Cooling in Granular Gases

by

Sanjay Puri

Overview

(a) Introduction

(b) Granular Gases

(c) Viscoelastic Granular Gases: Freely-evolving and Heated

(d) Frictional Cooling of Granular Gases

(e) Conclusion

(a) Introduction

Granular materials or powders (e.g., sand, glass) are macroscopic assemblies of inelastic particles.

- Typical size 10 μm – 1 cm.
- Particles dissipate energy on collision.
- Scale of assembly >> Particle size

Various properties depend upon shape and size of particles, so less universality at macro-level.

1) H.M. Jaeger, S.R. Nagel and R.P. Behringer, Rev. Mod. Phys. 68, 1259 (1996).
2) *Powders and Grains* conference proceedings.
3) N.V. Brilliantov and T. Poschel, *Kinetic Theory of Granular Gases*, OUP, Oxford (2004).

Dynamical problems for granular materials

- Continuously input energy to compensate energy loss due to inelastic collisions,

  e.g., rotation in a drum

  vertical or horizontal vibration

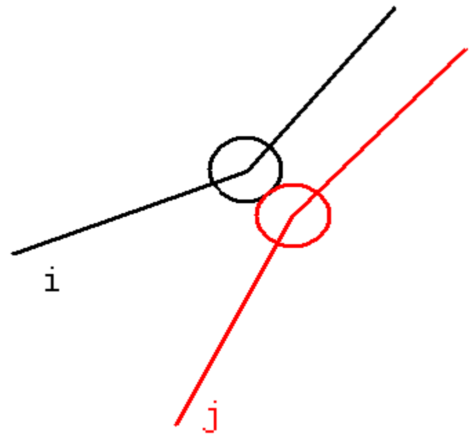
  flow and pouring, etc.

- Free evolution of an initially homogeneous granular gas.

1) P.K. Haff, J. Fluid. Mech. 134, 401 (1983).
2) I. Goldhirsch and G. Zanetti, Phys. Rev. Lett. 70, 1619 (1993).

(b) Granular Gases

Granular collisions



$$\vec{v}_i' = \vec{v}_i - \frac{(1+e)}{2}\left[\hat{n}\cdot\left(\vec{v}_i - \vec{v}_j\right)\right]\hat{n}$$

$$\vec{v}_j' = \vec{v}_j + \frac{(1+e)}{2}\left[\hat{n}\cdot\left(\vec{v}_i - \vec{v}_j\right)\right]\hat{n}$$

Constant restitution coefficient e=1 (elastic)

e<1 (inelastic)

- Density and momentum are conserved during collision.
- Magnitude of normal velocity is reduced for e<1.

Loss of energy (Haff's cooling law)

$$T(t) = \frac{T_0}{[1+\varepsilon\omega(T_0)t/2d]^2}$$

Parallelization of velocities (correlations build up)

Fraction of energy lost per collision $\boxed{= \frac{1-e^2}{d} = \frac{\varepsilon}{d}}$

Temperature $T = \frac{2E}{d}$ and $\boxed{\frac{dT}{dt} = -\frac{\varepsilon}{d}\omega(T)T}$

Collision frequency
$$\omega(T) \propto n\chi(n)\sigma^{d-1}\sqrt{T}$$
$$= \omega(T_0)\sqrt{\frac{T}{T_0}}$$

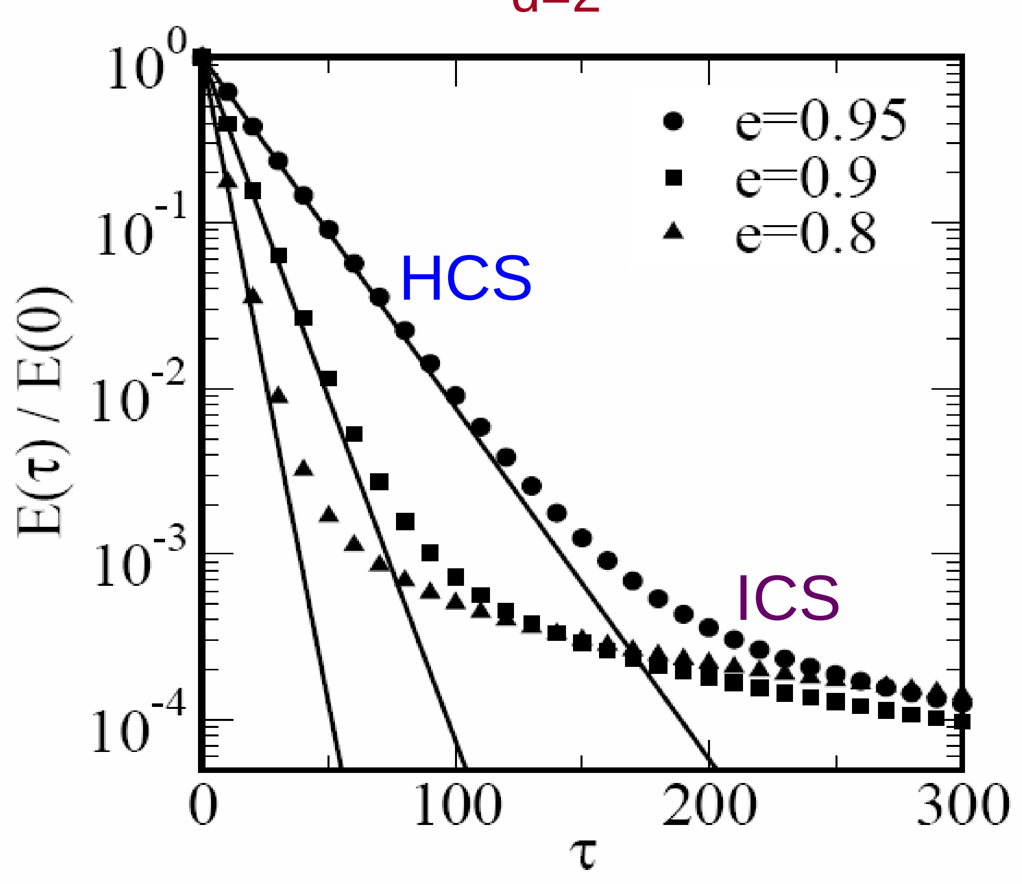
Haff's cooling law $\boxed{T(t) = \frac{T_0}{[1+\varepsilon\omega(T_0)t/2d]^2}}$

