Summer Research Program on
Dynamics of Complex Systems
Discussion Meeting 21-23 July 2016

The global modeling technique

Background & Applications

Sylvain MANGIAROTTI
Researcher at IRD

UMR 5126
(UPS-CNES-CNRS-IRD)

Theory of nonlinear dynamical systems

- Henri Poincaré

probably the first one to understand

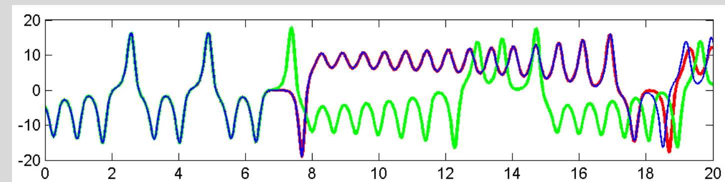
how a system can be both

Deterministic & unpredictable at long term

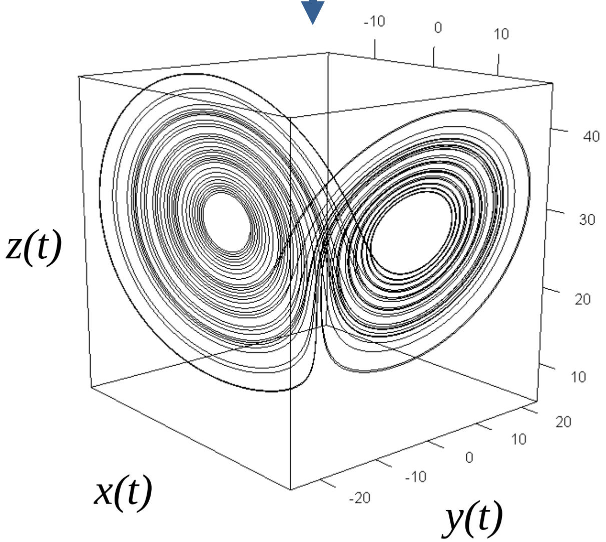
Lorenz system (1963)

$$
\begin{cases}
\dot{x} = \sigma\left(y - x\right) \\
\dot{y} = -xz + \rho x - y \\
\dot{z} = xy - \beta z
\end{cases}
$$

$x(t)$



$x(t),\ y(t),\ z(t)$



The *Phase Space* (or State space) :

a space that provides the complete solutions

Theory of nonlinear dynamical systems

- Henri Poincaré

- E.N. Lorenz-1963, 1st chaotic system

Simple systems, unpredictable at long term

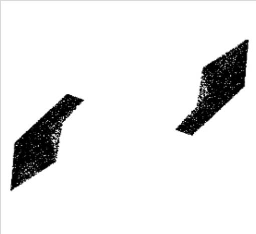
Followed by Rössler-76, Rössler-79 .. Malasoma-2000

E. Lorenz

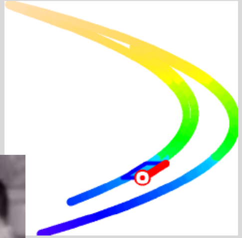
O. Rössler

J-M Malasoma

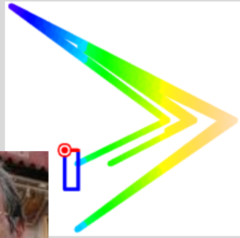
Discrete systems: Mira-69, Hénon-76, Lozi-78, Rössler-79



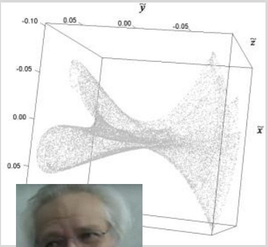
C. Mira



M. Hénon



R. Lozi



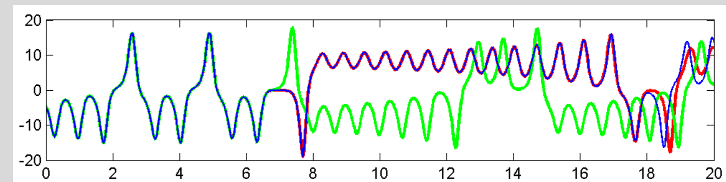
Theoretical background

F. Takens

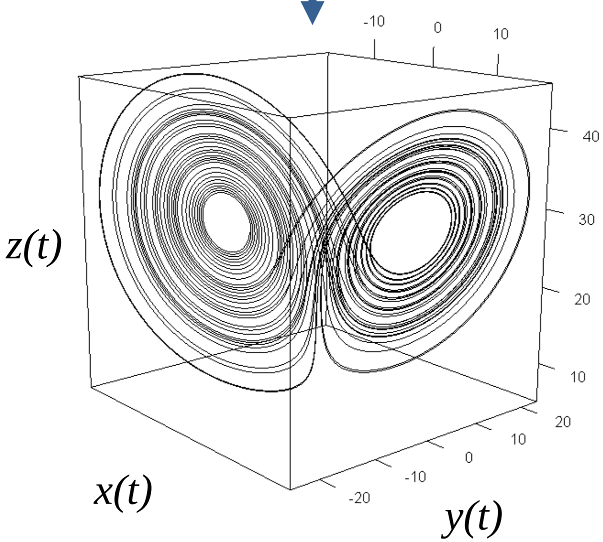
Takens Theorem (1981)

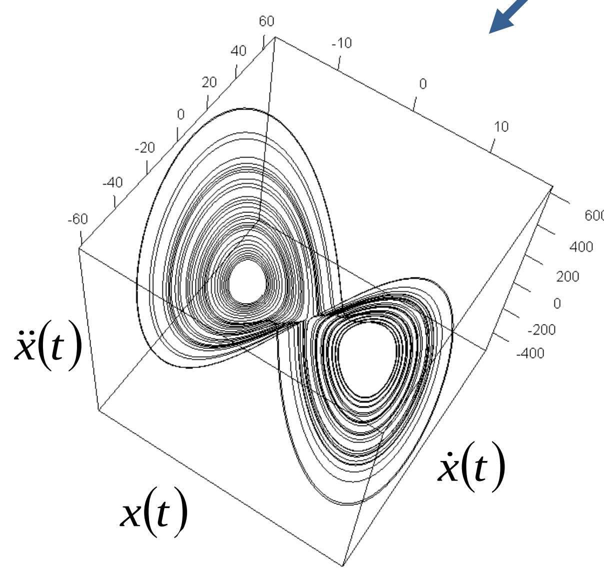
$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = -xz + \rho x - y \\ \dot{z} = xy - \beta z \end{cases}$$

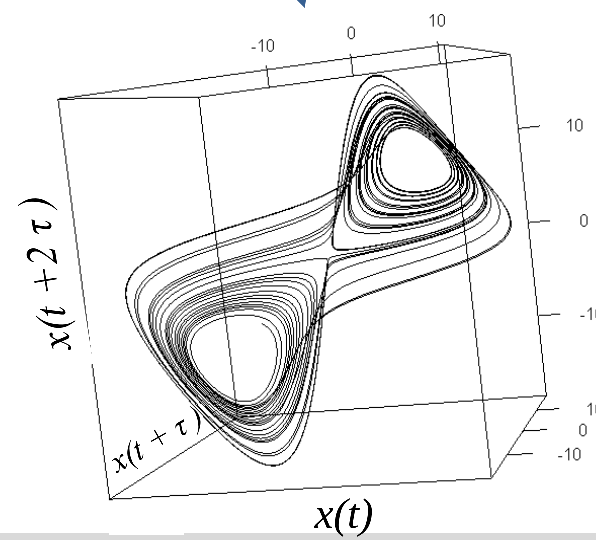
$x(t)$



$x(t),\ y(t),\ z(t)$







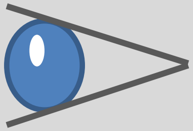
Nonlinear invariants are conserved

Global modeling

- multivariate

x_i is observed

$$\begin{cases} \dot{x}_1 = f_1\left(x_1, x_2, \ldots, x_n\right) \\ \dot{x}_2 = f_2\left(x_1, x_2, \ldots, x_n\right) \\ \ldots \\ \dot{x}_n = f_n\left(x_1, x_2, \ldots, x_n\right) \end{cases}$$



?

Equations from one single observed variable?

J. Crutchfield

Crutchfield & McNamara (1987)

Global modeling

G. Gouesbet

Gouesbet (1991)

- multivariate

$$\begin{cases} \dot{x}_1 = f_1\left(x_1, x_2, \ldots, x_n\right) \\ \dot{x}_2 = f_2\left(x_1, x_2, \ldots, x_n\right) \\ \ldots \\ \dot{x}_n = f_n\left(x_1, x_2, \ldots, x_n\right) \end{cases}$$

x_i is observed

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \ldots \\ \dot{X}_n = F\left(x_i, X_2, \ldots, X_n\right) \end{cases}$$

Global modeling

G. Gouesbet

C. Letellier

Gouesbet & Letellier (1994)

- multivariate

$$\begin{cases} \dot{x}_1 = f_1\left(x_1, x_2, \ldots, x_n\right) \\ \dot{x}_2 = f_2\left(x_1, x_2, \ldots, x_n\right) \\ \ldots \\ \dot{x}_n = f_n\left(x_1, x_2, \ldots, x_n\right) \end{cases}$$

x_i is observed

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \ldots \\ \dot{X}_n = F\left(x_i, X_2, \ldots, X_n\right) \end{cases}$$

Polynomial approximation

$$F\left(x_i, X_2, \ldots, X_n\right) = P\left(x_i, X_2, \ldots, X_n\right)$$

Good results for some variables of some systems / but not all …

Global modeling

C. Letellier

L. Aguirre

- multivariate

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, .., x_n) \\ \dot{x}_2 = f_2(x_1, x_2, .., x_n) \\ \dots \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) \end{cases}$$

« Problem » of observability

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \dots \\ \dot{X}_n = F(x_i, X_2, .., X_n) \end{cases}$$

Letellier & Aguirre (2001)

Global modeling

C. Letellier

L. Aguirre

- multivariate

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, .., x_n) \\ \dot{x}_2 = f_2(x_1, x_2, .., x_n) \\ \dots \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) \end{cases}$$

« Problem » of observability

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \dots \\ \dot{X}_n = F(x_i, X_2, .., X_n) \end{cases}$$

Letellier & Aguirre (2001)

Sophus Lie

Lie derivatives

$$\mathcal{L}_f f_i(\boldsymbol{x}) = \frac{\partial f_i(\boldsymbol{x})}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}) = \sum_{k=1}^{m} \frac{\partial f_i(\boldsymbol{x})}{\partial \boldsymbol{x}} f_k$$

Global modeling technique

- Empirical approach

  Few *a priori* knowledge required

  Directly applies to time series

- Well adapted to low-dimensional systems

  Can bring strong argument for determinism

  All the conditions and properties of chaos in a consistent fashion

- Strong and rich theoretical background

  Global solutions

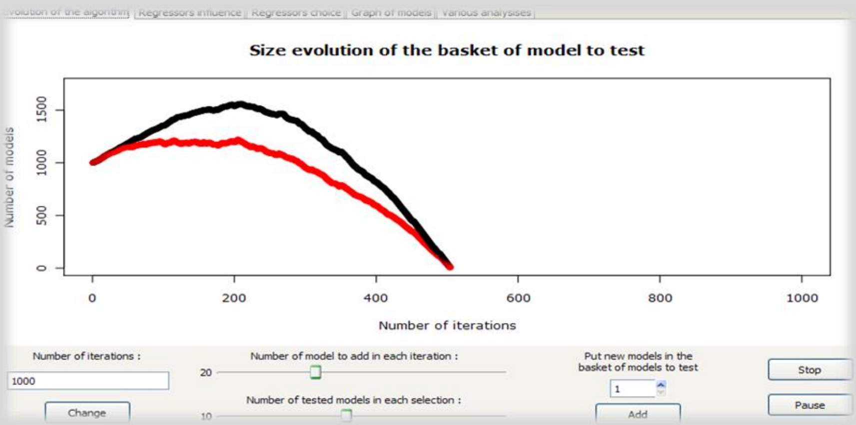
  Theory of nonlinear dynamical systems

  (Poincaré’s work, Takens’ Theorem, topology of chaos, etc.)

