Chimera states in mechanical oscillator networks

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‘We have neither Grammar nor Dictionary, neither Chart nor Compass, to guide us through the wide sea of Words.’

Simon Winchester, The Surgeon of Crowthorne

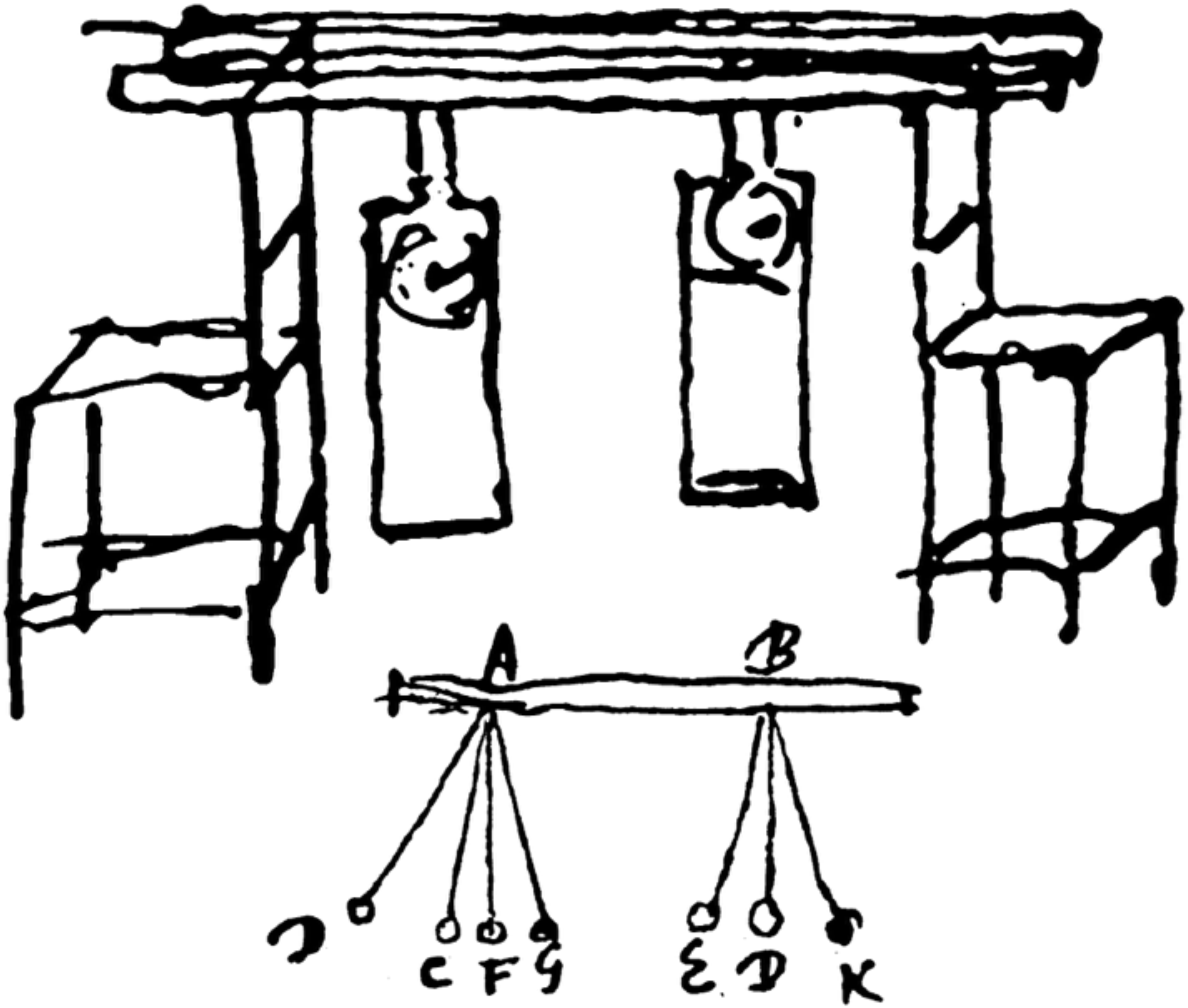
'We have neither Grammar nor Dictionary, neither Chart nor Compass, to guide us through the wide sea of Words.'

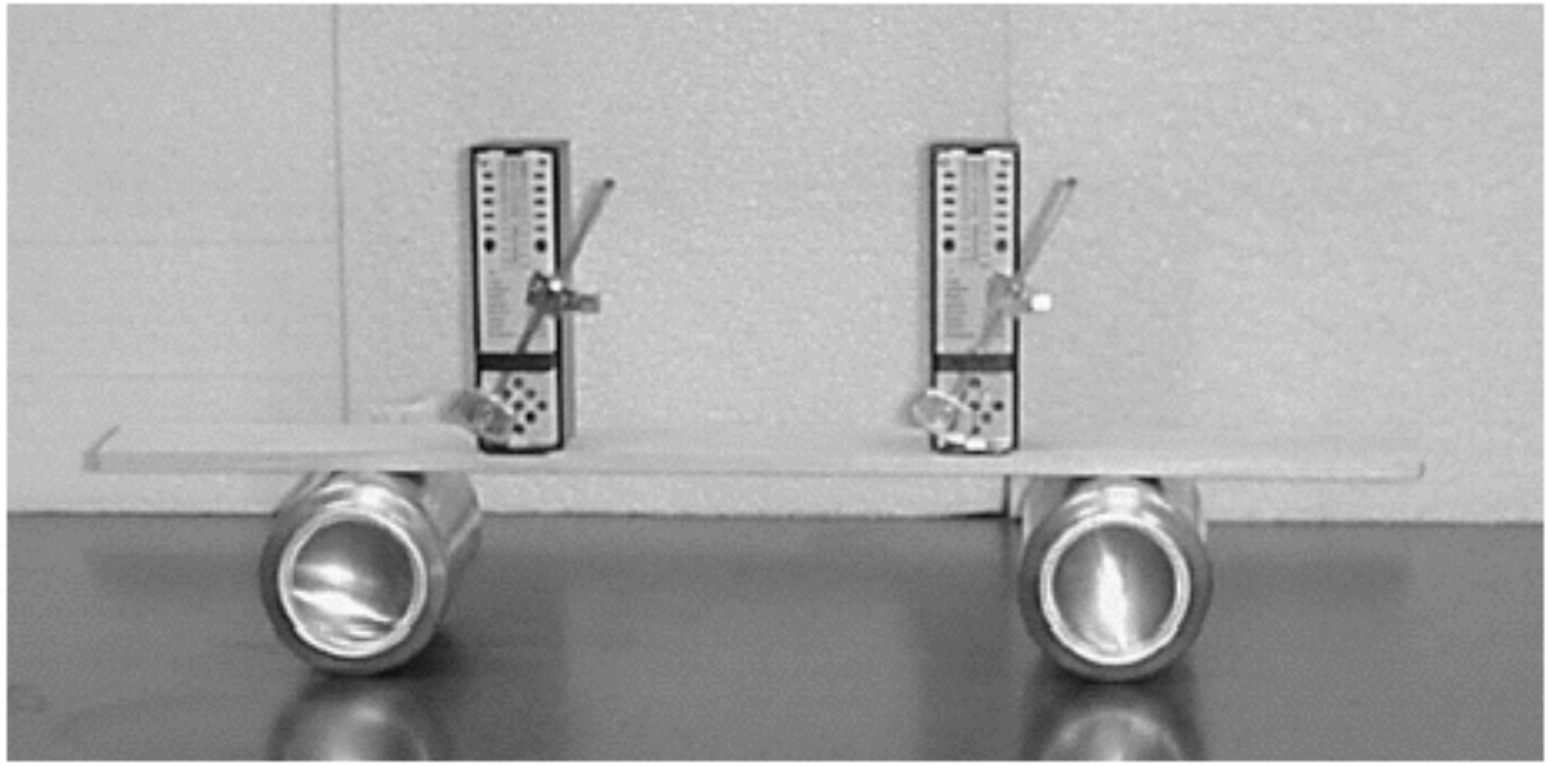
The language should be accorded the same dignity and respect as those other standards that science was at that time also defining. Physicists were wondering, what is blue, or yellow? How hot is boiling water? How long is a yard? How should what musicians knew as 'middle C' be defined? What, indeed, of the longitude of a ship? Enormous efforts were being made in this particular field at just the same time as the debate over the national language: a Board of Longitude had been set up by the government, funds were being disbursed, prizes offered, just so that a clock could be invented that would go to sea on a ship and be only almost imperceptibly inaccurate. Longitude was vitally important: so great a trading nation as Britain needed to have her ships' masters know exactly where they were.

Simon Winchester, The Surgeon of Crowthorne

Longitude measurement

Spontaneous synchrony - an ‘odd sympathy’

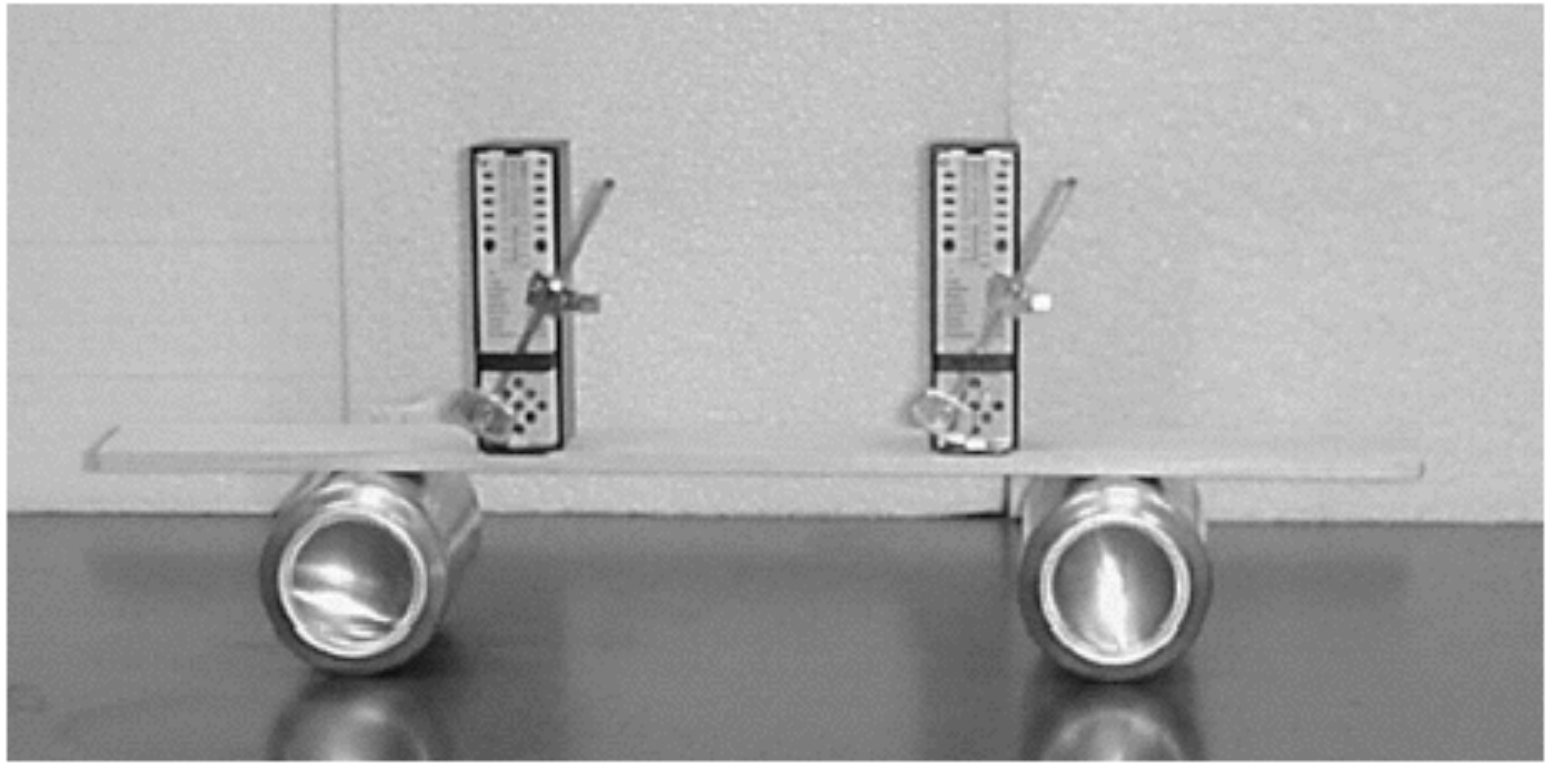




$$\frac{d^2\theta}{dt^2}+\frac{mr_{\text{c.m.}}g}{I}\sin\theta+\epsilon\left[\left(\frac{\theta}{\theta_0}\right)^2-1\right]\frac{d\theta}{dt}+\left(\frac{r_{\text{c.m.}}m\cos\theta}{I}\right)\frac{d^2x}{dt^2}=0,$$