Collision time
$$\tau(t) = \int_0^t dt'\omega(t')$$
$$= \frac{2d}{\varepsilon}\ln\left[1+\frac{\varepsilon\omega(T_0)}{2d}t\right]$$

$\boxed{T(\tau) = T_0 \exp\left(-\frac{\varepsilon}{d}\tau\right)}$ Valid only in the homogeneous cooling state (HCS).

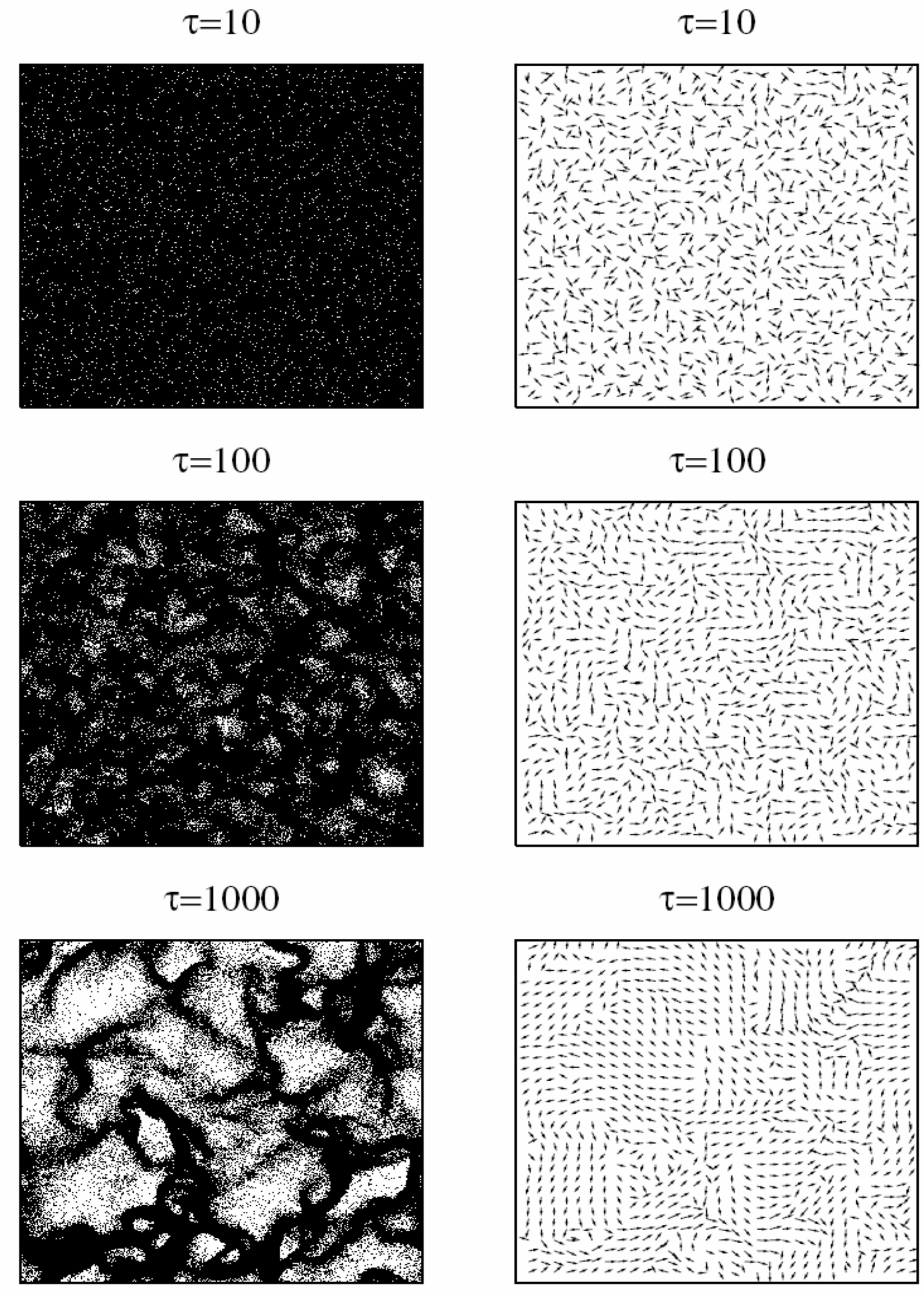
Event-driven simulations in d=2,3
N=10^6, number fraction=0.2

d=2



S.R. Ahmad and S. Puri, Europhys. Lett. 75, 56 (2006);
Phys. Rev. E 75, 031302 (2008).

