What can we learn about climate change from energy balance models?

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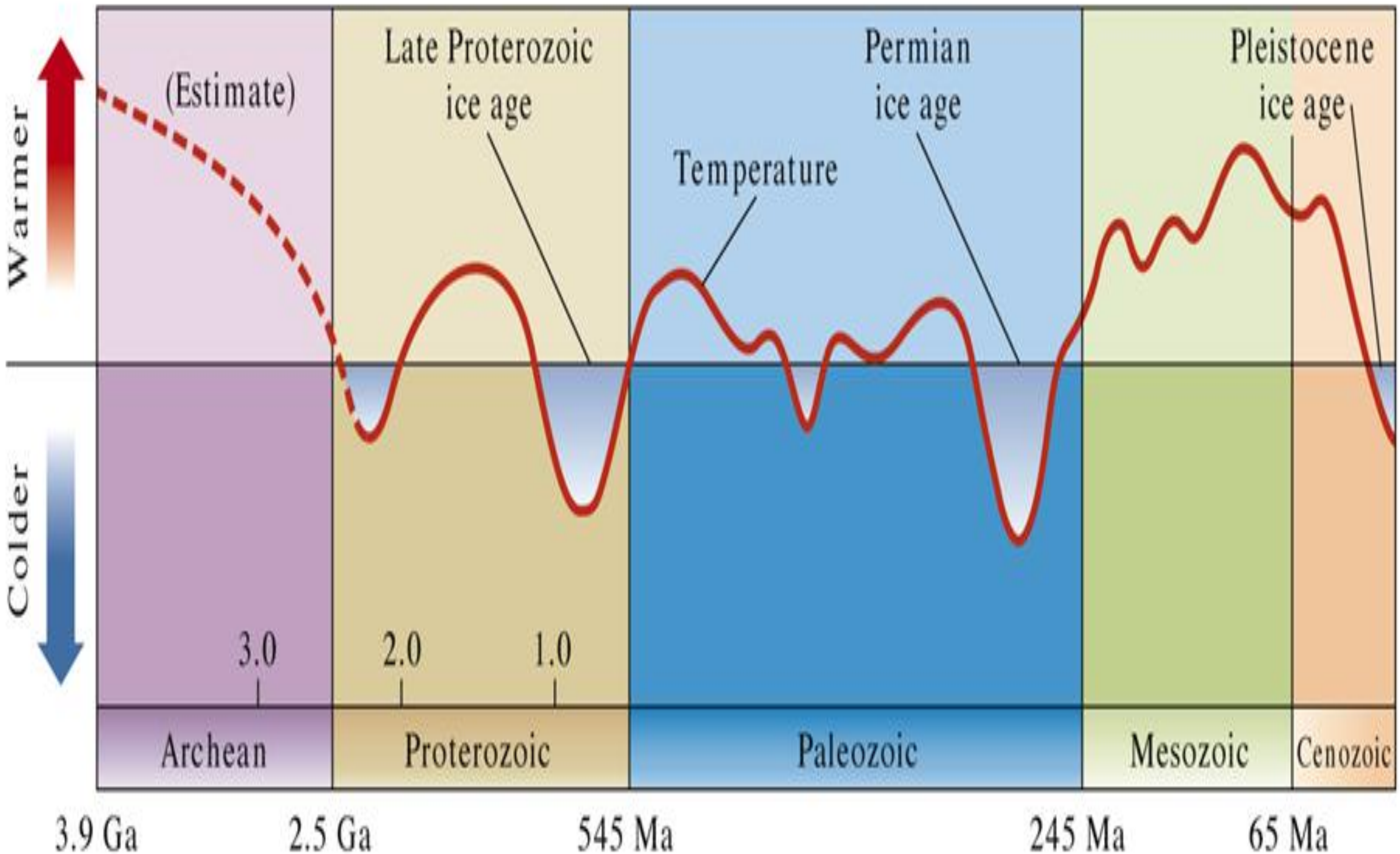
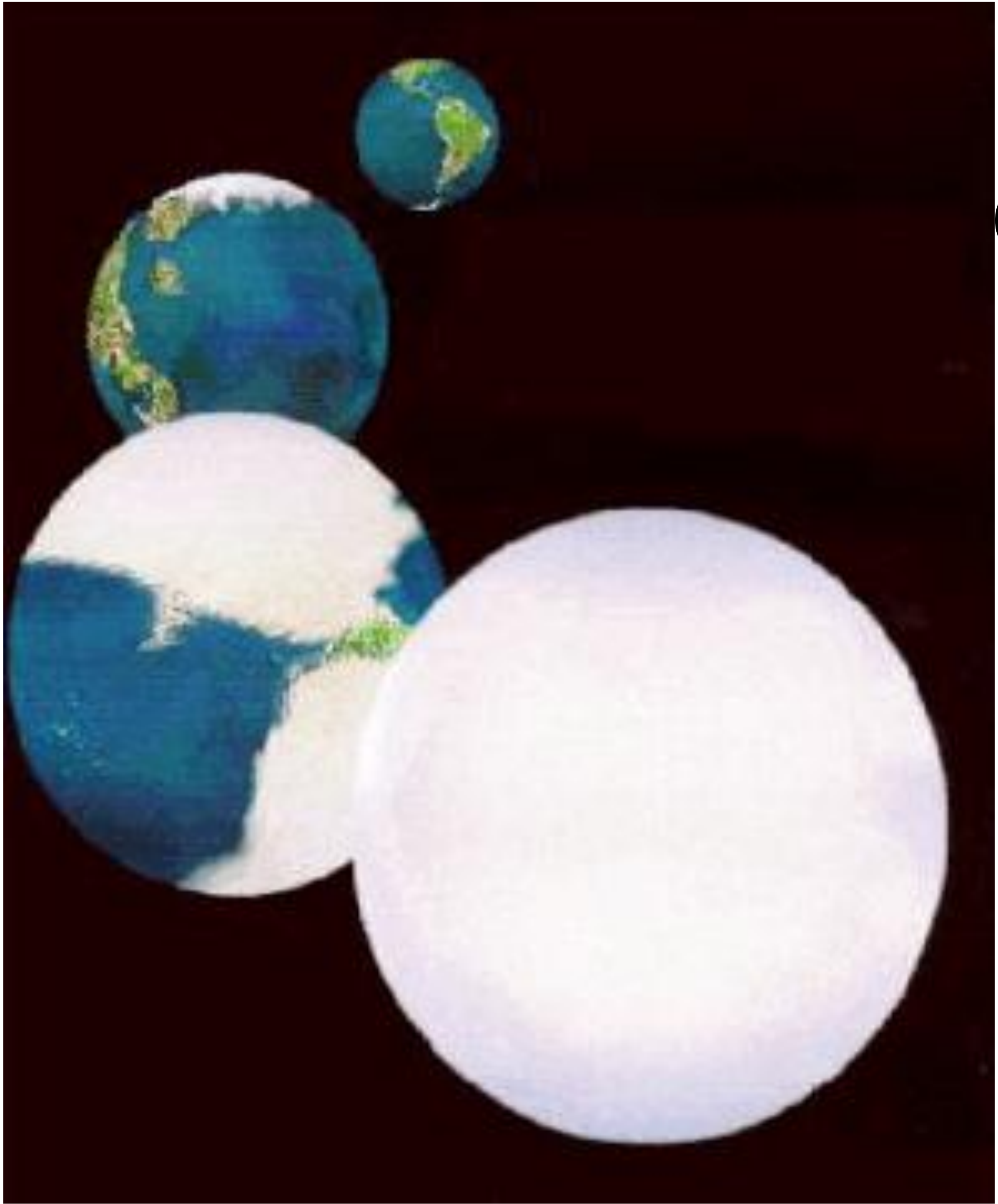
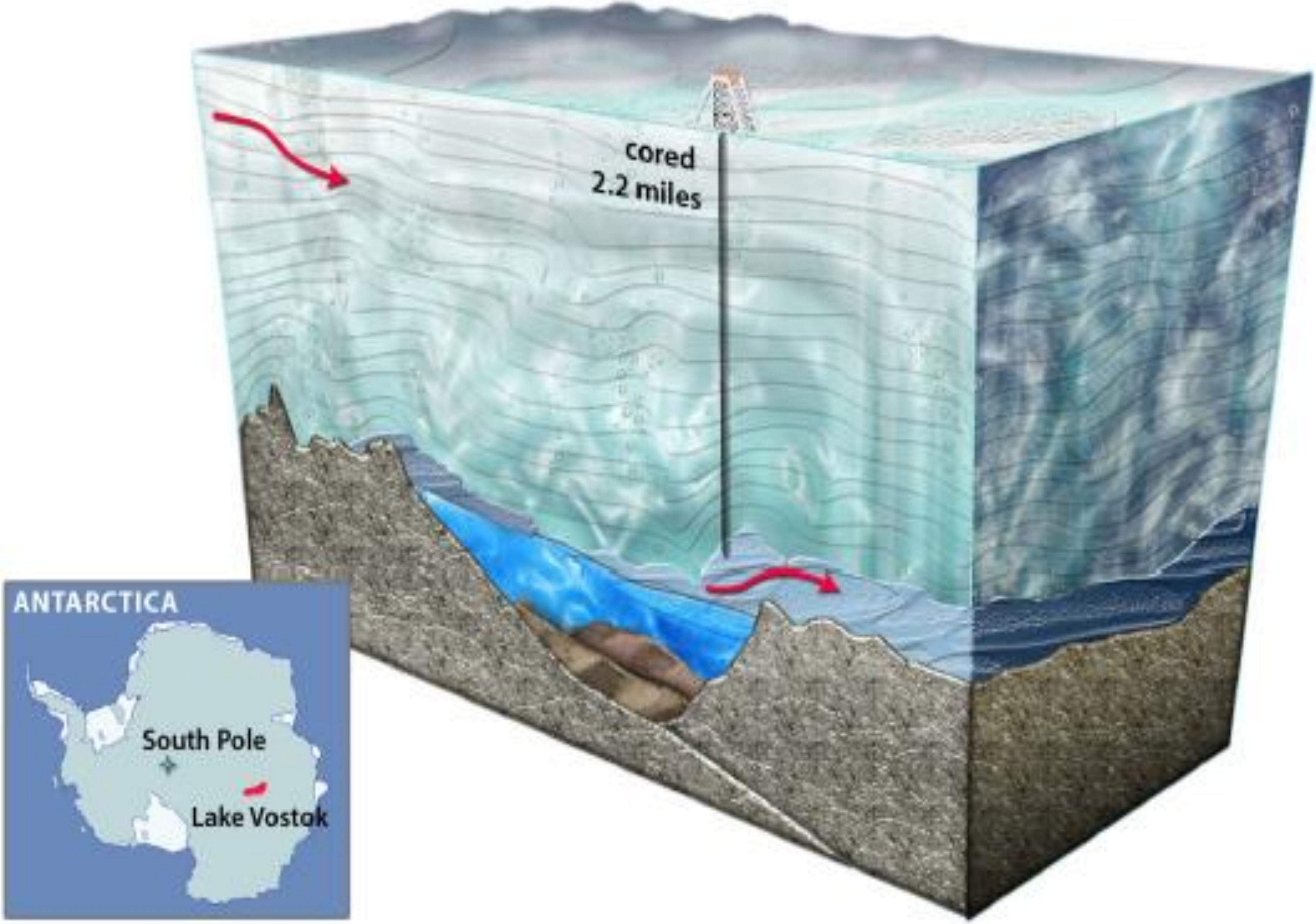
Global Climate History



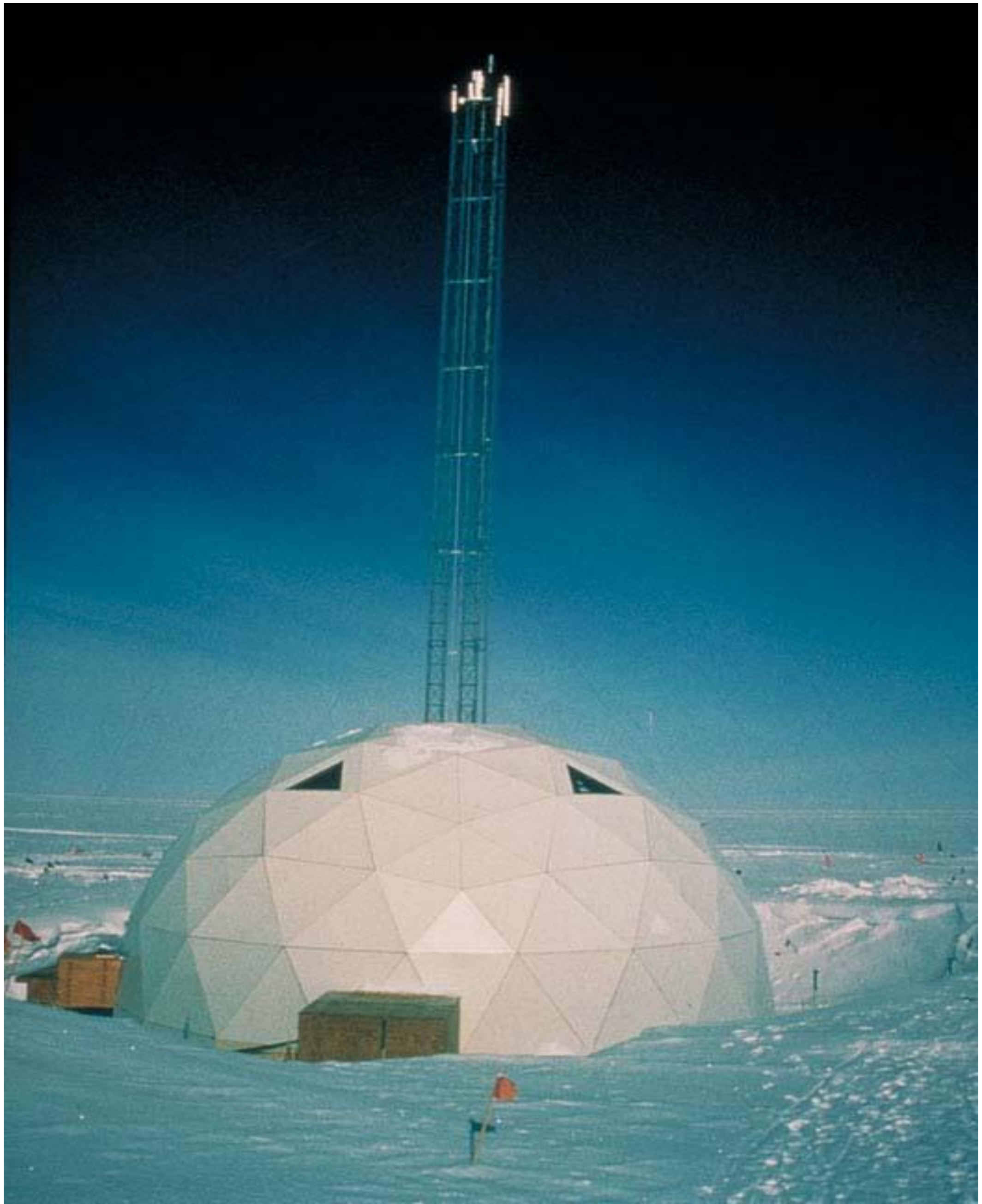


Oscillations between ice- free and ice-covered earth

Lake Vostok, Antarctica

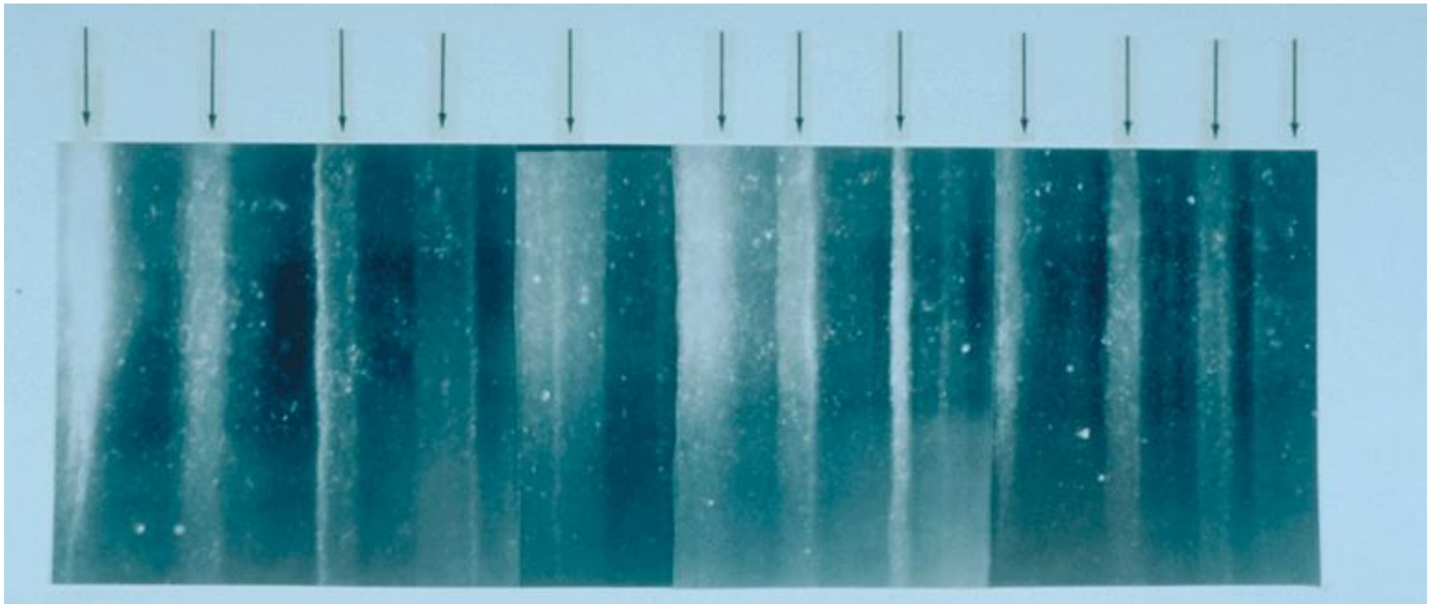


GISP2 Drilling Project

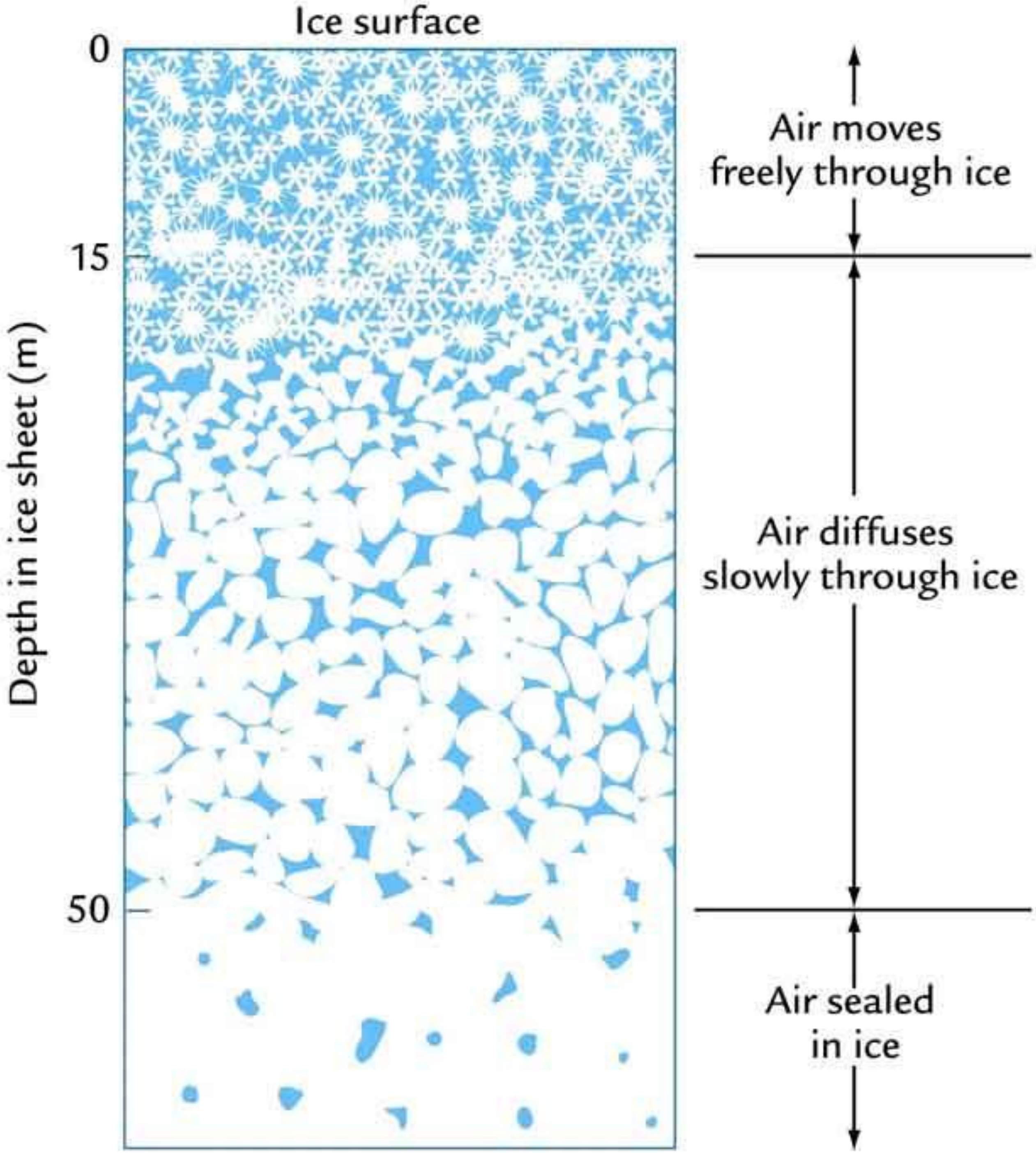


Extracting An Ice Core

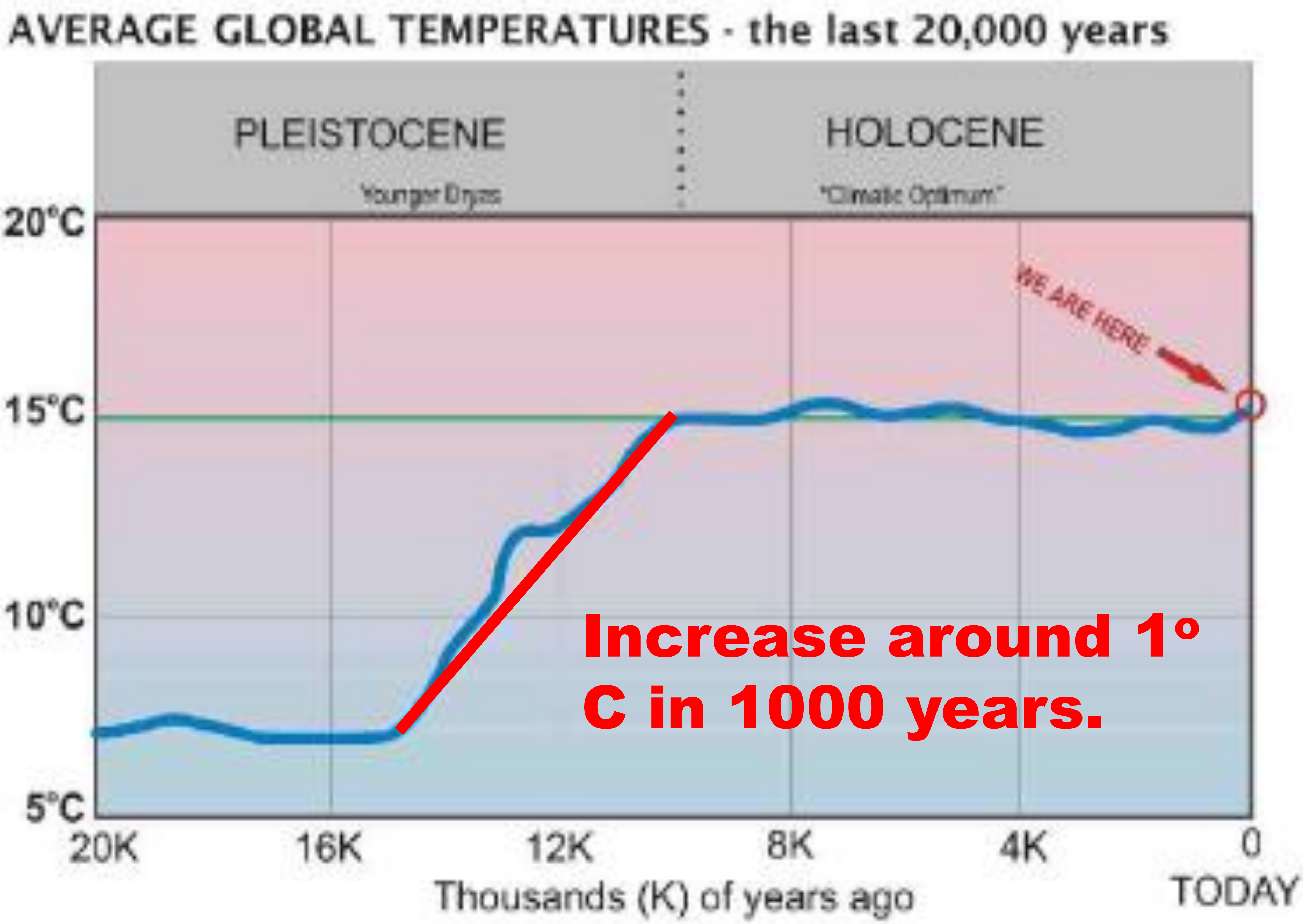
Annual Layers In Ice Core

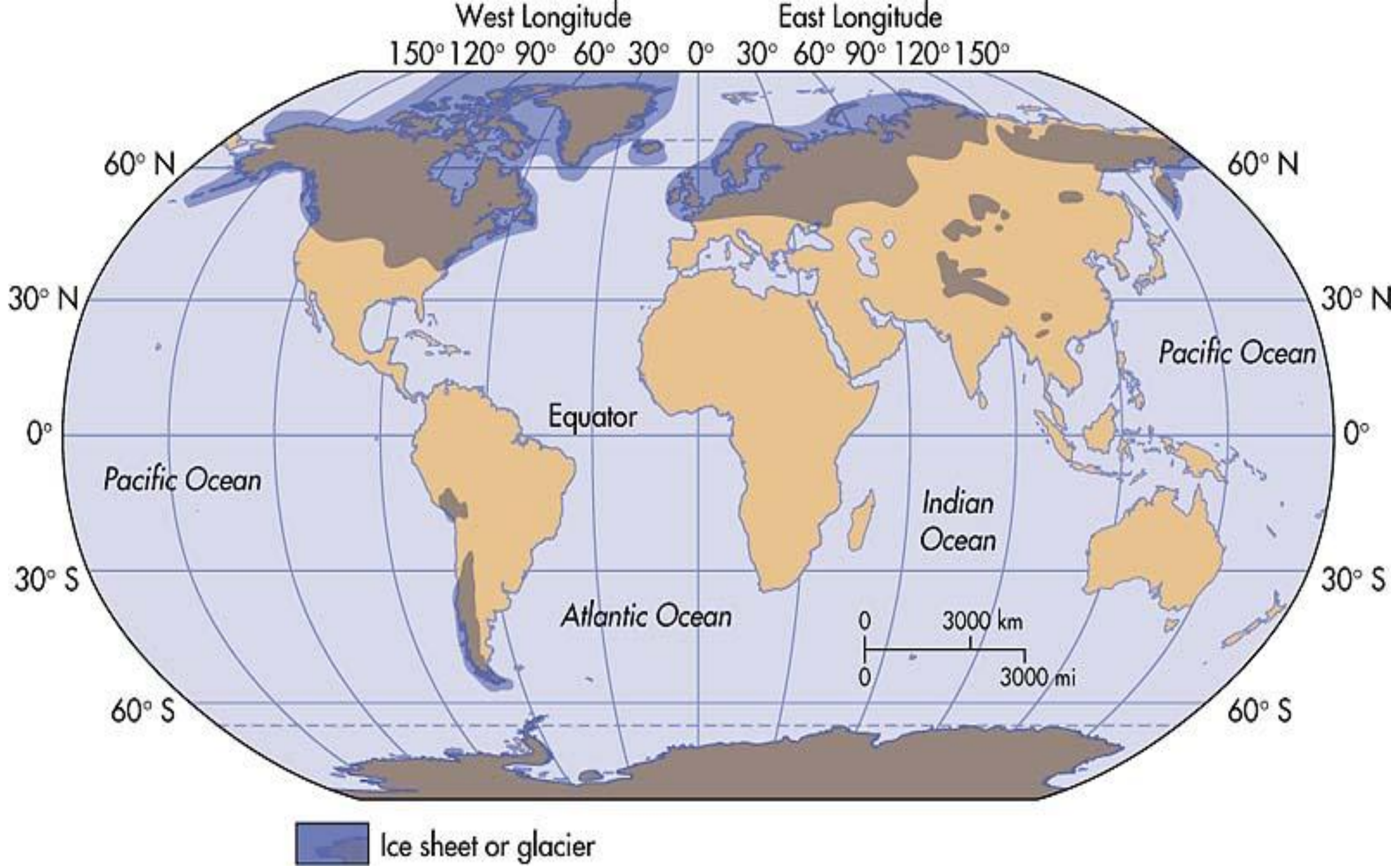


19 cm long section of GISP 2 ice core from 1855 m showing annual layer structure illuminated from below by a fiber optic source. Section contains 11 annual layers with summer layers (arrowed) sandwiched between darker winter layers.



