Two-dimensionalisation of Rotating 3D Turbulence Revisited

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Outline

3D Turbulence vs 2D Turbulence

3D Turbulence

- Inviscid conserved quantities: Energy, Helicity
- Forward energy cascade
- Energy spectrum: $E(k) \sim k^{-5/3}$

2D Turbulence

- Inviscid conserved quantities: Energy, Enstrophy ($Z = \langle \omega^2 \rangle / 2$)
- backward energy cascade corresponding to enstrophy cascade
- Energy spectrum: $E(k) \sim k^{-3}$

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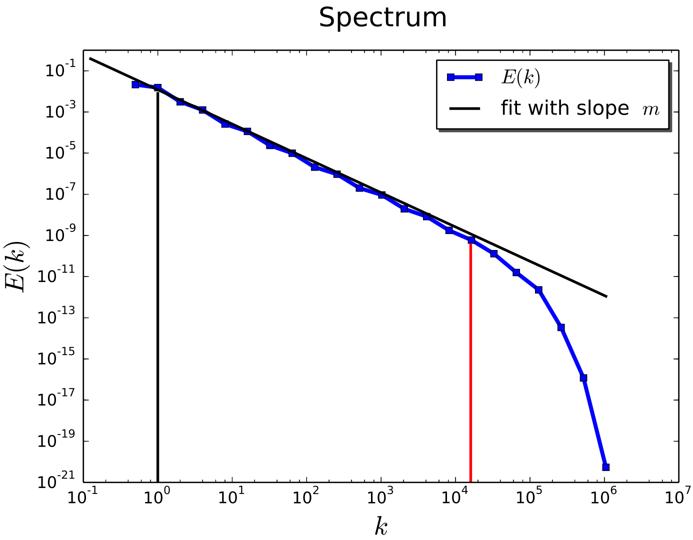
Rotating turbulence: Energy spectrum $E(k) \sim k^m$ where $m \approx -2$

(Baroud et.al., PRL 88(11):114501 March 2002)

Spectrum



Spectrum
$E(k)$
fit with slope m
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-21}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
$E(k)$
k

Intermittency

Strucure Functions: For $l_d \ll l \ll L$

$$S_p(l) \equiv \left\langle \left[\delta u_{\|}(l) \right]^p \right\rangle \sim l^{\zeta_p}$$

where

$$\delta u_{\|}(l) \equiv \left| \vec{u}(\vec{r} + \vec{l}) - \vec{u}(\vec{r}) \right| \cdot \frac{\vec{l}}{l} \ .$$

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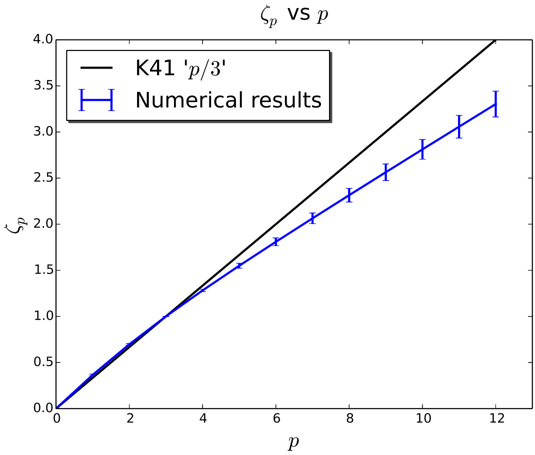
In K41 theory: $\delta u_{\|}(l) \sim (\bar{\epsilon}\ l)^{1/3}$,

$$S_p(l) \sim l^{p/3} \qquad \text{i.e.} \qquad \zeta_p = p/3$$

Intermittency continues...



Shell Model

NSE for incompressible flow:

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial_{jj} u_i + f_i \tag{1}$$

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$$\Big\downarrow \text{Fourier transform}$$

$$\partial_t u_i(\mathbf{k}) = -\iota k_j \int \left(\delta_{il} - \frac{k_i k'_l}{k^2} \right) u_j(\mathbf{k}') u_l(\mathbf{k} - \mathbf{k}') \mathrm{d}\mathbf{k}' - \nu k^2 u_i(\mathbf{k}) + f_i(\mathbf{k}) \tag{2}$$

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A generic shell model:

$$\left(\partial_t + \nu k_n^2 \right) u_n = k_n (NL)_n [u, u] + f_n \tag{3}$$

The GOY Shell Model

The evolution equation for the GOY shell model:

$$\left[\frac{d}{dt} + \nu k_n^2\right] u_n = \iota(a_n u_{n+1} u_{n+2} + b_n u_{n-1} u_{n+1} + c_n u_{n-1} u_{n-2})^* + f_n$$

where

1. $k_n = k_0 2^n$, where $k_0 = 1/16$;
2. $a_n = k_n$, $b_n = -\delta k_{n-1}$, $c_n = -(1-\delta) k_{n-2}$, where $\delta = 1/2$.