H. Poincaré

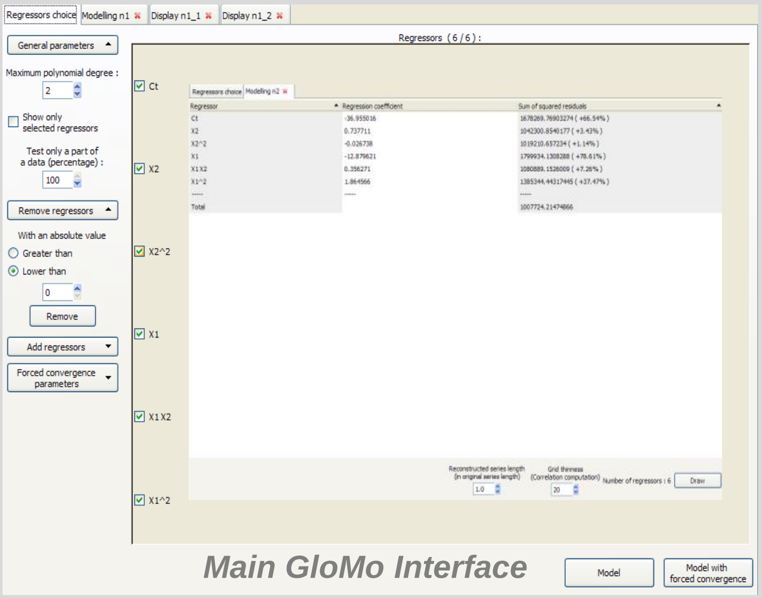
Algorithmic tools *R-packages*

PoMoS



Models selection algorithm interaction

GloMo



Main GloMo Interface

L. Drapeau
Ing. IRD

M. Huc
Ing. CNRS

R. Coudret

F. Le Jean

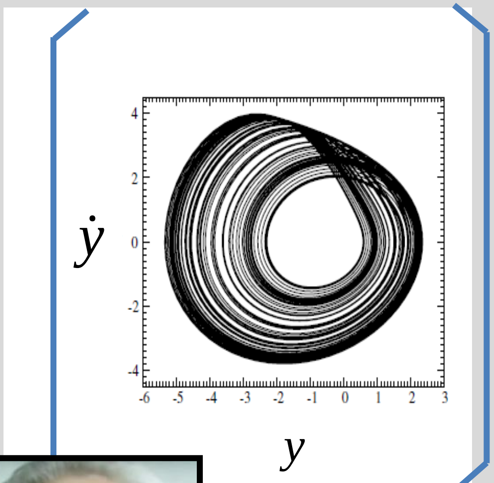
M. Chassan

Mangiarotti et al.
Phys. Rev. E (2012)

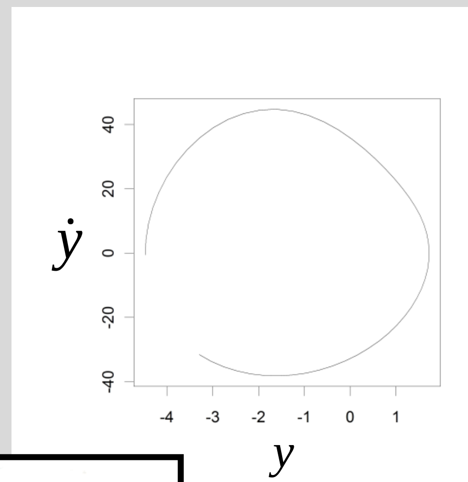
GPoM platform, the potential of the approach

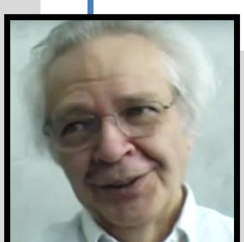
Rössler-y

Original system



Unstable periodic orbits (e.g. period 1)





O. Rössler

Letellier, Mangiarotti & Aguirre (under revision)

Letellier et al., Entropie 1997

GPoM platform, the potential of the approach

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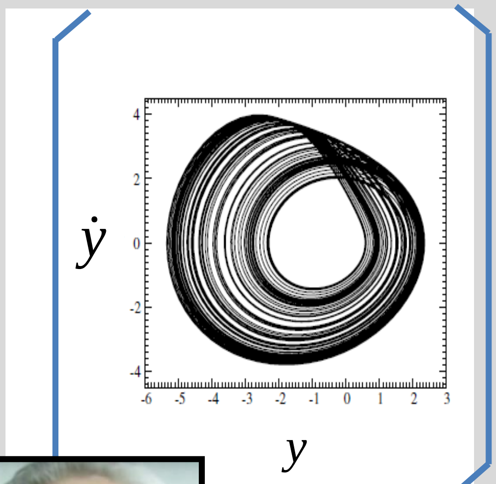
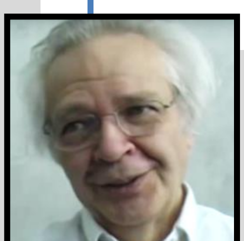
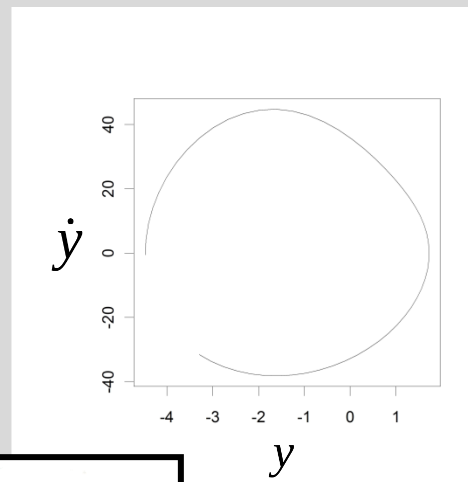
Original system



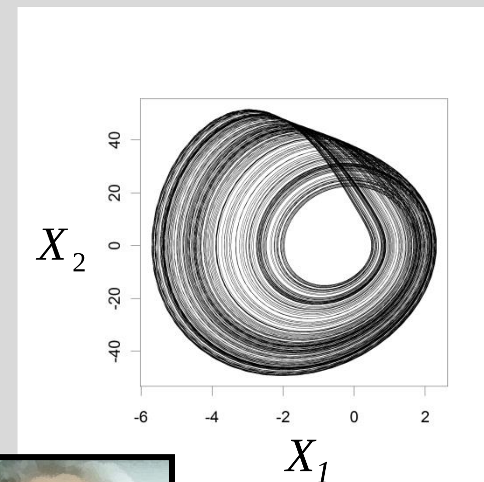


O. Rössler

Unstable periodic orbits (e.g. period 1)



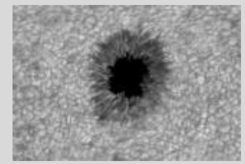
Global model



Letellier, Mangiarotti & Aguirre (under revision)
Letellier et al., Entropie 1997

Global modeling applied to real observations univariate

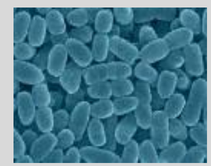
NARMA



Aguirre et al. 2008

- sun • sunspot cycles
- epidemiology • whooping caugh

NARMA



Boudjema et Cazelles 2001

- ecological • Canadian lynx

ODE

Maquet et al. 2007

- agricultural • cereal crops

ODE

Mangiarotti et al.
2011 RNL; 2014 *Chaos*

- climatic • snow cycles

.. systems

ODE

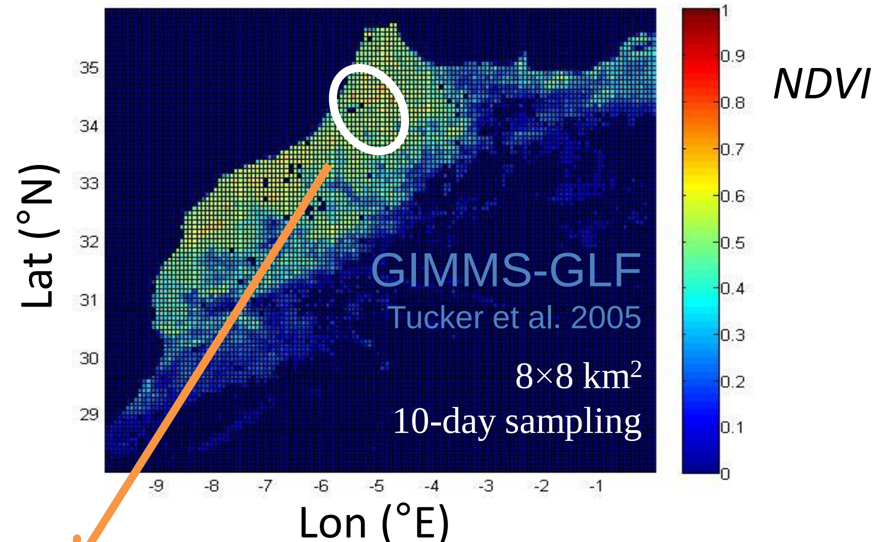
Mangiarotti 2014 *HDR*

Normalized Difference Vegetation Index

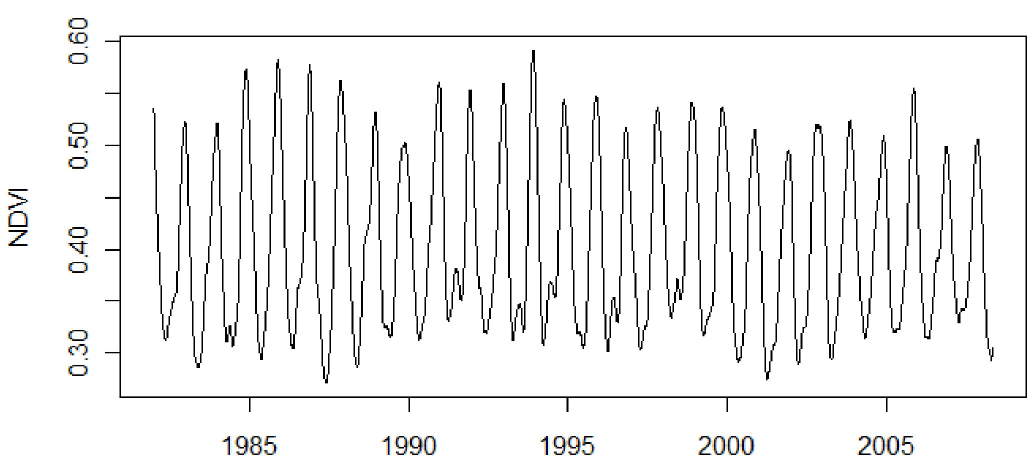
$$\mathrm{NDVI} = \frac{\rho_{PIR} - \rho_R}{\rho_{PIR} + \rho_R}$$

0.1 0.7

NDVI map (Morocco, February 2000)



Time series (North Morocco)

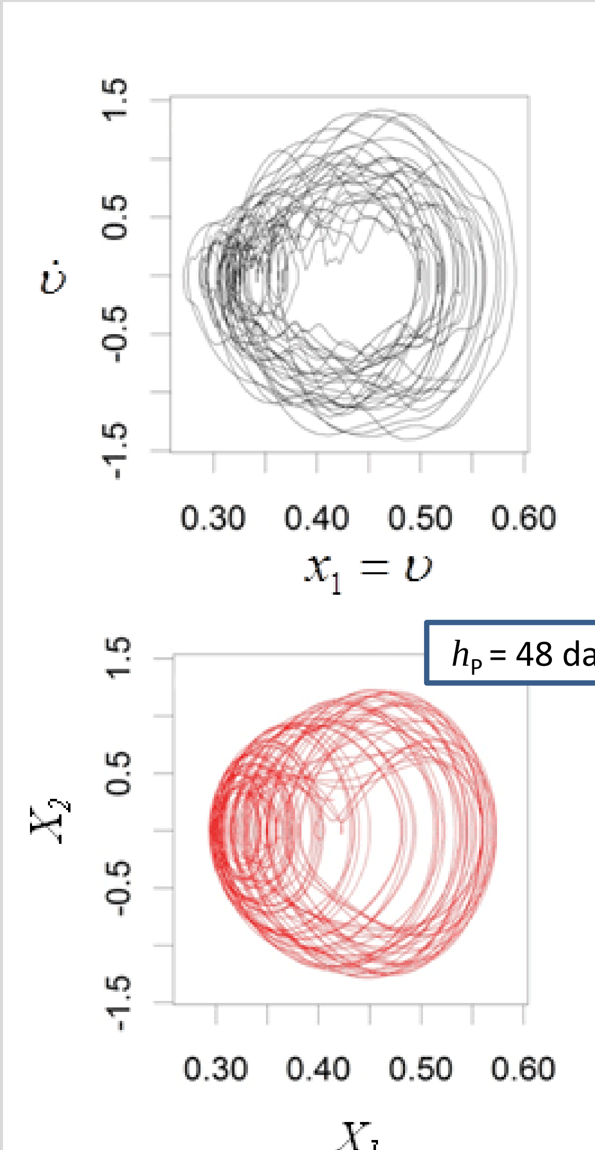


A global model for *cereal crops*

- Complex structure
- Very few previous cases
- Never directly obtained from data

