The shape of quantum fluctuations

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with

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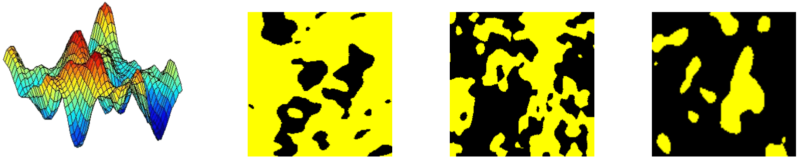
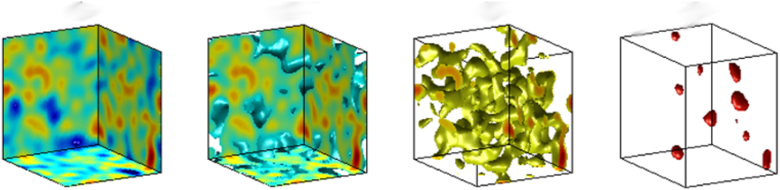
Excursion sets of random fields - overview

2 dimensions:



3 dimensions:



3D picture: R. Adler

Rich geometrical and topological structure

Excursion sets of random fields - overview

Morse theory - from differentiable functions to topology

Connectivity, Extrema counts - maximas, minimas, saddles

Searches for primordial non-Gaussianity, characterize cosmic web, typical scales of filamentary structures,...

Integral geometry

Minkowski functionals and Betti numbers: Volumes, boundary, structure counts

Searches for primordial non-Gaussianity, residual foreground contamination,...

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More Integral geometry

Q. Anisotropy and alignment information?

Q. What properties of the universe can be probed by this information?

Quantum fluctuations of inflaton field

$\Downarrow$

Universe with fluctuating energy+matter density.

Φ

Very large scales : space filled with matter or radiation must expand

Intermediate scales : gravitational attraction

Very small scales : Electromagnetic + weak + strong forces

Interactions at different scales modulate the "shape" of the primordial Φ.

The shape of Φ

Effect of scattering

Scattering decreases the amplitude of fluctuations. Homogenizes.

Effect of gravitational attraction

Gravity is isotropic. But the mass distribution on smaller scales is not isotropic. Hence tidal forces will distort the shape of Φ.

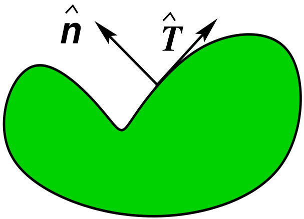
Effect of expansion

If the expansion is isotropic, the size of structures gets scaled, but shape does not change.

Quantifying shapes – 2D field

McMullen 1997
Alesker 1999
Hug, Schneider & Schuster 2008,

PC, Yogendran, Joby et al 2017



Contour Minkowski Tensor

$$\mathcal{W}^{(2)} = \int_{\partial Q} \hat{T} \otimes \hat{T} \ \mathrm{d}\ell$$

$$\left(\hat{T} \otimes \hat{T}\right)_{ij} \equiv \frac{1}{2}\left(\hat{T}_i \hat{T}_j + \hat{T}_j \hat{T}_i\right)$$

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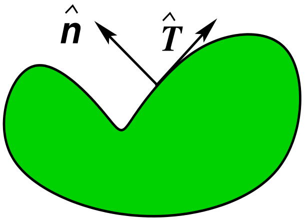

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Shape parameter : $\beta \equiv \lambda_1/\lambda_2$, $\lambda_1 < \lambda_2$

Schroeder-Turk et al 2010

$0 < \beta \leq 1.$ m-fold symmetry, $m \geq 3 \rightarrow \beta = 1.$

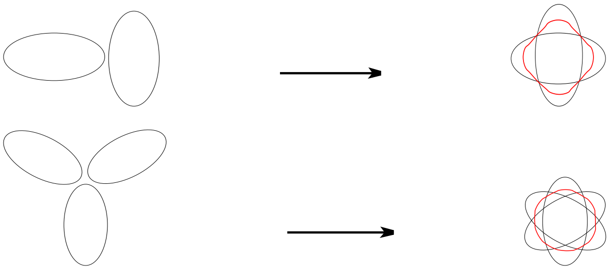
Alignment of many curves - α

Schroeder-Turk et al 2010

Sum all $\mathcal{W}^{(2)} \quad \longrightarrow \quad \widetilde{\mathcal{W}}^{(\mathbf{2})} \quad \longrightarrow \quad \mathbf{\Lambda_1}, \mathbf{\Lambda_2}$

