Dark Energy

- **Varun Sahni**

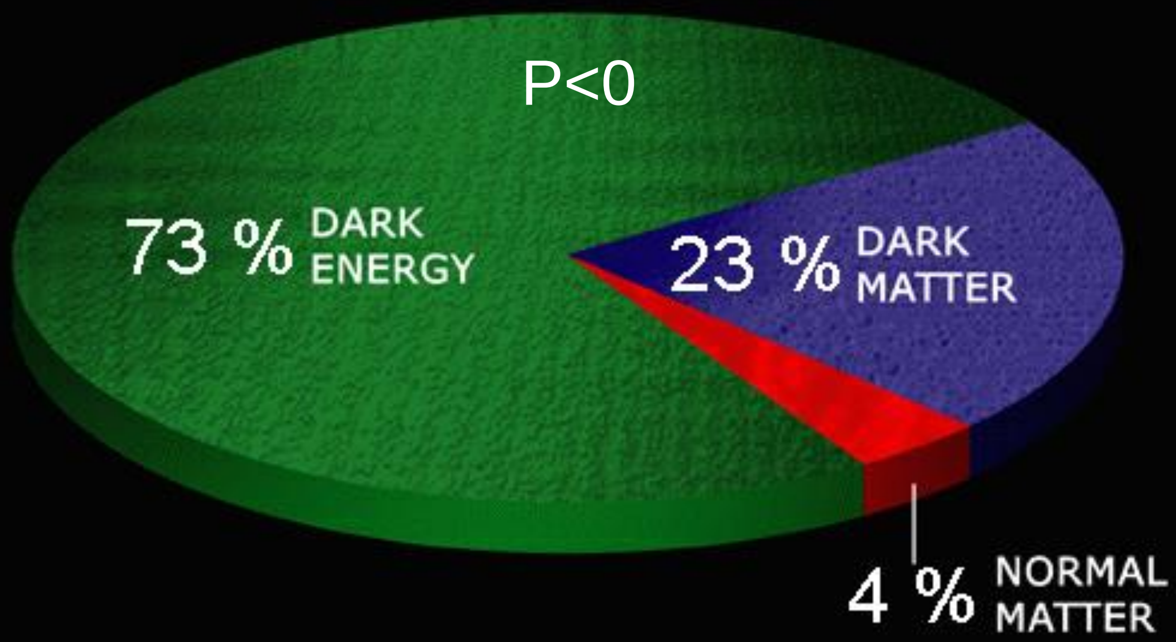
One of the most EXCITING observational discoveries of the past decade is that the Universe is Accelerating !

The source responsible for Cosmic Acceleration is presently unknown and has been called

Dark Energy

Dark Energy has large negative pressure and could account for up to 70% of the total matter density in the Universe !!

Dark Energy has negative pressure and can make the Universe Accelerate !



The beginning !

Inflation

Accelerating Universe

$$p \simeq -\rho \ !$$

Initial conditions for structure formation

Dark matter

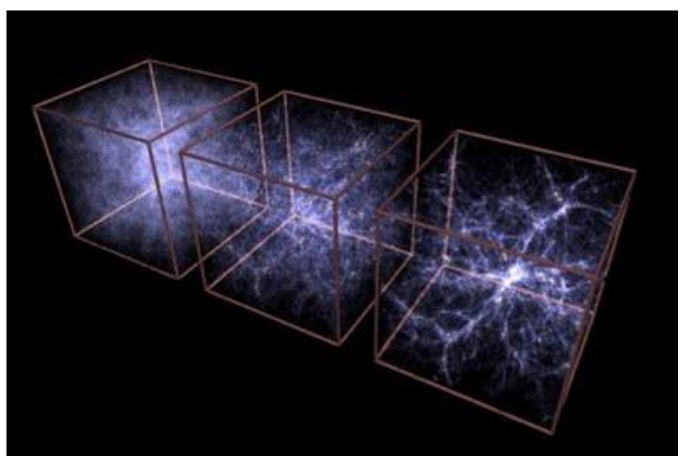
Dark energy

Speeds up structure formation

Cosmic Web

Slows down structure formation

Nature of dark matter:
Hot, warm, cold
affects cosmic web
profoundly



The present !

ASTRONOMY

Exploding Stars Point to a Universal Repulsive Force

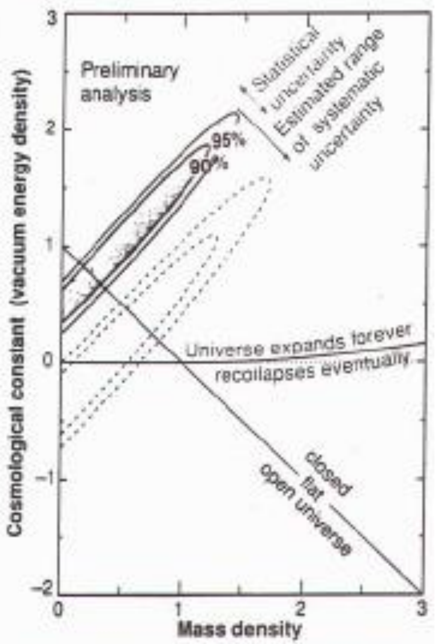
By now, even newspaper readers with a casual interest in astronomy may have heard the unsettling message delivered by distant, exploding stars called supernovae: The universe will likely expand infinitely, growing ever more tenuous. Now a new batch of supernovae has lent support to a strange picture of just what the universe is made of. A preliminary analysis may provide the first strong evidence that the universe could be permeated by a large-scale repulsive force. The reservoir of energy fueling that force could be anything from a quantum-mechanical shimmer in empty space, called the cosmological constant, to even more exotic possibilities that go by names like X-matter and quintessence.

At the meeting of the American Astronomical Society in Washington, D.C., earlier this month, Saul Perlmutter of Lawrence Berkeley National Laboratory in Berkeley, California, announced that he and an international team of observers have now studied a total of 40 far-off supernovae, using them as beacons to judge how the cosmic expansion rate has changed over time. Not only did the results support the earlier evidence that the expansion rate has slowed too little for gravity ever to bring it to a stop; they also hinted that something is nudging the expansion along. If they hold up, says Perlmutter, "that would introduce important evidence that there is a cosmological constant."

"It would be a magical discovery," adds Michael Turner of the University of Chicago. "What it means is that there is some form of energy we don't understand." Other observers had already found signs that the universe contains far less mass than the mainstream theory of the big bang predicts, which left open the possibility that some form of energy in empty space could be making up the deficit. The cosmological constant—also called lambda—is a longtime candidate for serving as this energy reservoir. But the new

was sparked when a fleck of the primordia vacuum underwent a chance fluctuation tha filled it with something much like a colossally intense cosmological constant. This "scalar," or directionless, field drove the patch into an exponential growth spurt. As the patch expanded and cooled, energy from the scalar field fed an explosion of material particles: The material universe was born—"creating everything from nothing," as the theory's creator, Alan Guth of the Massachusetts Institute of Technology, puts it.

During the exponential growth spurt, inflation would have ironed out any primordial



What the stars show. A preliminary analysis of 40 distant supernovae, reported by the Supernova Cosmology Project, offers strong evidence for an energy density in empty space, if space is "flat." The green regions indicate statistical uncertainties; the dashed lines show the preliminary estimates (now being refined) if all the systematic uncertainties added up in one direction.

Dawn of Dark Energy

Based on observations of distant Type Ia Supernovae.

Science 30 January 1998

[Based on Perlmutter, et al., Ap J (1999); also see Riess, et. al. Astron. J (1998)]

Perlmutter, Riess and Schmidt were awarded the 2011 Nobel prize for this discovery.

- Supernovae are amongst the brightest objects in our universe. Type Ia supernovae are standardized candles and can therefore be used to probe the universe on the largest scales.

- Over 1000 supernovae have been discovered up to a redshift of 1.7. This corresponds to a distance of several thousand Megaparsec (Gpc).

- Distant supernovae are systematically fainter than they would appear if the universe were decelerating.

- This effect cannot be accounted for by light absorption in an intervening (dusty) medium.

- Therefore the dimming of light from distant supernovae must have a cosmological origin.

The Universe is *ACCELERATING* !

- In Newtonian Gravity the gravitational potential is determined solely by the density of matter through the Poisson equation: $$\nabla^2 \phi = 4\pi G \rho$$
  - In general relativity this is replaced by $$G_{ik} = 8\pi G T_{ik}$$

``Matter tells space how to curve,
Space tells matter how to move'' *J.A. Wheeler*

In a homogeneous and isotropic setting $$T_i^k = diag(\rho, -P, -P, -P)$$

and the Poisson equation changes to $$\nabla^2 \phi = 4\pi G (\rho + 3P)$$

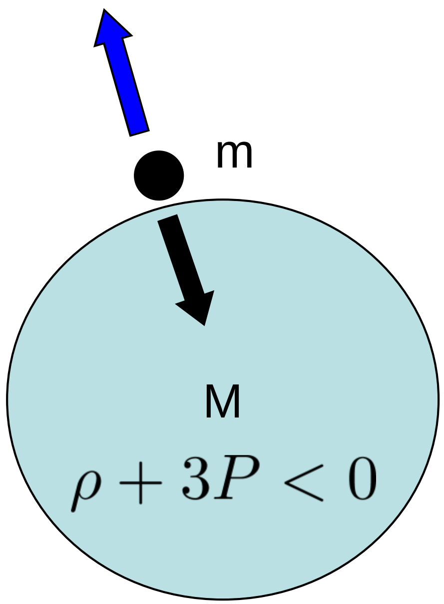
In other words `pressure carries weight' in Einstein's gravity.
The dynamics of space-time is influenced not only by
the density of matter but also by its PRESSURE !

If $$\rho + 3P < 0$$

then gravity becomes REPULSIVE !!

The acceleration of a mass m under the influence of a larger mass M is

$$\ddot{R} = -\frac{GM}{R^2} \quad \text{where} \quad M = \frac{4}{3}\pi R^3 \rho \quad \text{and } \rho \text{ is the density.}$$



Since M is always positive it follows that $\ddot{R} < 0$
i.e. the object m will decelerate and be attracted to M.

$$\rho \rightarrow \rho + 3P$$

Newton **Einstein**

In Einstein's General Relativity the formula for the mass changes to

$$M = \frac{4}{3}\pi R^3(\rho + 3P)$$

Where P is the pressure. So if the **pressure is negative** and $\rho + 3P < 0$ then M < 0 and the acceleration is positive: $\ddot{R} > 0$.
So m will feel a repulsive force which will cause it to accelerate away from M instead of being attracted to it !

An important candidate for Dark Energy is the **cosmological constant** Λ :

$$T_i^k = \frac{\Lambda}{8\pi G} g_i^k \,, \quad \text{where } g_i^k = \delta_i^k$$

$$\text{Since } T_i^k = \text{diag}(\rho, -p, -p, -p) \;\Rightarrow\; p = -\rho = -\frac{\Lambda}{8\pi G}$$

$$\text{Also } g_i^k{}_{;k} = 0 \;\Rightarrow\; T_i^k{}_{;k} = 0 \;\Rightarrow\; \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

conservation condition

- Substituting $p = -\rho$ gives $\rho =$ constant

Lorentz invariant equation of state: remains the same in all frames

- $p = 0 \;\Rightarrow\; \rho \propto a^{-3}$ for pressureless matter

The density in the cosmological constant remains constant as the universe evolves, while the density in other forms of matter/radiation decreases.
Therefore the cosmological constant can dominate the universe at late times !

The Einstein equations in the presence of the cosmological constant:

$$R_{\rm ik} - \frac{1}{2} g_{\rm ik} R = \frac{8\pi G}{c^4} T_{\rm ik} + \Lambda g_{\rm ik}$$

In a homogeneous and isotropic FRW Universe consisting of dust and Λ :

$$\ddot{a} = -\frac{4\pi G}{3} a \rho_{\rm m} + \frac{\Lambda}{3} a \qquad (R \equiv a)$$

This equation can be rewritten as a force law:

$$\mathcal{F} = -\frac{GM}{R^2} + \frac{\Lambda}{3} R\,, \quad \text{where } R \equiv a,\ \ M = \frac{4}{3} \pi R^3 \rho_{\rm m}$$

which demonstrates that the cosmological constant gives rise to a **repulsive force** whose value increases with distance.

FRW metric: $$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

Einstein equations: $$\bullet\ H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{\kappa}{a^2}$$

$$\bullet\ \frac{\ddot a}{a} = -\frac{4\pi G}{3}\sum_i(\rho_i + 3p_i) = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i), \quad \text{where } w_i = p_i/\rho_i$$

$$q = -\frac{\ddot a}{aH^2} = \sum_i \frac{\Omega_i}{2}(1+3w_i)\,, \text{ where } \Omega_i = \frac{8\pi G\rho_i}{3H^2}$$

$$\boxed{\ddot a > 0 \Rightarrow q < 0}$$

In a two component flat Universe with dust $(w_m = 0)$ and DE : $\Omega_{\rm m} + \Omega_{\rm DE} = 1$

$$q = \frac{1}{2}[1 + 3w_{\rm DE}\Omega_{\rm DE}]\,.\ \text{Therefore } q < 0 \Rightarrow w_{\rm DE} < -\frac{1}{3\Omega_{\rm DE}}$$

$$\boxed{\Rightarrow\ w_{\rm DE} < -\frac{1}{3(1-\Omega_{\rm m})} \text{ required for acceleration}}$$

$$\Rightarrow\ w_{\rm DE} < -\frac{1}{3} \text{ for } \Omega_{\rm m} = 0 \text{ (empty universe)}$$ ⇨ **Necessary condition for cosmic acceleration**

$$\Rightarrow\ w_{\rm DE} < -\frac{1}{2} \text{ for } \Omega_{\rm m} = 1/3 \text{ (realistic case)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_{\rm m} - 2\rho_{\Lambda}\right] \;\text{ if } p_{\Lambda} = -\rho_{\Lambda} = -\frac{\Lambda}{8\pi G}$$

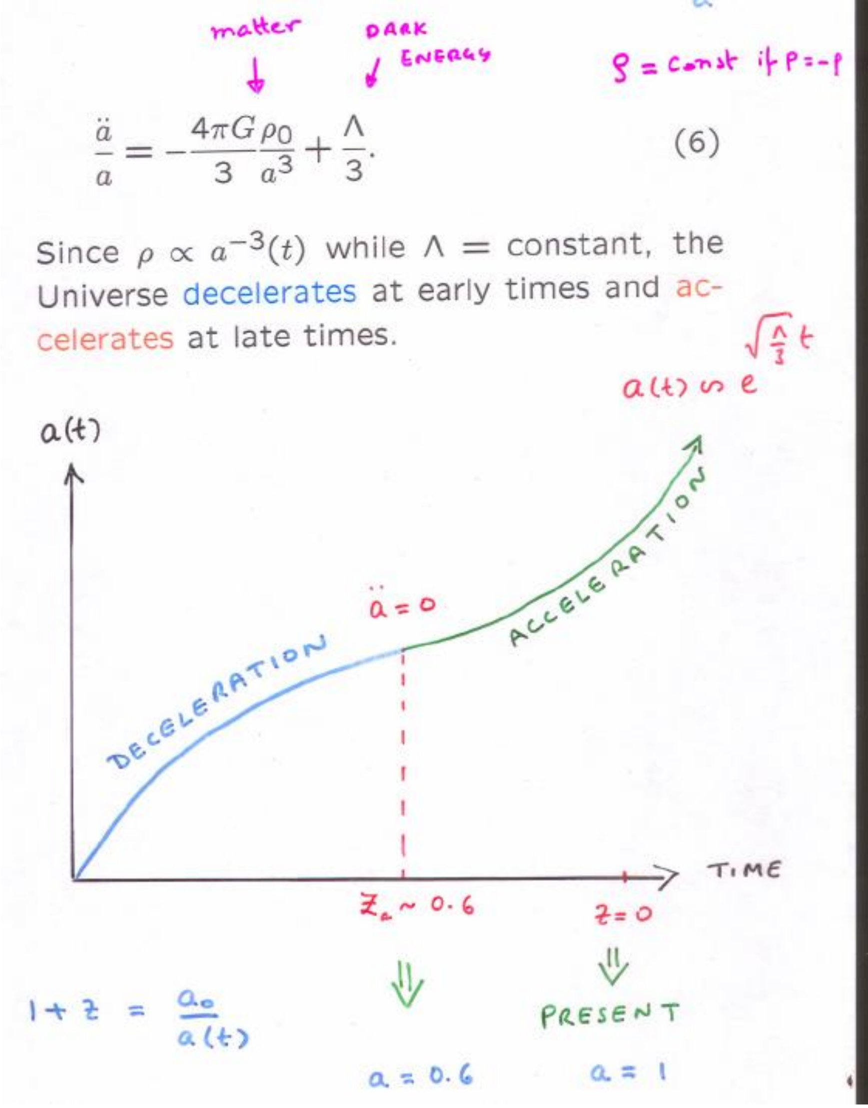
$$q = -\frac{\ddot{a}}{aH_0^2} = \frac{4\pi G}{3H_0^2}\left[\rho_{0\rm m}(1+z)^3 - \frac{\Lambda}{4\pi G}\right]$$

$$\Rightarrow \;\; q = \frac{\Omega_{0m}}{2}(1+z)^3 - \Omega_{\Lambda}$$

where $\Omega_{0m} = \dfrac{8\pi G \rho_{0m}}{3H_0^2}, \;\; \Omega_{\Lambda} = \dfrac{\Lambda}{3H_0^2}$

$\ddot{a} = 0$ when $q = 0 \Rightarrow$ $\boxed{(1+z_a)^3 = \dfrac{2\Omega_{\Lambda}}{\Omega_{0m}}}$

$z_a \simeq 0.6$ for $\Omega_{0m} = 1/3, \; \Omega_{\Lambda} = 2/3$



The universe began accelerating **recently** !

The universe was about **half its present size** when it began to accelerate.

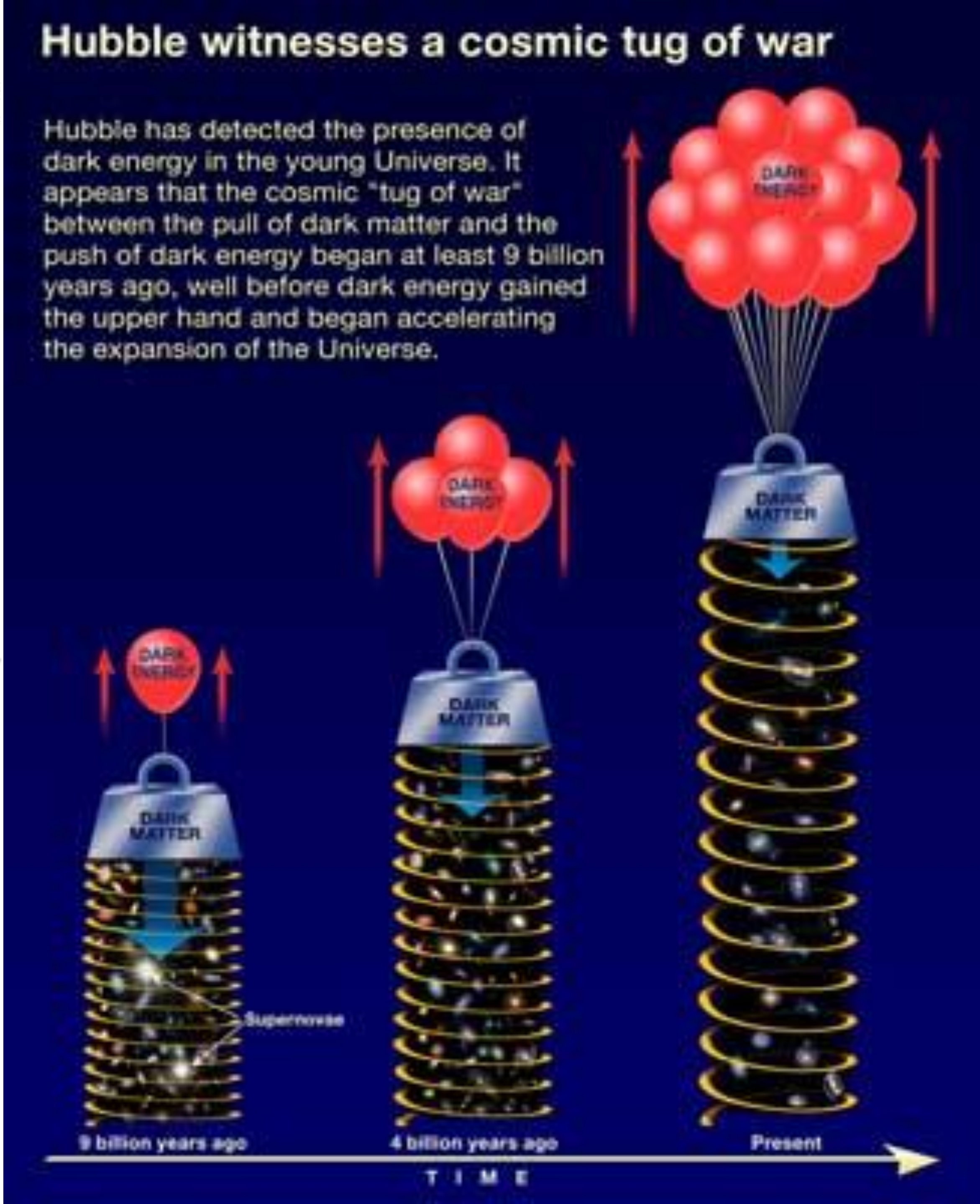
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\frac{\rho_{0m}}{a^3} + \frac{\Lambda}{3}$$

The influence of the cosmological constant grows as the universe expands and becomes larger !

We appear to be living during an epoch when the densities in dark matter and dark energy are comparable !!

$$\rho_{\rm DE} \simeq 2\rho_m$$

Could this be viewed as a Cosmic Coincidence ?



In flat Euclidean space the flux of light from a distant source is

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi r^2}$$

BUT in an expanding universe: $$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}$$

where $$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$ Is the luminosity distance,

and $$H^2(z) = \frac{8\pi G}{3}[\rho_{\mathrm{baryons}} + \rho_{\mathrm{radiation}} + \rho_{\mathrm{DM}} + \rho_{\mathrm{DE}}]$$

is the expansion rate: V = HR. The light source is at redshift $z = \frac{R_0}{R(t)} - 1$

and R(t) is the expansion factor of the universe. $H = \frac{\dot{R}}{R}$

Thus the dimming of light by a distant supernova is caused by the total energy density of the universe including: baryons, leptons, dark matter and dark energy.

The luminosity distance in cosmology

The luminosity flux $\mathcal{F}$ reaching the observer from a light source with luminosity $\mathcal{L}$ is

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}, \quad d_L^2 \neq x^2 + y^2 + z^2 ,$$

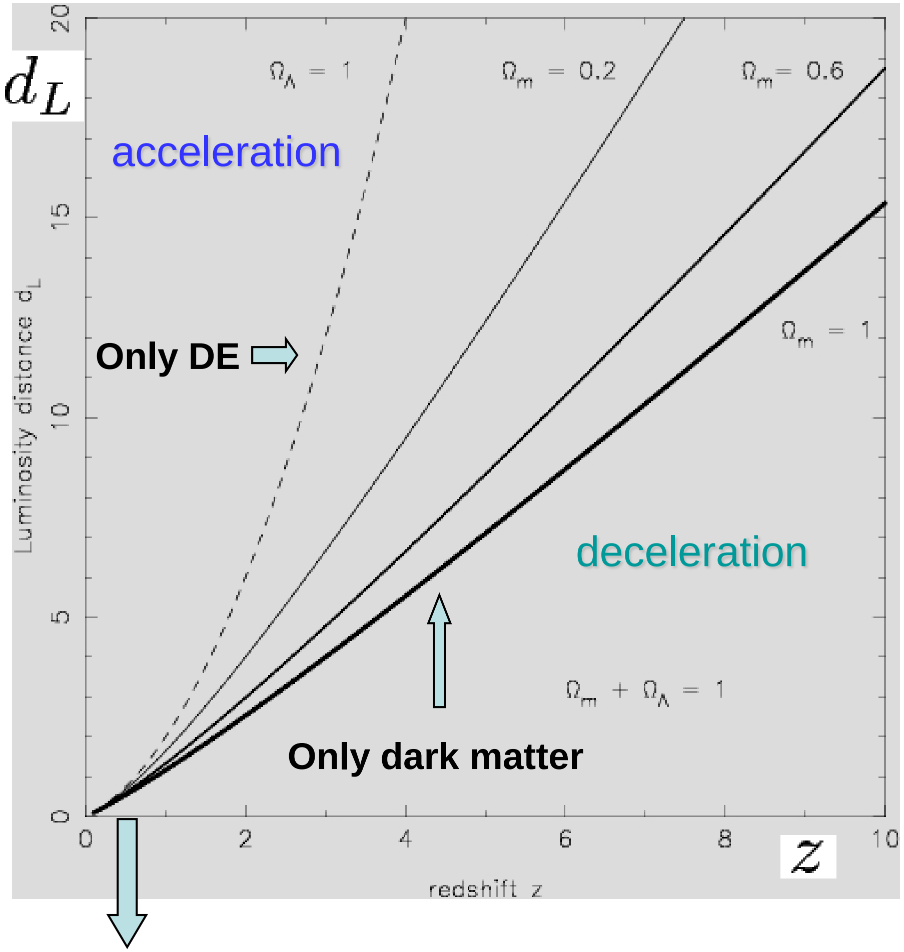
where d_L is the Luminosity Distance. In a flat universe ($\Omega_m + \Omega_\Lambda = 1$, $\Omega_\Lambda = \Lambda/3H^2$)

$$\frac{d_L}{1+z} = \int_0^z \frac{dz'}{H(z')}$$
$$= H_0^{-1} \int_0^z \frac{dz'}{\left[\Omega_m(1+z')^3 + \Omega_\Lambda\right]^{1/2}}$$

The luminosity distance depends upon the expansion history of the Universe (upto that redshift) and hence upon the properties of Dark Matter and Dark Energy.

Presence of Λ causes d_L to increase, as a result distant supernovae appear fainter in a Λ-dominated universe.

A QSO at z=3 is 9 times fainter in an DE dominated accelerating universe than in a matter dominated universe.

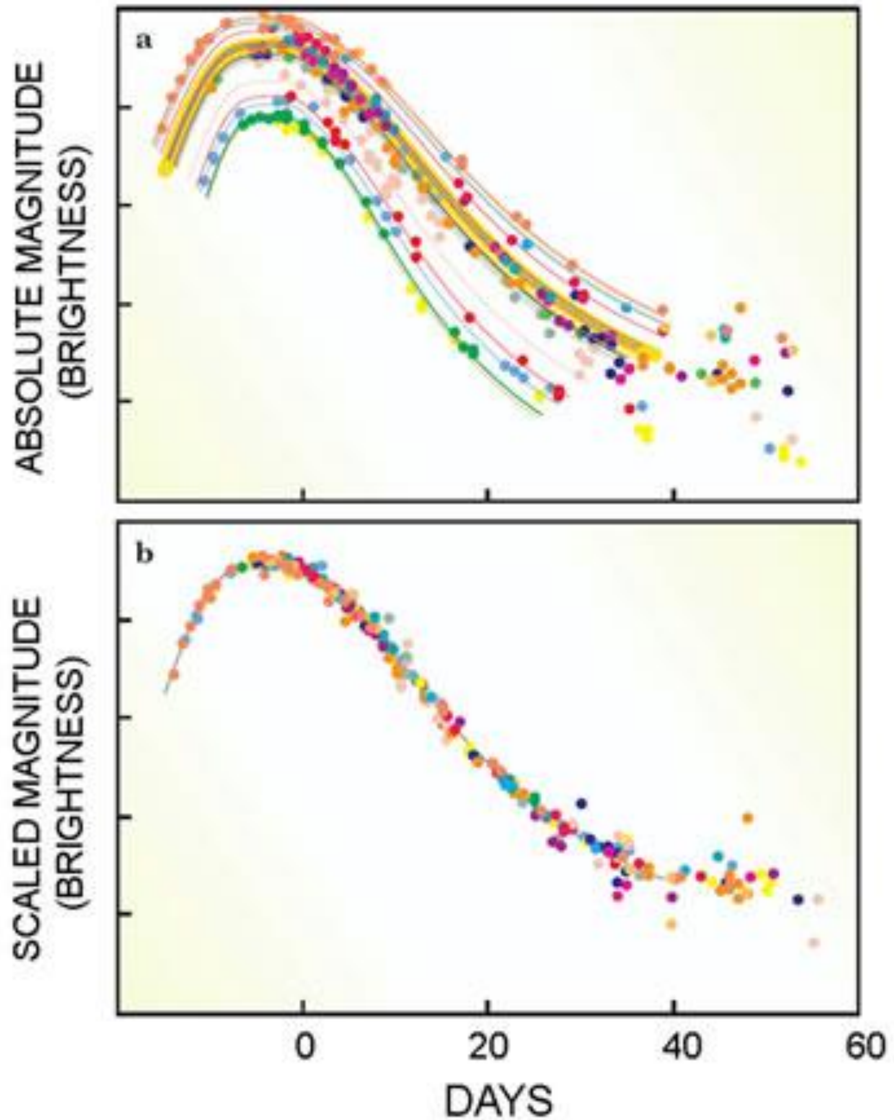


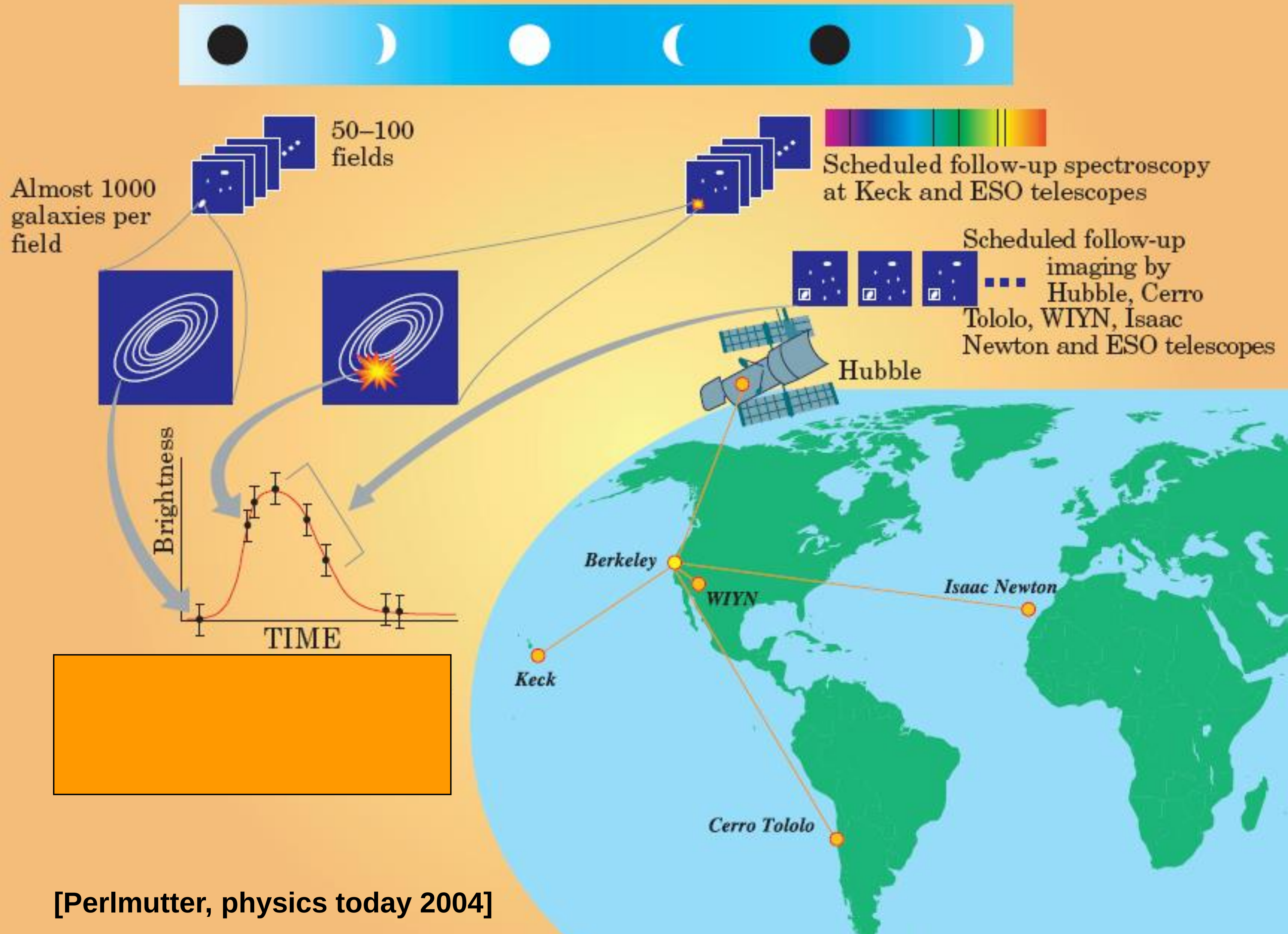
$$\lim_{z\to 0} d_L(z) \simeq \frac{cz}{H_0} \Rightarrow \text{Hubble expansion}$$

Type Ia supernovae are rare events – they occur only a few times in a thousand years in a galaxy like the Milky Way.

SN Ia take about 20 days to rise from obscurity to maximum light.
Therefore each part of the sky is observed twice, one month apart.

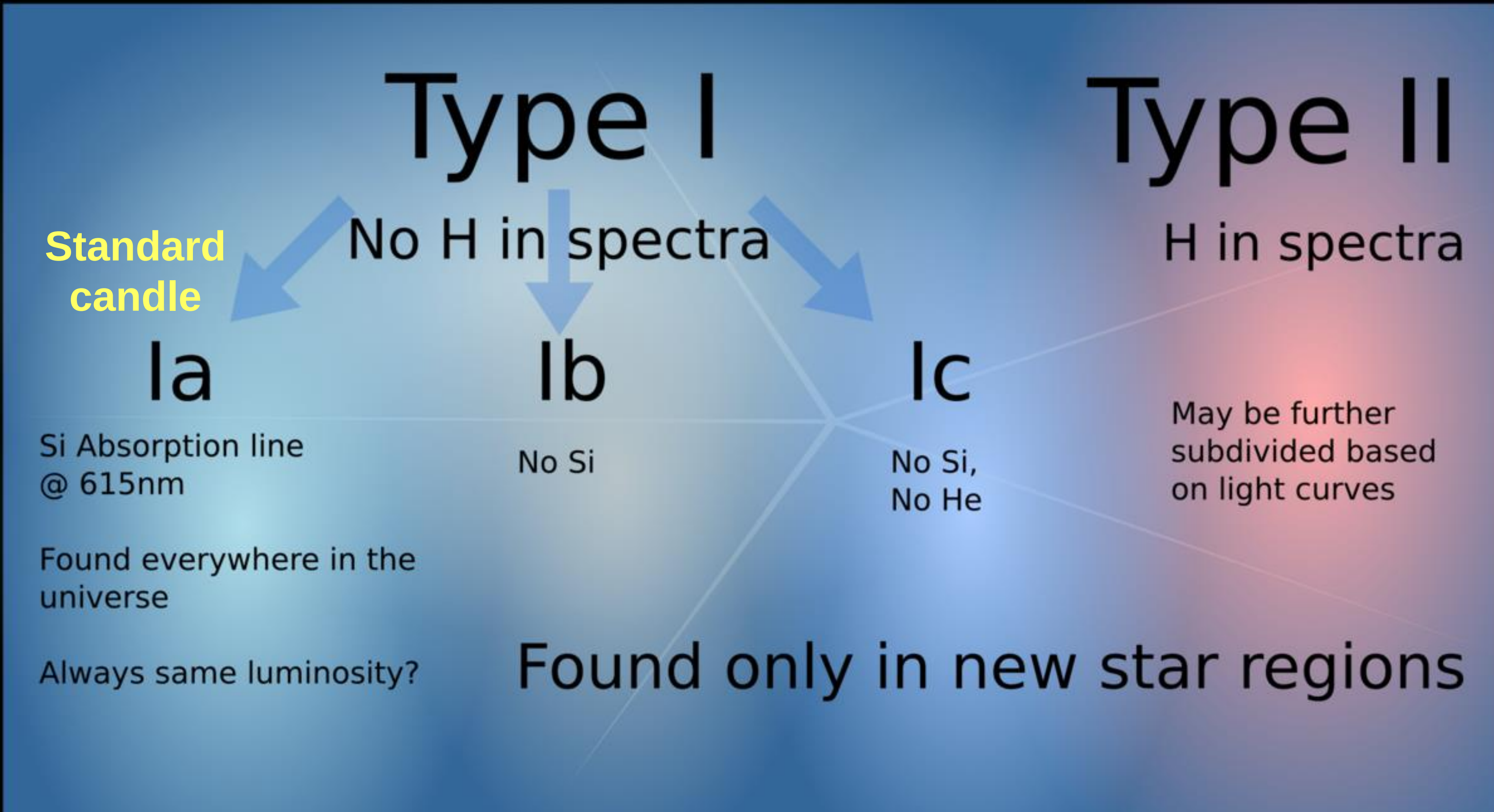
The light curve shape of SN Ia is correlated with its luminosity. Brighter supernovae take longer to fade. This observation can be used to make type Ia supernovae into standardized candles [Phillips (1993)].





[Perlmutter, physics today 2004]

Supernova Types



Evolutionary effects (if any) are of singular importance since they have in the past severely restricted the potency of tests aimed at determining the deceleration parameter and hence also the equation of state.

In the case of Type 1a supernovae: "A luminosity evolution of $\sim 25\%$ over a lookback time of $\sim 5 \times 10^9$ years would be sufficient to nullify the cosmological conclusions" and therefore the dark energy hypothesis.

(Riess et al, astro-ph/9907038)

Other tests to probe the existence of DARK ENERGY are therefore ESSENTIAL.

While SNIa provide examples of standard candles, important Evidence for Dark Energy also comes from standard rulers: Baryon Acoustic Oscillations (BAO).

1. Luminosity distance:

Known (standard candle)

measured ⇐ $$\mathcal{F} = \frac{L}{4\pi D_L^2}, \text{ where } D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

and L is a standard candle

2. Angular size distance:

Known (standard ruler)

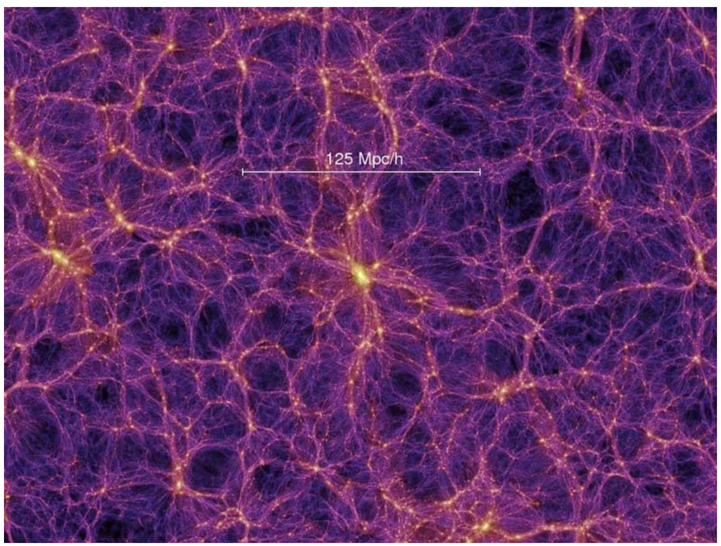
measured ⇐ $$\Delta\theta = \frac{d}{D_A}, \text{ where } D_A(z) = \frac{1}{1+z}\int_0^z \frac{dz'}{H(z')}$$

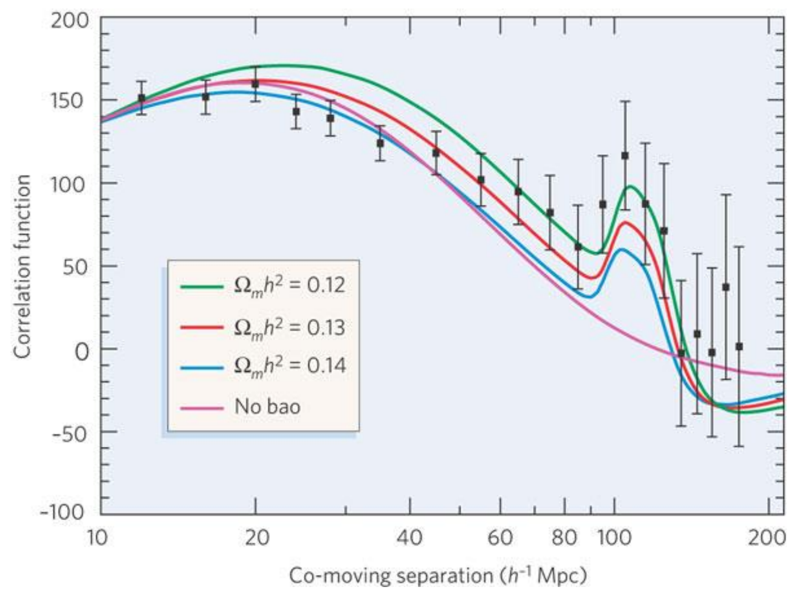
and d is a standard ruler

Baryon Acoustic Oscillations (BAO)

The galaxy distribution contains an imprint of the primordial fluctuations in the photon-baryon plasma. Prior to photon decoupling $(z \sim 1100)$ gravity creates oscillations in the photon-baryon plasma. After decoupling these oscillations correspond to a characteristic scale $\sim 150 Mpc$ (comoving horizon at recombination). This scale behaves like a standard ruler and can be used to determine the nature of DE.

Sunyaev & Zeldovich (1970) *Peebles & Yu (1970)*





Large length scale, better understood systematics.

Nature 440, 1126 (2006)

Galaxy clustering is anisotropic and the BAO scale can be measured both in the radial and the transverse direction. Radial direction gives expansion rate: $H = \dot{a}/a$.

The Cosmological Constant provides excellent agreement with observations !

$$\Omega_\Lambda \simeq 2/3, \;\; \Omega_m \simeq 1/3$$

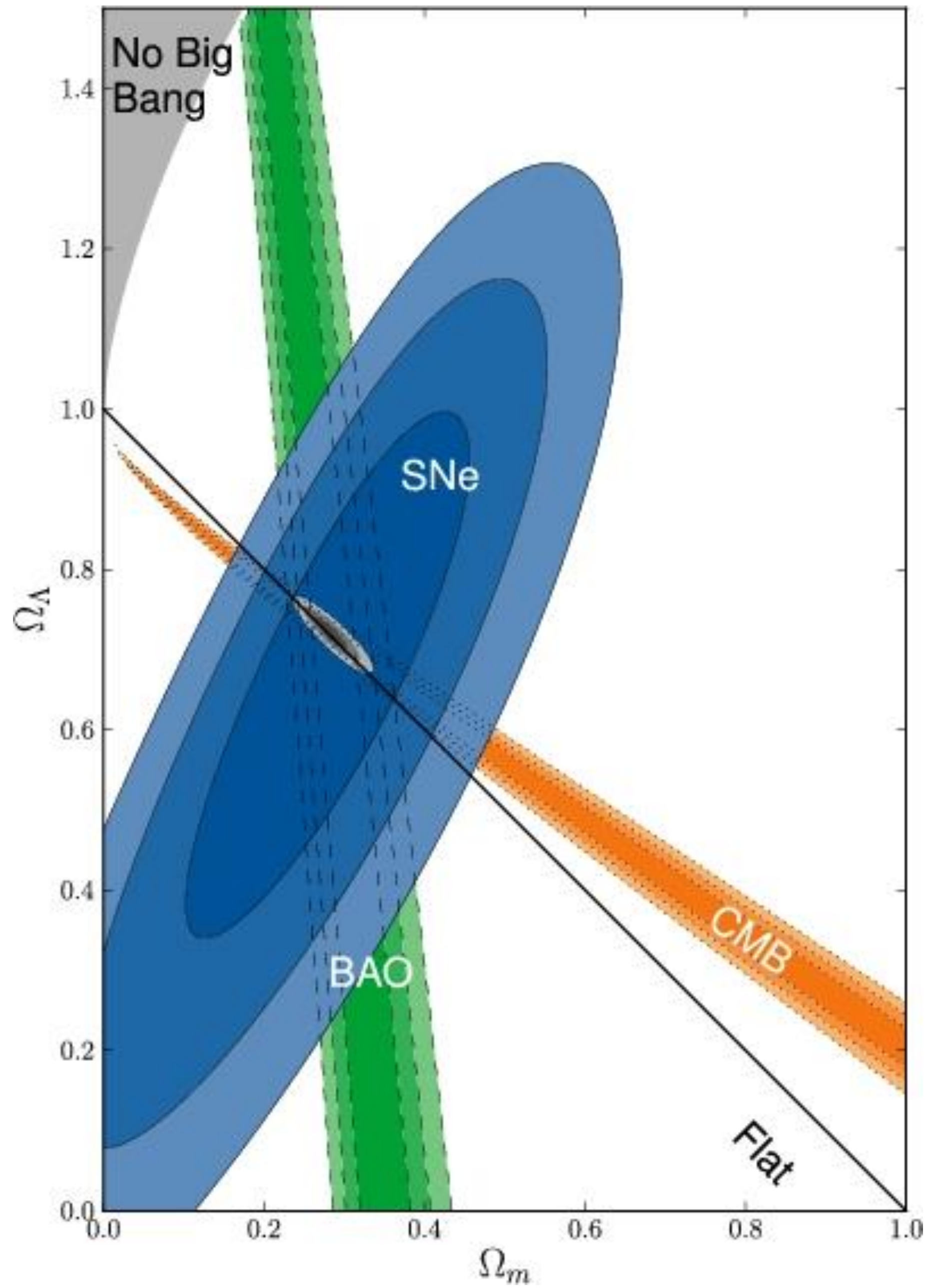
$$\text{where} \;\; \Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \;\; \Omega_m = \frac{8\pi G\rho}{3H_0^2}$$

$$\Omega_m + \Omega_\Lambda \simeq 1$$

$$\Rightarrow \;\; \Lambda \simeq 3H_0^2(1-\Omega_m)$$

⇩

Λ can be determined if one knows H_0 and Ω_m



[Suzuki et al arXiv:1105.3470]

Indirect evidence for an accelerating Universe also comes from its **AGE**

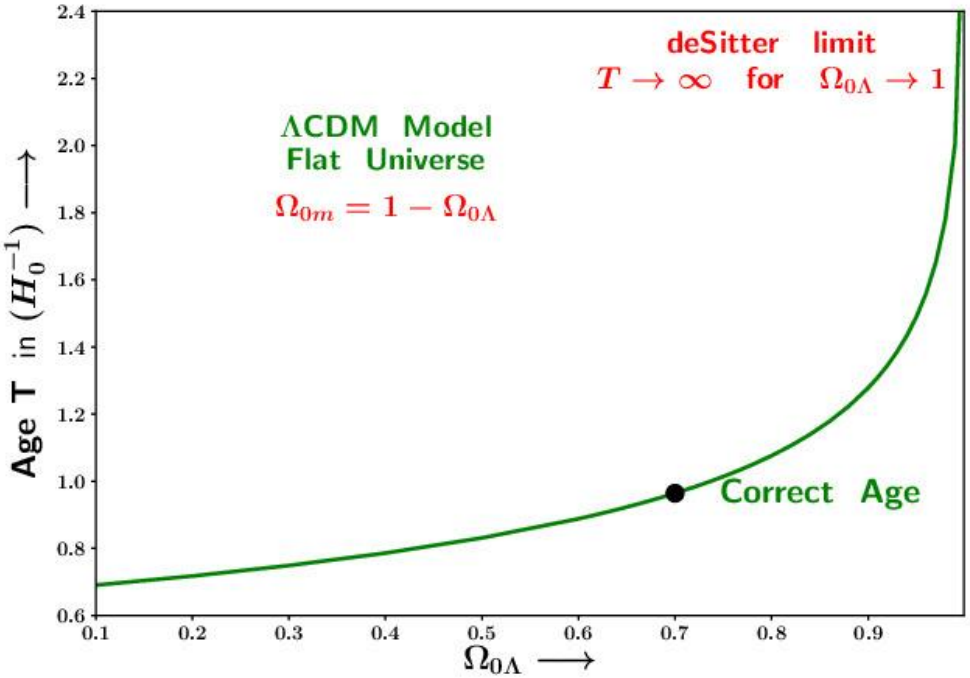
The presence of a cosmological constant leads to an **older** Universe !