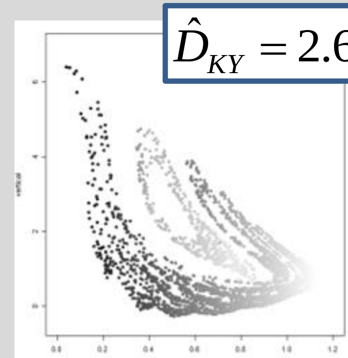
data

model



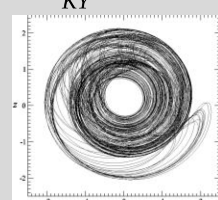
h_P = 48 days

Poincaré section



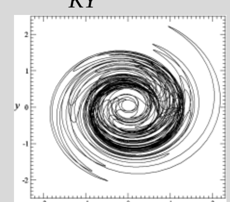
$\hat{D}_{KY} = 2.68 >> 2$

$\hat{D}_{KY} = 2.39$



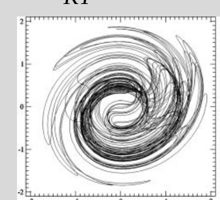
Lorenz-84

$\hat{D}_{KY} = 2.76$



Wieczorek 1999

$\hat{D}_{KY} = 2.54$



Chlouverakis 2002

Snow surface cycles

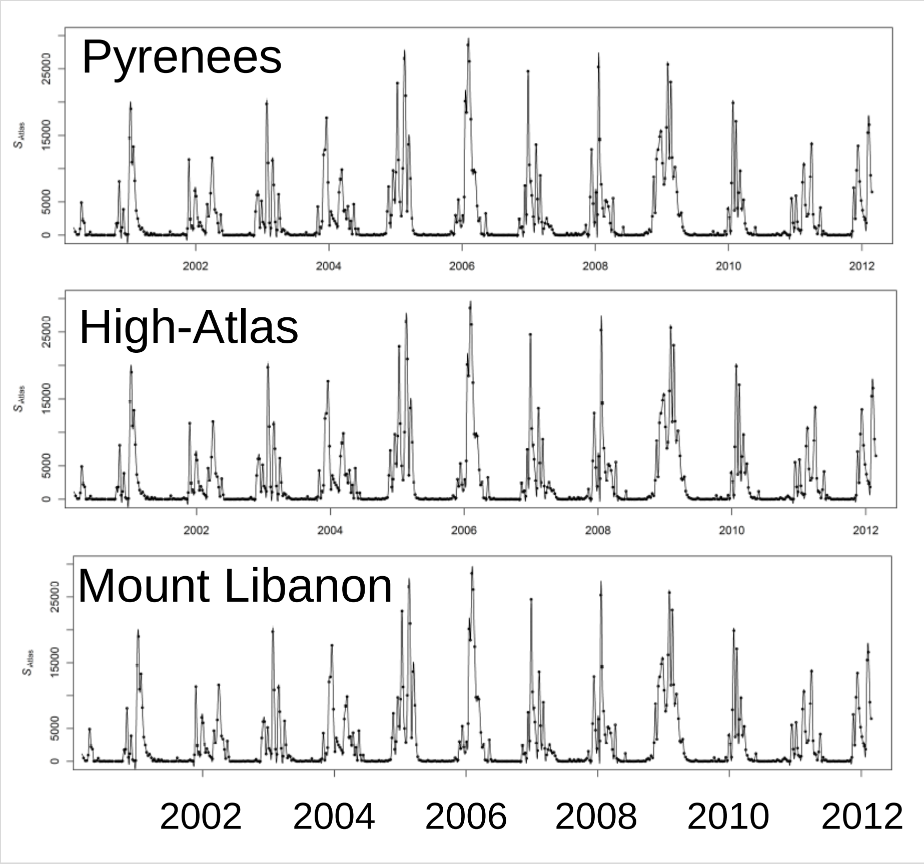
Snow surface estimated from satellite (NDSI)

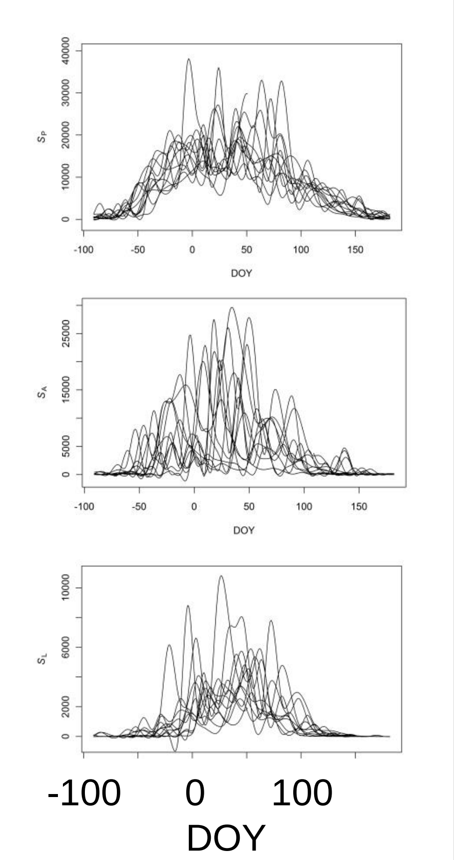
- daily MODIS product used: MOD10A1
- missing alues are interpolated (in time and space)

(Gascoin et al. *HESS* 2015)

S. Gascoin

L. Drapeau





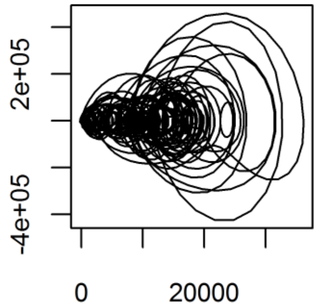
Snow surface cycles

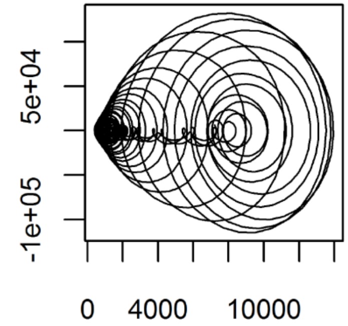
Original data (2000-2012)

Global Models

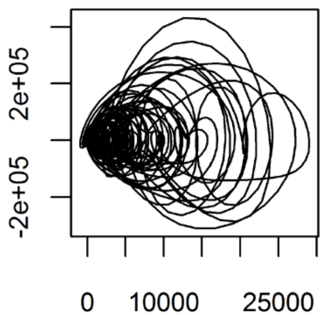
Phase portraits

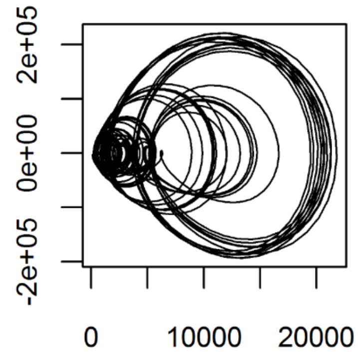
Pyrenees



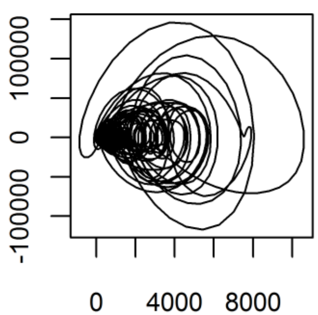


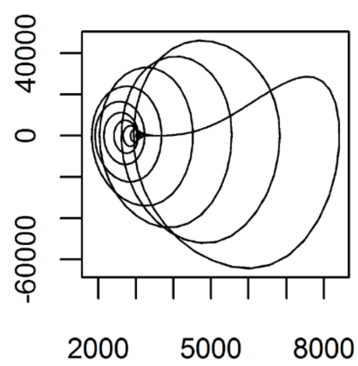
High-Atlas





Mount Libanon

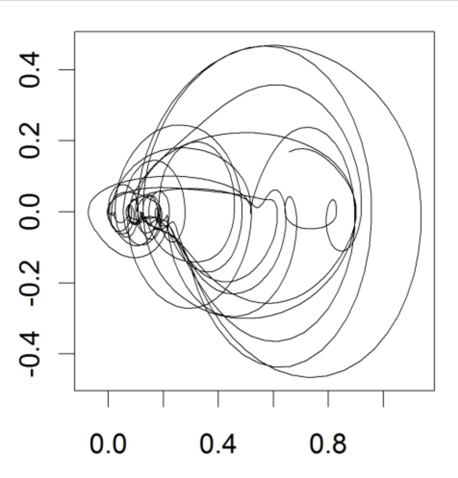




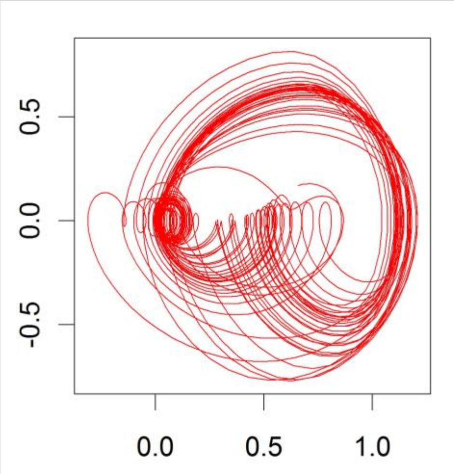
Autres analyses univariées

- Soil Evaporative Efficiency
  (thèse Vivien Stephan, Dir. O. Merlin)

Original data
(from TEC model)



4D Global Model



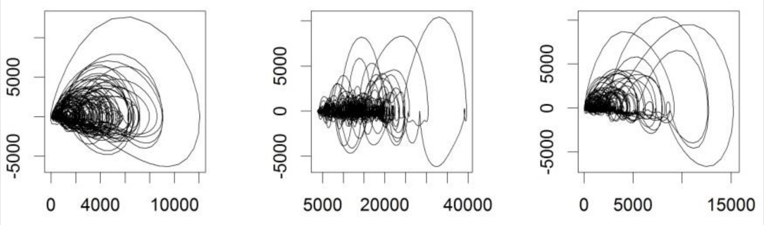
Autres analyses univariées

le Doubs

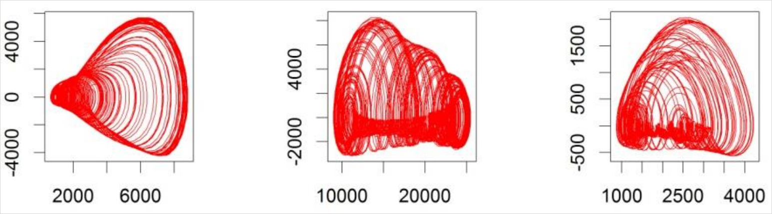
la Touvre

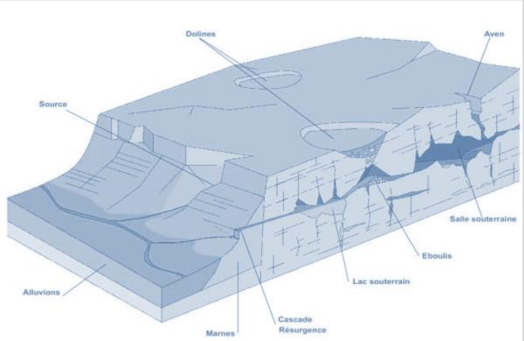
le Lez

Original data



Global models





M. Leblanc

Y. Zhang

- The discharge of karstic springs (in France)

data HYDRO (http ://www.hydro.eaufrance.fr/)