Alignment parameter : $\alpha \equiv \frac{\mathbf{\Lambda_1}}{\mathbf{\Lambda_2}}, \quad 0 < \alpha \leq 1$

Geometrical interpretation of α PC, Yogendran, Joby et al 2017



α **is the shape/anisotropy parameter of the locus curve.**

Quantifying shapes – 3D field

Single structure : $\mathcal{W}^{(3)} = \int_{\partial Q} \hat{n} \otimes \hat{n} \ \mathrm{d}a, \quad \lambda_{\mathbf{1}}, \lambda_{\mathbf{2}}, \lambda_{\mathbf{3}}$

Shape parameters : $\beta_{\mathbf{1}} \equiv \frac{\lambda_{\mathbf{1}}}{\lambda_{\mathbf{2}}}, \qquad \beta_{\mathbf{2}} \equiv \frac{\lambda_{\mathbf{2}}}{\lambda_{\mathbf{3}}}$

Many structures : $\widetilde{\mathcal{W}}^{(3)} \longrightarrow \quad \mathbf{\Lambda_1}, \mathbf{\Lambda_2}, \mathbf{\Lambda_3}$

Alignment parameters :

$$\alpha_{\mathbf{1}} \equiv \frac{\mathbf{\Lambda_1}}{\mathbf{\Lambda_2}}, \qquad \alpha_{\mathbf{2}} \equiv \frac{\mathbf{\Lambda_2}}{\mathbf{\Lambda_3}}$$

$\widetilde{\mathcal{W}}^{(2,3)}$ for Gaussian isotropic fields

PC, Yogendran, Joby, et al 2017;

Appleby, PC, Park et al 2018

$$\textbf{2D fields}: \quad \langle \widetilde{\mathcal{W}}^{(2)}(\nu) \rangle \quad \propto \quad \frac{1}{r_c} e^{-\nu^2/2} \times I \times \text{Area}$$

$$\textbf{3D fields}: \quad \langle \widetilde{\mathcal{W}}^{(3)}(\nu) \rangle \quad \propto \quad \frac{1}{r_c} e^{-\nu^2/2} \times I \times \text{Volume}$$

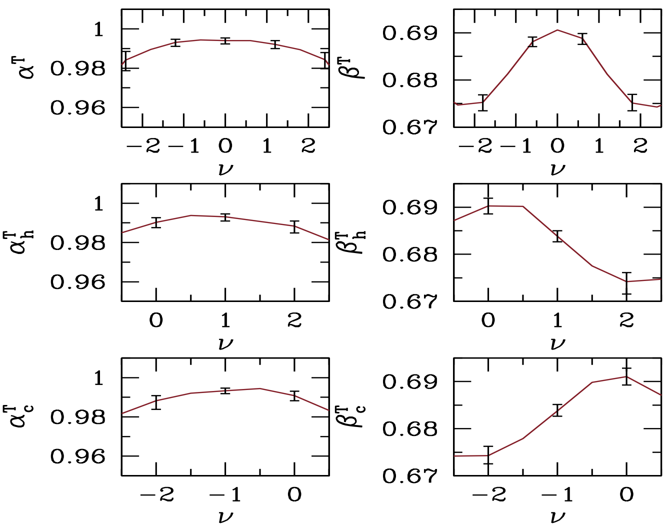
$$r_c \equiv \frac{\sigma_0}{\sigma_1} \qquad \longrightarrow \quad \text{correlation length.}$$

Statistical isotropy :

2D: The locus curve must have $\alpha = 1$.

3D: The locus surface must have $\alpha_1 = \alpha_2 = 1$.