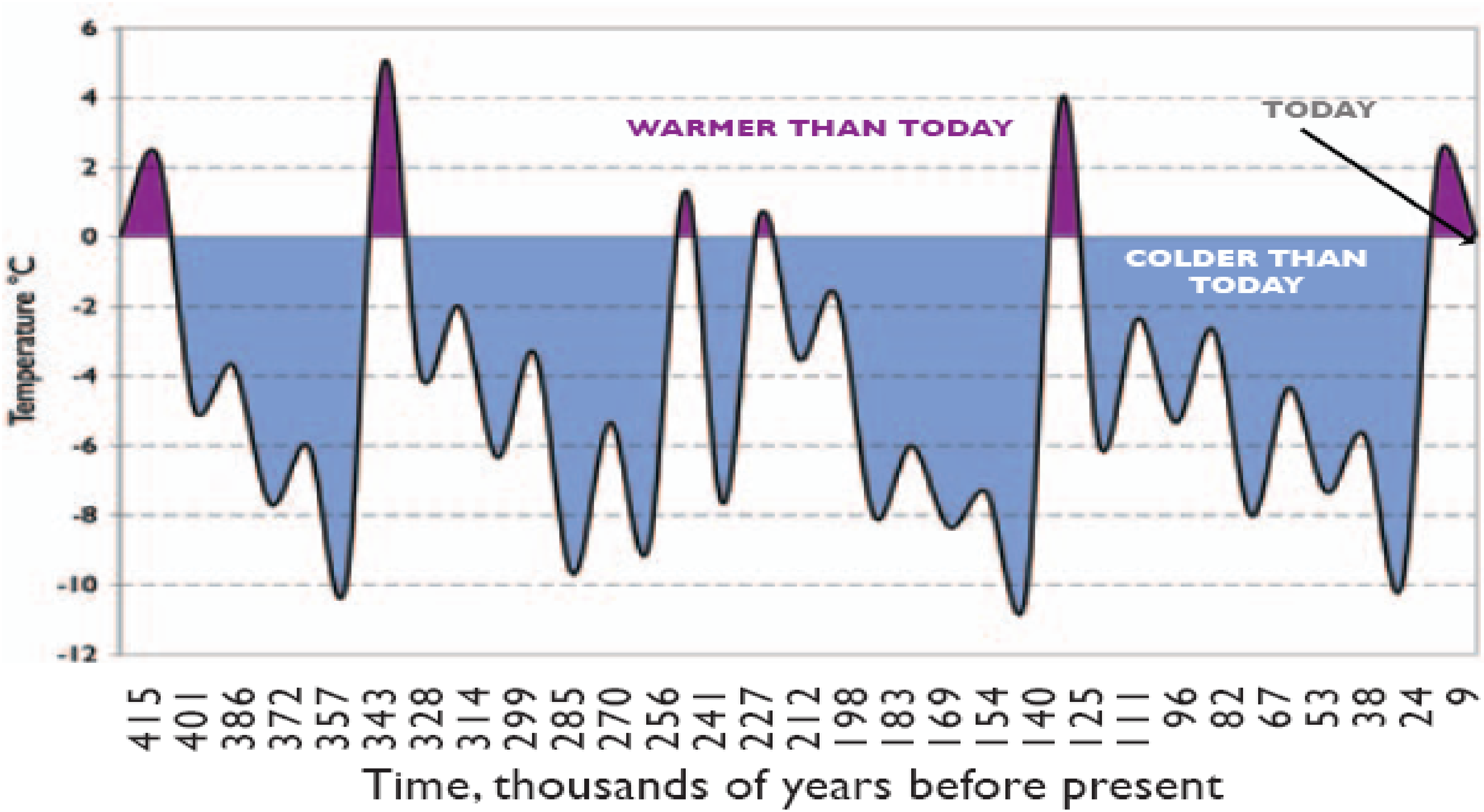
Ice surface
0
15
50
Depth in ice sheet (m)
Air moves freely through ice
Air diffuses slowly through ice
Air sealed in ice


AVERAGE GLOBAL TEMPERATURES - the last 20,000 years
PLEISTOCENE
HOLOCENE
Younger Dryas
"Climatic Optimum"
WE ARE HERE
20°C
15°C
10°C
5°C
20K
16K
12K
8K
4K
0
TODAY
Thousands (K) of years ago
Increase around 1° C in 1000 years.



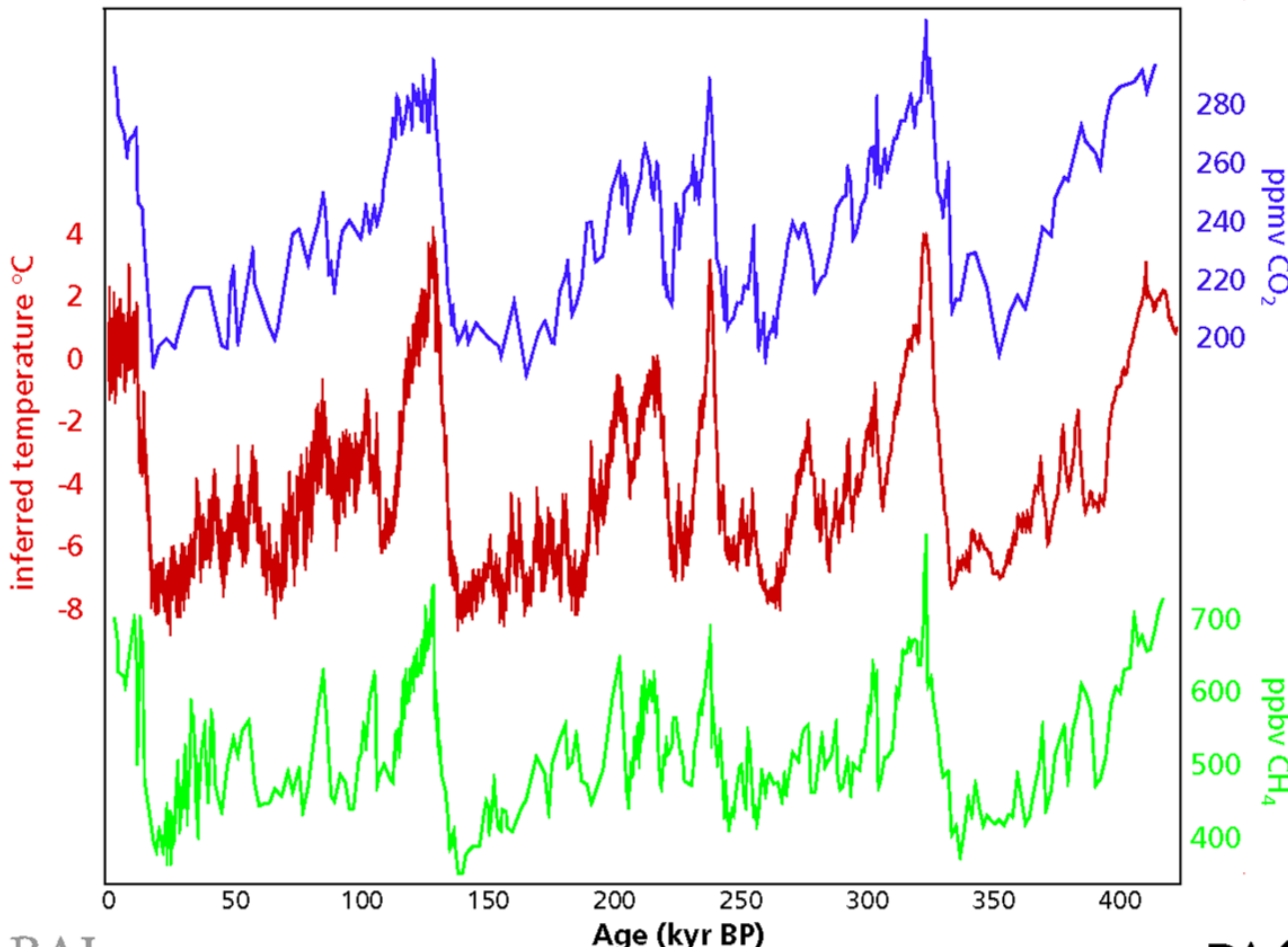
30% of earth covered with ice 20,000 yrs ago. Today only 11%

Figure 1: Long-term climatic (Milankovitch) cycles over the last 415,000 years from the Vostok ice core



Source: Salamatin A.N., (*et al.*) 'Ice core age dating and paleothermometer calibration based on isotope and temperature profiles from deep boreholes at Vostok station (East Antarctica)', *Journal of Geophysical Research*, 1998, vol. 103, no D8, pp. 8963-8977.

4 glacial cycles recorded in the Vostok ice core

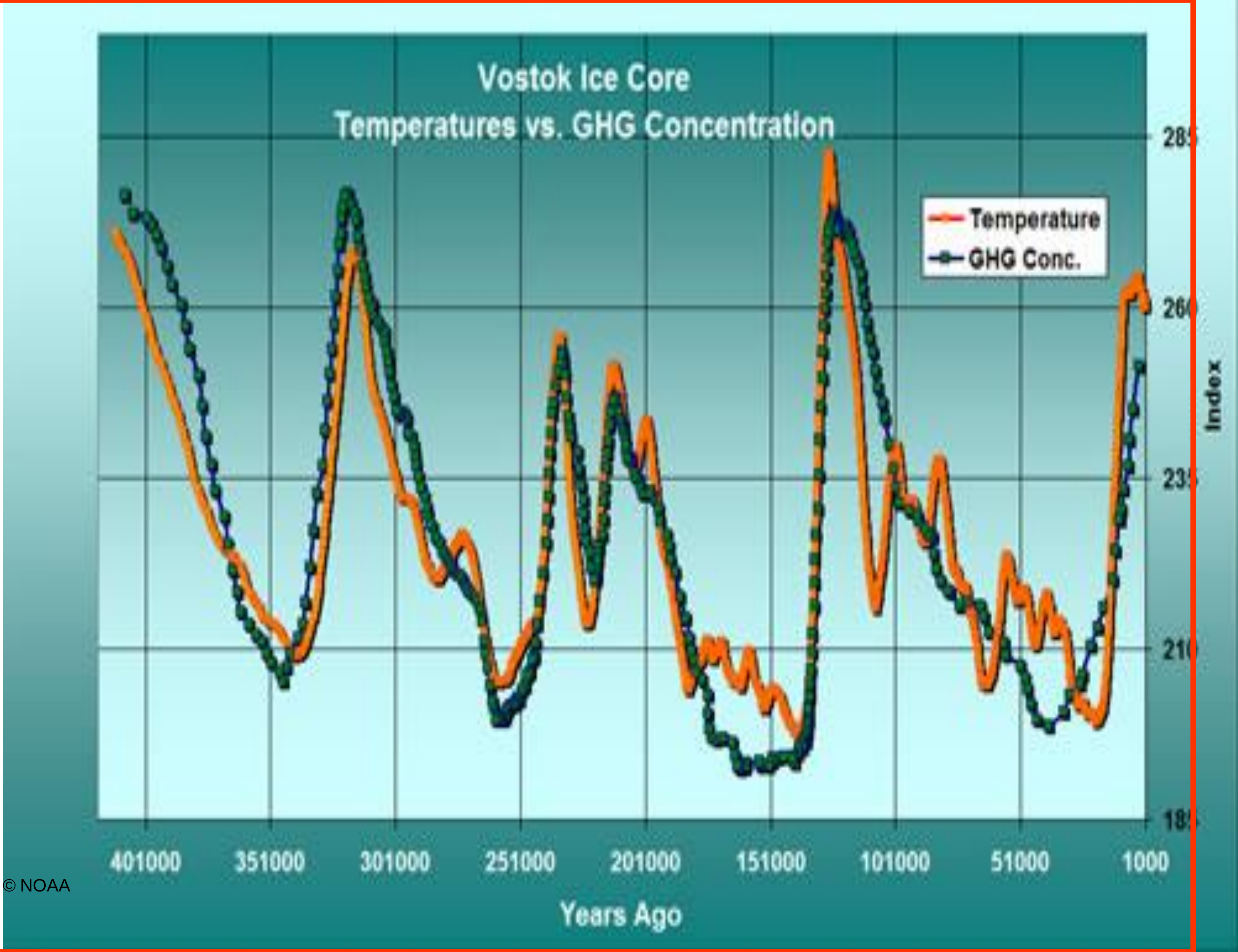


GLOBAL IGBP CHANGE

J.R. Petit et al., *Nature,* **399**, 429–36, 1999.

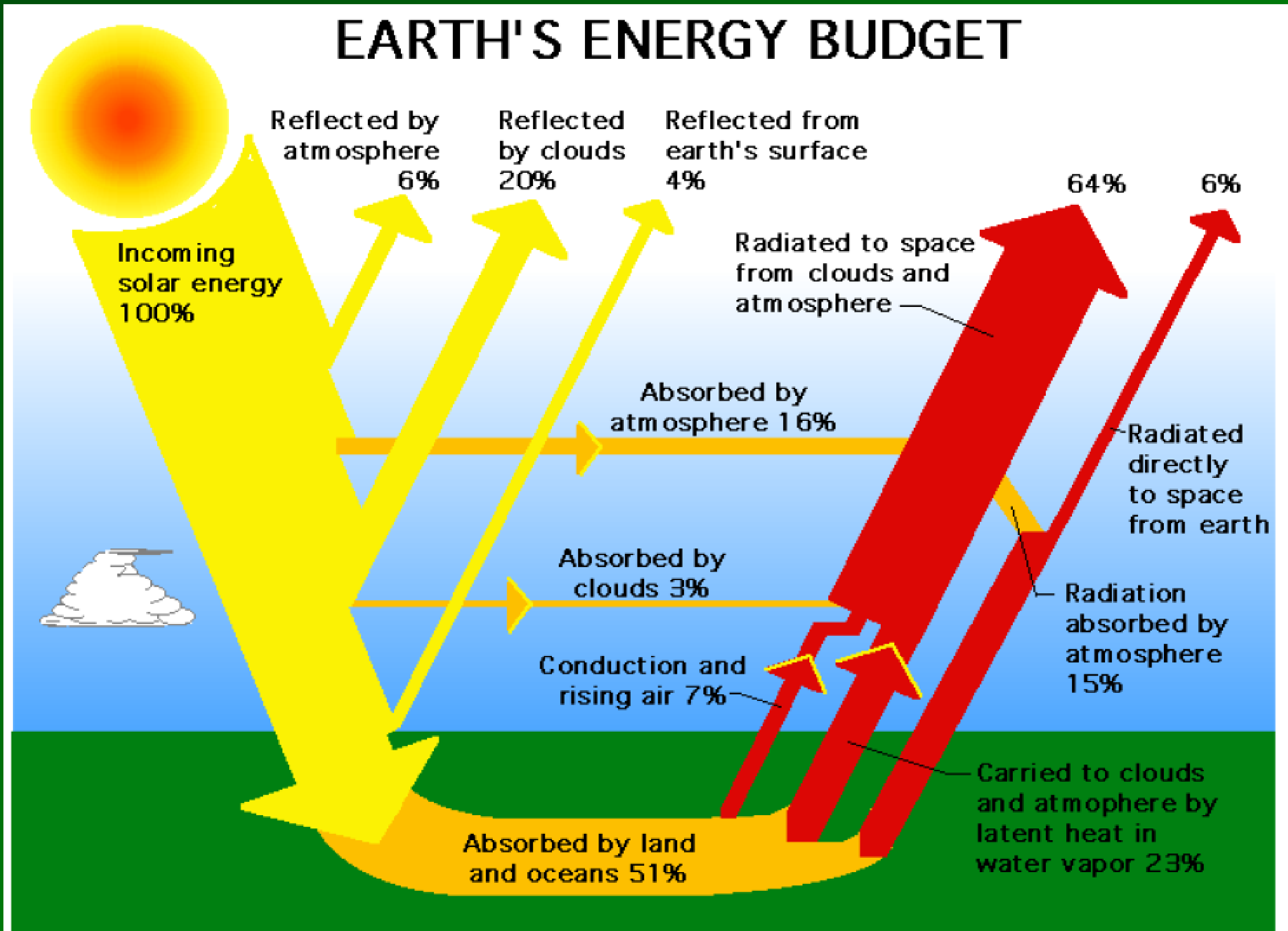
PAGES PAST GLOBAL CHANGES

6.1

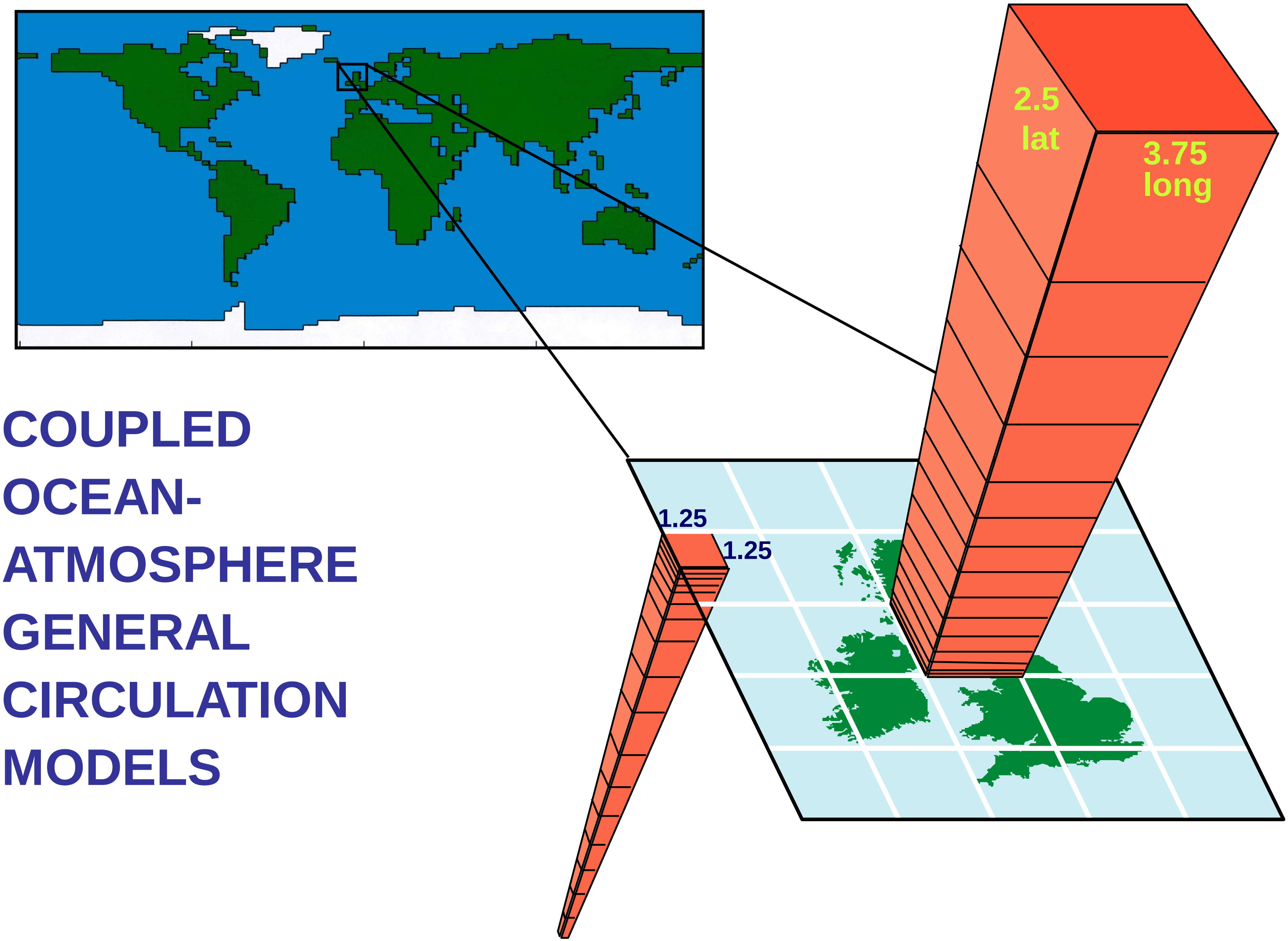


Vostok Ice Core
Temperatures vs. GHG Concentration
Temperature
GHG Conc.
285
260
235
210
185
Index
401000
351000
301000
251000
201000
151000
101000
51000
1000
Years Ago