Density field   Velocity field

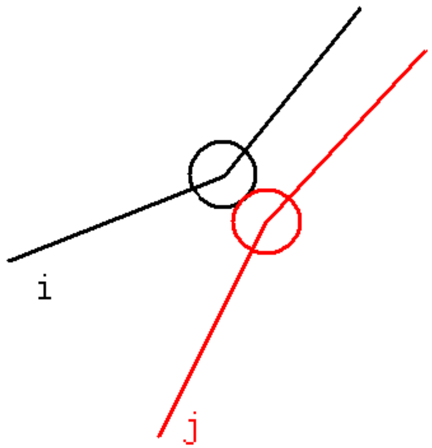


e=0.9
n=0.2

(c) Viscoelastic Granular Gases: Freely-evolving and Heated

A. Bodrova, A. Dubey, S. Puri and N. Brilliantov,
Phys. Rev. Lett. 109, 178001 (2012);
Phys. Rev. E 87, 062202 (2014).

Freely-Evolving



$$\vec{v}_i' = \vec{v}_i - \frac{(1+e)}{2}\left[\hat{n}\cdot\left(\vec{v}_i - \vec{v}_j\right)\right]\hat{n}$$

$$\vec{v}_j' = \vec{v}_j + \frac{(1+e)}{2}\left[\hat{n}\cdot\left(\vec{v}_i - \vec{v}_j\right)\right]\hat{n}$$

Velocity-dependent restitution coefficient

N.V. Brilliantov and T. Poschel, Phys. Rev. E 61, 5573 (2000).

$$e = 1 - C_1 A\kappa^{2/5}|\hat{n}\cdot\left(\vec{v}_i - \vec{v}_j\right)|^{1/5} + C_2 A^2\kappa^{4/5}|\hat{n}\cdot\left(\vec{v}_i - \vec{v}_j\right)|^{2/5}$$

$$\delta = A\kappa^{2/5}$$

- Inelastic kinetic theory for granular gases

  N.V. Brilliantov and T. Poschel, *Kinetic Theory of Granular Gases*, OUP, Oxford (2004).

- Boltzmann-Enskog (BE) equation

$$\boxed{\frac{\partial P(\vec{v},t)}{\partial t} = \chi(n) I(P,P)}$$

- Dimensionless form of BE equation

$$\frac{\mu_2}{3}\left(3 + c\frac{\partial}{\partial c}\right)F(\vec{c},t) + B^{-1}\frac{\partial}{\partial t}F(\vec{c},t) = \tilde{I}(F,F)$$

$$\mu_p = -\int d\vec{c}\, c^p \tilde{I}(F,F)$$ Moments of collision integral

$$B = v_T \chi(n) \sigma^2 n$$

Collision integral

$$\tilde{I}(F,F) = \int d\vec{c_2} \int d\hat{n}\, \Theta\left(-\vec{c_{12}} \cdot \hat{n}\right)\left[\frac{1}{e^2} F(\vec{c''_1},t) F(\vec{c''_2},t) - F(\vec{c_1},t) F(\vec{c_2},t)\right]$$

- Velocity distribution in terms of Sonine coefficients.

$$P_{MB}\left(\vec{v}\right)=\left(\frac{1}{\pi {v_0}^2}\right)^{d/2}\exp\left(-\frac{\vec{v}^2}{{v_0}^2}\right)$$

$$P_g\left(\vec{v},t\right)=\left(\frac{1}{\pi v_0(t)^2}\right)^{d/2}F\left(\frac{\vec{v}}{v_0(t)}\right)$$

$$F\left(\vec{c}\right)=\exp\left(-c^2\right)\sum_{n=0}^{\infty}a_n S_n\left(c^2\right)$$

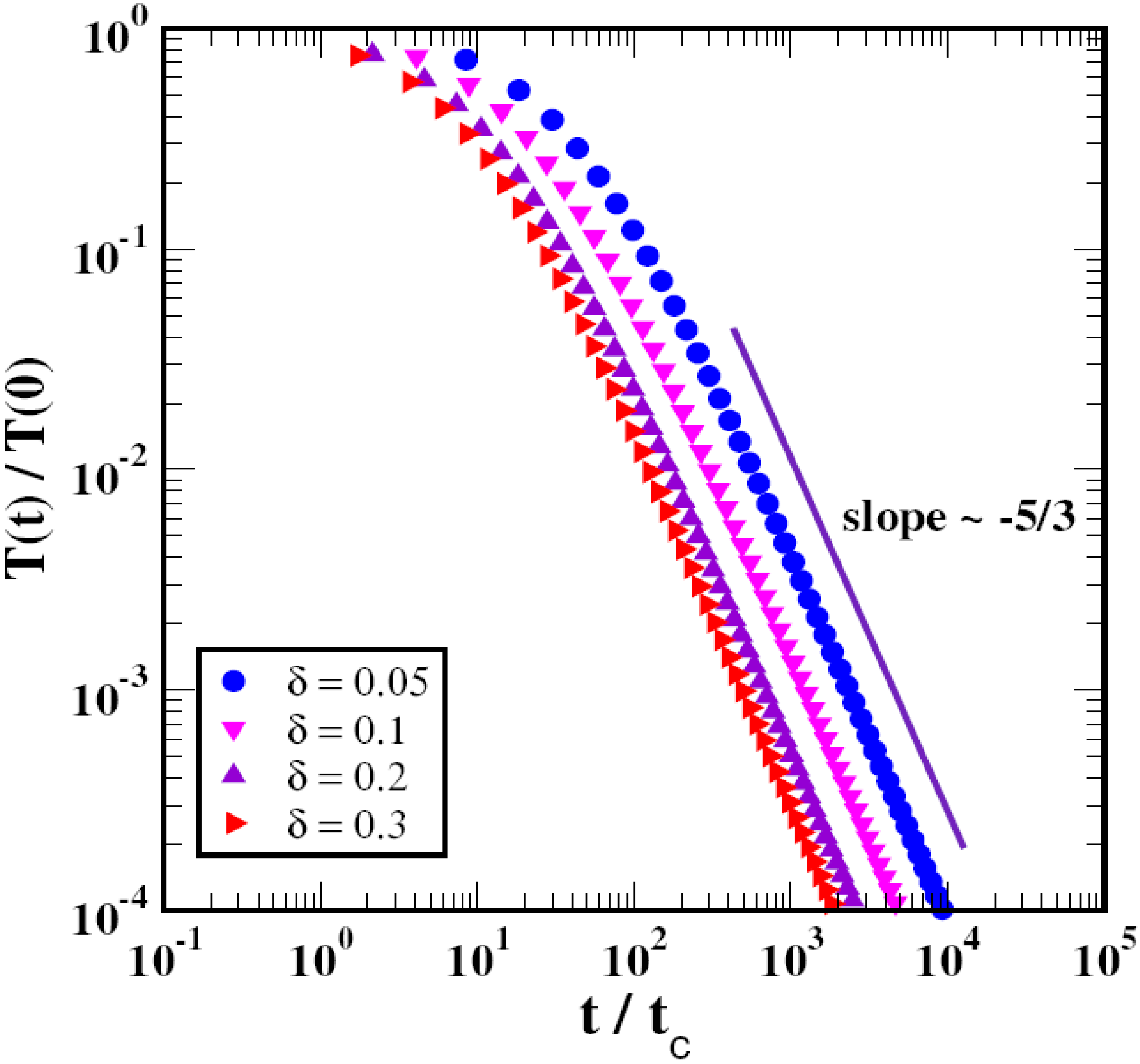
- Evolution equations for dimensionless temperature and Sonine coefficients.