- In the absence of Λ, $a(t) \propto t^{2/3} \Rightarrow t = \dfrac{2}{3H(t)} \Rightarrow t_0 \simeq 9\,\mathrm{Gyrs}$, for $H_0 \simeq 70\, km/s/Mpc$

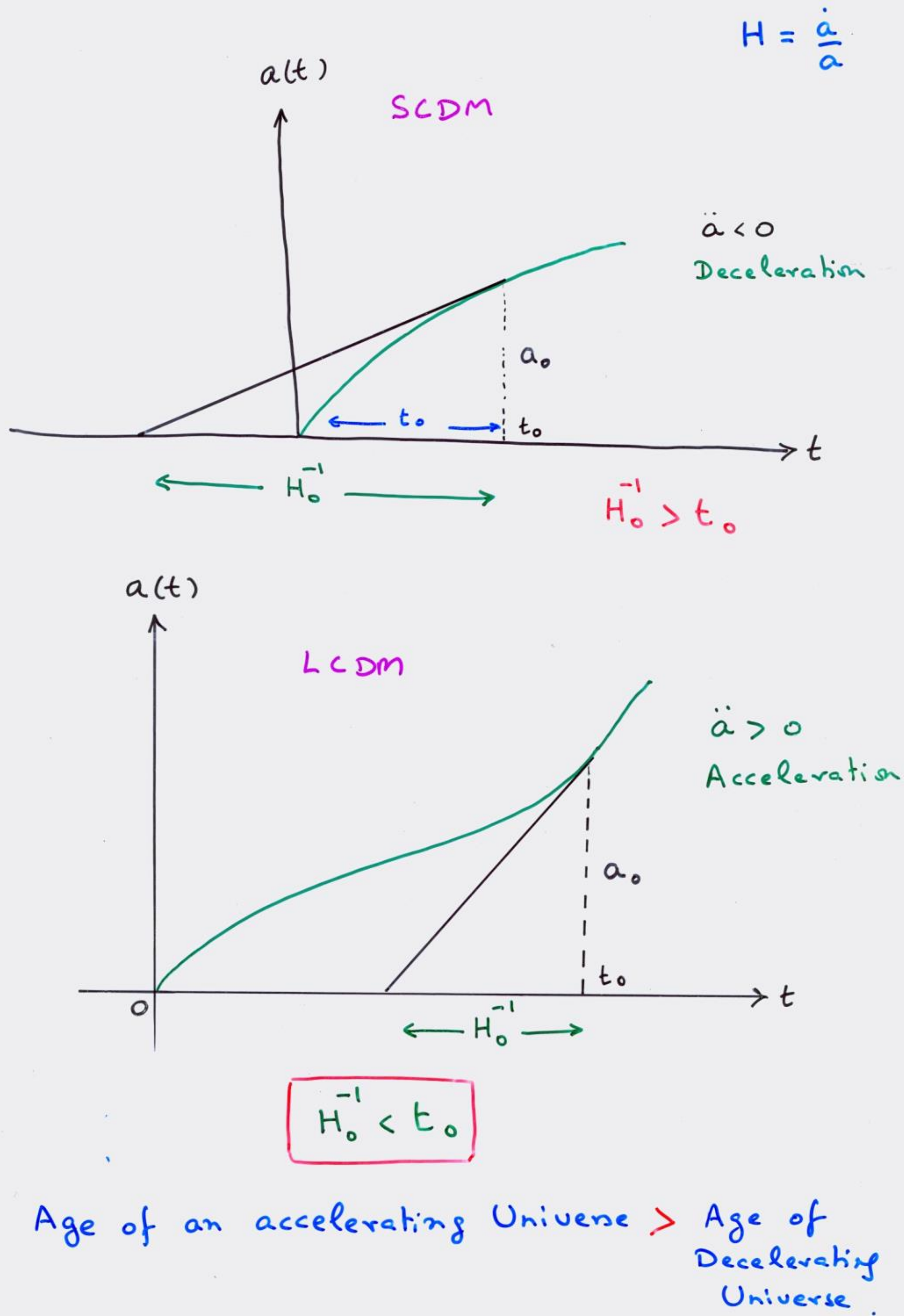
But this is too short since globular clusters have ages > 11 Gyrs !

$$\text{Also } t(z) = \frac{2}{3H(z)} = \frac{2}{3H_0(1+z)^{3/2}} = t_0\,(1+z)^{-3/2}$$

Discovery of old high-z objects provides a way of constraining models.



$$\text{Age of } \Lambda\text{CDM Universe} > \frac{2}{3H_0}$$



$$\text{slope} = \dot{a} \equiv \frac{a}{T} \Rightarrow T = \frac{a}{\dot{a}} = H_0^{-1}$$

$$\Rightarrow t_0 < H_0^{-1} \text{ in decelerating Univ.}$$

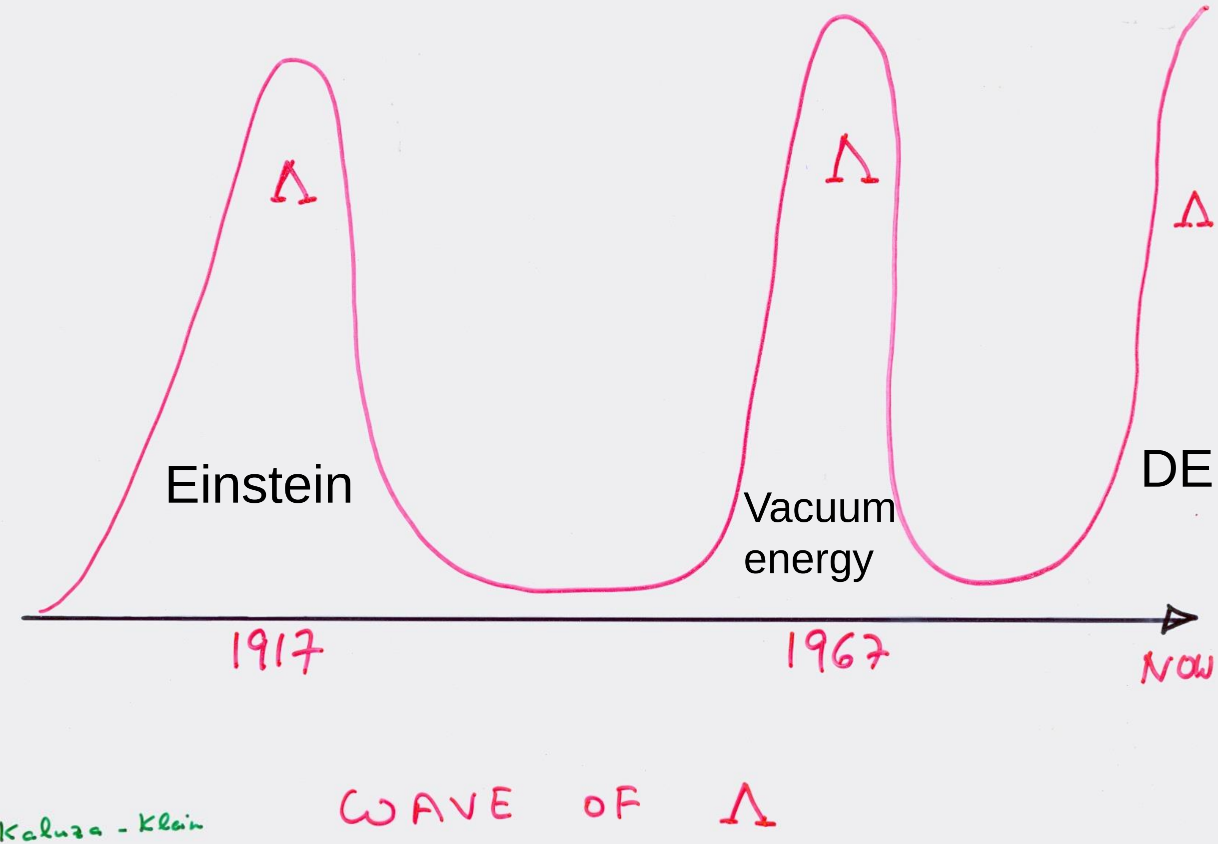
$$\Rightarrow t_0 > H_0^{-1} \text{ in accelerating Univ.}$$

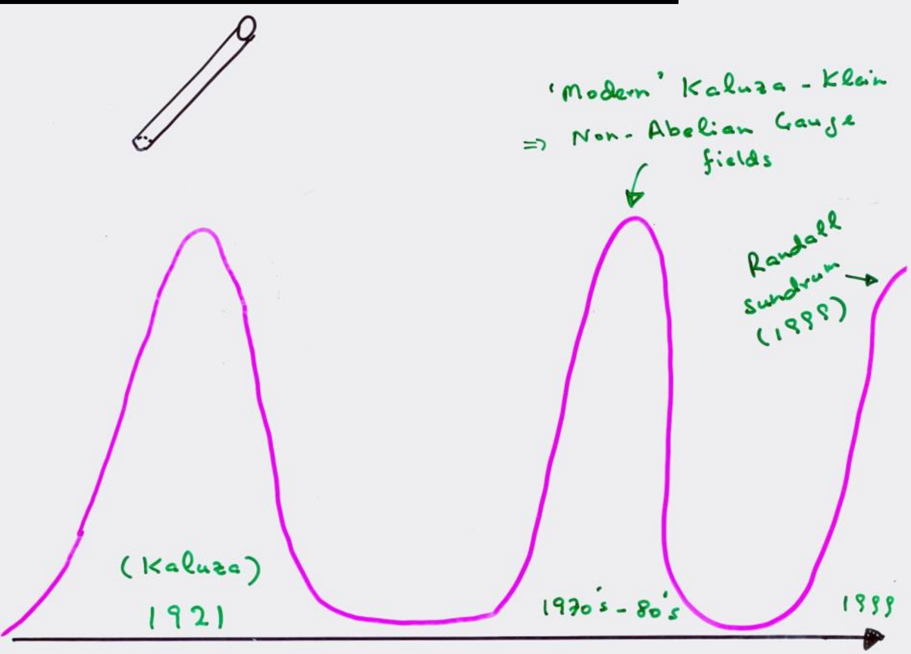
Accelerating universe is
older
than a decelerating universe.

BUT

What is the Cosmological Constant ?

$$\Lambda$$





`Wave' of extra dimensions

Brief History

- In 1917 Einstein proposes the Cosmological Constant `Λ' and constructs a closed quasi-static universe using:

$$G_{ik} = 8\pi G\, T_{ik} + \Lambda g_{ik}$$

In a letter to Ehrenfest Einstein writes

``I have again perpetrated something relating to the theory of gravitation that might endanger me of being committed to a madhouse''.

- In 1917 de Sitter presents vacuum solutions of $G_{ik} = \Lambda g_{ik}$

- In 1922 Alexander Friedman constructs a matter dominated expanding universe without Λ.

- In 1923, in a letter to Weyl, Einstein says

``If there is no quasi-static world, then away with the cosmological term ! ''

- *``Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life.'' -- George Gamow, My World Line, 1970*

THE COSMOLOGICAL CONSTANT AND THE THEORY OF ELEMENTARY PARTICLES

Ya. B. ZEL'DOVICH

Institute of Applied Mathematics, USSR Academy of Sciences

Usp. Fiz. Nauk 95, 209–230 (May, 1968)

Interest in gravitation theory with a cosmological constant was revived in 1967. Three papers were published, by Petrosian, Salpeter, and Szekeres in the USA[1] and by Shklovskii[2] and Kardashev[3] in the USSR, in which evolutionary universe models[1] in such a theory (the Λ models) are considered. The stimulus for the revival of the theory was provided by new observational data on remote quasistellar sources (quasars and quasags, QSR and QSG in the English-language literature).* It turned out, first of all, that for these objects the connection between the brightness and the red shift does not fit the simple models without a cosmological constant (and without assumptions concerning the evolution of the quasars!). In addition, as noted by the Burbidges[4], in ten quasars whose spectra have revealed absorption lines the red shift of these lines $z = (\lambda - \lambda_0)/\lambda_0$ lies in the narrow range $1.94 < z < 1.96$ or even $1.945 < z < 1.955$. This phenomenon will henceforth be referred to briefly as $z = 1.95$.

The Λ models were introduced in[1] to explain the observed relation between the red shift and the brightness; the explanation of $z = 1.95$ in the absorption spectrum was touched upon casually. References 2 and 3 are devoted entirely to the explanation of $z = 1.95$: the absorption lines are ascribed to galaxies lying along the path of the light ray arriving from the quasar. The predominant appearance of one value of z is attributed by the authors to the fact that with this z[2] the expansion of the universe was greatly slowed down both compared with the preceding period ($z > 1.95$) and compared with the succeeding period ($z < 1.95$ up to $z = 0$, corresponding to the present time). The slowed-down expansion leads to an increase of the path traversed by the ray in the corresponding interval of z, and increases the probability that the quasar light ray will encounter a galaxy and that absorption lines with precisely this value of z, i.e. about 1.95, will be imprinted in it.[3]

An expansion law with a sharp deceleration at a definite value of z is possible only for the Λ models; it is necessary here to satisfy with great accuracy the relation between the total amount of matter in the universe and the value of the cosmological constant Λ. The discussed model is closed in its three dimensional geometrical structure. As shown by Kardashev[3], the assumption

The cosmological constant was placed on a firm physical foundation by Zeldovich who showed that

$$\langle T_{ik} \rangle_{\rm vac} = \Lambda g_{ik}$$

ie. the vacuum had properties reminiscent of a Λ term !

Zeldovich (1968)

Pauli (1950-51)

``The Genie [cosmological constant] has been let out of the bottle, and it is no longer possible to force it back in''. – Zeldovich (1968)

The vacuum should be Lorentz invariant →

Let us explicitely demonstrate the Lorenz invariance of the equation of state $P = -\rho$ by considering the transformation properties of the energy-momentum tensor $T_{ik} = \Lambda g_{ik}$.

Consider a reference frame O' moving with a velocity $\mathbf{v} = (v, 0, 0)$ with respect to O, the components of the symmetric tensor T_{ik} transform as

$$
\begin{aligned}
T'_{00} &= \frac{1}{1 - v^2/c^2}\Big[T_{00} - \frac{2v}{c}T_{01} + (\frac{v}{c})^2 T_{11}\Big] \\
T'_{11} &= \frac{1}{1 - v^2/c^2}\Big[T_{11} - \frac{2v}{c}T_{10} + (\frac{v}{c})^2 T_{00}\Big] \\
T'_{01} &= \frac{1}{1 - v^2/c^2}\Big[\{1 + (\frac{v}{c})^2\}T_{01} - \frac{v}{c}(T_{00} + T_{11})\Big] \\
T'_{02} &= \frac{1}{\sqrt{1 - v^2/c^2}}[T_{02} - \frac{v}{c}T_{12}] \\
T'_{03} &= \frac{1}{\sqrt{1 - v^2/c^2}}[T_{03} - \frac{v}{c}T_{13}] \\
T'_{12} &= \frac{1}{\sqrt{1 - v^2/c^2}}[T_{12} - \frac{v}{c}T_{02}] \\
T'_{13} &= \frac{1}{\sqrt{1 - v^2/c^2}}[T_{13} - \frac{v}{c}T_{03}] \\
T'_{23} &= T_{23}, \quad T'_{22} = T_{22}, \quad T'_{33} = T_{33}
\end{aligned} \tag{1}
$$

Substituting $T_{ik} = \mathrm{diag}(\Lambda, -\Lambda, -\Lambda, -\Lambda)$, we find $T'_{ij} = 0$ if $i \neq j$ and

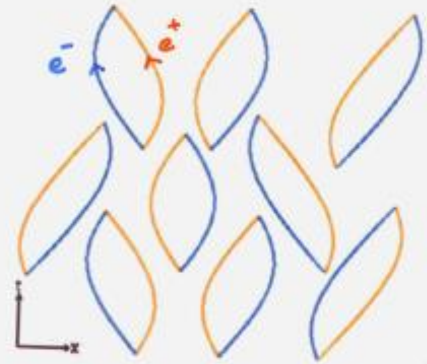
$$
\begin{aligned}
T'_{00} &= \frac{1}{1 - v^2/c^2}[T_{00} + (\frac{v}{c})^2 T_{11}] = \Lambda \\
T'_{11} &= \frac{1}{1 - v^2/c^2}[T_{11} + (\frac{v}{c})^2 T_{00}] = -\Lambda \\
T'_{22} &= T_{22} = -\Lambda \\
T'_{33} &= T_{33} = -\Lambda
\end{aligned} \tag{2}
$$

thus

$$T'_{ik} = \mathrm{diag}(\Lambda, -\Lambda, -\Lambda, -\Lambda)$$

demonstrating that the equation of state $P = -\rho$ is Lorentz invariant.

$$\Delta E \, \Delta t \gtrsim \hbar$$



The vacuum is a sea of virtual particle-antiparticle pairs continuously being created and destroyed.

The vacuum is seething with activity!

conducting plates

$\int \rightarrow \Sigma$

Boundaries change the wave-function and lead to a force of attraction between the plates

$$\langle T_{ik} \rangle_{vac} = \Lambda g_{ik}$$

$$F = -\frac{\hbar c \pi^2}{240 L^4}$$

↓

Casimir Effect confirms the existence of vacuum fluctuations

Measured to an accuracy of $\lesssim 5\%$!

Lambrecht and Reynaud Seminaire Poincaré 1 (2002) 79.

Classical fields can be written in terms of 'normal modes' which satisfy an oscillator-type equation. For instance the Electromagnetic potential can be written as

$$\mathbf{A} = \sum_k (\mathbf{a_k} e^{i\mathbf{kr}} + \mathbf{a_k}^* e^{-i\mathbf{kr}})$$

where $\mathbf{a_k} \sim e^{-iw_k t}$, $w_k = ck$. This results in the following expression for the Electromagnetic field energy

$$\begin{aligned} \mathcal{E} &= \tfrac{1}{8\pi} \int (\mathbf{E}^2 + \mathbf{H}^2) dV \ , \\ \mathcal{E} &= \textstyle\sum_k \mathcal{E}_k, \ \mathcal{E}_k = \frac{k^2 V}{2\pi} \mathbf{a_k} \mathbf{a_k}^* \ . \end{aligned}$$

Now we rewrite $\mathbf{A}$ in terms of the 'canonical variables' $\mathbf{Q_k}$ and $\mathbf{P_k}$:

$$\begin{aligned} \mathbf{Q_k} &= \sqrt{\tfrac{V}{4\pi c^2}} (\mathbf{a_k} + \mathbf{a_k}^*) \ , \\ \mathbf{P_k} &= -iw_k \sqrt{\tfrac{V}{4\pi c^2}} (\mathbf{a_k} - \mathbf{a_k}^*) = \dot{\mathbf{Q}}_k \ , \end{aligned}$$

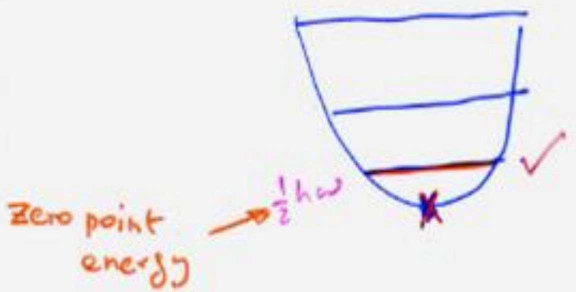
which leads to the Hamiltonian

$$H = \sum_k H_k = \sum_k \frac{1}{2} (\mathbf{P_k}^2 + w_k^2 \mathbf{Q_k}^2) \ ,$$

and the equations of motion $\partial H / \partial \mathbf{Q_k} = -\dot{\mathbf{P}}_k$ become

$$\ddot{\mathbf{Q}}_k + w_k^2 \mathbf{Q_k} = 0 \ .$$

The electromagnetic field is a sum of harmonic oscillators !



Absolute Rest is not allowed

Classical oscillator equation:

$$\ddot{x} + w^2 x = 0$$

Hamiltonian:

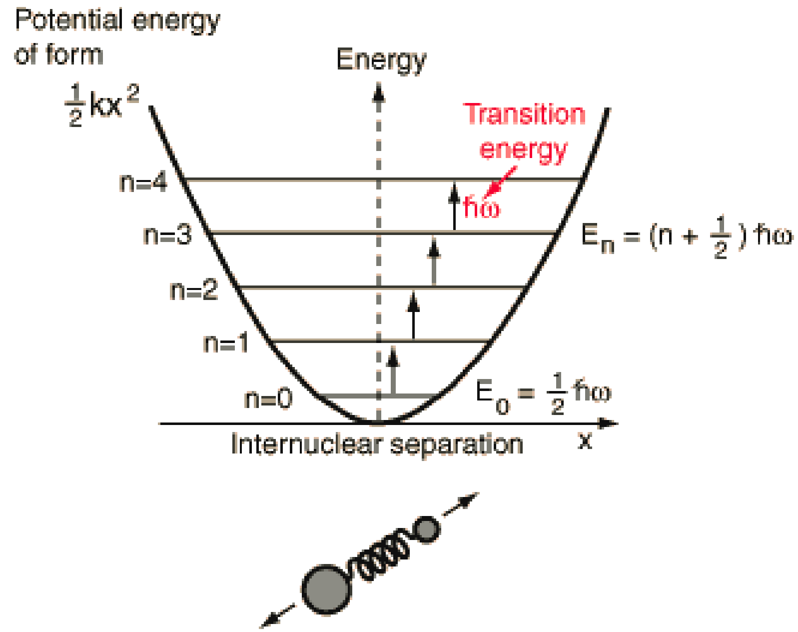
$$H = \frac{P^2}{2m} + \frac{1}{2} m w^2 Q^2$$

Quantizing the classical oscillator leads to an infinite sequence of energy levels

$$E_n = (n + \frac{1}{2})\hbar w, \quad n = 0, 1, 2, \ldots$$

The ground state energy level $E_0 = \frac{1}{2}\hbar w$ is called the zero-point energy. It exists because the uncertainty principle $\Delta p \Delta x \geq \hbar$ forbids a state of rest for the system.

Therefore a classical E-M field, when quantized, has zero-point vacuum energy. $\langle T_{00} \rangle \propto \sum \frac{1}{2}\hbar\omega$



x=0 represents the equilibrium separation between the nuclei.

The cosmological constant generated by vacuum fluctuations of particles with mass m and spin j is

$$\langle T_{00} \rangle_{\text{vac}} = (-1)^{2j}(2j+1) \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} ,$$

where $\omega_k = \sqrt{k^2 + m^2}$.

This leads to the Cosmological Constant Problem since

$$\langle T_{00} \rangle_{\text{vac}} \propto k^4 \to \infty, \ as \ k \to \infty.$$

The vacuum state has infinite energy !

A cutoff at the Planck scale ($m_{\text{pl}} = \sqrt{hc/G}$) yields

$$\langle T_{00} \rangle_{\text{vac}} \simeq m_{\text{pl}}^4 \sim 10^{93} \text{ g/cm}^3,$$

BUT

$$\rho_{\text{DE}} \simeq 10^{-30} \text{ g/cm}^3$$

Therefore

$$\rho_{\text{DE}} \simeq 10^{-123} \times \langle T_{00} \rangle_{\text{vac}}$$

Q. How does one account for such a small value of Λ today ?

Bosons and fermions contribute with opposite sign to the vacuum energy density

$$\mathcal{E}_{\text{bosons}} = +\frac{1}{2}\sum_k \omega_k, \quad \mathcal{E}_{\text{fermions}} = -\frac{1}{2}\sum_k \omega_k ,$$

where $\omega_k = \sqrt{k^2 + m^2}$. The discovery of supersymmetry in the 1970's led to the hope that the cosmological constant problem may be resolved by a judicious balance between bosons and fermions in nature. However supersymmetry (if it exists) is broken at the low temperatures prevailing in the universe today, therefore the cosmological constant should vanish in the early universe, but reappear during late times when the temperature has dropped below T_{SUSY}. But this is the very opposite of what one is looking for, since, a large value of Λ at early times is advantageous from the viewpoint of inflation, whereas a small current value of Λ today is in agreement with observations. Therefore

$$\langle T_{00}\rangle_{\text{vac}} \simeq m^4_{\text{SUSY}} \sim 10^{30}\ \text{g/cm}^3 \gg 10^{-30}\ \text{g/cm}^3 \ \ !$$

Energy of the quantum vacuum

Observed:	$\rho \simeq 10^{-30}$	$\mathrm{g\ cm^{-3}}$
Quantum field theory:	$\rho = \infty$	$\mathrm{g\ cm^{-3}}$
Quantum gravity:	$\rho \approx 10^{+90}$	$\mathrm{g\ cm^{-3}}$
Supersymmetry:	$\rho \approx 10^{+30}$	$\mathrm{g\ cm^{-3}}$
Higgs potential:	$\rho \approx -10^{+25}$	$\mathrm{g\ cm^{-3}}$
Other sources:	$\rho \approx \pm 10^{+20}$	$\mathrm{g\ cm^{-3}}$

[Weinberg, Rev. Mod. Phys. 1989]

The Cosmological Constant Problem

As I was going up the stair
I met a man who wasn't there
He wasn't there again today
I wish, I wish he'd stay away

Hughes Mearns

SN

pre-1998 – why is $\Lambda = 0$?

post-1998 – why is Λ so small ?

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \simeq 10^{-47} GeV^4 \text{ ?!}$$

So is Λ a new fundamental constant in the Einstein equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} + \Lambda$$

Or can it be derived from the known constants of physics ?

- Zeldovich (1968) suggested that after the removal of divergences,

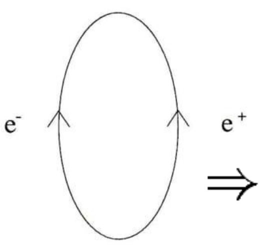
the `regularized' vacuum polarization contributed by a particle of mass m is

$$\epsilon_\Lambda \equiv \rho_{\rm vac} c^2 = \frac{G m^6 c^4}{\hbar^4}.$$

For $m = m_\pi$ one gets $\epsilon_\Lambda = \dfrac{\Lambda}{8\pi G} \simeq 10^{-47}\, GeV^4$

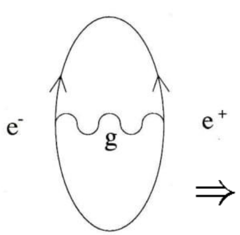
The gravitational interaction of particle-antiparticle pairs separated by $\lambda = \dfrac{\hbar}{mc}$ is

$$\epsilon_{\rm vac} = \rho_{\rm vac}\, c^2 \sim \frac{G m^2/\lambda}{\lambda^3} = G m^6 c^4/\hbar^4$$



$\Rightarrow \langle T_{\rm ik}\rangle_{\rm vac} = 0$

(a)



$\Rightarrow \langle T_{\rm ik}\rangle_{\rm vac} = \text{finite}$

(b)

- The fine structure constant $\alpha = e^2/\hbar c \simeq 1/137$

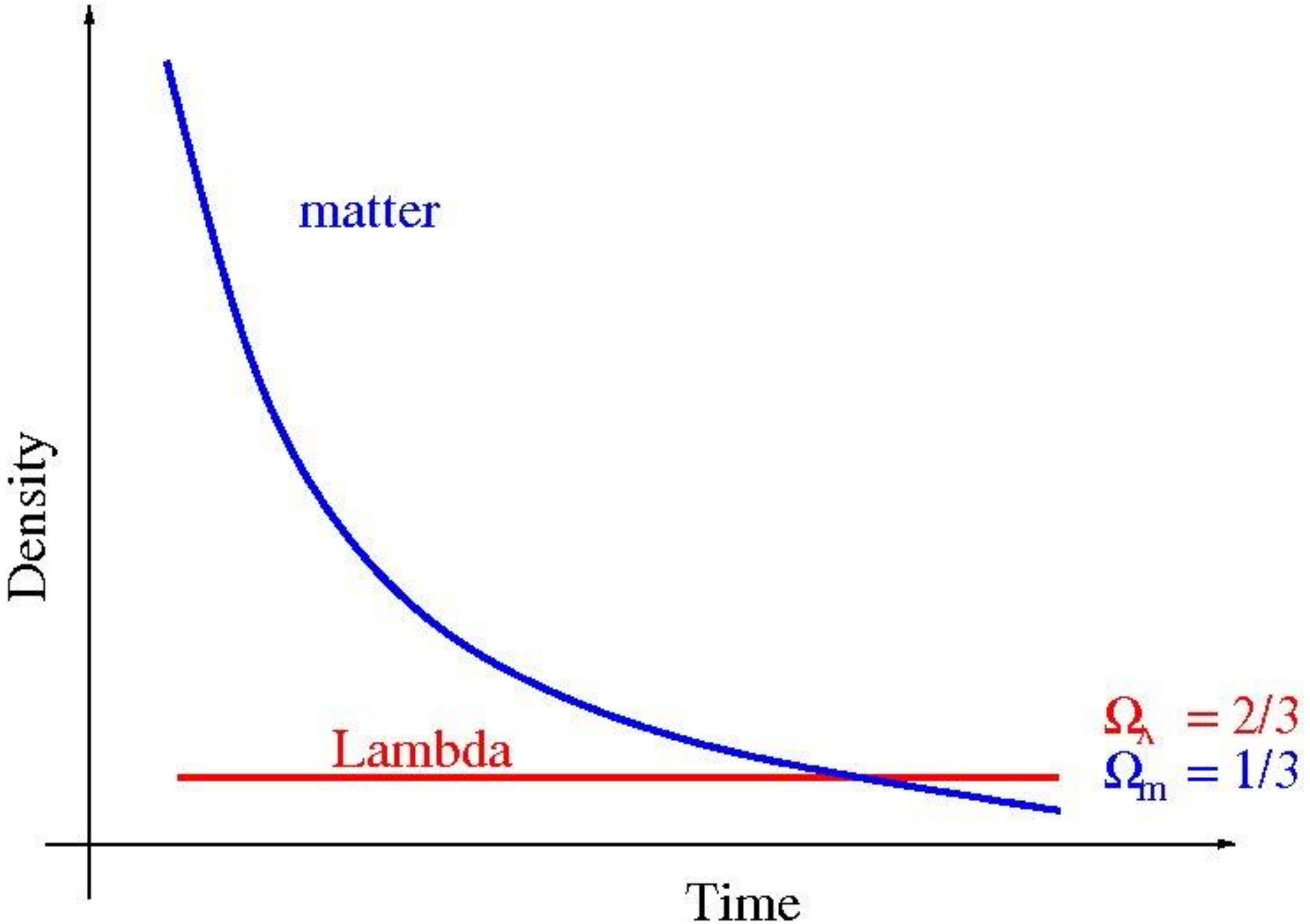
combined with the Planck constant, ρ_p where

$\rho_p = \dfrac{c^5}{G^2 \hbar} \sim 5 \times 10^{93} g/cm^3$ can lead to the relation

$$\rho_\Lambda = \frac{\rho_p}{(2\pi^2)^3} e^{-2/\alpha} \simeq 10^{-123} \rho_p = 10^{-47}\, GeV^4$$

$\rho_\Lambda / \rho_m \sim 10^{-44}$ at the electroweak scale if $\rho_{0m} \sim \rho_\Lambda$ now.

→ Initial conditions need fine tuning ?



Cosmic coincidence ?

Fine tuning problem for Λ

CMB temperature was higher in the past $T \propto 1/a(t)$

$$\rho_{\rm rad} = \rho_{0r}\left(\frac{a_0}{a}\right)^4 = \rho_{0r}\left(\frac{T}{T_0}\right)^4$$

$T_0 = 2.728^\circ$ K $\equiv 2.35 \times 10^{-13}$ GeV.

BUT

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = \text{constant !}$$

Therefore

$$\frac{\rho_\Lambda}{\rho_{\rm rad}} = \frac{\rho_\Lambda}{\rho_{0r}}\left(\frac{T_0}{T}\right)^4 \equiv \frac{\Omega_\Lambda}{\Omega_{0r}}\left(\frac{T_0}{T}\right)^4,$$

where

$$\Omega_{0r} = \rho_{0r}/\tfrac{3H^2}{8\pi G} = 2.48 \times 10^{-5}\ h^{-2},$$

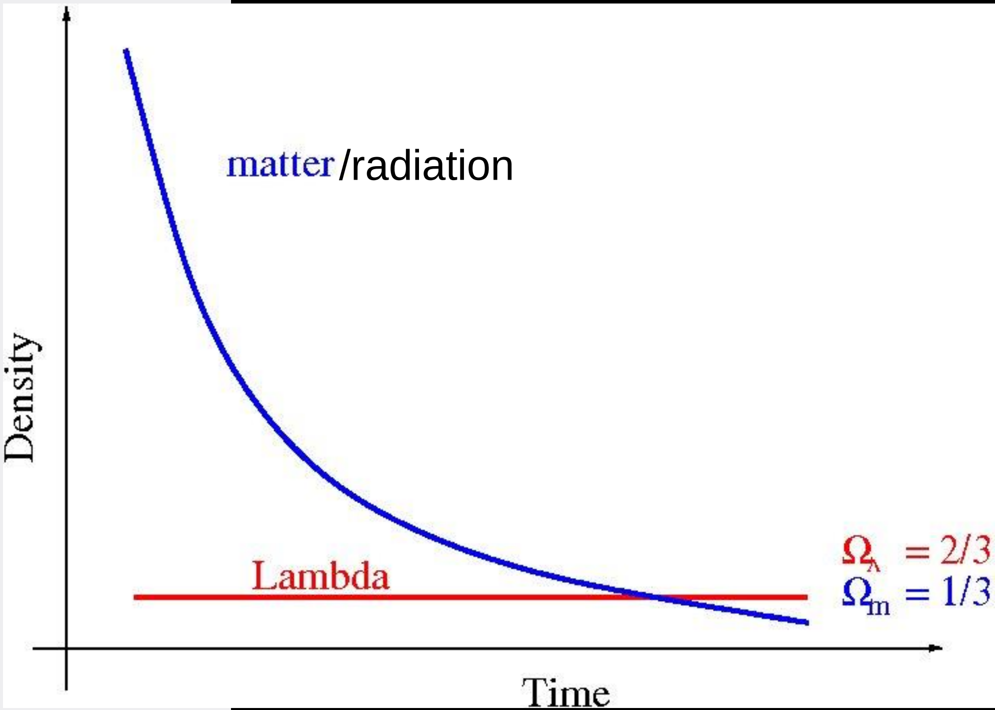
$$\Omega_\Lambda = \tfrac{\Lambda}{8\pi G} \simeq 0.7$$

At the Electroweak scale ($T = T_{EW} = 100$ GeV)

$$\frac{\rho_\Lambda}{\rho_{\rm rad}} \simeq 8.6 \times 10^{-55},$$

at the Planck scale ($T = T_{Pl} = 1.2 \times 10^{19}$ GeV)

$$\frac{\rho_\Lambda}{\rho_{\rm rad}} \simeq 4 \times 10^{-123}.$$



The dilemma of a cosmological constant prompted researchers to look for dynamical models of Dark Energy.

Dynamical relaxation

$$\Lambda_{\rm large} \longrightarrow \Lambda_{\rm small}$$

Early example: A massless non-minimally coupled scalar field in de Sitter space:

$$\mathcal{L} = \frac{1}{2}\left(\phi^{,l}\phi_{,l} - \xi R\phi^2\right),\ \xi < 0,\ \text{satisfies } \Box\phi + \xi R\phi = 0$$

$$\text{Einstein eqn}: H^2 = \frac{\Lambda}{3} + 8\pi G\,\xi H^2\phi^2 + \cdots$$

In the presence of a cosmological constant $\phi(t)$ is unstable and grows !

[Dolgov 1983]

$$\text{Therefore } H^2 \simeq \frac{\Lambda_{\rm eff}}{3} = \frac{\Lambda}{3\left(1 - 8\pi G\xi\phi^2\right)} \to 0$$

The value of $\Lambda_{\rm eff}$ declines rapidly!

- Perhaps it can solve the cosmological constant problem ?

Problem: Quenching mechanism won't work in the presence of matter since

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_\phi\right) + \frac{\Lambda}{3}$$

where $\rho_\phi = 3\xi H^2\phi^2 + \cdots\,,\ \xi < 0\,,$ and $\phi(t)$ grows with time

$$\Rightarrow\ H^2 = \frac{8\pi G_{\rm eff}}{3}\rho_m + \frac{\Lambda_{\rm eff}}{3}$$

where $G_{\rm eff} = \dfrac{G}{1 - 8\pi G\,\xi\phi^2} \to 0,\ \ \Lambda_{\rm eff} = \dfrac{\Lambda}{1 - 8\pi G\,\xi\phi^2} \to 0$ as $t \to \infty$

As a result, G becomes time dependent : $\dfrac{\dot{G}_{\rm eff}}{G_{\rm eff}} \simeq -2/t \sim -10^{-10}\,yr^{-1}$

which contradicts upper limits from lunar laser ranging experiments.

See: Dolgov 1983, Weinberg 1989, Sahni & Starobinsky 2000, Charmousis et al. 2011, Sola et al. 2017, for a discussion of self-tuning mechanisms for the cosmological constant.

Dynamical Dark Energy

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - V(\phi) \ , \ T_{ik} = \phi_{,i}\phi_{,k} - g_{ik}\mathcal{L}$$

$$\phi \equiv \ \text{inflaton/quintessence} \ \ (\phi_{,\mu} \equiv \partial\phi/\partial x^\mu)$$

For a homogeneous scalar field: $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \ P = \frac{1}{2}\dot{\phi}^2 - V(\phi).$

$$w = P/\rho = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

- $KE \gg PE \Rightarrow \frac{1}{2}\dot{\phi}^2 \gg V \Rightarrow w \simeq +1$
- $KE \ll PE \Rightarrow \frac{1}{2}\dot{\phi}^2 \ll V \Rightarrow w \simeq -1$

EOS lies in the interval $\boxed{-1 \leq w \leq +1}$

The acceleration of the universe is described by:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = \frac{8\pi G}{3}(V - \dot{\phi}^2)$$

An empty universe will accelerate ($\ddot{a} > 0$) if $\boxed{\dot{\phi}^2 < V(\phi)}$

Necessary, but not sufficient condition for acceleration.

Unfortunately potentials which work for inflation **do not work** for dark energy.

Inflation described by the slow roll parameters : $\epsilon = \frac{m_p^2}{16\pi}(V'/V)^2, \;\; \eta = \frac{m_p^2}{8\pi}(V''/V)$

$$\epsilon,\, \eta \ll 1 \quad \Rightarrow \quad \text{necessary condition for slow roll inflation}$$

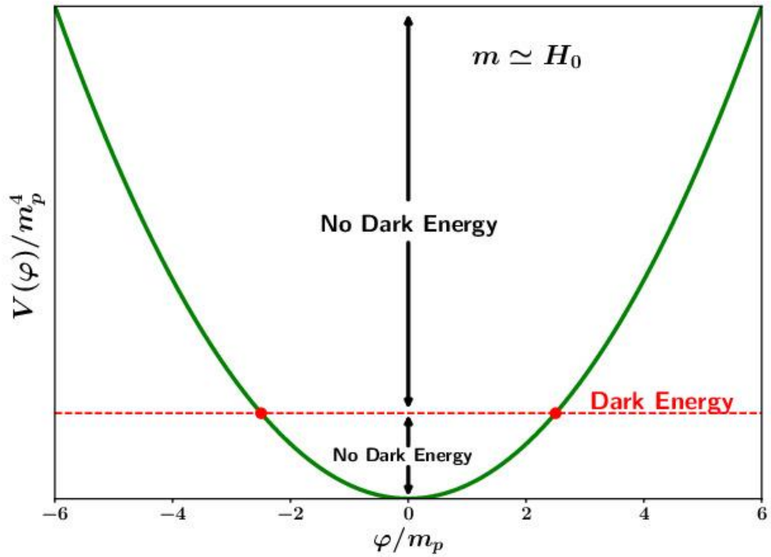
$$\text{For } V = \frac{1}{2}m^2\phi^2, \quad \epsilon,\, \eta < 1 \;\Rightarrow\; \phi \geq m_p$$

$$\text{Now } H_0^2 \simeq \frac{8\pi}{3m_p^2}(\rho_{\rm DE} + \rho_m) \simeq \frac{4\pi}{m_p^2}\rho_{\rm DE} \simeq 2\pi\, m^2\left(\frac{\phi}{m_p}\right)^2, \;\; \text{where } \rho_{\rm DE} \simeq \frac{1}{2}m^2\phi^2$$

$$\text{Since } \phi \geq m_p \;\Rightarrow\; m \leq H_0 \sim 10^{-33}\, eV\, !$$

$$\lambda_c = \frac{h}{mc} \sim cH_0 \sim 3000h^{-1}\, Mpc$$

Therefore one requires an ultra-light scalar field and an enormous fine-tuning of initial conditions !



Dark energy from tracker potentials

Example: the inverse power law potential $V = M^4 \left(\frac{M}{\phi}\right)^p$

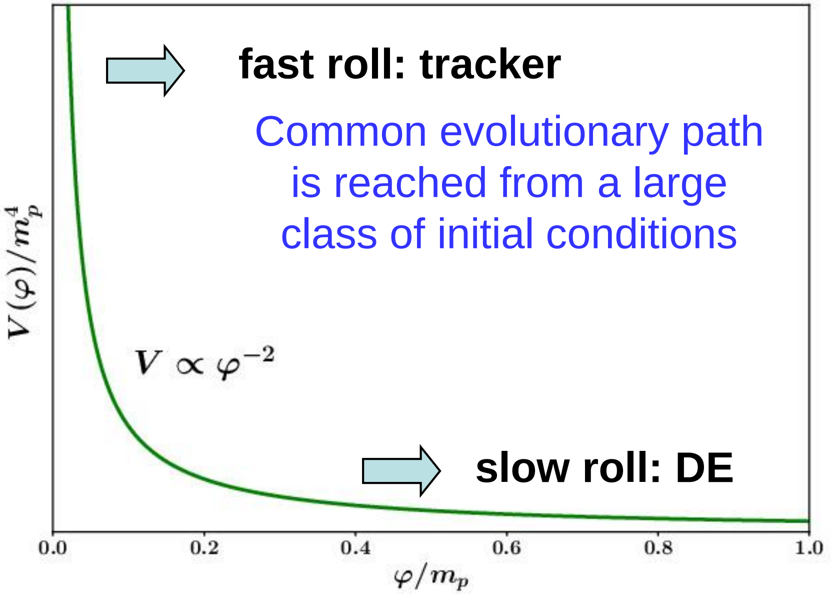
$$\text{For } \epsilon = \frac{m_p^2}{16\pi}(V'/V)^2 < 1, \;\; \eta = \frac{m_p^2}{8\pi}(V''/V) < 1 \;\; \Rightarrow \;\; \phi \geq m_p$$

$$\text{Currently } \rho_{\rm DE} \simeq V(\phi) = M^4 \left(\frac{M}{\phi}\right)^p \simeq 10^{-47}\,\text{GeV}^4 \sim H_0^2$$

$$\text{If } \phi \simeq m_p \simeq 10^{19}\,\text{GeV}, \text{ then } M \sim 0.1\,\text{GeV for } p = 2$$

Good news: more realistic parameter values in this model of DE.

