Dynamics of Piecewise Smooth maps

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LECTURE 2

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The continuous map (+ unimodal) case 1980-2000: sample results

Zero entropy (non-chaotic)

- Attractors periodic orbits of period 2^n or Cantor set (with infinite set of orbits period 2^n)
- Existence of an orbit not a power of two implies chaos
- Universal period-doubling cascades for differentiable families

Chaotic case

- Sharkovskii ordering
- Positive measure of parameters with chaotic attractors
- Inductive decomposition of the non-wandering set into possibly infinite set (bands within bands)
- Simple ‘full’ families (do everything possible)

The continuous map (+ unimodal) case 1980-2000: techniques

- Markov partitions
- Bifurcation theory
- Kneading theory
- Ergodic theory
- Renormalization (induced maps)
- Ideas of expansion

- To what extent can these be generalized in the PWS maps e.g. described by Viktor yesterday?

- I’ll not go into detail on bifurcations today.

Non-chaotic maps: two definitions

- Say the map is chaotic if it has a topological horseshoe.
- (e.g. symmetric tent map with slope greater than two)
- Say a fixed point is orientation reversing (an ORFP) if the map is decreasing there (e.g. negative slope).

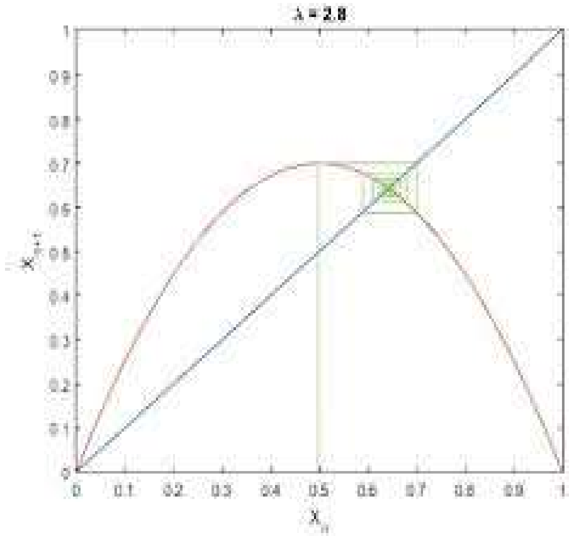


Figure 2 With $\lambda < 3$, only one stable state is observed. The 45° line, where $X_n = X_{n+1}$, is utilized to switch to the next iterate.

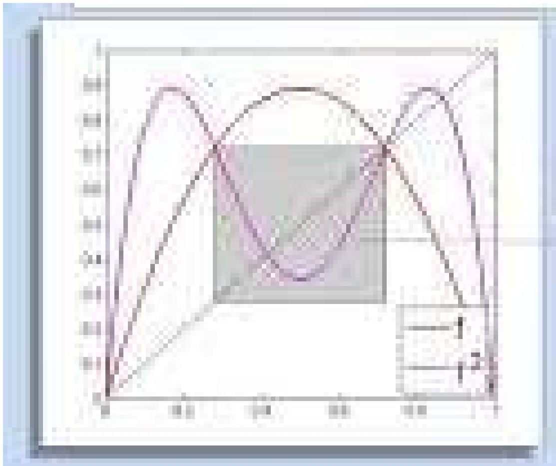
Dichotomy I: either there is an ORFP or not!

- No ORFP

All solutions tend to a fixed point

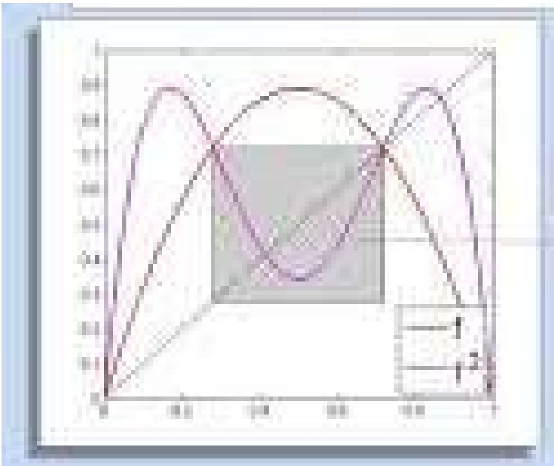
- ORFP

Look at the second iterate



Dichotomy II: either f^2 maps box into itself or not

- If maps box into itself



Does f^2 have an ORFP?
If it does it is period 2 for f.

