Business Ecosystems – Competition, Cooperation and Evolution

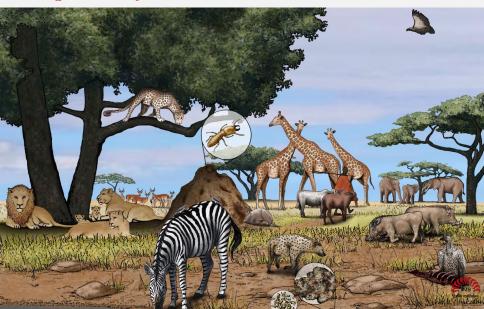
Rajeev R. Tripathi IIM Bangalore

Discussion Meeting
Summer Program on Dynamics of Complex Systems 2019
International Centre for Theoretical Sciences, Bengaluru

July 12, 2019



Biological Ecosystem



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- Ecosystems vary widely in their robustness and resilience e.g. Costa Rica vs Hawaii



Business Ecosystem

A framework that has emerged in recent years to illuminate how businesses function in today's highly *dynamic*, *disruptive* and *networked environment*

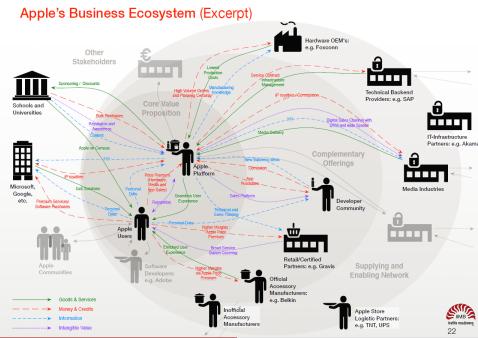


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"Business ecosystems, using biological ecosystems as a metaphor, can be defined as dynamic and evolving communities of diverse industries and organizations who create and capture values through both cooperation and competition"





• Species:



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- Constituents of an ecosystem: all the forces that have an impact on the fitness of the firm or industry, such as competitors, suppliers, macro-economic conditions, government organizations, social institutions, etc.
- An ecosystem, biological or business, is about **dynamic** interactions among entities and evolution over time











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Tesla Model S became car of the year in 2013, and the top selling electric car worldwide in 2015 & 2016







Case 2 Amazon's story

NETFLIX



► Amazon's Timeline on CNN



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- 2019: the most valuable company on the planet (\approx \$810 billion)

What is the business relationship of Amazon and Netflix?



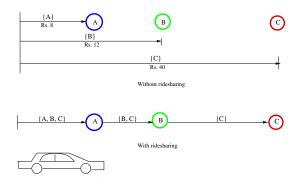
"On any given day, AT& T might find Motorola to be a supplier, a buyer, a competitor, and a partner" – (Hamel and Prahalad, 1994)



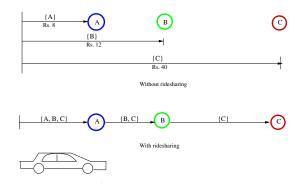
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"Business is cooperation when it comes to creating a pie and competition when it comes to dividing it up" – (Brandenburger and Nalebuff, 1996)



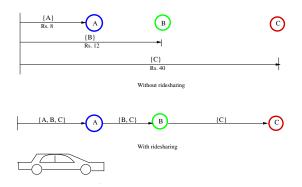






Game representation: Modeling it as a cooperative game (N, v)

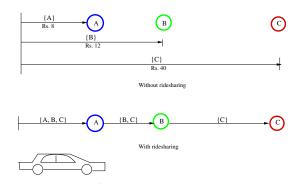




Game representation: Modeling it as a cooperative game (N, v)

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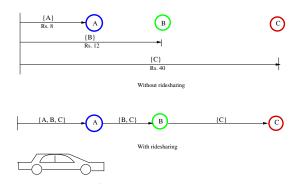




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- Payoff function: v(A) = 8, v(B) = 12, v(C) = 40,v(AB) = 12, v(BC) = v(AC) = v(ABC) = 40





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Solution concept: How to allocate 40 among A, B and C?

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Some solution concepts:

• Equal – (13.33, 13.33, 13.33)



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- Proportional -(5.34, 8, 26.66)



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- Proportional (5.34, 8, 26.66)
- Core $(0 \le x_A \le 8,$ $0 \le x_B \le 12, 28 \le x_C \le 40)$ a closed convex polytope, can be empty



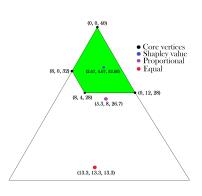
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Example 2 – linear production game

Consider the following linear production game with $N = \{1, 2, 3\}$:

Coalition	Rese	ource	bundle	Chairs	Tables	Profit,
(C)	M_1	M_2	M_3	(x_1)	(x_2)	v(C)
{1}	36	50	60	3	9	330
{2}	39	40	42	6	5	270
{3}	36	50	40	8	4	280
{1,2}	75	90	102	12	13	630
$\{2, 3\}$	75	90	82	14	9	550
$\{1, 3\}$	72	100	100	11	13	610
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How to allocate v(N) among players 1, 2 and 3?