Global modeling applied to real observations multivariate

- biomedical
- eco-epidemiology

• human breath

NARMA

Letellier et al. 2013

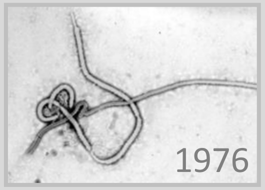
• bubonic plague (1896-1911)

ODE

Mangiarotti 2015

• Ebola Virus Disease (2014-2016)

ODE



Huc & Mangiarotti 2016

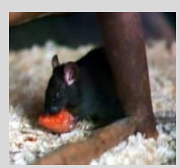
Multivariate analysis

Projet MoMu
2015-2016
(AO **LEFE-Manu**)

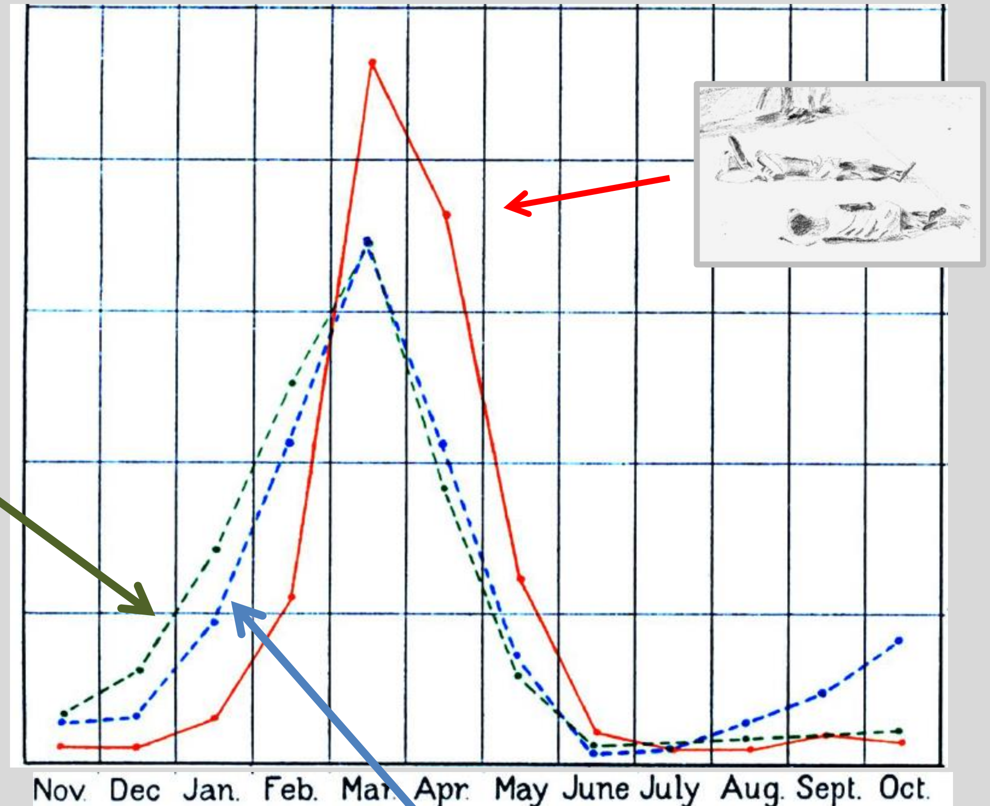
Multivariate global modeling

?

$$\begin{cases} \dot{x}_1 = P_1(x_1, x_2, x_3) \\ \dot{x}_2 = P_2(x_1, x_2, x_3) \\ \dot{x}_3 = P_3(x_1, x_2, x_3) \end{cases}$$



Prevalence of plague (in percent) 1906-07



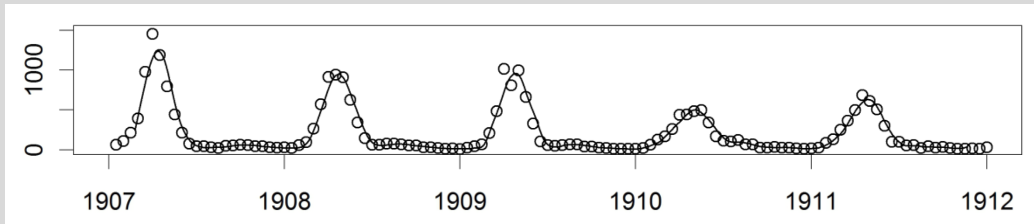
Human plague

Plague in M. decumanus

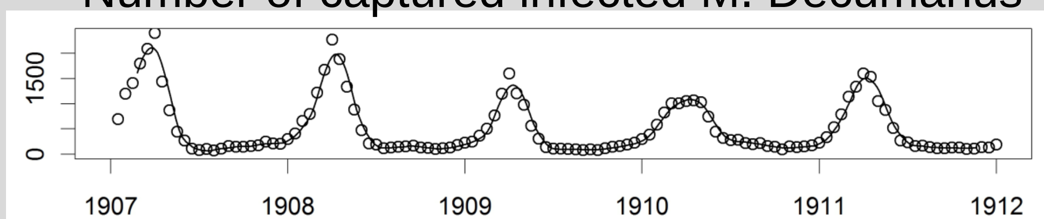
Plague in M. rattus

Reproduced from
« Report on Plague investigations in India »
J. of Hygienne 8(2) 1908

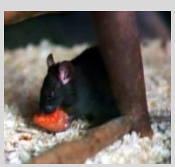
Plague death per half-month

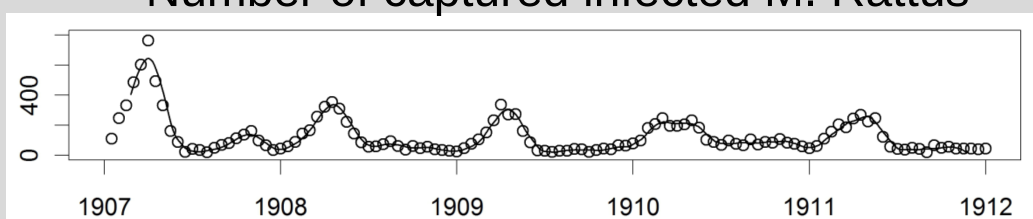


Number of captured infected M. Decumanus



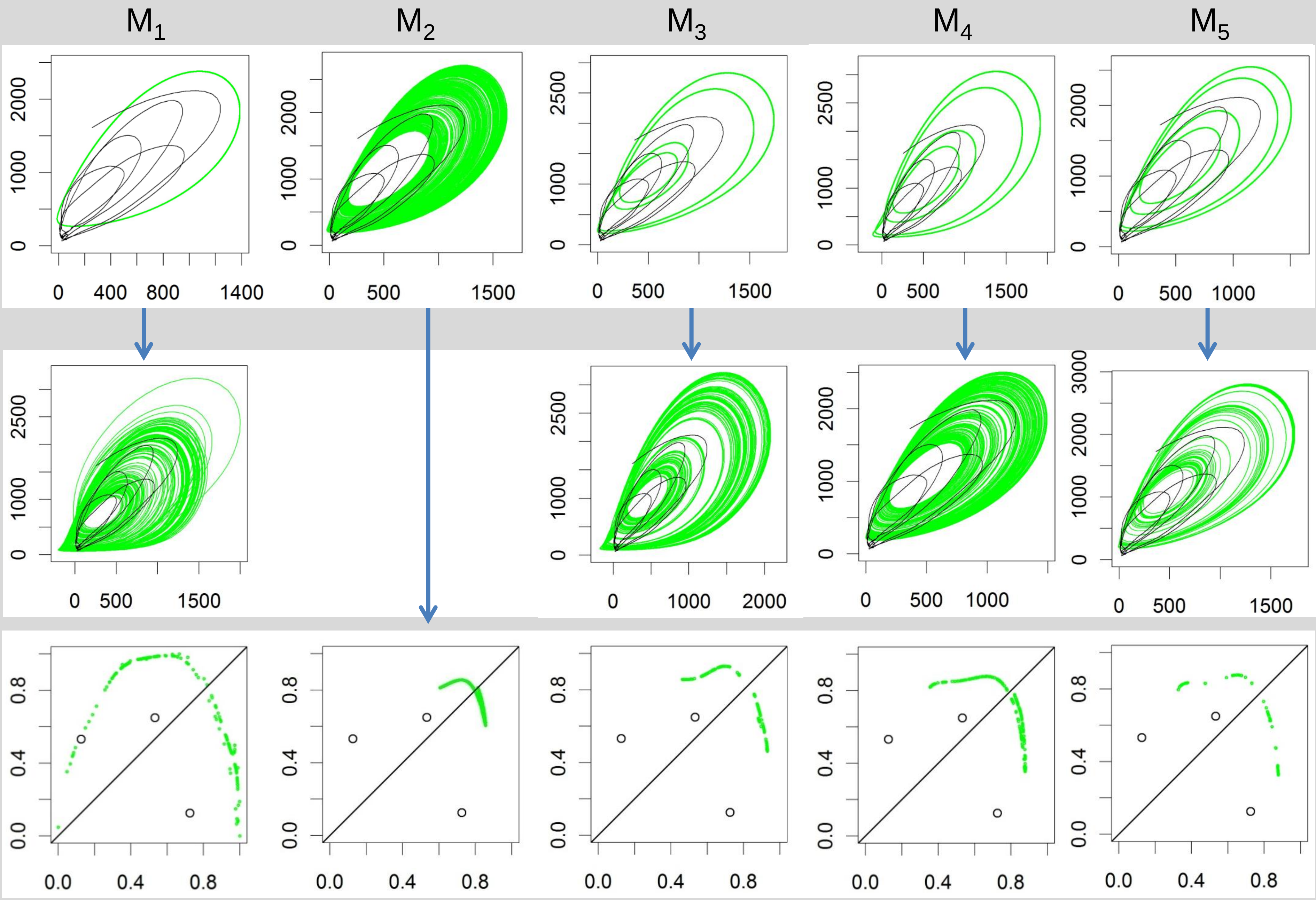
Number of captured infected M. Rattus



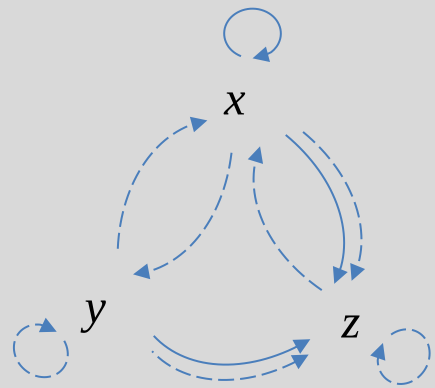


Original data from The Advisory Committee
"Reports on plague investigation in India", J. Hyg. 12 (1912).

