# Internal Waves.

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01/06/2016

# Internal Gravity Waves

Propagating disturbances in density stratification $\rho(z)$ in a stably stratified fluid

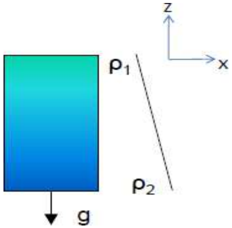


Figure: Stable Stratification

Density stratifications are present both in atmosphere and ocean

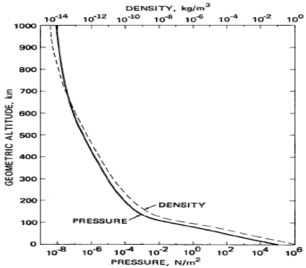


Figure: US Standard atmosphere(1976)

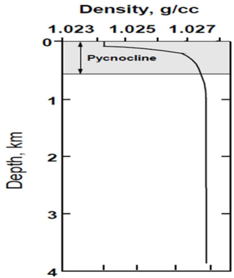


Figure: Pycnocline,(Pond & Pickard,1983)

## Signature of internal waves in environment

Figure: Morning glory waves in Gulf of Carpentaria(Source:astronoo)

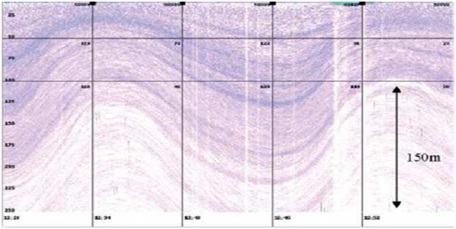


Figure: Lombok Strait, Susanta.et.al(2005)


Figure: Sulu Sea(Source:NASA GSFC)

# Brunt Vaisalla Frequency

Fluid parcel displaced from 2 to 1.
Restoring force proportional to product of gravity and density difference between the two layers.

$$
\begin{aligned}
F &= Vg(\rho_1 - \rho_2) = Vg\frac{d\rho}{dz}Z \\
F &= -mN^2 Z \\
N &= \sqrt{\frac{-g}{\rho}\frac{d\rho}{dz}}
\end{aligned}
$$

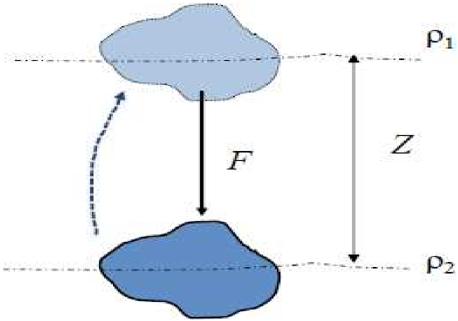

Figure: Fluid parcel displaced in stable stratification

N is Brunt Vaisalla frequency.
Internal waves propagate due to restoring action of buoyancy force in stable stratifications.

# Clear Air Turbulence(CAT)

- Internal waves are generated in atmosphere by flow over mountains.
- As they propagate, they break down due to various instabilities leading to CAT.

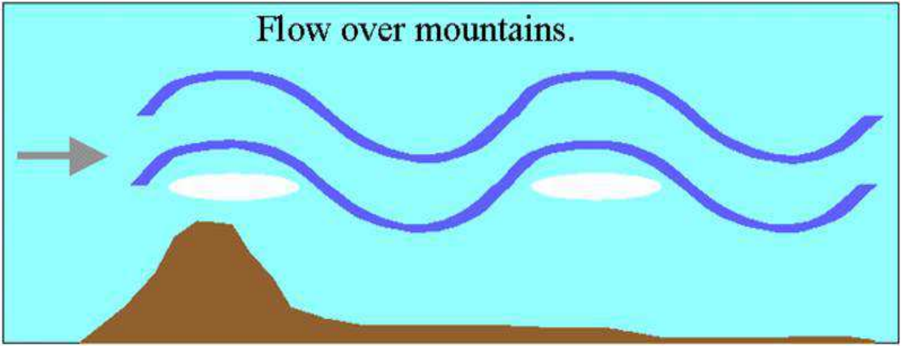


Figure: Hocking(2001)

# DC-8 cargo jet accident(1992)

Figure: Clark et.al (2000)

- Severe structural damage due to Clear Air Turbulence(CAT).
- Numerical simulations carried out using observation data.
- CAT due to interaction of jet streams and mountain-forced internal Gravity waves.

# Dead water Phenomenon

- Fritjof Nansen, a Norwegian explorer in his epic attempt to reach the North Pole experienced it in 1892.
- Energy from the boat gets mainly transmitted to interfacial waves between the two layers. Energy remaining for the motion of the boat is drastically decreased(Ekman,1904.