Gaussian isotropic random field

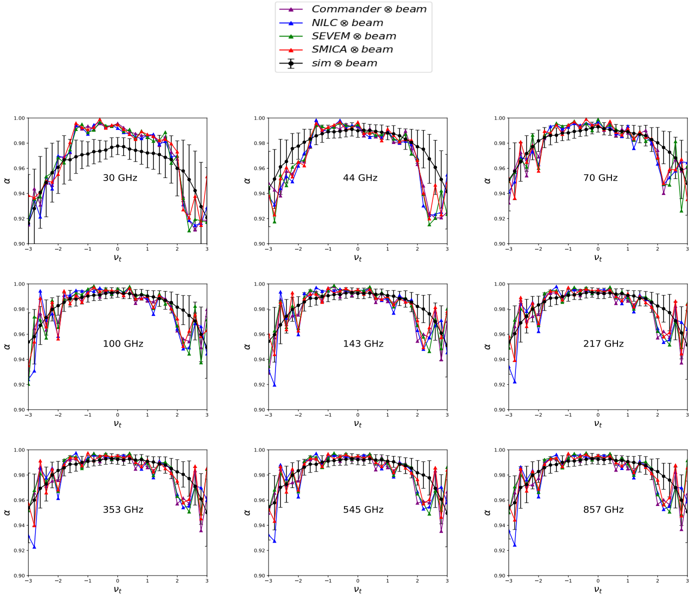


Application to cosmological fields

- **Statistical isotropy test using PLANCK data**
- **Probing length and times scales of the EoR**

Search for statistical anisotropy using 2015 PLANCK data

Joby, PC, T. Ghosh *et al.*, 2018



Search for statistical anisotropy using PLANCK data

Joby, PC, T. Ghosh *et al.*, 2018

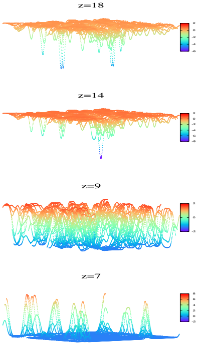
- **Conclusion** : PLANCK 2015 temperature data exhibits no statistically significant deviation from SI.
- 30 GHz data for all cleaning methods show mild decrepancy at $\sim 2\sigma$.
- Small increase of discrepancy when analyzed at higher resolution using $Nside = 1024$.

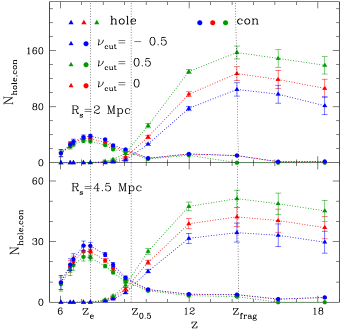
  $\Rightarrow$ discrepancy is likely due to improper noise estimates for 30 GHz.
- 2015 polarization data for all cleaning methods found to have higher than 3σ deviation from SI. Vidhya Ganesan & PC 2017
- Full probe of 2018 PLANCK data for both temperature and polarization ongoing.

Epoch of Reionization - ionization history

Akanksha Kapahtia, PC, Appleby et al 2018

21cmFAST simulations   Messinger, Furlanetto & Cen 2010



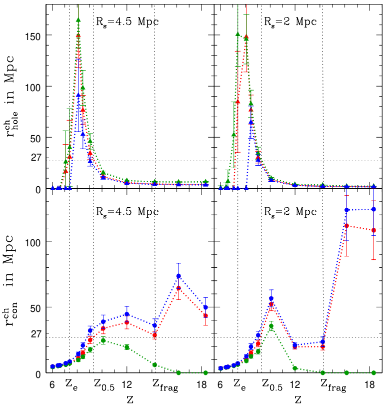


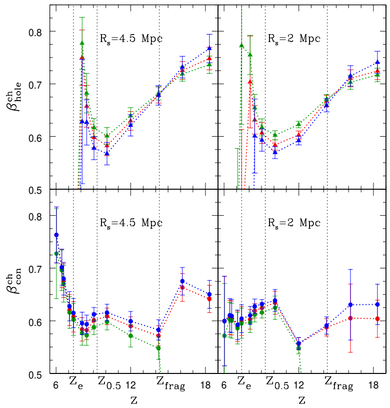
Important time scales: $\mathbf{z}_{\rm frag} \sim 14$, $\mathbf{z}_{0.5} \sim 9$, $\mathbf{z}_{\rm e} \sim 7$

$$r \equiv (\lambda_1 + \lambda_2)/2\pi$$

Calculate $n_{\rm hole}$, $n_{\rm con}$, $r_{\rm hole}$, $r_{\rm con}$, $\beta_{\rm hole}$, $\beta_{\rm con}$.

EoR - ionization field





Ionized bubbles grow to size of 27 Mpc by $z_{0.5}$.

Maximum merger happens before $z_{0.5}$.

Summary

- The rich geometry and topology of excursion sets of cosmological fields not fully exploited yet.
- Anisotropy and alignment information can be quantified using the contour and area Minkowski tensors.
- Provide tests of statistical isotropy and Gaussianity using models CMB, matter fields.
- Can be used to probe the EoR and distinguish different models of reionization.