Global Energy Balance

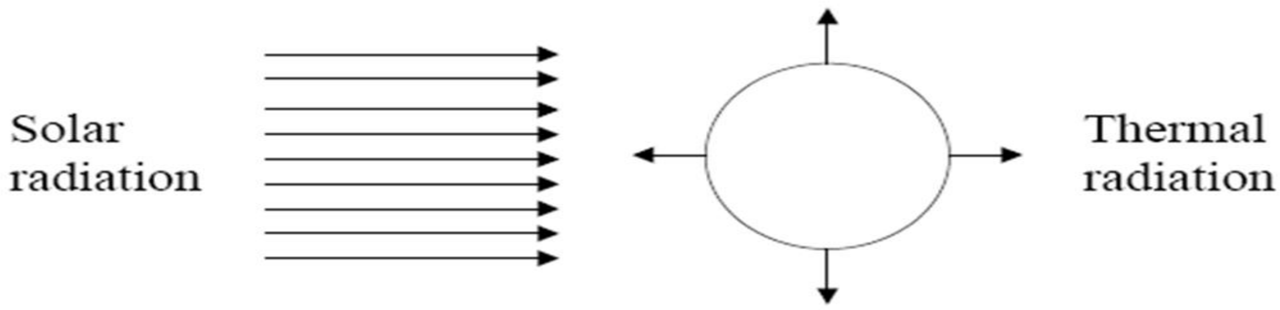


COUPLED OCEAN-ATMOSPHERE GENERAL CIRCULATION MODELS



SIMPLE ANALYTICAL MODEL FOR GLOBAL MEAN SURFACE TEMPERATURE OF THE EARTH

Overall energy balance of the Earth



$$(1-\alpha)\, S_o\, \pi r^2 = 4\, \pi r^2\, \sigma\, T^4_{eff}$$

Simplifying, we find that;

$$\sigma\, T^4_{eff} = (1-\alpha)\, S_o/4$$

and hence

$$T_{eff} \approx 255 \text{ K}$$

for S = 1365 W/m2 , α = 0.3

ABSORBED SOLAR RADIATION =OUTGOING LONGWAVE RADIATION

240 W/m^2 **240 W/m^2**

Absorbed Solar

Earth's emission

ATMOSPHERE

σ T^4=390 W/m^2 for T$_s$ =288K

Greenhouse effect=390 -240=150

GREENHOUSE EFFECT

GHE = RADIATION EMITTED BY PLANET'S SURFACE – RADIATION LEAVING THE PLANET

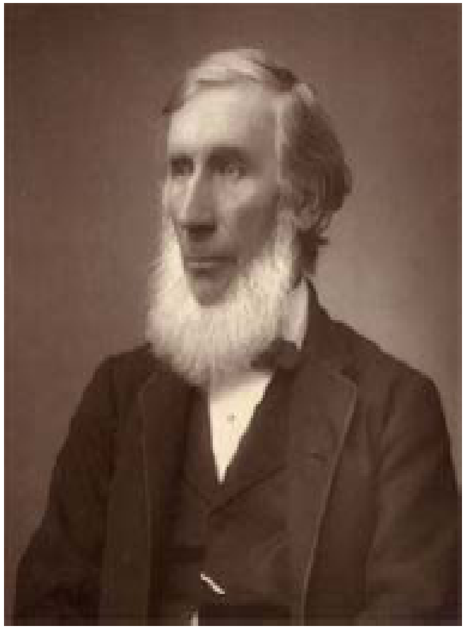
EARTH = 390 -240 =250 W/m^2

VENUS=16100-200=15,900 W/m^2

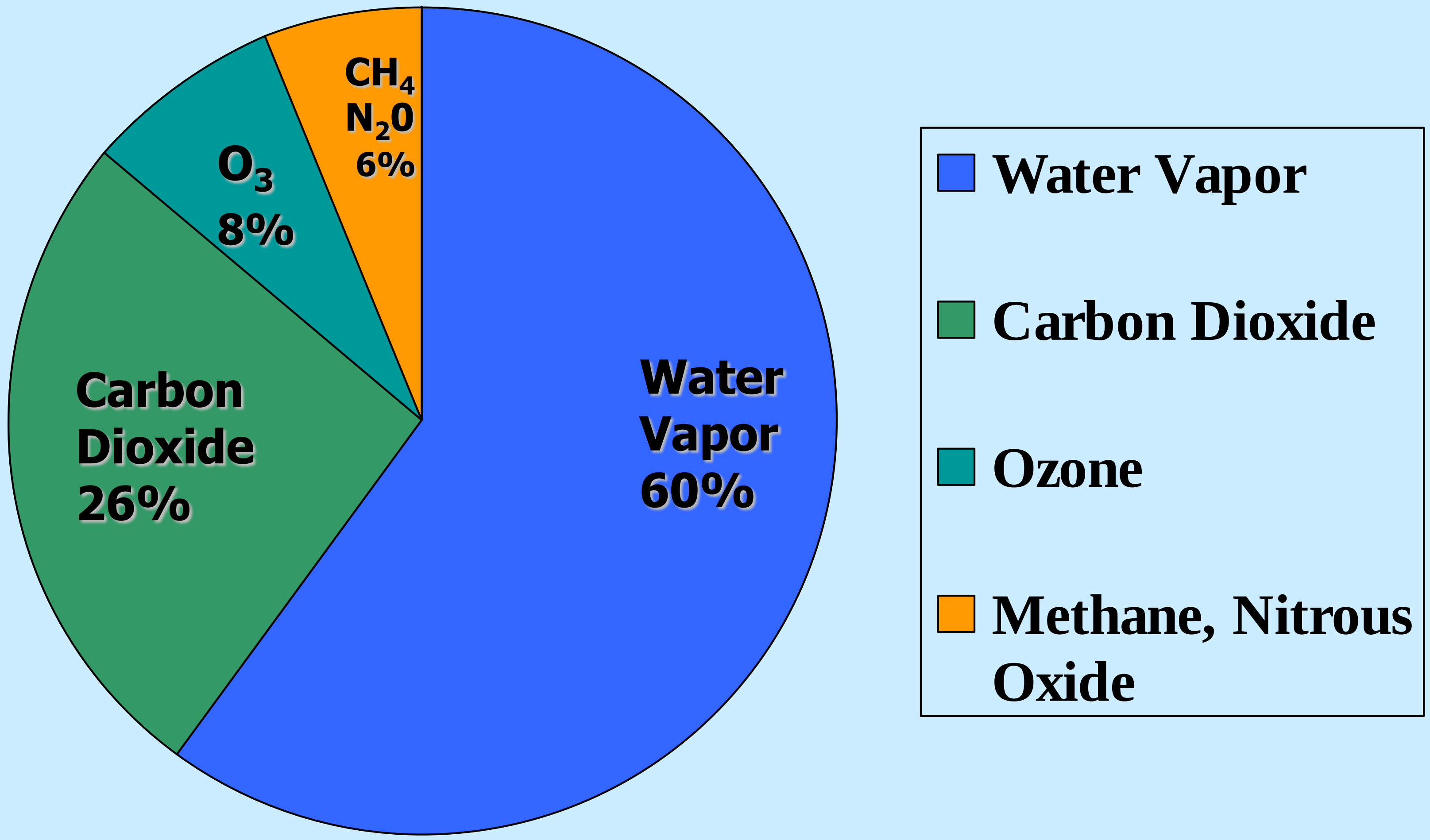
The Greenhouse Effect: Tyndall

John Tyndall*, 1862 (river analogy):

"As a dam built across a river causes a local deepening of the stream, so our atmosphere, thrown as a barrier across the terrestrial rays, produces a local heightening of the temperature at the Earth's surface."



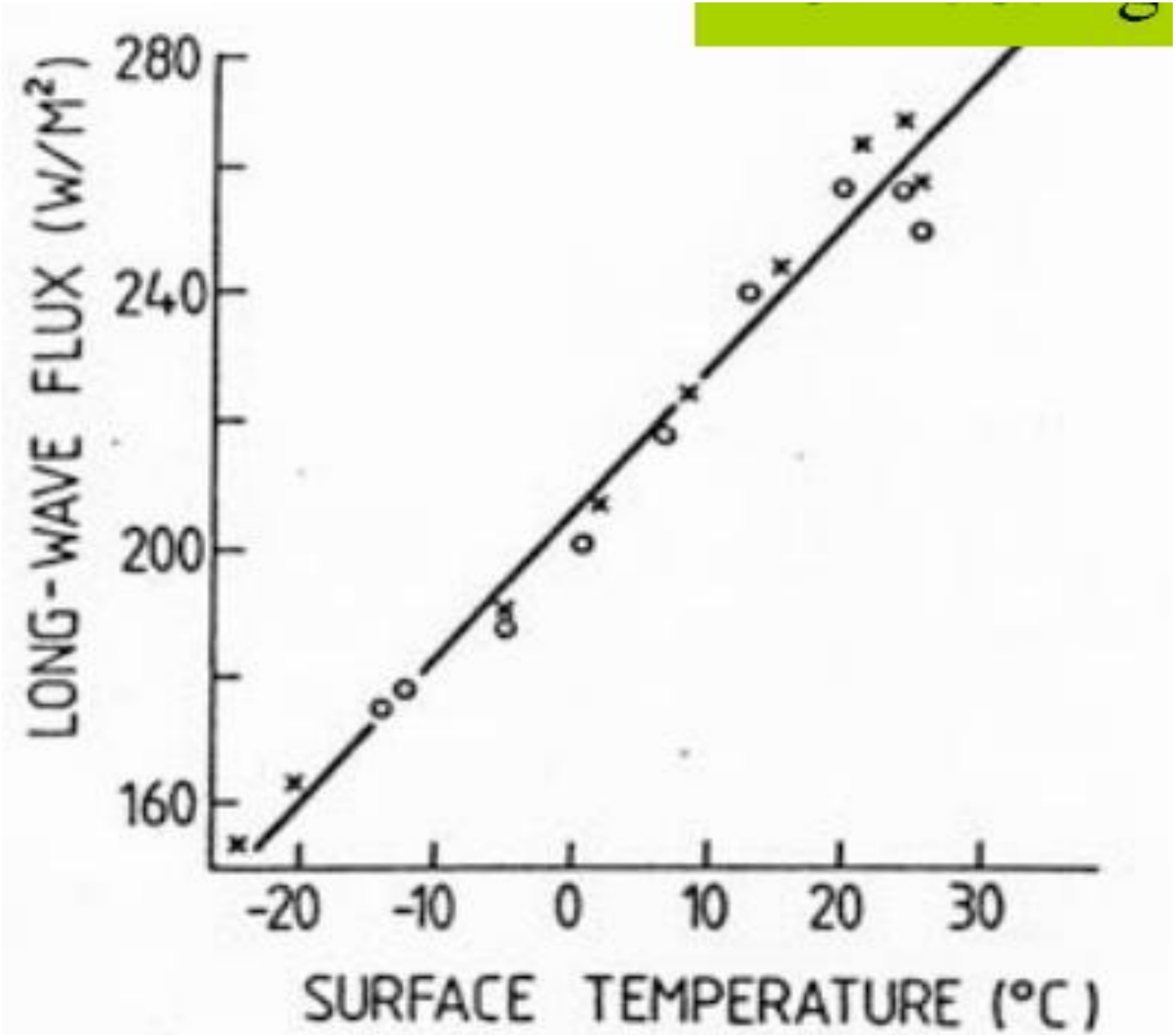
6.2 The Natural Greenhouse Effect: clear sky



Clouds also have a greenhouse effect

Kiehl and Trenberth 1997

LINEARIZATION OF OUTGOING LONGWAVE RADIATION



$$OLR = A + BT$$

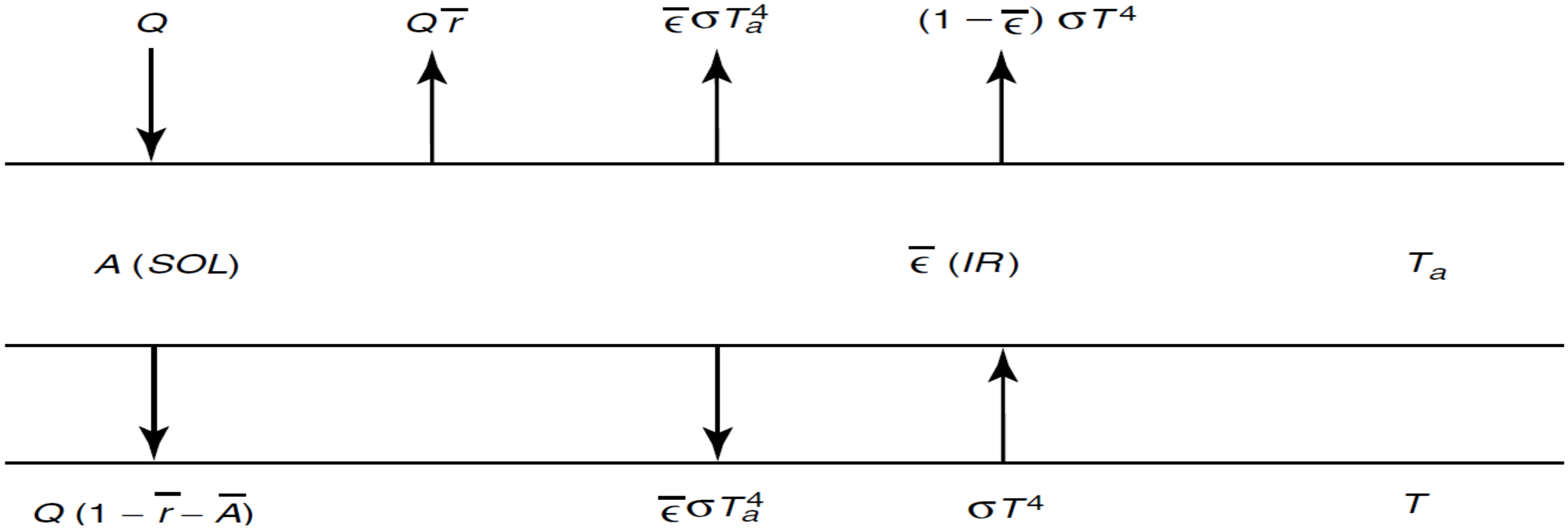
$$S/4\ (1 - \alpha) = A + B*T$$

A and B include Greenhouse effect

$$I = A + BT - 5.397 \ln \left(\frac{c_a}{c_{a,P}} \right)$$

A = 203.34 W /m²

B = 2.09 W /m² K



$$Q(1-\bar{r})-\bar{\varepsilon}\sigma T_a^4-(1-\bar{\varepsilon})\sigma T^4=0,$$

$$Q(1-\bar{r}-\bar{A})+\bar{\varepsilon}\sigma T_a^4-\sigma T^4=0.$$

$$T^4=Q[2(1-\bar{r})-\bar{A}]/[\sigma(2-\bar{\varepsilon})],$$

$$T_a^4=Q[\bar{A}+\bar{\varepsilon}(1-\bar{r}-\bar{A}]/[\sigma\bar{\varepsilon}(2-\bar{\varepsilon})].$$

Where Q= S/4 , r =albedo

A= solar absorptivity ε = atmos emissivity

For r=0.3 , A=0.2, ε= 0.95

T = 288 K close to the observation

Limiting Cases :

No atmosphere : A= 0 and ε= 0

$T^4 = S/4 \{ 1 - r\}$ **for r =0.3 T=255 K**

No atm solar absorp, ε = 1, & r=0.3

$T^4 = S/4 \{2 (1 - r)\}$ **T =303 K**

SENSITIVITY

$$\frac{1}{T_s}\frac{\partial T_s}{\partial S} = \frac{1}{4S}$$

Δ S=.01, ΔT_s= 0.75K

$$\frac{1}{T_s}\frac{\partial T_s}{\partial A} = \frac{-S/4}{4\sigma T_s^4(2-\bar{\epsilon})}$$

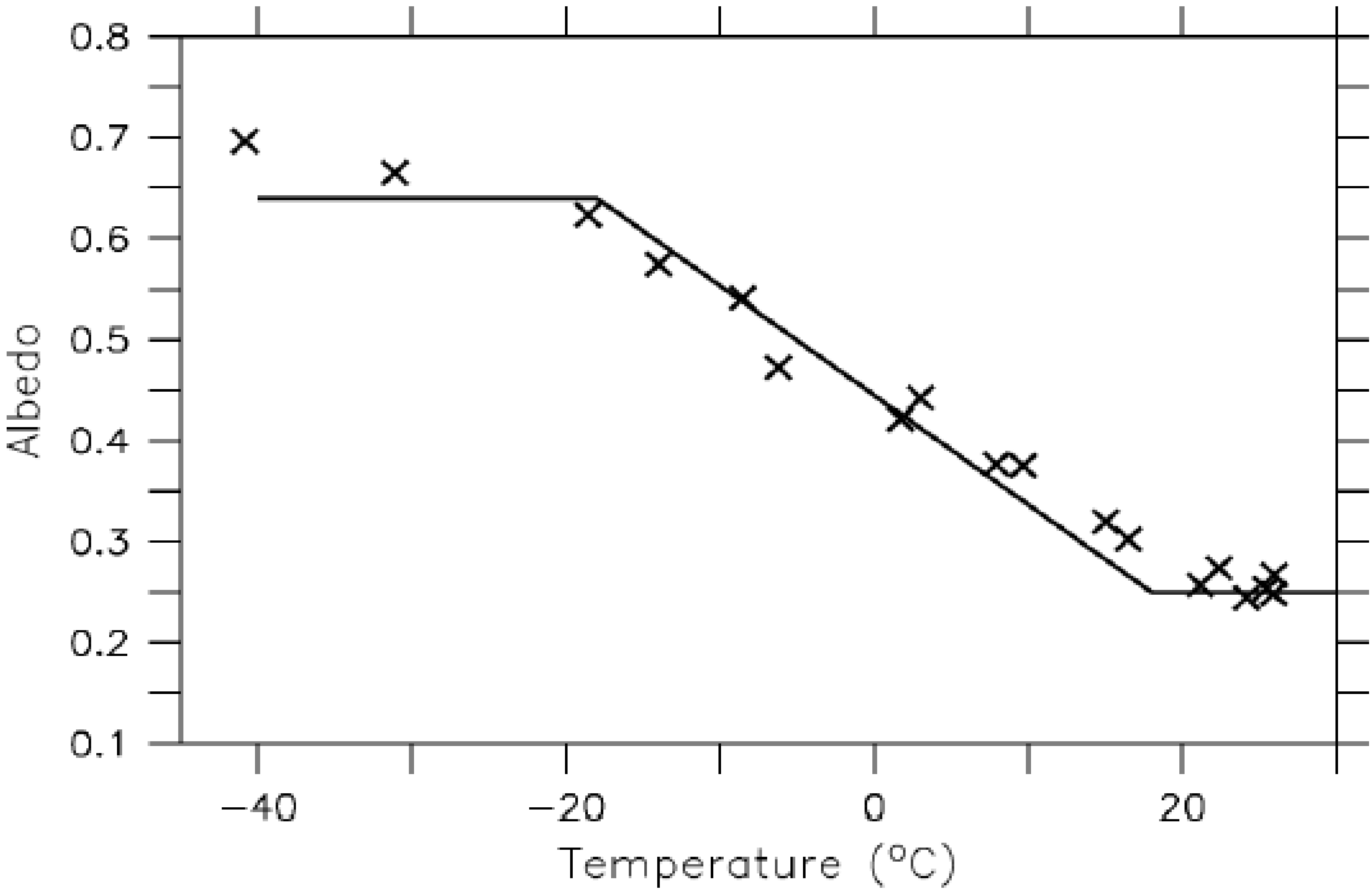
ΔA = 0.01 ΔT_s= -0.60K

$$\frac{1}{T_s}\frac{\partial T_s}{\partial \bar{r}} = \frac{-S/2}{4\sigma T_s^4(2-\bar{\epsilon})}$$

Δr = 0.01 ΔT_s= -1.20K

$$\frac{1}{T_s}\frac{\partial T_s}{\partial \bar{\epsilon}} = \frac{1}{4(2-\bar{\epsilon})}$$

$\Delta\varepsilon$ = 0.01 ΔT_s= 0.75K



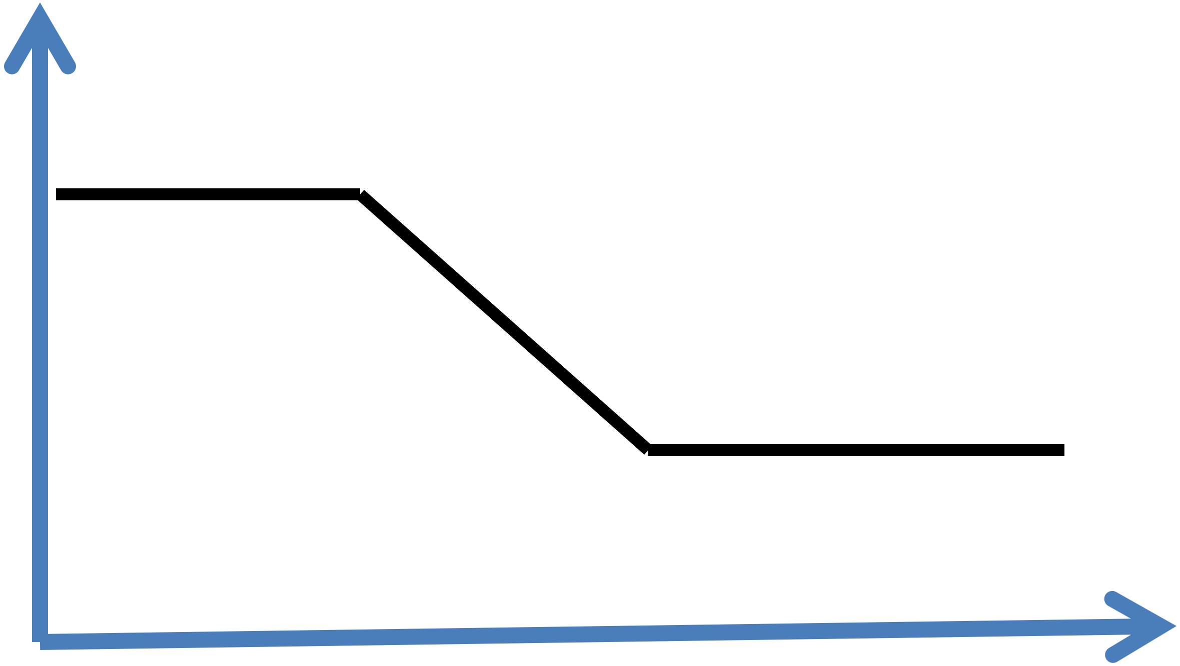
ALBEDO FROM SATELLITE (ERBE) DATA

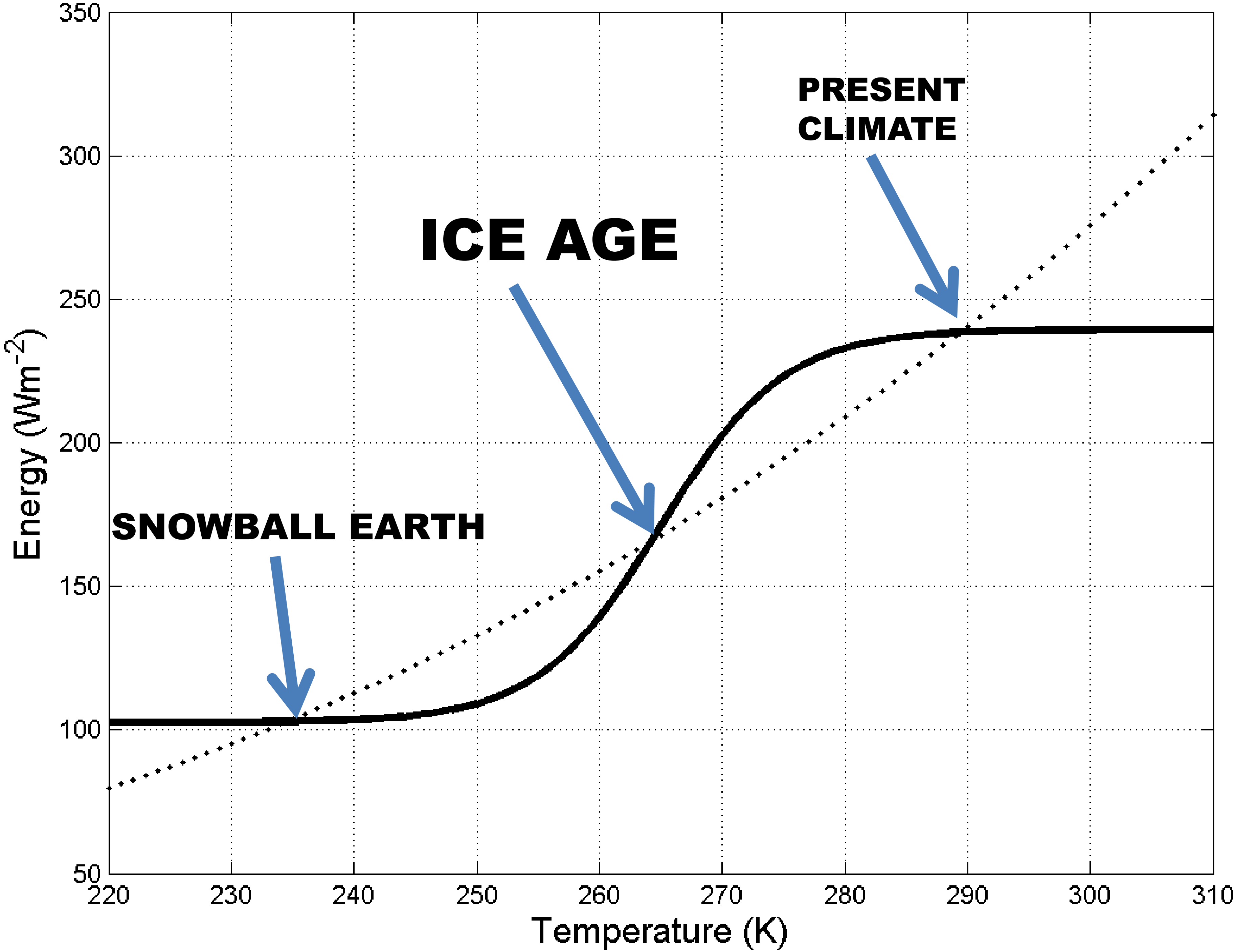
$$S/4\,\{1 - \rho(T)\} = A + B*T$$

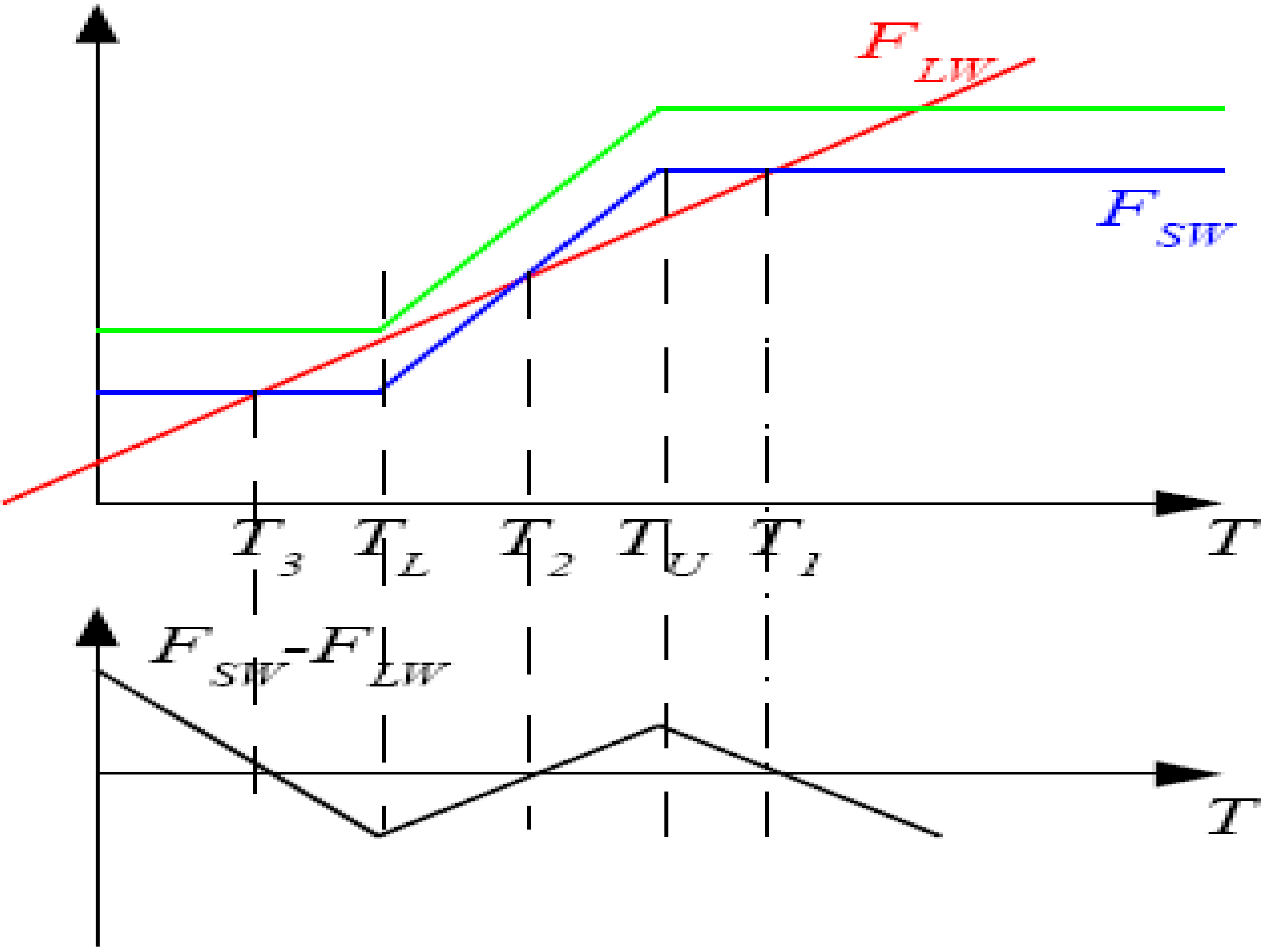
$$\rho(T) = \rho_o \quad \text{for } T < T_o$$