Bad news: $p < 0.5$ suggested by recent observations which turns *M* into a very small quantity !



[Ratra & Peebles, 1988]

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - V(\phi) \ , \ T_{ik} = \phi_{,i}\phi_{,k} - g_{ik}\mathcal{L}$$

The inverse power law model provides one example of a **tracker** field

If $\boxed{V''V/V'^2 \geq 1}$ the scalar field tracks the dominant matter component.

The exponential potential $V(\phi) = V_0 \exp\left(-\lambda\phi/M_P\right)$ also results in tracker behavior

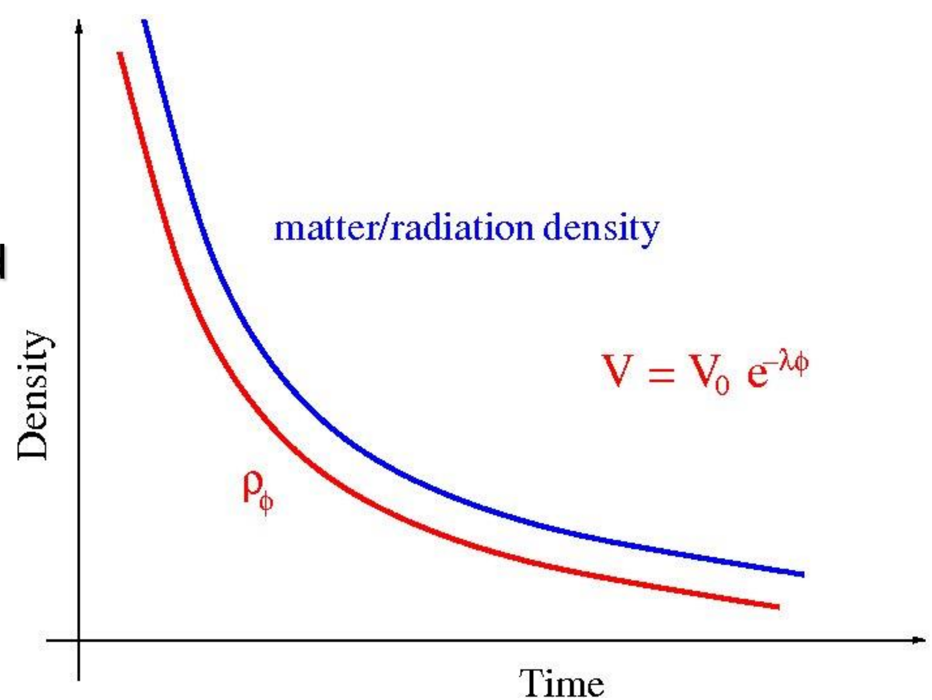
$$\frac{\rho_\phi}{\rho_{\rm total}} = \frac{3(1+w_B)}{\lambda^2} = constant$$

where w_B is the EOS of the background

and $M_P = 1/\sqrt{8\pi G}$.

A tracker can alleviate the initial conditions problem faced by the cosmological constant

Ratra and Peebles (1988)
Wetterich (1988)



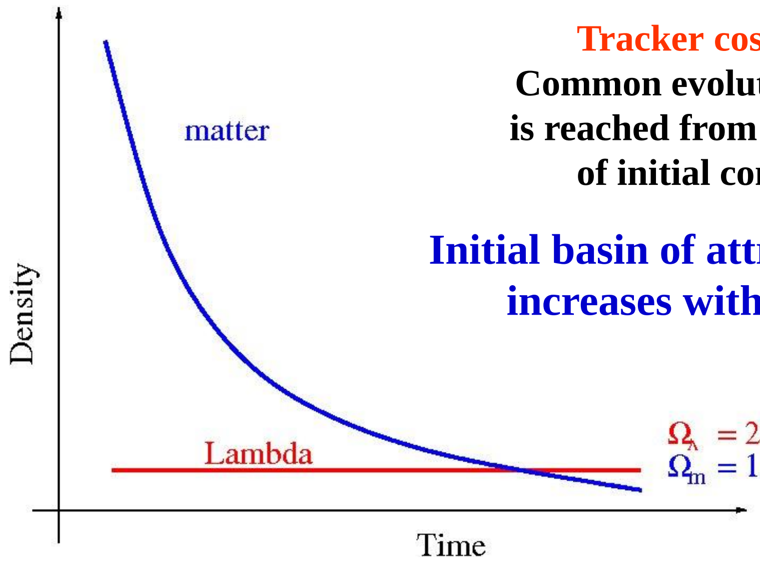
Tracker models of dark energy and dark matter

The cosmological constant fits the data quite well, but is associated with a substantial fine-tuning of initial conditions.

Tracker models can remedy this situation.

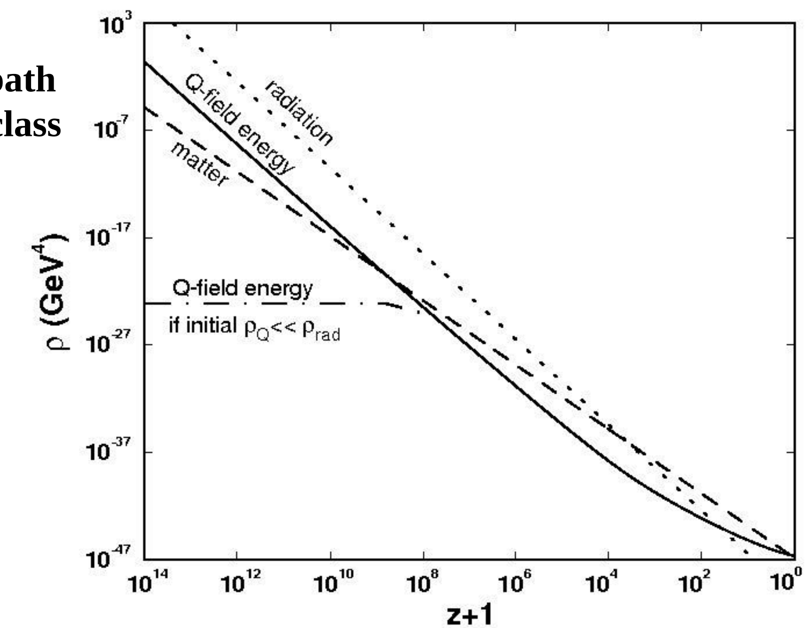
[Ratra & Peebles 1988, Zlatev, Wang & Steinhard 1999, Barreiro, Copeland & Nunes 2000, etc.]

For $V = V_0/\phi^p$, $w_\phi = \dfrac{p w_B - 2}{p+2}$ and $\dfrac{\rho_\phi}{\rho_B} \propto t^{\frac{4}{2+p}}$ during tracking.



Tracker cosmology:
Common evolutionary path is reached from a large class of initial conditions.

Initial basin of attraction increases with p



Unfortunately observations suggest p < 0.5 in $V \propto \phi^{-p}$ which considerably decreases the initial basin of attraction. [Park and Ratra, arXiv:1807.07421]

Tracker models based on the α attractors can remedy this situation.

Kallosh, Linde and Roest (2013) introduced the α-attractor family of potentials following the prescription

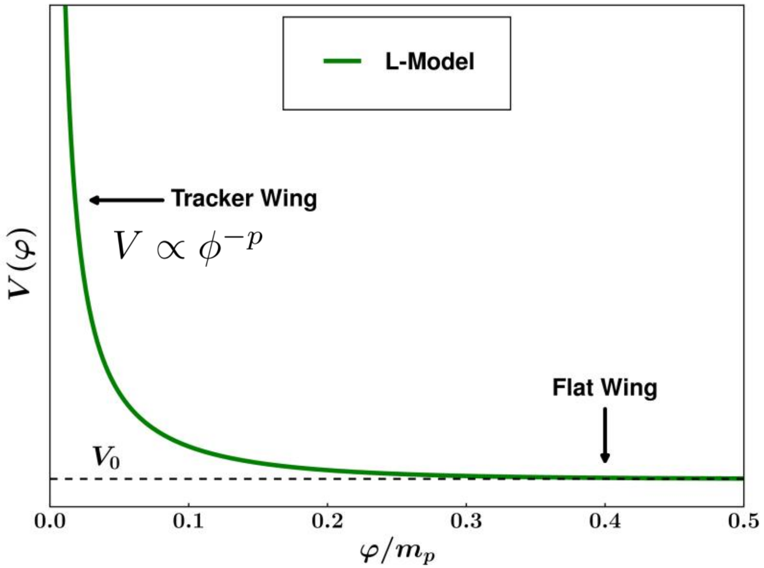
$$V(\varphi) = m_p^4 F\left(\tanh \frac{\varphi}{\sqrt{6\alpha}\, m_p}\right)$$

The simplest potential $V(\varphi) = V_0 \tanh^p(\lambda\varphi), \ \lambda = \frac{1}{\sqrt{6\alpha}}$ includes many important inflationary models.

Interestingly the α-attractor family can also describe **dark energy** and **dark matter** !

The L-model $V(\varphi) = V_0 \tanh^{-p}(\lambda\varphi) \equiv \coth^p(\lambda\varphi)$ describes **tracker DE**

$$V(\varphi) = V_0 \coth^p(\lambda\varphi)$$



For $\lambda\varphi \ll 1$ $\quad V \propto \varphi^{-p}$

Inverse power law (IPL)

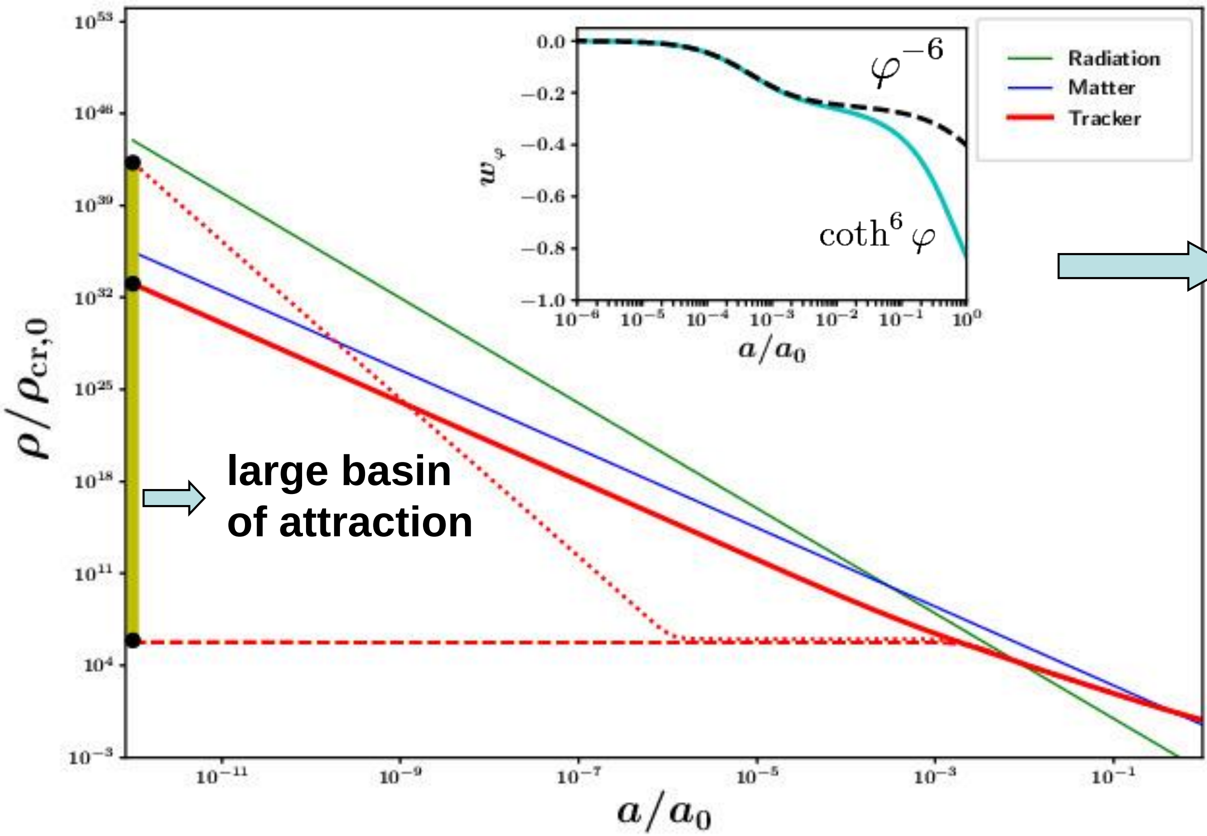
Tracker wing

For $\lambda\varphi \gg 1, \quad V \simeq V_0$

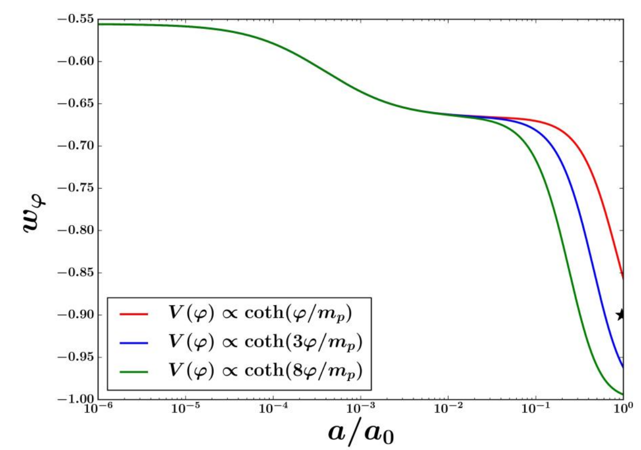
Flat wing

Bag, Mishra, Sahni, arXiv:1709.09193 JCAP

$$V(\varphi) = V_0 \coth^p(\lambda\varphi)$$



At late times the EOS in $\coth^p\varphi$ falls below the EOS in φ^{-p}.



The large basin of attraction allows equipartition initial conditions for DE.

Increasing λ in $\coth(\lambda\varphi)$ makes w_φ drop to even more negative values.

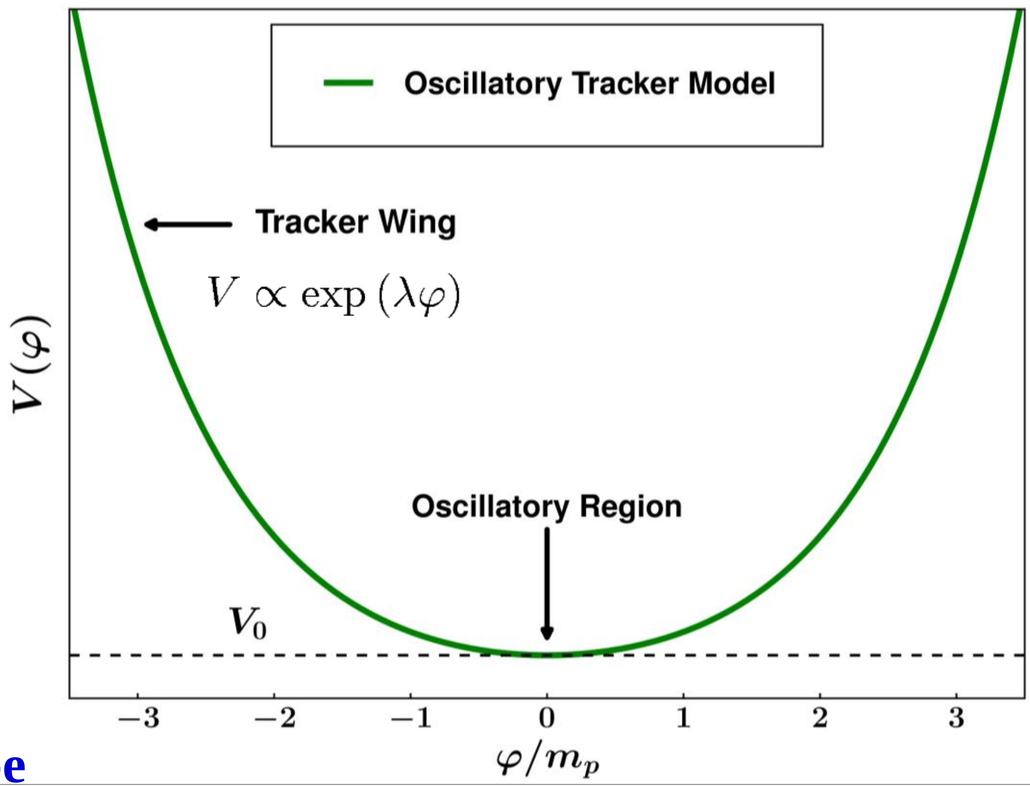
The coth potential can lead to $w_0 \sim -1$ from a **larger** initial basin of attraction than φ^{-p}

Another α attractor potential : $$V = V_0 \cosh(\lambda\varphi)$$

- **Early times:**

For $\lambda|\varphi| \gg 1, \quad V \propto \exp(\lambda\varphi)$

Tracker wing



An exponential potential leads to tracker behavior:

$$\frac{\rho_\phi}{\rho_{\rm total}} = \frac{3(1+w_B)}{\lambda^2} = constant$$

A common evolutionary path can be reached from a large class of initial conditions.

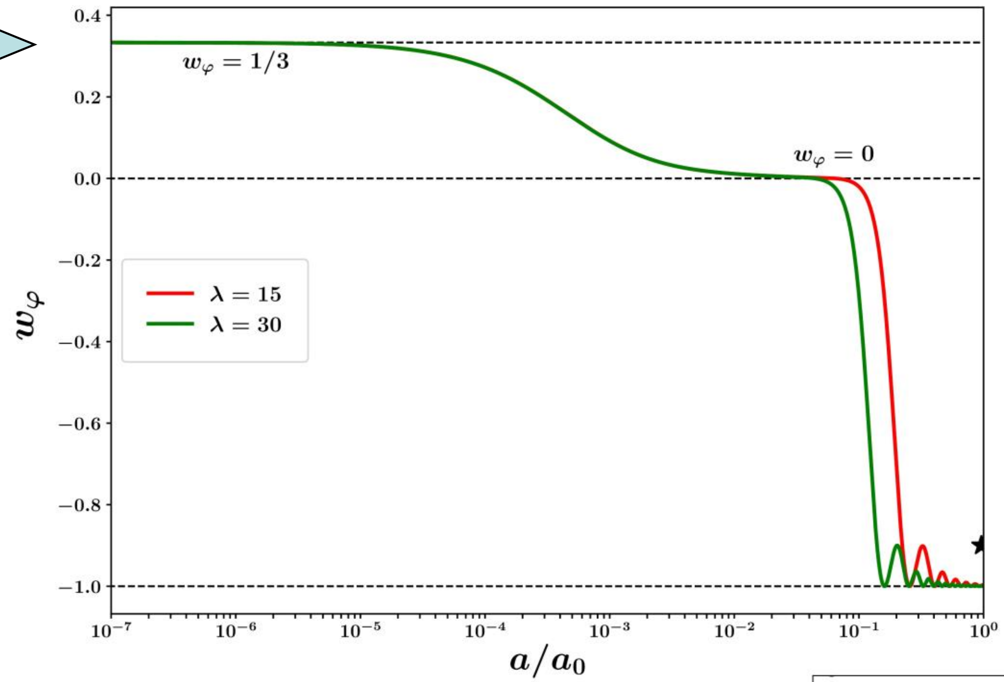
- **Late times:**

For $\lambda|\varphi| \ll 1\,, \quad V \simeq V_0\left[1 + \frac{1}{2}(\lambda\varphi)^2\right]$ **Oscillatory region**

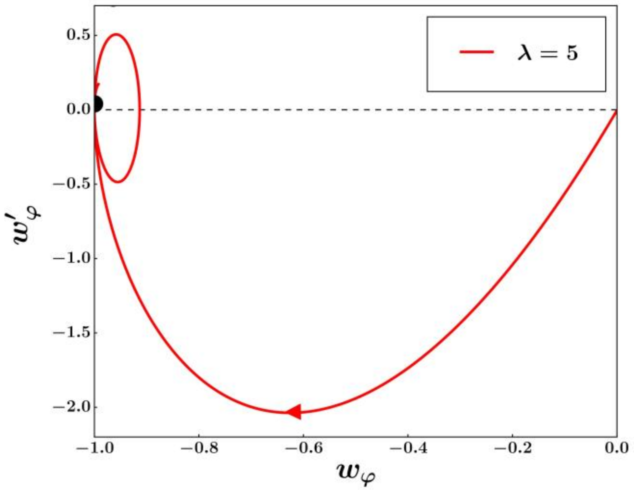
The LCDM asymptote $w_\varphi = -1$, is reached via small oscillations.

$$V = V_0 \cosh(\lambda\varphi)$$

Tracker



Bag, Mishra, Sahni, arXiv:1709.09193 JCAP



At late times $w_\varphi(t) \simeq -1 + \dfrac{\dot{\varphi}^2(t)}{V_0}$

oscillations

The LCDM asymptote $w_\varphi = -1$, is reached via small oscillations.

[A] Analogy between canonical scalar field Lagrangian and non-relativistic point particle Lagrangian in classical mechanics:

$$\text{Point particle Lagrangian}: \; \mathcal{L} = \tfrac{1}{2} m\dot{r}^2 - U(r)$$

$$\text{Scalar field Lagrangian}: \; \mathcal{L} = \tfrac{1}{2}\dot{\phi}^2 - V(\phi), \quad \phi \to r$$

[B] Analogy between Chaplygin gas and relativistic particle Lagrangian:

$$\text{Relativistic Lagrangian}: \; \mathcal{L} = -mc^2\sqrt{1-\frac{v^2}{c^2}} \qquad v = \dot{r}$$

Chaplygin gas: $\mathcal{L} = -V_0\sqrt{1-\phi_{,\mu}\phi^{,\mu}} \qquad \phi \to r$

2. Chaplygin Gas

The Born-Infeld lagrangian density $\mathcal{L} = -V_0\sqrt{1-\phi_{,\mu}\phi^{,\mu}}$

leads to the Chaplygin gas $(A = V_0^2)$ $p = -\dfrac{A}{\rho}$ < 0 !

The conservation equation

$$dE = -pdV \Rightarrow d(\rho a^3) = -pd(a^3)$$ gives $$\rho = \sqrt{A + \frac{B}{a^6}}$$

(B is a constant of integration.)

So that $\rho \propto a^{-3}$ at early times (like matter)

while $\rho \longrightarrow \mathrm{constant}$ at late times -- just like Λ !!

The Chaplygin gas behaves like pressureless matter at early times and like a cosmological constant during late times !!

Q. Can Chaplygin gas unify dark matter and dark energy ? **NO**

[Kamenshchik, Moschella, & Pasquier (2001)]

Note that the equation of state may not define the DE Lagrangian uniquely !

• The Chaplygin gas which has $p = -A/\rho$ can be described by a minimally coupled scalar field with the potential

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh(2\sqrt{6\pi G}\phi) + \frac{1}{\cosh(2\sqrt{6\pi G}\phi)}\right) ,$$

and associated with the Lagrangian density $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.

[Kamenschik, Moschella and Pasquier, PLB, 2001, Gorini et al, PRD 2005]

• However the Chaplygin gas can also be modeled completely differently using a scalar field with the Born-Infeld kinetic term:

$$\mathcal{L} = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} .$$

[Bilic et al, PLB 2002, Frolov et al, PLB 2002]

This illustrates the fact that the equation of state $w(z)$ does not uniquely define an underlying field-theoretic model !

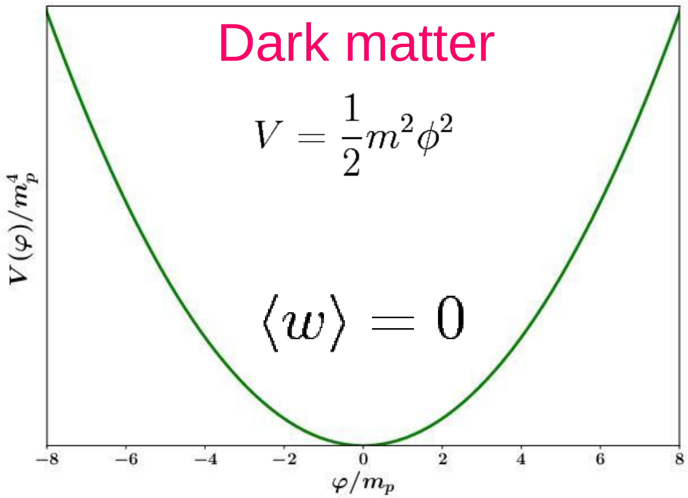
Models of dark matter and dark energy can also be constructed from an **oscillating** scalar field.

If $V(\phi) \sim \phi^{2p}$ then the mean EOS during oscillations is given by

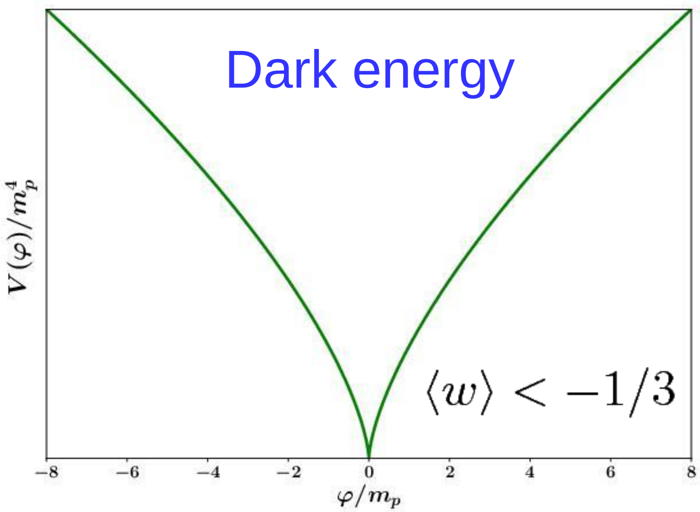
$$\langle w \rangle = \frac{p-1}{p+1}$$

[Turner 1983]

- p = 1 corresponds to **dark matter** with $\langle w \rangle = 0$
- p < ½ corresponds to **dark energy** with $\langle w \rangle < -1/3$



Commonly associated with Monodromy Inflation



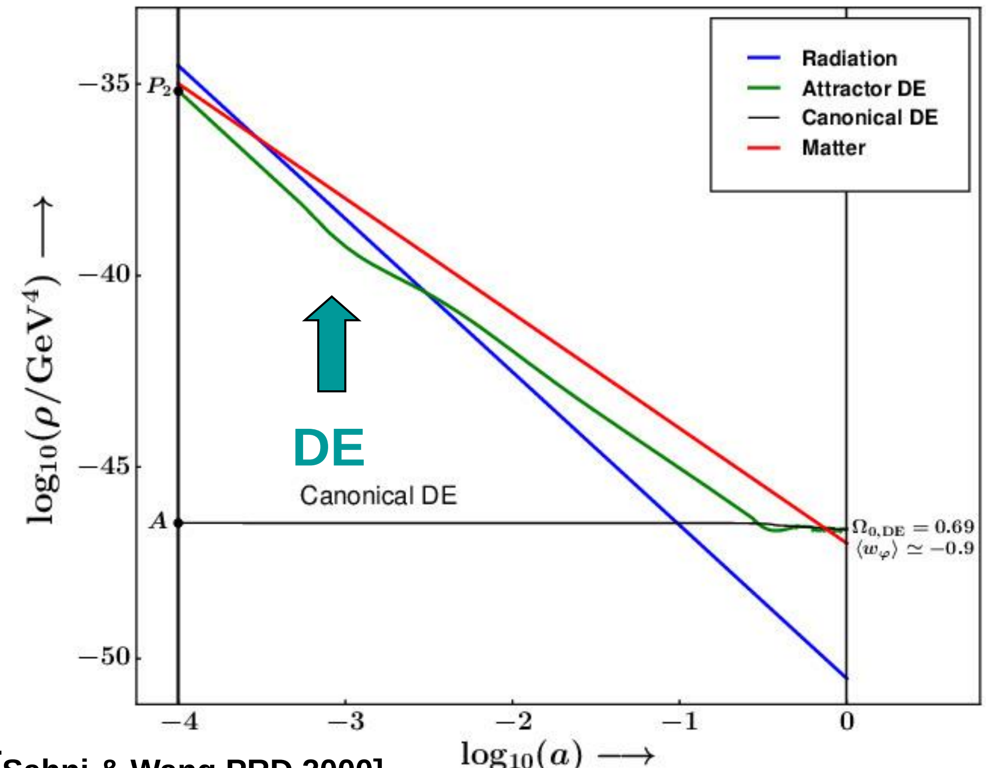
However a simple power law potential such as $V \propto \phi^{2p}$ suffers from a severe fine tuning problem since the initial conditions need to be finely tuned in order to get $\Omega_{\rm m,DE} \sim O(1)$ today.

This is avoided in the potential $V = V_0 \sinh^{2p} \lambda\phi$ which has the asymptotic form :

$$[A] \quad V(\phi) \sim e^{-p\lambda\phi} \quad \text{for } |\lambda\phi| \gg 1\,; \quad [B] \quad V(\phi) \sim \phi^{2p} \quad \text{for } |\lambda\phi| \ll 1$$

⇩

Ensures that at early times the field tracks the background: $\frac{\rho_\phi}{\rho_{\rm total}} = \frac{3(1+w_{\rm B})}{p^2\lambda^2}$



[Sahni & Wang PRD 2000]
[Mishra et al. JCAP 2018]

Large initial basin of attraction:
A common evolutionary path is reached from a large range of initial conditions.

- p = 1 corresponds to **dark matter**
- p < ½ corresponds to **dark energy**

Same potential can describe dark matter and dark energy !

Yet another possibility: the equation of state of dark energy might be super-negative: w < -1

[Caldwell 2002]

Phantom dark energy

• Dark energy requires the violation of the strong energy condition: $\rho + 3p \geq 0$

• Phantom DE requires the violation of the weak energy condition: $\rho + p \geq 0$

$$\Rightarrow w = p/\rho < -1$$

$$\text{Simple prescription}: \mathcal{L}_P = -\frac{1}{2}\dot{\phi}^2 - V$$

The sign of the kinetic term is flipped relative to $\mathcal{L}_Q = \frac{1}{2}\dot{\phi}^2 - V$

$$\text{standard formula } w_Q = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \text{ changes to}: \ w_P = \frac{-\frac{1}{2}\dot{\phi}^2 - V}{-\frac{1}{2}\dot{\phi}^2 + V} < -1$$

BUT …… $\dot{H} = 4\pi G\left(\rho_m - 4\pi G\dot{\phi}^2\right) \simeq -4\pi G\dot{\phi}^2$

$$\Rightarrow H \text{ grows at late times!}$$

→ **Universe encounters a Big Rip future singulatity !**

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A phantom menace?
Cosmological consequences of a dark energy component with super-negative equation of state

R.R. Caldwell
Department of Physics & Astronomy, Dartmouth College, Hanover, NH 03755, USA

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- But Phantom models run into problem such as ghosts and **singularities**:

PHYSICAL REVIEW D **70**, 043543 (2004)

The phantom menaced: Constraints on low-energy effective ghosts

James M. Cline,* Sangyong Jeon,* and Guy D. Moore*
Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8

(Received 24 November 2003; published 31 August 2004)

- Phantom DE runs into a `Big Rip' future singularity at which

$$H(t_{\rm BR}) \to \infty, \;\; \text{when } t_{\rm BR} = \frac{w}{1+w} t_{\rm eq}, \quad \text{where } \rho_P(t_{\rm eq}) = \rho_m(t_{\rm eq})$$

- A phantom-like EOS arises in some **modified gravity** theories: $w_{\rm eff} < -1$

It is also supported by BAO observations. No BIG RIP singularity !

Two ways of making the Universe ACCELERATE:

- modify the MATTER sector:

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

This leads to Physical models of DE such as Quintessence, Chaplygin Gas, Phantom matter etc.

- modify the GRAVITY sector:

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

The cosmological constant introduced by Einstein in 1917 was the first model of this kind since

$$G_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

This leads to Geometrical models of DE such as higher dimensional (Braneworld) Gravity, scalar-tensor gravity, string/M-theory inspired models, f(R) gravity, etc.

Cosmic acceleration from modified gravity

The Universe can accelerate even without the presence of additional fields which violate the SEC $\rho + 3p \geq 0$, if one modifies the gravity sector.

An early example is provided by the Starobinsky model of Inflation for which

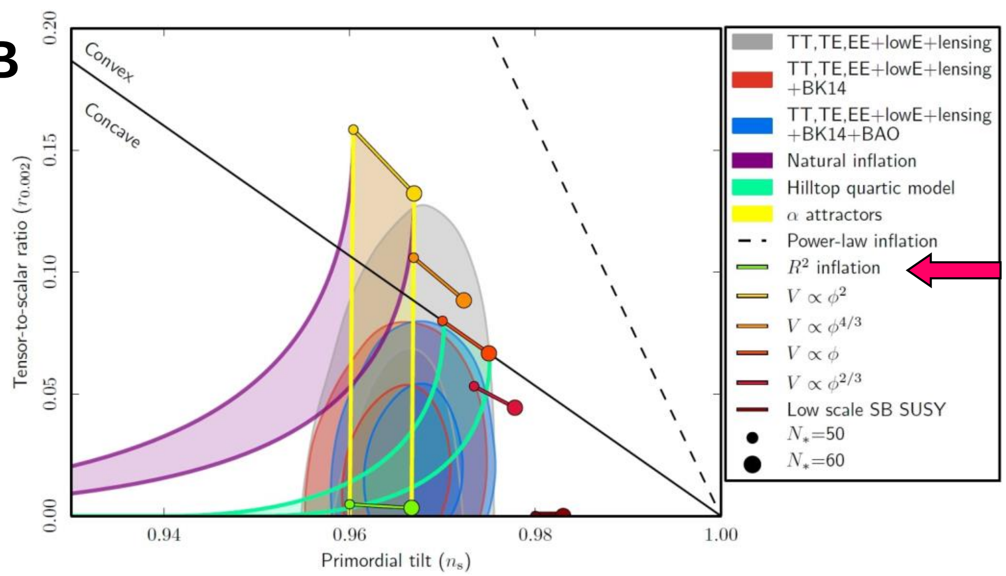
$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \frac{1}{6m^2}R^2\right] \quad \Rightarrow R^2 \text{ term leads to } \Box R \text{ term in eqn. of motion}$$

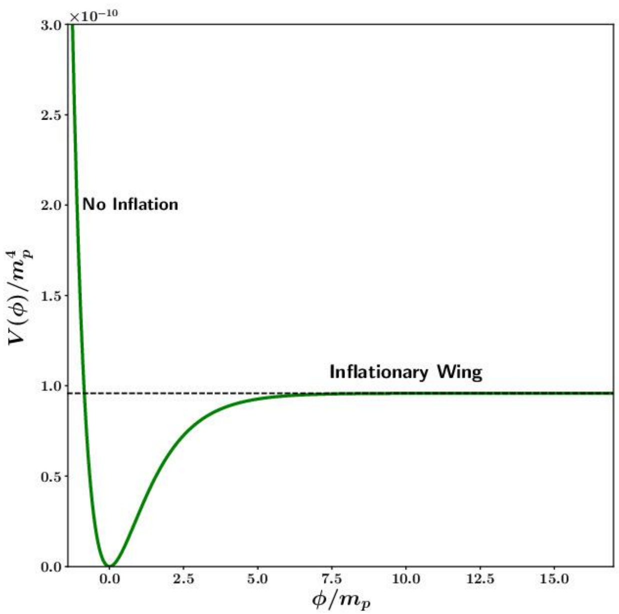
and to <u>fourth order</u> gravity !

The corresponding action in the Einstein frame is given by [Whitt 1984, Maeda, 1988]

$$S_E = \int d^4x\sqrt{-g}\left[\frac{\hat{R}}{16\pi G} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \quad \text{where } V(\phi) = \frac{3}{4}m^2 m_p^2\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{m_p}}\right)^2$$

CMB





So if $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \frac{1}{6m^2}R^2\right]$ gives rise to **early** acceleration when **R** is large,

perhaps $S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\left(R - \frac{\mu^4}{R}\right) + \int d^4x\sqrt{-g}\,\mathcal{L}_M\,,\quad \mu \sim H_0 \simeq 10^{-42}GeV$

can give rise to **late-time** acceleration when **R** has dropped to a small value [Carroll et al. (2004), Capozziello et al. (2004)].

Unfortunately this model develops strong instabilities and does not pass solar system tests [Chiba 2003, Dolgov & Kawasaki 2003].

However instability-free modified gravity models with late-time acceleration can be constructed using

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R + f(R)\Big],\quad \text{where } f''(R) > 0$$

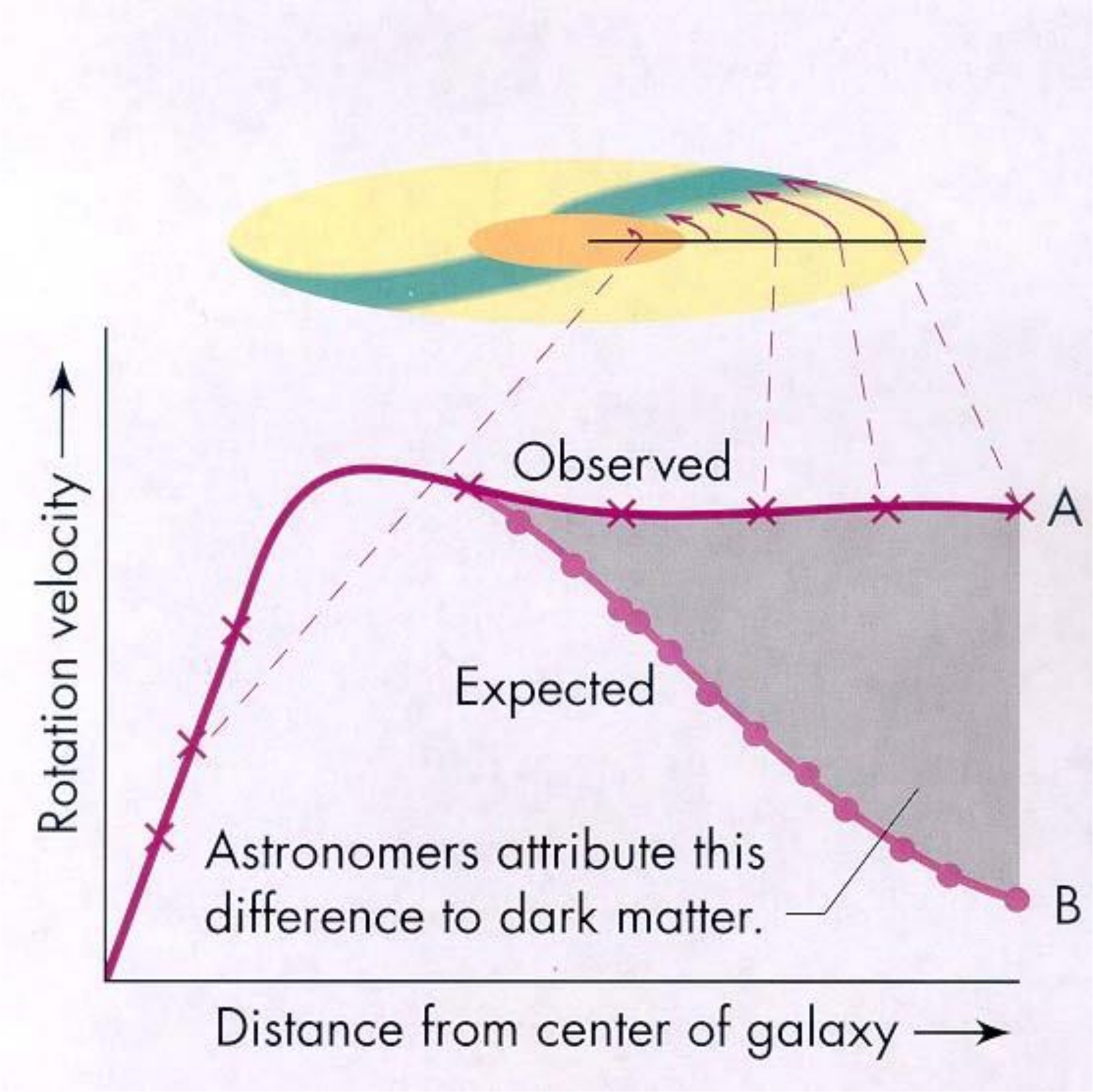
An example is $f(R) = -m^2\frac{c_1(R/m^2)^n}{c_2(R/m^2)^n + 1}\,,\quad n > 0$

[Hu & Sawicki PRD 76 064004, 2007]

for which $\lim_{R\to\infty} f(R) = \text{const},\quad \lim_{R\to 0} f(R) = 0,\quad$ and $m \sim (10^4 Mpc)^{-1}$

[Also see: Appleby and Battye, PLB, 654, 7, 2007, Starobinsky, JETP Lett, 86, 157, 2007, Tsujikawa, PRD 78, 023507, 2008]

The idea of modifying either the law of gravity or the laws of motion to explain the `dark sector' has earlier been tried for **dark matter**.



Kepler's law:

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}}$$

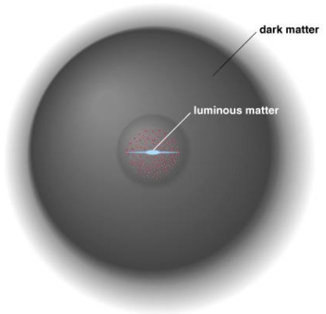
Therefore for M = constant

$$v \propto \frac{1}{\sqrt{R}}$$

But $v =$ constant is observed !

$$\Rightarrow M \propto R$$

Mass in a galaxy grows with Radius !!



An alternative to dark matter may be constructed if we are willing to give up on Newtonian gravity/GR on large scales

OR

In regions experiencing a Low Acceleration.

Milgrom (1983)

Conventional approach:

$$F = mg = ma_N, \quad \text{where } g = \frac{GM}{r^2}$$

$$\Rightarrow a_N = g = \frac{GM}{r^2}$$

$$\text{Centripetal acceleration } a_c = \frac{v^2}{r}$$

$$a_c = a_N \Rightarrow \frac{v^2}{r} = \frac{GM}{r^2} \Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

MOND – Modified Newtonian Dynamics:

$$F \neq ma$$

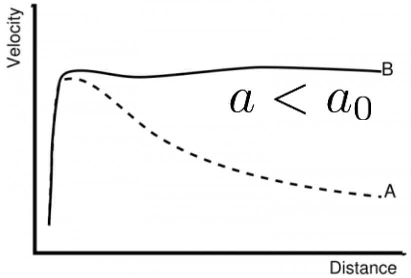
$$\text{For } a \leq a_0 = 10^{-8} cm/s^2 \sim cH_0$$

$$F = mg = ma\left(\frac{a}{a_0}\right) \Rightarrow a^2 = a_0 g \Rightarrow a = \sqrt{a_0 g} = \sqrt{\frac{a_0 GM}{r^2}}$$

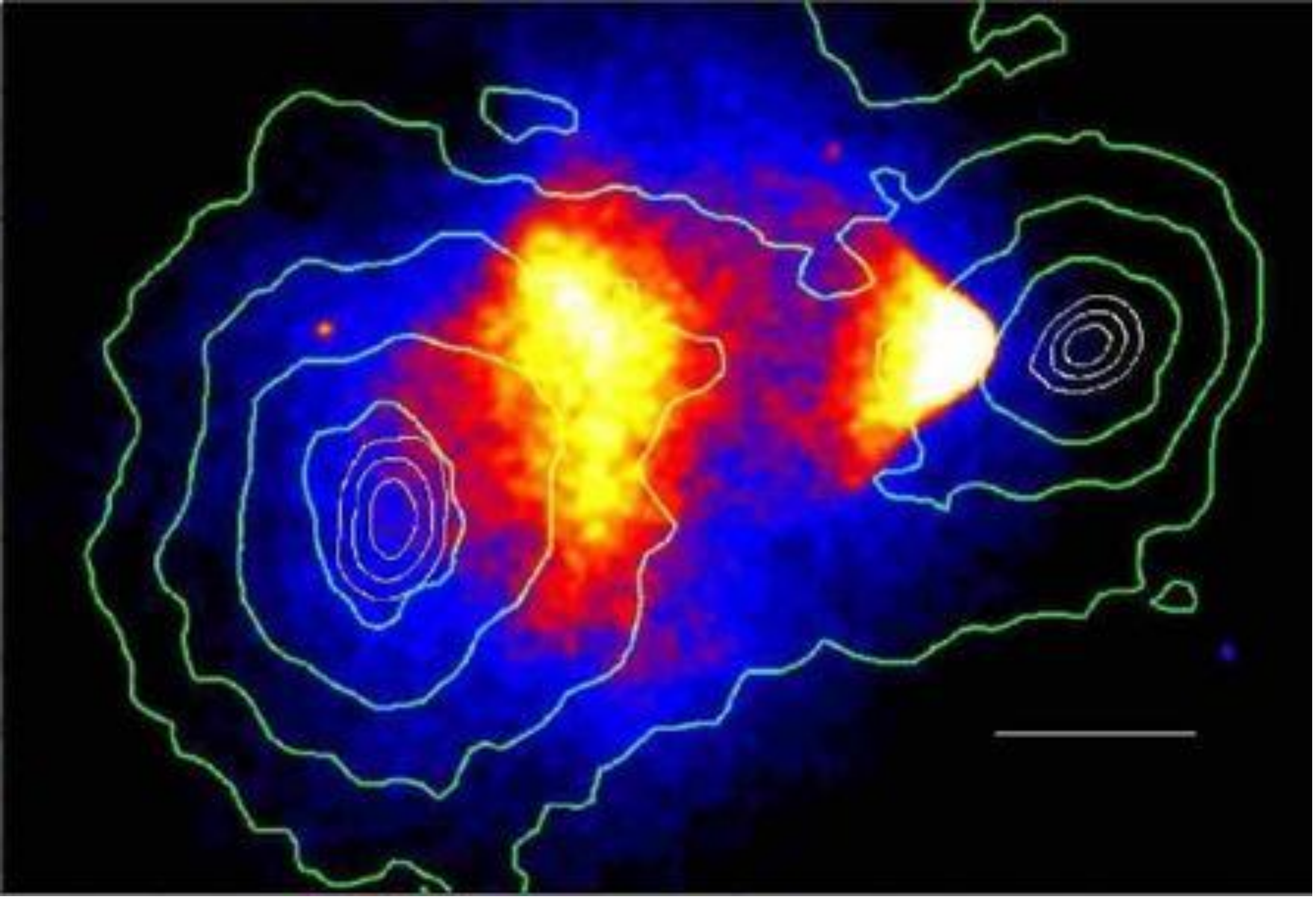
$$\text{Centripetal acceleration } a_c = \frac{v^2}{r} = a \Rightarrow \frac{v^2}{r} = \sqrt{\frac{a_0 GM}{r^2}}$$

$$\Rightarrow \boxed{v^2 = \sqrt{a_0 GM}}$$

Flat rotation curves !