- If not the f is chaotic (f^2 has a topological horseshoe).

Now use INDUCTION on f, f^2, f^4, f^8,… having no topological horseshoe and an ORFP

Dichotomy III: either induction process terminates or it does not.

- If terminates then f^{2^m} does not have an ORFP and the only periodic points are those with periods 1,2, … , 2^{m} and possibly 2^{m+1}.

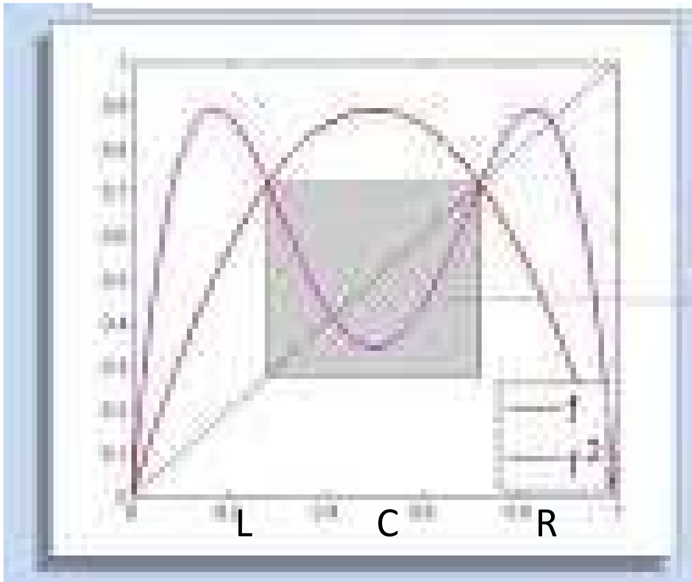
- If it does not terminate then f has orbits with period 2^k all k, and an attracting Cantor set (‘the accumulation of period-doubling’).

And that’s it!

Approximate beginning of kneading theory

Use C, L and R to indicate iterates at the critical point on its left and on its right respectively.

Kneading invariant is the sequence obtained from the critical point, and with order
L < C < R if agree on even # of Rs
R < C < L if agree on even # of Rs
before they disagree, then
the shift of the kneading invariant is the largest of all shifts of the kneading invariant (furthest to the right).

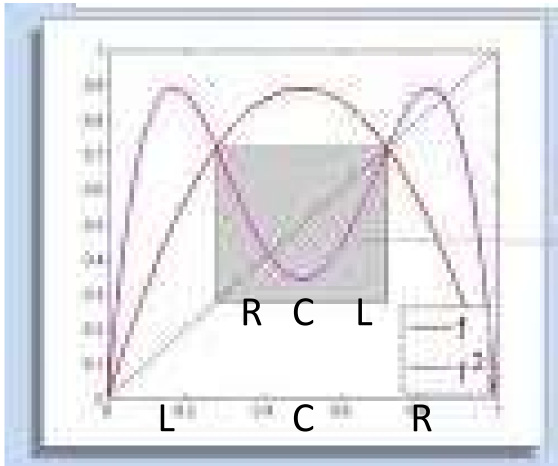


Full families and Accumulation of doubling

- Say a family is a full family if every possible consistent kneading invariant exists in the family.

- Collet-Eckmann: the quadratic family is full for mu between 0 and 4.

- Doubling:



C→CR, L→RR, R → LR

C→CR→CRLR→CRLRRRLR→CRLRRRLRLRLRRRLR.....