Multivariate analysis



Multivariate analysis

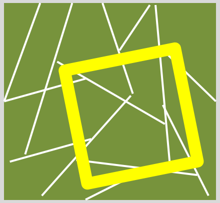


$$
\begin{cases}
\dot{x} = a_1 \, z(y - \alpha \, z) - a_3 \, x \\
\dot{y} = b_1 \, y^2 - b_2 \, xy \\
\dot{z} = c_1 z^2 + c_2 \, y - c_3 \, xz + c_5 \, x(y - \beta)
\end{cases}
$$

- Multivariate ODE directly obtained from observed time series
- All the terms could be interpreted
- A good tool for coupling detection

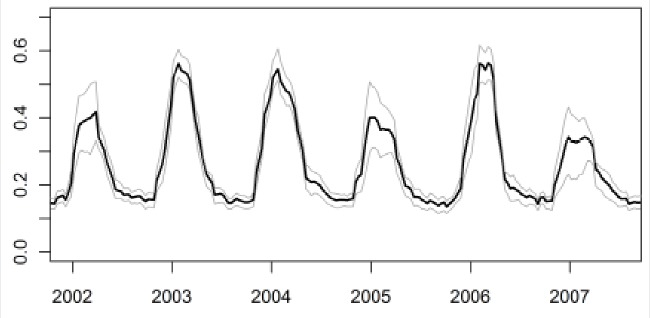
Mangiarotti CSFrac 2015

Aggregated signal



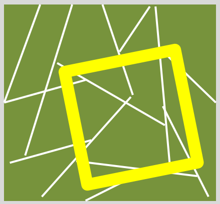
Phase synchronisation

AVHRR data (8km, aggr. 100km)



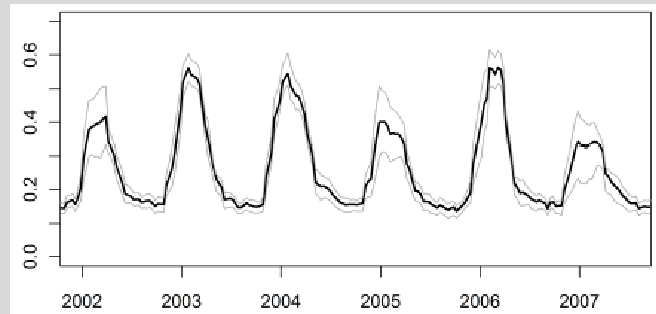
Safi province (Morocco)

Aggregated signal

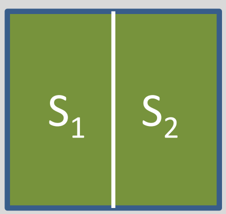


Phase synchronisation

AVHRR data (8km, aggr. 100km)



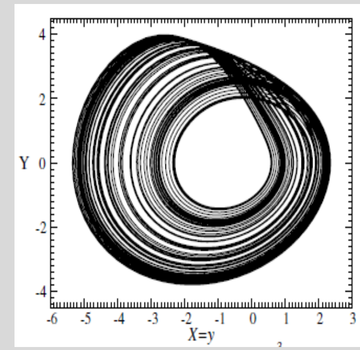
Safi province (Morocco)



What can be the effect of aggregation?

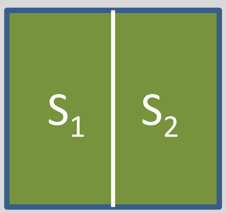
Analytical point of vue

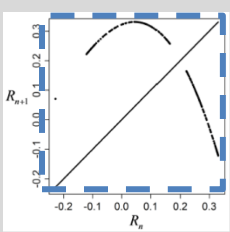
$$\begin{cases} \dfrac{dx}{dt} = -y - z \\ \dfrac{dy}{dt} = x + ay \\ \dfrac{dz}{dt} = b + z(x - c) \end{cases}$$



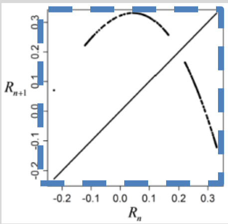
O. Rössler

Rössler-y





Analytical point of vue

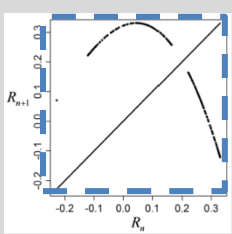


Rössler S

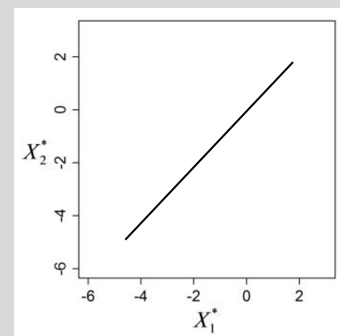
synchro.-y
full

Rössler-y

S_1 | S_2

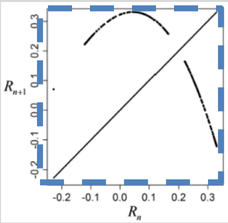


Cross portrait



$$y_k = y$$

Analytical point of vue

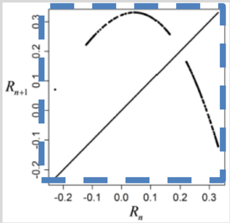


Rössler S

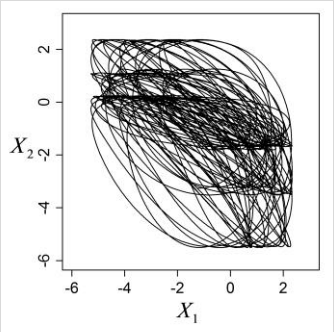
synchro.-*y*
no

Rössler-*y*

S_1 | S_2

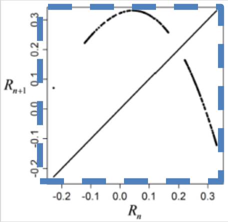


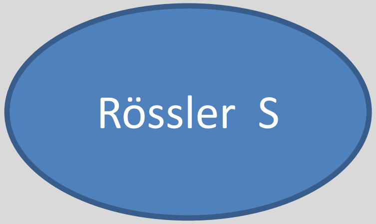
Cross portrait



$$y_k = \beta_k(t)\, y$$

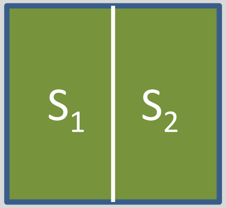
Analytical point of vue

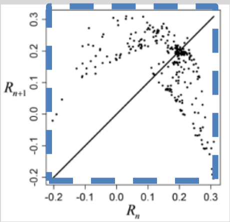




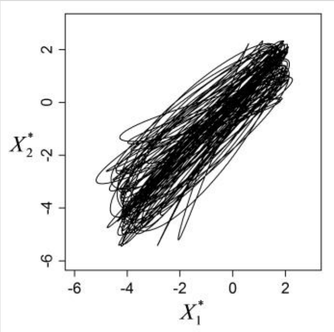
synchro.-y
in phase

Rössler-y



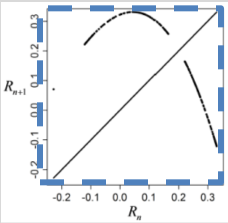


Cross portrait



$$y_k \approx \beta_k \; y$$ locally

Analytical point of vue



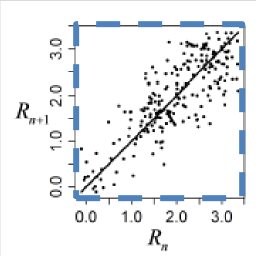
Rössler S

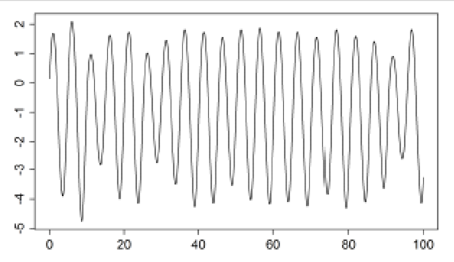
synchro.-y
in phase

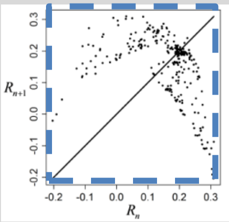
Rössler-y

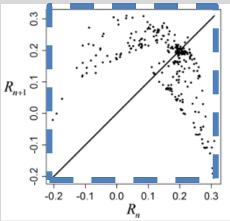
S_1 S_2

aggregation



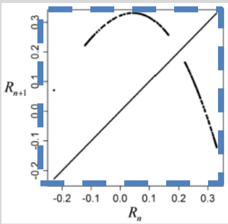






Aggregated system ?

Analytical point of vue



Rössler S

synchro.-*y*
in phase

Rössler-*y*

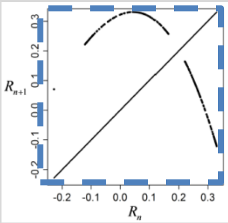
S_1 S_2

aggregation

?

$$S_k \quad \begin{cases} \dot{x}_k = -y_k - z_k \\ \dot{y}_k = x_k + a_k y_k + \alpha_k (y_k - y) \\ \dot{z}_k = b + z_k (x_k - c) \end{cases}$$

Analytical point of vue



Rössler S

synchro.-y
in phase

Rössler-y

S_1 S_2

aggregation

?

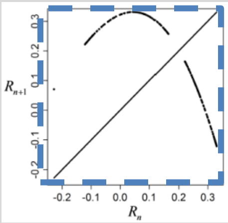
$$\approx \begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = 2b + z[(1-\gamma)x - c] \end{cases}$$

$$S_k \begin{cases} \dot{x}_k = -y_k - z_k \\ \dot{y}_k = x_k + a_k y_k + \alpha_k (y_k - y) \\ \dot{z}_k = b + z_k (x_k - c) \end{cases}$$

$\Longrightarrow$ décalage dans le diagramme de bifurcation

$\Longrightarrow$ seuls les invariants topologiques peuvent être conservés

Analytical point of vue



Rössler S

synchro.-*y*
in phase

Rössler-*y*

S_1 S_2

aggregation

?

$$\approx \begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = 2b + z[(1-\gamma)x - c] \end{cases}$$

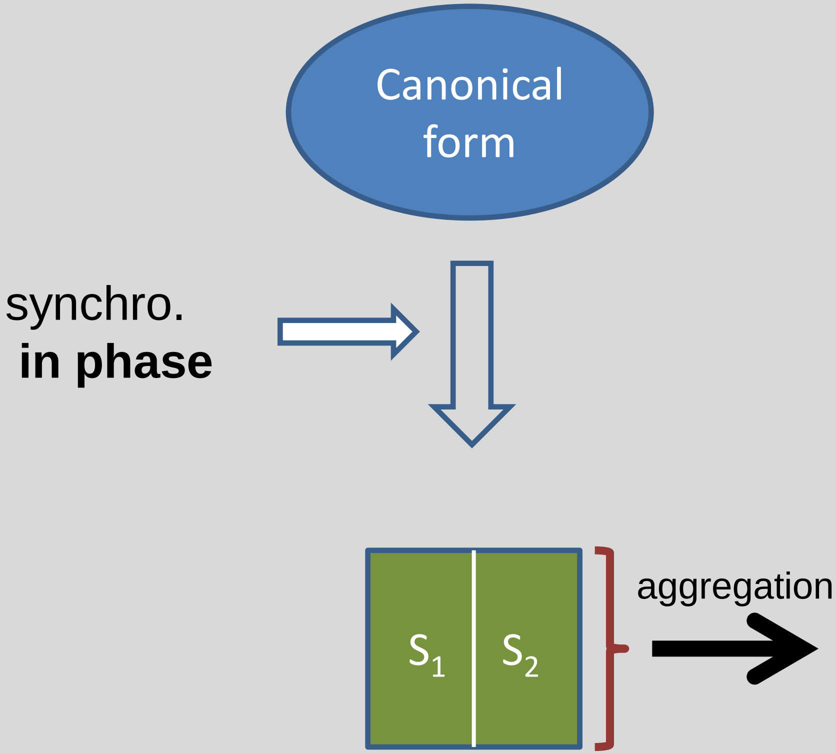
$\Rightarrow$ décalage dans le diagramme de bifurcation

$\Rightarrow$ seuls les invariants topologiques peuvent être conservés

Can be generalized?

$$S_k \begin{cases} \dot{x}_k = -y_k - z_k \\ \dot{y}_k = x_k + a_k y_k + \alpha_k(y_k - y) \\ \dot{z}_k = b + z_k(x_k - c) \end{cases}$$

Analytical point of vue



Can be generalized?