$$\frac{\mathrm{d}u}{\mathrm{d}\tau}=-\frac{\sqrt{2}\mu_2}{6\sqrt{\pi}}u^{\frac{3}{2}}$$

$$\frac{\mathrm{d}a_2}{\mathrm{d}\tau}=\frac{\sqrt{2}\sqrt{u}}{3\sqrt{\pi}}\mu_2\left(1+a_2\right)-\frac{\sqrt{2}}{15\sqrt{\pi}}\mu_4\sqrt{u}$$

$$\frac{\mathrm{d}a_3}{\mathrm{d}\tau}=\frac{\sqrt{u}}{\sqrt{2\pi}}\mu_2(1-a_2+a_3)-\frac{\sqrt{2}}{5\sqrt{\pi}}\mu_4\sqrt{u}+\frac{2\sqrt{2u}}{105\sqrt{\pi}}\mu_6.$$

Event-driven simulations in d=3
N=4.096x10^6, number fraction=0.028



Heated

Let us now thermostat the granular gas by uniform heating. For this, we introduce a diffusive Fokker-Planck term in the Boltzmann-Enskog equation.

$$\boxed{\frac{\partial f(\mathbf{v},t)}{\partial t} = g_2(\sigma) I(f,f) + \frac{\xi_0^2}{2} \frac{\partial^2}{\partial \mathbf{v}^2} f(\mathbf{v},t).}$$

Insert the Sonine polynomial expansion for the velocity distribution function and truncate at fifth order.

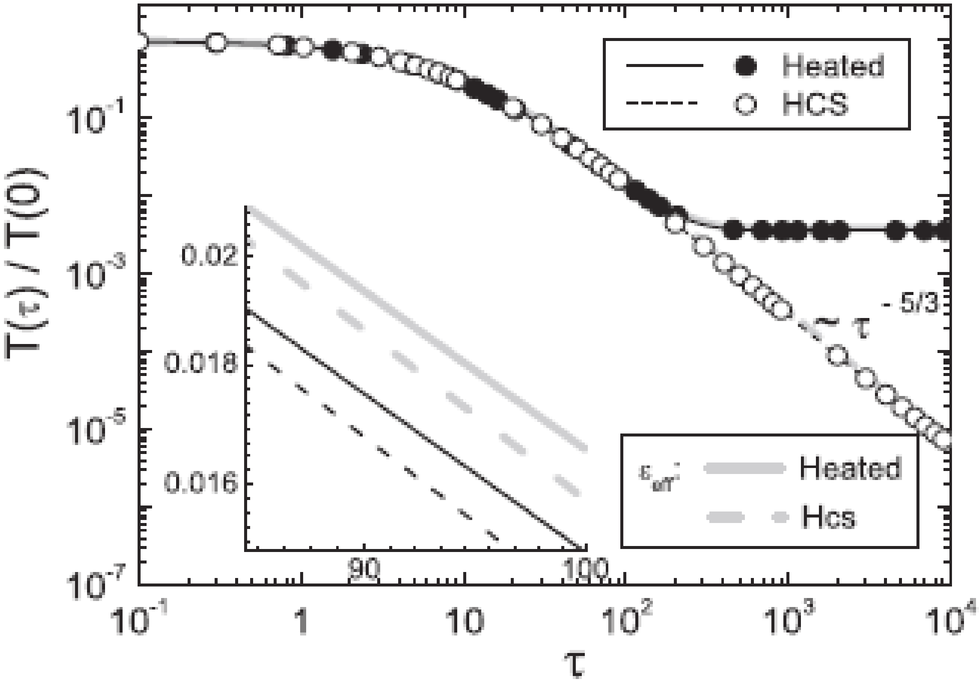
Coupled equations for Sonine coefficients

$$\frac{\partial\langle c^2\rangle}{\partial t} + \frac{1}{T}\frac{dT}{dt}\langle c^2\rangle = -\sqrt{\frac{2T}{m}}g_2(\sigma)\sigma^2 n\mu_2 + 3\frac{m\xi_0^2}{2T},$$