$$\rho(T) = \rho_1 \quad \text{for } T > T_1$$

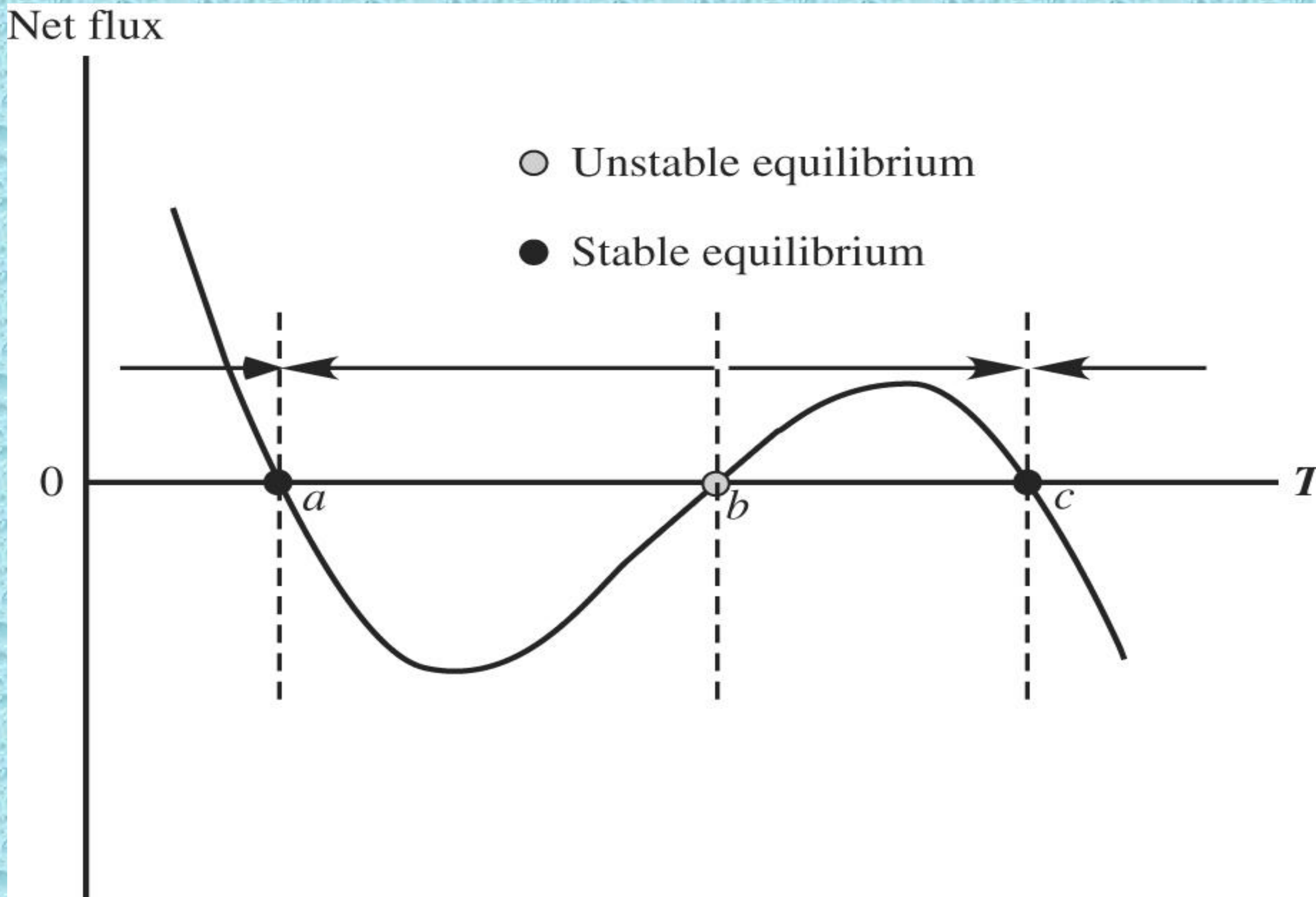
$$\rho(T) = \rho_o + (\rho_1 - \rho_o)(T - T_o)/(T_1 - T_o)$$

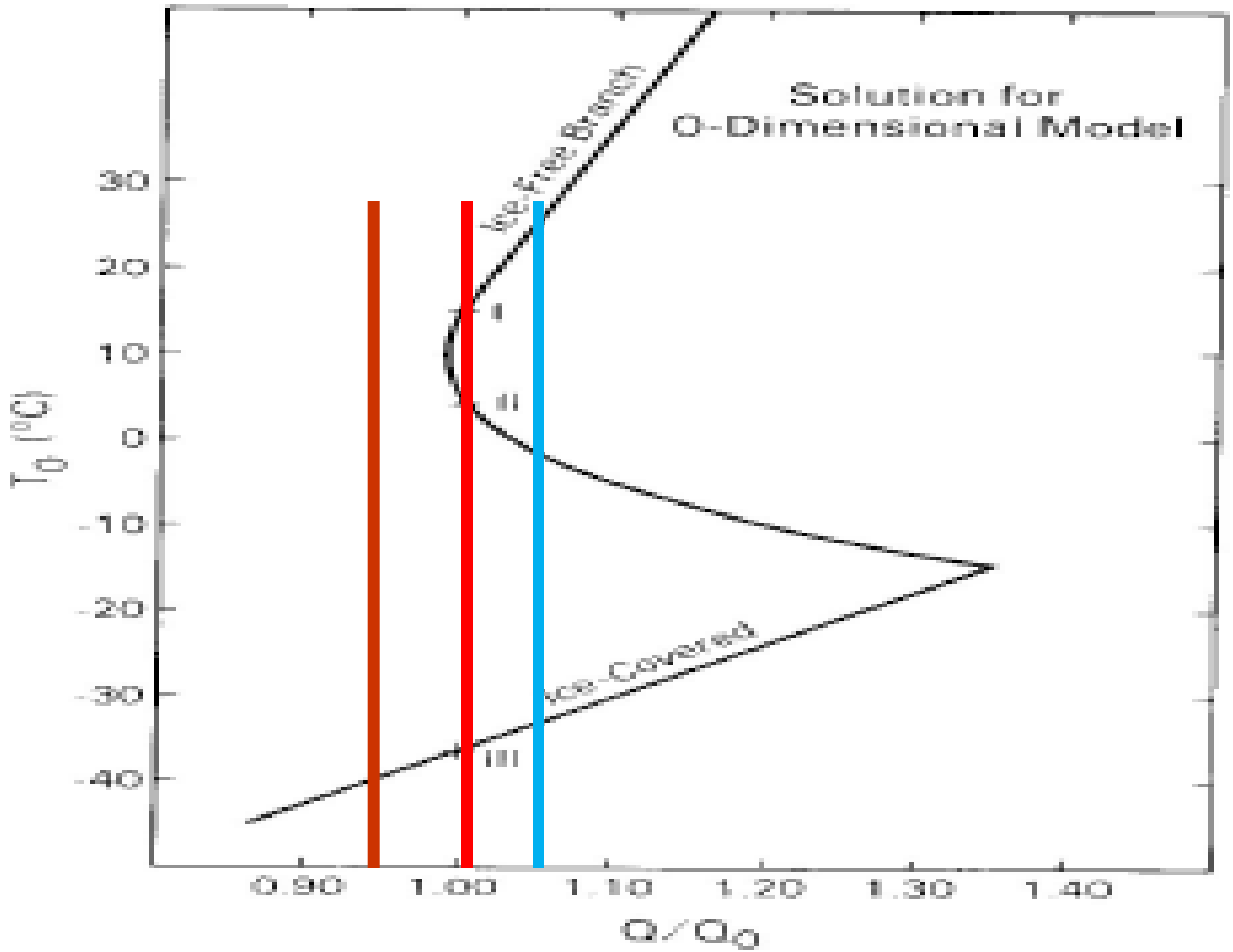



PRESENT
CLIMATE
ICE AGE
SNOWBALL EARTH
350
300
250
200
150
100
50
220
230
240
250
260
270
280
290
300
310
Energy (Wm-2)
Temperature (K)

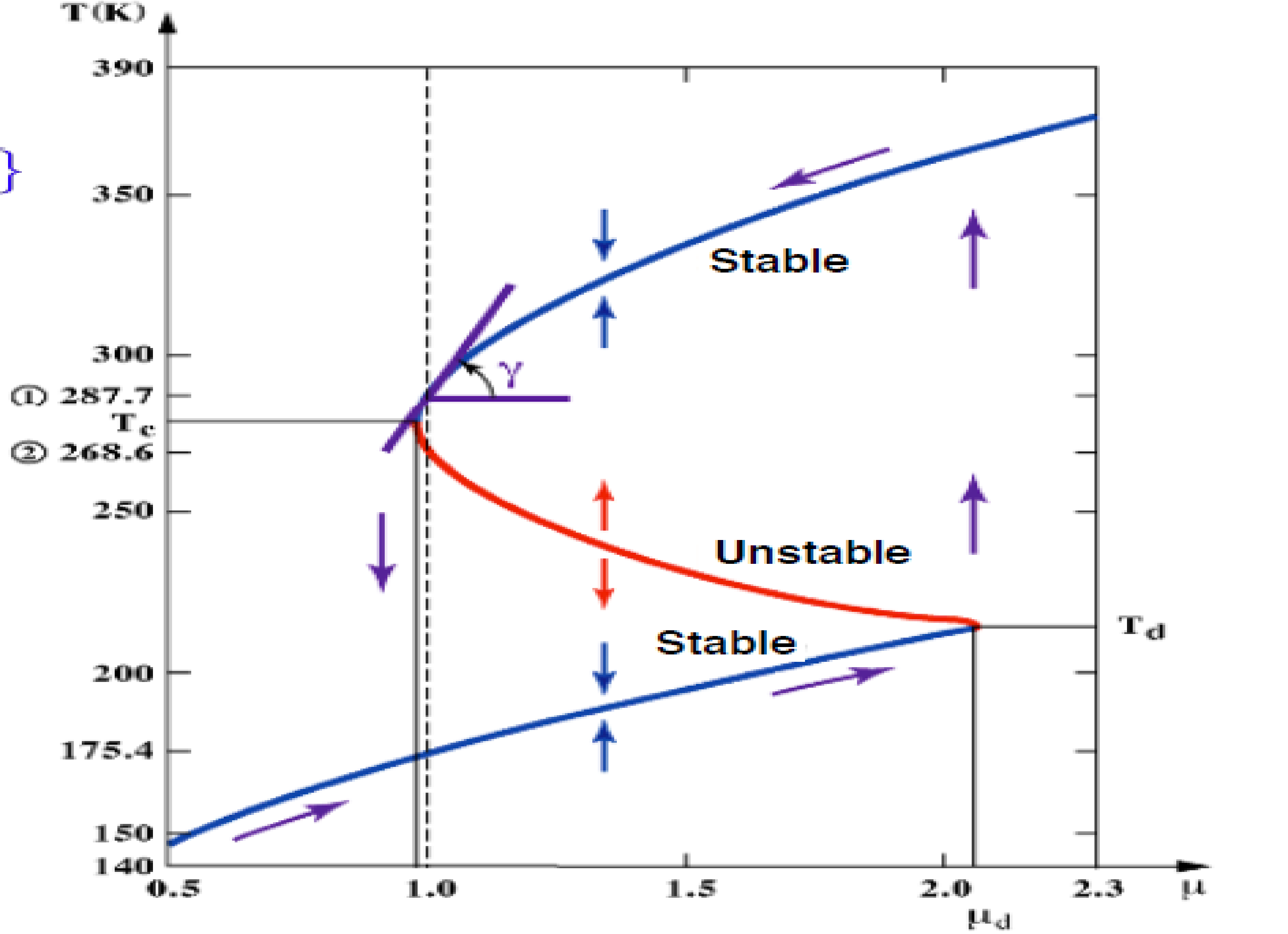


F_{LW}
F_{SW}
T_3
T_L
T_2
T_U
T_1
T
F_{SW}-F_{LW}
T


Net flux
Unstable equilibrium
Stable equilibrium
0
T
a
b
c



Solution for
0-Dimensional Model
Ice-Free Branch
I
II
III
Ice-Covered
30
20
10
0
-10
-20
-30
-40
T_0 (°C)
0.90
1.00
1.10
1.20
1.30
1.40
Q/Q_0



T(K)
390
350
300
① 287.7
T_c
② 268.6
250
200
175.4
150
140
0.5
1.0
1.5
2.0
2.3
μ
μ_d
T_d
γ
Stable
Unstable
Stable

Stability characteristics

The stability of these equilibria can be investigated by **linearizing** the equation

$$C\frac{dT}{dt} = Q\left(1-\alpha(T)\right) - a^* - b^* T$$

***C* is the heat capacity of the system per unit surface area**.
This is a nonlinear first order differential equation. The standard method of investigating the stability of the equilibrium solution is to add a perturbation:

$$T(t) = T_{eq} + T'(t)$$

The albedo may be written as:

$$\alpha(T) \cong \alpha\left(T_{eq}\right) + \frac{d\alpha}{dT}T'$$

Inserting the above two equations into the top equation we obtain

$$\boxed{\frac{dT'}{dt} = -\lambda T'}$$ with $$\boxed{\lambda = \frac{b^* + Q\dfrac{d\alpha}{dT}}{C}}$$

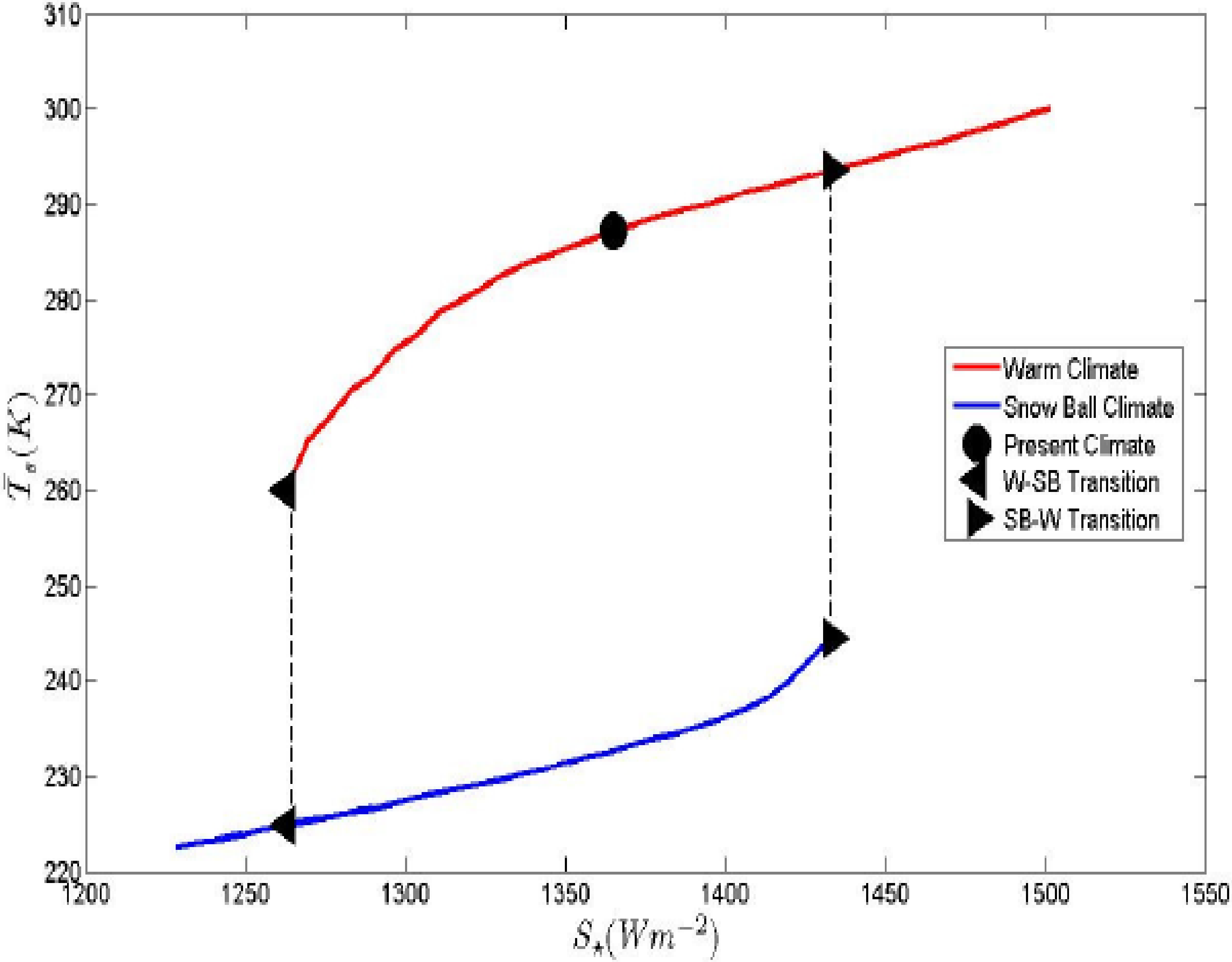
The solution of this differential equation is $T' = T'(0)\exp(-\lambda t)$

Therefore there is decay back to equilibrium if $\lambda > 0$ and **exponential growth of the amplitude of the perturbation (instability) if $\lambda < 0$**. The sign of λ is determined by the sign of $d\alpha/dT$.

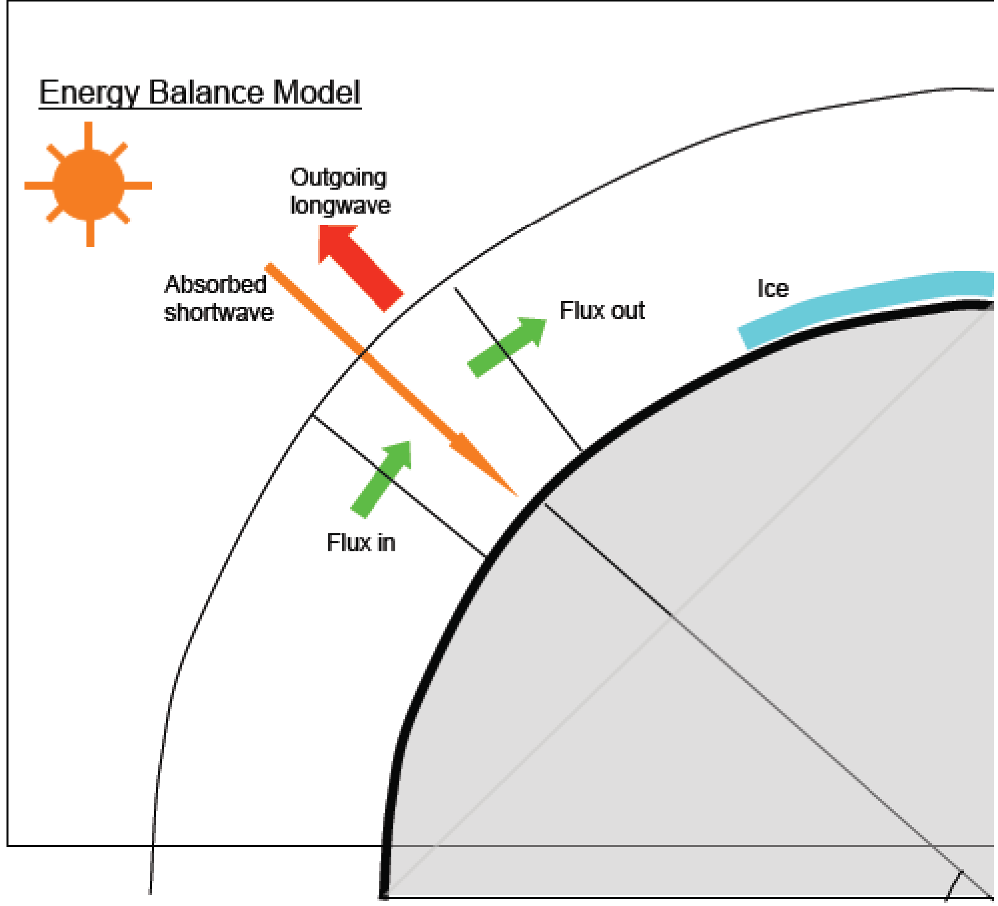
For $T > T_1$ or $T < T_0$ $d\alpha/dT = 0$. Therefore, the solution for these equilibria are stable (because $b^* > 0$).