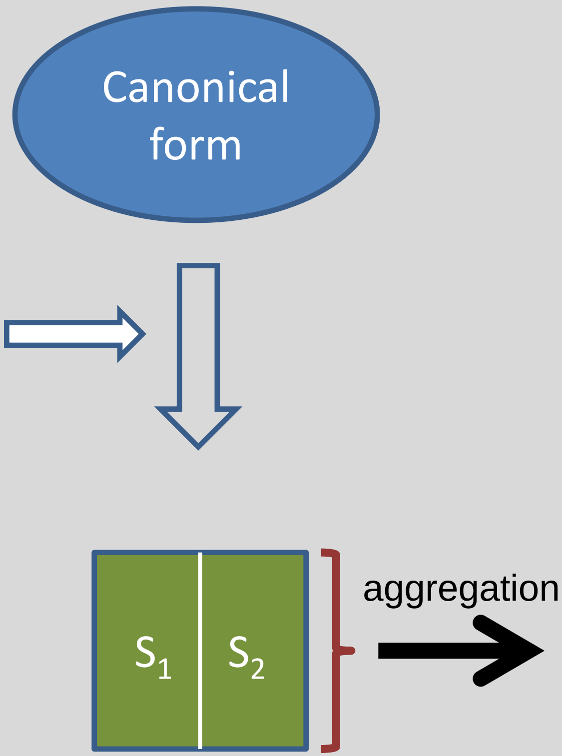
S_k

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = X_3 \\ \dot{X}_n = F(X_1, X_2, .., X_n) \end{cases}$$

$$F(X,Y,Z) = \alpha_{000} + \sum_{i+j+k=1} \alpha_{ijk} X^i Y^j Z^k + \sum_{2 \leq i+j+k \leq q} \alpha_{ijk} X^i Y^j Z^k$$

Analytical point of vue

synchro.
in phase



Can be generalized?

S_k

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = X_3 \\ \dot{X}_n = F(X_1, X_2, \ldots, X_n) \end{cases}$$

$$\begin{cases} \dot{X} = Y \\ \dot{Y} = Z \\ \dot{Z} = 2\alpha_{000} + \sum_{i+j+k=1} \alpha_{ijk} X^i Y^j Z^k \\ \quad +(1-\nu_2) \sum_{i+j+k=2} \alpha_{ijk} X^i Y^j Z^k + \ldots \\ \quad +(1-\nu_n) \sum_{i+j+k=n} \alpha_{ijk} X^i Y^j Z^k + \ldots \\ \quad +(1-\nu_q) \sum_{i+j+k=q} \alpha_{ijk} X^i Y^j Z^k. \end{cases}$$

with $\nu_n = \sum_{i=2}^{n-1} r_i \beta^i / (1+\beta)^n$

$$F(X,Y,Z) = \alpha_{000} + \sum_{i+j+k=1} \alpha_{ijk} X^i Y^j Z^k + \sum_{2 \le i+j+k \le q} \alpha_{ijk} X^i Y^j Z^k$$

Analytical point of vue

Canonical form

synchro.
in phase

Can be generalized to
- N attractors
- n dimensions
- any polynomial form (of any degree q)

S_1 S_2

aggregation

$$\begin{cases} \dot{X} = Y \\ \dot{Y} = Z \\ \dot{Z} = 2\alpha_{000} + \sum_{i+j+k=1} \alpha_{ijk} X^i Y^j Z^k \\ \quad +(1-\nu_2) \sum_{i+j+k=2} \alpha_{ijk} X^i Y^j Z^k + \ldots \\ \quad +(1-\nu_n) \sum_{i+j+k=n} \alpha_{ijk} X^i Y^j Z^k + \ldots \\ \quad +(1-\nu_q) \sum_{i+j+k=q} \alpha_{ijk} X^i Y^j Z^k . \end{cases}$$

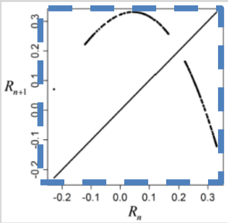
S_k

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = X_3 \\ \dot{X}_n = F(X_1, X_2, \ldots, X_n) \end{cases}$$

with $\nu_n = \sum_{i=2}^{n-1} r_i \beta^i / (1+\beta)^n$

$$F(X,Y,Z) = \alpha_{000} + \sum_{i+j+k=1} \alpha_{ijk} X^i Y^j Z^k + \sum_{2 \le i+j+k \le q} \alpha_{ijk} X^i Y^j Z^k$$

Practical point of vue



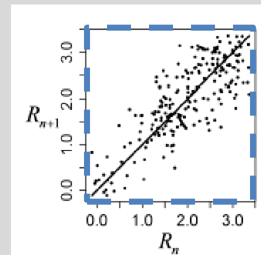
Rössler S

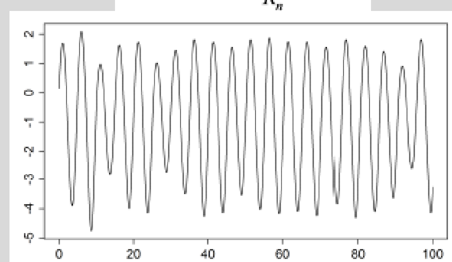
synchro.
in phase

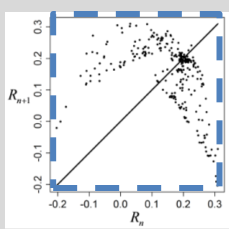
Rössler

S_1 … S_N

aggregation

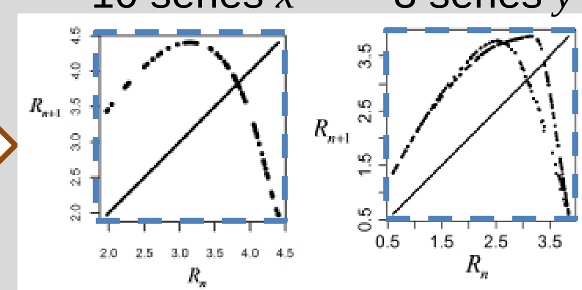






Global
modeling

10 series x 3 series y



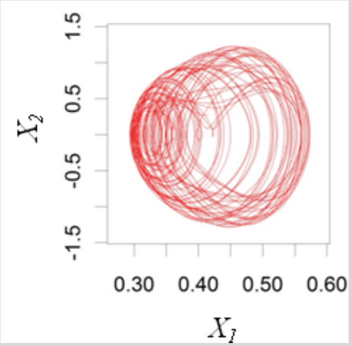
Good approx. of
the local dynamics!

Mangiarotti et al. CSFrac 2016

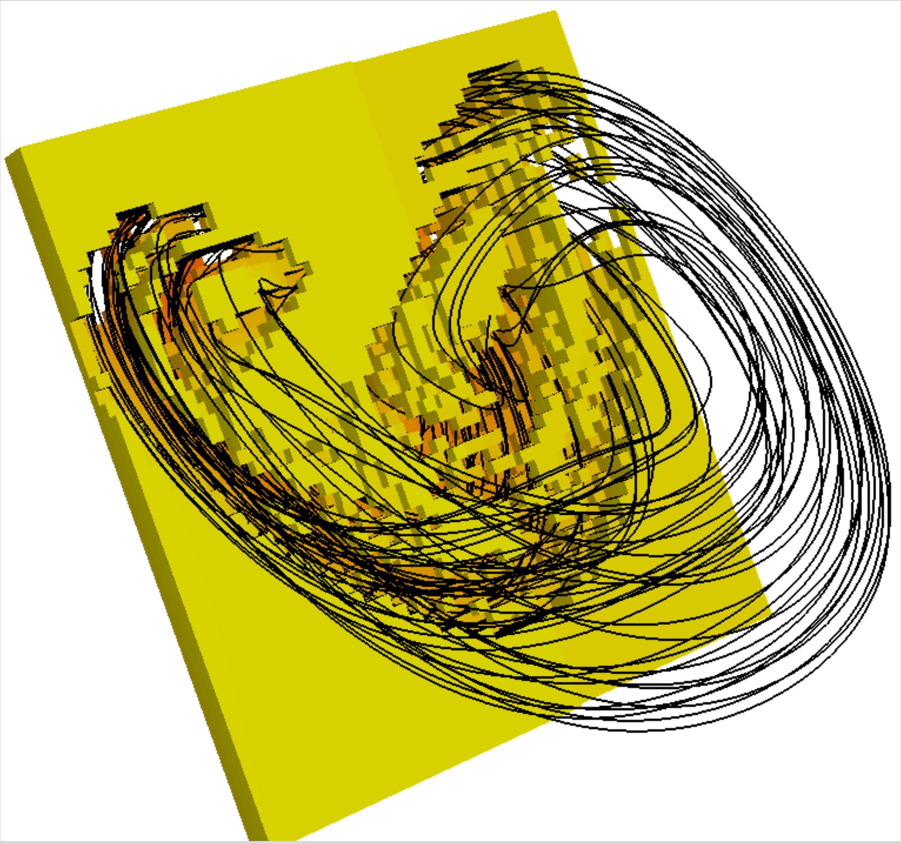
Topology of chaos

Phase portrait

model



Cereal crops attractor



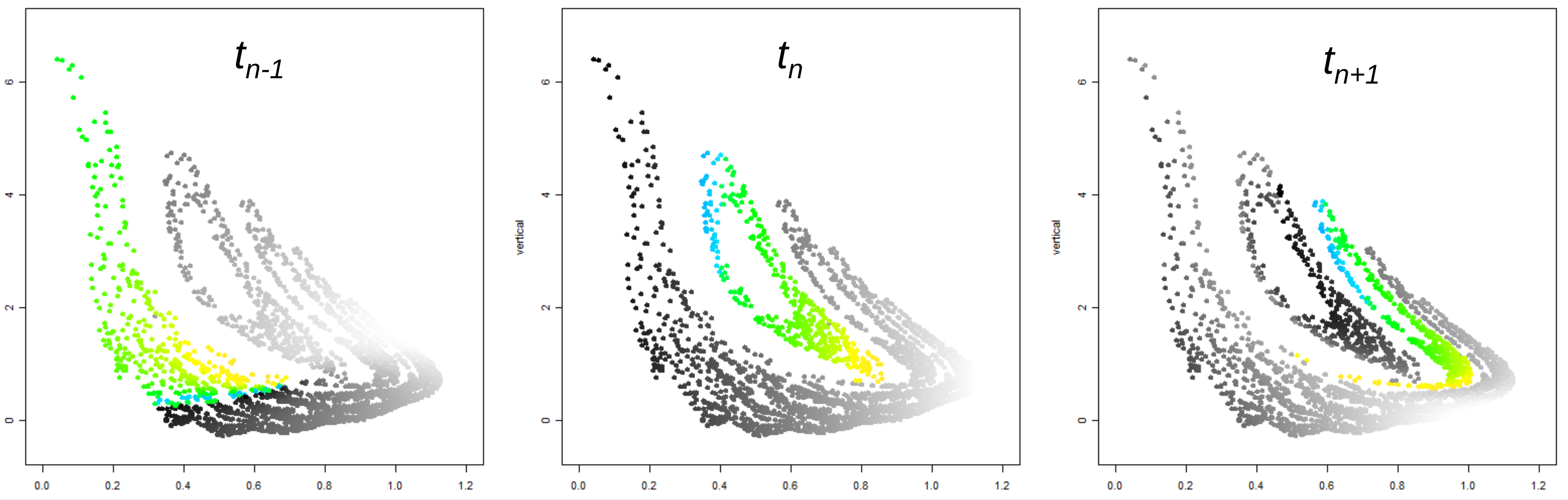
To understand the dynamics of a chaotic system, it requires to understand the organisation of the flow in the phase space