$$x=-\frac{m}{M+2m}r_{\text{c.m.}}(\sin\theta_1+\sin\theta_2),$$

Pantaleone, Am. J. Phys (2002)



$$\frac{d^2\theta_1}{d\tau^2}+(1+\Delta)\sin\theta_1+\mu\left(\left(\frac{\theta_1}{\theta_0}\right)^2-1\right)\frac{d\theta_1}{d\tau}$$
$$-\beta\cos\theta_1\frac{d^2}{d\tau^2}(\sin\theta_1+\sin\theta_2)=0,$$

$$\frac{d^2\theta_2}{d\tau^2}+(1-\Delta)\sin\theta_2+\mu\left(\left(\frac{\theta_2}{\theta_0}\right)^2-1\right)\frac{d\theta_2}{d\tau}$$
$$-\beta\cos\theta_2\frac{d^2}{d\tau^2}(\sin\theta_1+\sin\theta_2)=0.$$

$$\tau=\omega t$$
$$\omega^2=\frac{mr_{\mathrm{c.m.}}g}{I}$$
$$\omega_{\mathrm{pen}}\approx\omega(1-\tfrac{3}{2}\gamma)$$

$$\Delta\approx\frac{\omega_1-\omega_2}{\omega}$$
$$\beta=\left(\frac{mr_{\mathrm{c.m.}}}{M+2m}\right)\left(\frac{r_{\mathrm{c.m.}}m}{I}\right)$$

Pantaleone, Am. J. Phys (2002)

Heuristics : synchronisation

$$\frac{d}{d\tau}\left[\frac{d\delta}{d\tau}+\mu\left(\sigma^2+\frac{1}{3}\delta^2-1\right)\delta\right]+\delta=0 \qquad \delta=\frac{\theta_1-\theta_2}{2\theta_0}$$

$$\frac{d}{d\tau}\left[\frac{d\sigma}{d\tau}+\mu\left(\frac{1}{3}\sigma^2+\delta^2-1\right)\sigma\right]+(1+2\beta)\sigma=0. \qquad \sigma=\frac{\theta_1+\theta_2}{2\theta_0}$$

$$\frac{d^2\sigma}{d\tau^2}+\mu(\sigma^2-1)\frac{d\sigma}{d\tau}+(1+2\beta)\sigma=0$$

$$\sigma(\tau)\approx 2\cos((1+\beta)\tau) \qquad \delta(t)\approx\delta(0)e^{-\mu\tau/2}\cos(\tau+\chi)$$

Two-timing, amplitude-phase equations

$$\theta_1 = A\,\theta_0 \cos(\tau + \phi) \qquad \theta_2 = B\,\theta_0 \cos(\tau + \xi)$$

$$\frac{dA}{d\tau} = \frac{1}{8}[\mu A(4 - A^2) + 4\beta B \sin\psi] \qquad \frac{dB}{d\tau} = \frac{1}{8}[\mu B(4 - B^2) - 4\beta A \sin\psi]$$

$$\frac{d\psi}{d\tau} = \frac{1}{8}\left[-3\gamma(A^2 - B^2) + 8\Delta + 4\beta\left\{\frac{B}{A} - \frac{A}{B}\right\}\cos\psi\right] \qquad \psi = \phi - \xi$$

$$\frac{d\psi}{d\tau} \approx \left[\Delta - (3\gamma + \beta)\frac{\beta}{\mu}\sin\psi\right]$$

N oscillators

$$\frac{dA_i}{d\tau} = \mu\left(\frac{A_i}{2}\right)\left\{1-\left(\frac{A_i}{2}\right)^2\right\} + \frac{\tilde{\beta}}{N}\sum_{j=1}^{N}\left(\frac{A_j}{2}\right)\sin[\phi_i - \phi_j]$$

$$\frac{d\phi_i}{d\tau} = \frac{\Delta_i}{2} - \frac{3}{2}\gamma\left(\frac{A_i}{2}\right)^2 + \frac{\tilde{\beta}}{2N}\sum_{j=1}^{N}\left(\frac{A_j}{A_i}\right)\cos[\phi_i - \phi_j]$$

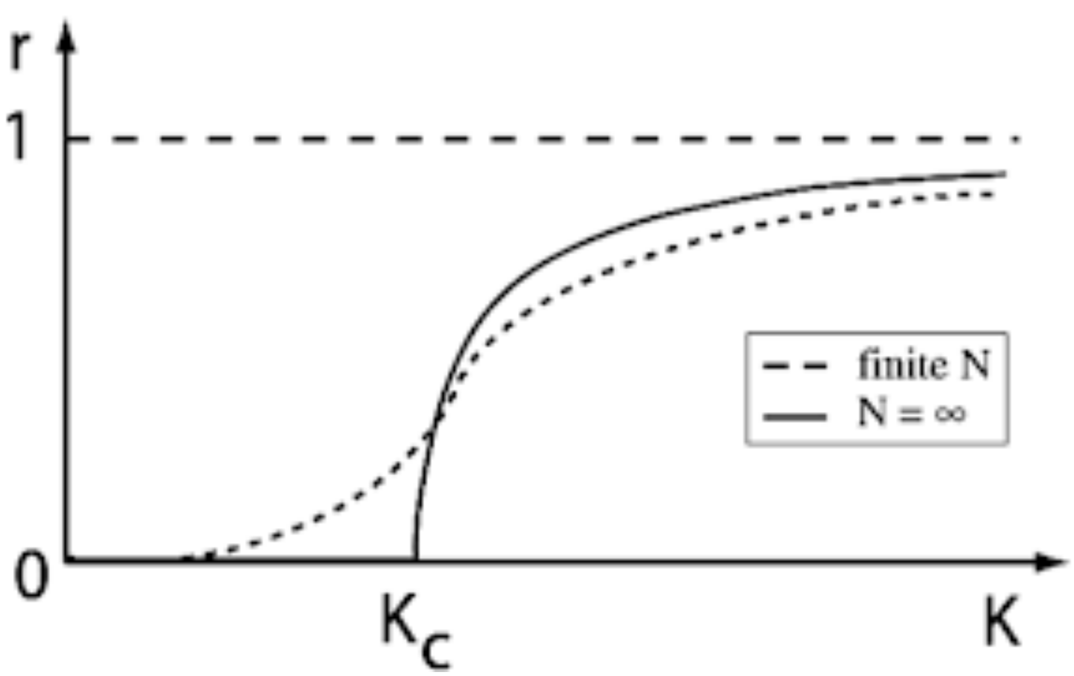
$$\frac{d\phi_i}{d\tau} = -\frac{3}{2}\gamma + \frac{\Delta_i}{2} - \sqrt{\left(3\frac{\tilde{\beta}}{\mu}\right)^2 + \left(\frac{\tilde{\beta}}{\gamma}\right)^2}\left(\frac{\gamma}{2N}\right) \times \sum_{j=1}^{N}\sin[\phi_i - \phi_j - \xi],$$

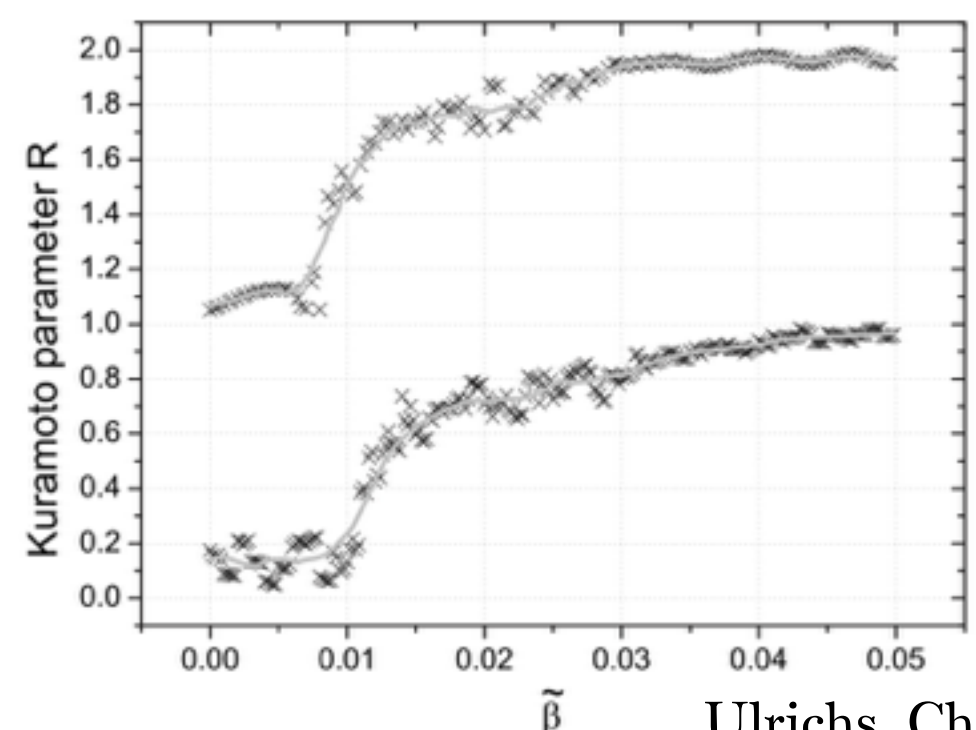
$$\tilde{\beta} = \left(\frac{m}{\mathcal{M}}\right)\left(\frac{r_{\mathrm{c.m.}}^2 m}{I}\right)$$

The Kuramoto Model

$$\frac{d\phi_i}{d\tau} = -\frac{3}{2}\gamma + \frac{\Delta_i}{2} - \sqrt{\left(3\frac{\tilde{\beta}}{\mu}\right)^2 + \left(\frac{\tilde{\beta}}{\gamma}\right)^2}\left(\frac{\gamma}{2N}\right)$$
$$\times \sum_{j=1}^{N} \sin[\phi_i - \phi_j - \xi],$$

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$$





Ulrichs, Chaos (2009)