For $T_0 < T < T_1$ $d\alpha/dT < 0$. Therefore, the solution for this intermediate equilibrium state is unstable if

$$\frac{d\alpha}{dT} < \frac{b^*}{Q}$$



310
300
290
280
270
260
250
240
230
220
1200
1250
1300
1350
1400
1450
1500
1550
$\overline{T}_s(K)$
$S_\star(Wm^{-2})$
Warm Climate
Snow Ball Climate
Present Climate
W-SB Transition
SB-W Transition



$$\underbrace{Q_0/4(1-\alpha)S(\phi)}_{\text{Net heat absorbed from the sun}} = \underbrace{(A+BT)}_{\text{Longwave emitted to space}} - \underbrace{D\frac{d^2T}{dy^2}}_{\text{Heat divergence: net heat transported out of the box by atmosphere and ocean.}}$$

$$-\frac{d}{dx}D(1-x^2)\frac{dT(x)}{dx}+A+BT(x)=QS(x)a(x,x_s)$$

DIFFUSIVE

OR

CONVECTIVE

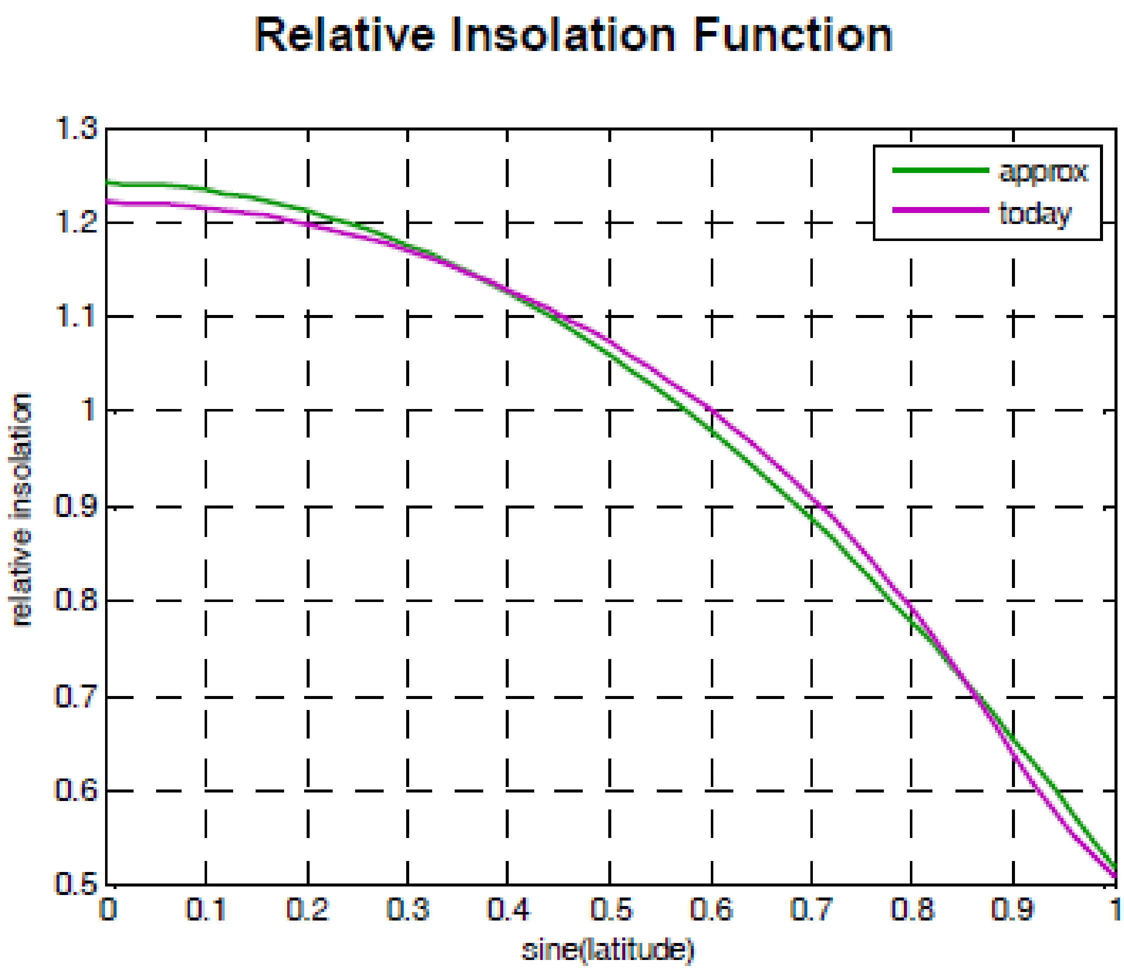
$$\gamma[T(x)-T_0]$$

Energy Balance Models

Inhomogeneous Earth

green = quadratic approximation (Tung and North)

mauve = formula using obliquity of 23.5°



Variation of incident solar radiation with latitude

$$341\,(0.53 + 0.71\,\mathrm{Cos}^2\,\theta)$$

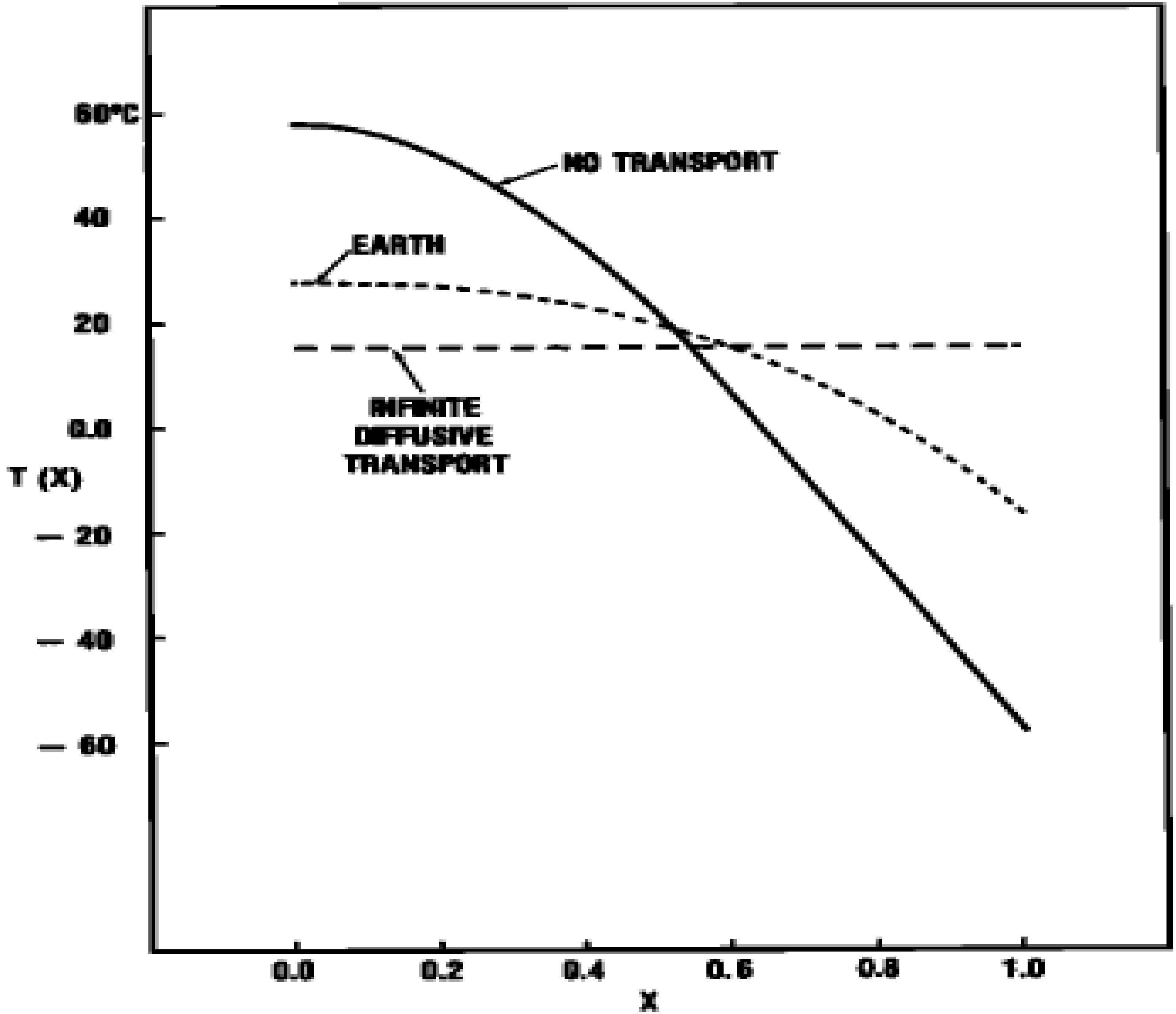
$$\mathrm{Albedo} = 0.6 - 0.4\,\mathrm{Cos}\,\theta$$

$$\mathrm{OLR} = 216 + 1.58\,T$$

$$\text{Meridional Heat transfer} = C\,(T - T_{av})$$

$$C = 3.8\ \mathrm{W/m^2\ {}^oC}$$

$$T = \{341\,(0.53 + 0.71\,\mathrm{Cos}^2\,\theta)(0.4 + 0.4\,\mathrm{Cos}\,\theta) - A - C\}/\{A+B\}$$



60°C
40
20
0.0
T (X)
— 20
— 40
— 60
NO TRANSPORT
EARTH
INFINITE
DIFFUSIVE
TRANSPORT
0.0
0.2
0.4
0.6
0.8
1.0
X

$$-\frac{d}{dx}D(x)(1-x^2)\frac{d}{dx}T(x)+A+BT(x)=QS(x)a(x)$$

Boundary Conditions:

$$\sqrt{(1-x^2)}\frac{d}{dx}T(x)\Big|_{x=0,1}=0$$

(No Heat Flux into Poles or across Equator)

$S(x)$=Distribution of Solar Insolation

Very Simple Solution:

$$If \quad a(x) = a_0$$

Legendre Polynomial
$P_2(x)$

$$Then \quad T(x) = T_0 + T_2 \cdot \overline{\frac{1}{2}(3x^2 - 1)}$$

$$T_0 = \frac{Qa_0 \cdot 1 - A}{B}, \quad T_2 = \frac{Qa_0 S_2}{6D + B}$$

$$T_0 \approx 15^0 C, \quad T_2 \approx -28^0 C$$

Energy Balance Models

Homogeneous Earth

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

T = global mean temperature (°C)
Q = mean solar input (W/m²)
α = mean albedo
$A+BT$ = outward radiation (linear approximation)
R = heat capacity of Earth's surface

Tung's values:

T = global mean temperature (°C)
Q = 343 W/m²
A = 202 W/m²
B = 1.9 W/(m² °C)

$\alpha = \alpha_1 = 0.32$ (water and land)
$\alpha = \alpha_2 = 0.62$ (ice)

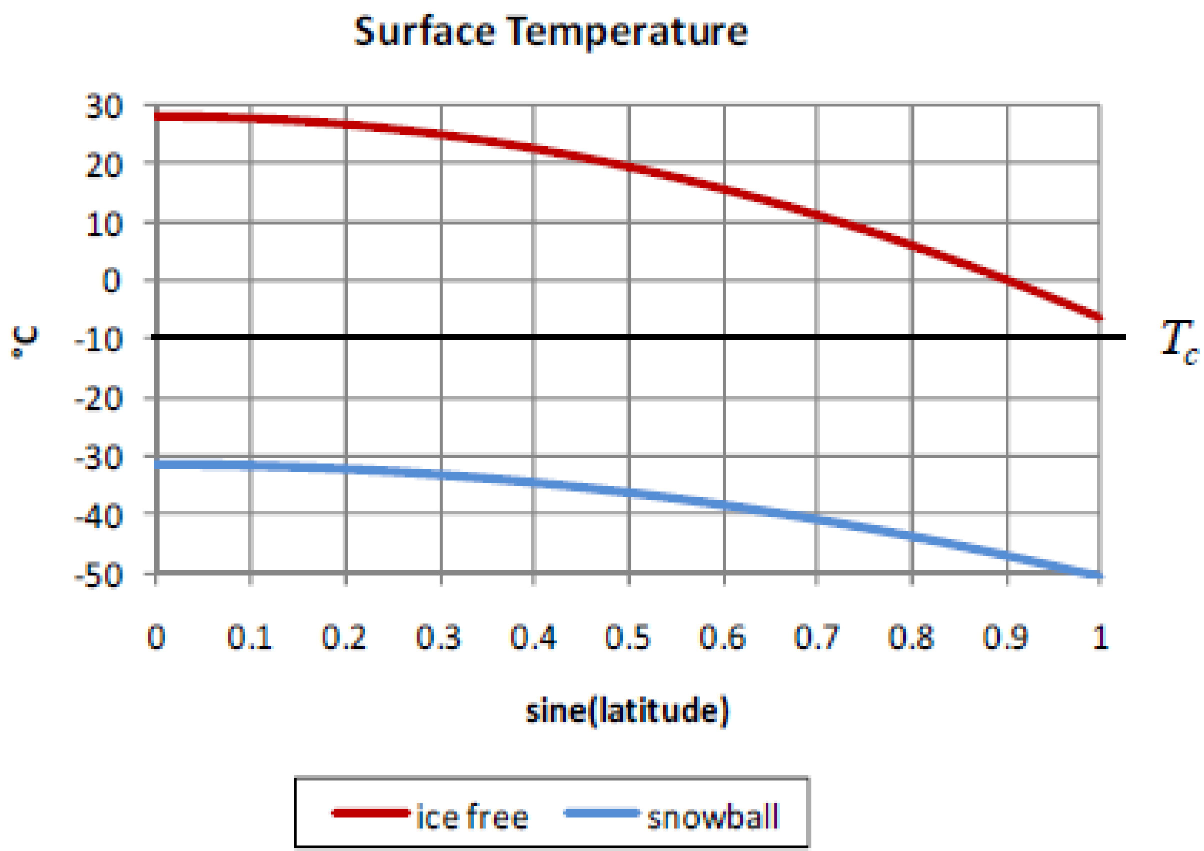
Energy Balance Models

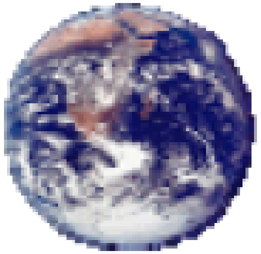
Inhomogeneous Earth

Assume that $\alpha(y) = \alpha_i$ (constant).

$$T^*(y) = \frac{1}{B+C}\left(Qs(y)(1-\alpha_i) - A + C\overline{T^*}\right)$$

Both ice-free ($\alpha = \alpha_1 = 0.32$) and snowball ($\alpha = \alpha_2 = 0.62$) states look pretty stable.



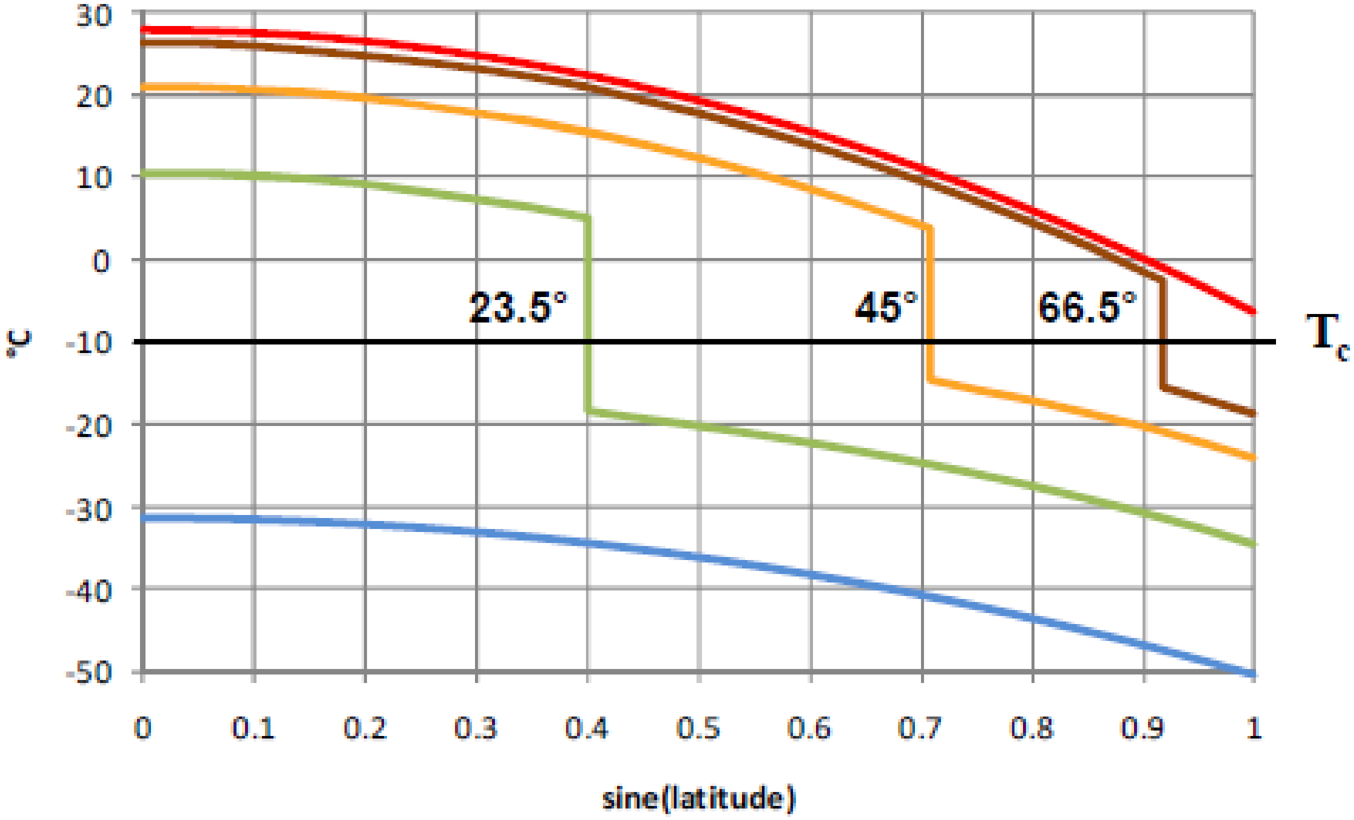


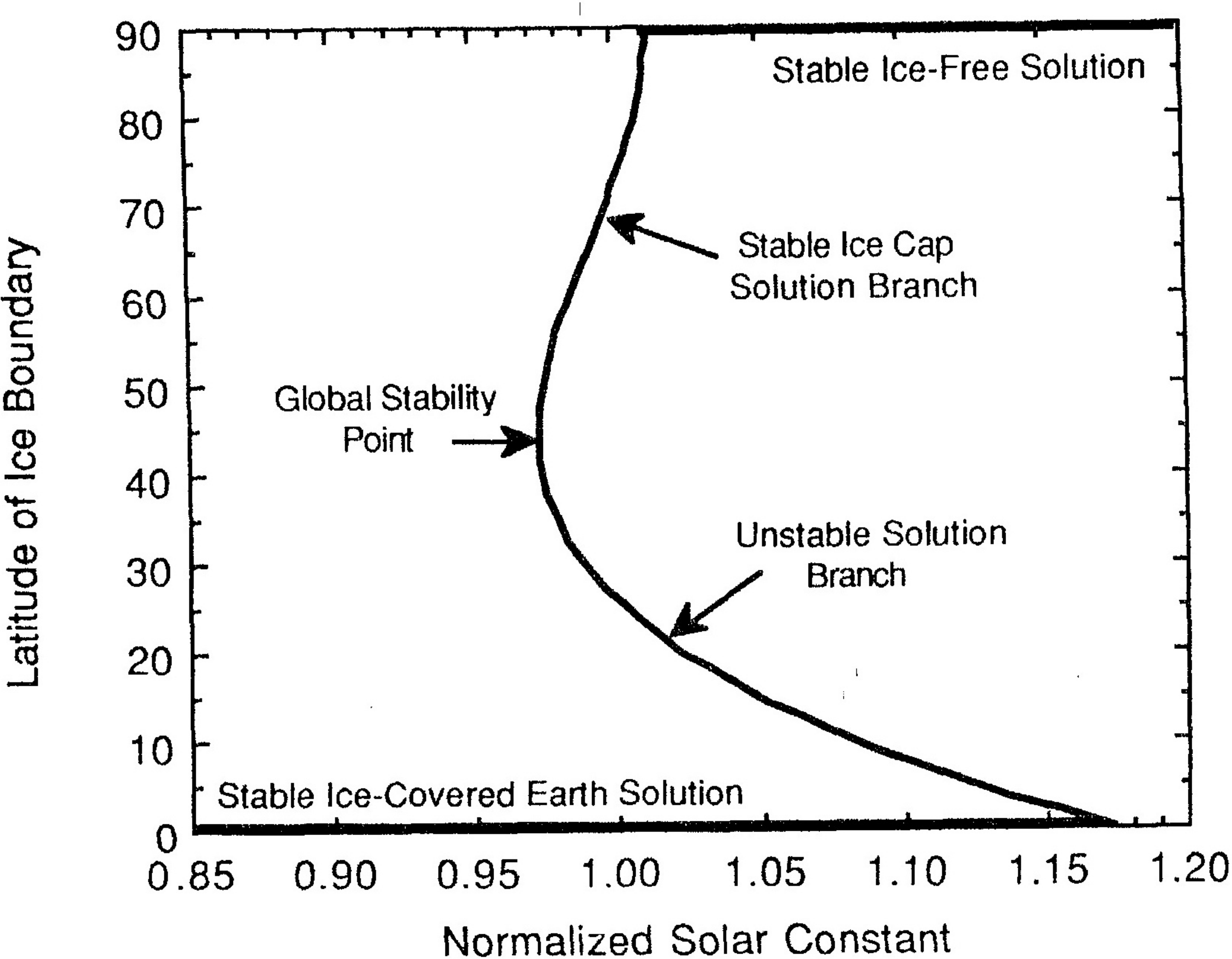
Energy Balance Models

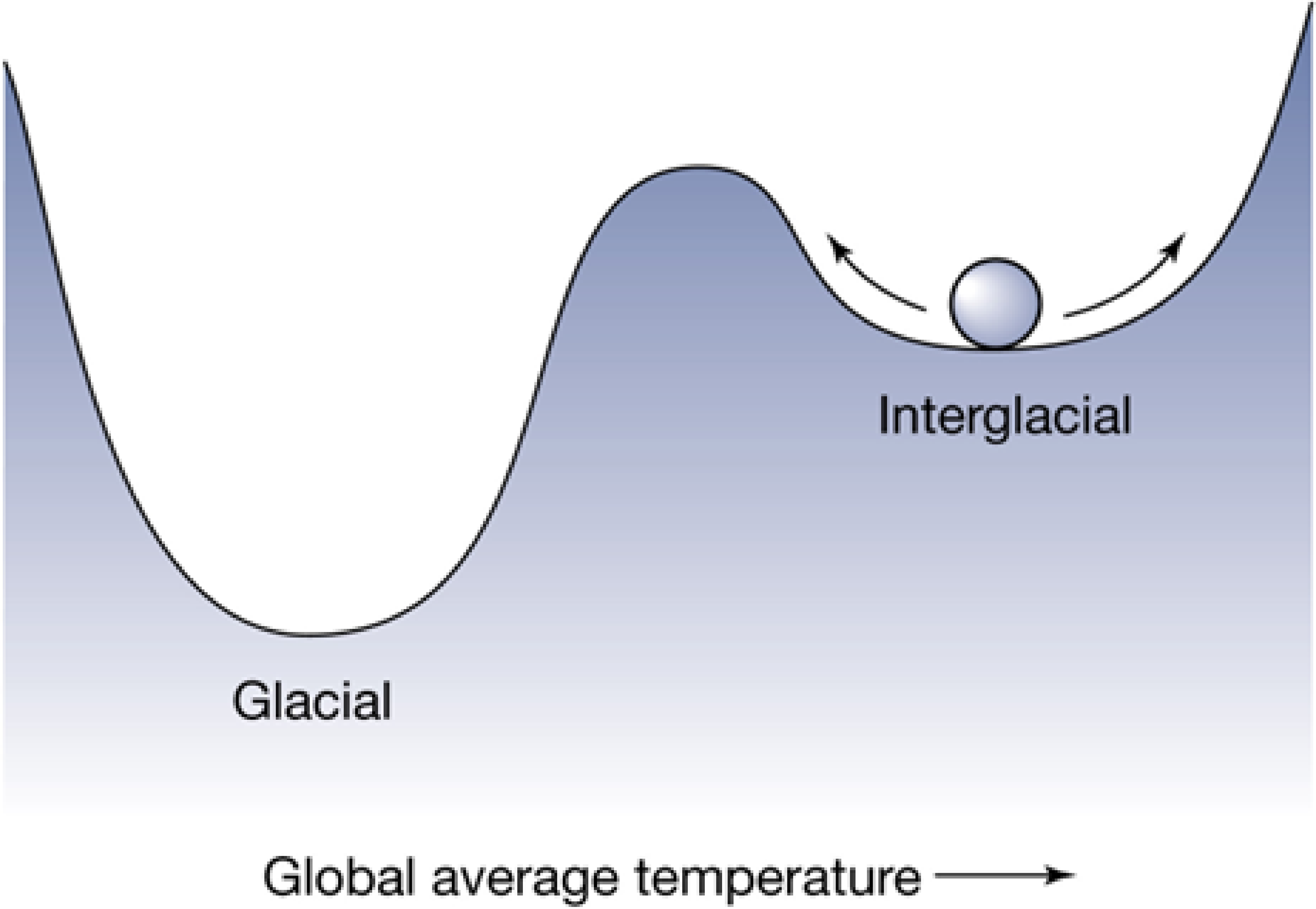
Inhomogeneous Earth

Temperature profiles for various ice lines

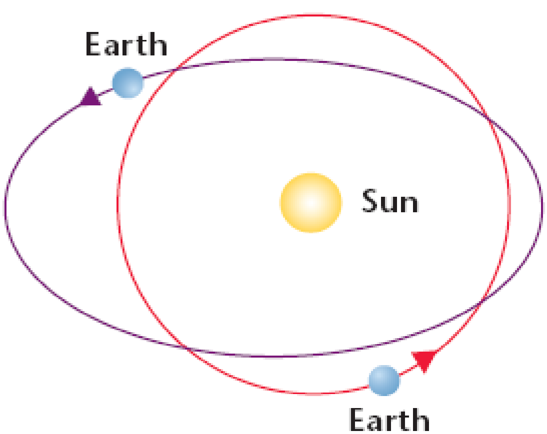
The standard argument seems to imply that these are all stable.