Player 2
$$NC C$$

Player 1 $C (1,1) (5,0)$
 $C (0,5) (4,4)$



Player 2

$$\begin{array}{c|cccc}
 & NC & C \\
\hline
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For cooperation to be attained,



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For cooperation to be attained,

$$\therefore 4 + 4\delta + 4\delta^2 + 4\delta^3 + \dots \ge 5 + 1\delta + 1\delta^2 + 1\delta^3 + \dots$$

$$\Rightarrow 4 \cdot (\delta + \delta^2 + \delta^3 + \dots) \ge 5 + 1 \cdot (\delta + \delta^2 + \delta^3 + \dots)$$

$$\Rightarrow 4 \cdot \frac{1}{1 - \delta} \ge 5 + \frac{\delta}{1 - \delta} \quad \therefore \boxed{\delta \ge \frac{1}{4}}$$



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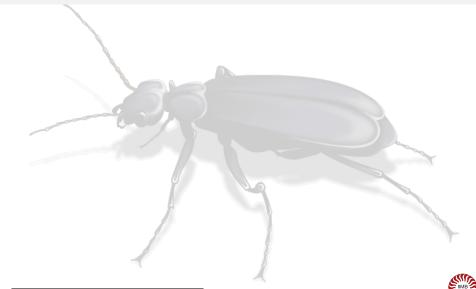
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It means that the discount factor needs to be sufficiently high.



Example 4: Evolutionary game among beetles



¹Large beetles need more food to maintain their metabolism

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- Consider a population of beetles with *small* body size.
- Mutation is introduced into the population, causing beetles with the mutation to grow a significantly *larger* body size¹.



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- We now have two types of beetles *small* and *large*.



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- The beetles in this population compete with each other for food.
 - Small beetle vs small beetle equal share of the food
 - Large beetle vs large beetle equal share of the food
 - \bullet Large beetle vs small beetle large beetle gets more food



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²The numerical payoffs in the matrix satisfy the fitness criteria outlined earlier

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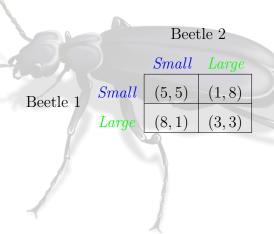
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Interaction among the beetles in this population can be thought of as $a\ two-player\ game\ given\ below^2$:

		Beetle 2	
		Small	Large
Beetle 1	Small	(5,5)	(1,8)
Bootie	Large	(8,1)	(3,3)

A beetle is not asking, "What do I want my body size to be in this interaction?"

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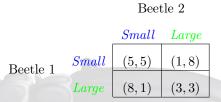
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Question: Which body size is evolutionarily stable – large or small?

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Beetle	\mathcal{L}

Beetle 1 Small (5,5) (1,8) Large (8,1) (3,3)







Beetle 2

Beetle 1 $\begin{array}{c|c} Small & Large \\ \hline Small & (5,5) & (1,8) \\ \hline Large & (8,1) & (3,3) \\ \end{array}$



Beetle 2

		Small	Large
Beetle 1	Small	(5,5)	(1,8)
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When large invades small at x:

(i) Expected fitness of a small beetle = $5 \cdot (1 - x) + 1 \cdot x = 5 - 4x$



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- (ii) Expected fitness of a large beetle = $8 \cdot (1 x) + 3 \cdot x = 8 5x$



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	Large	(8,1)	(3,3)

When large invades small at x:

- (i) Expected fitness of a small beetle = $5 \cdot (1-x) + 1 \cdot x = 5 4x$
- (ii) Expected fitness of a large beetle $= 8 \cdot (1 x) + 3 \cdot x = 8 5x$ $\therefore 5 - 4x \ge 8 - 5x$, *small* is not evolutionarily stable.

- (i) Expected fitness of a small beetle = $(1-x) + 5 \cdot x = 1 + 4x$
- (ii) Expected fitness of a large beetle = $3 \cdot (1 x) + 8 \cdot x = 3 + 50$

Beetle 2

		Small	Large
Beetle 1	Small	(5,5)	(1,8)
Decile 1	Large	(8,1)	(3,3)

When large invades small at x:

- (i) Expected fitness of a small beetle = $5 \cdot (1-x) + 1 \cdot x = 5 4x$
- (ii) Expected fitness of a large beetle $= 8 \cdot (1 x) + 3 \cdot x = 8 5x$
- $\therefore 5 4x \geqslant 8 5x$, *small* is not evolutionarily stable.

- (i) Expected fitness of a small beetle = $(1-x) + 5 \cdot x = 1 + 4x$
- (ii) Expected fitness of a large beetle = $3 \cdot (1 x) + 8 \cdot x = 3 + 56$
- 3 + 5x > 1 + 4x

Beetle 2

		Small	Large
Beetle 1	Small	(5,5)	(1, 8)
20000	Large	(8,1)	(3,3)

When large invades small at x:

- (i) Expected fitness of a small beetle = $5 \cdot (1-x) + 1 \cdot x = 5 4x$
- (ii) Expected fitness of a large beetle $= 8 \cdot (1 x) + 3 \cdot x = 8 5x$
- $\therefore 5 4x \ge 8 5x$, *small* is not evolutionarily stable.

- (i) Expected fitness of a small beetle = $(1-x) + 5 \cdot x = 1 + 4x$
- (ii) Expected fitness of a large beetle = $3 \cdot (1 x) + 8 \cdot x = 3 + 5x$
- $\therefore 3 + 5x > 1 + 4x$, *large* is evolutionarily stable.

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