How can one differentiate between Dark Matter and MOND ?



``A direct empirical proof of the existence of Dark matter !"

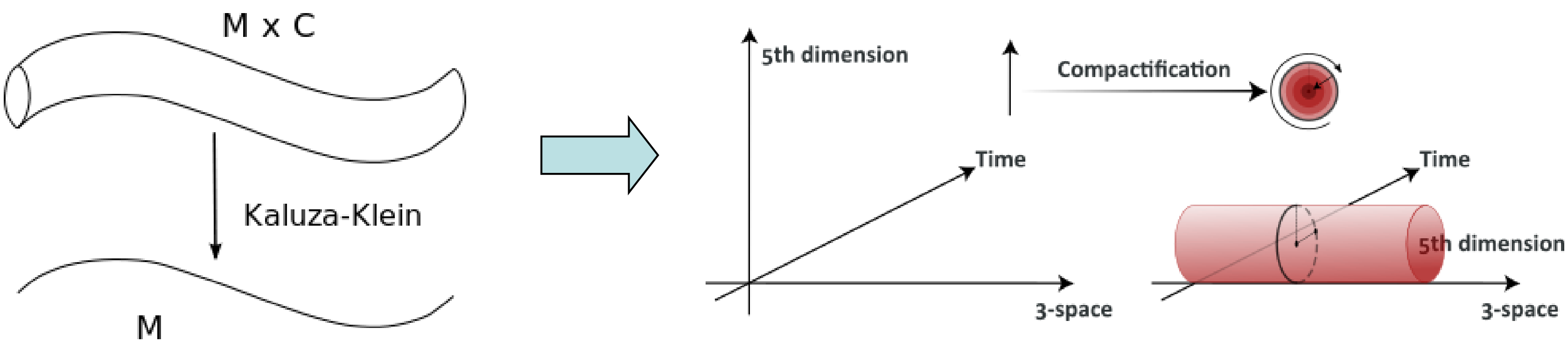
Clowe, et al. ApJ Lett. astro-ph/0608407

Contours show Mass location obtained by Gravitational lensing.

Due to a collision which occurred about 100 Myrs ago the two clusters of galaxies participating in this merger have moved ahead of their respective plasma clouds. Weak lensing maps (of background galaxies) show that **the gravitational potential is not associated with the baryonic matter** in plasma clouds which lag behind the center of mass. The primary mass component, therefore, is likely to be dark and non-baryonic !

A completely different means of sourcing cosmic acceleration is by means of **extra dimensions**.

The idea of the universe having extra dimensions goes back to the early work of Kaluza (1921) and Klein (1926) who showed that the U(1) gauge symmetry of Electromagnetism could be associated with a **compact fifth dimension**.



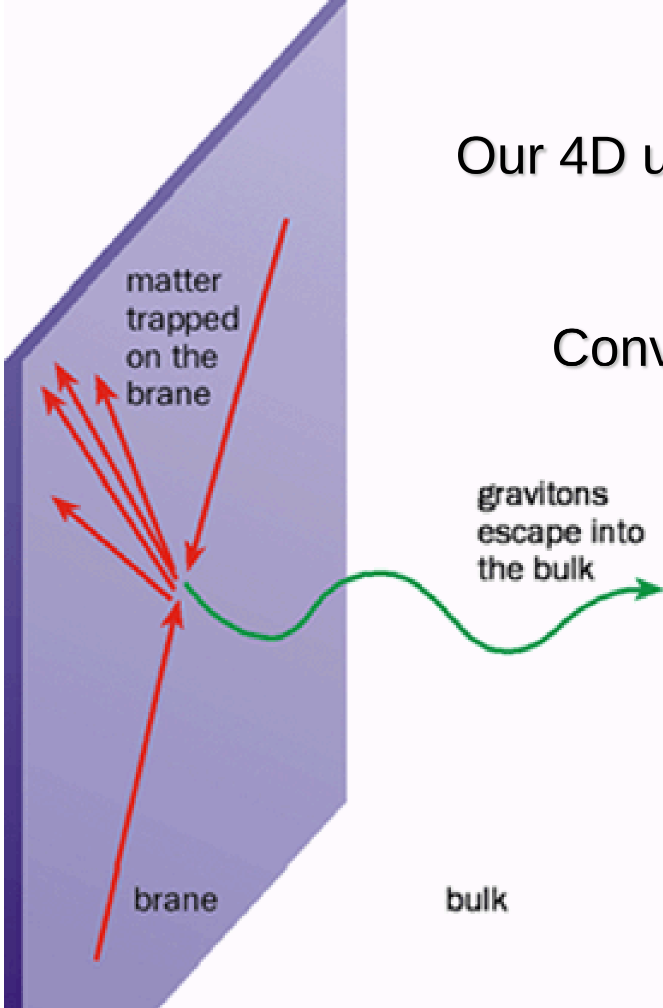
$$S^{(5)} = \frac{1}{16\pi G^{(5)}} \int \sqrt{-g^{(5)}} R^{(5)} d^4x\, dy$$

Gravity in 5D

$$S^{(4)} = \frac{1}{16\pi G^{(4)}} \int \sqrt{-g^{(4)}} \left(R^{(4)} + \frac{1}{4}\phi\, F^{\mu\nu} F_{\mu\nu} + \frac{1}{6\phi^2} \partial^\mu \phi \partial_\mu \phi \right) d^4x$$

Extended to include standard model fields in the 1960's and 70's.

GR **EM** Dilaton/radion



Recent developments:

Our 4D universe is a mem-brane embedded in a higher dimensional bulk space-time.

Conventional expansion is modified either during: early times (UV), or late times (IR).

- Changes in the expansion law at high energies (early times) can: modify Inflationary dynamics and even ameliorate the Big Bang singularity – replacing it with a Big Bounce !

- A change in the gravity-law at low energies (IR) may cause the universe to accelerate at late times, alleviating the need for dark energy !

Modified gravity models in 4D can also lead to cosmic acceleration:

$$S = \int \sqrt{-g} f(R) d^4x \ , \ \ f(R) = R + R^2 \quad (\text{Starobinsky}, 1980)$$

⇒ **Starobinsky Inflation**

The Randall-Sundrum model: $S = M^3 \left[\int_{\rm bulk} \mathcal{R} - 2 \int_{\rm brane} K \right] - 2 \int_{\rm brane} \sigma + \int_{\rm brane} L(\rm matter)$

$M -$ five dimensional Planck mass, $\sigma -$ brane tension

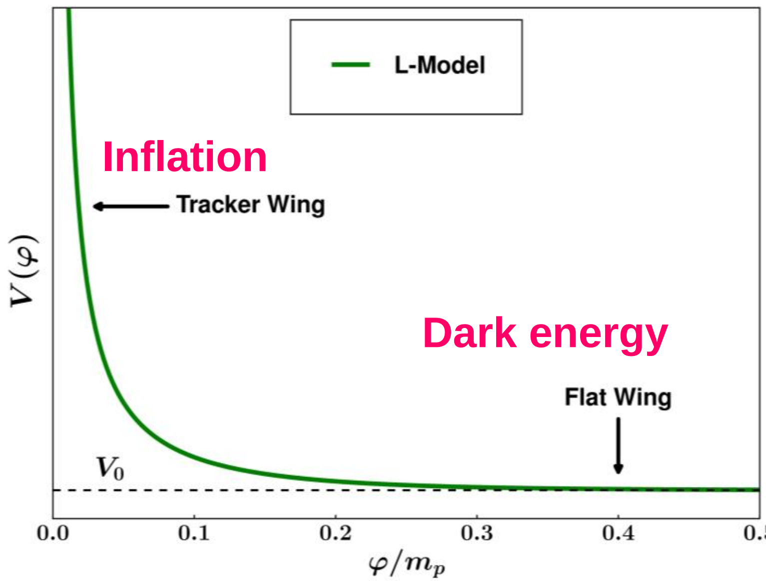
$R-$ scalar curvature in 5D, $K-$ trace of extrinsic curvature of brane embedded in bulk

Consequently, the Einstein equations `on the brane' are modified to

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\rho^2}{2\sigma}\right) + \ldots \text{ where } G = \frac{\sigma}{12\pi M^6} \text{ is no longer a fundamental constant!}$$

The new term $\rho^2/2\sigma$ increases **H** thereby increasing the damping on the inflaton field as it rolls down its potential via $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

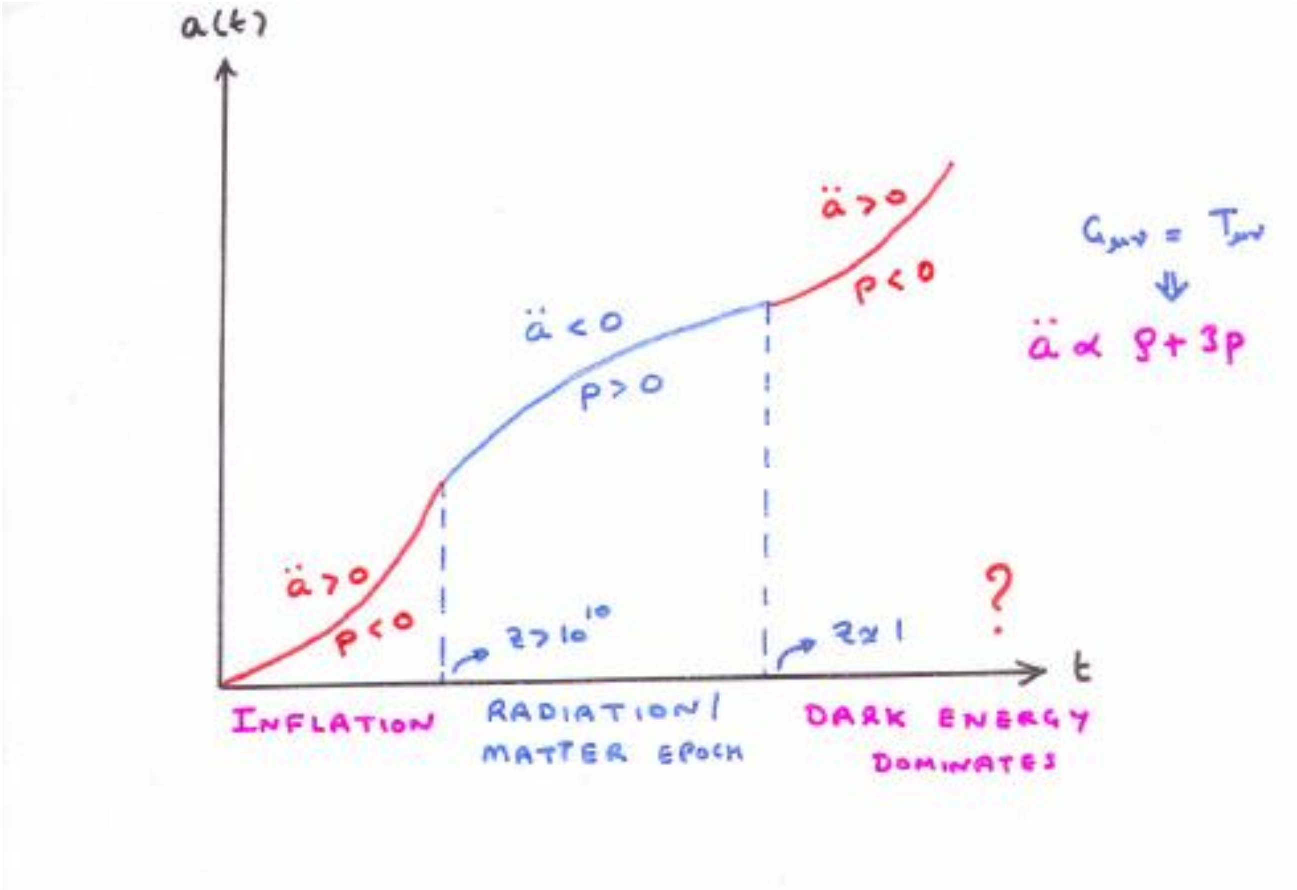
Consequently inflation can be driven by **steep potentials** usually associated with tracker models of dark energy.



Therefore models based on extra dimensions may help unify Inflation (early acceleration) with dark energy (late acceleration).

Quintessential Inflation: Peebles & Vilenkin 1999, Copeland et al. 2001, Sahni et al. 2002, etc.

Quintessential inflation would address an important issue:



The universe seems to accelerate twice: (i) at early times (Inflation), (ii) again at late times (Dark Energy).

Are Inflation and Dark Energy related ?
(Nature should be economical !)

The Randall-Sundrum equation

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\rho^2}{\rho_c}\right)$$

has a **dual**, which arises when the extra dimension is time-like

$$H^2 = \frac{8\pi G}{3}\left(\rho - \frac{\rho^2}{\rho_c}\right)$$

This equation can lead to **singularity avoidance** at early times.

Big Bang singularity may be prevented by Quantum effects or if our Universe has a time-like extra dimension !

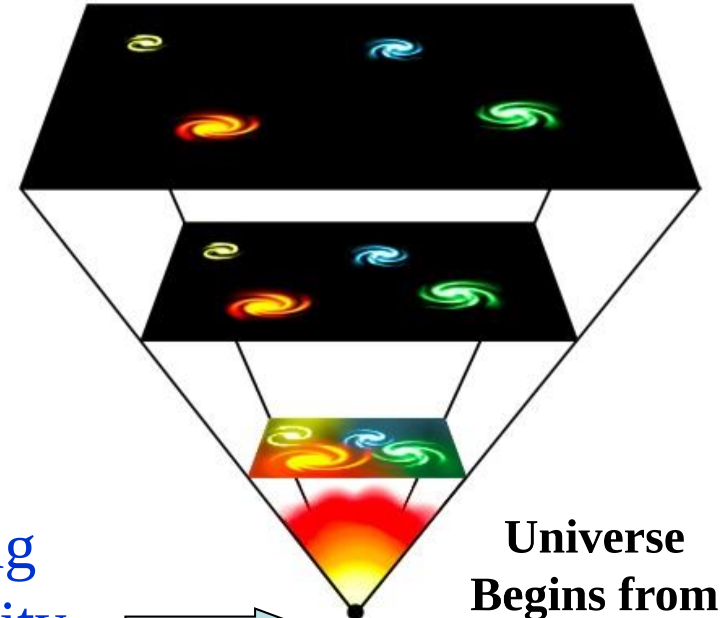
In GR:

$$H^2 = \frac{8\pi G}{3}\rho$$

As the density ρ increases so does the expansion rate H !

Big Bang singularity when:

$$\rho \to \infty, \ H \to \infty$$



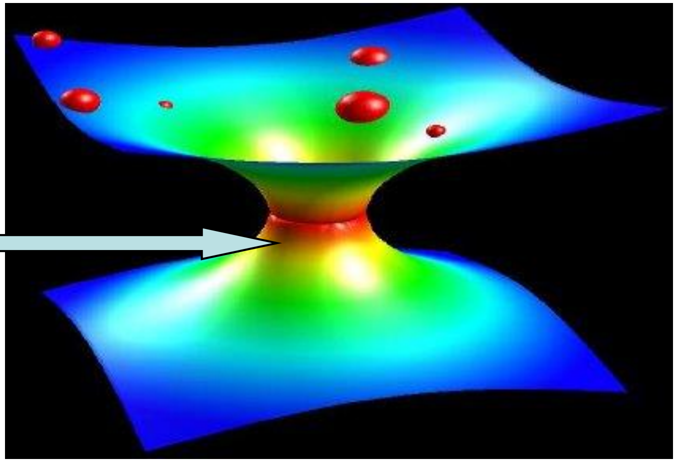
Big Bang singularity

Universe Begins from a point !

But with quantum effects/extra dimensions:

$$H^2 = \frac{8\pi G}{3}\left(\rho - \rho^2/\rho_c\right)$$

Big Bounce replaces Big Bang

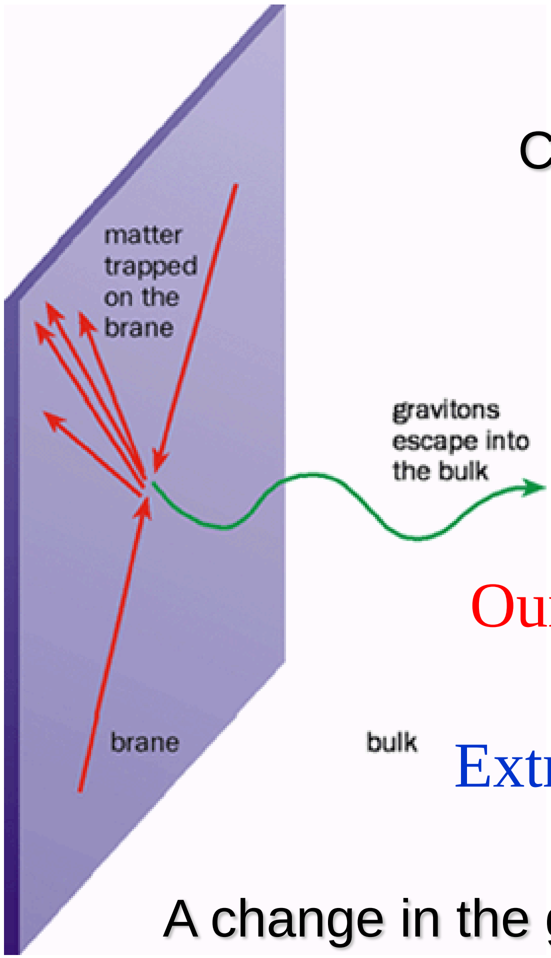


So $H = 0$ when $\rho = \rho_c$ [Shtanov & Sahni, 2003]

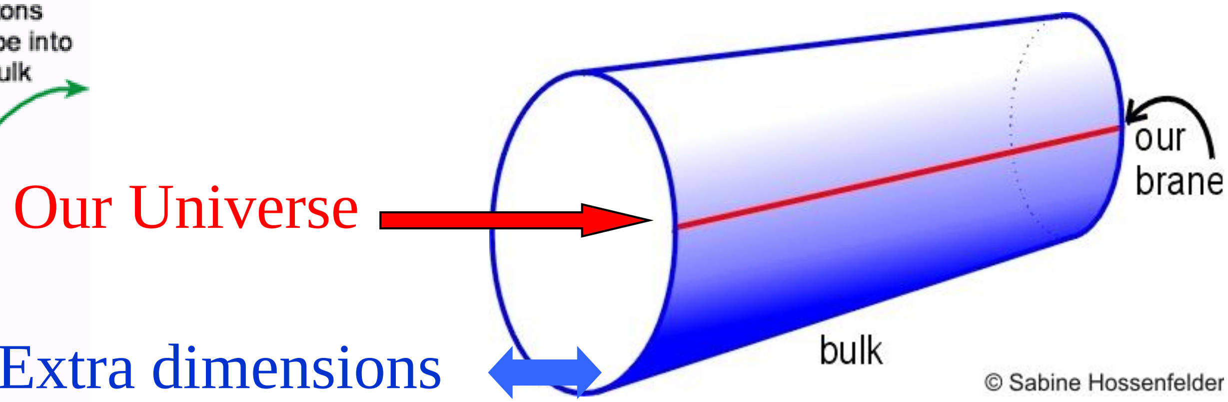
[Ashtekar et al., 2006]

No singularity !!

BRANEWORLD: Our 4D universe is a mem-brane embedded in a higher dimensional bulk space-time.



Conventional expansion is modified either during: early times (UV), or late times (IR).



A change in the gravity-law at low energies (IR) may cause the universe to accelerate at late times, alleviating the need for dark energy !

Modified gravity models in 4D can also lead to cosmic acceleration:

$$S = \int \sqrt{-g} f(R) d^4x \ , \ \ f(R) = R + R^2 \quad \text{(Starobinsky, 1980)}$$

⇨ **Starobinsky Inflation**

For instance a Braneworld embedded in 5D can accelerate !

$$S = M^3 \int_{\rm bulk} \mathcal{R} + m^2 \int_{\rm brane} R \ + \ S_{\rm matter}$$

[Dvali, Gabadadze and Porrati, 2000]

$$H = \sqrt{\frac{8\pi G \rho_{\rm m}}{3} + \frac{1}{\ell^2}} + \frac{1}{\ell}\ , \quad \ell = m^2/M^3$$

is a new macroscopic length scale.

For $M \simeq 10 - 100 \ {\rm MeV}, m \simeq 10^{19}{\rm GeV}, \ell \sim cH_0^{-1}$!

Compare with $\Lambda CDM : \ H = \sqrt{\frac{8\pi G \rho_{\rm m}}{3} + \frac{\Lambda}{3}}$

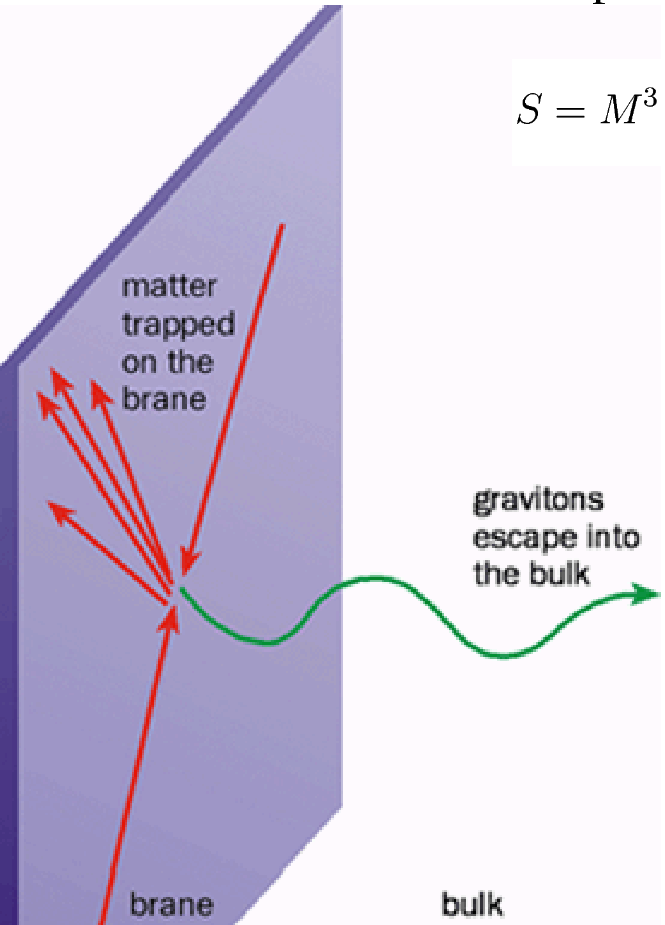
As in ΛCDM a single parameter ℓ controls late time acceleration giving rise to acceleration without dark energy !

BUT DGP has ghosts [Gregory etal arXiv:0707.2666]. Braneworld models without ghosts have the distinctive feature $w < -1$ [Sahni and Shtanov, 2002].

Braneworld Dark Energy may be a Phantom !!

One possible way of achieving a phantom equation of state $w_{\rm eff} < -1$ without running into instabilities is in models in which dark energy is screened.

An example is provided by Braneworld models (Sahni & Shtanov 2002).



$$S = M^3\left[\int_{\rm bulk}(\mathcal{R} - 2\Lambda_{\rm b}) - 2\int_{\rm brane} K\right] + \int_{\rm brane}(m^2 R - 2\sigma) + \int_{\rm brane} L\,(h_{ab}, \phi)$$

(Includes GR, DGP and Randall-Sundrum as subclasses)

$$h^2 = \Omega_{0m}(1+z)^3 + \underbrace{\Omega_\Lambda - f(z)}_{\text{Screened DE}}$$

Screened DE

$$f(z) = 2\sqrt{\Omega_\ell}\sqrt{\Omega_{0m}(1+z)^3 + \Omega_\sigma + \Omega_\ell}$$

⇓

Screening term, whose value increases with redshift !

⇒ $w_{\rm eff} < -1$

$$\Omega_\sigma = \frac{\sigma}{3m^2 H_0^2}\,,\ \Omega_\ell = \frac{1}{\ell^2 H_0^2}$$

where $\Omega_\Lambda = \Omega_\sigma + 2\Omega_\ell$

$\ell = 2m^2/M^3$ is a new length scale !

The DGP model, and Quintessence models have difficulty in accommodating recent high z measurements of H(z).

The value obtained by Delubac et al: $H(z = 2.34) = 222 \pm 7$ km/sec/Mpc

is lower than the LCDM value: $H(z = 2.34) = 238$ km/sec/Mpc

• This can happen in models in which the cosmological constant is **screened**

$$H^2(z) = \underbrace{\frac{\Lambda}{3} - f(z)}_{\Lambda_{\text{eff}}/3} + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

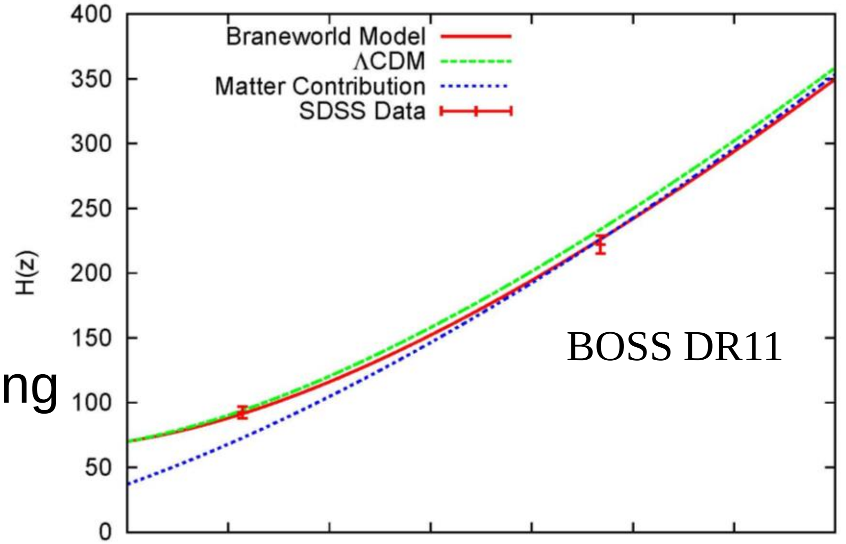
$f(z)$ increases with z $\Lambda_{\text{eff}}/3$ ⇒ Λ_{eff} **grows with time !**

• Or in phantom models with w < -1.

In this case $\rho_{\text{DE}} \propto a^{-3(1+w)} = a^{3|1+w|}$

So the DE density grows with time !

But phantom models usually possess instabilities. A problem-free means of getting w<-1 is provided by the Phantom brane.

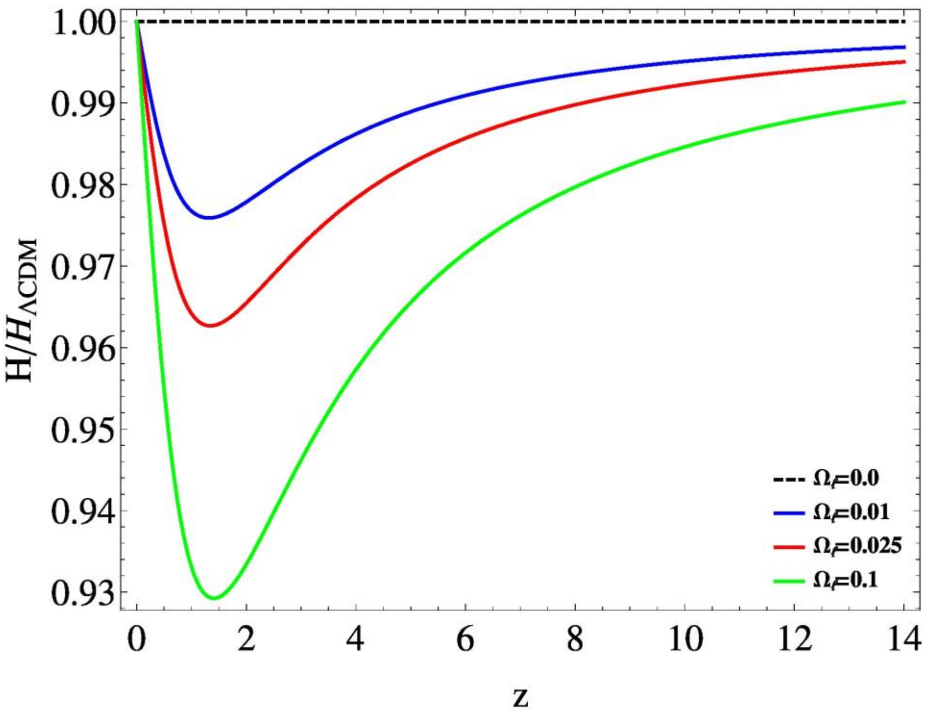


BOSS DR11

Three properties of Screened Dark Energy

$$H^2(z) = \frac{\Lambda}{3} - f(z) + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

H(z) is lower than in LCDM. This will affect cosmological quantities.



Luminosity distance: D_L increases

$$\frac{D_L(z)}{1+z} = \int_0^z \frac{dz'}{H(z')}$$

$$F = \frac{\mathcal{L}}{4\pi D_L^2}$$

decreases !

• SNIa are fainter than in LCDM.

Age of the Universe:

$$T(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

• Universe containing Screened DE is older than LCDM at a given redshift.

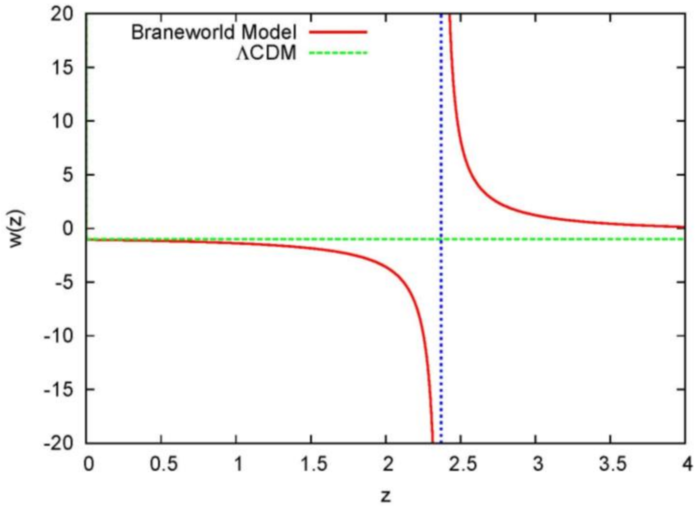
• Will also affect angular size distance, optical depth to electron scattering, AP test, etc.

The cosmological constant is screened in braneworld models.

$$H^2(z) = \underbrace{\frac{\Lambda}{3} - f(z)}_{\Lambda_{\rm eff}/3} + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

Since $h^2 = \Omega_\Lambda - f(x) + \Omega_{0m}x^3 \quad \Rightarrow \quad 1 + w = -\frac{x}{3}\frac{f'}{\Omega_\Lambda - f(x)}$

- The EOS has a **pole** at which $w(z_p) \to \infty$!



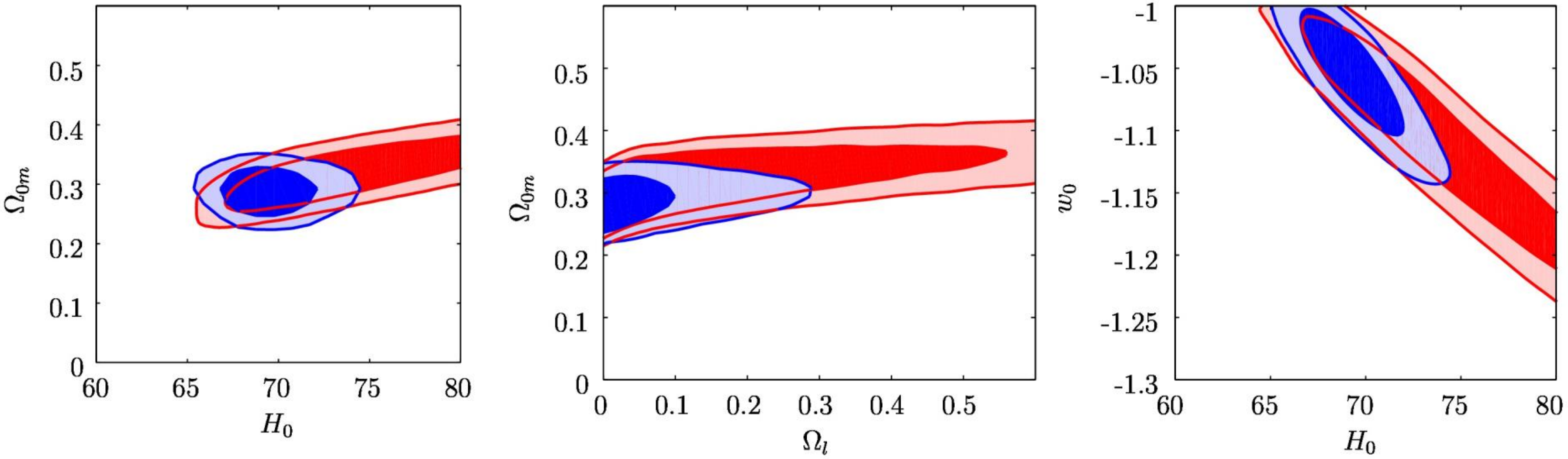
Pole occurs when $\Lambda_{\rm eff} = 0$.

Smoking gun test of screened models.

$$w(z) = \frac{\frac{2x}{3}\frac{d}{dx}\log H - 1}{1 - \left(\frac{H_0}{H}\right)^2 \Omega_{0m}x^3}, \quad x = 1 + z$$

Observational constraints on the phantom brane

Alam, Bag & Sahni, PRD 2017 [arXiv:1605.04707]



Red contours show 1 and 2σ CL's for SNe+BAO data, while blue contours show results for SNe+BAO+CMB.

(Union2.1 with 580 SNIa; BAO: SDSS11,SDSS12)

$w_0 < -1$ is preferred

Larger values of H_0 imply a more negative equation of state for dark energy.

Phantom braneworld can reconcile the larger value $H_0 = 73.24 \pm 1.74\ km/s/Mpc$ obtained from HST observations with CMB observations [Riess et al., **arXiv:1604.01424]**

⇨ CMB and HST derived H_0 are in tension at 3.4σ in ΛCDM

Reconstructing Dark Energy

Numerous Dark Energy models have been suggested to account for an accelerating Universe:

(i) Cosmological constant

(ii) Quiessence with w = constant < -1/3, (cosmic strings/walls), the cosmological constant Λ (w = -1) is a special member of this class;

(iii) Quintessence models;
(iv) The Chaplygin gas;
(v) Phantom DE (w < -1);
(vi) Oscillating DE;
(vii) Models with interactions between DE and dark matter;
(viii) Scalar-tensor DE models;
(ix) Modified gravity models:
(x) Dark energy driven by quantum effects;
(xi) Higher dimensional braneworld models, etc.

Faced with the increasing proliferation of DE models a cosmologist can proceed in either of two ways:

(i) Test each and every model against observations.

(ii) Reconstruct properties of dark energy in a model independent manner.

Model independent reconstruction of Dark Energy: two approaches

[A] Study cosmic expansion using geometrical parameters:

$$H = \dot{a}/a, \;\; q = -\ddot{a}/aH^2, \;\; r = \dddot{a}/aH^3, \;\; \text{etc.}$$

which arise in the Taylor expansion the expansion factor

$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\big|_0}{6}(t - t_0)^3 + \cdots$$

[B] Study cosmic expansion via physical parameters such as $(x = 1 + z)$

$$\rho_{\rm DE} = \frac{3H^2}{8\pi G} - \rho_m \,, \qquad w \equiv p_{DE}/\rho_{DE} = \frac{2q(x) - 1}{3(1 - \Omega_m(x))}$$

Note that the definition of physical parameters, $\rho_{\rm DE}$, w , is based on the validity of general relativity. Consequently, $\rho_{\rm DE}$, $w(z)$, as defined above could show very unusual behaviour in modified gravity theories !

Model independent reconstruction of Dark Energy

Let us define Dark Energy through the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(\sum_a T_{\mu\nu}^{(a)} + T_{\mu\nu}^{DE} \right)$$

then, in a spatially flat universe

$$H^2 = \frac{8\pi G}{3} \left(\sum_a \rho_a + \rho_{DE} \right) \;, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\sum_a (\rho_a + 3p_a) + \rho_{DE} + 3p_{DE} \right)$$

If one neglects radiation, then these equations give $(q \equiv -\ddot{a}/aH^2)$

$$\rho_{\rm DE} = \frac{3H^2}{8\pi G}(1 - \Omega_{\rm m}) \;, \quad \text{where } \Omega_m = \frac{8\pi G \rho_m}{3H^2}, \quad p_{\rm DE} = \frac{H^2}{4\pi G}(q - \frac{1}{2}) \;,$$

From where we obtain the effective equation of state of dark energy

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x) - 1}{3(1 - \Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1 - (H_0/H)^2 \Omega_{0m} x^3} \;, \; x = 1 + z \; .$$

So one can obtain the eqn. of state, **w(z)** from the expansion history **H(z).**

Cautionary note ....

Geometrical models of dark energy (in which the LHS of the Einstein equation is modified) such as the Braneworld model

$$H = \sqrt{\frac{8\pi G\rho_{\rm m}}{3} + \frac{1}{l_c^2}} + \frac{1}{l_c}\ , \tag{5}$$

do not conform to the Einsteinian representation of DE

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm m} + \rho_{DE})\ .$$

Consequently the equation of state $w \equiv p_{DE}/\rho_{DE}$ is an effective quantity which may still be useful for descriptive purposes but which no longer represents any fundamental physical property of an accelerating universe. Indeed, instances are known when $w_{\rm eff} < -1$ even when matter itself satisfies the weak energy condition $\rho + P \geq 0$ [Boisseau et al., 2000; Sahni and Shtanov, 2002; Gannouji et al., 2006].

In this case it is better to define acceleration through more fundamental geometrical quantities which depend upon the space-time metric.

Such as:

$$H = \dot{a}/a,\ q = -\ddot{a}/aH^2,\ r = \dddot{a}/aH^3,\ \text{etc.}$$

(i) Top-down approach to reconstruction: from $D_L \to w_{\rm DE}(z)$

Observational tests of Dark Energy usually rely on an accurate measurement of either the angular size distance or the luminosity distance:

$$\mathcal{F} = \frac{L}{4\pi D_L^2} \; , \quad D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \; .$$

One can then reconstruct the Hubble parameter through

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1} \; .$$

Differentiating a second time we can reconstruct the equation of state of DE

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x)-1}{3(1-\Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1-(H_0/H)^2\Omega_{0m}x^3} \; , \; x = 1+z \; .$$

A good ansatz for luminosity distance is: **[Saini et al. PRL 2000]**

$$\frac{H_0 D_L(z)}{1+z} = 2\left[\frac{x - A_1\sqrt{x} - 1 + A_1}{A_2 x + A_3\sqrt{x} + 2 - A_1 - A_2 - A_3}\right] \; , \quad x = 1+z \; ,$$

(ii) Bottom-up approach: Fit the equation of state, go from $w_{\mathrm{DE}}(z) \to D_L$

Fitting functions to the equation of state. (A) The simple Taylor expansion

$$w(z) = \sum_{i=1}^{N} w_i z^i \ ,$$

is of limited utility since its only valid for $z \ll 1$.

(B) A much more versatile ansatz is

$$w(a) = w_0 + w_1(1-a) = w_0 + w_1 \frac{z}{1+z} \ ,$$

CPL ansatz

where the parameters $w_0, \ w_1$ are obtained after substituting into:

$$H^2(z) = H_0^2[\Omega_m(1+z)^3 + \Omega_{\mathrm{DE}}]^2 \ , \ \Omega_{\mathrm{DE}} = (1-\Omega_m) \exp\left\{3\int_0^{x-1} \frac{1+w(z,a_i)}{1+z} dz\right\} .$$

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \ .$$

$$\mathcal{F} = \frac{L}{4\pi D_L^2} \ ,$$

SNIa

[Chevalier & Polarski 2001; Linder 2003]

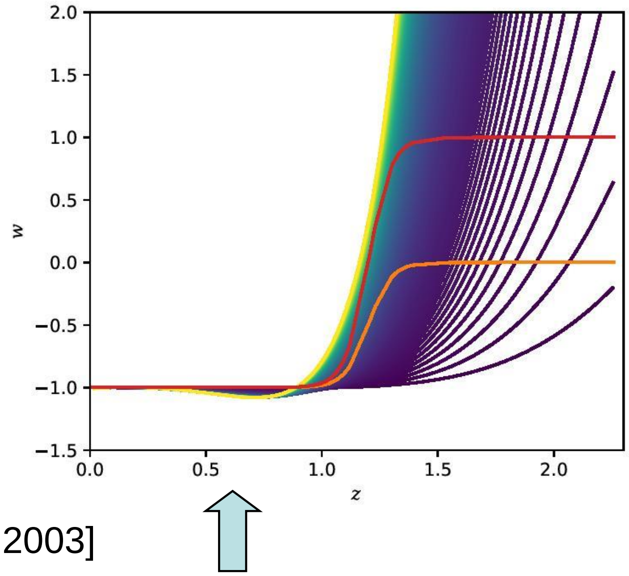
For an ansatz to be successful it should embrace within its fold the behaviour of a reasonably large class of Dark Energy models.

An ansatz involving only 2 free parameters can describe DE whose equation of state evolves gradually with redshift. It is quite clear that these simple fits cannot be used to rule out models with rapidly evolving $w(z)$.

To accommodate models with a fast transition in the EOS one might try:

$$1.\quad w(z) = w_i + \frac{w_f - w_i}{1 + \exp(\frac{z - z_t}{\Delta})} ,$$

$$2.\quad w(z) = -\frac{1 + \tanh\left[(z - z_t)\Delta\right]}{2} .$$



[Bassett *et al* MNRAS 336, 1217, 2002; Corasaniti *et al*, PRL 90, 091303, 2003]

[Shafieloo *et al*, PRD 80, 101301, 2009; Ishida *et al*, 2008; L'Huillier et al. arXiv:1812.03623]

One should note that while increasing the number of parameters increases the accuracy of reconstruction of the `best fit', this is often accompanied by severe degeneracies which limit the utility of introducing a large number of free parameters.

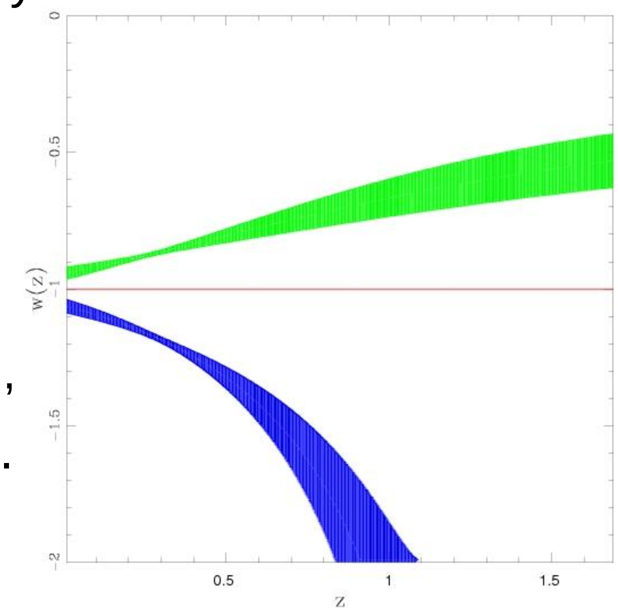
Errors and pitfalls in cosmological reconstruction

1. The (mythical) influence of Dark Matter on Dark Energy.

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1} . \qquad w(x) = \frac{(2x/3)d\log H/dx - 1}{1-(H_0/H)^2\,\Omega_{0m}x^3}\ , \ x = 1+z$$

An uncertainty in Ω_{0m} propogates into $w(z)$ even if $H(z)$ has been reconstructed quite accurately !

