm NL}$ in matter bounce for $n_{\rm B} = 2$ (left) and $n_{\rm B} = 0$ (right).

The squeezed limit and the consistency relation

Since the amplitude of a long wavelength mode freezes on super-Hubble scales during inflation, such a mode can be treated as a background as far as the smaller wavelength modes are concerned.

Consequently, the three-point function can be expressed in terms of the two-point functions involving the perturbations through the so-called consistency relation.

In the case of de Sitter inflation, in the squeezed limit (*i.e.* when $k_1 \to 0$ and $\boldsymbol{k}_2 = -\boldsymbol{k}_3 = \boldsymbol{k}$), we obtain[6]

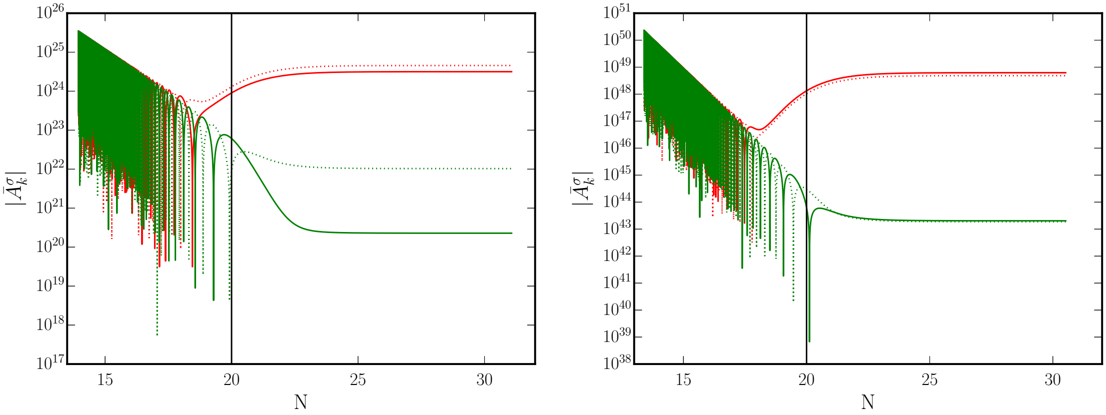
$$\lim_{k_1 \to 0} b_{_{\rm NL}}(\boldsymbol{k}_1, \boldsymbol{k}, -\boldsymbol{k}) = 2\, n_{_{\rm B}} - 8,$$

where $n_{_{\rm B}}$ is the spectral index of the power spectrum of the magnetic field.

Therefore, for a scale invariant magnetic power spectrum (*i.e.* $n_{_{\rm B}} = 0$), we must have $b_{_{\rm NL}} = -8$ in the squeezed limit, for the consistency relation to be valid.

[6] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, arXiv:1807.05530 [astro-ph.CO], to appear in JCAP.

Helical electromagnetic modes in inflation



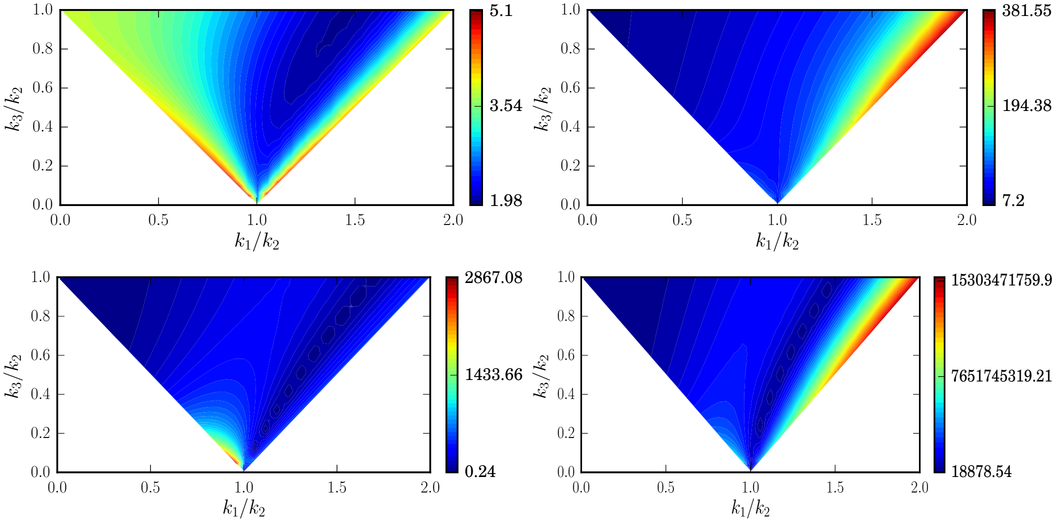
Evolution of $\bar{A}_k^\sigma$ for $n_{\rm B} = 2$ (left) and $n_{\rm B} = 0$ (right), and for $\sigma = -1$ (red) and $\sigma = 1$ (green).

We have considered the following action[7] with $I = J = (\eta/\eta_{\rm e})^{-n}$:

$$S_{\rm em}[A^\mu, \phi] = -\frac{1}{16\,\pi} \int {\rm d}^4x \sqrt{-g} \left[J^2(\phi)\, F_{\mu\nu} F^{\mu\nu} - \frac{\gamma}{2}\, I^2(\phi)\, F_{\mu\nu} \tilde{F}^{\mu\nu}\right].$$

[7] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **1810**, 031 (2018).

Non-Gaussianity parameter $b_{\rm NL}$ with helical electromagnetic modes



Top: Density plots of $b_{\rm NL}$ in non-helical case for $n_{\rm B} = 2$ (left) and $n_{\rm B} = 0$ (right).
Bottom: Density plots of $b_{\rm NL}$ in helical case for $n_{\rm B} = 2$ (left) and $n_{\rm B} = 0$ (right)[8].

[8] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **1810**, 031 (2018).

Summary

- We have obtained scale invariant magnetic fields of relevant strengths over the scales of cosmological interest in a matter bounce.
- The shapes of the spectra are preserved across the bounce.
- The cross-correlation between the primordial magnetic fields and scalar perturbations in a matter bounce leads to large non-Gaussianities and violates the consistency relation.
- We have numerically evaluated the cross-correlation between the primordial helical magnetic fields and the perturbations in an auxiliary scalar field in de Sitter inflation.
- Helicity significantly boosts the amplitude of non-Gaussianities.

Thank you!