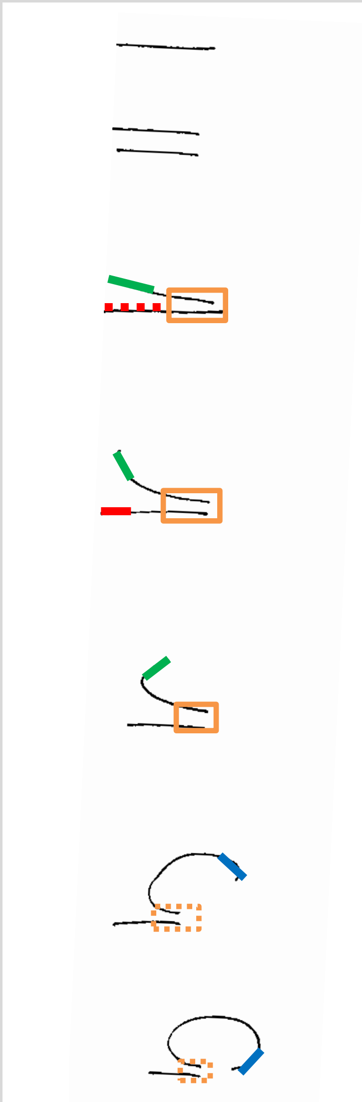
TOPOLOGY OF CHAOS

Poincaré section

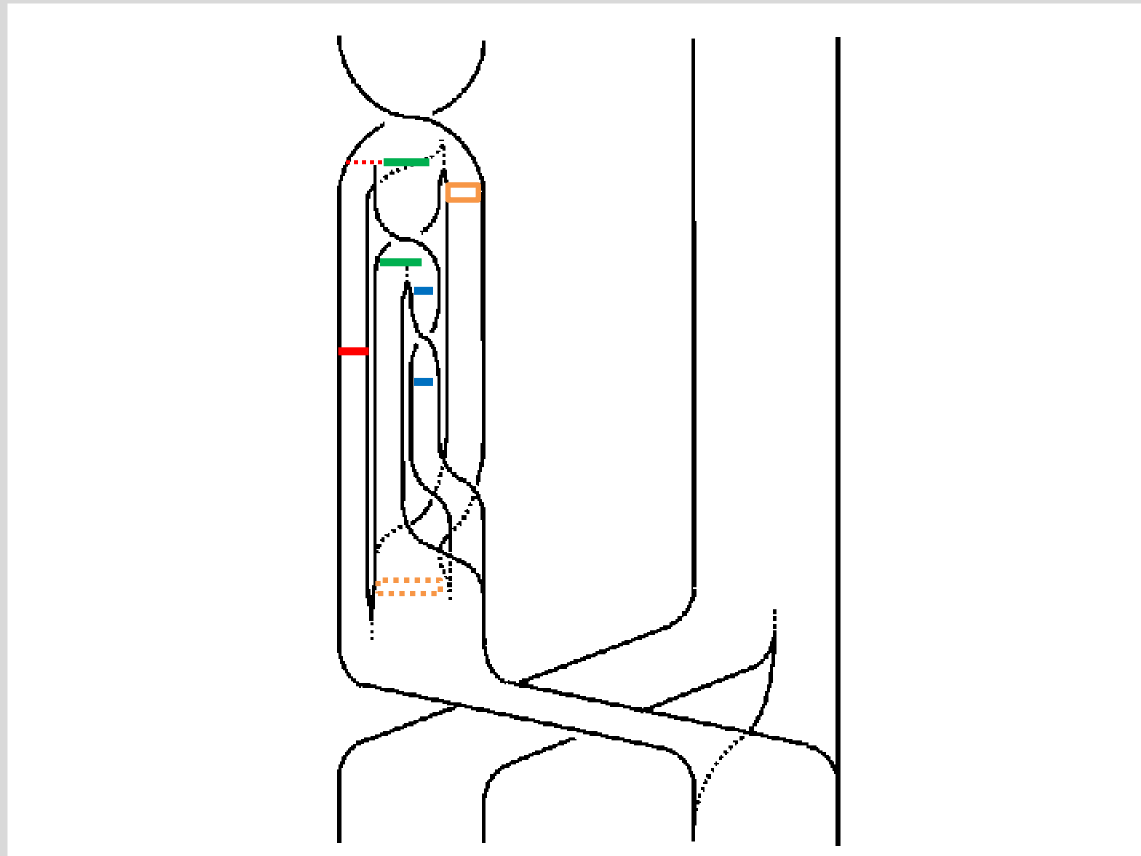
with color tracer

Back direction ⟵ ⟶ Forth direction





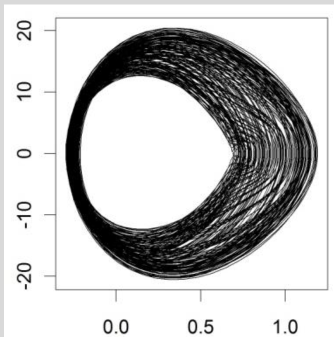
Template

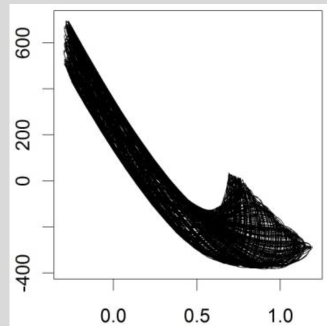


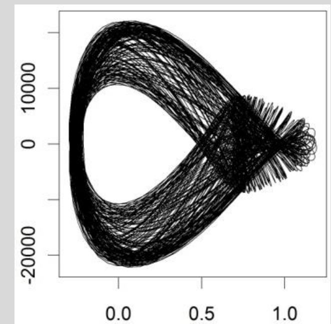
4D–models

Cereal crops (Al Ismailia)

