Research at the Interface of Computer Science and Economics

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IIT Kanpur

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Research Areas and The Tools

- **Voting** – choosing an alternative collectively

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- Game Theory
- Non-linear optimization
- Mechanism design
- Real analysis
- Approximation algorithms
- Probabilistic concentration bounds

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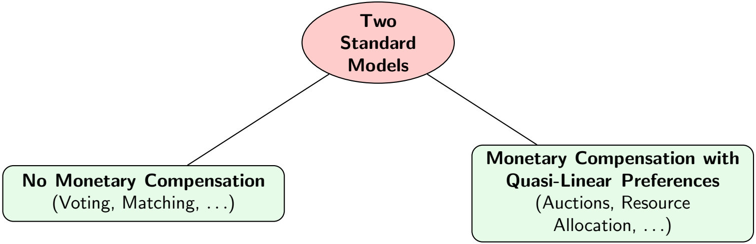

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Answered by Roberts (1979)

- Dictatorial SCFs are strategyproof - transfers are not required
- Efficient SCFs are strategyproof - with VCG payments
- Roberts' answer: affine maximizers

Affine Maximizers

Definition (Affine Maximizer)

An SCF $F : V^n \to A$ is an *affine maximizer* if there exists $w_i \geq 0, i \in N$, not all zero, and a function $\kappa : A \to \mathbb{R}$ such that,

$$F(v) \in \arg\max_{a \in A} \left(\sum_{i \in N} w_i v_i(a) + \kappa(a) \right).$$

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- $w_i = 1, \forall\ i$ and $\kappa \equiv 0$: **allocatively efficient** SCF
- $w_d = 1$, for some d, $w_i = 0, \forall\ i \neq d$ and $\kappa \equiv 0$: **dictatorial** SCF

Roberts' Theorem

Theorem (Roberts 1979)

Let the allocation space A be arbitrary and finite with $|A| \geq 3$. If the space of valuations V is unrestricted, *then an onto and strategyproof SCF $F : V^n \to A$ is an affine maximizer.*

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Payments are of the following form: for all $i \in N$

$$p_i(v_i, v_{-i}) = \begin{cases} \frac{1}{w_i} \left(\sum_{j \neq i} w_j v_j(F(v)) + \kappa(F(v)) + h_i(v_{-i}) \right), & w_i > 0 \\ 0 & w_i = 0 \end{cases}$$

Roberts' Theorem Does Not Apply For Selfish Valuations

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- Value differences cannot be arbitrary - if two distinct alternatives x and y have identical i-th row, i.e., $x_i = y_i$, then $u_i(x) - u_i(y) = 0$ for all $u \in U^n$

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  - *with an additional assumption*
  - *and a different proof technique*

Affine Maximizers for Selfish Valuations[1]

Theorem

If $n \geq 3$*, every* onto, allocation non-bossy *and* strategyproof *SCF* $F : U^n \to A$ *is an affine maximizer. If* $n = 2$*, the result holds without the allocation non-bossiness assumption.*

[1] Nath and Sen, "Affine Maximizers in Domains with Selfish Valuations", in **ACM Transactions on Economics and Computation (TEAC)**, 2015.

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- Roberts' kind of argument works everywhere except the points where at least one component of the allocations are identical
- It uses the continuity argument to claim the result to hold even at those points (the points form a measure zero space)

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Preferences and Domains

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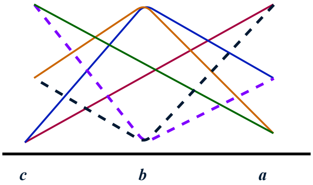

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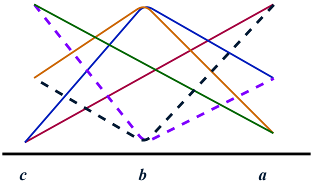

Single-peakedness provides a number of nice properties, including non-manipulability.

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Most of these problems become *efficiently solvable* in a single peaked domain (Cornaz and Spanjaard (2013))

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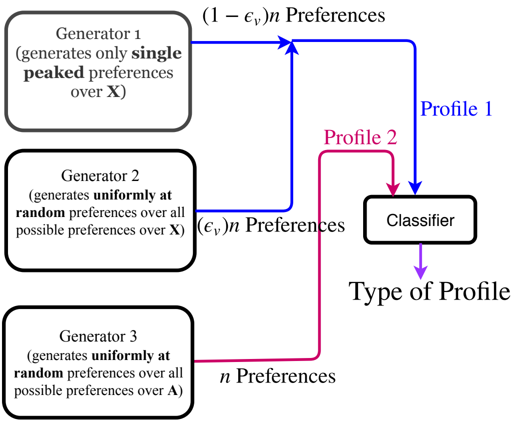
- Real world profiles do not occur perfectly from a domain – can be **nearly** single peaked

Sampling and Near Single Peakedness[2]

Problem 1: $(\epsilon_v, \epsilon_a, \delta, D)-$ Random Outliers vs Random Profile Test



[2] Dey, Nath, and Shakya, "Testing Preferential Domains Using Sampling", in **Autonomous Agents and Multiagent Systems (AAMAS)**, 2019.

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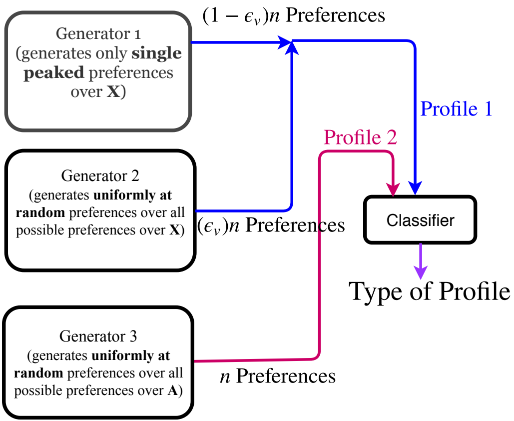

$X \subseteq A$, $|X| = (1 - \epsilon_a)|A|$. D is the domain (e.g. single peaked). Classification is correct with error probability at most δ.

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Results Summary

Input profile: Possibility 1	Input profile: Possibility 2	Sample complexity
$\epsilon_v n$ random preferences away	random	$\mathcal{O}(\frac{1}{(1-\epsilon_v)^2} \log \frac{1}{\delta})$
$\epsilon_v n$ arbitrary preferences away	random	$\mathcal{O}(\frac{1}{(1-3\epsilon_v)^2} \ln \frac{1}{\delta})$ for $\epsilon_v < 1/3$
$\epsilon_a m$ alternatives away	random	$\mathcal{O}(\log \frac{\log_{1/\epsilon_a} 1/\delta}{\delta} \times \log_{1/\epsilon_a} \frac{1}{\delta} \log \log_{1/\epsilon_a} 1/\delta)$
$\epsilon_v n$ arbitrary preferences away	$\epsilon'_v n$ arbitrary preferences away	$\mathcal{O}(\frac{1}{(\epsilon'_v - \epsilon_v)^2}(2^m m^2 \log^2 m + \log 1/\delta))$
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The results use concentration bounds asymptotic in n to find the sample complexity bounds

Thank you! Questions?