非線形問題解決軍団
池口研究室
NONLINEAR DYNAMICAL SYSTEM

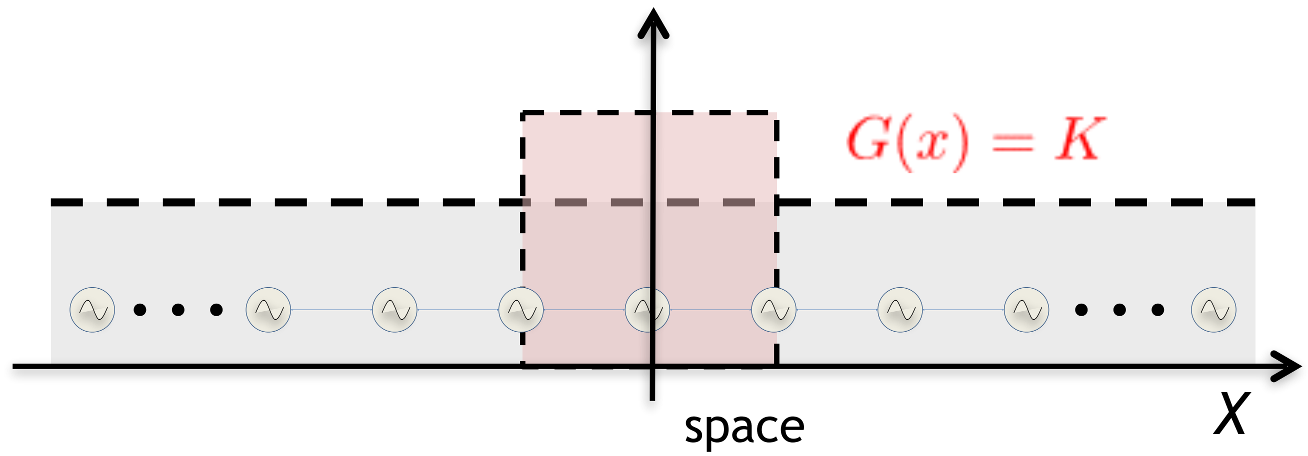
メトロノーム同期 (32個)

Synchronization of thirty two metronomes

2012年09月14日，池口研究室前廊下にて撮影

Filmed at Ikeguchi Laboratory, on September 14, 2012.

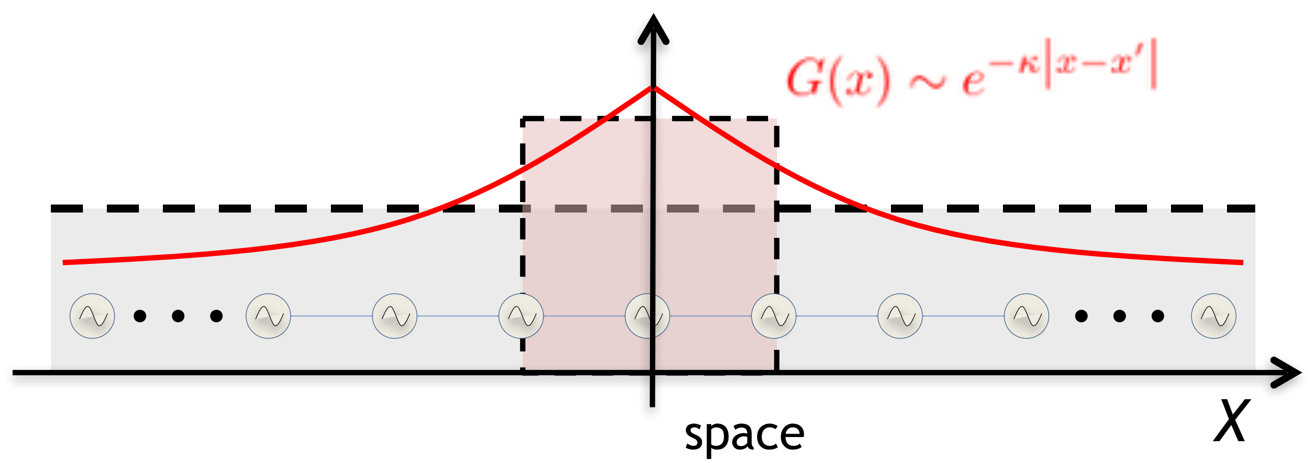
Global/local coupling : synchrony OR desynchrony



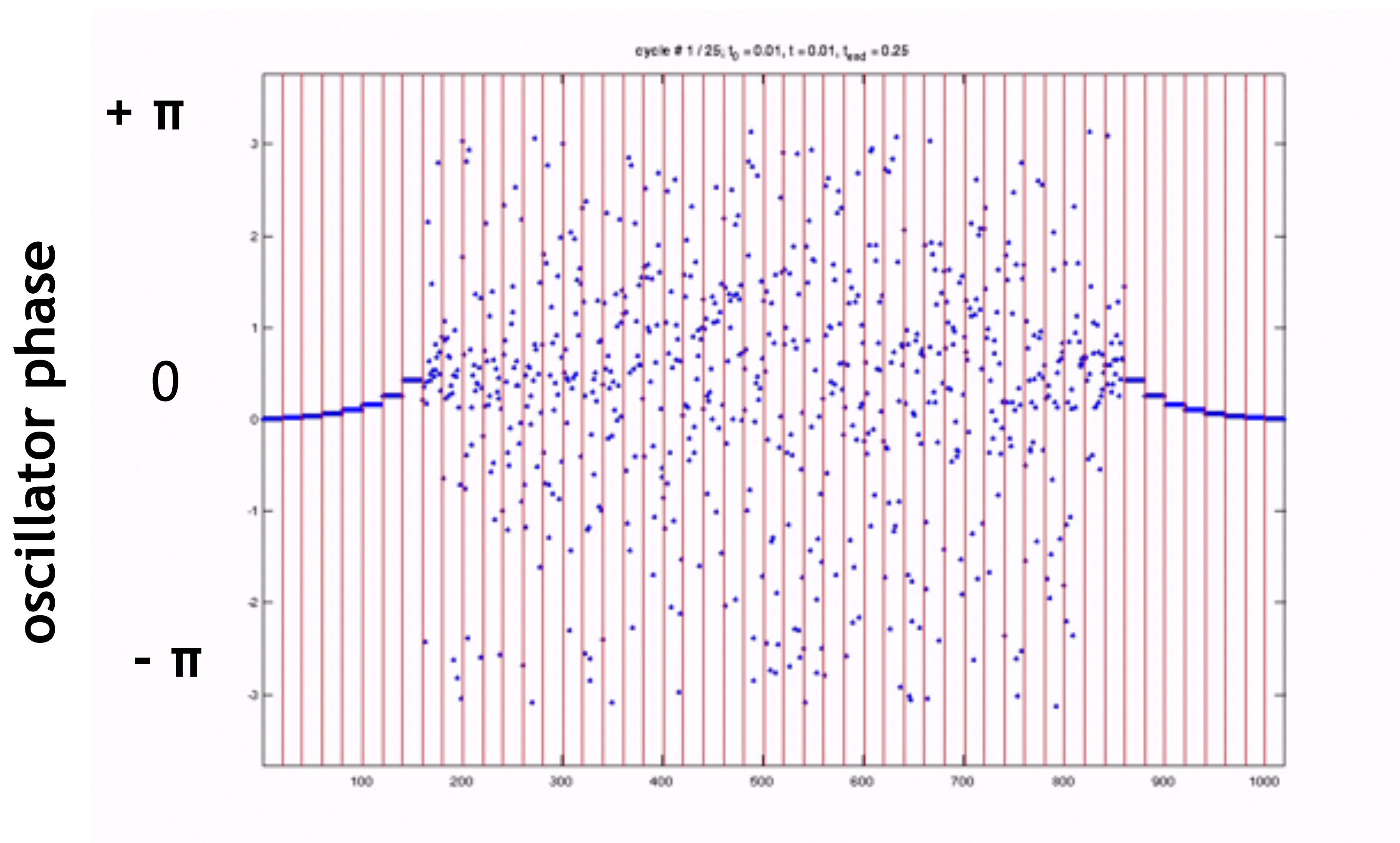
$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$$

identical oscillators $\omega_i = \omega$

Nonlocal coupling : synchrony AND desynchrony



kuramoto and battagtokh, 2002

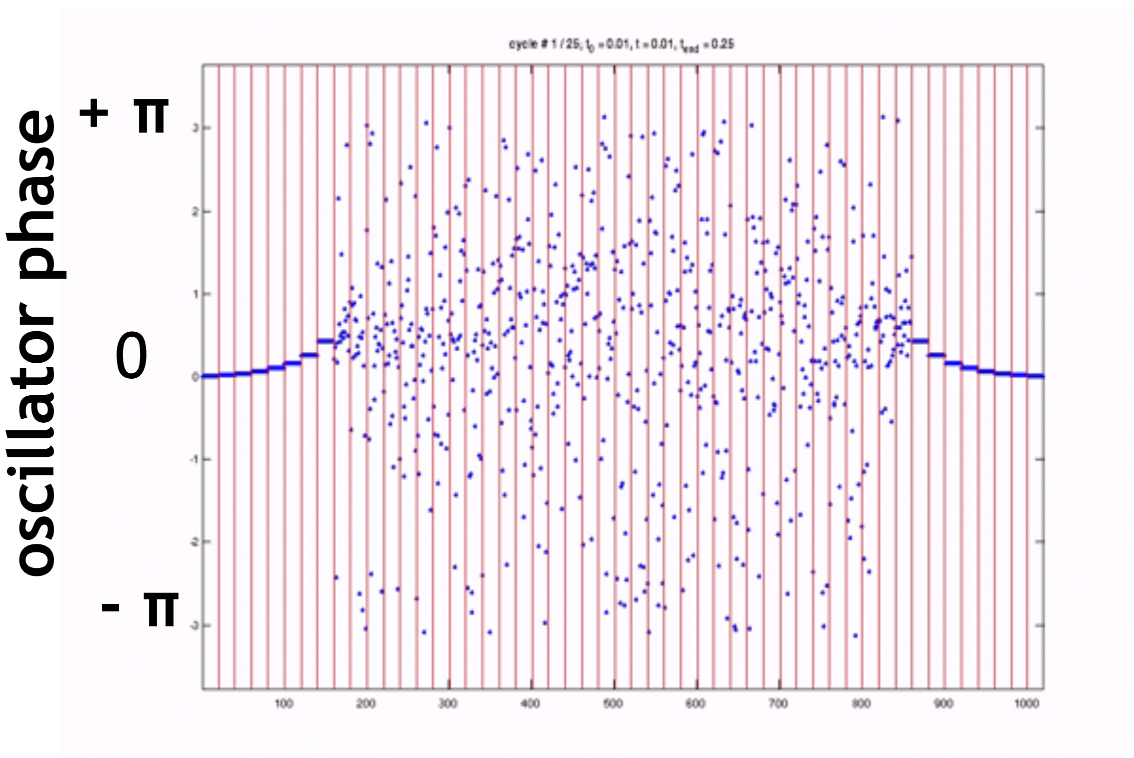


identical oscillators $\sigma = 0$ $\omega_i = \omega$

abrams and strogatz, prl, 2004

Chimera state in oscillator populations

co-existence of synchrony and asynchrony in a population of **identical** coupled oscillators



abrams and strogatz, prl, 2004

Extensive theoretical analysis

Battogtokh & Kuramoto (2002); Shima (2004), Abrams et al. (2004,2008),
Omel'chenko et al. (2008), Setha et al., (2008), Martens (2010), Laing (2008, 2010) …

chimeras occur in a variety of settings and are robust:

- **different oscillator systems**

delay coupled oscillators, planar oscillators
Fitz-Hugh-Nagumo oscillators, reaction diffusion