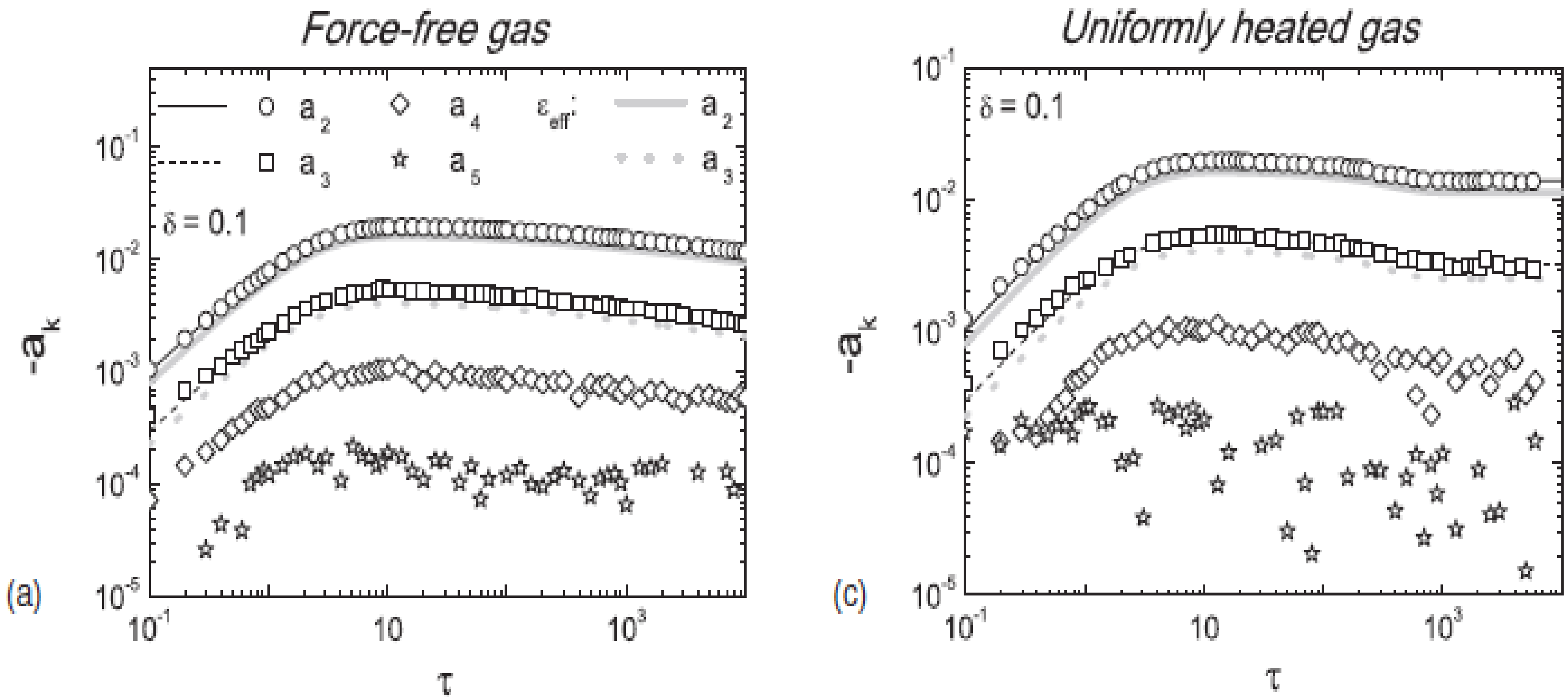
$$\frac{\partial\langle c^4\rangle}{\partial t} + \frac{2}{T}\frac{dT}{dt}\langle c^4\rangle = -\sqrt{\frac{2T}{m}}g_2(\sigma)\sigma^2 n\mu_4 + 10\langle c^2\rangle\frac{m\xi_0^2}{2T},$$

$$\frac{\partial\langle c^6\rangle}{\partial t} + \frac{3}{T}\frac{dT}{dt}\langle c^6\rangle = -\sqrt{\frac{2T}{m}}g_2(\sigma)\sigma^2 n\mu_6 + 24\langle c^4\rangle\frac{m\xi_0^2}{2T}$$

Cooling of heated granular gas



Evolution of Sonine coefficients



(d) Frictional Cooling of Granular Gases

P. Das, S. Puri and M. Schwartz (2015)

Molecular dynamics (MD) simulations of frictional particles with the goal of studying the role of plugs in a continuum theory of dense granular matter.

$$
\begin{aligned}
V(r) &= \infty, \qquad r < R_1 \\
&= V_0 \left(\frac{r - R_2}{r - R_1} \right)^2, \quad R_1 < r < R_2 \\
&= 0, \qquad r > R_2.
\end{aligned}
$$