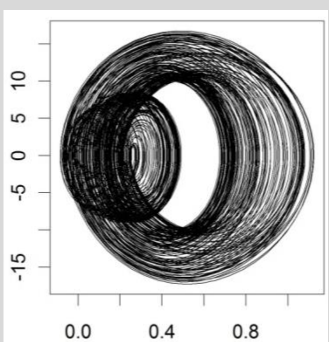
model 1

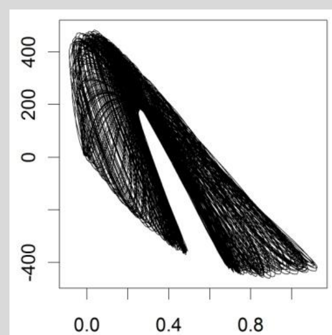


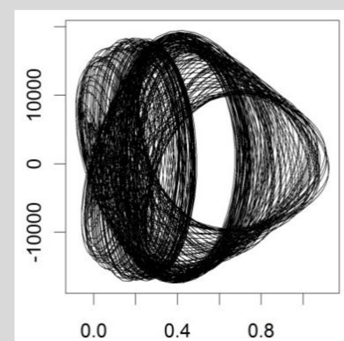




model 2

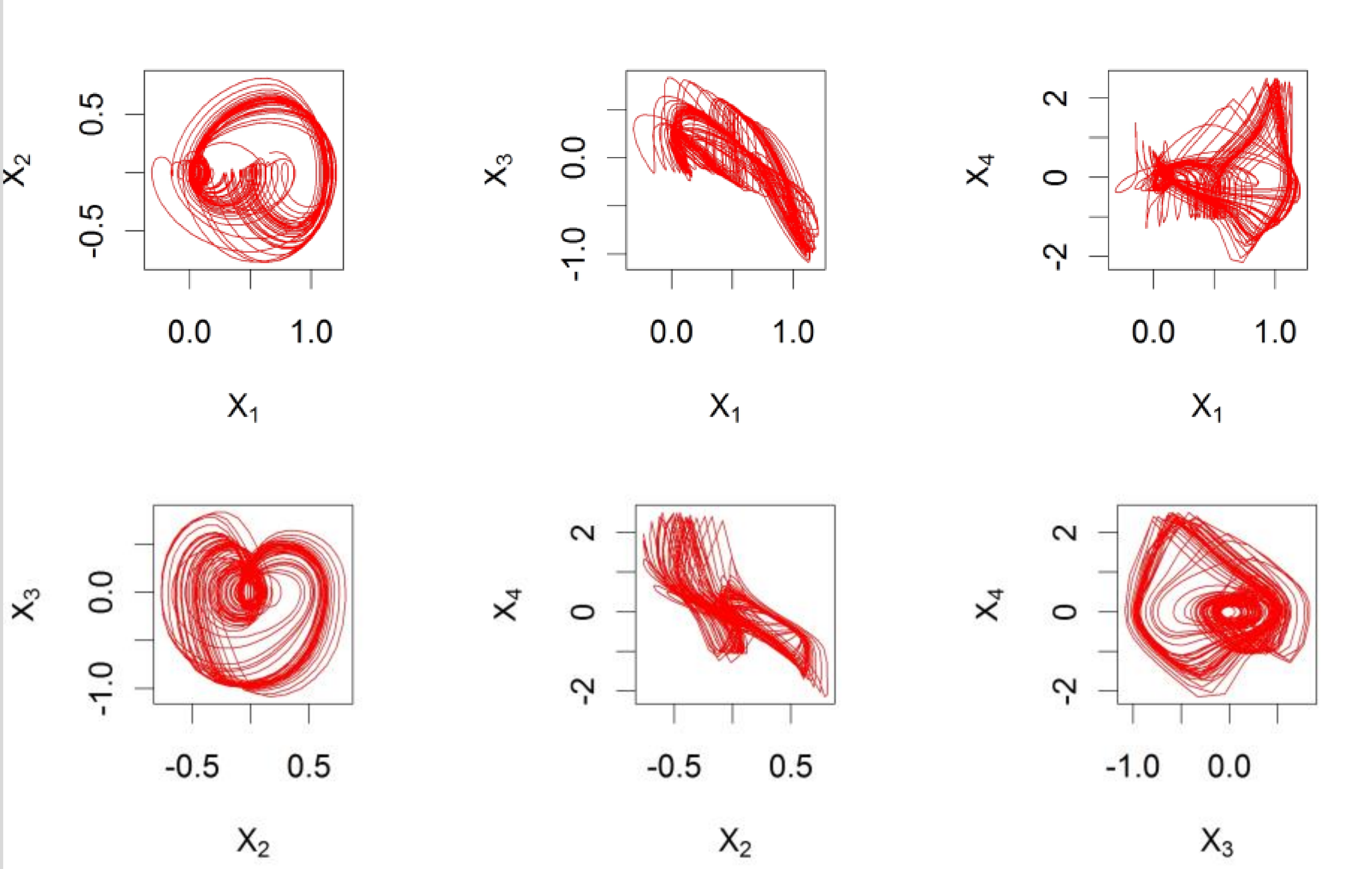






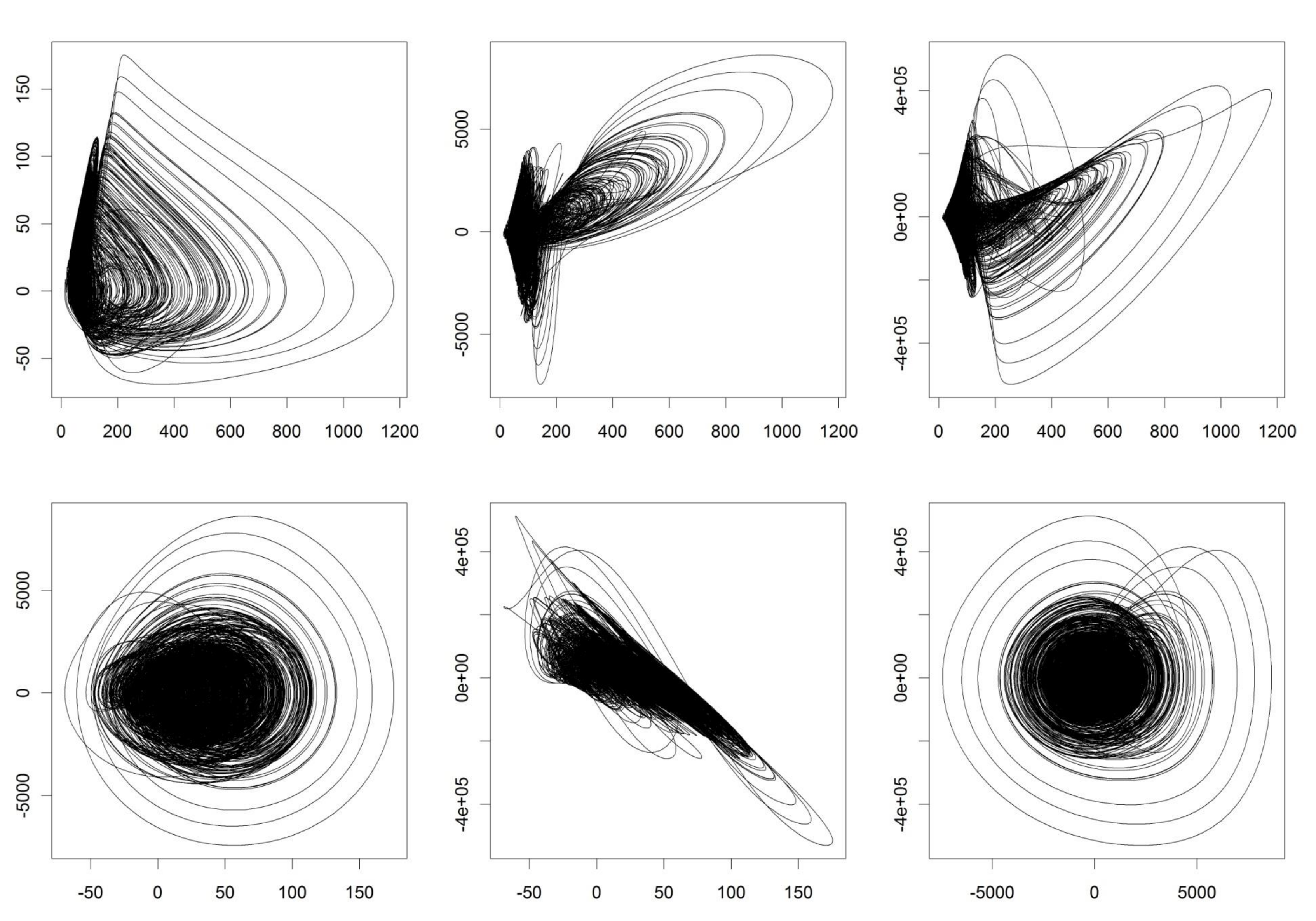
4D–models

E/E_0



4D–models

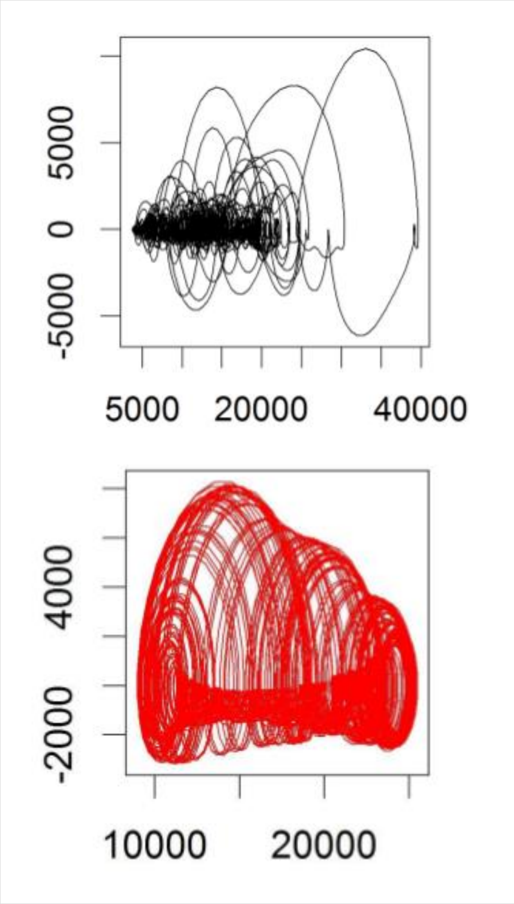
Ebola attractor



4D–models

Karstic springs

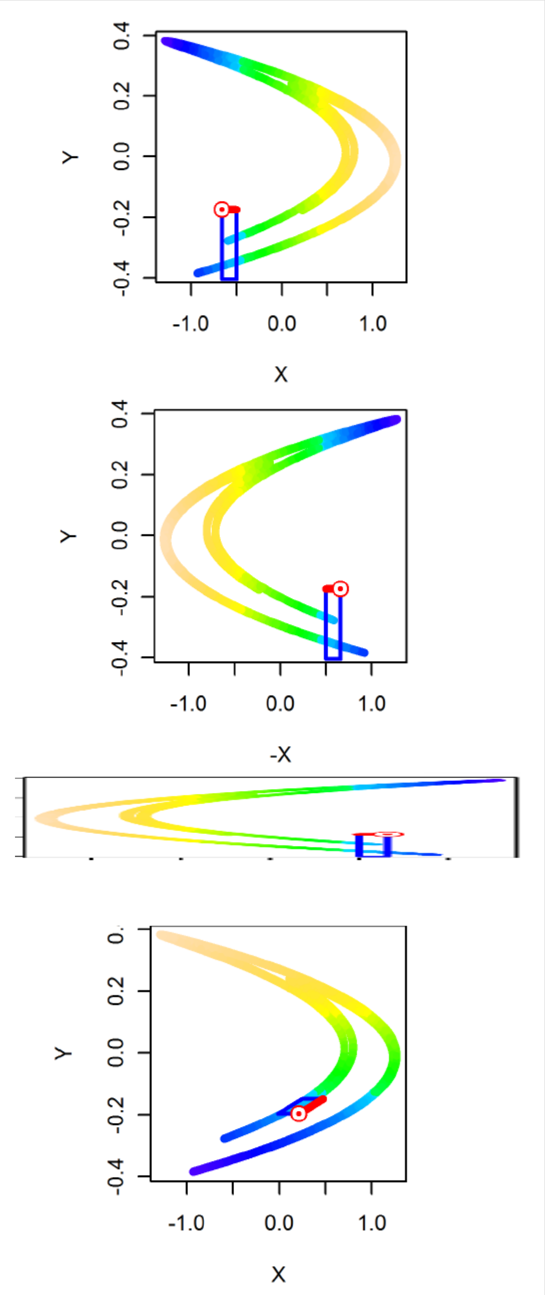
Touvre

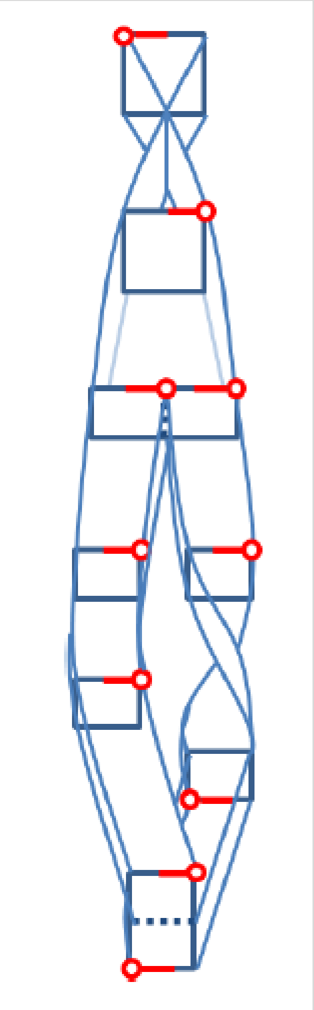


Application to the Hénon attractor (1976)

M. Hénon

$$\left\{ \begin{aligned} X_{i+1} &= Y_i + 1 - aX_i^2 \\ Y_{i+1} &= bX_i . \end{aligned} \right.$$





Mangiarotti HDR 2014

chronique of the torsions:

$$\left\{ (tz)^{\pm 4}, \left[\begin{array}{cc} (tx)^{-4} & (tx)^{-2} \\ (tx)^{-2} & 1 \end{array} \right] \right\}$$

Topological analysis for designing a suspension of the Hénon map

Mangiarotti & Letellier, Phys. Letters A 2015

Physics Letters A 379 (2015) 3069–3074

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Topological analysis for designing a suspension of the Hénon map

Sylvain Mangiarotti [a,*], Christophe Letellier [b]

[a] Centre d'Études Spatiales de la Biosphère, UPS-CNRS-CNES-IRD, Observatoire Midi-Pyrénées, 18 avenue Édouard Belin, 31401 Toulouse, France
[b] CORIA-UMR 6614 – Normandie Université, CNRS-Université et INSA de Rouen, Campus Universitaire du Madrillet, 76800 Saint-Étienne du Rouvray, France

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ABSTRACT

A suspension of a map consists of the flow for which the Poincaré section is that map. Designing a suspension of a given map remains a non-trivial task in general. The case of suspending the Hénon map is here considered. Depending on the parameter values, the Hénon map is orientation preserving or reversing; it is here shown that while a tridimensional suspension can be obtained in the former case, a four-dimensional flow is required to suspend the latter. A topological characterization of the three-dimensional suspension proposed by Starrett and Nicholas for the orientation preserving area is performed. A template is proposed for the four-dimensional case, for which the governing equations remain to be obtained.

1. Introduction

The concept of Poincaré section was introduced by Henri Poincaré to reduce the stability analysis of periodic orbits to the analysis of a single point and its associated neighborhood [1]. In the context of chaos theory, the Poincaré section is used in a global sense since this is a set of intersections of a chaotic trajectory with it (and not only in a restricted neighborhood of a single periodic point) that is investigated. Lorenz used such a global Poincaré section to describe how the trajectory was switching from one wing to the other within the eponym attractor [2]. It was later shown that the so-called Lorenz map results in fact from a two-component Poincaré section [3]. Rössler was one of the very first to use Poincaré sections and their related first-return map to distinguish the different type of chaos he constructed [4–7]. It is now used as a very useful step for performing a topological analysis of chaotic attractors [8–10]. Practically, the Poincaré section of a flow is numerically obtained by collecting the intersections of a trajectory with a half-plane. There is no analytical procedure for extracting the equations of a Poincaré section associated with a given flow.

The inverse procedure, namely constructing a flow corresponding to a given Poincaré section, is even much more difficult to perform. The flow associated with a given map was named the "suspension" of that map by Smale [11]. To the best of our knowledge, there are only two attempts to provide a suspension of the Hénon map. The very first one was proposed by Mayer-Kress and Haken [12] for parameter values for which the Hénon map is orientation preserving. Their suspension works on the cylinder $\mathbb{R}^2 \times [0,1]$, $t \in [0,1]$ by interpolating the initial and final conditions of the Hénon map. This flow cannot, strictly speaking, be considered as autonomous since the third variable is the time itself (see [13] for an extended discussion of that point). The second attempt was recently provided by Starrett and Nicholas for the Hénon map with specific parameter values [14]. The second suspension of the Hénon map was also provided when it is orientation preserving; this is a three-dimensional autonomous flow. Starting from a globalization of the local tangent space to suspended periodic orbits of the Hénon map, a three-dimensional autonomous differential system was obtained by a least square fit of the non-autonomous differential equations associated with the exact suspension. Since, for certain parameter values, the Hénon map can be also orientation reversing [15], we addressed the question whether it was possible to construct a suspension of the Hénon map or not.

The subsequent part of this brief report is organized as follows. Section 2 discusses the existence conditions concerning the dimensionality of the embedding space used for constructing a suspension of a given map. Section 3 is devoted to the case of the Hénon map. Section 4 gives some conclusions.

2. Existence conditions for a suspension

Among the constraints a flow must obey, a particularly important one guarantees the underlying determinism: the trajectory to the flow cannot intersect with itself. Let us designate as the Poincaré set the collection of intersections of the flow with a Poincaré section S. The latter is a hypersurface of dimension $d-1$ being transverse to the flow $\phi_t : \mathbb{R}^d \to \mathbb{R}^d$ of the system studied. Let $\dot{\phi}_t(\tilde{X}_0)$ be the unit vector at time t of the flow ϕ_t issuing from

* Corresponding author.
E-mail address: sylvain.mangiarotti@ird.fr (S. Mangiarotti).

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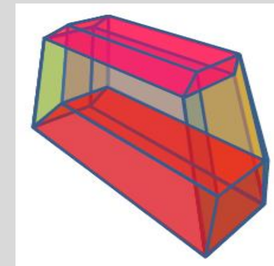
Existence condition for a suspension

4D objects



- Visualization tools

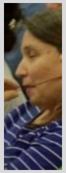
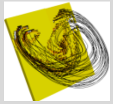
- Extraction of multi-dimensional structures

Collaboration with J.-P. Jessel at IRIT (UPS)

Modelling platform GPoM





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