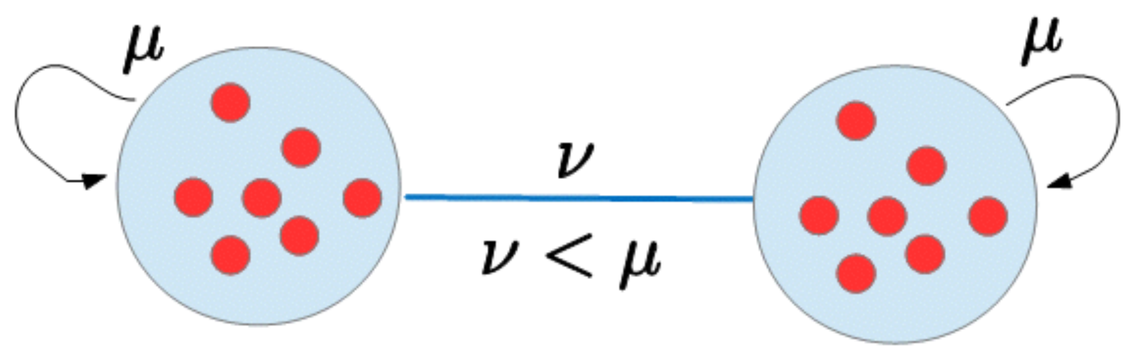
- **various network topologies & coupling**

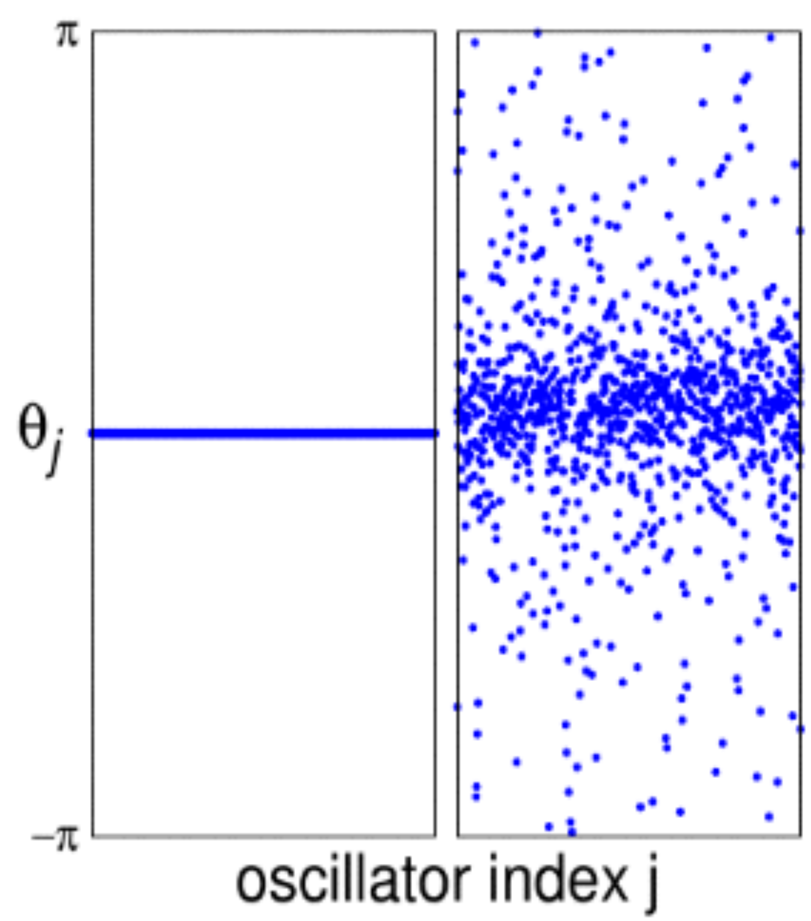
ring, planar lattice, **hierarchical networks**, torus, sphere

- **exist even in heterogeneous systems**
  - frequencies
  - connectivity
  - coupling strength
  - noise

The simplest hierarchical network
two identical coupled populations

$$\frac{d\theta_i^\sigma}{dt} = \omega + \sum_{\sigma'=1}^{2} \frac{K_{\sigma\sigma'}}{N_{\sigma'}} \sum_{j=1}^{N_{\sigma'}} \sin(\theta_j^{\sigma'} - \theta_i^\sigma - \alpha)$$

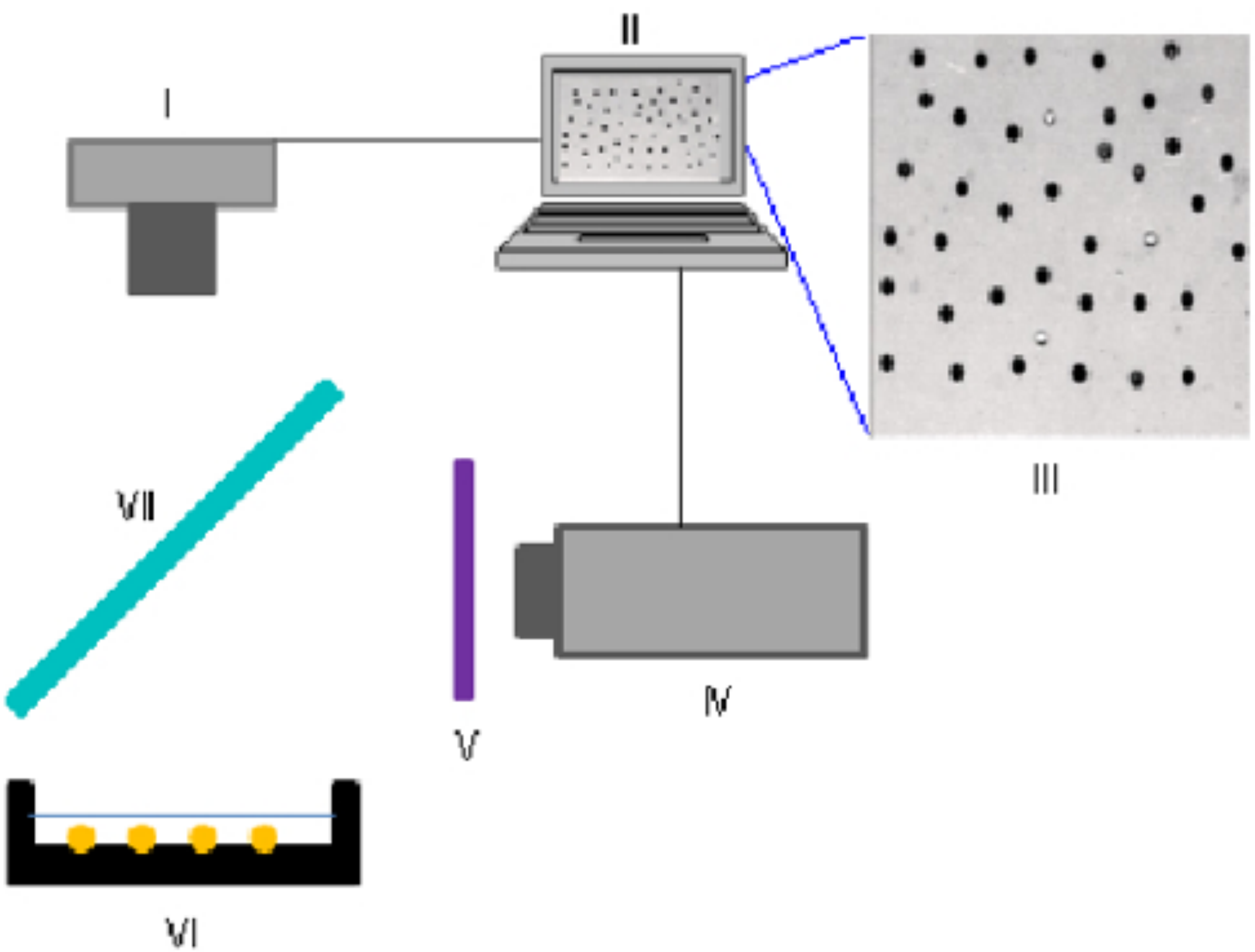




abrams et. al., prl, 2004

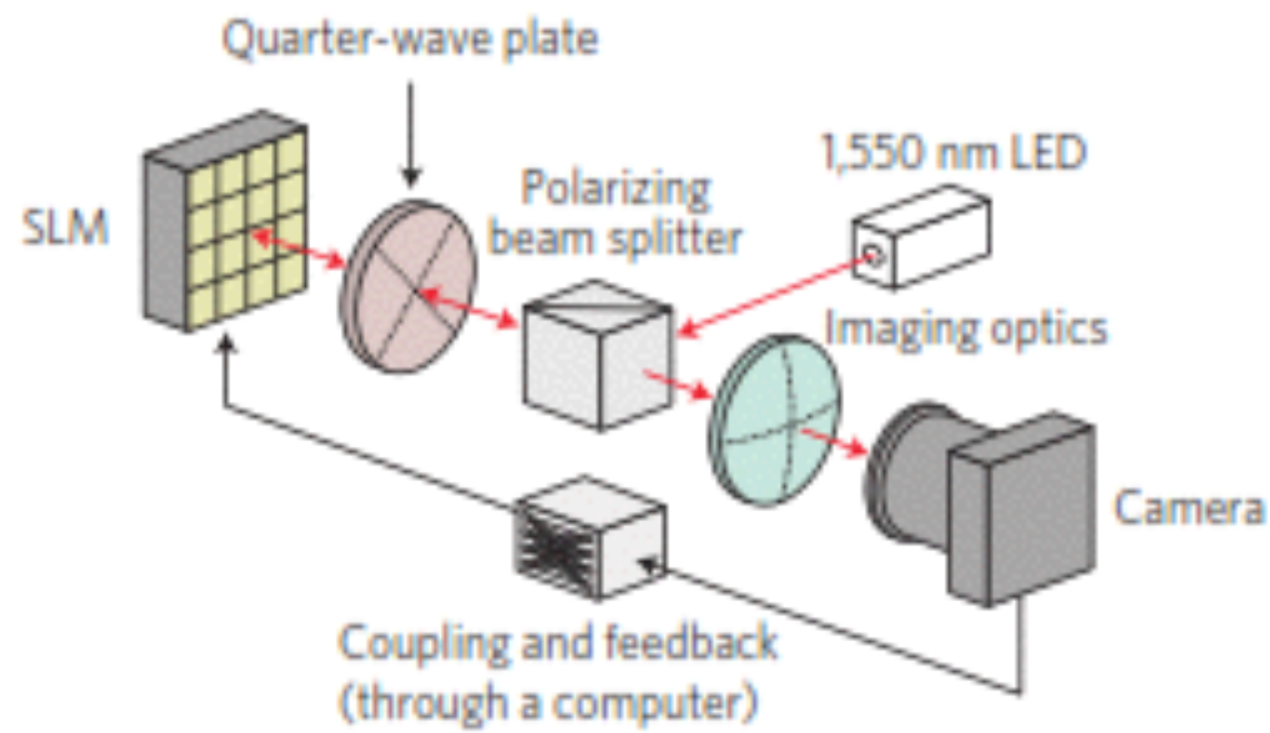
A couple of 'experiments'