$$\vec{F}_{ij}^{n}(r) = -\vec{\nabla} V(r)$$

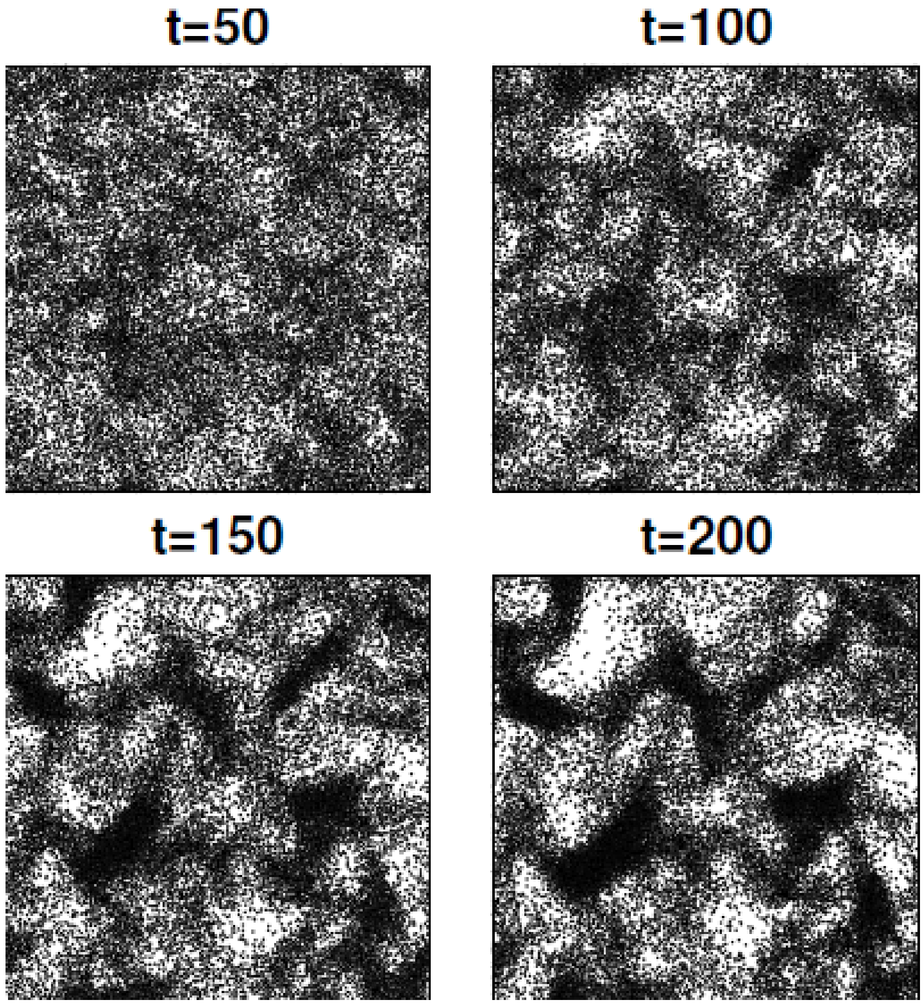
Normal force

$$\vec{F}_{ij}^{f}(r) = \mu |F_{ij}^{n}| \frac{\vec{v}_1 - \vec{v}_2}{|\vec{v}_1 - \vec{v}_2|}$$