In the figure a fiducial $\Lambda\mathrm{CDM}$ model (w=-1) has been reconstructed using an incorrect value of Ω_{0m} .



True value: $\Omega_{0m}^{true} = 0.27$.

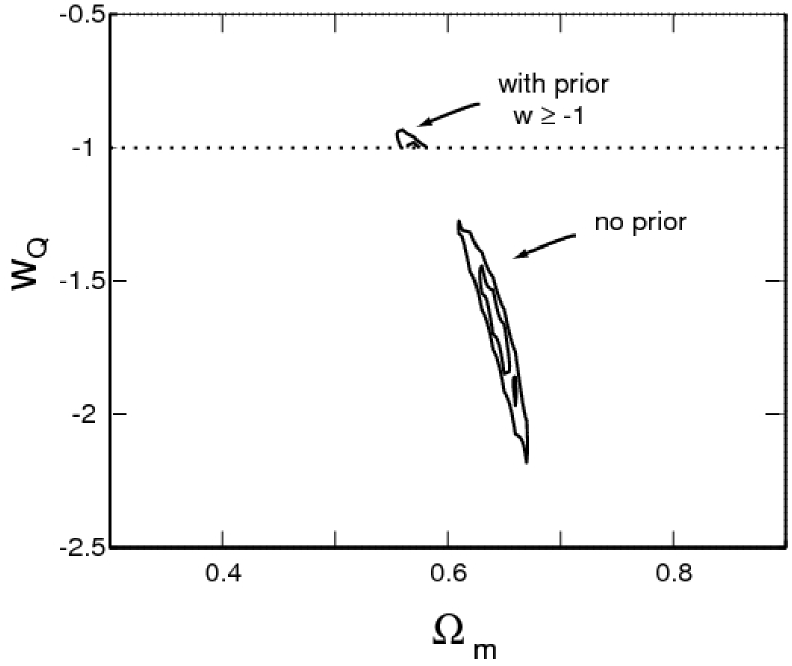
For $\Omega_{0m}^{false} = 0.22$, $w(z)$ is Quintessence-like (w > -1),
while for $\Omega_{0m}^{false} = 0.32$, $w(z)$ is a Phantom (w < -1).

If $w(z) = w_0 + w_1 \dfrac{z}{1+z}$ is used in

$$H^2(z) = H_0^2[\Omega_m(1+z)^3 + \Omega_{\rm DE}]^2\ ,\ \Omega_{\rm DE} = (1-\Omega_m)\exp\left\{3\int_0^{x-1}\frac{1+w(z,a_i)}{1+z}dz\right\} .$$

If Ω_{0m} is incorrectly chosen then the DE parameters w_0, w_1 will adjust to make H(z) as close to its real value as possible, leading in an incorrect reconstruction of $w(z)$.

Pitfall No. 2: Erroneous priors on the equation of state of dark energy.



[Maor et al, PRD 65, 123003, 2003]

True fiducial model is evolving DE: $\Omega_{0m} = 0.3\ ,\ w(z) = -0.7 + 0.8\,z\ .$

$$\Rightarrow\ w(z) > -1$$

A. $w(z) =$ constant gives the large lower contour: DE is erroneously a Phantom !

B. The additional constraint $w \geq -1$ results in the small upper contour which gives $w = -1$ as the best fit: DE is erroneously a cosmological constant !

Idea behind reconstruction

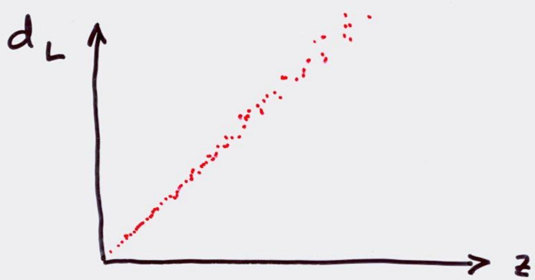
Type Ia supernovae determine the properties of dark energy by means of the relation

$$\mathcal{F} = \frac{L}{4\pi d_L(z)^2}$$

where the *luminosity distance*

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

is a noisy quantity



Physical quantities are determined from $d_L(z)$ via differentiation:

- Differentiating once we obtain the Hubble parameter

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1} ,$$

- Differentiating twice we recover the equation of state of dark energy

$$w(z) = \frac{[2(1+z)/3]\ H'/H - 1}{1\ -\ (H_0/H)^2\Omega_m\ (1+z)^3} .$$

Differentiating a noisy observable such as the luminosity distance increases the noise !

Therefore one must smoothen

$$D_L(z)\ or\ H(z)$$

before differentiating.

This can be done either by approximating $D_L(z)\ or\ H(z)$ by an ansatz, or by smoothing the data directly.

Non-parametric reconstruction

Prescription: smoothen the noisy observable $D_L(z)\ or\ H(z)$ directly.

For instance, a smooth quantity $D^S(\mathbf{x})$ is constructed from a fluctuating raw quantity $D(\mathbf{x})$ using a low pass filter **F** with a smoothing scale Δ

$$D^s(\mathbf{x}, \Delta) = \int D(\mathbf{x}')\, F(|\mathbf{x} - \mathbf{x}'|; \Delta)\, d\mathbf{x}' \ ,$$

in studies of large scale structure D is the density field δ, whereas for cosmological reconstruction D could be either $D_L(z)\ or\ H(z)$

A commonly used filter is the Gaussian filter:

$$F_G \propto \exp\left(-\frac{|\mathbf{x} - \mathbf{x}'|^2}{2\Delta^2}\right) \ .$$

Other non-parametric methods discussed in: Huterrer & Starkman, PRL, 2003; Wang & Lovelace, ApJ, 2001; Saini, MNRAS 2003; Daly & Djorgovsky, ApJ, 2003, 2004; Wang & Tegmark, PRL, 2004, PRD 2005; Espana-Bonet & Ruiz-Lapuente PRD 2006; Huterer & Cooray, PRD 2005; Shafieloo et al MNRAS 2006; Shafieloo & Clarkson, arXiv: 0911.4858, Clarkson & Zunckel, arXiv:1002.5004, Holsclaw et al arXiv:1104.2041,etc. See VS & Starobinsky IJMP 15, 2105 (2006) for a review.

Since we wish to smooth the noise and not the signal we proceed as follows:

$$\ln d_L(z,\Delta)^s = \ln\ d_L(z)^g + N(z)\sum_i\left[\ln d_L(z_i) - \ln\ d_L(z_i)^g\right]\times\exp\left[-\frac{\ln^2\left(\frac{1+z_i}{1+z}\right)}{2\Delta^2}\right],$$

$d_L(z)^g$ is a guess model, Eg. LCDM

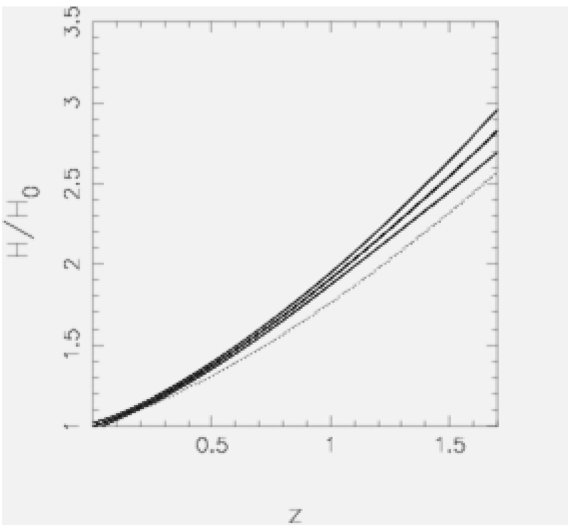
$$N(z)^{-1} = \sum_i \exp\left[-\frac{\ln^2\left(\frac{1+z_i}{1+z}\right)}{2\Delta^2}\right].$$

Final value of $d_L(z)$ is obtained iteratively

Using $d_L^{(s)}(z)$ we obtain the smoothed expansion rate and look back time:

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1},$$

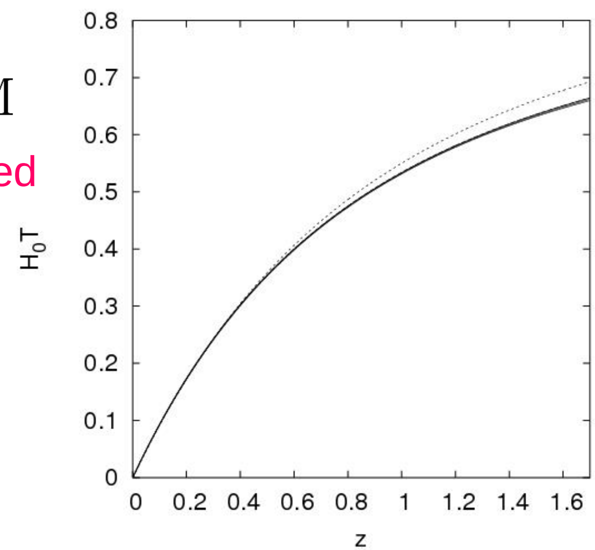
$$T(z) = t(z=0) - t(z) = H_0^{-1}\int_0^z \frac{dz'}{(1+z')H(z')}.$$



Dotted line: ΛCDM

is easily distinguished from fiducial model:

$$w(z) = -\frac{1}{1+z}$$



$H(z)$ reconstructed to 2 % accuracy, look back time to 0.2 % accuracy using mock data !

[Shafieloo, Alam, VS, Starobinsky, MNRAS, 2006]

Advantages: reconstruction is sensitive to the presence of features in the EOS w(z).

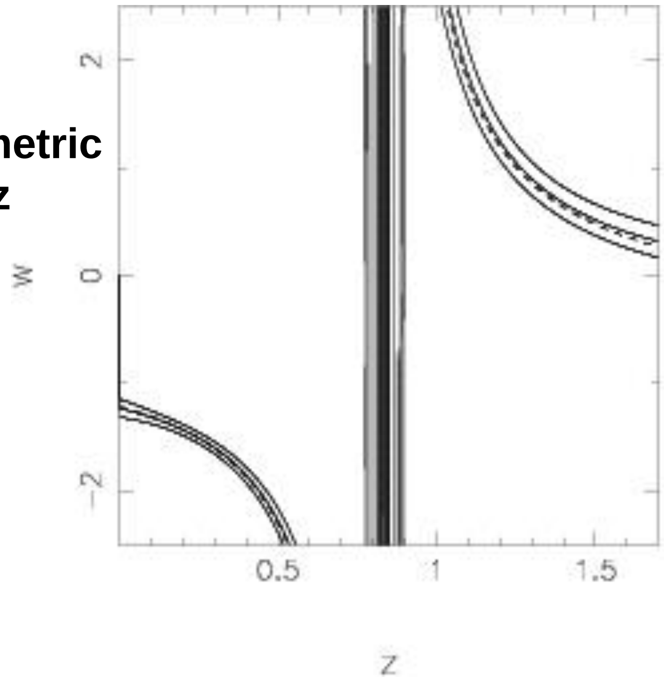
Observations suggest that a primordial Λ term may be dynamically screened :

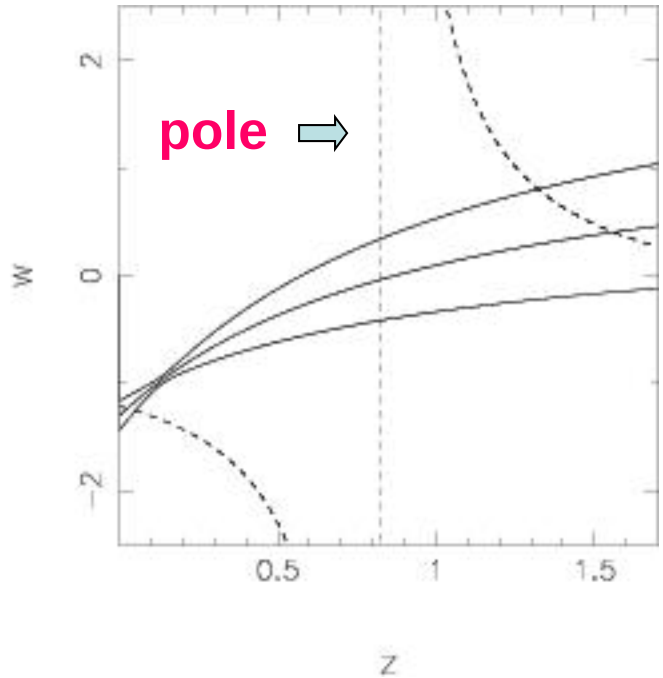
$$h^2 = \Omega_\Lambda - f(x) + \Omega_{0m}x^3 \quad \Rightarrow \quad 1 + w = -\frac{x}{3}\frac{f'}{\Omega_\Lambda - f(x)} \qquad (x = 1 + z)$$

which leads to a pole in w(z) even though the deceleration parameter is well behaved !

[Linder hep-th/0410017; VS & Shtanov, PRD 2005; Grande et al JCAP 2006; Zhou et al, JCAP 2009, etc.]

Non-parametric ansatz





Parametric ansatz

$$w(z) = w_0 + w_1(1 - a)$$

A smoothing approach can find the pole (left) while a parametric ansatz (right) cannot !

Of all Dark Energy models the cosmological constant is single out by its elegance and simplicity:

$$T_i^k = \Lambda \delta_i^k \ .$$

So, as a first step, its logical to find tests which could falsify

The Cosmological Constant hypothesis

$$\Downarrow$$

NULL tests for the cosmological constant Λ .

[VS, Saini, Starobinsky, Alam JETP 2003; VS, Shafieloo, Starobinsky PRD 2008, Zunckel & Clarkson PRL 2008, Arabsalmani & VS, PRD 2011, VS, Shafieloo, Starobinsky, ApJLett 2014, etc.]

Model independent reconstruction of Dark Energy:

The expansion factor $a(t)$ provides the most general information about expansion history:

$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\big|_0}{6}(t - t_0)^3 + \cdots$$

In 1970 Alan Sandage described observational cosmology as being ``a search for two numbers'': $H_0 = (\dot{a}/a)_0 \quad q_0 = -(\ddot{a}/aH^2)_0$.

In this era of `precision cosmology' let us define a third number $r = \frac{\dddot{a}}{aH^3}$

Surprisingly, $r = 1$ only in LCDM !

For all other dark energy models $r \neq 1$

Similarly define $s = \frac{r-1}{3(q-1/2)}$. s = 0 only in LCDM !

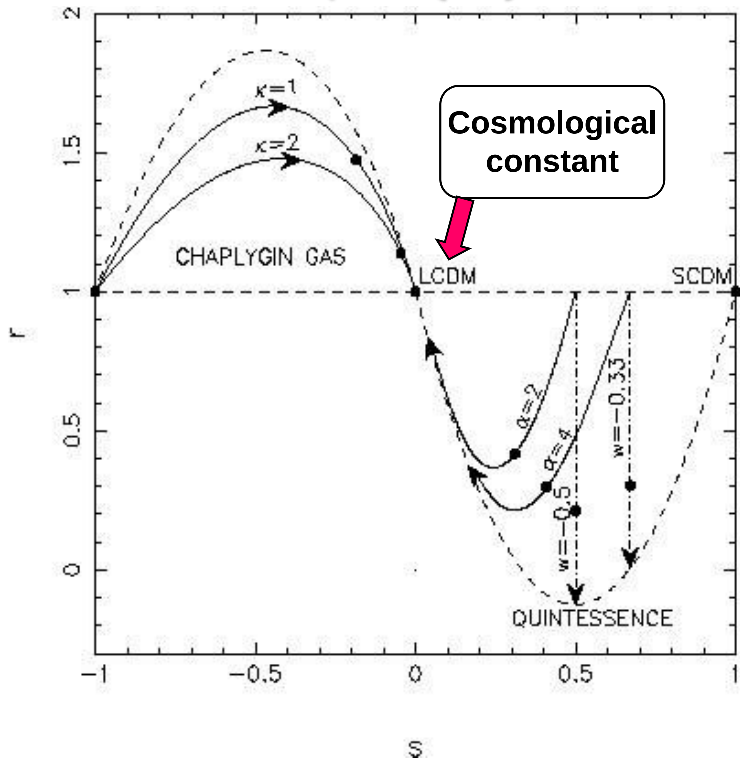
Therefore r,s are null diagnostics for the cosmological constant, since

$$\{r, s\} = \{1, 0\}$$ only for Λ

⇩

Statefinders

The Statefinder pair {r,s} is an excellent diagnostic of Dark Energy !



[VS, Saini, Starobinsky, Alam (2003)]

$$r = \frac{\dddot{a}}{aH^3}\ , \quad s = \frac{r-1}{3(q-1/2)}$$

r = 1, s = 0: fixed point for the cosmological constant !

Quintessence: $V(\phi) \propto \phi^{-\alpha}$

Statefinder provides a fingerprint of Dark Energy !

It can easily distinguish the cosmological constant from other models.

Observational data expected during the coming decade from space experiments: DES, Euclid, SKA, etc. will help determine whether or not dark energy is Einstein's cosmological constant.

But differentations amplify noise, so

Can one determine a null diagnostic only from H(z) ?

(with no differentiations)

Yes, its called the Om diagnostic.

[VS, Shafieloo, Starobinsky, PRD 2008]

The Om diagnostic – a null test for the Cosmological Constant.

$$Om(z) = \frac{h^2(z) - 1}{(1+z)^3 - 1}$$

$$h(z) = \frac{H(z)}{H_0}, \;\; H = \frac{\dot{a}}{a}$$

Om is constant only for the Cosmological Constant !

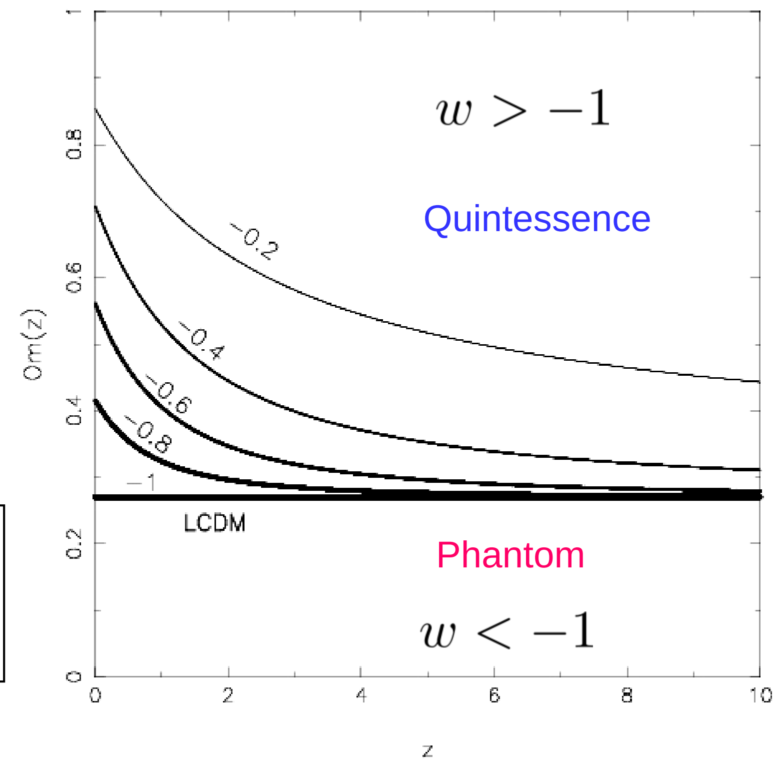
For all other Dark Energy models Om evolves with time.

$$Om(z) = \Omega_{0m} \text{ for } \Lambda\text{CDM}$$

$$Om(z) > \Omega_{0m} \text{ in Quintessence}$$

$$Om(z) < \Omega_{0m} \text{ in Phantom}$$

So if Om evolves with redshift then the Cosmological constant is ruled out !



[VS, Shafieloo, Starobinsky PRD 2008]

Advantages: the Om diagnostic depends only on the expansion rate:

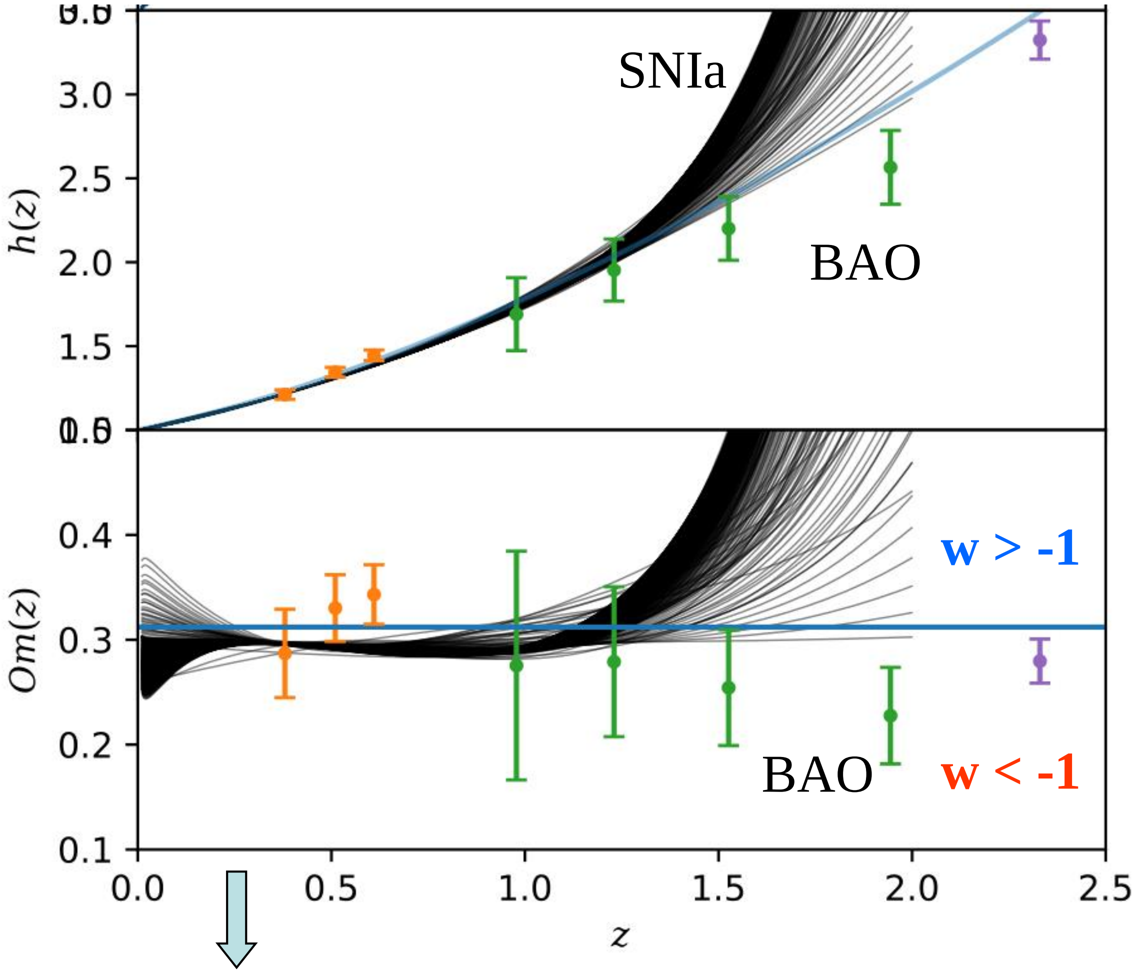
$$Om(z) = \frac{h^2(z) - 1}{(1+z)^3 - 1}$$

and not on the value of the matter density, Ω_{0m}

Unlike w(z) which depends upon a derivative of H(z) (and hence can be noisy) and Ω_{0m}

$$w(x) = \frac{(2x/3)d\log H/dx - 1}{1 - (H_0/H)^2\,\Omega_{0m}x^3}\ ,\ x = 1 + z$$

Therefore errors in the determination of Ω_{0m} do not propagate into Om.



High-z BAO data indicate:

$H(z) < H_{\Lambda\mathrm{CDM}}$

at $z \geq 1.5$

BAO data prefer a Phantom-like EOS at high z.

[Sahni, Shafieloo, Starobinsky, 2014]

[Shafieloo, L-Huillier, Starobinsky, arXiv:1804.04320]

There appears to be some tension between SNIa and BAO data, especially at high-z.
-- Unknown systematics ?

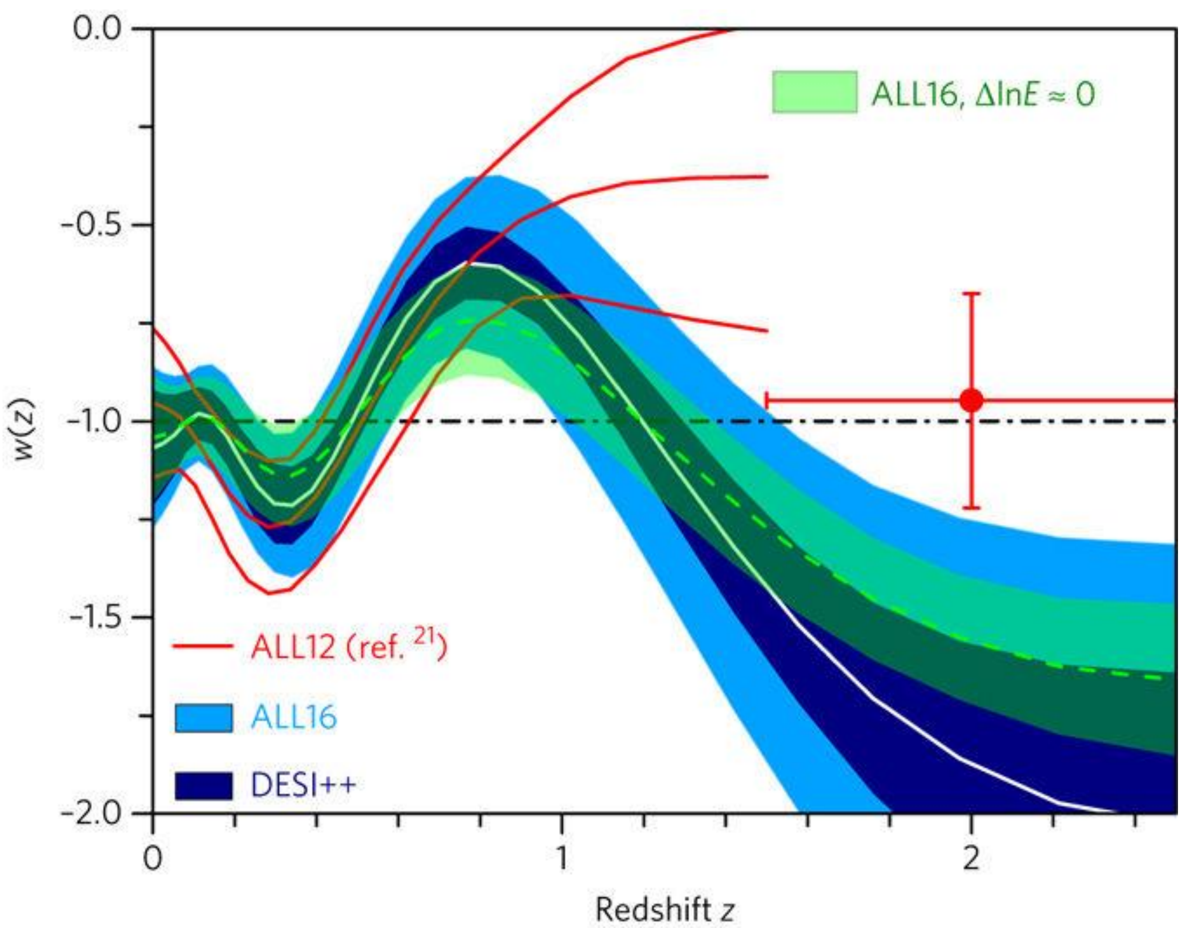
But the tension persists despite improvements in the quality and quantity of data.

[Zhao et al., Nature Astronomy Lett. 2017]

BUT perhaps nature is more complicated ……..

Reconstructed equation of state of Dark Energy

The reconstructed EOS appears to cross the phantom divide at least twice !



$$w = P/\rho$$

An oscillating phantom-like EOS seems to fit the combined data quite well, and reduces the tension between different data sets !

Zhao et al. Nature Astronomy Letters (2017)

The nature of dark energy can also be probed using the
COSMIC WEB

The cosmic web can help break any **degeneracy** that may arise between different DE models

One example of a degeneracy……….

For Quintessence, one can reconstruct the potential $V(\phi)$ from observations of $H(z)$.

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] , \quad \dot{H} = -4\pi G(\rho_m + \dot{\phi}^2)$$

which can be rewritten as

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_{0\mathrm{m}}\, x^3,$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_{0\mathrm{m}} x}{H^2}, \quad x \equiv 1+z \ .$$

Integrating, we determine $\phi(z)$. Inverting $\phi(z) \rightarrow z(\phi)$ and substituting into $V(z)$ allows us to reconstruct $V(\phi)$ from $H(z)$.

However, this reconstruction is valid only when $H^2(z) > H_0^2\left[1 + \Omega_{0m}(1+z)^3\right]$,

which is a restatement of the weak energy condition: $\rho_\phi + p_\phi \geq 0$,

satisfied by the scalar field.

[Saini et al, PRL 2000, VS & Starobinsky, IJMP 2006]

Cosmic Degeneracy: Different dark energy models may have the same expansion rate !

If we know $H(z)$ then we can determine the Quintessence potential from

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_{0\mathrm{m}}\, x^3, \quad \frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_{0\mathrm{m}} x}{H^2}, \quad x \equiv 1+z\ .$$

But $H(z)$ could equally be described by a modified gravity model such as the five dimensional DGP Braneworld:

$$H = \sqrt{\frac{8\pi G\rho_{\mathrm{m}}}{3} + \frac{1}{\ell^2}} + \frac{1}{\ell}\ , \qquad \textbf{DGP brane}$$

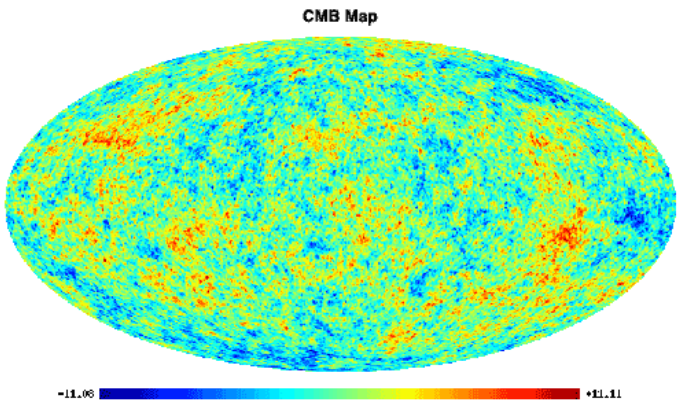
So two models: (i) Quintessence with $V(\phi)$ reconstructed from $H(z)$ and (ii) the Braneworld have exactly the same expansion history $H(z)$!

Q. How to break this degeneracy between the two models ?

Ans. The Cosmic Web comes to our rescue: the evolution of the Web will be different in Quintessence and the Braneworld !

Gravitational Instability and the Cosmic Web

Small Gaussian fluctuations exist when the universe is young



These fluctuations get amplified to form the galaxies we see today !

$$\frac{\rho-\bar{\rho}}{\bar{\rho}}=10^{-5}, \quad \text{when} \quad \frac{a_0}{a(t)} = 1 + z_{\rm rec} = 1100$$

$$\rho \gg \bar{\rho}$$

Today

[Bond, Kofman and Pogosyan, Nature 380, 603, 1996]

Galaxies are the building blocks of the Universe.

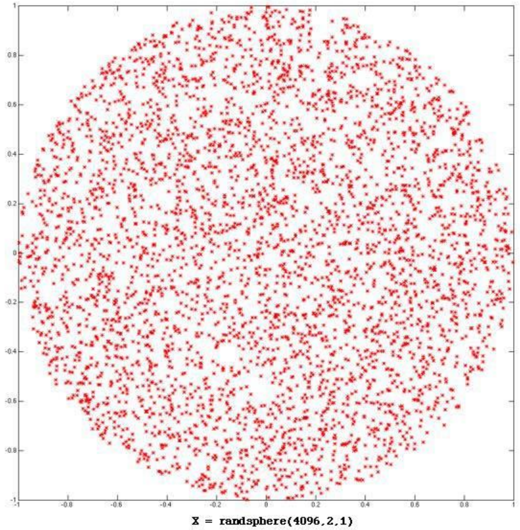
Galaxies also come in a bewildering range of shapes and sizes, ranging from

Dwarf galaxies $(10^7 M_\odot)$ to Giant Elliptical's $(10^{12} M_\odot)$

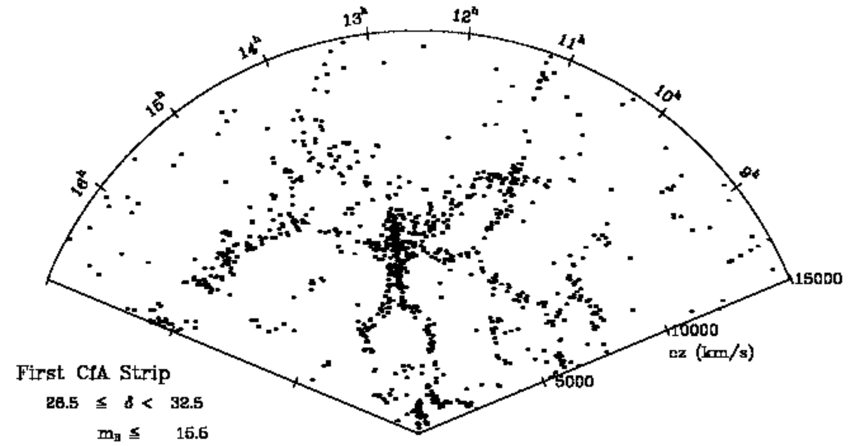
Two questions naturally arise:

1. Did galaxies always exist or did they come into existence during the course of cosmic expansion ?

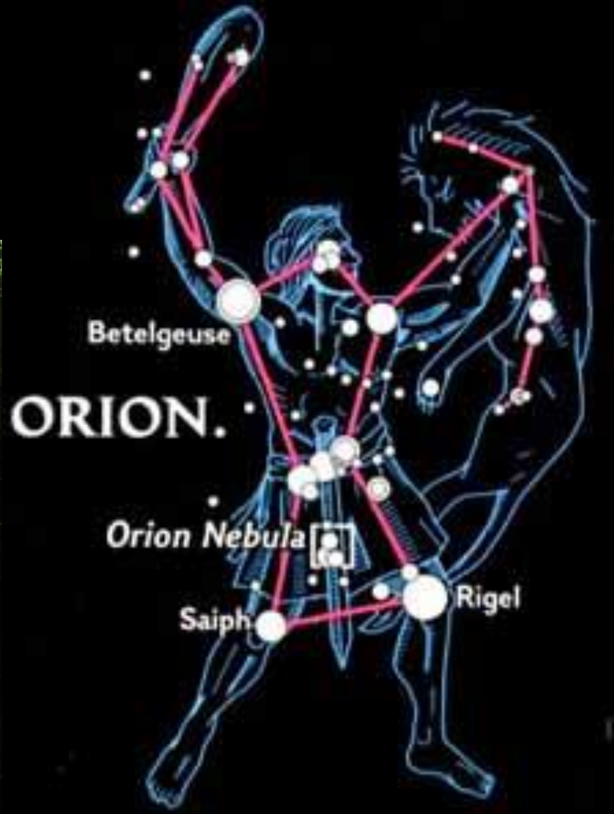
2. Are galaxies distributed randomly in space (like a Poisson distribution) or does the galaxy distribution show a pattern ?



OR

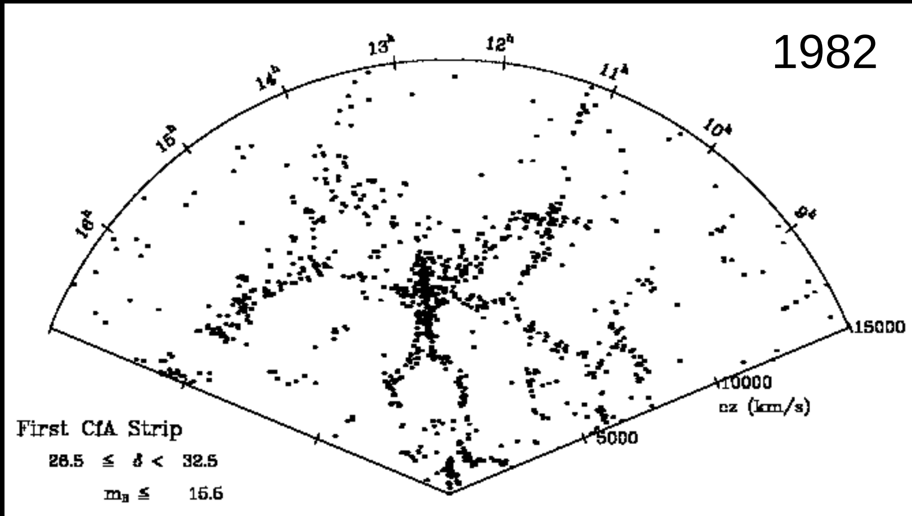


?

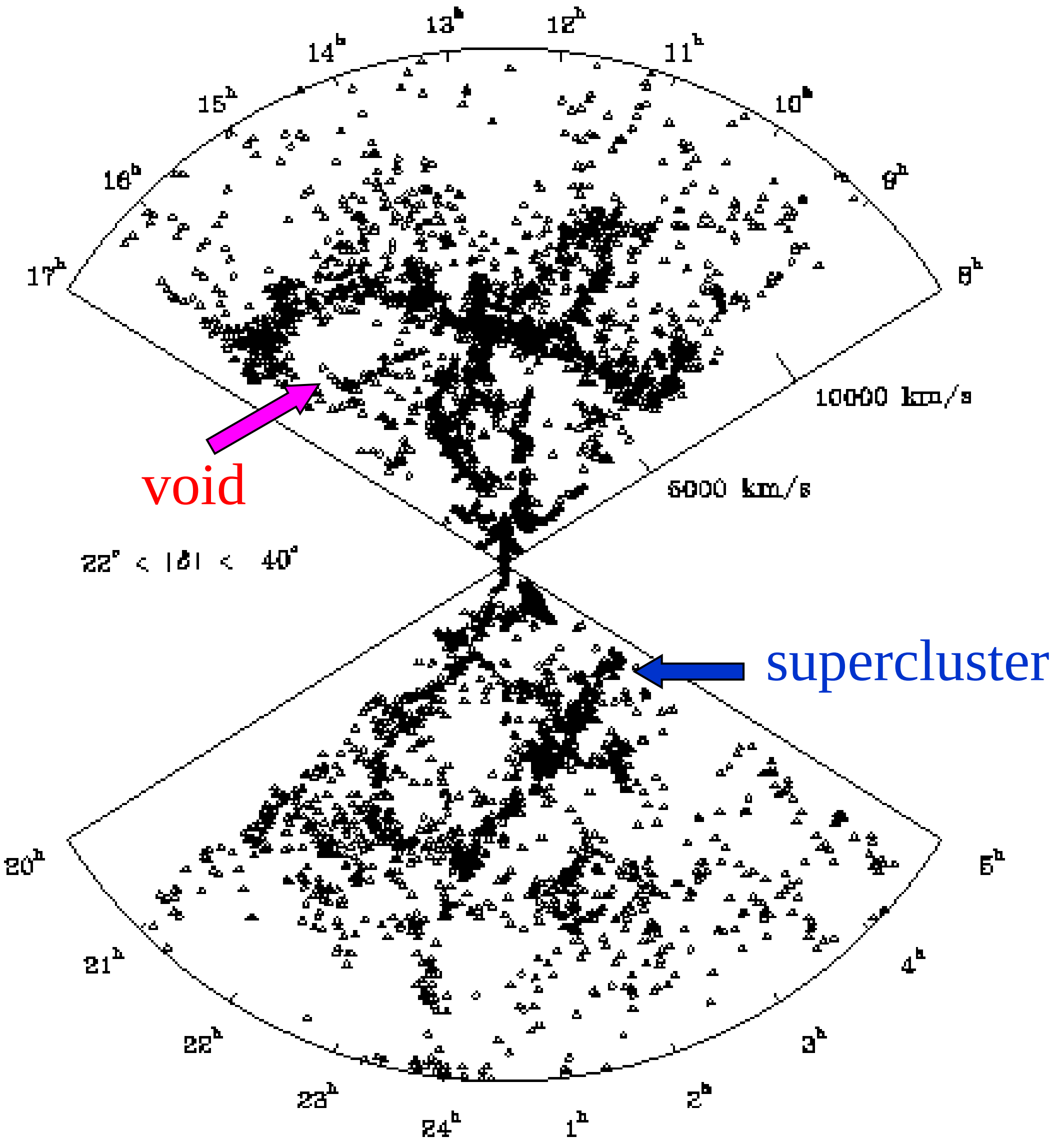


Our eye clearly likes to form patterns !

So is the filamentary distribution of galaxies on the right REAL ?



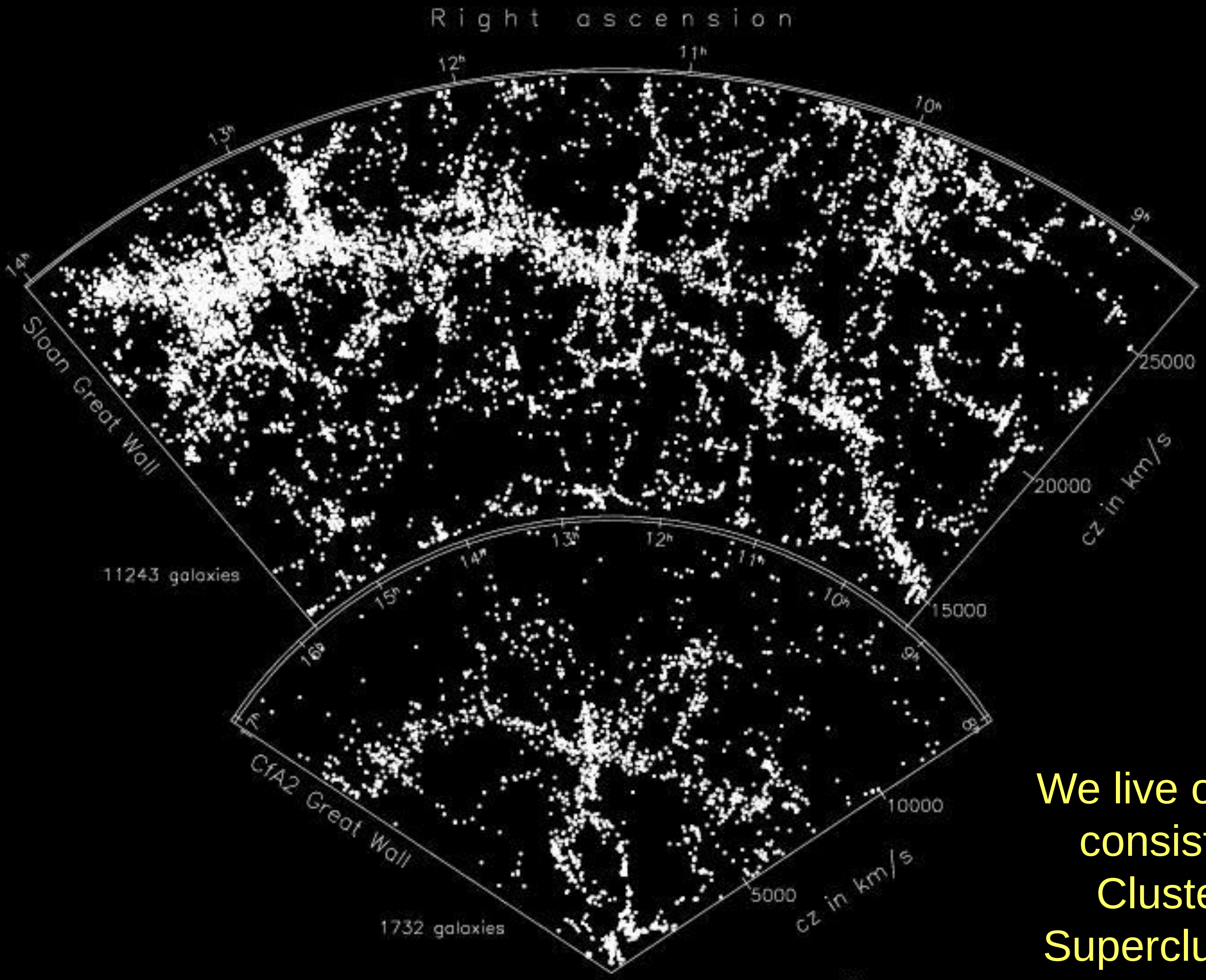
The Cosmic Web is REAL !



Until about a decade ago, the CfA Great wall was the largest structure in the Universe. Its size, roughly 100 Mpc (0.3 billion light years) was of the same order as the survey extent, so one could not know whether this was a `typical’ object, or whether larger superclusters existed !

The reality of coherent structures can be probed by geometrical and morphological tools including:
Minimal Spanning Trees,
Percolation analysis, and
Minkowski functionals.