tinsley et. al., nature physics 2012



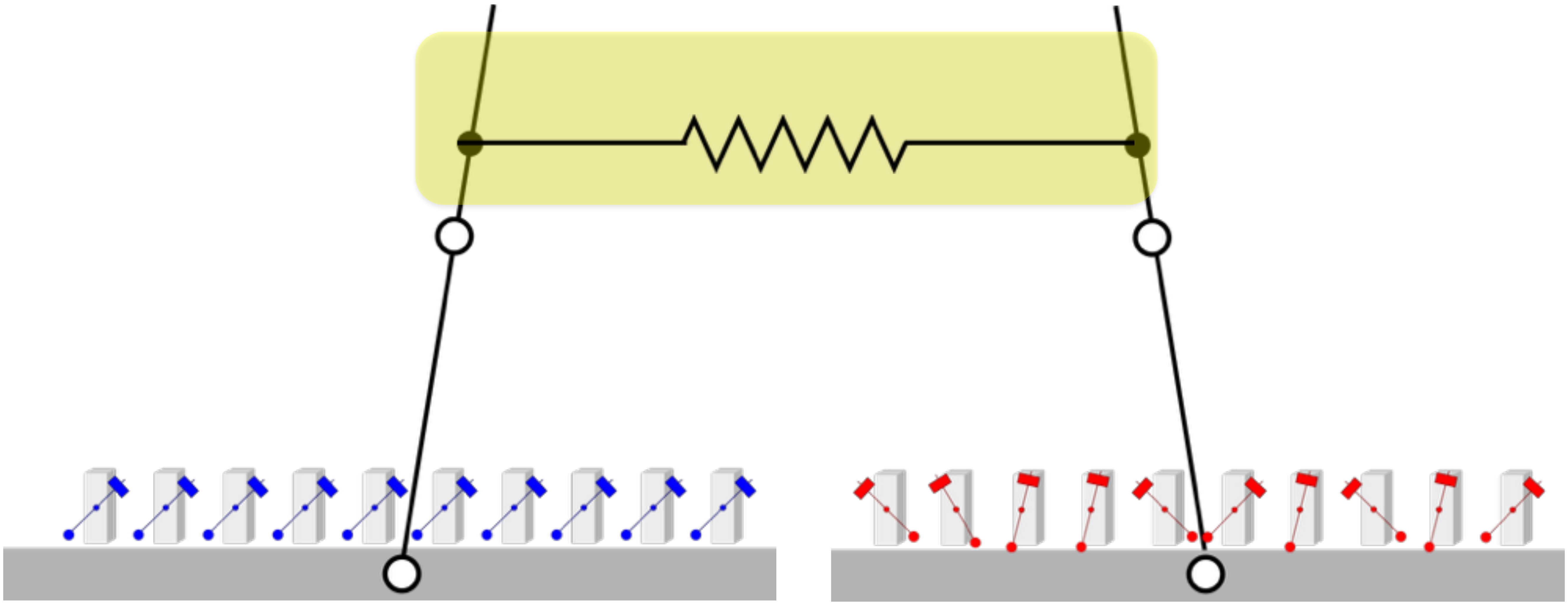
chemical oscillators

hagerstrom et. al., nature physics 2012

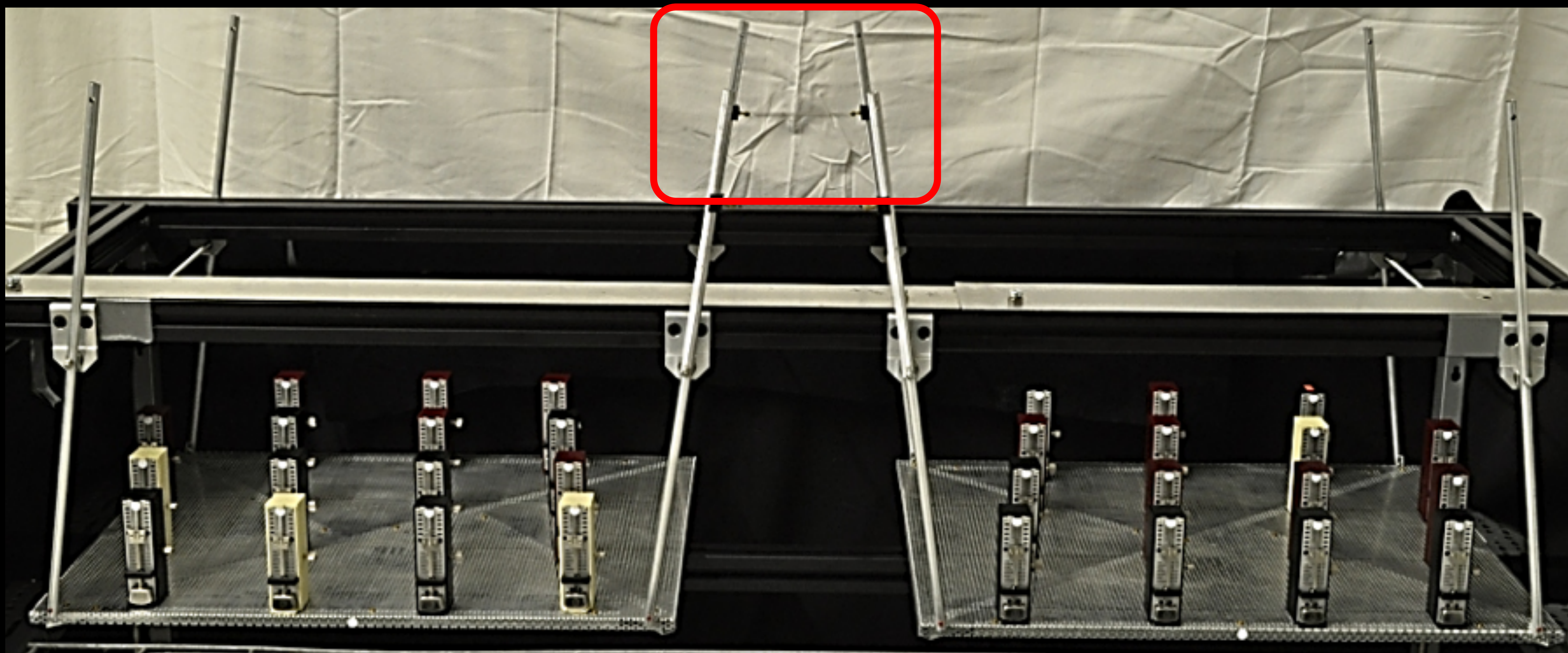


nonlinear optics

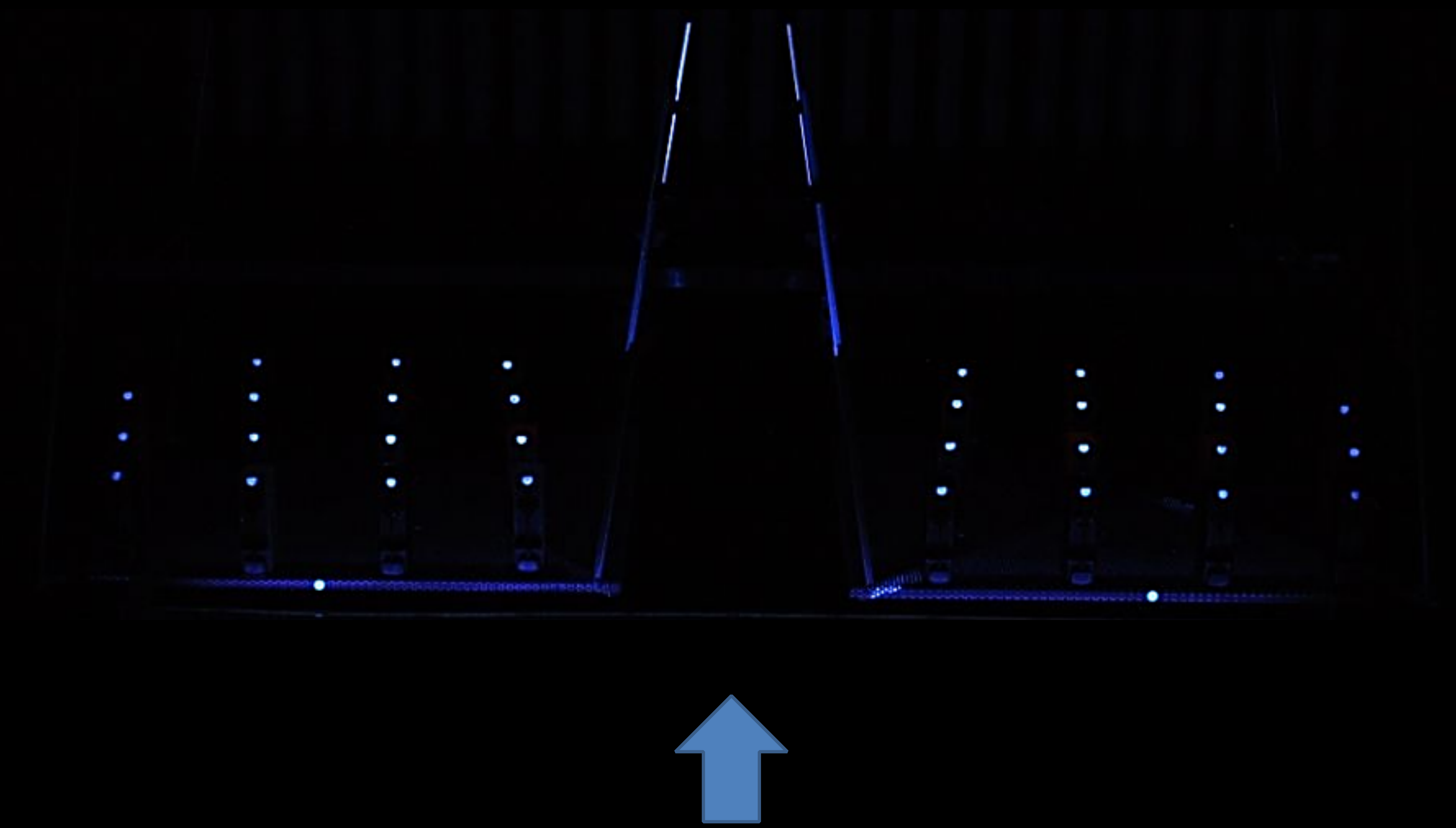
Experimental design



Springs, swings and metronomes

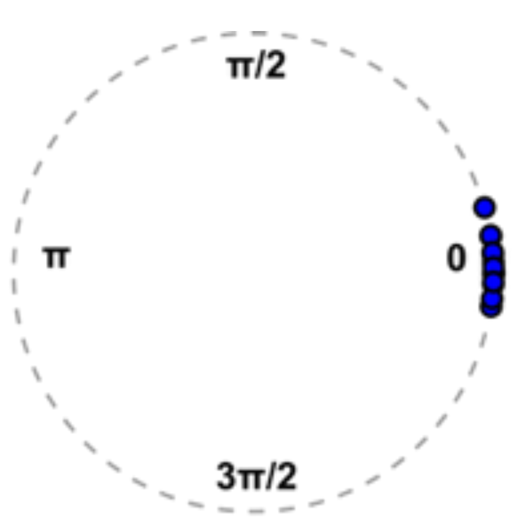


Flourescence Macroscopy

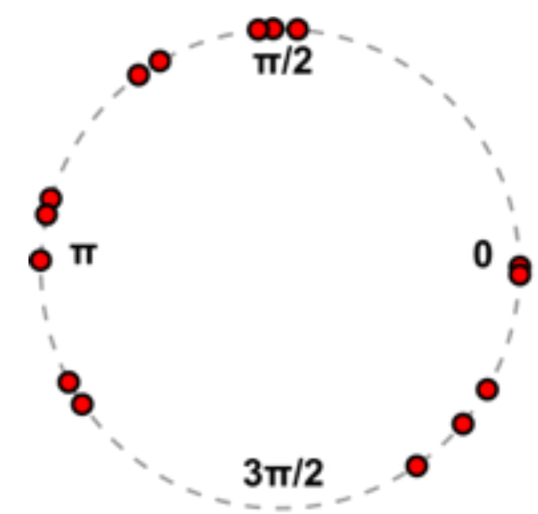


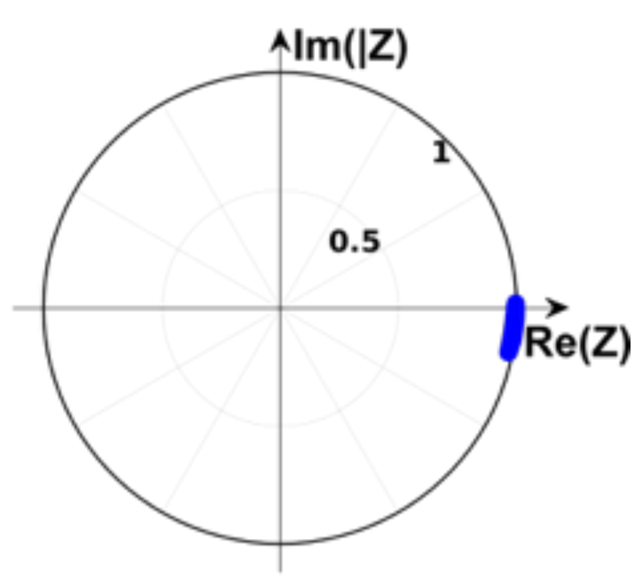

Increasing spring stiffness

$$H\left[x_k^{(p)}(t) - X^{(p)}(t) - \left\langle \left(x_k^{(p)}(t) - X^{(p)}(t)\right\rangle_t\right] = \pi^{-1} \int_{-\infty}^{\infty} \left[x_k^{(p)}(t') - X^{(p)}(t') - \left\langle x_k^{(p)}(t') - X^{(p)}(t')\right\rangle_t\right] \Big/ (t-t')\, dt'$$