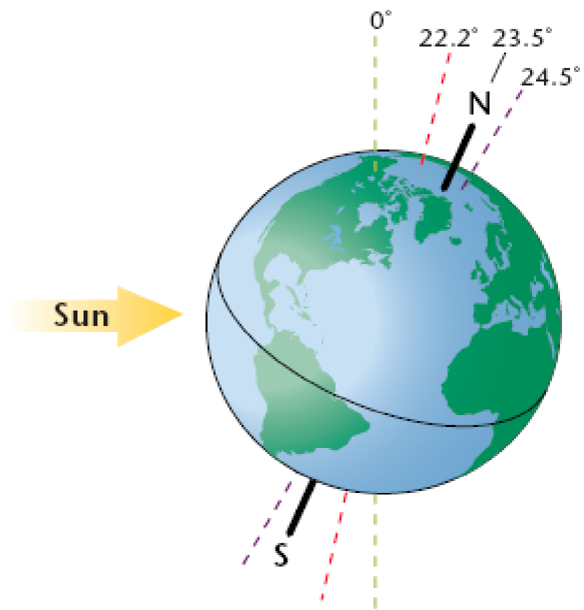

Stable Ice-Free Solution
Stable Ice Cap
Solution Branch
Global Stability
Point
Unstable Solution
Branch
Stable Ice-Covered Earth Solution
Latitude of Ice Boundary
Normalized Solar Constant
0
10
20
30
40
50
60
70
80
90
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20


Interglacial
Glacial
Global average temperature

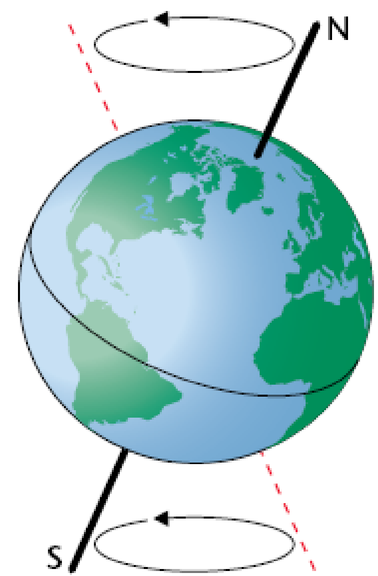
Changes in Earth's ORBIT around the Sun can Change the Climate



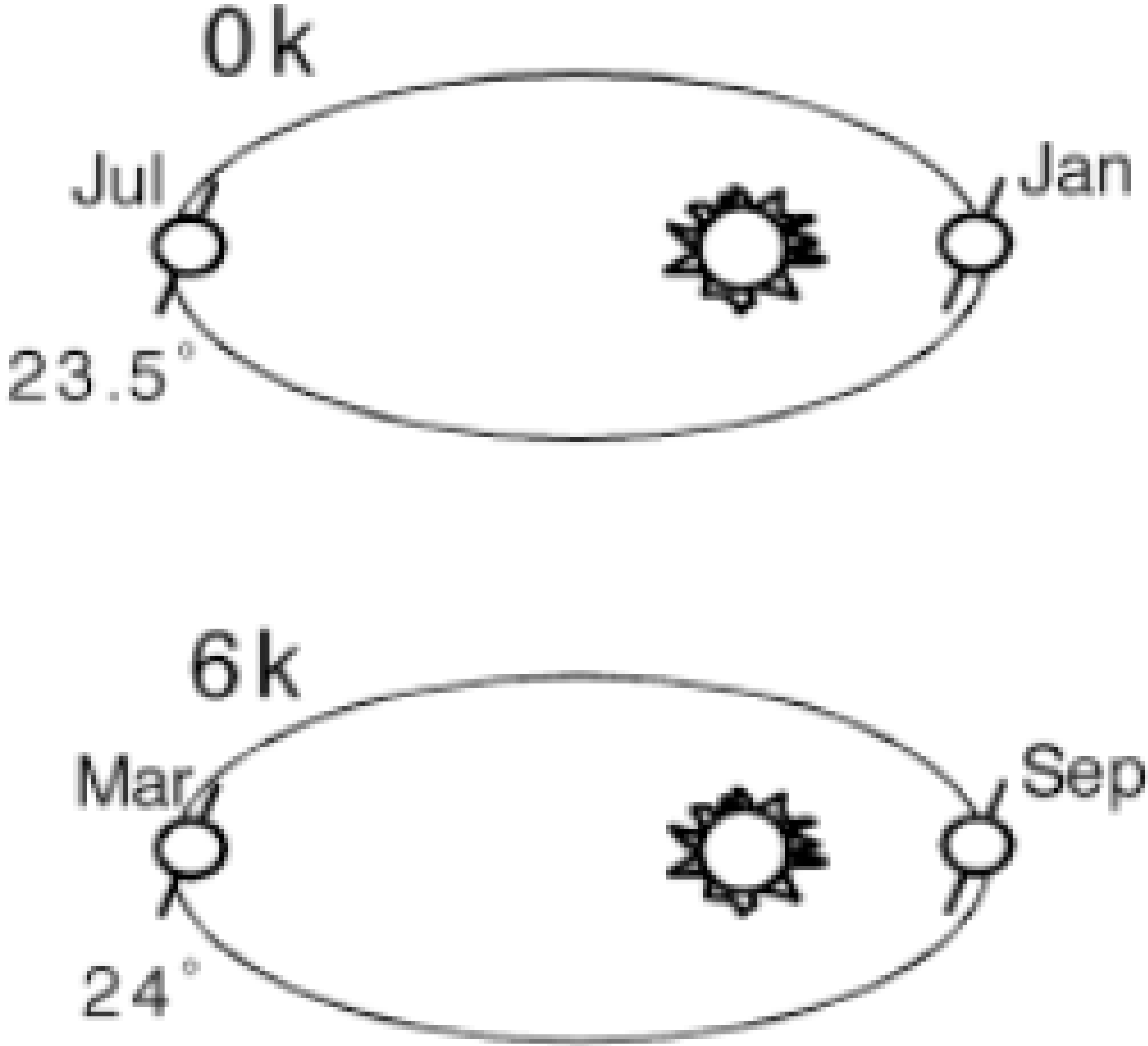
Eccentricity Changes in orbital eccentricity cause an increase in seasonality in one hemisphere and reduce seasonality in the other hemisphere.



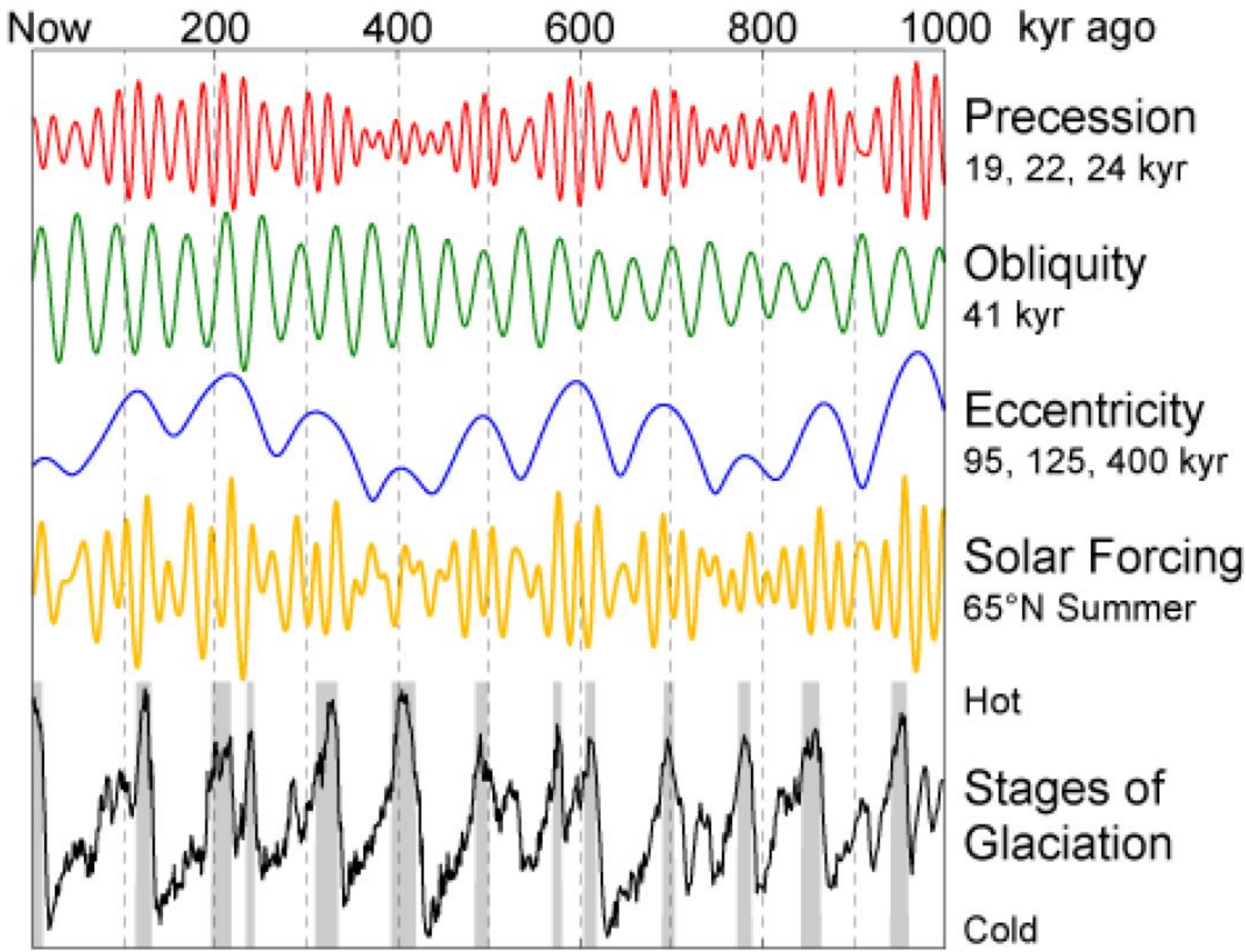
Tilt Over a period of 41,000 years, the tilt of Earth's axis varies between 22.2° and 24.5°. The poles receive more solar energy when the tilt angle is greater.

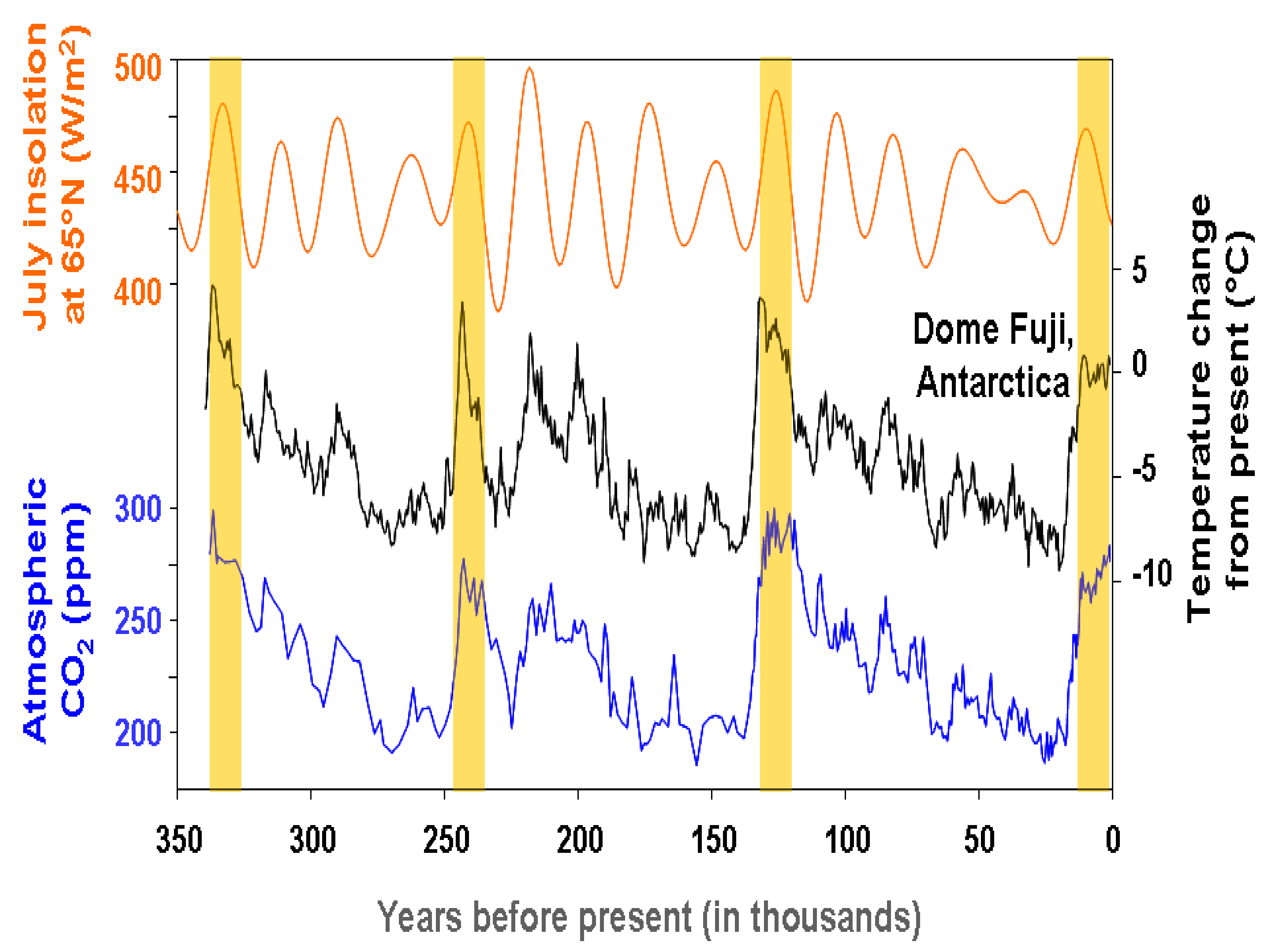


Precession The wobble of Earth's axis affects the amount of solar radiation that reaches different parts of Earth's surface at different times of the year.

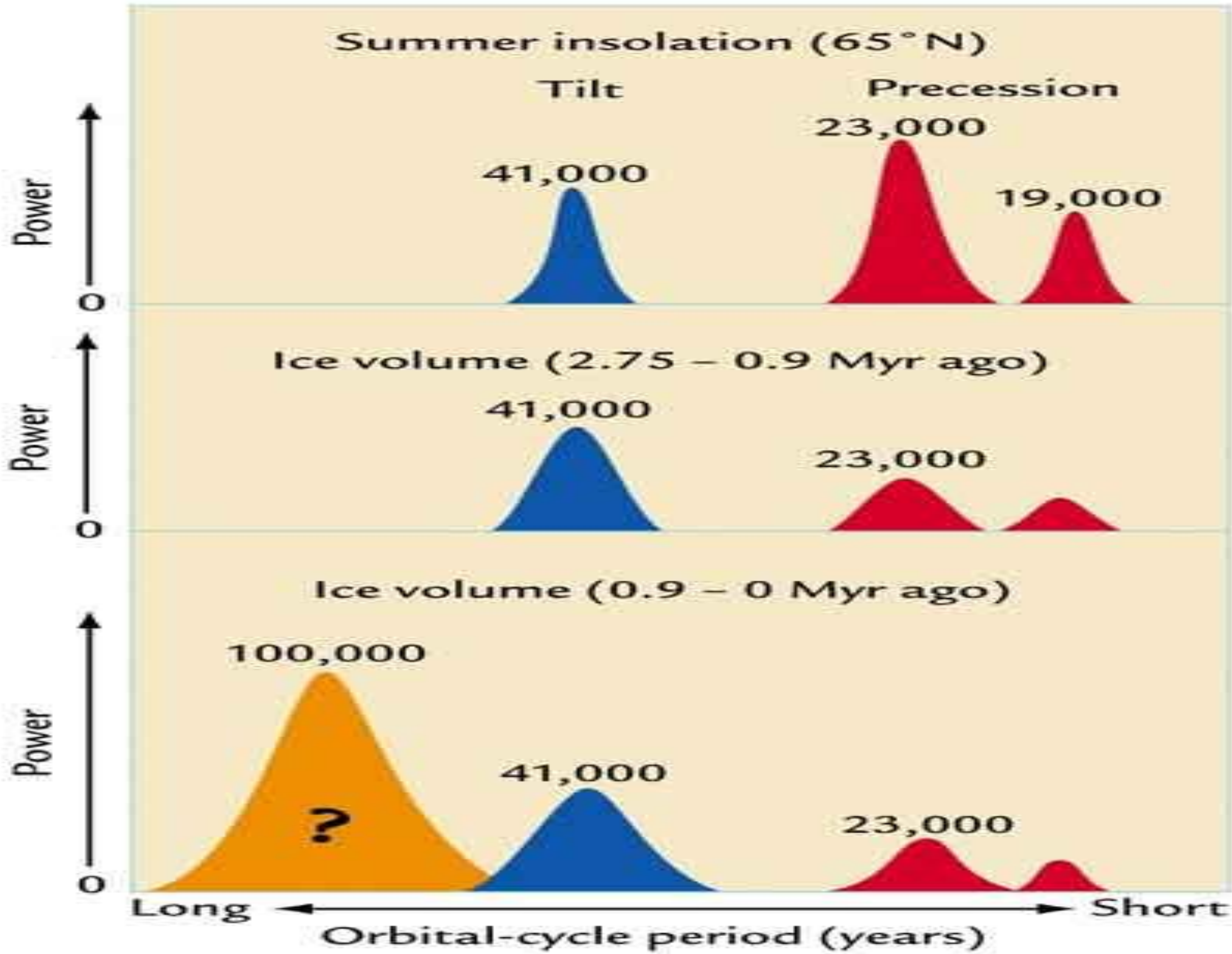


0 k
Jul
Jan
23.5°
6 k
Mar
Sep
24°

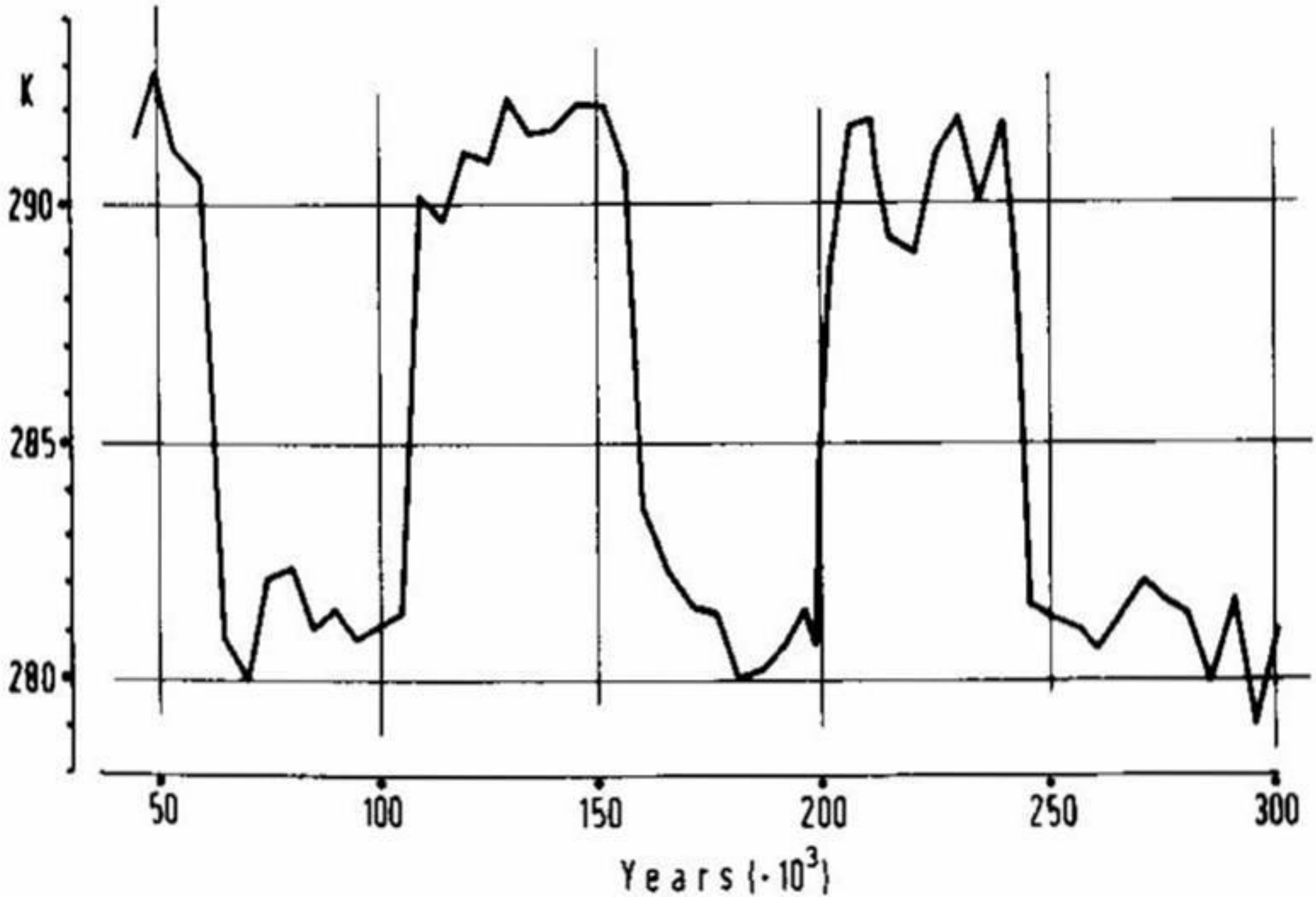

Now
200
400
600
800
1000
kyr ago
Precession
19, 22, 24 kyr
Obliquity
41 kyr
Eccentricity
95, 125, 400 kyr
Solar Forcing
65°N Summer
Hot
Stages of
Glaciation
Cold



July insolation at 65°N (W/m2)
500
450
400
Atmospheric CO2 (ppm)
300
250
200
Dome Fuji, Antarctica
Temperature change from present (°C)
5
0
-5
-10
350
300
250
200
150
100
50
0
Years before present (in thousands)


Summer insolation (65°N)
Tilt
Precession
23,000
41,000
19,000
Power
0
Ice volume (2.75 – 0.9 Myr ago)
41,000
23,000
Power
0
Ice volume (0.9 – 0 Myr ago)
100,000
?
41,000
23,000
Power
0
Long
Short
Orbital-cycle period (years)

STOCHASTIC RESONANCE



A zero-dimensional energy balance model

$$\frac{dT}{dt} = F(T)$$

Solutions of *F(T)*= 0 represent <u>steady</u> or <u>equilibrium states</u> .

To investigate the stability properties, introduce the the concept of <u>pseudo-potential</u>