[VS & Coles, Phys Rep. 1995; Klypin & Shandarin ApJ 1993; Bharadwaj et al, ApJ 2004; etc.]



The Sloan Great Wall is the largest known structure in the Universe.

It is 1.37 billion light years distant and one billion Light years in size.

So it is several times larger than the CfA wall.

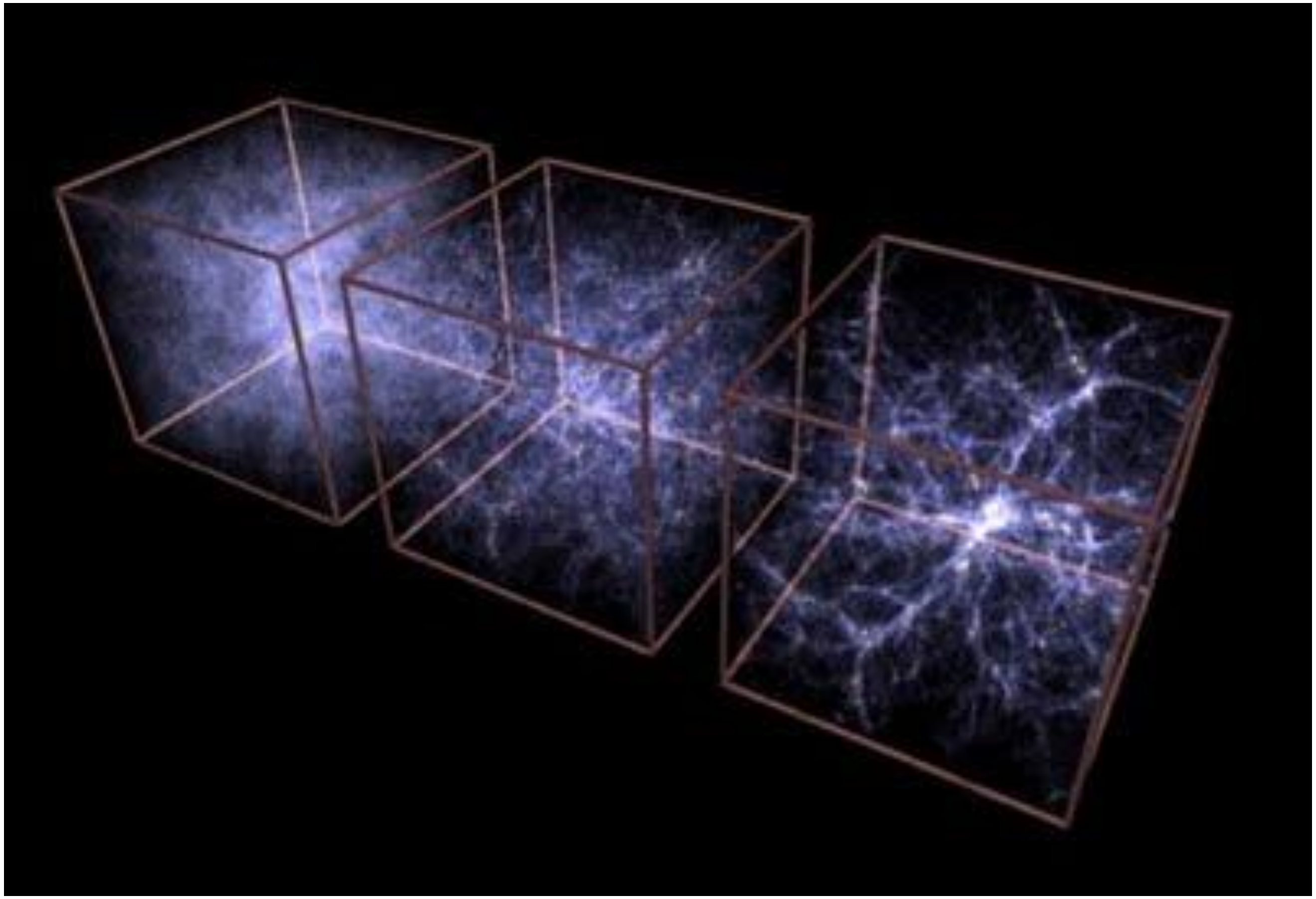
We live on a **Cosmic Web** consisting of galaxies, Clusters of galaxies, Superclusters of galaxies, separated by large empty regions called Voids.

Can such a large structure arise in a LCDM Universe ?

Tiny initial fluctuations (1 part in 100,000) are amplified by gravitational instability over a period of 13 billion years, to give rise to a percolating network of superclusters and voids known as the COSMIC WEB !

$$\frac{\delta\rho}{\rho} \simeq 10^{-5}$$

Gaussian density field initially



Non-Gaussian density field finally

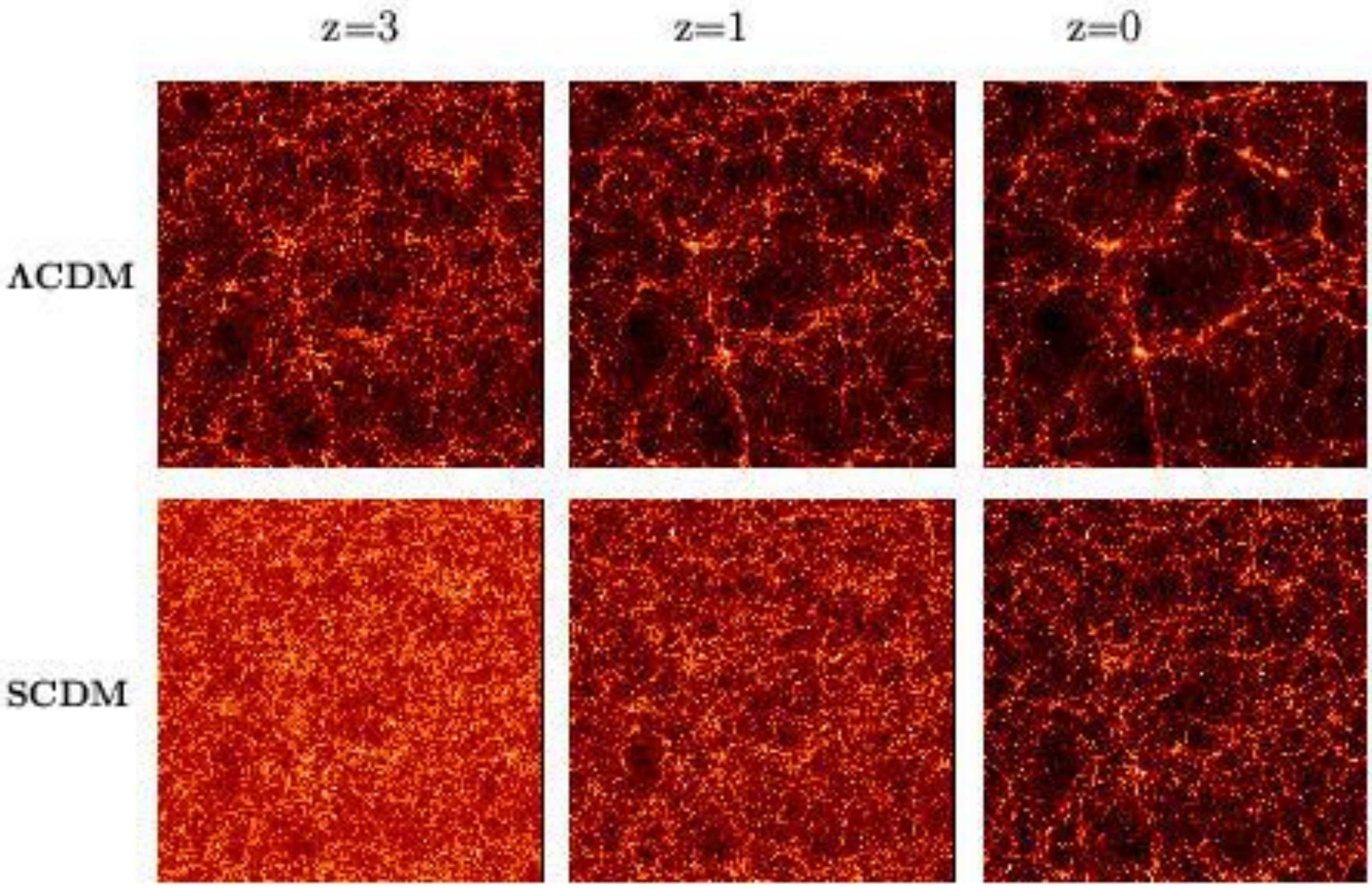
$$\frac{\delta\rho}{\rho} \simeq 10^{5}$$

TIME →

- The presence of a SMOOTH dark energy component in the universe, such as

the Cosmological Constant

slows down the rate of assembly of the Cosmic Web !



$\Omega_m \simeq 0.3, \Omega_\Lambda \simeq 0.7$

Dark energy + matter

$\Omega_m = 1, \Omega_\Lambda = 0$

No dark energy,
Only dark matter.

$\delta_m = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

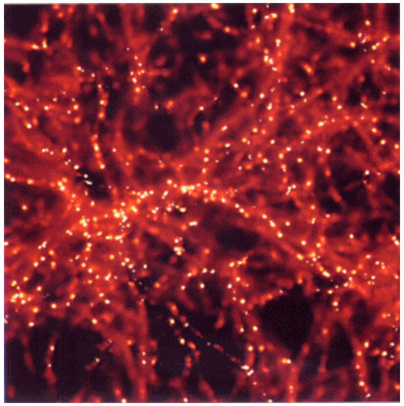
The damping term $2H\dot{\delta}_m$ in $\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G \rho_m \delta_m = 0$

causes perturbations to freeze at late times, as dark energy begins to dominate the universe:

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \rho_{\rm DE} + \cdots\right], \qquad \rho_{\rm DE} = \frac{\Lambda}{8\pi G}$$

The formation of high density regions in the distribution of matter on the cosmic web is similar to the formation of caustics in light !

Caustics form on the Cosmic Web

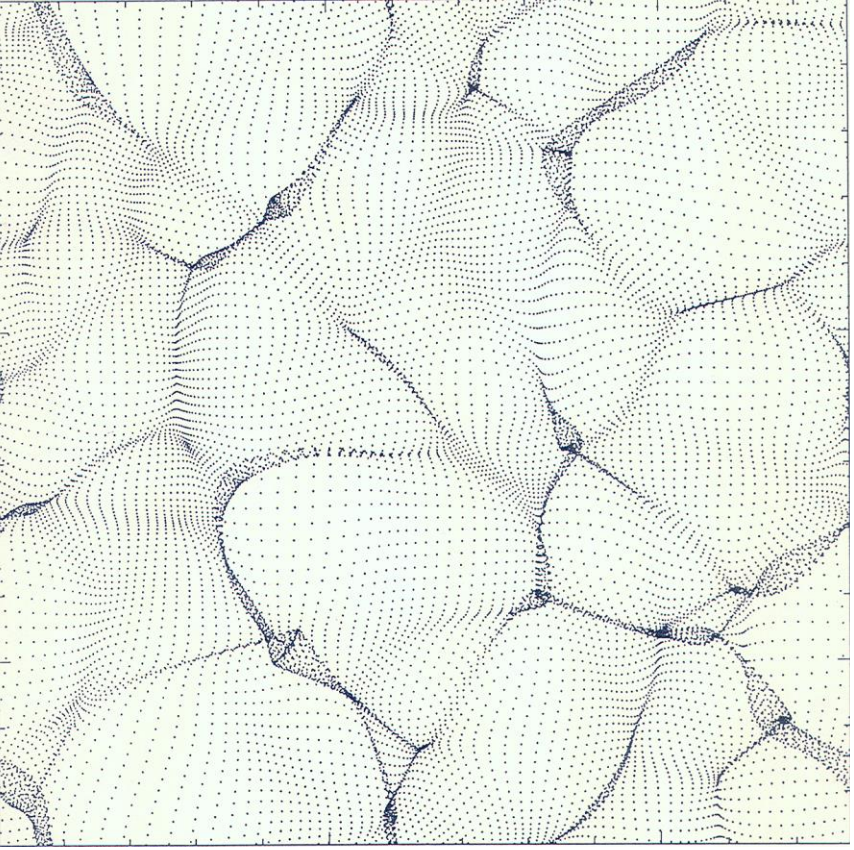


Trajectories of matter intersect to form the Cosmic web

After passing through glass/water neighboring light trajectories intersect to form caustics where the intensity of light is exceedingly bright !

Caustics in the distribution of light

Zeldovich, 1970



Caustics in water:

$$R(z, \mathbf{q}) = \mathbf{q} + \mathbf{s}z$$

$$s_i = -(n-1)\frac{\partial h(\mathbf{q})}{\partial q_i}$$

Caustics in Zeldovich approximation:

$$\mathbf{r} = \mathbf{q} + D(t)\mathbf{v}(\mathbf{q})$$

$$\mathbf{v}(\mathbf{q}) = -\nabla\phi$$

$$\frac{\phi}{c^2} \simeq 10^{-5} \Rightarrow \text{ CMB}$$

The gravitational potential $\phi(\mathbf{q})$ in ZA plays the same role as the plate/water thickness $h(\mathbf{q})$, in optics !

[Shandarin & Zeldovich, Rev Mod Phys 61, 185, 1989]

Structure formation is a key test for modified gravity

The perturbed FRW metric, in the longitudinal (quasi-Newtonian) gauge is

$$ds^2 = -(1+2\phi)dt^2 + (1-2\psi)a^2(t)d\vec{x}^2 \ ,$$

where $\phi = \psi$ only in GR (provided matter is free of anisotropic stress).

In GR, on sub-horizon scales, the linearized matter density contrast $\delta_m = \frac{\delta\rho}{\bar{\rho}}$,

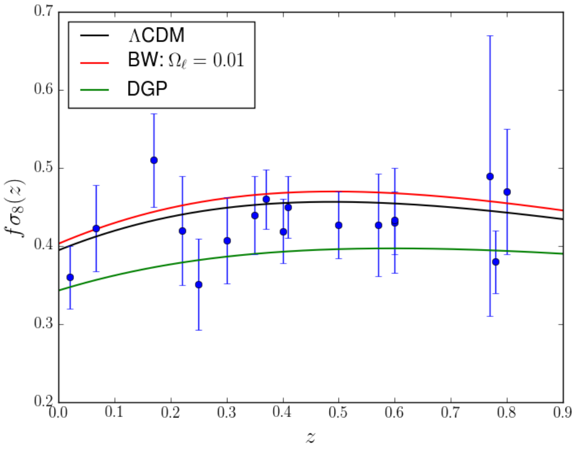
satisfies the equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \qquad \textit{(1)}$$

But (1) is only valid in GR. In modified gravity models the perturbation eqn is more complex since $\phi \neq \psi$!

$$f(z) = \frac{d\ln\delta}{d\ln a} \simeq \Omega_m^\gamma(z) \quad \begin{cases} \gamma \simeq 0.55 \ \ \text{in } \Lambda CDM \\ \text{but } \gamma \simeq 0.67 \ \ \text{in } DGP \ ! \end{cases}$$

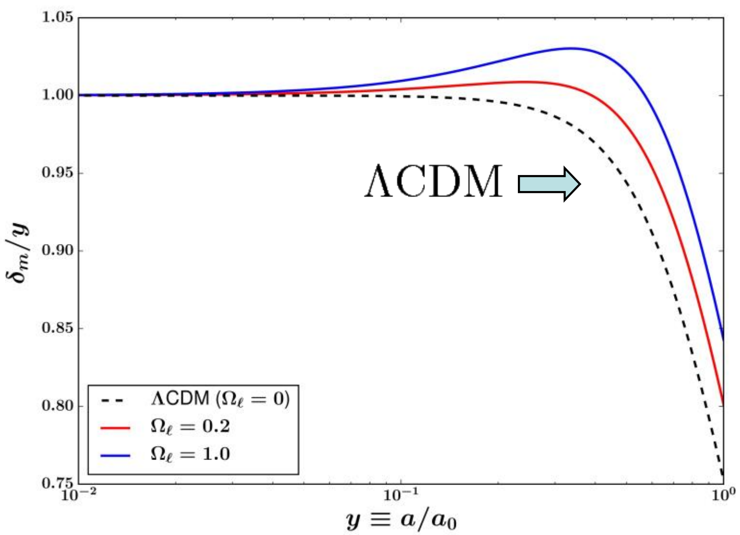
Weak lensing information also crucial: DES, LSST !



Density perturbations grow at a slower rate in Quintessence models and at a faster rate in braneworld models, compared to LCDM.

Observations of large scale structure should be able to distinguish between rival models of dark energy.

Braneworld

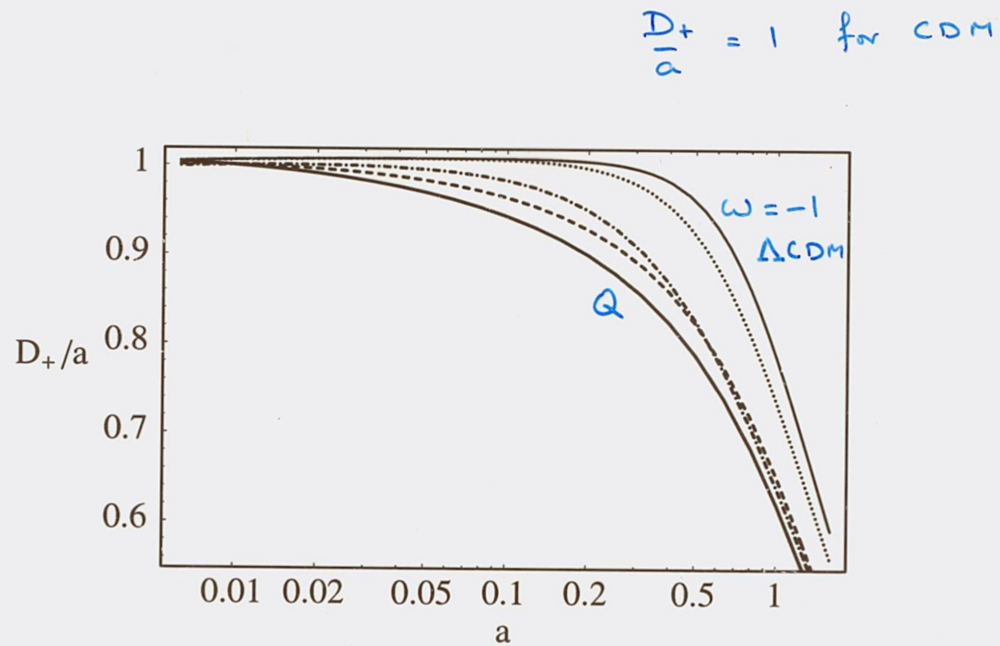


$$\delta_m \equiv D_+\,,\;\; y = a$$

$$D_+ \propto a(t) \text{ in SCDM } (\Omega_m = 1)$$

$$\Rightarrow \frac{\delta_m}{y} = 1 \text{ in SCDM}$$

Quintessence: $V(\phi) \propto \phi^{-p},\; etc.$



Quintessence scenario's with a tracking field show 20 - 30 % smaller growth rate with respect to ΛCDM.

[Benabed & Bernadeau astro-ph/0104371]

Can one unify **dark matter** and **dark energy** ?

Can one unify Dark Matter and Dark Energy ?

The Chaplygin gas described by $\mathcal{L} = -V_0\sqrt{1-\phi_{,\mu}\phi^{,\mu}}$

leads to the EOS $p = -\frac{A}{\rho}$ < 0 ! $(A = V_0^2)$

The conservation equation

$$dE = -pdV \Rightarrow d(\rho a^3) = -pd(a^3) \quad \text{gives} \quad \rho = \sqrt{A + \frac{B}{a^6}}$$

So that $\rho \propto a^{-3}$ at early times (like matter) (B is a constant of integration.)

while $\rho \longrightarrow \mathrm{constant}$ at late times -- just like Λ !!

The Chaplygin gas behaves like pressureless matter at early times and like a cosmological constant during late times !!

Q. Can Chaplygin gas unify dark matter and dark energy ?
[Kamenshchik, Moschella, & Pasquier (2001)]

No, perturbations in this model do not satisfy observations since the speed of sound grows rapidly as the universe expands. $\lambda_J \propto c_s/\sqrt{G\rho}$

Unifying Dark Matter and Dark Energy

Noncanonical scalar field Lagrangian : $\mathcal{L} = X^\alpha - V(\phi)$, $X = \frac{1}{2}\dot{\phi}^2$

[Mukhanov & Vikman, 2006]

dark matter (X^α) dark energy ($V(\phi)$)

$\alpha = 1 \Rightarrow$ Canonical scalar field Lagrangian: $\mathcal{L}(X, \phi) = X - V(\phi)$,

has been used to describe both Inflation and Dark energy.

Sound speed : $c_s = \frac{c}{\sqrt{2\alpha - 1}}$, Jeans length $\lambda_J \sim v_s/\sqrt{G\rho}$

Jeans instability:

Perturbations with wavelengths $\lambda < \lambda_J$ grow, while those with $\lambda \geq \lambda_J$ do not.

$[A]$ $c_s = c$ for $\alpha = 1 \Rightarrow$ No gravitational instability in this model

$[B]$ $c_s \to 0$ for $\alpha \gg 1 \Rightarrow$ field can cluster and behave like dark matter

Therefore for $\alpha \gg 1$ the non-canonical Lagrangian can, in principle, describe **Dark Matter**.

Recipe for unification of dark matter and energy.

Eqn. of motion: $$\ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha - 1} + \left(\frac{V'(\phi)}{\alpha(2\alpha-1)}\right)\left(\frac{2M^4}{\dot{\phi}^2}\right)^{\alpha-1} = 0$$

$\alpha = 1$

$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

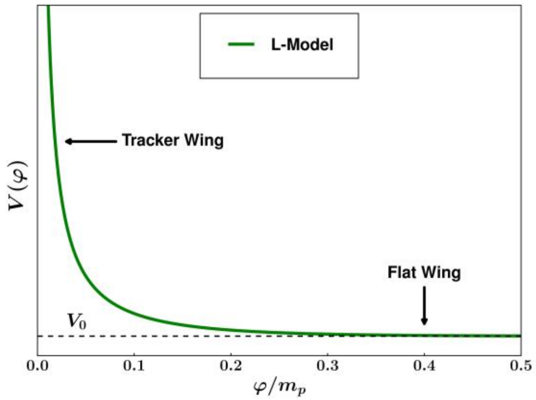
3rd term << 2nd term $\Rightarrow \ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha - 1} \simeq 0$ $\quad \rho_X \propto \dot{\phi}^{2\alpha}$

$$\Rightarrow \quad \dot{\phi} \propto a^{-\frac{3}{2\alpha - 1}} \quad \Rightarrow \quad \rho_X \propto a^{-3} \text{ for } \alpha \gg 1 \quad \Rightarrow$$ **Dark matter**

$c_s \to 0$

Potential should satisfy: $\dot{V} \ll 3H\rho_X \Rightarrow \left|\frac{dV}{dz}\right| \ll \frac{3\rho_X}{1+z}$

Since ρ_X is large at early times the potential can be quite **steep** initially and behave like a **tracker** !



$$\Rightarrow \quad V = V_0 \coth^2 \phi$$

[Mishra & Sahni, arXiv:1803.09767]

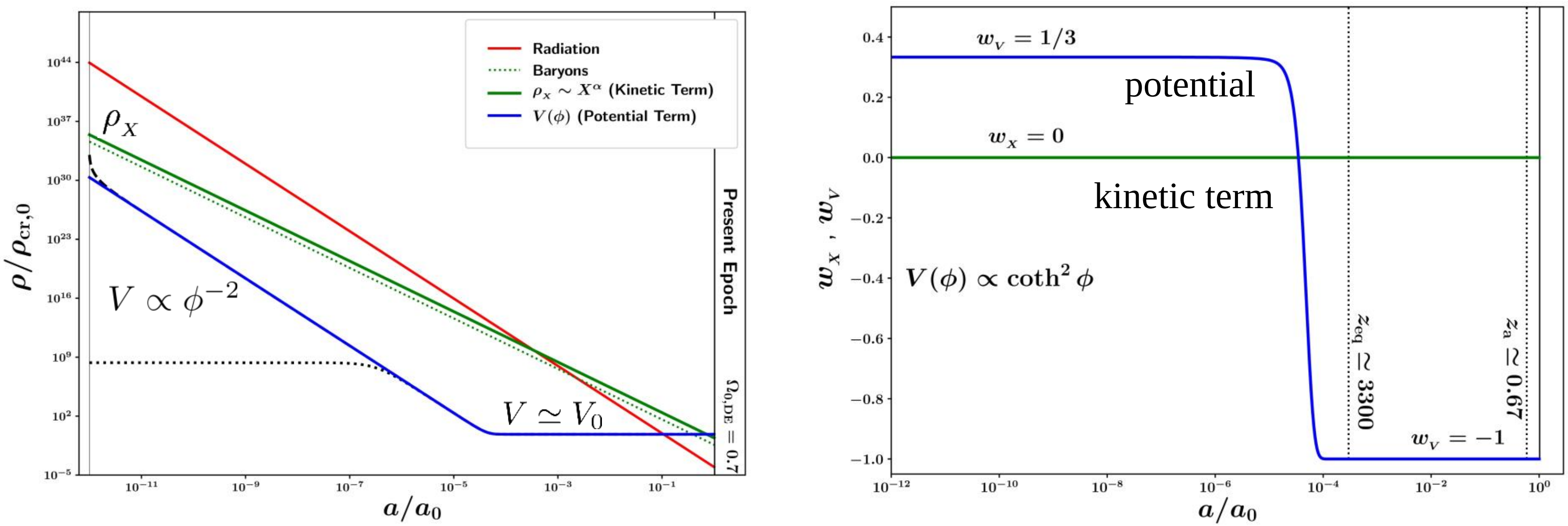
[Sahni & Sen, arXiv:1510.09010]

Unifying DM and DE with $V = V_0 \coth^2 \phi$

$$V(\phi) \simeq \frac{V_0}{(\phi/m_p)^2}, \quad \text{for} \quad \lambda\phi \ll m_p$$

$$V(\phi) \simeq V_0, \quad \text{for} \quad \lambda\phi \gg m_p$$

At early times V **tracks** the background density: $V \propto \rho_B$
At late times V behaves like a cosmological constant and drives acceleration.



Dotted curves show initial values of $V(\phi)$ which converge onto the scaling tracker.

- The kinetic term $\rho_X \propto X^\alpha$ behaves like dark matter $\rho_X \propto a^{-3}$
- The potential, V, behaves like tracker **dark energy**.

$$\mathcal{L} = X^\alpha - V(\phi)$$

DM **DE**

[Li & Scherrer, arXiv:1602.05065] [Mishra & Sahni, arXiv:1803.09767]

7 Open Questions for Dark Energy & Cosmology

1. Is Dark Energy a Cosmological Constant or is it something else ?

2. Does general relativity need to be extended to accommodate cosmic acceleration ?

3. Is late-time acceleration (dark energy) related to early-time acceleration (Inflation) ?

4. What is dark matter ? Are dark matter and dark energy related ?

5. Why is the density in dark matter almost the same as the density in dark energy: is this simply a cosmic coincidence ?

6. Did a Big Bounce precede the Big Bang ?

7. What is the size of the largest superclusters in the Universe ? Are there structures even larger than the Great WALL ?

If dark energy is the cosmological constant then a gloomy future awaits us because of the presence of an **event horizon**

The future of a Λ-dominated universe

Expansion of the universe rapidly approaches the exponential rate $a \propto \exp Ht$ where $H = H_\infty = \sqrt{\Lambda/3} = H_0\sqrt{1-\Omega_m}$.

The matter density will decline asymptotically to zero $\rho_m \propto a^{-3} \propto e^{-3Ht} \rightarrow 0$.

Density perturbations will freeze $\delta\rho/\rho \rightarrow constant$, if they are still in the linear regime. But the acceleration of the universe will not affect gravitationally bound systems on present scales of $R < 10\text{h}^{-1}$ Mpc (includes our own galaxy as well as galaxy clusters).

The universe will consist of islands of matter immersed in an accelerating sea of vacuum energy: 'Λ'.

The universe will soon develop an 'event horizon': The local neighborhood of an observer from which he/she is able to receive signals will eventually contract and shrink. Even those regions of the universe which are observable to us at present will eventually be hidden from view.

(This is analogous to what is observed for an object falling through the horizon of a black hole.)

"A universe with an event horizon poses a serious challenge for string theory" since "constructing a conventional S-matrix is not possible and one may have to ask what the observables are in a string theory that is descried by a finite dimensional Hilbert space." [Fishler, Kashani-Poor, McNees, Paban (2001); Hellerman, Kaloper, Susskind (2001)]

The presence of an event horizon implies that, at any given moment of time t_0, there is a 'sphere of influence' around our civilization. This sphere has an associated redshift z_H, and a celestial body having $z > z_H$ will be unreachable by any signal emitted by our civilization now or in the future; $z_H \simeq 1.8$ in ΛCDM cosmology with $\Omega_\Lambda \simeq 2\Omega_m \simeq 2/3$. Thus all celestial bodies with $z > 1.8$ lie beyond our event horizon and there is no possibility of causal contact with any of them.

Linearised density perturbations freeze in a ΛCDM cosmology, since

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\delta = 0 \ ,$$

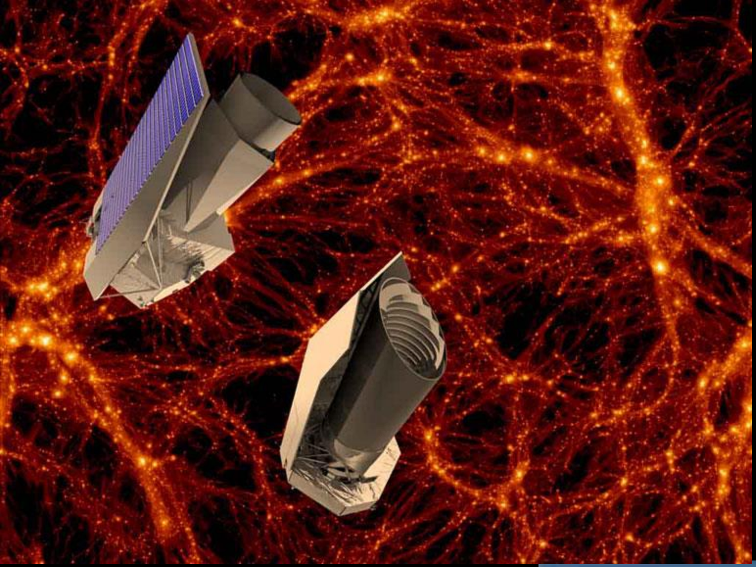
Since $H \to \sqrt{\Lambda/3}$ = constant, while $\rho \propto a^{-3} \to 0$

$$\ddot{\delta} + 2H\dot{\delta} \simeq 0 \Rightarrow \dot{\delta} \propto a^{-2}$$

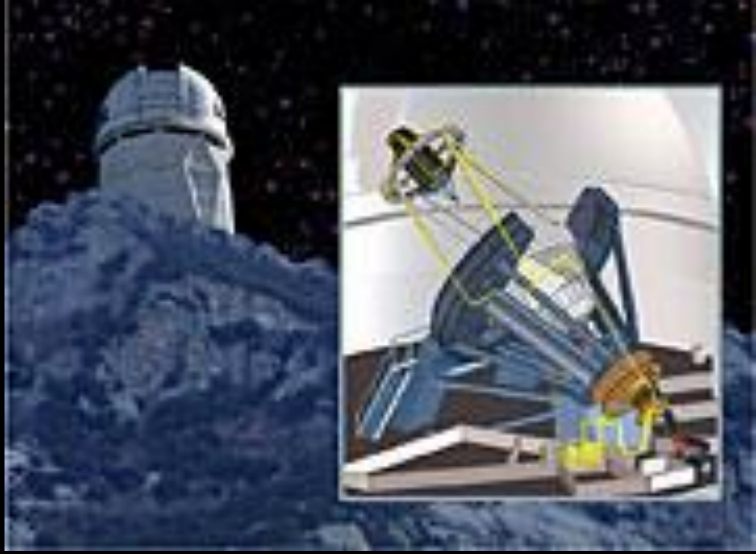
So the density perturbation freezes at late times (δ = constant) if it is in the linear regime.

Only large overdensities $\delta\rho/\rho \sim 17$ have enough gravitational attraction to withstand the repulsive effects of the cosmological constant and to remain gravitationally bound [Lokas & Hoffman (2002)]. Weaker overdensities will be pulled apart by the rapid acceleration of the Universe. Since the local group has $\delta\rho/\rho > 17$ it will survive. An N-body simulation tracking the future of an LCDM universe has shown that ~ 100 billion years from now the observable universe will consist of only a single massive galaxy within our event horizon – the merger product of the Milky Way and Andromeda galaxies; [Nagamine & Loeb (2002)].

The Future Universe is very boring ! – The end of Astronomy ?



Big breakthroughs await us …. 100,000,000 galaxy redshifts soon !



Fresh insights into Dark Energy from DES and Euclid

Epoch of recombination from SKA !

Precision cosmology may be just around the corner !!

The significant problems we have cannot be solved at the same level of thinking with which we created them.

--- Albert Einstein

Perhaps this is also true for Dark Energy !

Dark Energy Reviews

1. S. Weinberg, 1989, Rev. Mod. Phys. **61**, 1.
2. V. Sahni, and A.A. Starobinsky, 2000, *IJMP D* **9**, 373 [astro-ph/9904398].
3. S.M. Carroll, 2001, Living Rev.Rel. **4** 1 [astro-ph/0004075];
4. P.J.E. Peebles, and B. Ratra, 2002, Rev.Mod.Phys. **75**, 559 [astro-ph/0207347];
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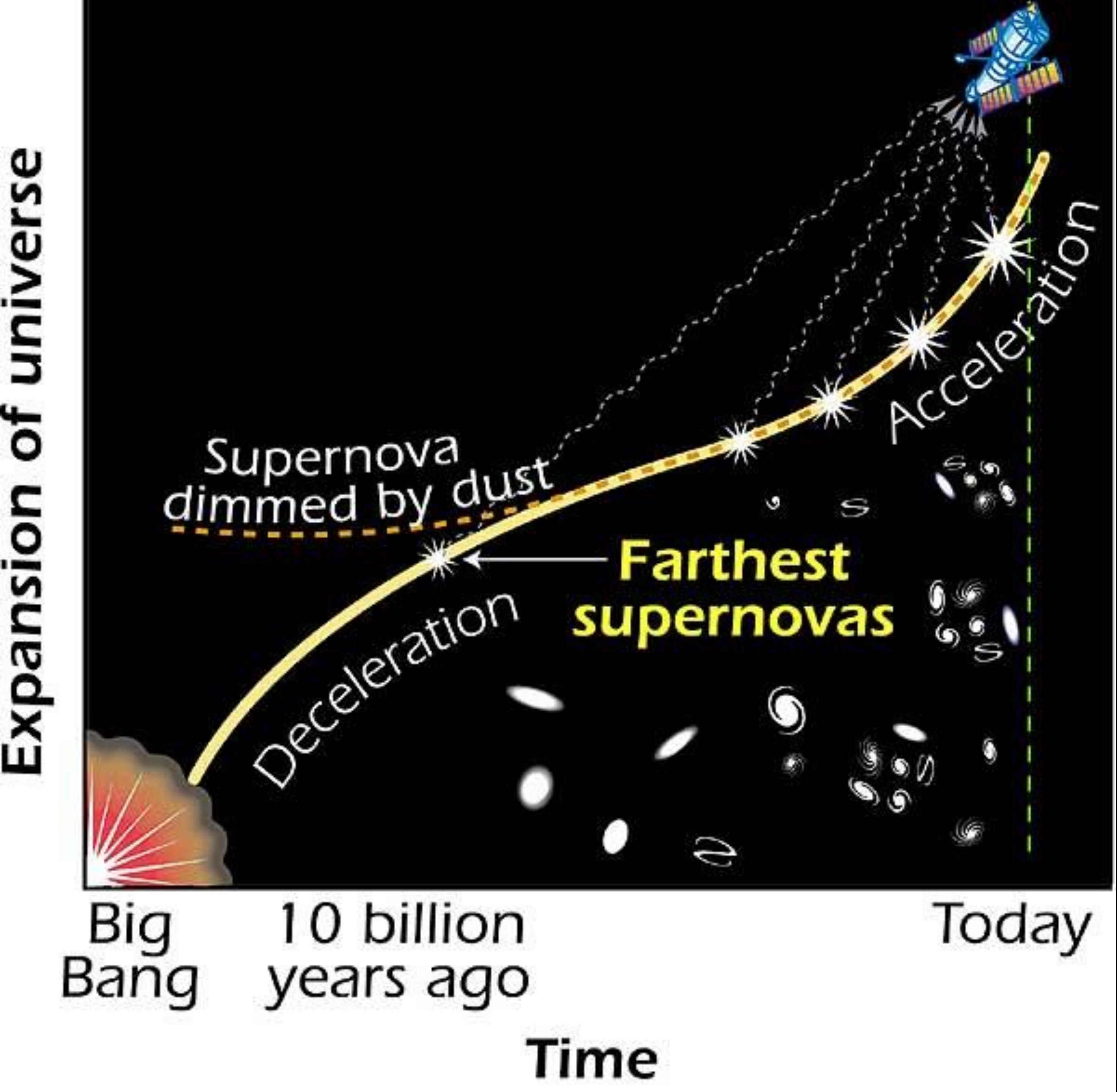
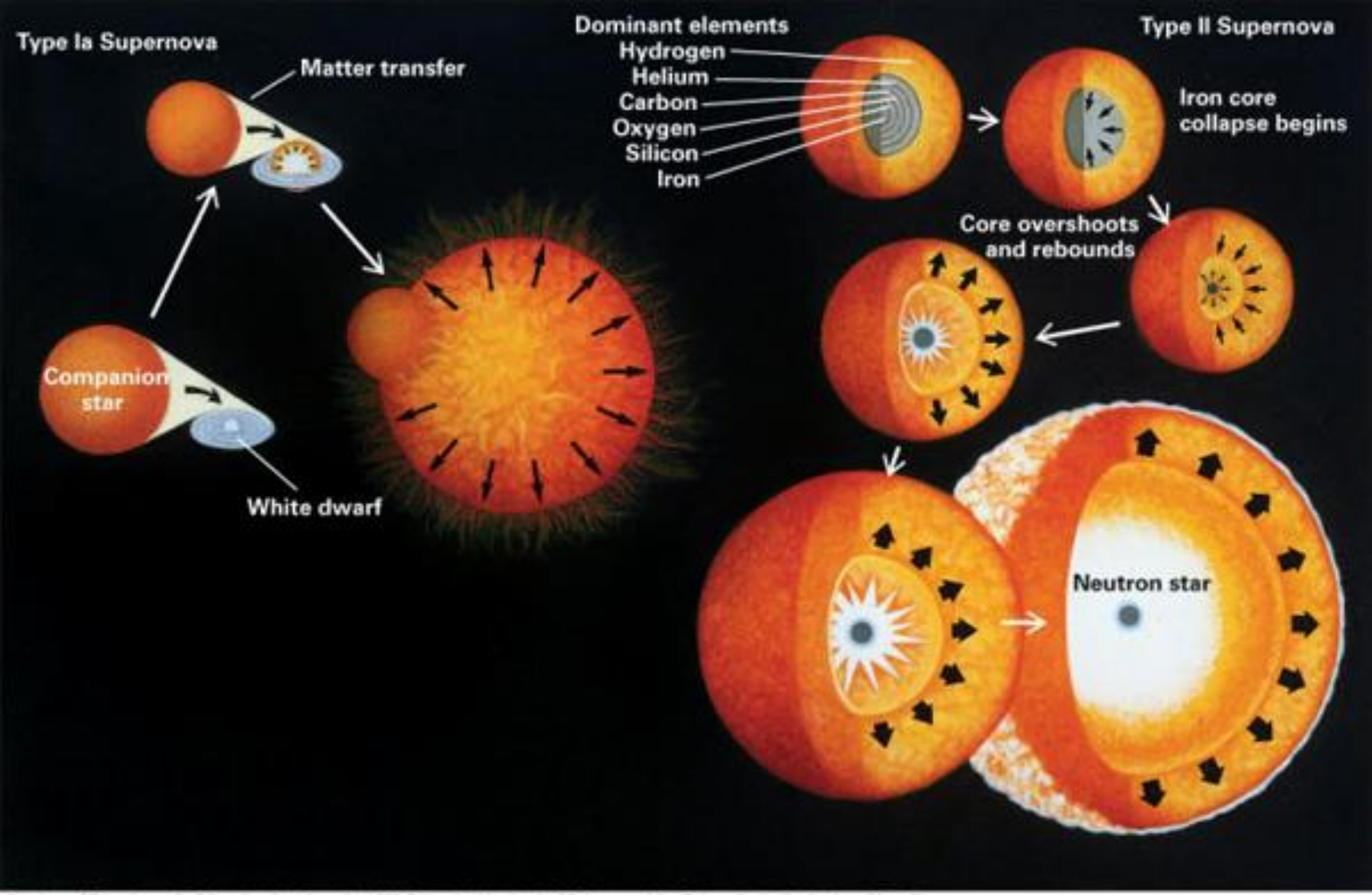
6. V. Sahni, astro-ph/0403324, astro-ph/0502032;
7. S. Nojiri and S. D. Odintsov, hep-th/0601213;
8. R. P. Woodard, astro-ph/0601672.
9. E. J. Copeland, M. Sami, S. Tsujikawa, hep-th/0603057.
10. V. S and A. Starobinsky IJMP (D) 15, 2105 (2006) astro-ph/0610026

Thank You !!

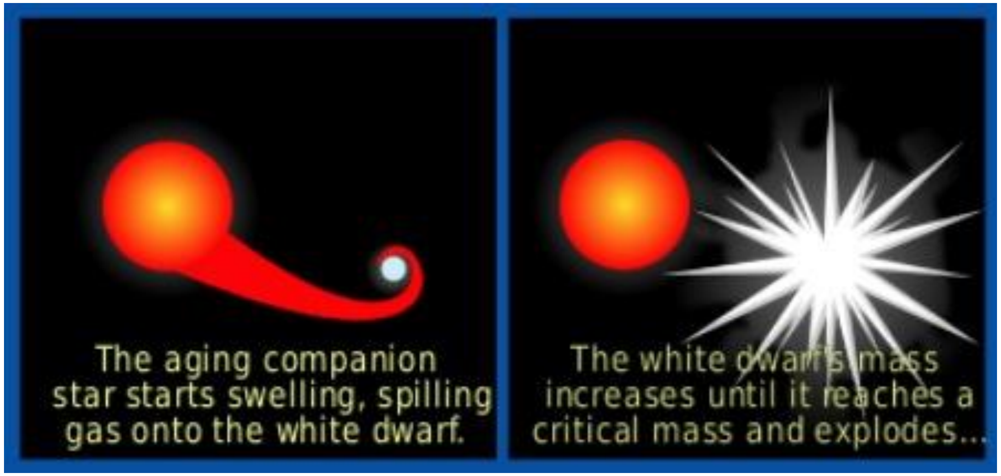


Cosmic acceleration
may be a recent
Phenomenon !



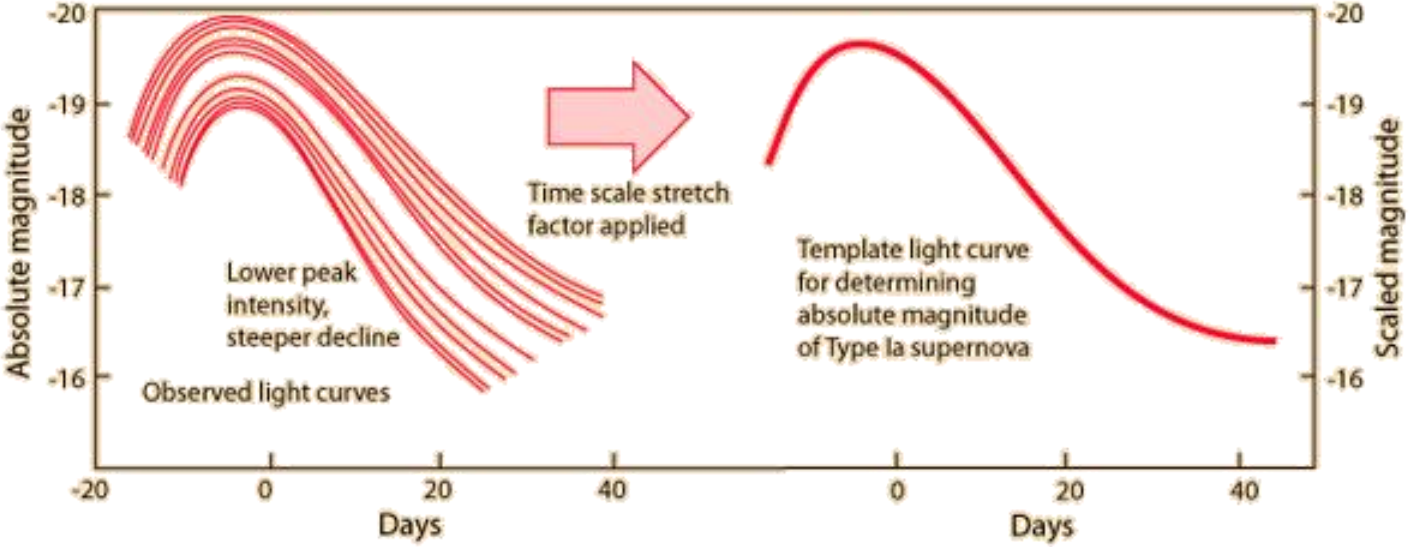
Courtesy of Encyclopaedia Britannica, Inc.; from the 1989 Britannica Yearbook of Science and the Future; illustration by Jane Meredith

Birth of a type Ia supernova (SNIa)



The characteristic light curve of SNIa allows it to be used as a standard candle with which one can probe the rate of expansion of the Universe.