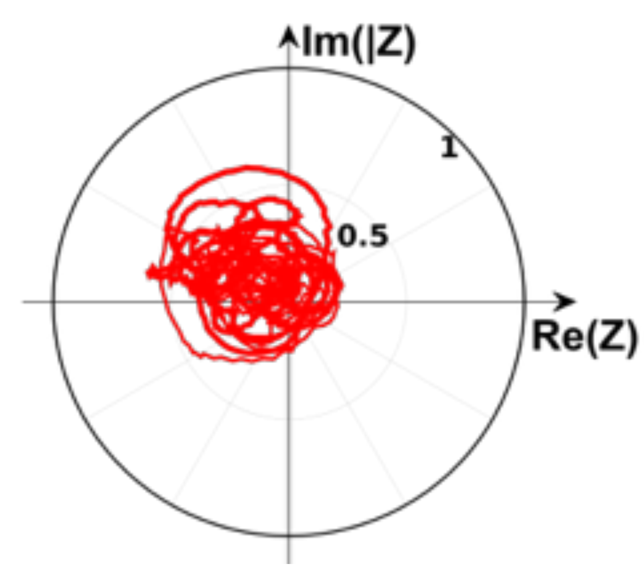


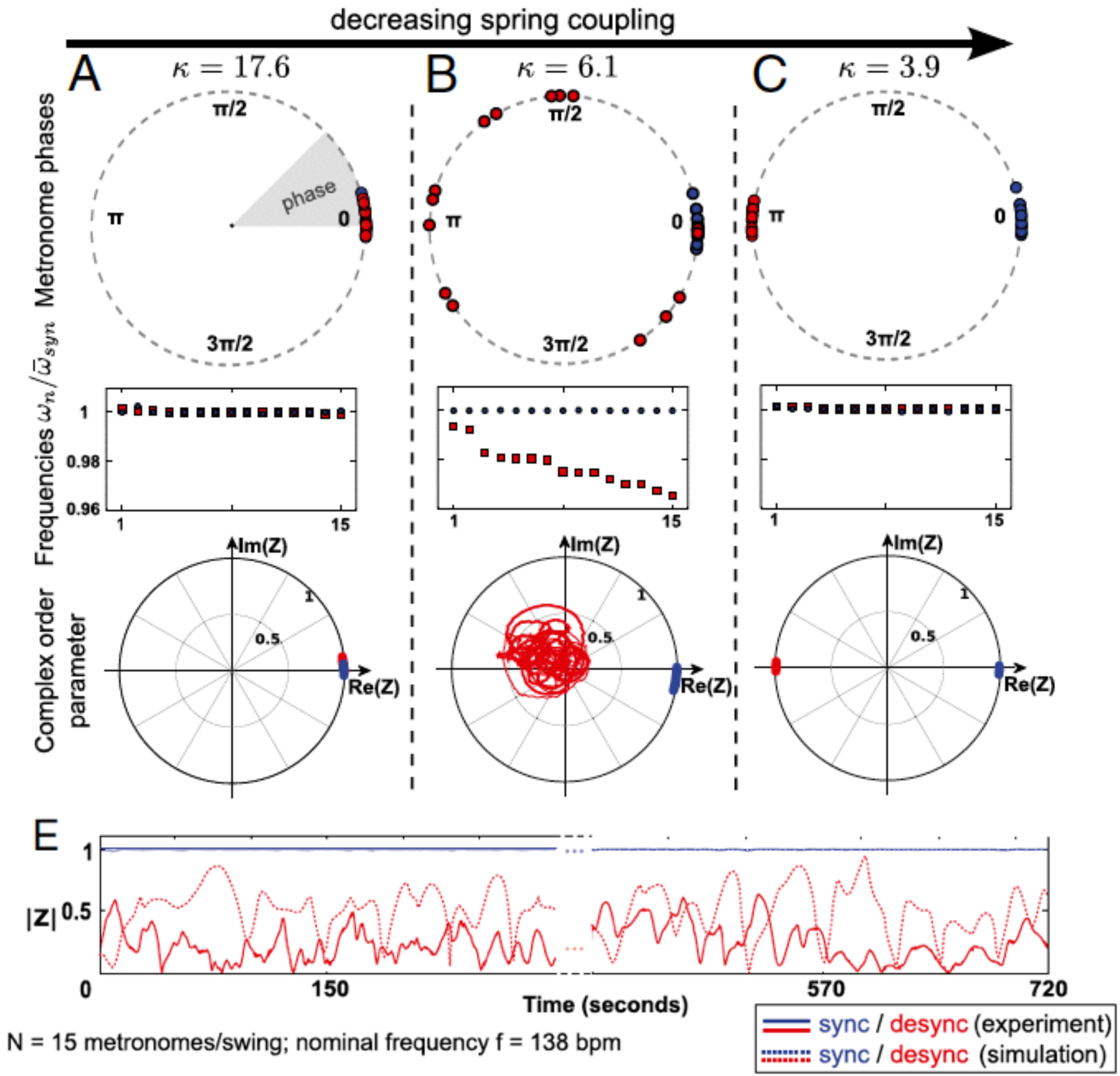
Phase angles





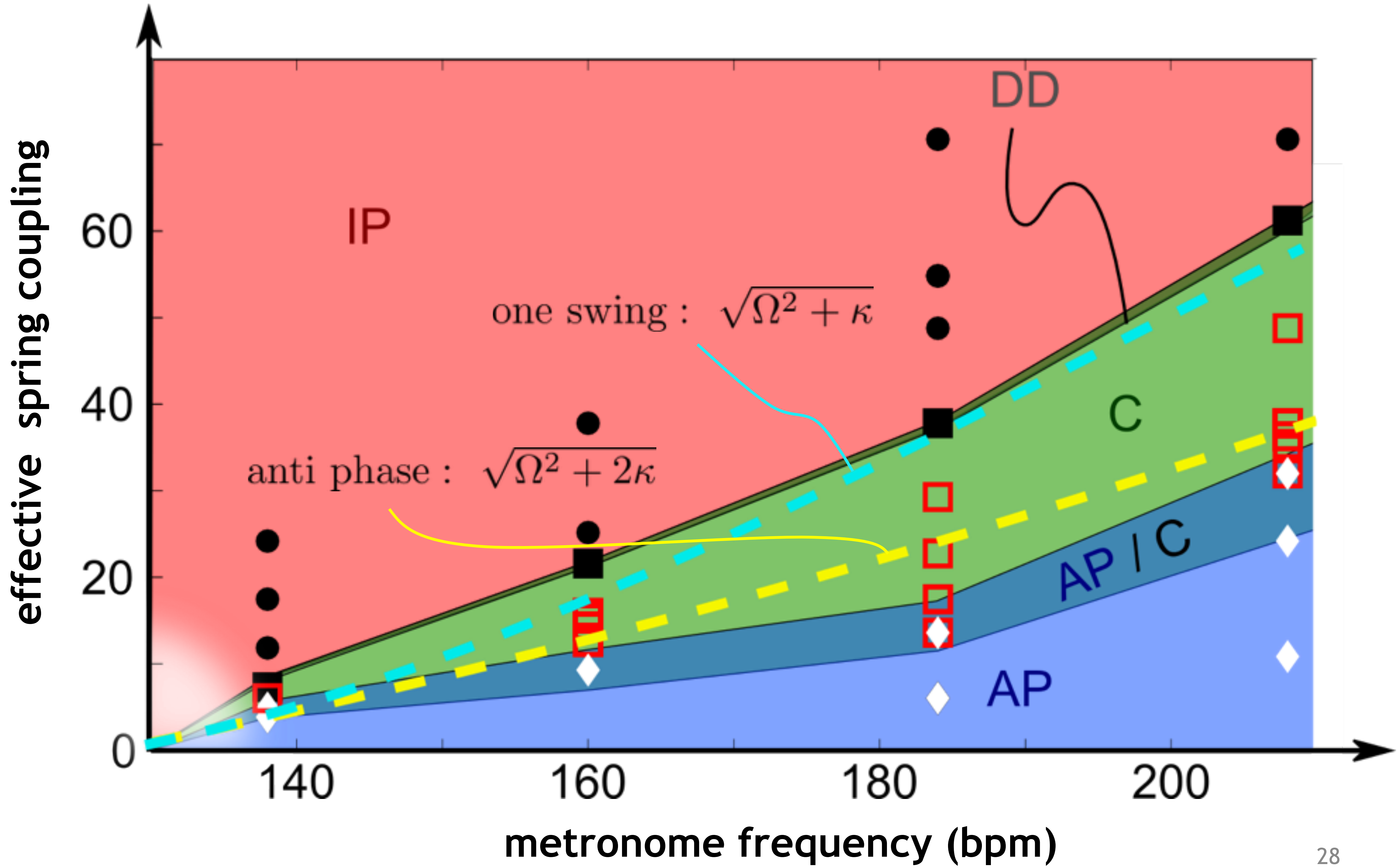
Order parameter



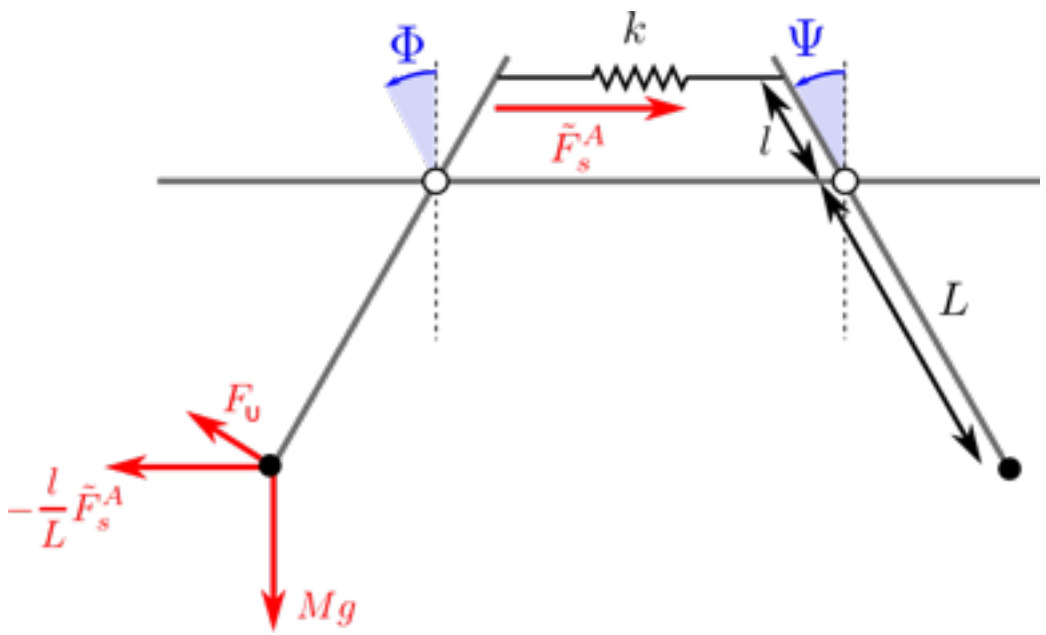

decreasing spring coupling
A
$\kappa = 17.6$
B
$\kappa = 6.1$
C
$\kappa = 3.9$
Metronome phases
π/2
π
0
3π/2
phase
Frequencies $\omega_n/\bar{\omega}_{syn}$
1
0.98
0.96
15
Complex order parameter
Im(Z)
Re(Z)
0.5
E
|Z|
150
570
720
Time (seconds)
sync / desync (experiment)
sync / desync (simulation)