$$\Phi = -\int F(T)dT$$

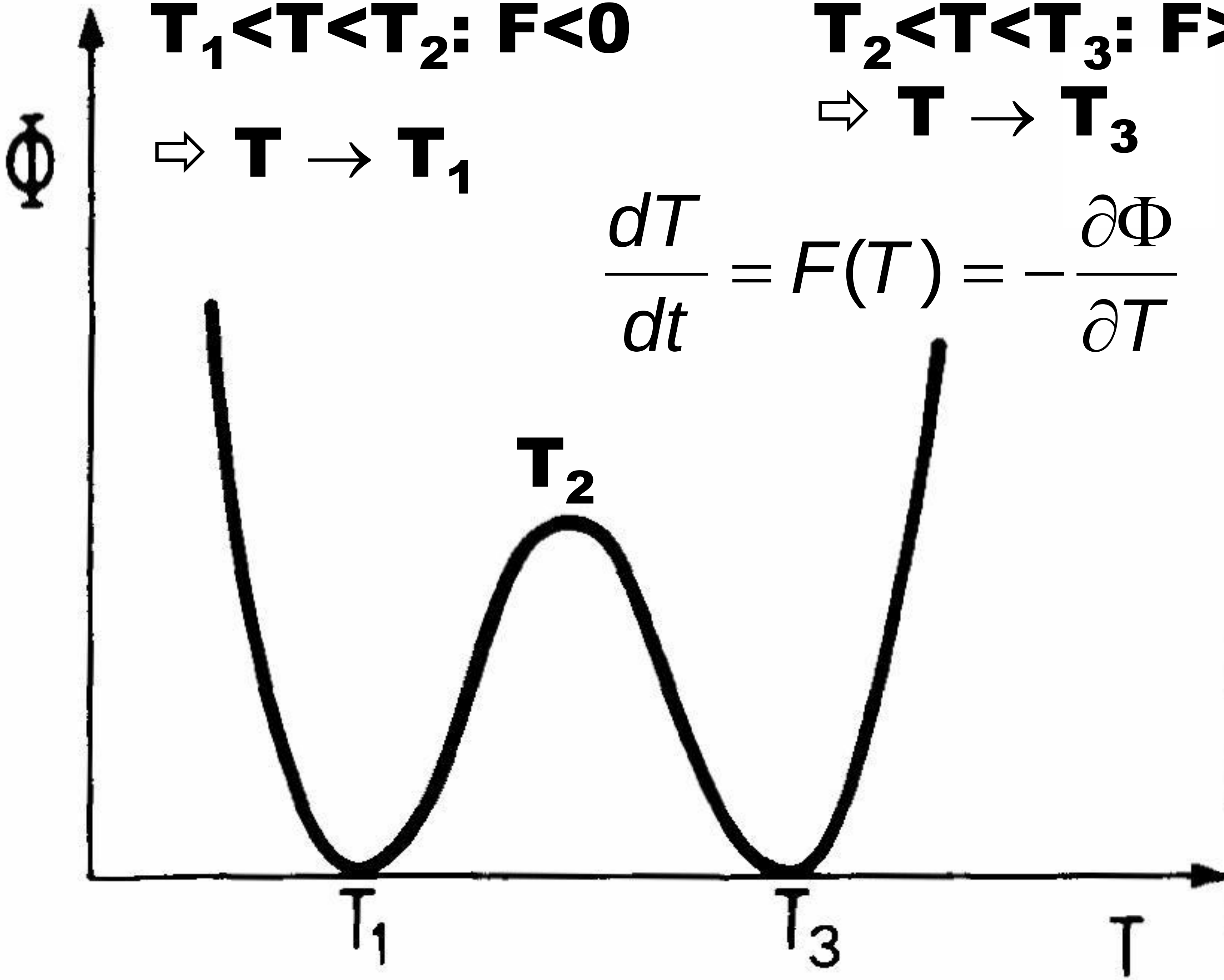
$T_1<T<T_2$: $F<0$

⇨ $T \to T_1$

$T_2<T<T_3$: $F>0$

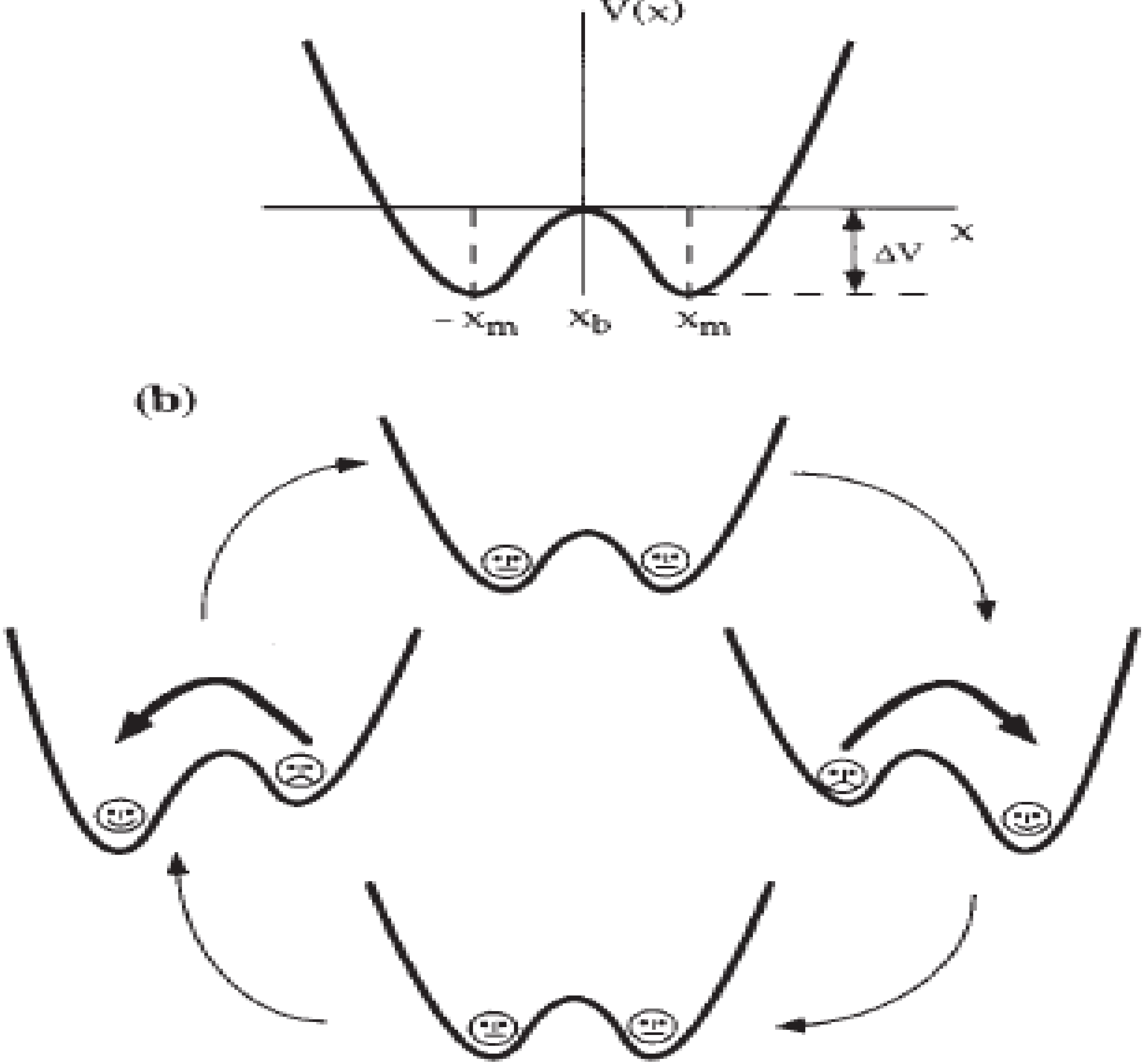
⇨ $T \to T_3$

$$\frac{dT}{dt} = F(T) = -\frac{\partial \Phi}{\partial T}$$

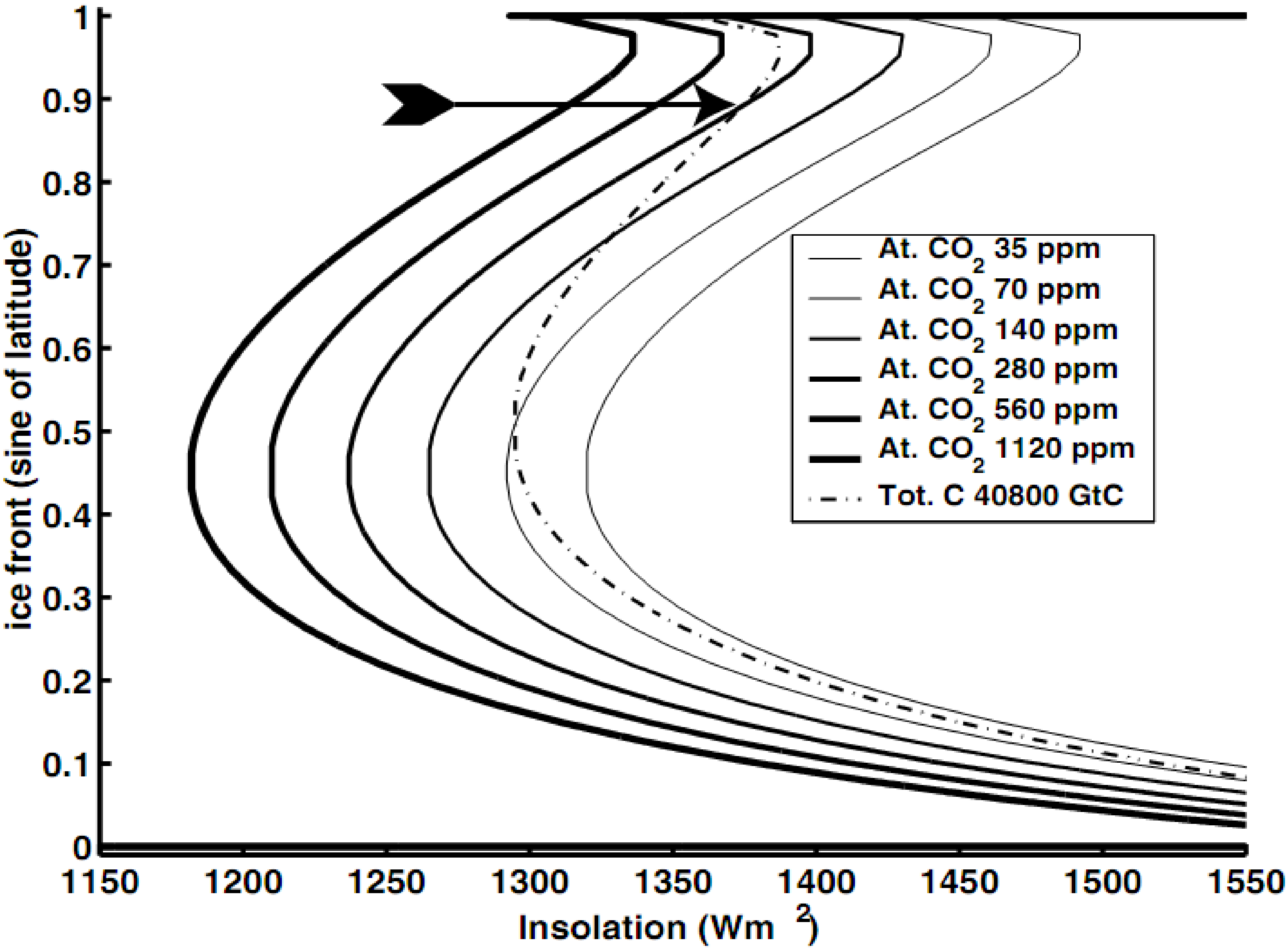


$$\widetilde{F}(T,t) = F(T) \times (1 + 0.0005 \cos \varpi t)$$

$$\varpi = 2\pi / 10^5 \text{years}$$


V(x)
x
ΔV
− x_m
x_b
x_m
(b)

- **Orbital forcing of a simple energy balance model results in the right spectrum, but the amplitude is too small.**
- **Noise added to a simple model with prescribed stable equilibrium states results in the right amplitude, but the spectrum shows no peak.**
- **Combination of both is able to explain both amplitude and frequency of observed climate shifts.**



ice front (sine of latitude)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1150
1200
1250
1300
1350
1400
1450
1500
1550
Insolation (Wm⁻²)
At. CO_2 35 ppm
At. CO_2 70 ppm
At. CO_2 140 ppm
At. CO_2 280 ppm
At. CO_2 560 ppm
At. CO_2 1120 ppm
Tot. C 40800 GtC

SALTZMAN'S MODEL

$$\frac{dX}{dt} = -\alpha_1 Y - \alpha_2 Z - \alpha_3 Y^2$$

$$\frac{dY}{dt} = -\beta_0 X + \beta_1 Y + \beta_2 Z - \left(X^2 + 0.004Y^2\right)Y + F_Y$$

$$\frac{dZ}{dt} = X - \gamma_2 Z$$

where in this particular case X, Y and Z are the ice mass, deep ocean temperature and atmospheric carbon dioxide.

where X is ice mass,

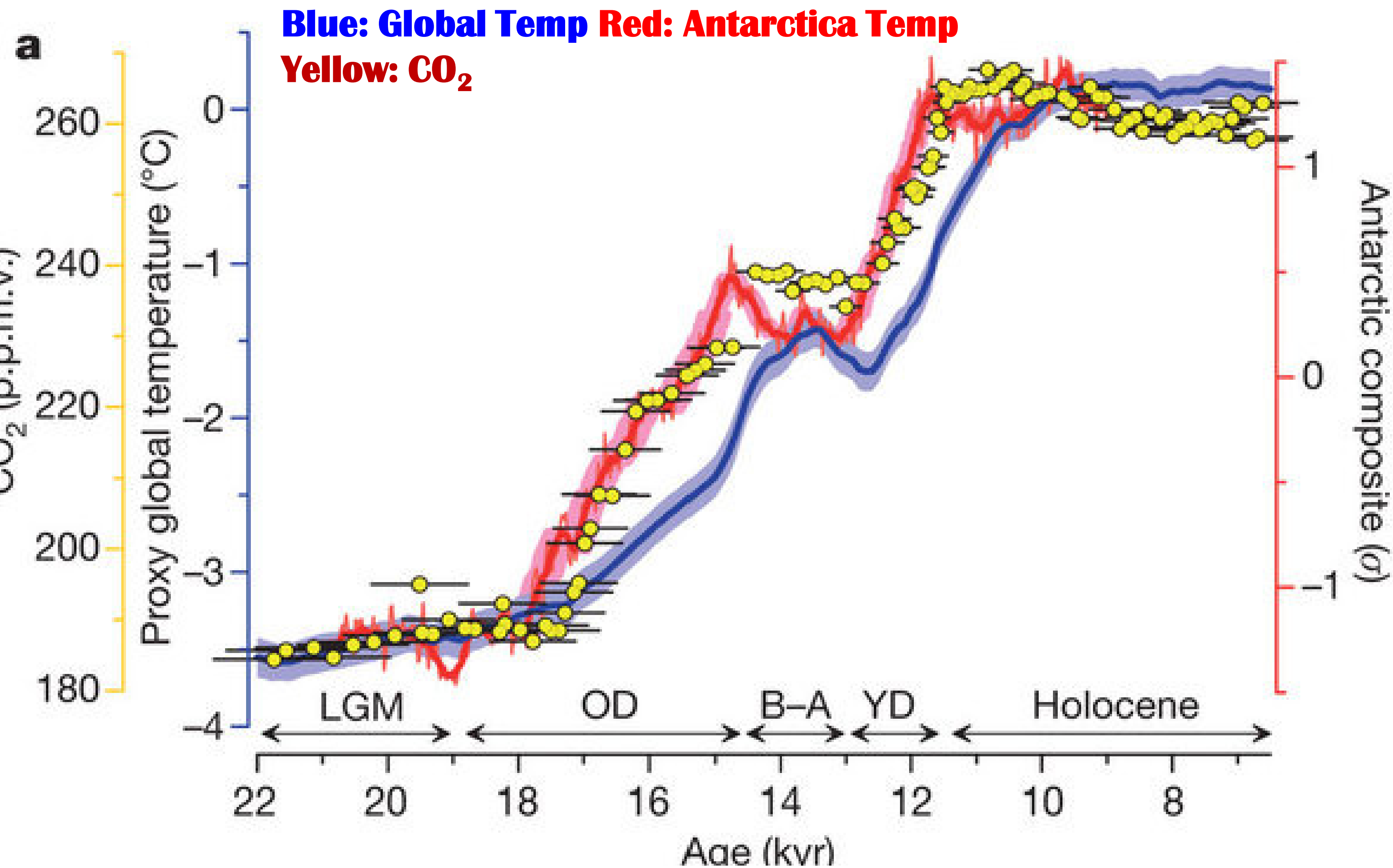
Y is ocean temperature

Z is CO_2

Common Misconception

CO_2 is a very minor constituent of the atmosphere and hence cannot influence climate change

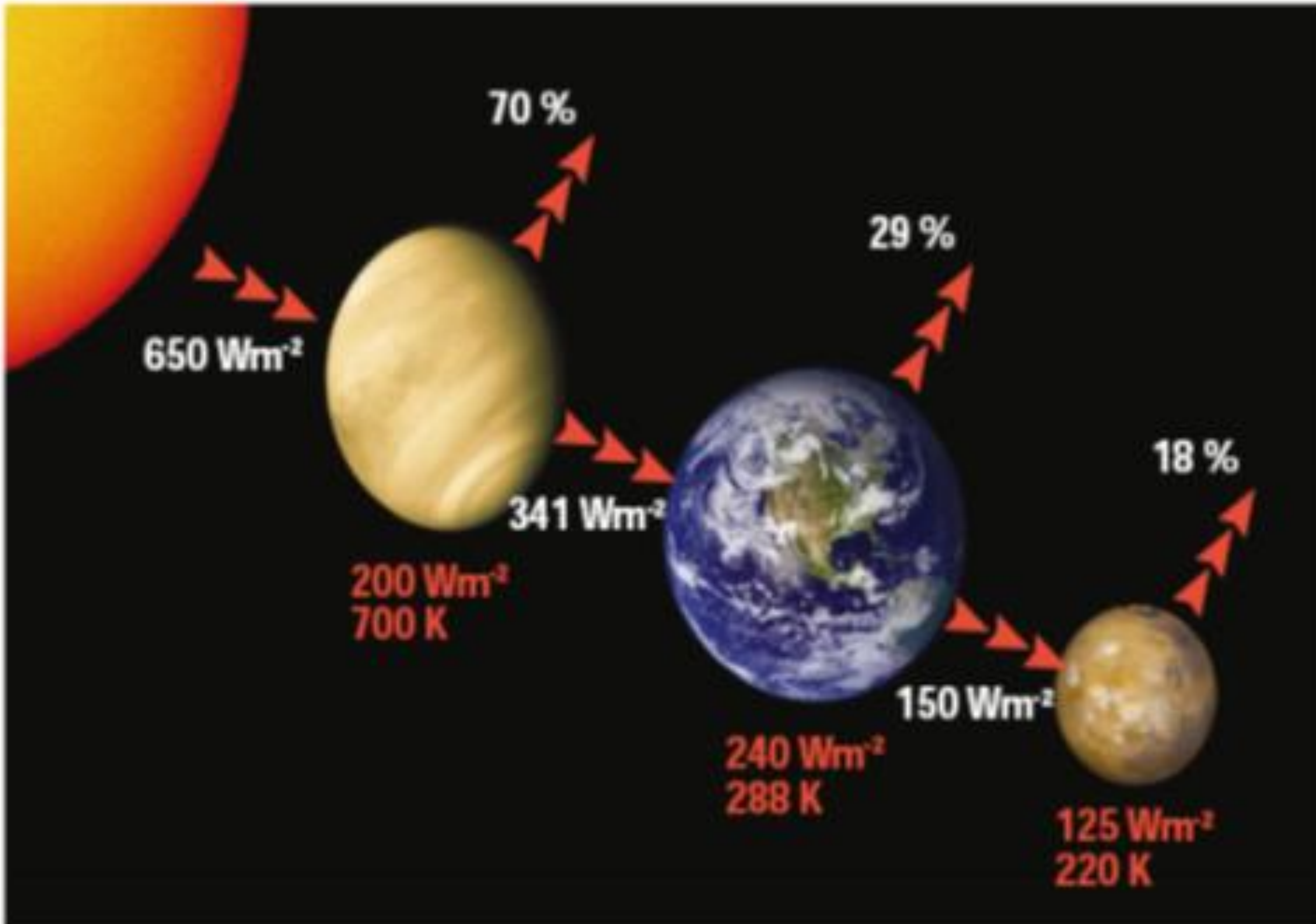
J.Shakun et al. NATURE,484, 49-54, 5 April 2012



The global proxy temperature stack (blue) as deviations from the early Holocene (11.5–6.5kyr ago) mean, an Antarctic ice-core composite temperature record[42] (red), and atmospheric CO_2 concentration (refs 12, 13; yellow dots).

Earth vs Venus

- Although Earth and Venus started with similar composition
  - Earth evolved such that carbon safely buried in early sediments
    - Avoiding runaway greenhouse effect
- Venus built up CO_2 in the atmosphere
  - Build-up led to high temperature



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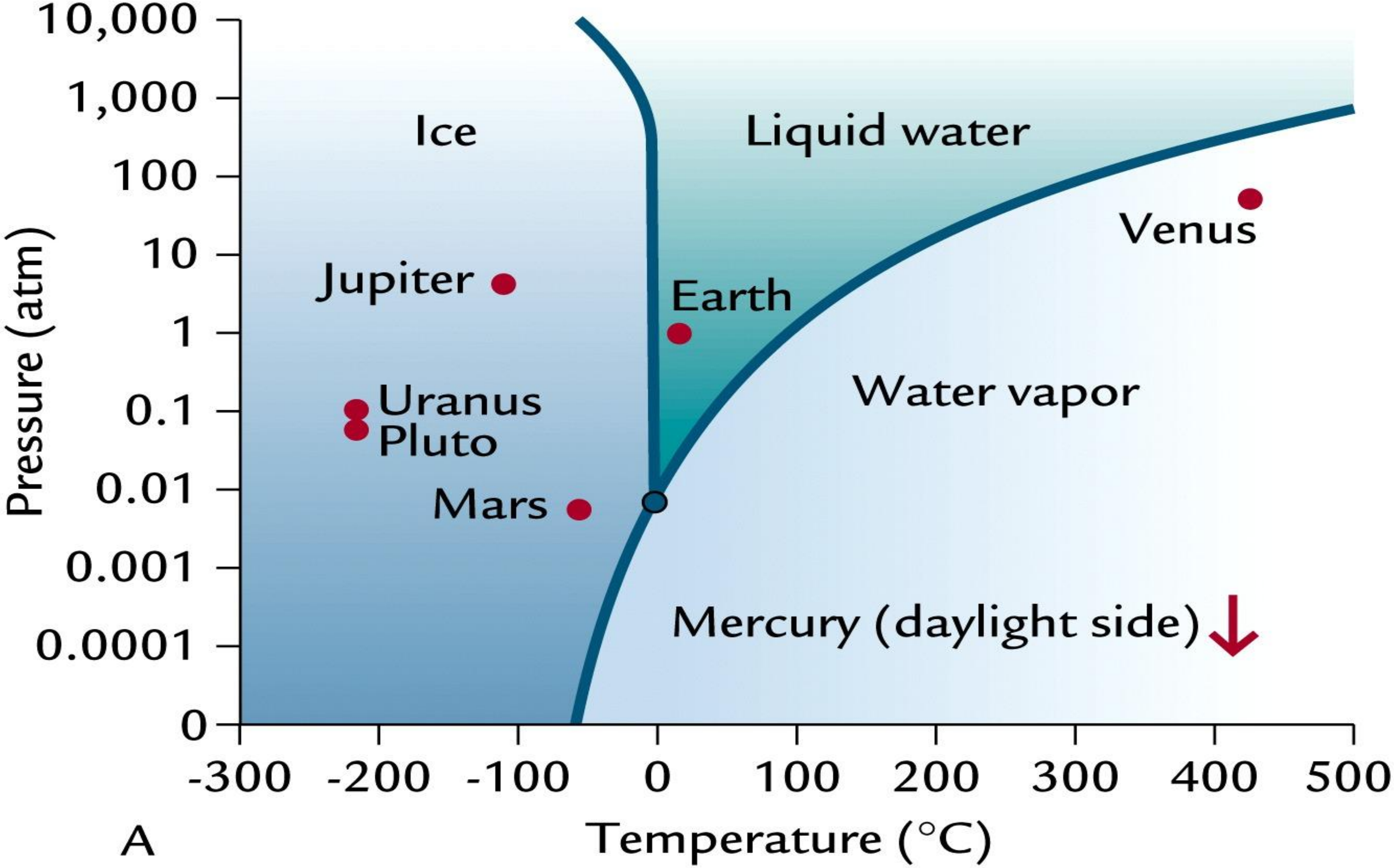


Water exists in solid, liquid and gaseous form only on earth

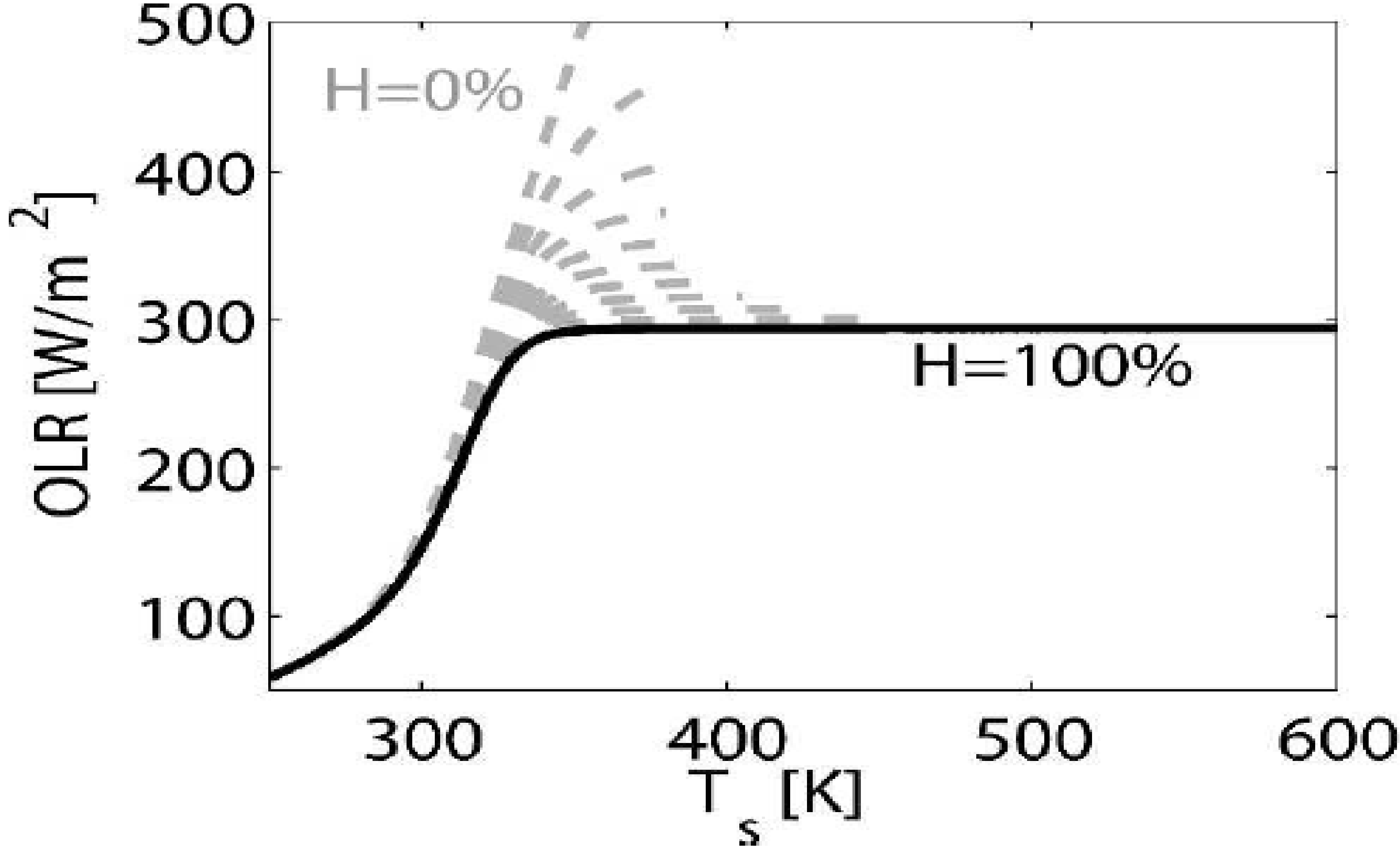
"If we burn all our reserves of fossil fuels, there is a substantial chance that we will initiate the runaway greenhouse"

James Hansen

"Storms of my grand children"

2009 ,New York

Note that OLR $\neq$ σ T ^{4}



Runaway Greenhouse

$$T_s^4 = S_o/4(1 - \alpha)(1 + 0.75\delta_a)$$

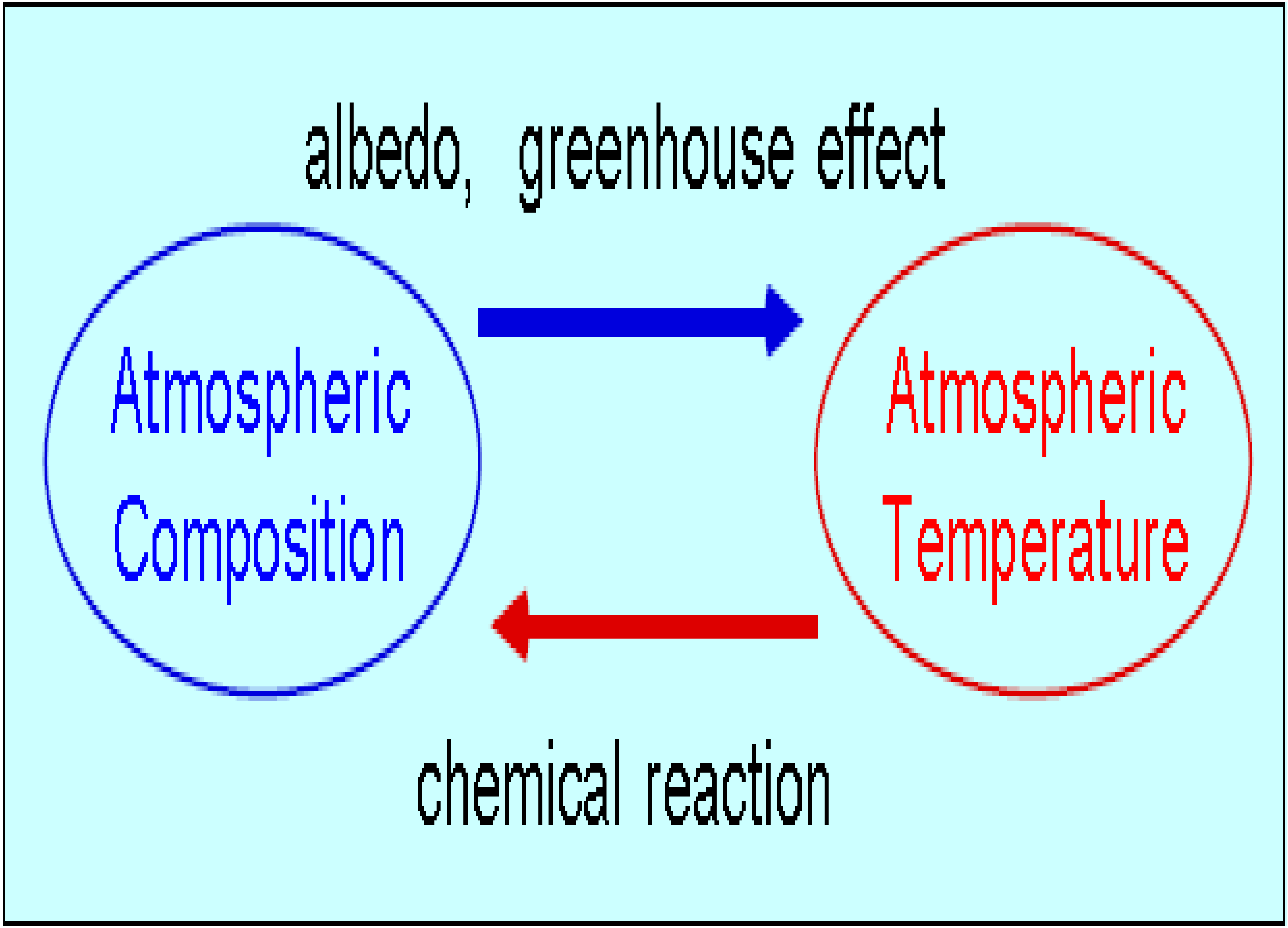
δ_a =optical depth= $\int K_{abs} \; \rho_v \, dz$

$$\delta_a = K_{abs} P_s (T_s) M_v/(M_a g)$$

If the atmosphere is saturated with vapor, then the optical depth will increase monotonically with surface temperature