Brighter supernovae take longer to fade !



Practically the Statefinders can be determined from observations of

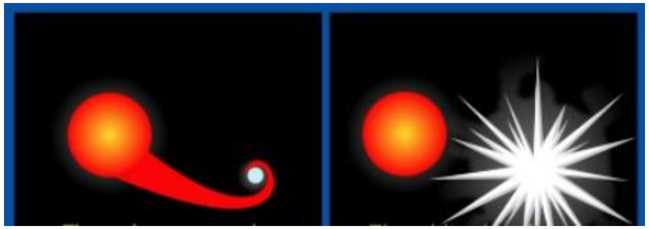
Standard Candles (type Ia Supernovae) and Standard Rulers

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2} \qquad d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

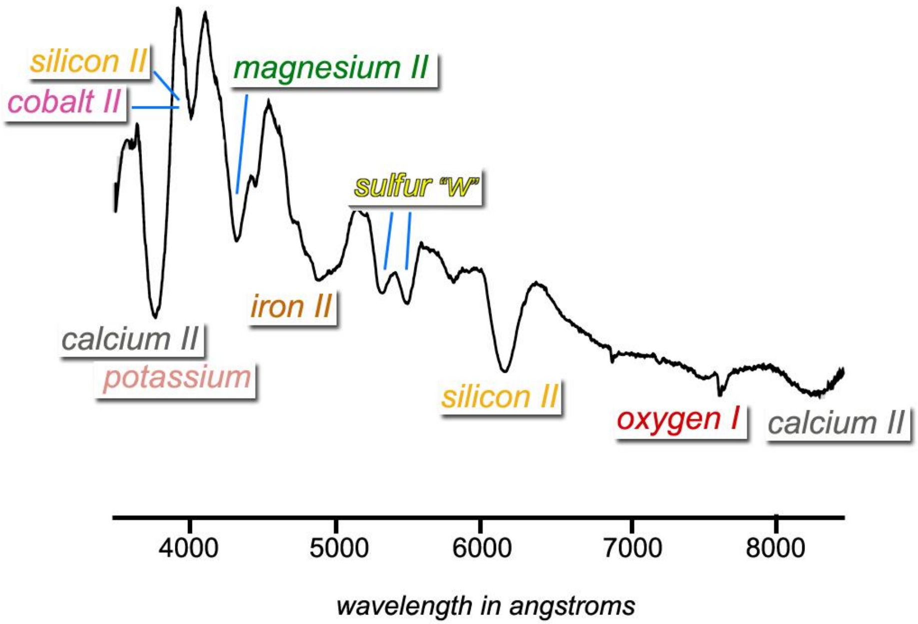
The observed quantity d_L needs to be differentiated thrice to determine the Statefinder

Since $r = \frac{\dddot{a}}{aH^3}$, and $H = \dot{a}/a$. But this is a noisy operation, since errors increase on differentiating a noisy quantity -- d_L

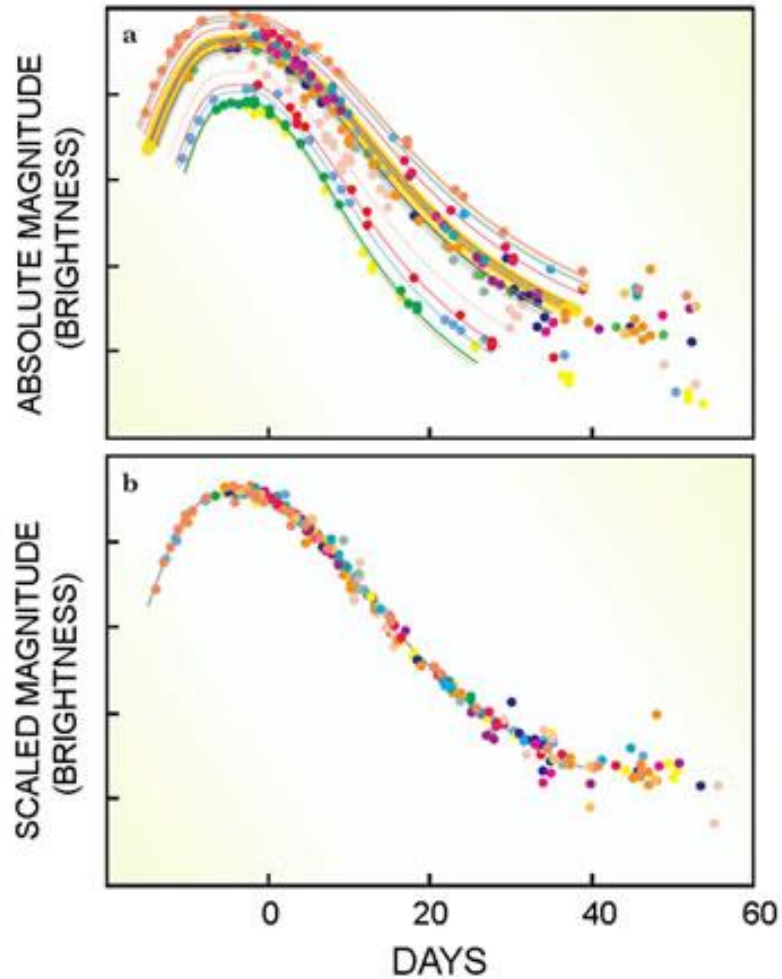
Also type Ia supernovae involve unknown systematics !

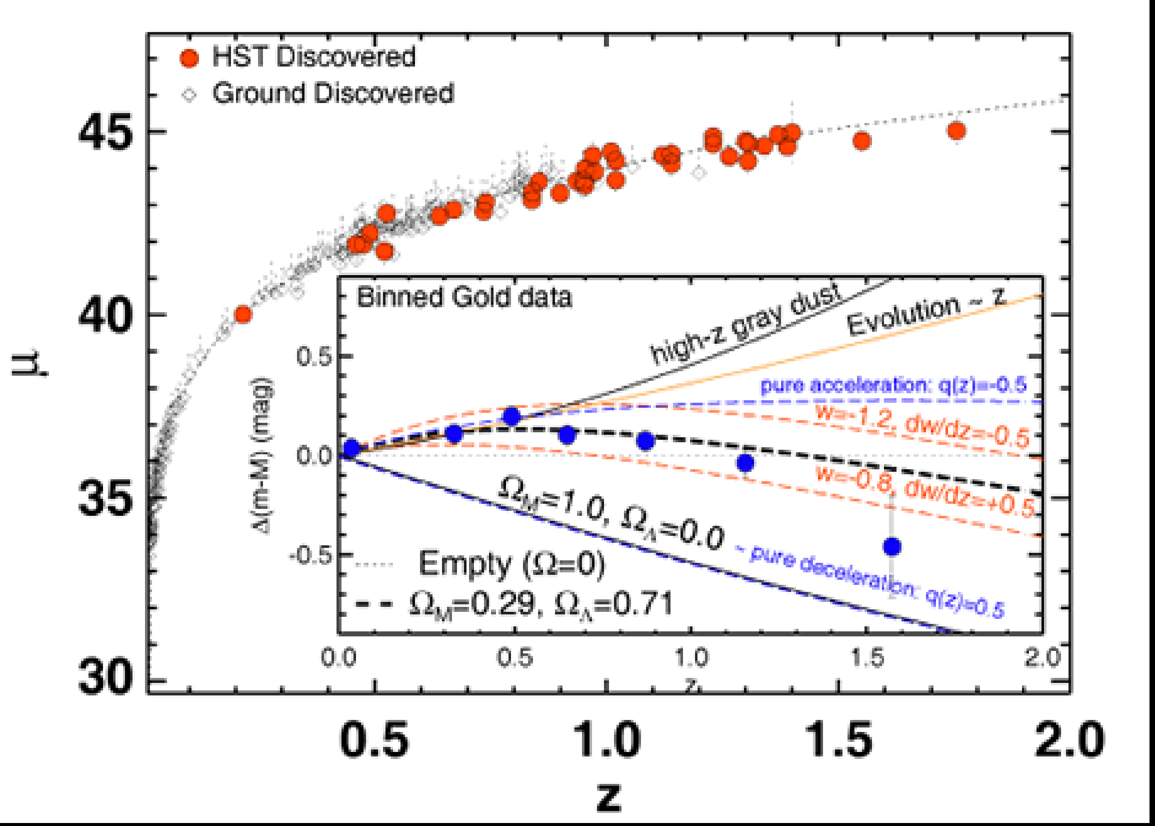


Highly non-linear system, difficult to simulate in the laboratory.



Spectrum of a Type Ia supernova





Q. Could the dimming of SN Ia be due to some form of dust or evolution in Supernovae with redshift ?

Ans. Probably not ! Distant supernovae (z > 1) appear brighter than they would if dimming were due to absorption by `grey' dust.

Grey dust:

Dimming without reddening

Horizons

The local neighborhood of an observer from which he/she is able to receive signals will eventually contract and shrink. Even those regions of the universe which are observable to us at present will eventually be hidden from view.

Ask the question: an observer at $r = r_1$, $t = t_1$ sends a light signal to an observer at $r = 0$. Will the signal ever reach the observer ? Suppose it does and let its time of arrival be t, then

$$ds^2 = 0 \Rightarrow \int_0^{r_1} \frac{dr}{\sqrt{1 - \kappa r^2}} = \int_{t_1}^{t} \frac{dt'}{a(t')}.$$

This relation determines t for any r_1 provided the integral on the left is large enough to match that on the right. Now it could happen that as $t \to \infty$ the $\int dt$ integral converges to a finite value which corresponds to a value of the integral $\int_0^{r_1}$ for $r_1 = r_H$, say. In this case the above relation is not possible to satisfy for $r_1 > r_H$. In other words the signal from the observer at $r_1 > r_H$ will never reach the observer at $r = 0$. Thus no observer beyond a proper distance

$$R_H = a_1 r_1 = a_1 \int_{t_1}^{\infty} \frac{dt'}{a(t')} \ ,$$

at $t = t_1$ can communicate with another observer. This limit is called the event horizon. The event horizon does not exist in FRW models for which $\int dt$ integral diverges for $a(t) \propto t^p$, $p < 1$, therefore an observer at $r = 0$ will be able to receive signals from any event provided s'he waits long enough.

There is a problem however, the *vacuum expectation value* $\langle T_{00}\rangle$ is formally *infinite*, resulting in the *Cosmological Constant problem*.

We demonstrate this for a massive scalar field in Minkowski space with Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\eta^{ij}\Phi_{,i}\Phi_{,j} - m^2\Phi^2) \tag{1}$$

its equation of motion is given by the Klein-Gordon equation

$$(\Box + m^2)\Phi = 0 \tag{2}$$

where $\Box \equiv \eta^{ik}\partial_i\partial_k$.

To quantize the system we treat the field Φ as an operator

$$\Phi(x) = \sum_{\mathbf{k}}[a_k\phi_k(\mathbf{x},\eta) + a_k^\dagger\phi_k^*(\mathbf{x},\eta)] \tag{3}$$

where a_k, $a_k^\dagger$ are annihilation and creation operators $[a_k, a_{k'}^\dagger] = \delta_{kk'}$, defining the vacuum state $a_k|0\rangle = 0\ \forall k$. An orthonormal set of solutions defined using periodic boundary conditions on a three dimensional torus of side L is

$$\begin{aligned} \phi_{\mathbf{k}} &= \frac{1}{\sqrt{2L^3\omega}}\exp(i\mathbf{kx} - i\omega_k t) \\ k_j &= \frac{2\pi n_j}{L}, \quad n_j \in I \end{aligned} \tag{4}$$

where $\omega_k^2 = k^2 + m^2$, and the field modes have been normalised using

$$(\phi_{\mathbf{k}}, \phi_{\mathbf{k}'}) = \delta_{\mathbf{kk}'} \tag{5}$$

where

$$(\phi_1, \phi_2) = -i\int[\phi_1\partial_t\phi_2^* - \phi_2^*\partial_t\phi_1]d^3x. \tag{6}$$

The energy-momentum tensor for the field is

$$T_{ij} = \Phi_{,i}\Phi_{,j} - \frac{1}{2}\eta_{ij}\eta^{kl}\Phi_{,k}\Phi_{,l} + \frac{1}{2}m^2\Phi^2\eta_{ij} \tag{7}$$

where T_{00} defines the energy density

$$T_{00} = \frac{1}{2}(\dot{\phi}^2 + \partial_\mu \phi \partial^\mu \phi + m^2\phi^2) \tag{8}$$

and $T_{0\alpha}$ the momentum density

$$T_{0\alpha} = \frac{\partial \phi}{\partial t}\frac{\partial \phi}{\partial x^\alpha} \tag{9}$$

Substituting from (3) & (4) into (7) one obtains for the Hamiltonian H

$$H \equiv \int T_{00} d^3x = \frac{1}{2}\sum_{\mathbf{k}} (a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + a_{\mathbf{k}} a^\dagger_{\mathbf{k}})\omega_{\mathbf{k}} \tag{10}$$

which can be further simplified using the commutation relation $[a_k, a^\dagger_{k'}] = \delta_{kk'}$ to

$$H = \sum_{\mathbf{k}} (a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \frac{1}{2})\omega_{\mathbf{k}} \tag{11}$$

A similar operation on the momentum density yields

$$P_\alpha \equiv \int T_{0\alpha} d^3x = \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} k_\alpha \tag{12}$$

Inspecting expressions (10) and (12) for the Hamiltonian H and momentum operator P_α we find, for the expectation value of these quantities in the vacuum state $|0\rangle$

$$\langle 0|\mathbf{P}|0\rangle = 0, \qquad \langle 0|H|0\rangle = \frac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}} \tag{13}$$

Transforming the sum $\sum_k$ to an integral we get

$$\langle 0|H|0\rangle = \frac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}} = \frac{1}{2}(\frac{L}{2\pi})^3 \int \omega(\mathbf{k}) d^3k = \frac{L^3}{4\pi^2}\int_0^\infty \sqrt{k^2+m^2}\, k^2 dk \tag{14}$$

we see that zero-point fluctuations are dominated by ultraviolet divergences which diverge as k^4 when $k \to \infty$. The vacuum state therefore has zero momentum and *infinite* energy !

Pauli

First three minutes after Big Bang: production of light elements.

$$n \to p + e^- + \bar{\nu}_e, \quad n + p \to D + \gamma$$

$$D + D \to T + p, \quad D + D \to He^3 + n$$

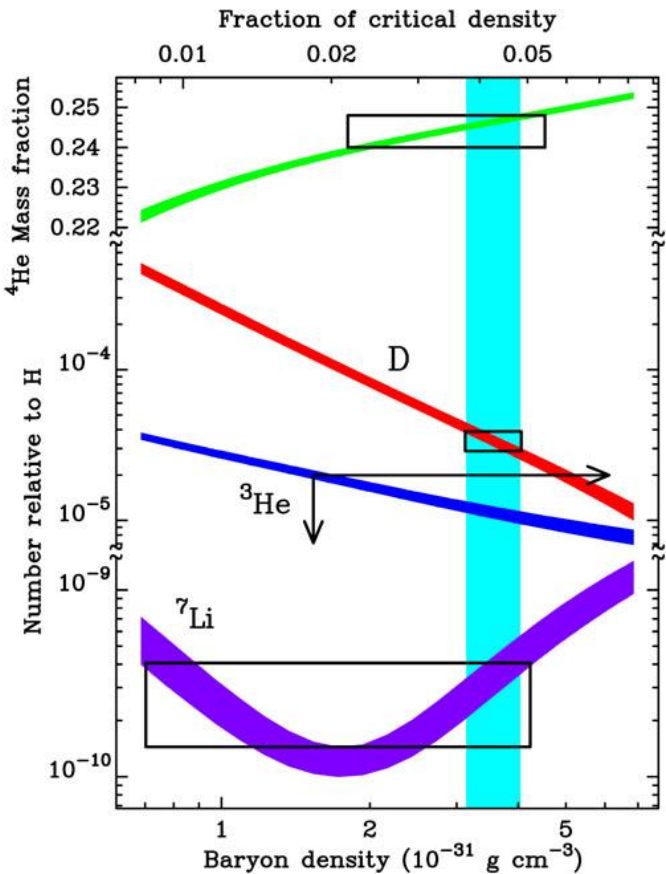
$$D + T \to He^4 + n, \quad He^3 + n \to T + p$$

$$H = p, \; D = np$$
$$T = nnp$$
$$He^3 = npp$$
$$He^4 = nnpp$$

Deuterium is difficult to make and easy to break: binding energy of Deuterium is only about 2 MeV, while its 28 MeV for Helium 4 !

Abundance of Deuterium is very sensitive to the baryon density.

BBN and CMB observations indicate $\Omega_b \simeq 0.04$



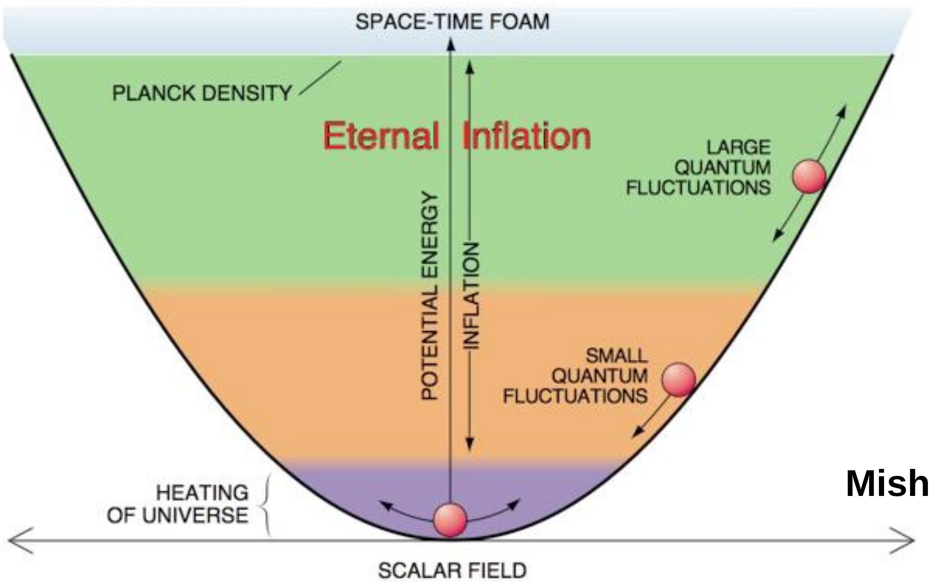
Therefore 96% of the matter content of the Universe is likely to be non-baryonic in nature !

$$\Omega_{\text{Dark matter}} \sim 0.3, \quad \Omega_{\text{DE}} \sim 0.7$$

Other possibilities : Scalar field dark matter

- Oscillating scalar fields can constitute dark matter !

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Mishra, Sahni, Shtanov, JCAP 2017

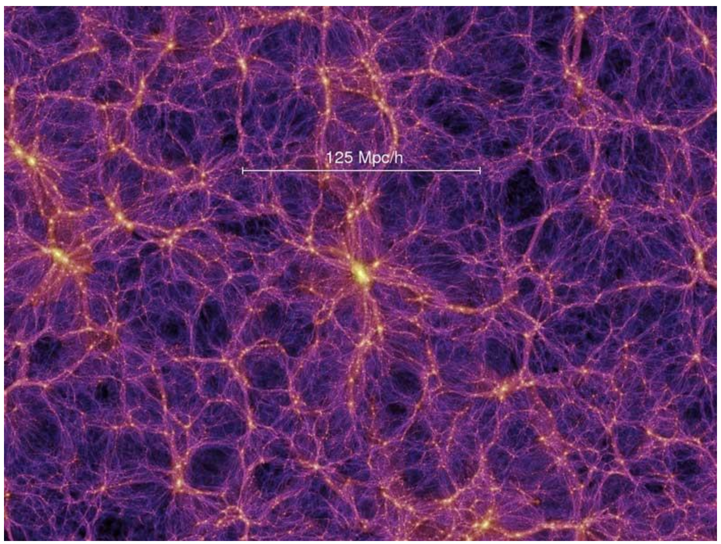
$$\text{Jeans length :} \quad \lambda_J = \pi^{3/4}(G\rho)^{-1/4}m^{-1/2}$$

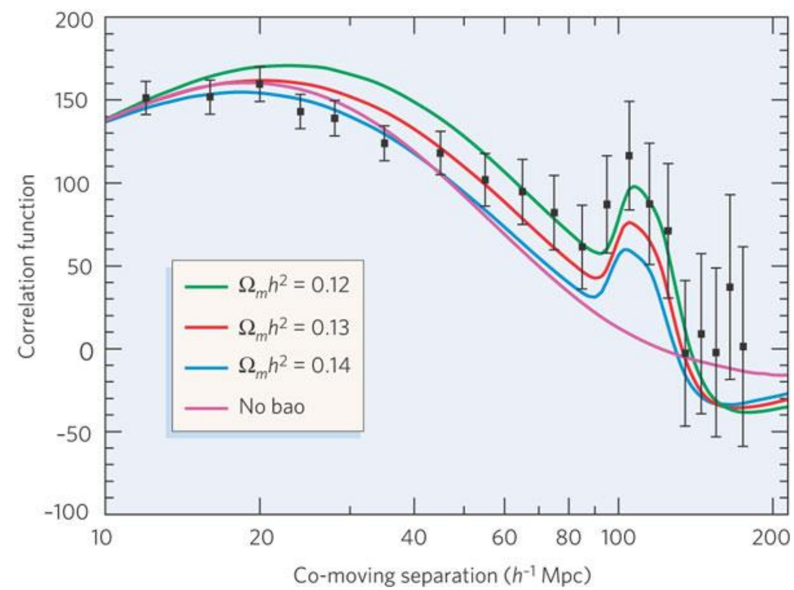
May resolve small scale problems of CDM: **Sahni & Wang, PRD 2000, Hu et al, PRL 2000**

Baryon Acoustic Oscillations (BAO)

The galaxy distribution contains an imprint of the primordial fluctuations in the photon-baryon plasma. Prior to photon decoupling $(z \sim 1100)$ gravity creates oscillations in the photon-baryon plasma. After decoupling these oscillations correspond to a characteristic scale $\sim 150 Mpc$ (comoving horizon at recombination). This scale behaves like a standard ruler and can be used to determine the nature of DE.

Sunyaev & Zeldovich (1970)   Peebles & Yu (1970)





Large length scale, better understood systematics.

Nature 440, 1126 (2006)

Galaxy clustering is anisotropic and the BAO scale can be measured both in the radial and the transverse direction. Radial direction gives $H = \dot{a}/a.$

H needs to be differentiated twice to get the Statefinder parameter $r = \frac{\dddot{a}}{aH^3}$

$$\text{If } V = a^3, E = \rho a^3, \text{ then}$$
$$dE = -p dV \Rightarrow d(\rho a^3) = -p d(a^3)$$
$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

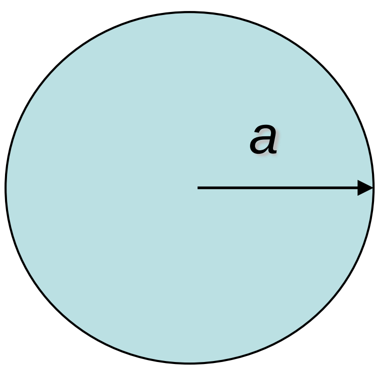
matter ↓ DARK ENERGY ↓ ρ = const if $p = -\rho$

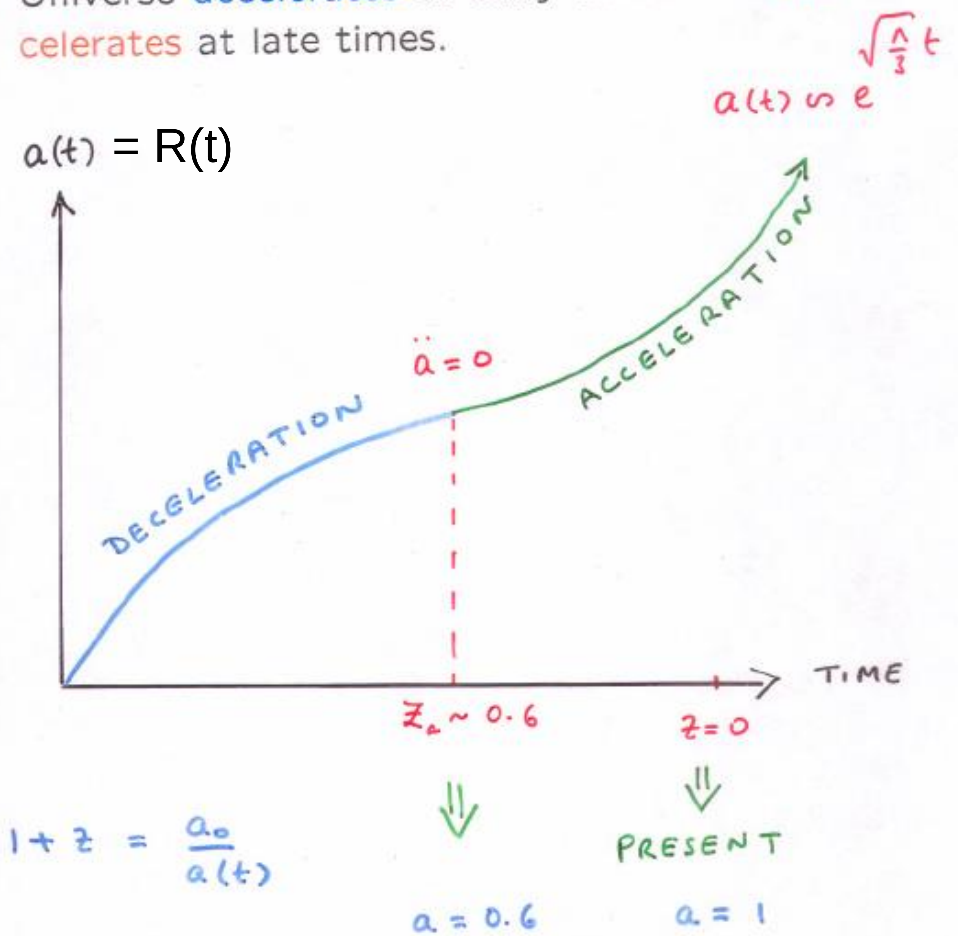
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\frac{\rho_0}{a^3} + \frac{\Lambda}{3}. \quad (6)$$

Since $\rho \propto a^{-3}(t)$ while Λ = constant, the Universe decelerates at early times and accelerates at late times.

If p = -ρ then ρ = constant = Λ.

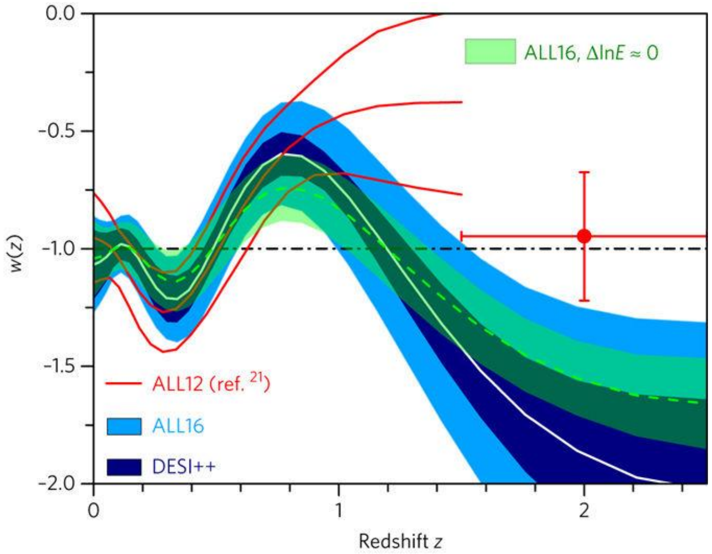
But for pressureless matter p = 0 and ρ $\propto 1/a^3$.





$$a(t) \propto \left(\sinh \tfrac{3}{2}\sqrt{\tfrac{\Lambda}{3}}ct\right)^{2/3}$$

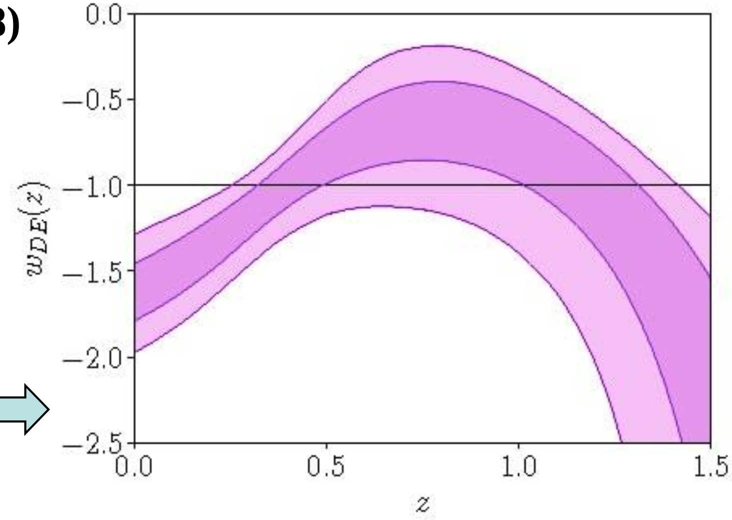
The universe was roughly half its present size when it began to accelerate !



Zhao et al. (2018)

Model independent reconstruction from **Observations**

Capozziello et al. arXiv:1806.03943



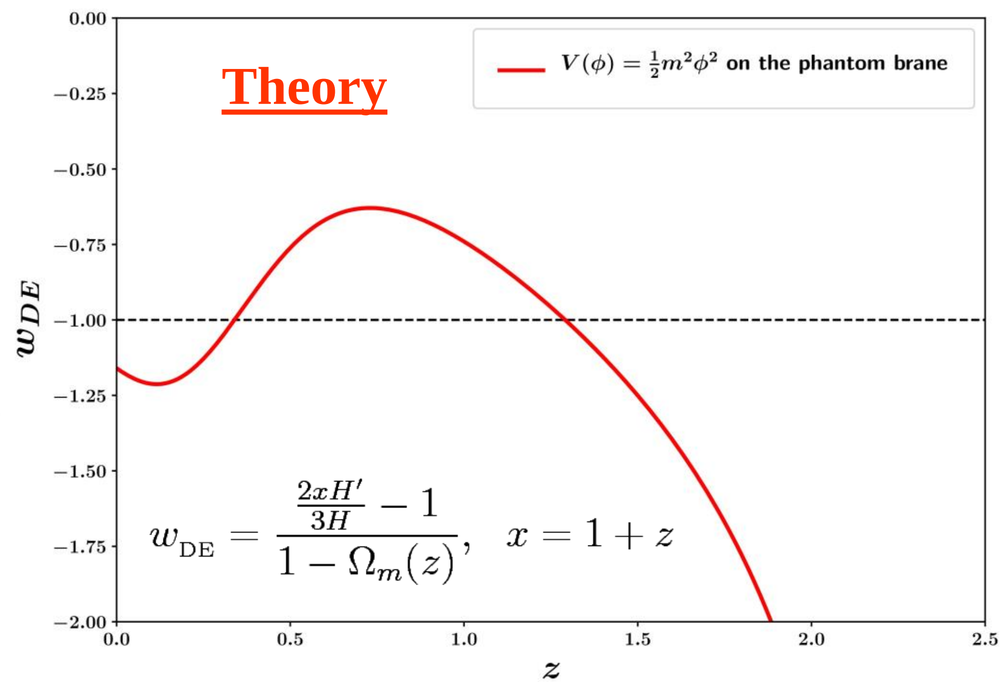
$$\text{Oscillations in } \phi(t) \text{ induce phantom} - \text{like oscillations in } w_{\rm DE}(z)$$

An ultra-light scalar field oscillating on the Phantom brane gives rise to phantom-like oscillations in w(z)

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

$$h(x) = \sqrt{\Omega_{0m}x^3 + \Omega_{DE}(x) + \Omega_\ell} - \sqrt{\Omega_\ell}$$

$$\Omega_\ell = 0.1, \quad m/H \sim O(1) \qquad x = 1 + z$$



[Mishra, Sahni, Shafieloo, Shtanov (2018)]

Numerous Dark Energy models have been suggested to account for an accelerating Universe:

(i) Cosmological constant

(ii) Quiessence with w = constant < -1/3, (cosmic strings/walls), the cosmological constant Λ (w = -1) is a special member of this class;

(iii) Quintessence models;
(iv) The Chaplygin gas;
(v) Phantom DE (w < -1);
(vi) Oscillating DE;
(vii) Models with interactions between DE and dark matter;
(viii) Scalar-tensor DE models;
(ix) Modified gravity models:
(x) Dark energy driven by quantum effects;
(xi) Higher dimensional braneworld models, etc.

Faced with the increasing proliferation of DE models a cosmologist can proceed in either of two ways:

(i) Test each and every model against observations.

(ii) Reconstruct properties of dark energy in a model independent manner.

However, Cosmological Reconstruction is NOT UNIQUE !

The same expansion history, $H(z)$, may result from two very different dark energy models !

Example 1. DE with a constant equation of state $-1 < w < 0$ is described by the potential:

$$V(\phi) = \frac{3H_0^2(1-w)(1-\Omega_{m0})^{1/|w|}}{16\pi G \Omega_{m0}^{\alpha}} \sinh^{-2\alpha}\left(|w|\sqrt{\frac{6\pi G}{1+w}}(\phi - \phi_0 + \phi_1)\right) ,$$

where

$$\alpha = \frac{1+w}{|w|}, \quad \phi_0 = \phi(t_0), \quad \phi_1 = \sqrt{\frac{1+w}{6\pi G}}\frac{1}{|w|}\ln\frac{1+\sqrt{1-\Omega_{m0}}}{\sqrt{\Omega_{m0}}} .$$

Consequently, a universe filled with such a scalar field will have properties which are **identical** to those of a different universe filled with a tangled network of cosmic strings $(w = -1/3)$ or domain walls $(w = -2/3)$.

7 Open Questions for Dark Energy

1. Is Dark Energy a Cosmological Constant or is it something else ?

2. Does general relativity need to be extended to accommodate cosmic acceleration ?

3. Is late-time acceleration (dark energy) related to early-time acceleration (Inflation) ?

4. Why is the density in dark matter almost the same as the density in dark energy is this simply a cosmic coincidence ?

5. Are dark matter and dark energy related ? Perhaps ! Dark matter and dark energy can be described in a unified setting by the:

$$\text{Noncanonical scalar field Lagrangian}: \; \mathcal{L} = X^\alpha - V(\phi) \quad (\alpha > 1)$$

X^α ⇩ **dark matter**; $V(\phi)$ ⇩ **dark energy**

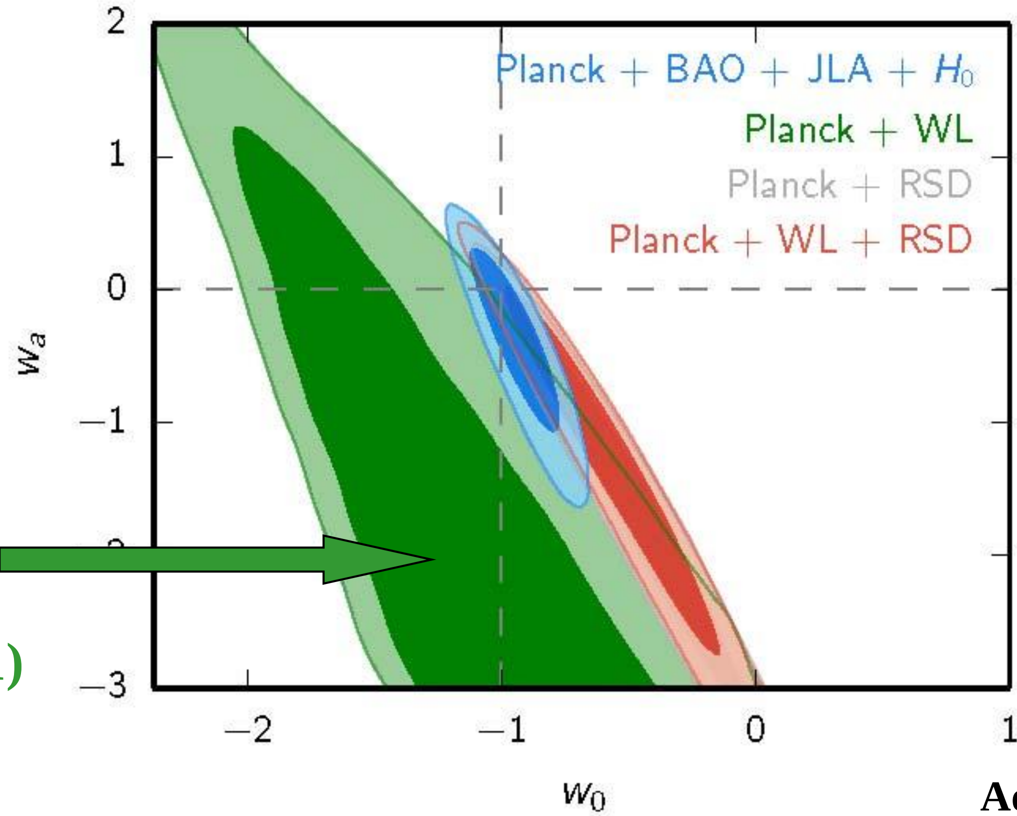
Sahni & Sen (2017), Mishra & Sahni (2018)

$$\text{Canonical scalar field Lagrangian}: \; \mathcal{L} = X - V(\phi), \; X = \frac{1}{2}\dot{\phi}^2$$

Some data sets are consistent with LCDM but others show tension with Λ

(Unknown systematics or evolving dark energy ?)

$$w(a) = w_0 + (1-a)w_a$$



$w_0 = -1, w_a = 0$

LCDM

Dynamical Dark energy
(Phantom: w< -1)

Ade et al. arXiv:1502.01590

Tension between Planck and weak lensing data is at over 2σ!

Joudaki et al. arXiv:1610.04606

Example 2. The Chaplygin gas which has $p = -A/\rho$ can be described by a minimally coupled scalar field with the potential

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh(2\sqrt{6\pi G}\phi) + \frac{1}{\cosh(2\sqrt{6\pi G}\phi)}\right) ,$$

and associated with the Lagrangian density $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.

[Kamenschik, Moschella and Pasquier, PLB, 2001, Gorini et al, PRD 2005]

However the Chaplygin gas can also be modeled completely differently using a scalar field with the Born-Infeld kinetic term:

$$\mathcal{L} = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} .$$

[Bilic et al, PLB 2002, Frolov et al, PLB 2002]

This once more illustrates the fact that the equation of state $w(z)$ does not uniquely define an underlying field-theoretic model !

Observational tests of Dark Energy usually rely on an accurate measurement of either the angular size distance or the luminosity distance:

$$\mathcal{F} = \frac{L}{4\pi D_L^2} \ , \quad D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \ .$$

One can then reconstruct the Hubble parameter through

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1} \ .$$

Differentiating a second time we can reconstruct the equation of state of DE

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x)-1}{3(1-\Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1-(H_0/H)^2\Omega_{0m}x^3} \ , \ x = 1+z \ .$$

BUT $w(z)$ will be a noisier quantity than $H(z)$ since two differentiations are needed for the reconstruction $D_L \to w(z)$ while a single suffices for $D_L \to H(z)$

Also note that $H(z)$ is independent of the value of Ω_{0m} while $w(z)$ is not !

Therefore uncertainties in Ω_{0m} affect the reconstruction of $w(z)$ much more significantly than the reconstruction of $H(z)$.

In practice observational quantities such as $D_L(z_i)$ are noisy and known only at discrete values of the redshift. Thus it is impossible to directly differentiate them. Therefore, to convert from $D_L(z_i)$ to $H(z)$ one requires some sort of smoothing procedure.

This is usually accomplished using either parametric or non-parametric methods.

Parametric reconstruction [A] Fitting functions to $D_L(z)$.

1. The simplest Taylor series: $\frac{D_L(z)}{1+z} = \sum_{i=1}^{N} a_i z^i$, does not work since to accurately determine $H(z), w(z)$ one must make N large which increases the errors of reconstruction [Huterrer and Turner, PRD 1999]. Better convergence is achieved by

$$D_L = \frac{c}{H_0}\left[y + Ay^2 + By^3 + \cdots\right] \ , \ y = \frac{z}{1+z} \ ,$$

[Cattoen & Visser CQG 2007,2008; Guimaraes & Lima, 2010]

2. A versatile 2 parameter ansatz is

$$\frac{H_0 D_L(z)}{1+z} = 2\left[\frac{x - A_1\sqrt{x} - 1 + A_1}{A_2 x + A_3\sqrt{x} + 2 - A_1 - A_2 - A_3}\right] \ , \quad x = 1+z \ ,$$

which exactly reproduces both CDM $(\Omega_m = 1)$ and the steady-state model $(\Omega_\Lambda = 1)$.

[Saini *et al* PRL 85, 1162, 2000]

B. Fitting function to the dark energy density:

$$\rho_{\rm DE} = A_1 + A_2 x + A_3 x^2 \ , \ \ x = 1 + z \ .$$

This leads to the following ansatz for H(z): [VS *et al* 2003, Barboza & Alcaniz, 2011]

$$H(x) = H_0 \left[\Omega_m x^3 + A_1 + A_2 x + A_3 x^2\right]^{1/2} \ .$$

C. Fitting functions to the equation of state. The simple Taylor expansion $w(z) = \sum_{i=1}^{N} w_i z^i$, with N=1 fares much better than the Taylor expansion for $D_L(z)$.

But $w(z) = w_0 + w_1 z$ is of limited utility since its only valid for $z \ll 1$.

A much more versatile ansatz is $w(a) = w_0 + w_1(1-a) = w_0 + w_1 \frac{z}{1+z}$,

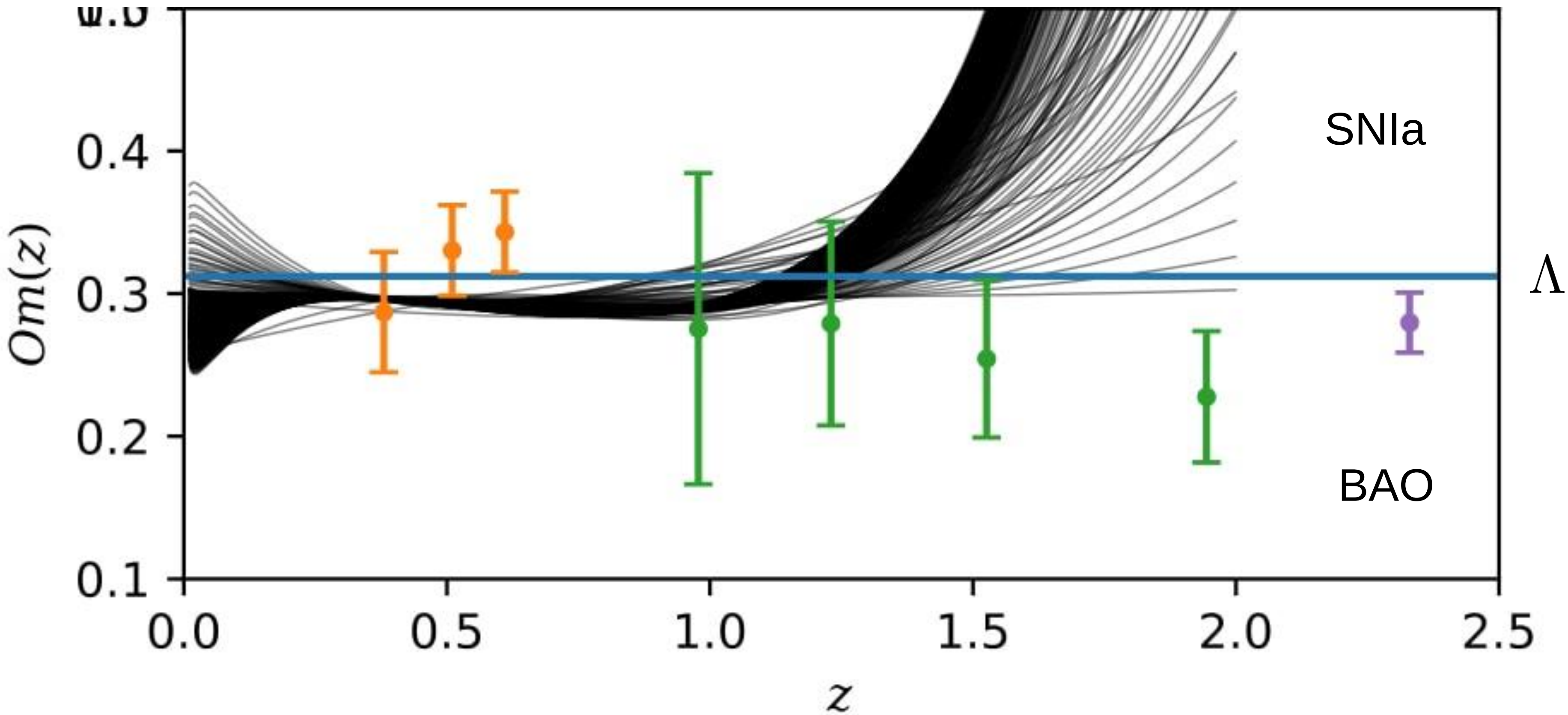
where the parameters $w_0, \ w_1$ are obtained after substituting into:

$$H^2(z) = H_0^2[\Omega_m(1+z)^3 + \Omega_{\rm DE}]^2 \ , \ \Omega_{\rm DE} = (1-\Omega_m) \exp\left\{3\int_0^{x-1} \frac{1+w(z,a_i)}{1+z} dz\right\} .$$

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \ .$$

[Chevalier & Polarski 2001; Linder 2003]

D. Fitting functions to the deceleration parameter q(z) have been discussed in:
Ishida, Reis, Toribio & Waga, Astropart. Phys., 2008 (and references therein).