N = 15 metronomes/swing; nominal frequency f = 138 bpm

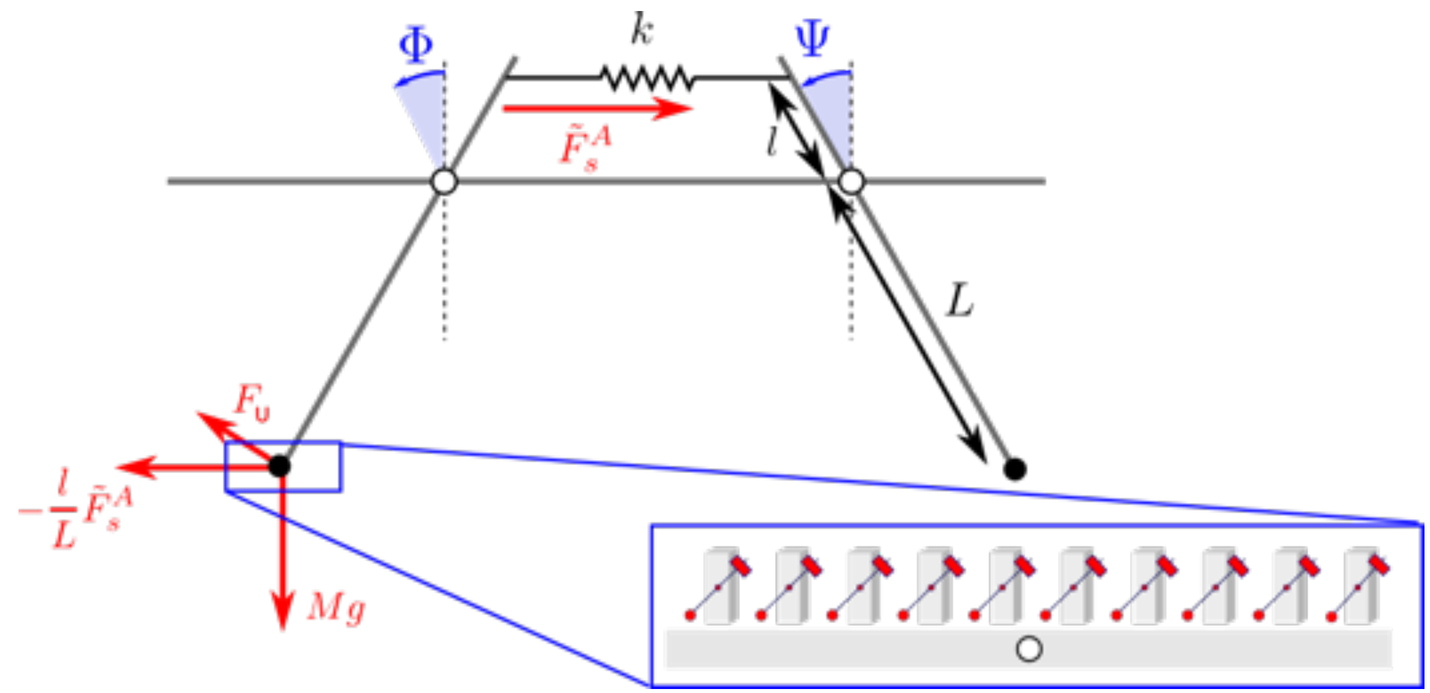
We've seen these before!

Where do the chimeras live?



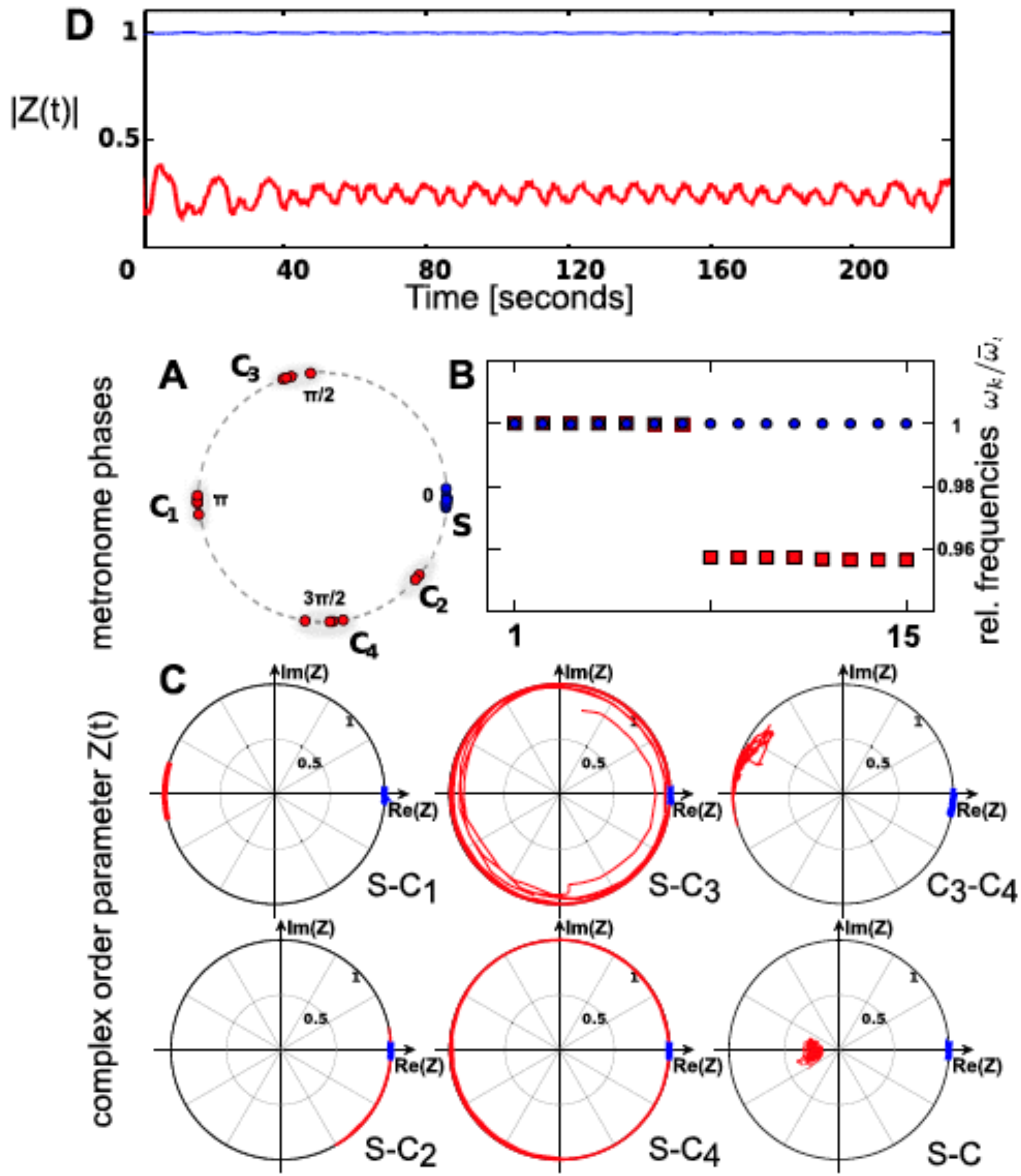
Extending Pantaleone
Generalising Huygens





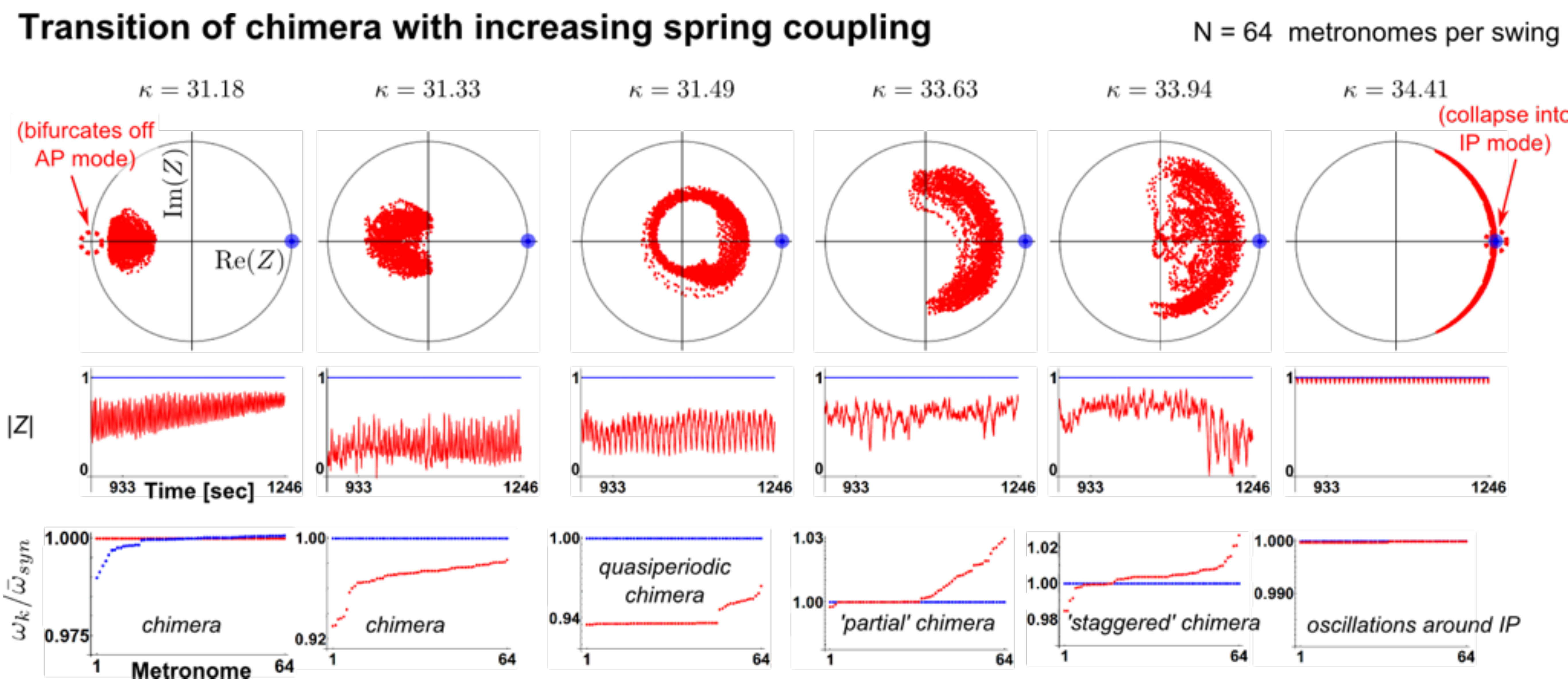
$$\ddot{\phi}_i + \sin\phi_i + \mu_m\,\dot{\phi}_i\; D(\phi_i) + \frac{\omega^2}{\Omega^2}\cos\phi_i\,\ddot{\Phi} = 0$$

$$\ddot{\Phi} + \Omega^2\Phi - \kappa(\Psi - \Phi) + \mu_s\dot{\Phi} + \frac{x_0}{L}\sum_{k=1}^{N}\partial_{\tau\tau}\,\sin\phi_k = 0$$



