# Introduction to Random Matrix Theory and its various applications

Outline of the course:

(1) Brief historical introduction to RMT: applications

Discussion of basic properties of matrices, different random matrix ensembles, rotationally invariant ensembles such as Gaussian ensembles etc.

(2) Gaussian ensembles: derivation of the joint probability distribution of eigenvalues, starting from the joint distribution of matrix entries.

(3) Analysis of the spectral properties of eigenvalues: given the joint distribution of eigenvalues, how to calculate various observables such as:

(i) Average density of eigenvalues ----Wigner semi-circle law
(ii) Counting statistics, spacings between eigenvalues etc.
(iii) Distribution of the extreme (maximum or minimum eigenvalues)

(4) Two complementray approaches to study spectral statistics: (a) Large N (for an NxN matrix) method by the Coulomb gas approach: saddle point method (b) finite N method: for Gaussian unitary ensemble: orthogonal polynomial method: Connection to the quantum mechanics problem of free fermions in a trap at zero temperature and application to cold atom physics.

(5) Tracy-Widom distribution: prob. distribution of the top eigenvalue. Its appearence in a large number of problems, universality and an associated third order phase transition.

(6) Perspectives, summary and other applications.

Suggested readings/references:

(1) ``Random matrices"...book by M. L. Mehta

(2) ``Log-gases and Random matrics" ...book by P.J. Forrester

(3) ``Introduction to Random Matrices - Theory and Practice", G. Livan, M. Novaes, P. Vivo: arXiv: 1712.07903

(4) S.N. Majumdar, Les Houches lecture notes (Complex systems, 2006), arXiv/cond-mat/0701193

(5) ``Extreme Value statistics of correlated random variables", lecture notes for the GGI (Florence, 2014) workshop by S.N. Majumdar (notes taken by a student A. Pal), arXiv: 1406.6768

(6) S.N. Majumdar, a book chpater in ``Handbook of random matrix theory" ed. by G. Akemann et.al. arXiv: 1005.4515

(7) Review: S.N. Majumdar and G. Schehr, ``Top eigenvalue of a random matrix: large deviations and third order phase transition", J. Stat. Mech. P01012 (2014), arXiv: 1311.0580

(8) Review article by Y. V. Fyodorov, arXiv: 0412017

(9) Review ``Random matrix theory of quantum transport" by C.W.J. Beenakker, Rev. of Mod. Phys. 69, 731 (1997).

See also two recent polpular articles:

(1) ``Equivalence Principle" by M. Buchanan, Nature Phys. 10, 543 (2014)

http://www.nature.com/nphys/journal/v10/n8/full/nphys3064.html?WT.ec_id=NPHYS-201408

(2) ``At the far ends of a new universal law" by N. Wolchover, Quanta magazine (October, 2014)

https://www.quantamagazine.org/20141015-at-the-far-ends-of-a-new-universal-law/