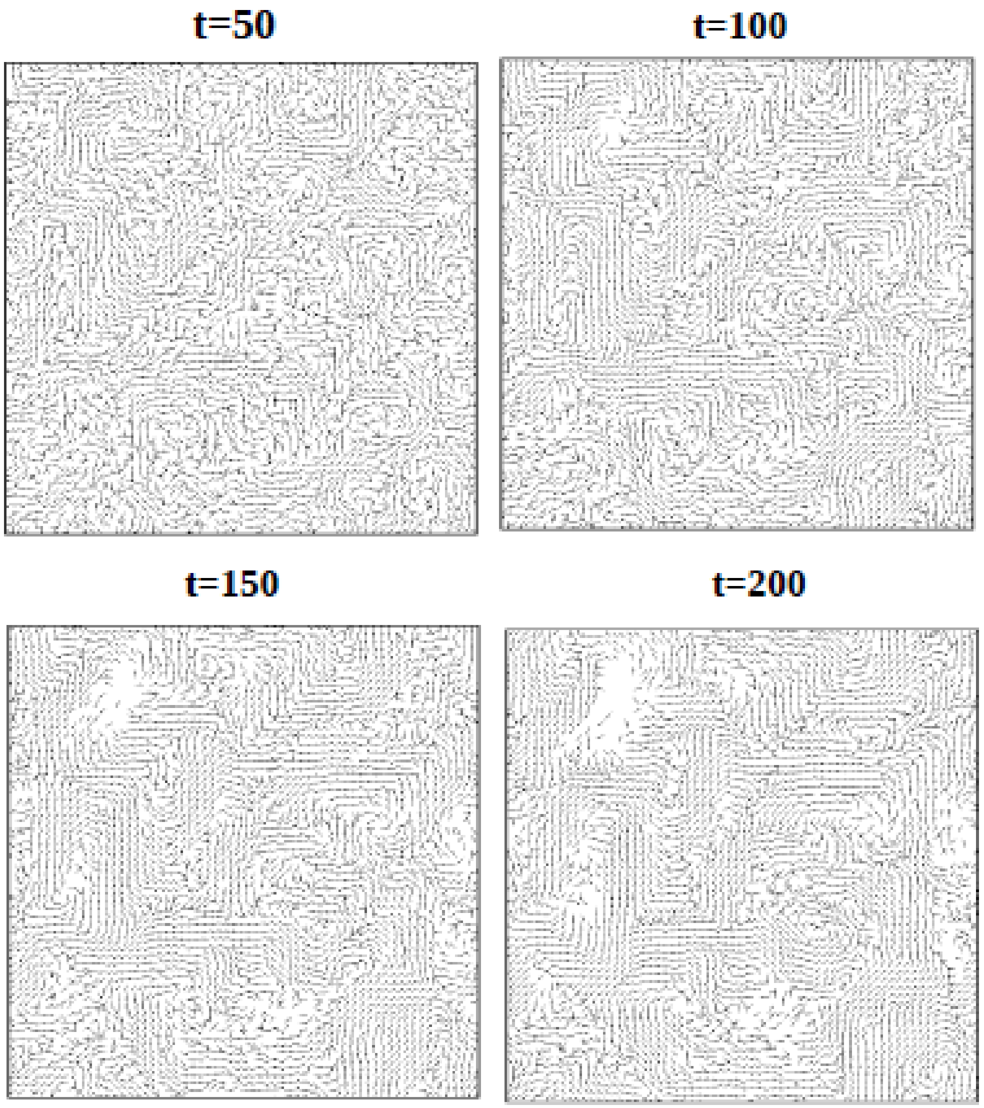
Frictional force

MD Simulations of frictional grains

N=19700, Packing fraction

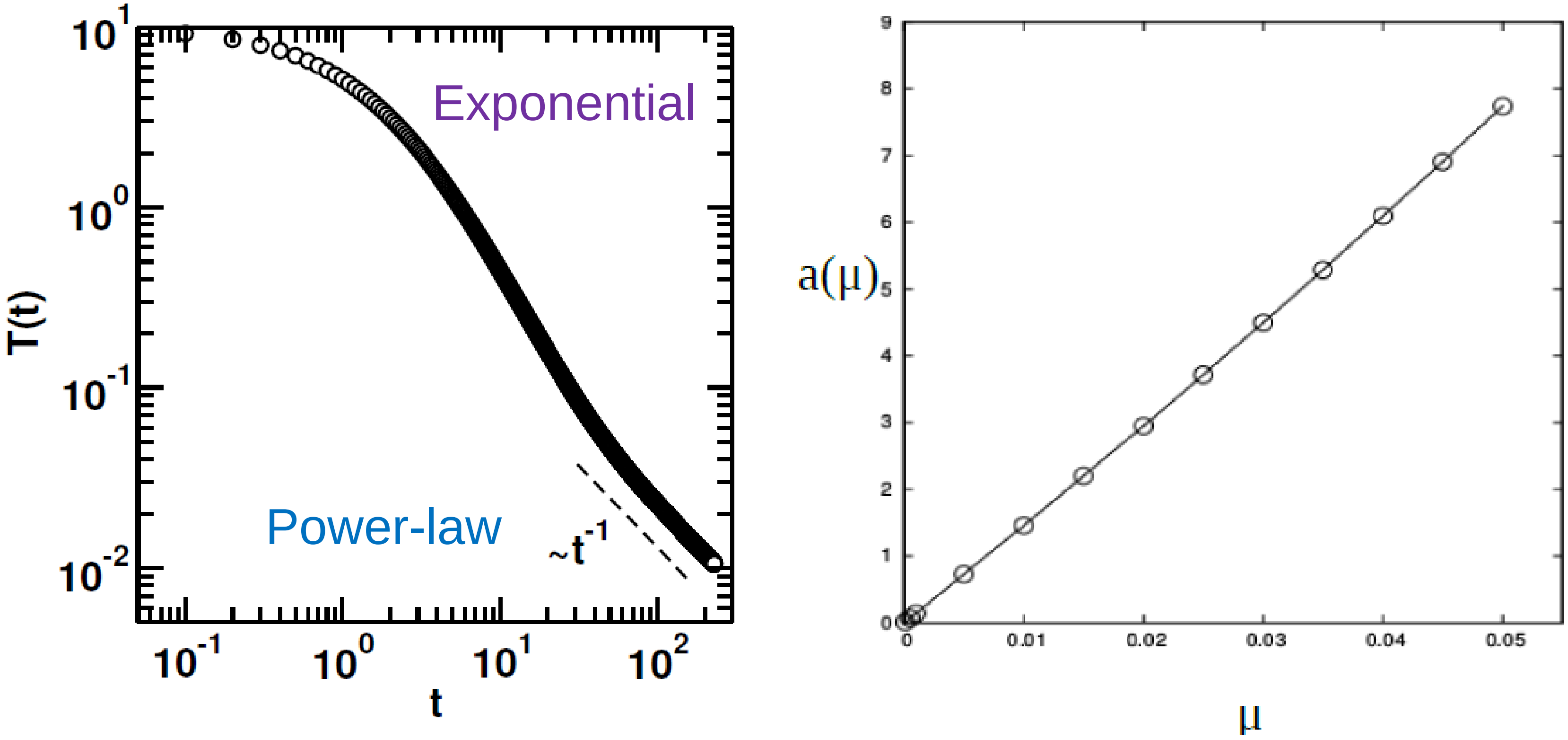


Density field

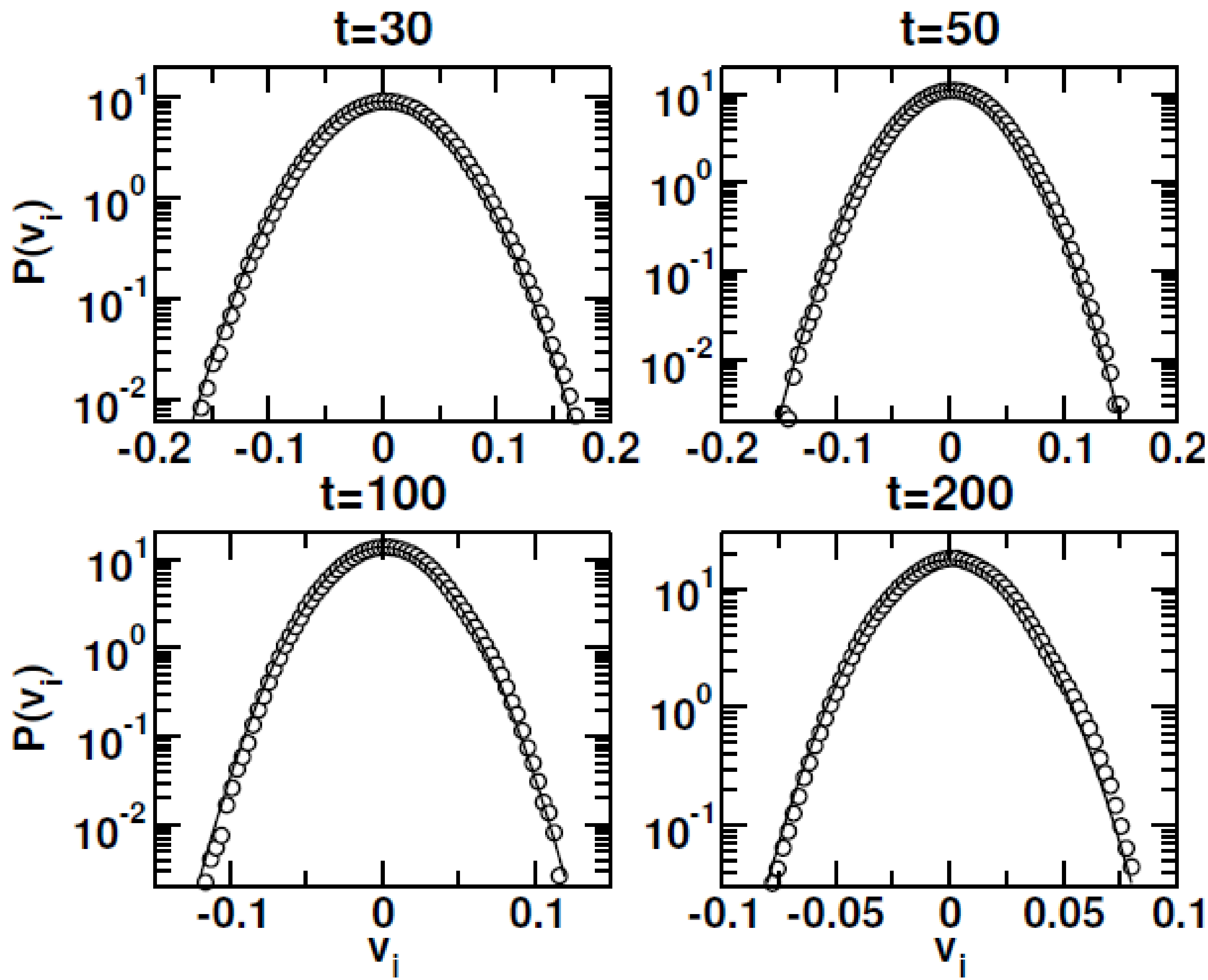


Velocity field

Frictional cooling

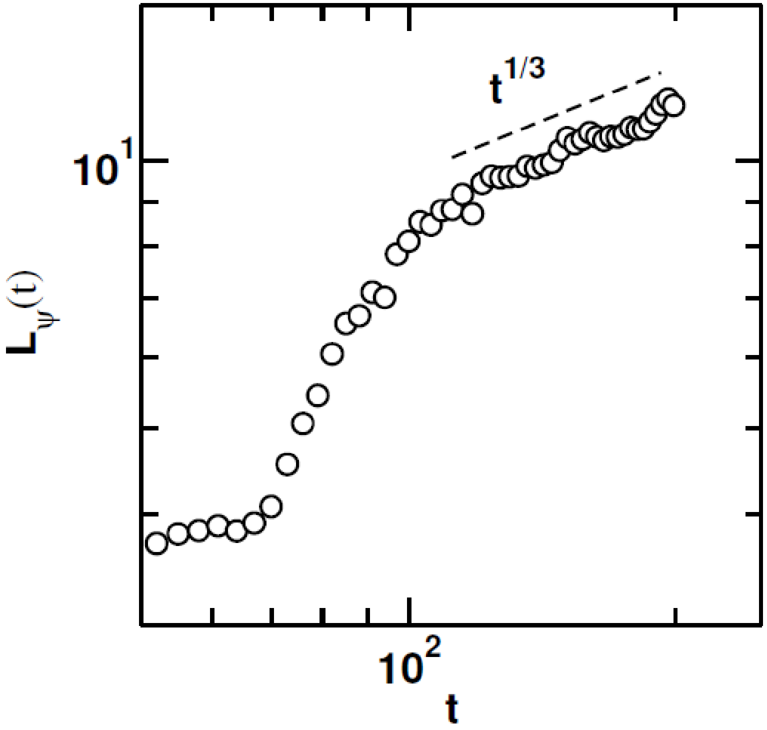


Velocity distributions

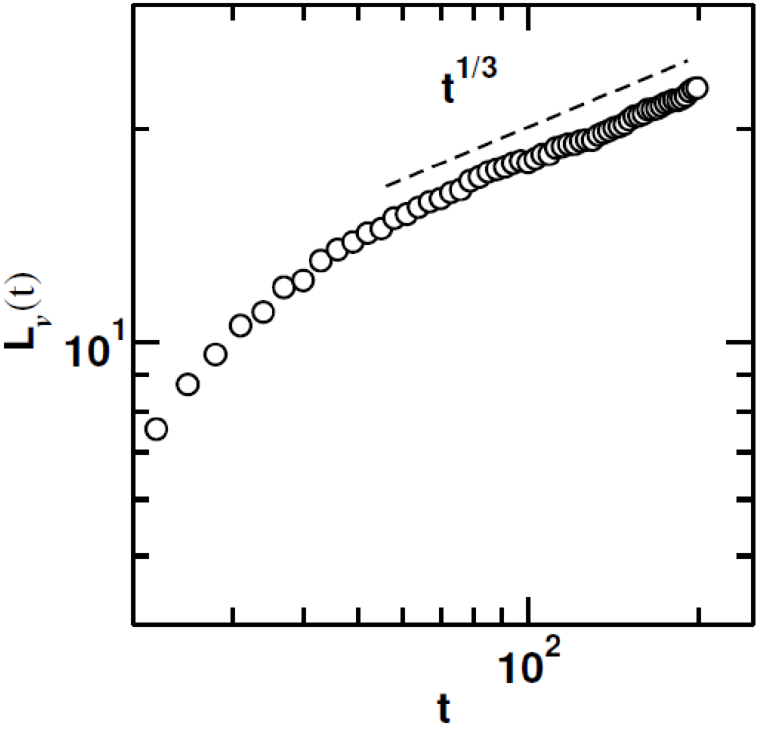


Time-dependence of length scale

Density field



Velocity field



(e) Conclusion

- Granular gases with inelastic collisions:

  Velocity distributions are non-Maxwellian in the HCS and can be obtained from the inelastic Boltzmann equation. We understand these for cases with and without heating. There is a crossover from the HCS to the ICS.

- Granular gases with friction:

  Similar phenomenology to case with inelastic collisions. Growth exponents are different.

- Important to introduce space-dependent heating for granular gases, e.g., from vibrating plate or rotating drum.

- Different set of tools needed to deal with dense granular fluids with multiple-grain collisions, prolonged contact, plugs, etc.

  P.Das, S. Puri and M. Schwartz (2015).