Composition of Venus atmosphere is controlled by chemical reaction on the planetary surface

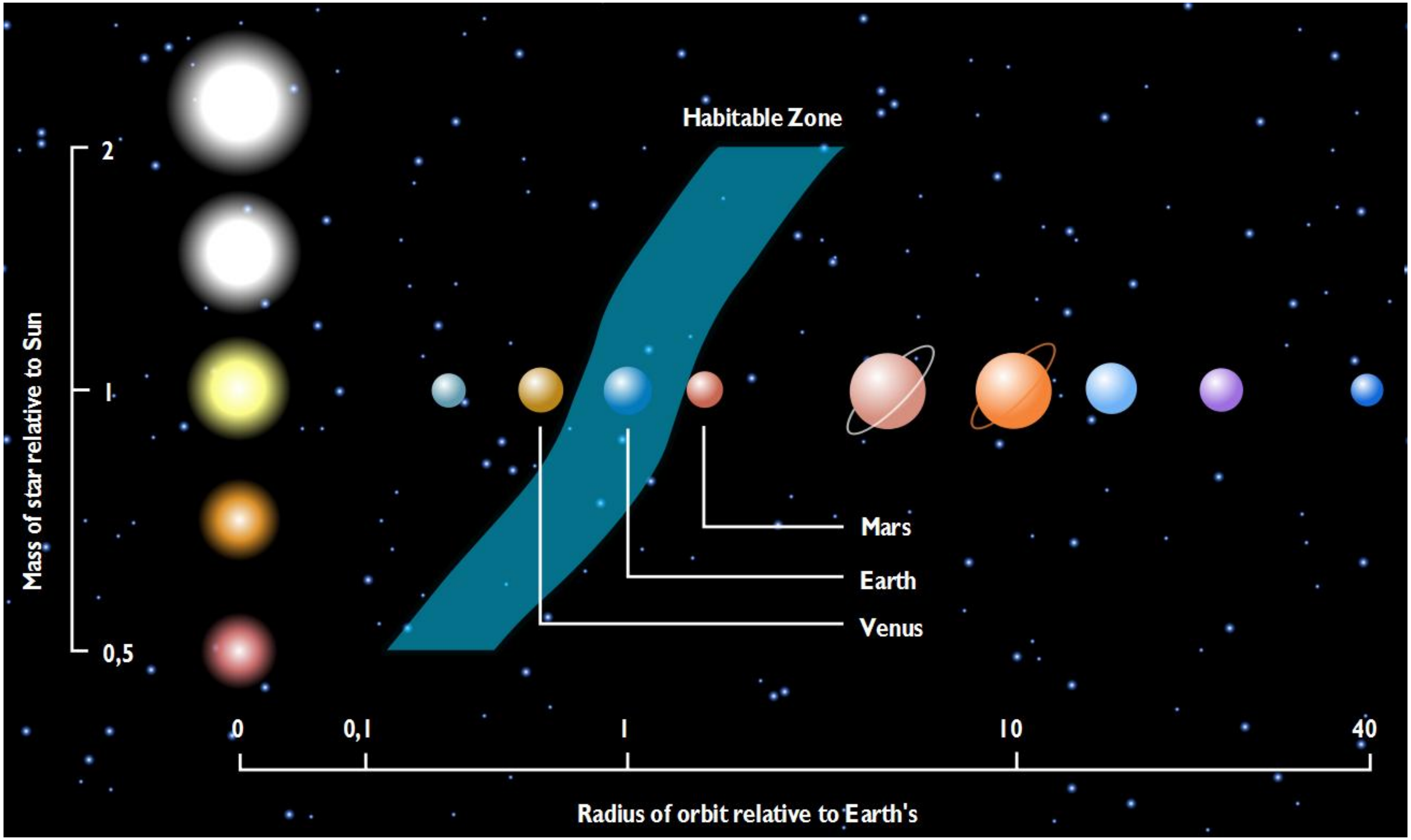
- surface temperature
- chemical composition of planetary surface

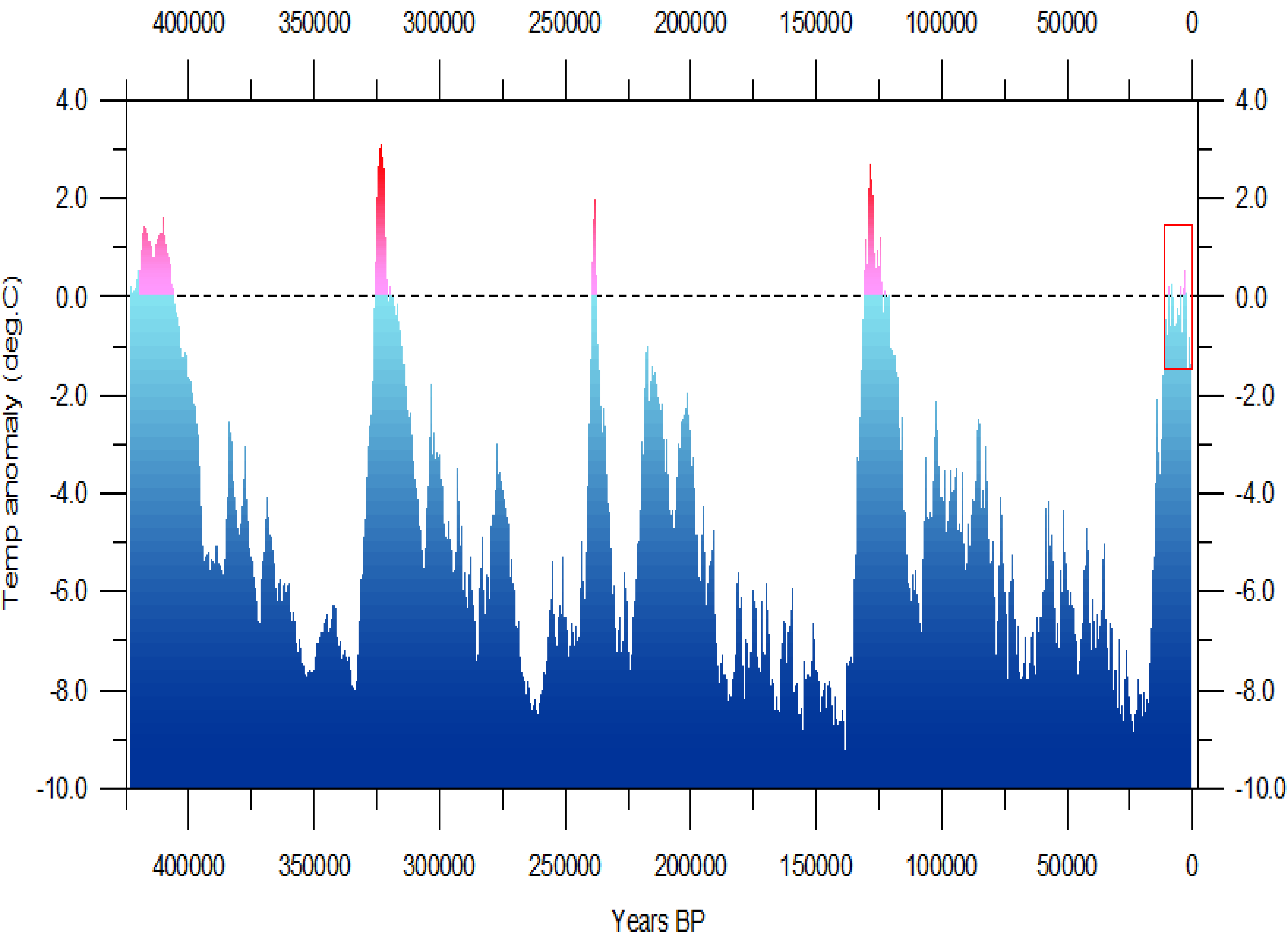

albedo, greenhouse effect
Atmospheric Composition
Atmospheric Temperature
chemical reaction

$CaSiO_3$ (wollastonite) + CO_2 $\rightarrow$ $CaCO_3$ (calcite) + SiO_2

Carbonate-silicate reaction shifted in the favour of carbonate formation below 300 °C but shifts to silicate formation above 300 °C

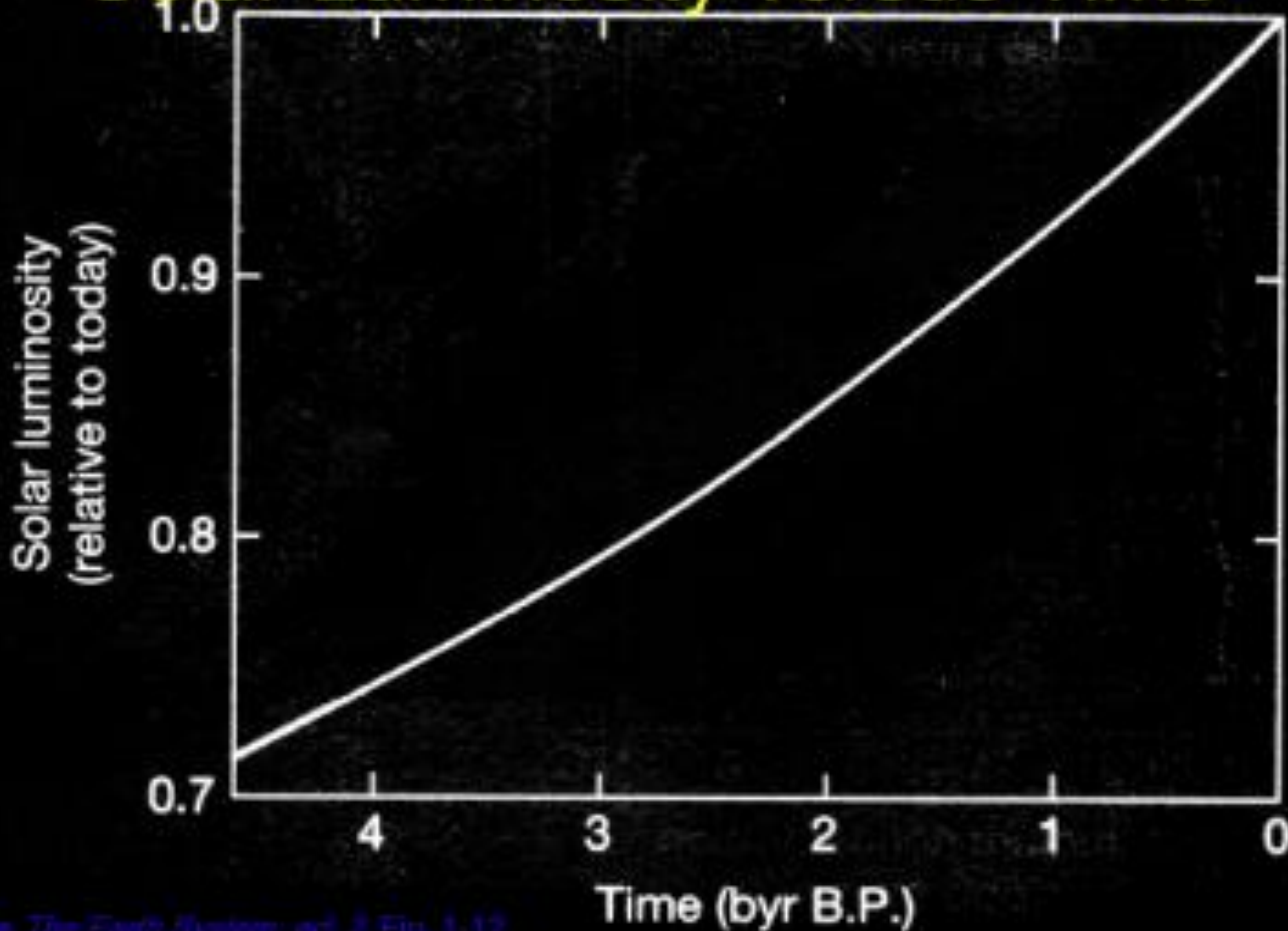
The (liquid water) habitable zone

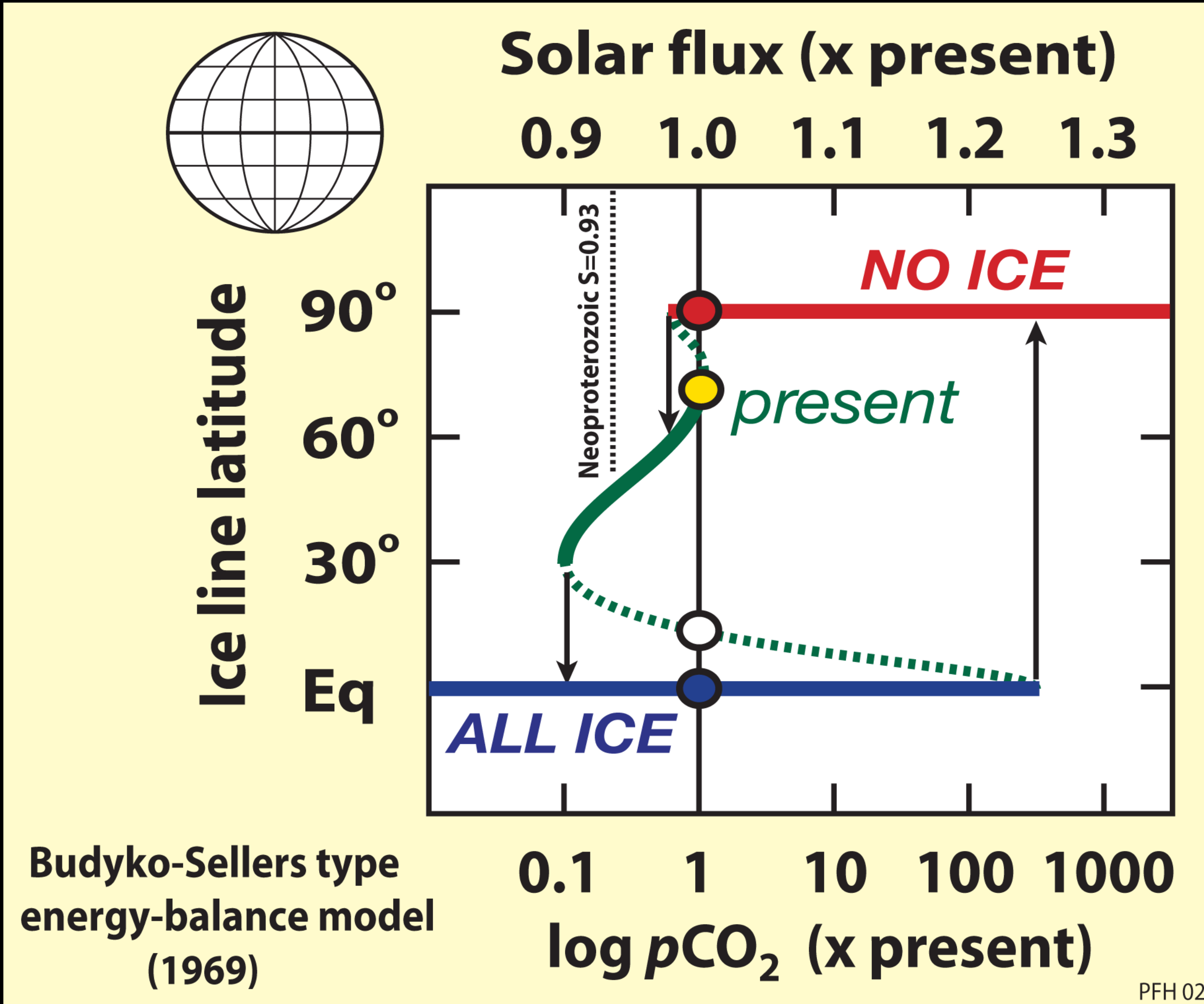




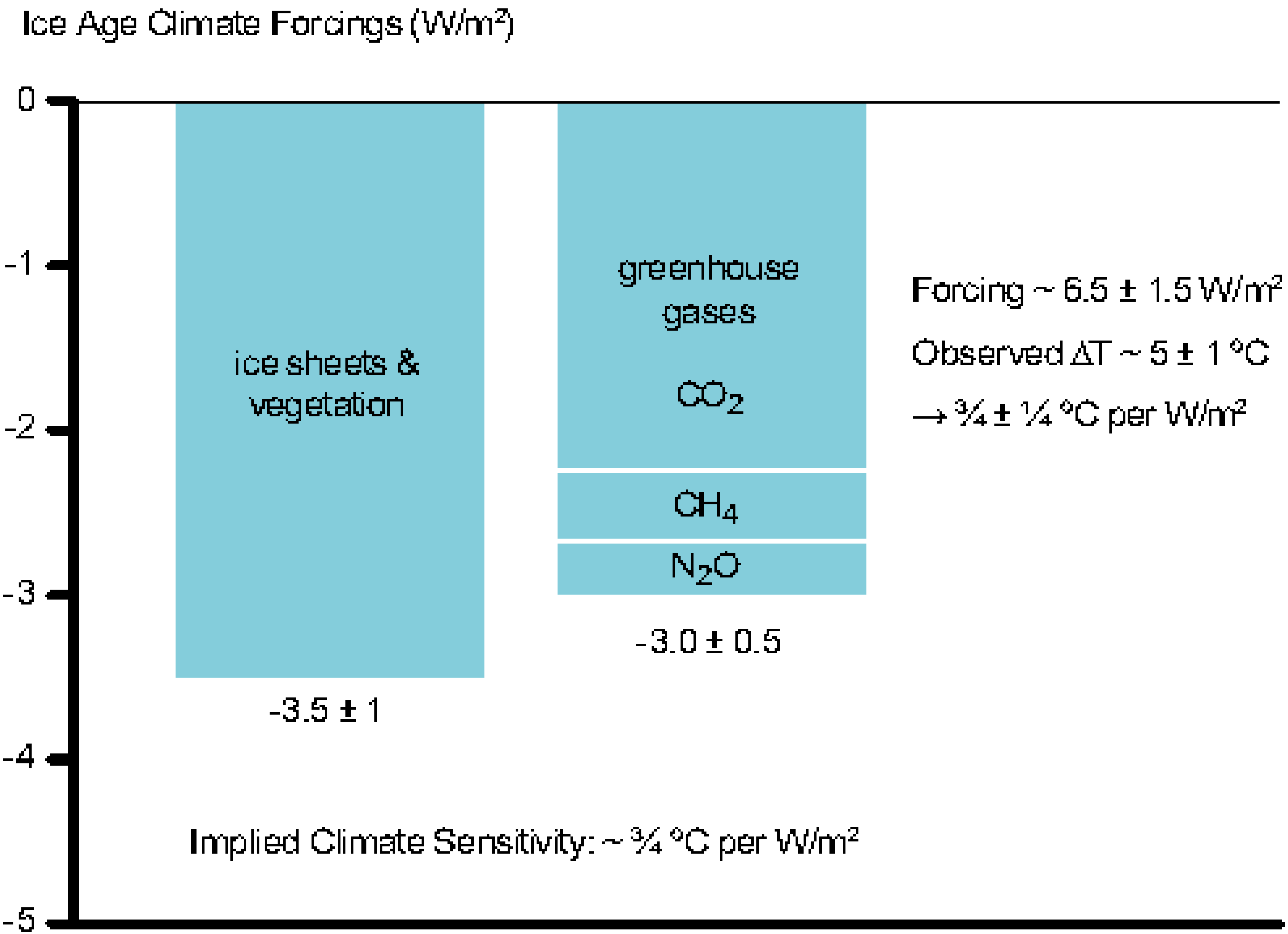
400000
350000
300000
250000
200000
150000
100000
50000
0
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
-10.0
Temp anomaly (deg.C)
Years BP

Solar Luminosity versus Time

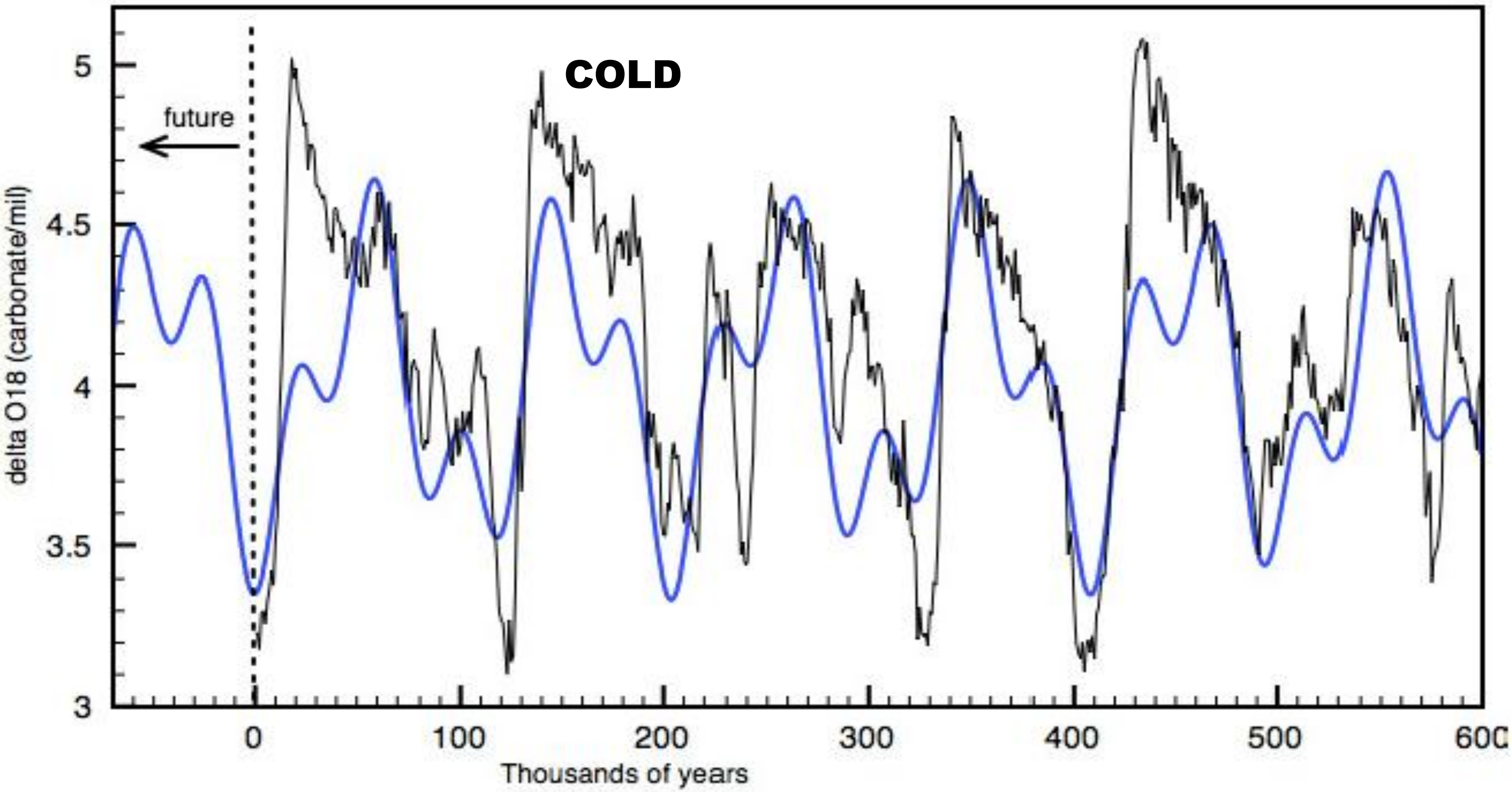




Solar flux (x present)
0.9
1.0
1.1
1.2
1.3
Neoproterozoic S=0.93
NO ICE
present
Ice line latitude
90°
60°
30°
Eq
ALL ICE
0.1
1
10
100
1000
log pCO2 (x present)
Budyko-Sellers type
energy-balance model
(1969)
PFH 02



Climate forcings during ice age 20 ky BP, relative to the present (pre-industrial) interglacial period. (Source: Hansen et al. 2008)



δO_{18} record from microfossils in the ocean. Blue line solar radiation at 65° N