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The GOY shell model for decaying turbulence with rotation strength Ω:

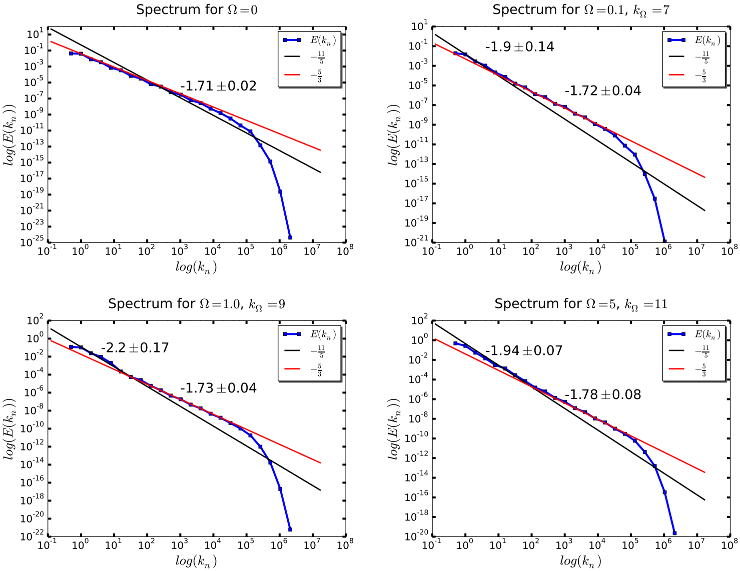
$$\left[\frac{d}{dt} + \nu k_n^2\right] u_n = \iota(a_n u_{n+1} u_{n+2} + b_n u_{n-1} u_{n+1} + c_n u_{n-1} u_{n-2})^* - \iota \Omega u_n. \tag{4}$$

Some Details

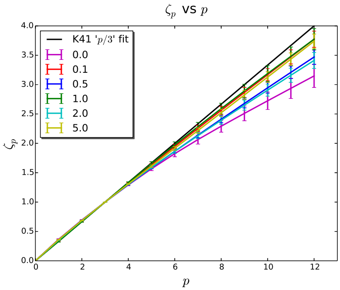
1. Scheme: slaved Adam-Bashforth
2. Number of shells = 28
3. Viscosity $\nu = 10^{-8}$
4. Timestep $\delta t = 10^{-4}$
5. Initial conditions:
   - At large length scales (for $n = 1, 2$), $u_n^0 = k_n^{1/2} e^{\iota\theta_n}$
   - And for $n = 3$ to 28, $u_n^0 = k_n^{1/2} e^{-{k_n}^2} e^{\iota\theta_n}$;

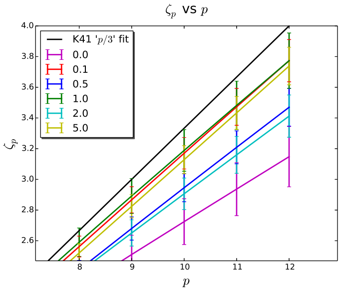
   where θ_n is a random phase angle distributed uniformly between 0 and 2π.
6. No. of statistically independent initial configurations = 10000

Two-dimensionalisation of Rotating 3D Turbulence



Intermittency in Rotating Turbulence





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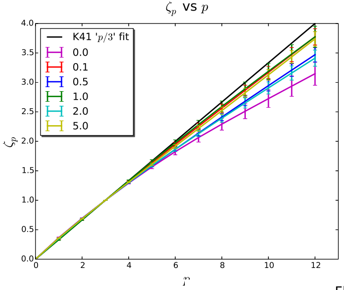

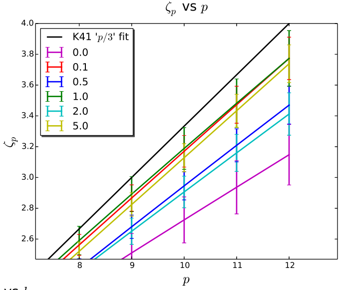

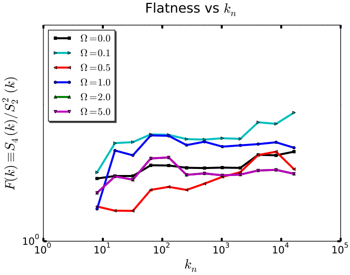

Conclusion

1. Dual scaling is observed in the energy spectrum of rotating turbulence. The inertial subrange (with no rotation) shrinks as rotation increases.
2. In first numerical investigation, intermittency does not seem to have some pattern with rotation strength. This needs to be understood with advanced models of intermittency.
3. This 2-dimensionalisation effect needs to be tested with energy flux and enstrophy.

References

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Thank you