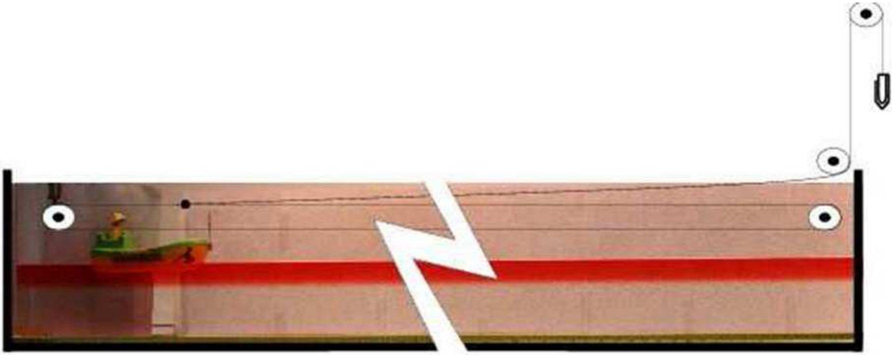

Figure: Mercier et.al(2011)

# Ocean Turbulence

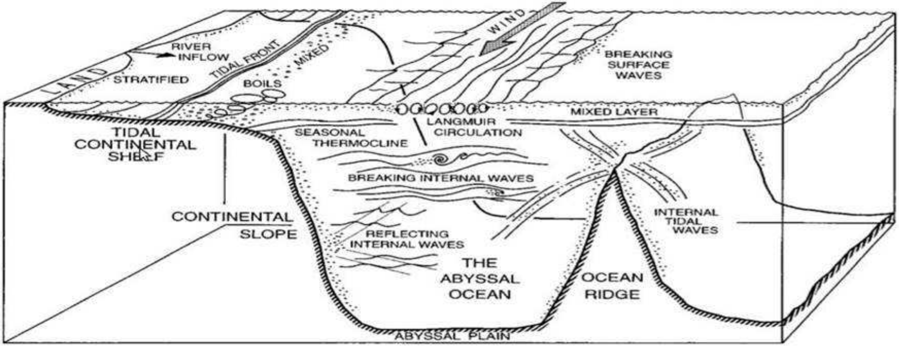


Figure: Various ocean processes,S.A.Thorpe(2004)

A cartoon of some of the processes that lead to oceanic turbulence. Internal waves are one of the dominant processes that lead to turbulence at scales of order 1 mm to 100m

# Governing equations

Under the assumptions of 2-d, inviscid, incompresible, f-plane, Boussinesq approximation, base flow $\vec{U} = 0$ and $\bar{\rho} = \bar{\rho}(z)$, we have

$$\begin{aligned} \rho^*(\frac{Du}{Dt} - f_0 v) &= -\frac{\partial p}{\partial x} \\ \rho^*(\frac{Dv}{Dt} + f_0 u) &= 0 \\ \rho^* \frac{Dw}{Dt} &= -\frac{\partial p}{\partial z} - g\rho \\ \frac{D\rho}{Dt} &= -w\frac{d\bar{\rho}}{dz} \end{aligned}$$

Neglecting the advection terms and expressing velocities in terms of stream function $(u, w) = (-\psi_z, \psi_x)$, linear internal wave equation is

$$\rho^*[\frac{\partial^2}{\partial t^2}(\nabla^2 \psi_1)] + f_0^2 \frac{\partial^2 \psi_1}{\partial z^2} + N^2 \frac{\partial^2 \psi_1}{\partial x^2} = 0$$

# Solution to linear internal wave equation

Solution in form of modes given by

$$\psi_1 = \sum_{n=1}^{\infty} a_n \Phi_n(z) \cos(k_n x - \omega t + \alpha_n)$$

where $\Phi_n(z)$ is obtained by solving the equation to satisfy no normal flow condition at $z = 0$ and $z = H$

$$\frac{d^2\Phi_n}{dz^2} + \frac{k_n^2(N^2 - \omega^2)}{\omega^2 - f_0^2}\Phi_n = 0$$

For uniform stratification i.e $N = N_0$ , $\Phi_n(z) = \sin(\frac{n\pi z}{H})$ and $k_n = \frac{n\pi}{H \cot\theta}$.

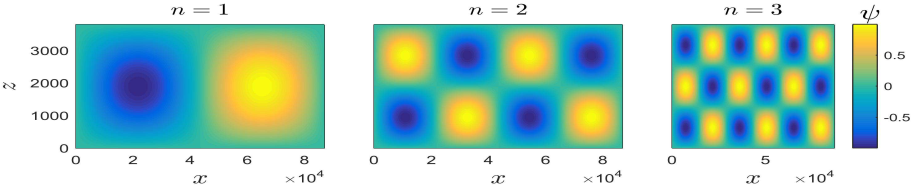


Figure: Stream function plots for modes

# Intriguing properties of internal waves

- The dispersion relation for the internal waves is given by

$$\cot\theta = \frac{N^2 - \omega^2}{\omega^2 - f_0^2} = \frac{m}{k}$$

  where $\theta$ is the angle the wave-vector(k,m) makes with the z-axis.
- Forcing frequency determines the wave direction.
- Group velocity is defined as

$$V_g = (\frac{\partial\omega}{\partial k}, \frac{\partial\omega}{\partial m})$$

- Direction of energy propagation is orthogonal to direction of phase propagation.

# St.Andrew cross Experiment

- Wave beams generated by vertically oscillating(frequency,$\omega$) cylinder in a uniformly stratified fluid.

- Green arrows showing the direction of energy propagation and yellow arrows showing the direction of phase propagation.
- The direction of phase propagation and the energy propagation are orthogonal to each other.

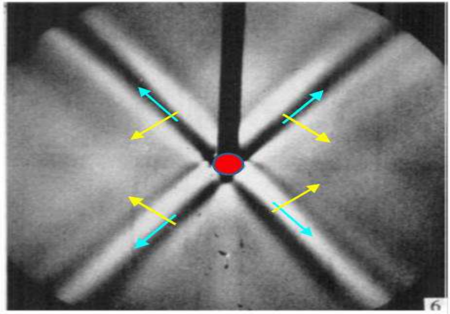


Figure: D.E.Mowbray and B.S.H Rarity(1967)