D
|Z(t)|
1
0.5
0
40
80
120
160
200
Time [seconds]
A
metronome phases
C3
π/2
C1
π
0
S
C2
3π/2
C4
B
rel. frequencies $\omega_k/\bar{\omega}_i$
1
0.98
0.96
1
15
C
complex order parameter Z(t)
Im(Z)
Re(Z)
1
0.5
S-C1
S-C3
C3-C4
S-C2
S-C4
S-C

The chimera zoo



PNAS, 2013

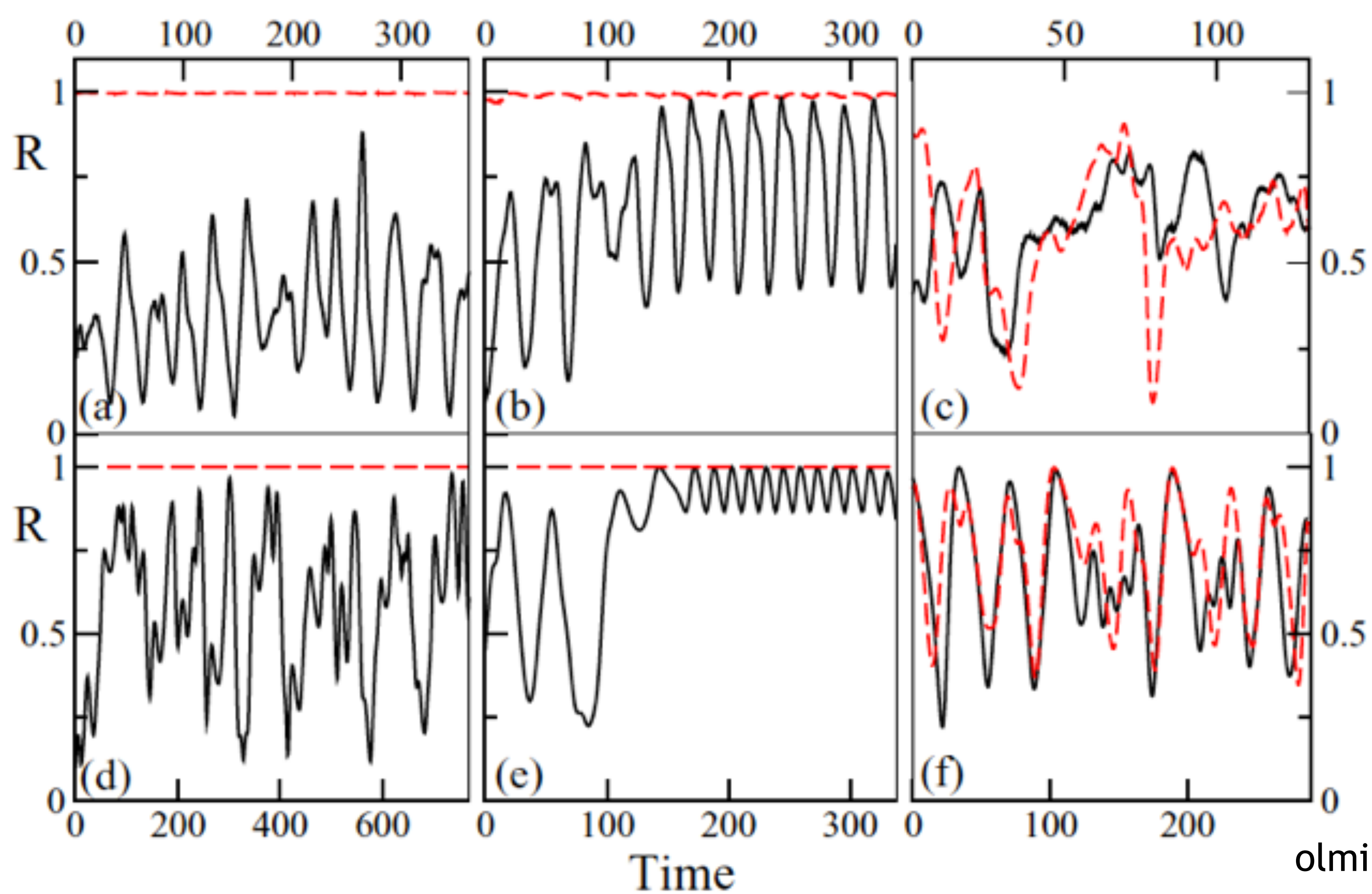
Coupled rotators

$$\frac{d\theta_i^\sigma}{dt} = \omega + \sum_{\sigma'=1}^{2} \frac{K_{\sigma\sigma'}}{N_{\sigma'}} \sum_{j=1}^{N_{\sigma'}} \sin(\theta_j^{\sigma'} - \theta_i^\sigma - \alpha)$$

abrams et. al., prl, 2004

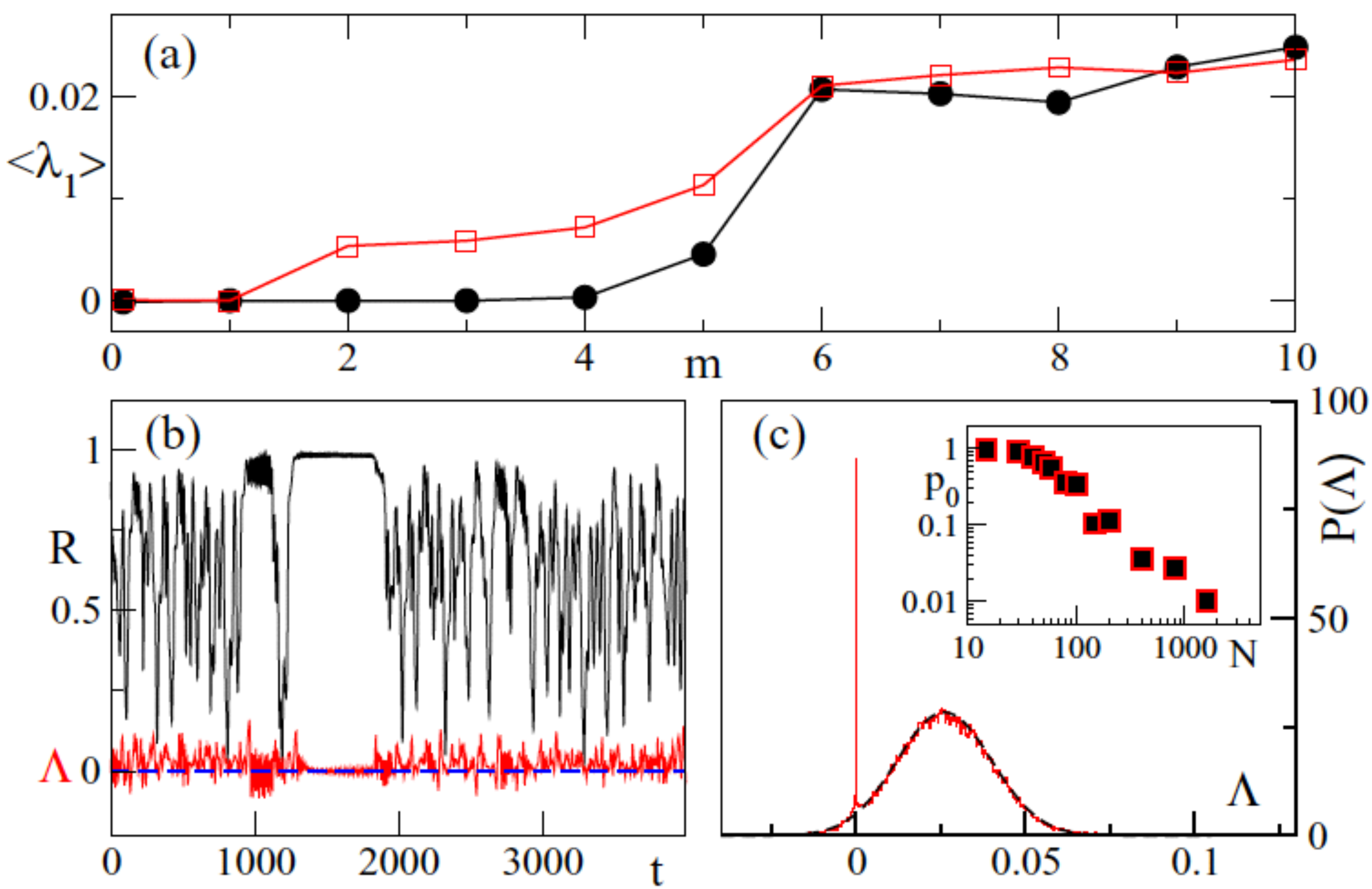
$$m\ddot{\theta}_i^{(\sigma)} + \dot{\theta}_i^{(\sigma)} = \Omega + \sum_{\sigma'=1}^{2} \frac{K_{\sigma\sigma'}}{N} \sum_{j=1}^{N} \sin\left(\theta_j^{(\sigma')} - \theta_i^{(\sigma)} - \gamma\right)$$

with Simona Olmi, Alessandro Torcini and Erik Martens



olmi et. al., PRE, 2015

Intermittency and intermittent chaotic chimeras

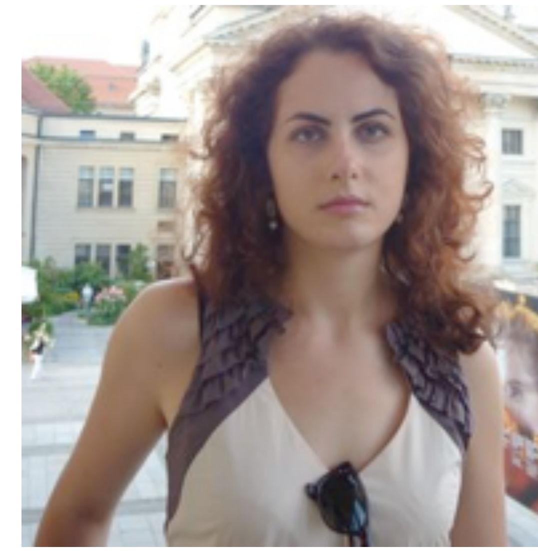


the fellowship of the (dis)order of the chimera

Oskar Hallatschek

Erik Martens

Antoine Fourrière



Simona Olmi

Alessandro Torcini