Recent results show some tension with the cosmological constant: arXiv:1804.04320

Phantom dark energy (or extra dimensions) appears to be preferred by high redshift data !

$$w = P/\rho < -1 \quad !$$

Of all Dark Energy models the cosmological constant is single out by its elegance and simplicity:

$$T_i^k = \Lambda \delta_i^k \; .$$

So, as a first step, its logical to find tests which could falsify

$$\Lambda\mathrm{CDM} \; .$$

$$\Downarrow$$

NULL tests for the cosmological constant Λ .

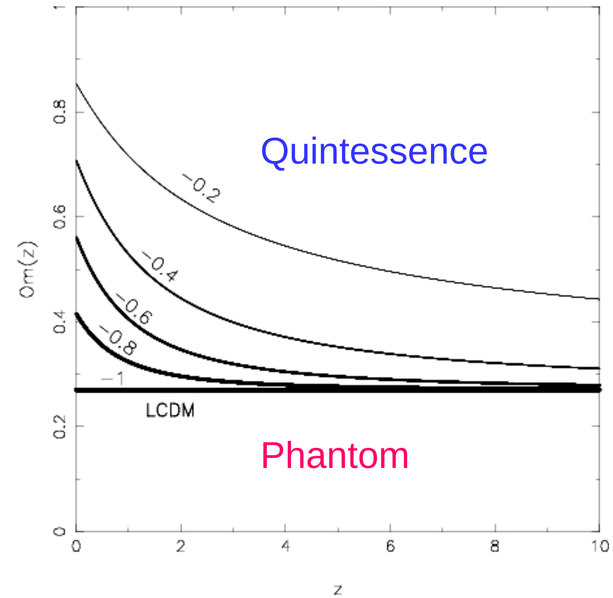
[VS, Saini, Starobinsky, Alam JETP 2003; VS, Shafieloo, Starobinsky PRD 2008, Zunckel & Clarkson PRL 2008, Arabsalmani & VS, PRD 2011, VS, Shafieloo, Starobinsky, ApJLett 2014, etc.]

The Om diagnostic – a null test for the Cosmological Constant.

$$Om(z) = \frac{\tilde{h}^2(z)-1}{(1+z)^3-1} \quad \text{or} \quad Om(z_1,z_2) = \frac{\tilde{h}^2(z_1)-\tilde{h}^2(z_2)}{(1+z_1)^3-(1+z_2)^3}$$

Om is constant only for the Cosmological Constant ! $\tilde{h} = H(z)/H_0, \; H = \dot{a}/a$

For all other Dark Energy models Om evolves with time.



$$Om(z) = \Omega_{0m} \text{ for } \Lambda \Rightarrow \text{LCDM}$$

$$\Rightarrow Omh^2 = \Omega_{0m}h^2 \text{ for } LCDM$$

$$\text{CMB determines } \Omega_{0m}h^2 \text{ very accurately :}$$

$$\Omega_{0m}h^2 = 0.1426 \pm 0.0025 \quad \text{(CMB)}$$

$$\Rightarrow Omh^2 = 0.1426 \pm 0.0025 \text{ for } LCDM$$

Null test for the cosmological constant. $h = H(z)/100\text{km/sec/Mpc}$

[VS, Shafieloo, Starobinsky, PRD 2008, ApJLett 2014]

Good news ! CMB determines $\Omega_{0m}h^2$ to great accuracy in LCDM cosmology:

$$\Omega_{0m}h^2 = 0.1426 \pm 0.0025$$

To test LCDM: [A] Determine Omh^2 from

$$Omh^2(z_i, z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}, \text{ where } h(z) = \frac{H(z)}{100\text{km/sec/Mpc}}$$

[B] check whether $Omh^2 = \Omega_{0m}h^2 = 0.1426 \pm 0.0025$

If [B] holds then dark energy = cosmological constant, if not surprise !

Independent measurements of H(z) are available at 3 redshifts: $z = 0, 0.57, 2.34$

$H(z=0) = 70.6 \pm 3.3,\ H(z=0.57) = 92.4 \pm 4.5,\ H(z=2.34) = 222 \pm 7$ km/sec/Mpc

[Efstathiou, 2014, Samshia et al, 2013 Delubac et al, 2014]
BOSS DR11

Leading to $Omh^2(z_1, z_2) = 0.124 \pm 0.045,\ Omh^2(z_1, z_3) = 0.122 \pm 0.010,$
$Omh^2(z_2, z_3) = 0.122 \pm 0.012$

Result: the model independent value $Omh^2 \simeq 0.122$ is stable and is in tension with the LCDM based value $Omh^2|_{\rm LCDM} \simeq 0.14$!

Tension with LCDM is at over 2σ !

[VS, Shafieloo, Starobinsky 2014]

So whats going on with dark energy ?

The value obtained by Delubac et al: $H(z = 2.34) = 222 \pm 7$ km/sec/Mpc

is much lower than the LCDM value $H(z = 2.34) = 238$ km/sec/Mpc

This can happen in models in which the cosmological constant is screened

$$H^2(z) = \underbrace{\frac{\Lambda}{3} - f(z)}_{\Lambda_{\rm eff}/3} + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

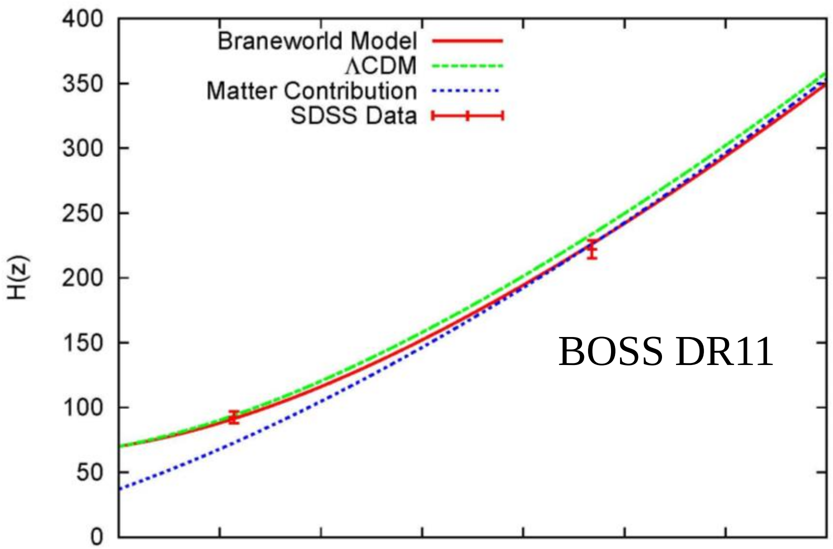
1. Λ relaxes from a large initial (bare) value through an adjustment mechanism

[Dolgov 1983, Brandenberger 2002, Bauer et al 2010]

2. Gauss-Bonnet gravity [Zhou et al 2009]

3. Braneworld models [VS & Shtanov 2003]

4. Modified gravity [Boisseau et al 2000]



The Dolgov mechansim for screening the cosmological constant:

Infrared instability of a massless scalar field coupling non-minimally to gravity.

$$3H^2 = \Lambda + 8\pi G\rho_\phi, \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + 3\xi H^2\phi^2 + \cdots$$

$$\Box\phi + \xi R\phi = 0 \;\Rightarrow\; \ddot{\phi} + 3H\dot{\phi} + 6\xi\left[\frac{\ddot{a}}{a} + H^2\right]\phi = 0$$

If $\xi < 0$ then $\phi(t)$ grows with time !

$$3H^2 \simeq \underbrace{\Lambda - 3|\xi|H^2\phi^2(t)}_{\Lambda_{\text{eff}}(t)\,\to\,0}$$

[Dolgov 1983]

The value of the cosmological constant decreases due to quenching by $\phi(t)$.

- The cosmological constant is dynamically screened.

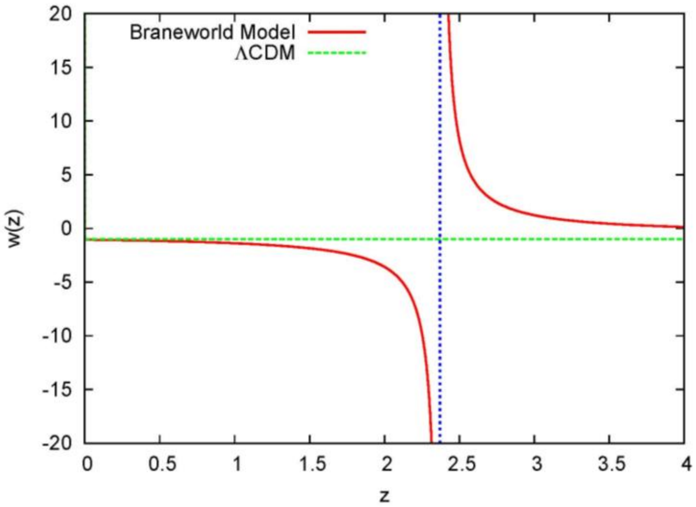
But, the mechanism does not work in practice….

• The cosmological constant can also be screened in braneworld models.

$$H^2(z) = \underbrace{\frac{\Lambda}{3} - f(z)}_{\Lambda_{\rm eff}/3} + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

Since $w \equiv p_{DE}/\rho_{DE} = \dfrac{2q(x)-1}{3(1-\Omega_m(x))} \equiv \dfrac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1 - (H_0/H)^2\Omega_{0m}x^3}$, $x = 1 + z$.

$w(z)$ will have a pole at which $w(z_p) \to \infty$!



Pole occurs when $\Lambda_{\rm eff} = 0$.

Smoking gun test of such models.

• Another possibility – interaction between dark matter and dark energy.

*Cosmological reconstruction. Step 1: Determine **expansion history** directly from observations :*

$$\mathcal{F} = \frac{L}{4\pi D_L^2}\ , \quad D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \longrightarrow H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1} .$$

Alternatively, H(z) can also be determined from radial BAO's, ages of passively evolving galaxies, the redshift drift, etc.

$$h(z) = H(z)/H_0$$

One can now construct a null test for the cosmological constant using

$$Om(z) = \frac{h^2(z) - 1}{(1+z)^3 - 1}$$

Om is constant only for the Cosmological Constant !

$$\Rightarrow Om(z) = \Omega_{0m}$$

For all other dark energy models Om(z) varies with redshift (time).

[VS, Shafieloo & Starobinsky, PRD 2008]

Step 2. Differentiate H(z) to get the equation of state:

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x) - 1}{3(1 - \Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1 - (H_0/H)^2\Omega_{0m}x^3}\ , \ x = 1+z\ .$$

$$w = -1 \Rightarrow \ \text{cosmological constant}$$

Q. Does the expansion history, H(z), uniquely determine a Dark energy model ?

Ans. No. Different dark energy models can have Identical expansion histories, and therefore identical equations of state !

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x) - 1}{3(1 - \Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1 - (H_0/H)^2\Omega_{0m}x^3} \ , \ x = 1 + z \ .$$

COSMIC DEGENERACY !

Unfortunately cosmological reconstruction is not unique since, given an expansion history H(z), one can reconstruct a corresponding potential $V(\phi)$, for dark energy by inverting the Einstein eqns:

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \quad , \quad \dot{H} = -4\pi G(\rho_m + \dot{\phi}^2)$$

which can be rewritten as

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_{0\mathrm{m}}\, x^3,$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_{0\mathrm{m}}x}{H^2}, \quad x \equiv 1+z\ .$$

Here $V(\phi)$ *corresponds to the Lagrangian density* $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

BUT *H(z) could arise from the Chaplygin gas with* $\mathcal{L} = -V_0\sqrt{1-\phi_{,\mu}\phi^{,\mu}}$

or even the DGP braneworld $H = \sqrt{\frac{8\pi G\rho_m}{3} + \frac{1}{r_c^2}} + \frac{1}{r_c}$ $\quad r_c = m^2/M^3$

with its 5 D action: $S = M^3\int_{\mathrm{bulk}}\mathcal{R} + m^2\int_{\mathrm{brane}}R + \int_{\mathrm{brane}}\mathcal{L}_{\mathrm{matter}}$ ***!***

So three completely different classes of models can have the same expansion history : H(z) . COSMIC DEGENERACY !!

Structure formation is a key test for modified gravity

The perturbed FRW metric, in the longitudinal (quasi-Newtonian) gauge is

$$ds^2 = -(1+2\phi)dt^2 + (1-2\psi)a^2(t)d\vec{x}^2 \ ,$$

where $\phi = \psi$ only in GR (provided matter is free of anisotropic stress).

In GR, on sub-horizon scales, the linearized matter density contrast $\delta_m = \dfrac{\delta\rho}{\bar{\rho}}$,

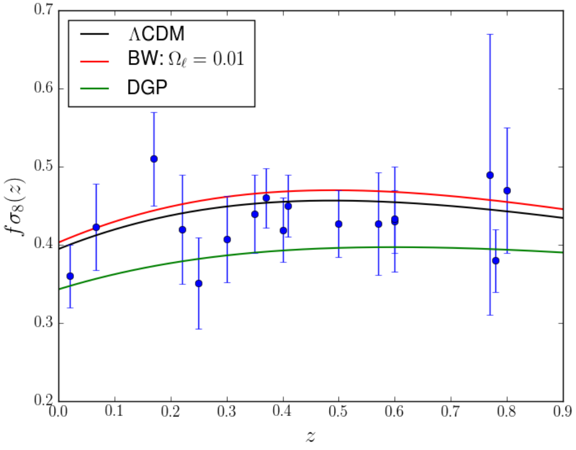
satisfies the equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \qquad \textit{(1)}$$

But (1) is only valid in GR. In modified gravity models the perturbation eqn is more complex since $\phi \neq \psi$!

$$f(z) = \frac{d\ln\delta}{d\ln a} \simeq \Omega_m^\gamma(z) \quad \begin{cases} \gamma \simeq 0.55 \ \ \text{in } \Lambda CDM \\ \text{but } \gamma \simeq 0.67 \ \ \text{in } DGP \ ! \end{cases}$$

Weak lensing information also crucial: DES, LSST !



The growth of gravitational instability can help distinguish between alternative causes for cosmic acceleration.

The perturbed FRW metric, in the longitudinal (quasi-Newtonian) gauge is

$$ds^2 = -(1+2\phi)dt^2 + (1-2\psi)a^2(t)d\vec{x}^2 \ ,$$

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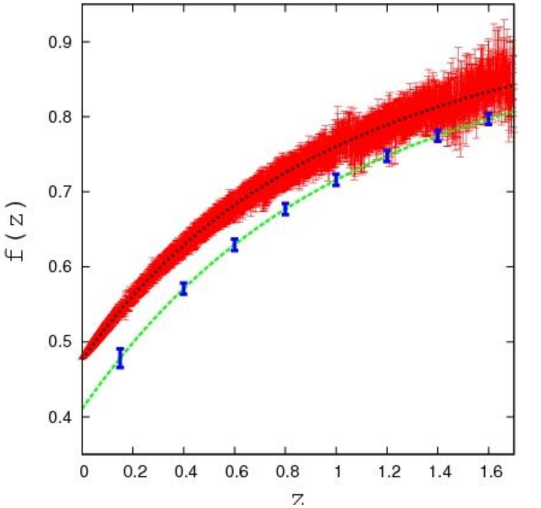
But (1) is valid only in GR. In modified gravity models the perturbation eqn can be more complex since $\phi \neq \psi$!

$$f(z) = \frac{d\ln\delta}{d\ln a} \simeq \Omega_m^\gamma(z) \quad \gamma \simeq 0.55 \ \text{ in } \Lambda CDM$$

$$\text{but } \gamma \simeq 0.67 \ \text{ in } DGP \ !$$



Black dotted line shows perturbation growth determined using DGP expansion history => (1) and JDEM data.
Green line shows correct DGP growth
and expected observational constraints from Euclid expt.
[Alam, VS, Starobinsky, ApJ 704, 1086, 2009.]

Weak lensing information also crucial: DES, LSST !

7 Questions for Dark Energy

1. Is Dark Energy a Cosmological Constant or is it dynamically evolving ?

(Was DE small originally or did it become small through evolution.)

2. Is DE a classical field or a quantum entity ?
(The cosmological constant problem)

3. Does the acceleration of the Universe arise because of amendments to the matter sector or to the gravity sector of general relativity ?

4. Is late-time acceleration (dark energy) related to early-time acceleration (Inflation) ?

5. Do dark matter and dark energy interact ?

6. Is the fact that $\rho_{\rm DE} \simeq 2\rho_{\rm DM}$ *merely a cosmic coincidence ?*

7. Is the Universe homogeneous and isotropic on very large scales ?

(What is the effect of inhomogeneity on dark energy ?)

Matter is moved from its initial location $(\mathbf{q})$ to its final position $(\mathbf{x})$ by means of the Zeldovich transformation

$$\mathbf{r} = \mathbf{q} + D(t)\mathbf{v}(\mathbf{q})$$

Where D(t) is the density contrast predicted by linear theory:

$D \propto t^{2/3}$ if the universe is flat and matter dominated.

$\mathbf{v}(\mathbf{q})$ is the initial velocity field of perturbations. If the particle flow is irrotational then, under some assumptions, one can relate the velocity field to the linearized gravitational potential

$$\mathbf{v}(\mathbf{q}) = -\nabla\phi$$

Introducing a new time coordinate: $T = D(t)$ we get

$$\mathbf{r} = \mathbf{q} + \mathbf{v}(\mathbf{q})T$$

The Zeldovich approximation is therefore equivalent to the simple inertial motion of particles !

The growth of gravitational instability can distinguish between alternative causes for cosmic acceleration such as:
(i) modified gravity theories (f(R) gravity, braneworld models, etc.)
and (ii) dark energy including: cosmological constant, quintessence, etc.)

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0$$

This equation is only valid in DE models based on General Relativity !
In modified gravity models the perturbation equations may be more complex.

The perturbed FRW metric, in the longitudinal (quasi-Newtonian) gauge is

$$ds^2 = -(1+2\phi)dt^2 + (1-2\psi)a^2(t)d\vec{x}^2 \ ,$$

where $\phi = \psi$ in GR (provided matter is free of anisotropic stress).

In GR the Newtonian potential ϕ and the matter density contrast $\delta_m = \frac{\delta\rho}{\bar{\rho}}$, are related via the linearised Poisson equation: $\triangle\phi = 4\pi G a^2 \rho_m \delta_m$

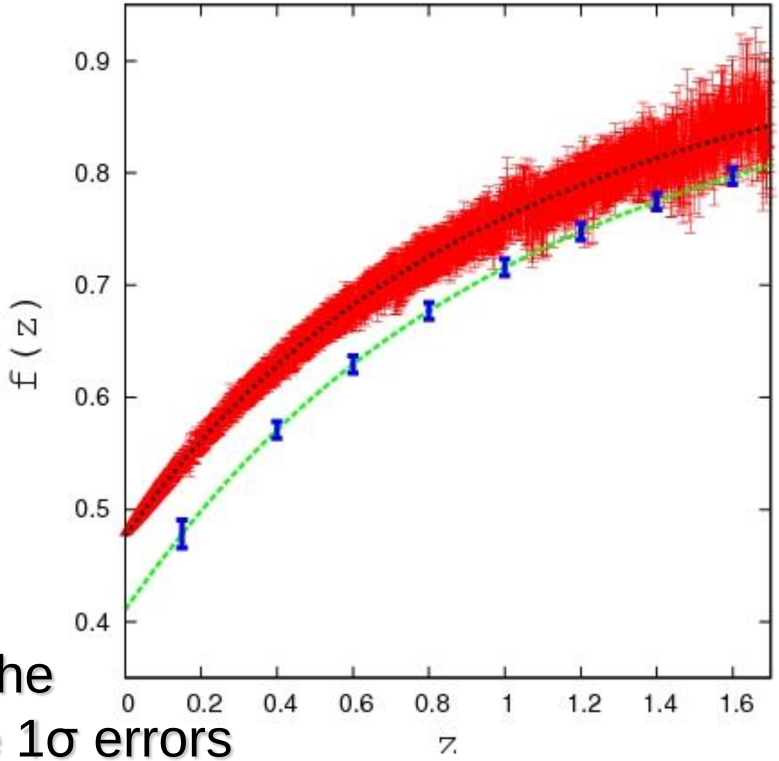
and the density contrast satisfies the equation $\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0$

The expansion history for the DGP brane is

$$H(z) = H_0 \left[\left(\frac{1-\Omega_{0\mathrm{m}}}{2} \right) + \sqrt{\Omega_{0\mathrm{m}}(1+z)^3 + \left(\frac{1-\Omega_{0\mathrm{m}}}{2} \right)^2} \right]$$

which, when substituted in

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G \rho_m \delta_m = 0$$



gives an incorrect reconstruction, shown by the black dotted line: the red solid lines show the 1σ errors for the integral reconstruction:

$$\begin{aligned} \delta(E) &= 1 + \delta_0' \int_0^E [1 + z(E_1)] dE_1 + \cdots \\ \delta'(E) &= \delta_0' [1 + z(E)] + \cdots \end{aligned}$$

The correct results from gravitational instability are shown by the green dashed line, where the blue vertical lines show the expected observational constraints from Euclid [Alam, Sahni & Starobinsky, ApJ (2009), Cimatti et al. (2008)]

$$f(z) = \frac{d\ln\delta}{d\ln a} \simeq \Omega_m^\gamma(z) \quad , \qquad \gamma \simeq 0.68 \ .$$

Cosmic Degeneracy: Different dark energy models may have the same expansion rate $H(z)$!

So I can take the Braneworld expression $H = \sqrt{\frac{8\pi G\rho_{\rm m}}{3} + \frac{1}{\ell^2}} + \frac{1}{\ell}$, and construct a scalar field potential $V(\phi)$ which will match it ! The Einstein equations

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \quad , \quad \dot{H} = -4\pi G(\rho_m + \dot{\phi}^2)$$

can be rewritten as

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_{0\rm m}\, x^3,$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_{0\rm m}x}{H^2}, \quad x \equiv 1 + z \ .$$

Integrating, we determine $\phi(z)$. Inverting $\phi(z) \rightarrow z(\phi)$ and substituting into $V(x)$ allows us to reconstruct $V(\phi)$ from $H(z)$.

Fluctuations in the potential are related to fluctuations in the density of matter through the Poisson equation:

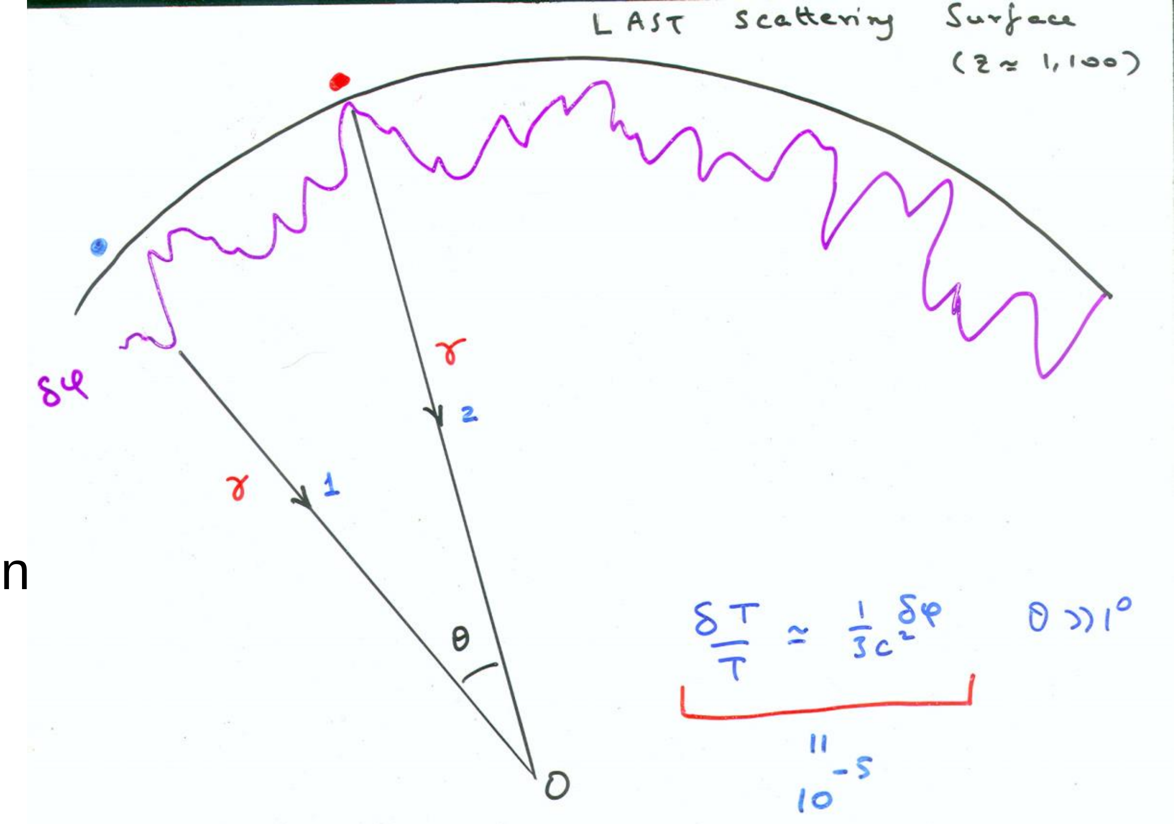
$$\nabla^2\phi = \frac{4\pi G}{a}\delta\rho/\rho$$

But during most of its expansion history the density perturbation simply evolves as (Lifshitz):

$$\frac{\delta\rho}{\rho} \propto a(t)$$

Consequently the potential remains frozen in time to its initial value $\phi \sim 10^{-5}$.

By contrast $\delta\rho/\rho$ grows very rapidly !



Photons travelling from "2" will be redshifted relative to those from "1" since they are surmounting a deeper gravitational potential (at last scattering).

⇒ leads to CMB fluctuations $\frac{\Delta T}{T}$.

- A famous example of dark energy is the cosmological constant Λ

Introduced by Einstein in 1917. The cosmological constant has the Lorentz invariant equation of state $P = -\rho = -\Lambda/8\pi G$. Consequently

$$T_i^k \propto \Lambda \delta_i^k \ , \text{and} \ \ \rho + 3P = -\frac{\Lambda}{4\pi G} < 0 \ .$$

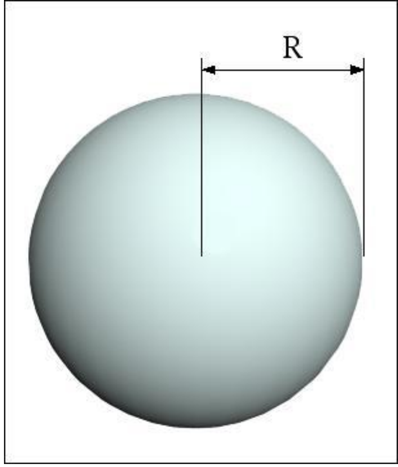
Λ has the interesting property that its energy density stays the same as the universe expands.

By contrast, the density of all \`normal' forms of matter and radiation get diluted during the Universe's expansion.

So if the universe contains the cosmological constant in addition to normal matter (visible + dark)
then the density in the former will eventually dominate the latter !!

This will cause the universe to accelerate at late times – as observed.

Consider a sphere of radius R(t) embedded in an expanding universe. The sphere encloses matter having density ρ and pressure $P = w\rho$ (w = 1/3 for radiation, w = 0 for pressure-less matter and w = -1 for the Λ-term).



$E=\rho V$ is the total Energy within the expanding sphere.

From thermodynamical considerations $dE = -PdV$ which implies

$$d\rho = -(\rho + P)\frac{dV}{V} = -(1+w)\rho\frac{3dR}{R}$$

Integrating we get $\rho(t) \propto R(t)^{-3(1+w)}$

As the universe expands, the density of radiation falls as $\rho_r \propto R^{-4}$, the density of matter falls as $\rho_m \propto R^{-3}$, while for the cosmological constant

$$\rho_\Lambda = constant \, .$$

The cosmological constant will eventually dominate the density of the universe even if its value was much smaller than the matter density at early times !

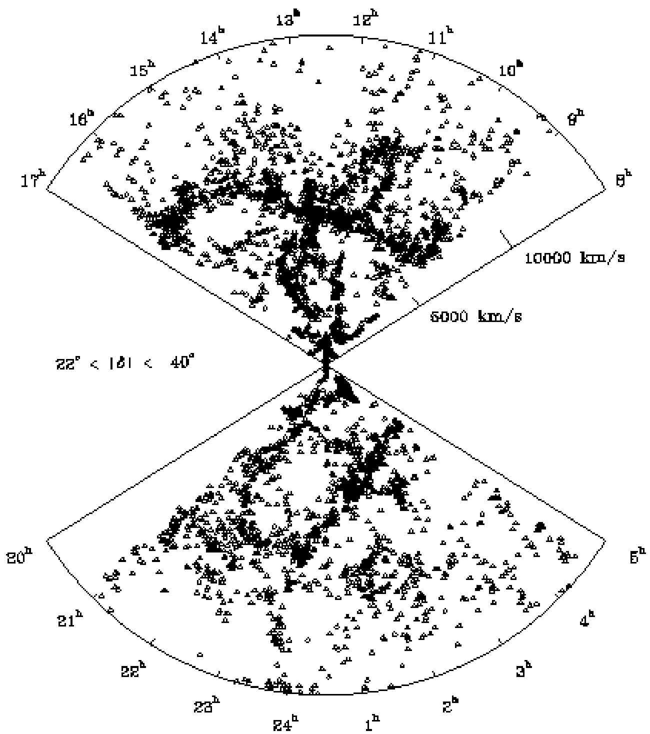
- Rotation curves to over 1000 galaxies have been measured.

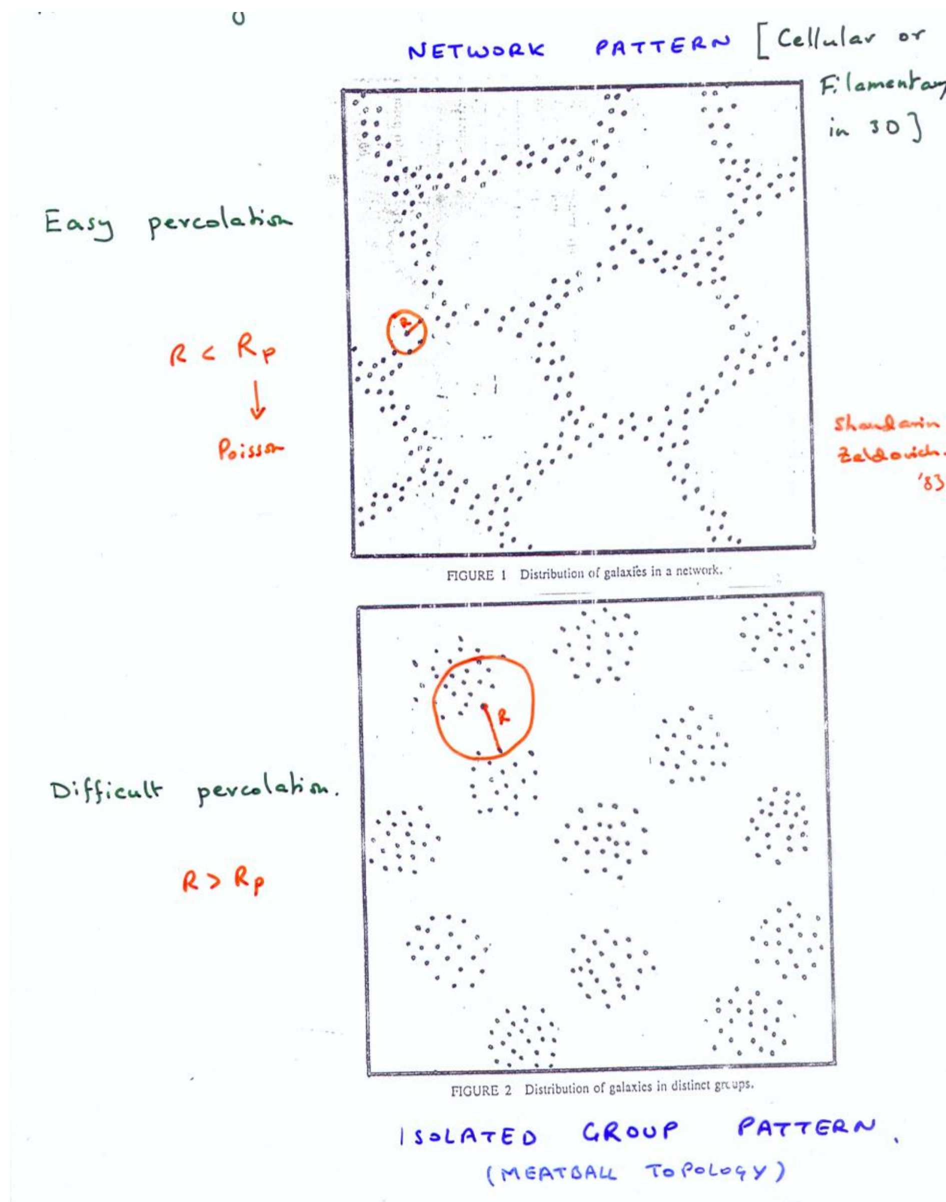
- The ratio of Dark Matter to visible matter is the most in dwarf galaxies. For instance the galaxy M33 has at least 50 times more dark matter than luminous matter !

- So all galaxies appear to be embedded in a halo of dark (non-baryonic) matter.

The morphology of the Cosmic Web can be quantified using percolation analysis in conjunction with the Minkowski functionals.

Percolation analysis reveals that galaxies appear to follow a network pattern than an isolated group pattern.

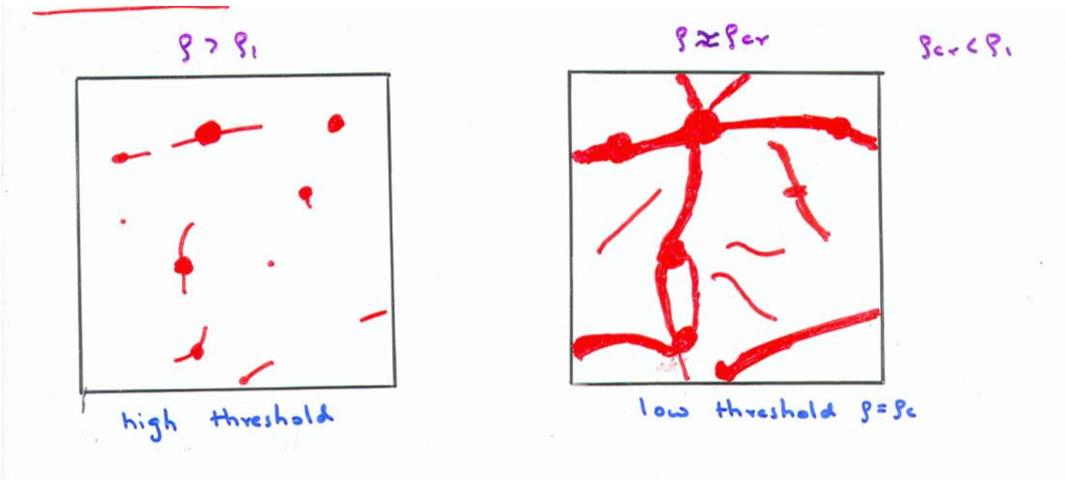




Ansatz: Lower the density threshold starting from some high value.
At a critical value $\rho = \rho_c$ over-dense regions will begin to percolate.

Filling fraction (FF): Fraction of volume in all over-dense regions.

$FF_c \Rightarrow$ Filling fraction at percolation.



Gaussian random fields percolate at $FF_c \simeq 0.16$.

A density field evolving under gravitational instability (such as $\Lambda\mathrm{CDM}$) percolates at a much lower value of the filling factor: $FF_c \simeq 0.05$.

Lower values of the filling factor favour a filamentary/sheet-like distribution since filaments occupy a smaller volume and percolate much more easily.

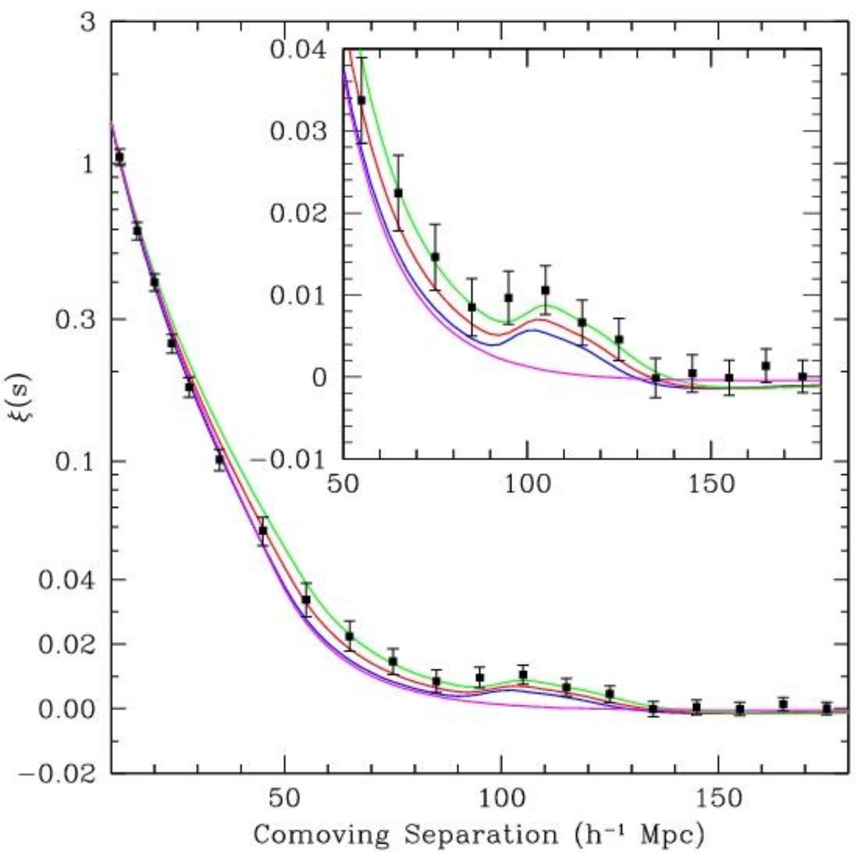
Klypin and Shandarin, ApJ 413, 48, 1993

A. Baryon Acoustic Oscillations (BAO)

A remarkable confirmation of the standard big bang cosmology has been the recent detection of a peak in the correlation function of luminous red galaxies in the Sloan Digital Sky Survey (SDSS). This peak, which is predicted to arise precisely at the measured scale of $100h^{-1}$ Mpc due to acoustic oscillations in the photon-baryon plasma prior to recombination, can provide a standard ruler with which to test dark energy models [Eisenstein et al. (2005)].

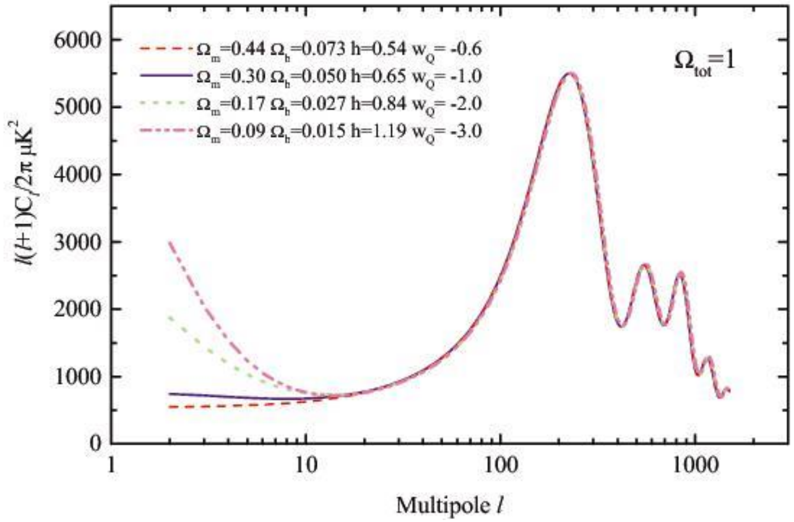
$$A = \frac{\sqrt{\Omega_{0\mathrm{m}}}}{h(z_1)^{1/3}}\left[\frac{1}{z_1}\int_0^{z_1}\frac{dz}{h(z)}\right]^{2/3}$$
$$= 0.469\left(\frac{n}{0.98}\right)^{-0.35} \pm 0.017\,,$$

$z_1 = 0.35$ is the redshift at which the acoustic scale has been measured.



But CMB does not determine the cosmological parameters $\Omega_m, \Omega_\Lambda, w_{\rm DE}$ independently !

Therefore different cosmological models can give almost identical CMB fluctuations !

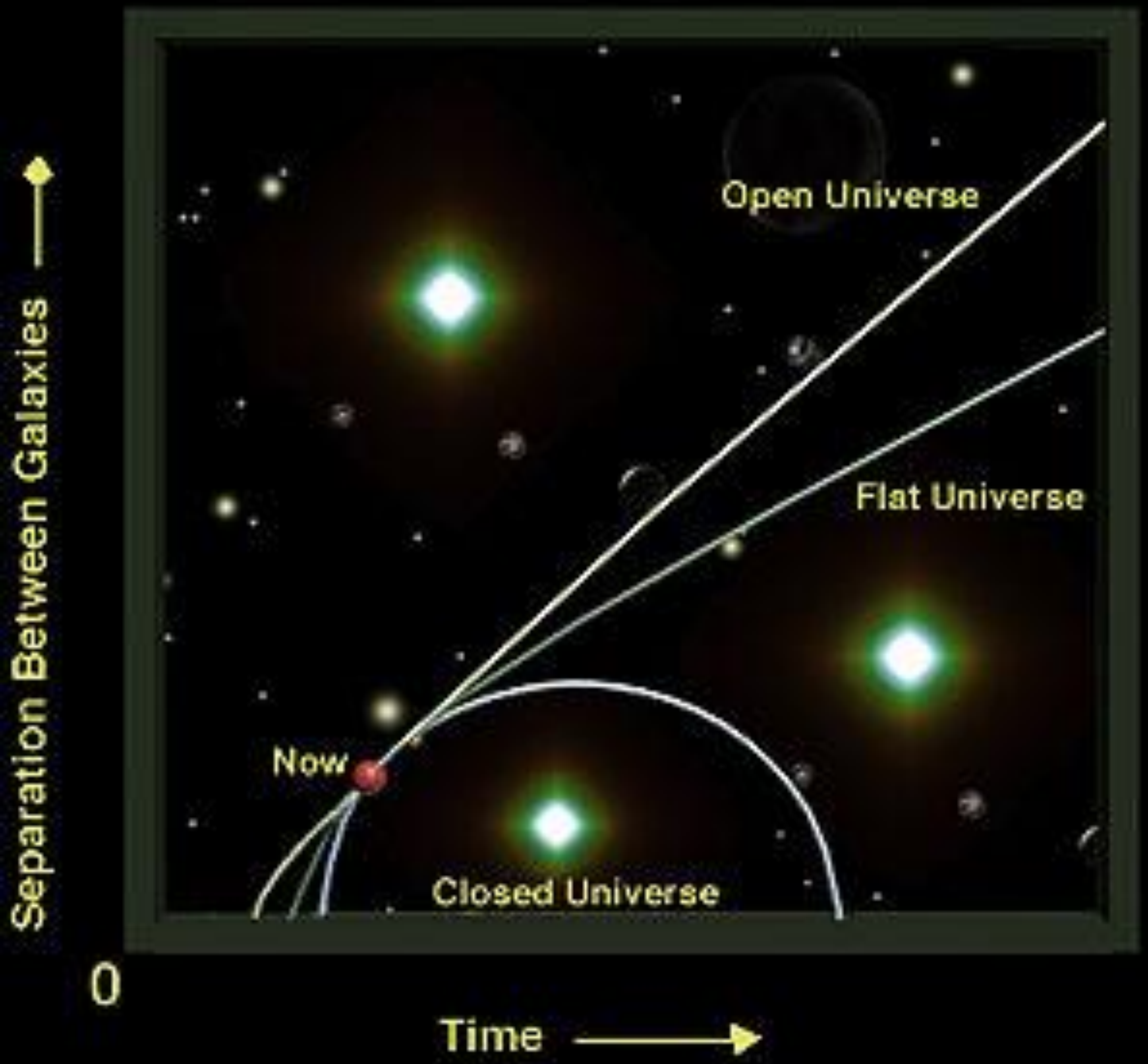


[From: Melchiorri, Mersini, Odman and Trodden, astro-ph/0211522]

For instance a change in CMB spectra from $w_{\rm DE}$ can easily be compensated by a change in the curvature. Even for flat models, the same CMB spectrum arises by decreasing $w_{\rm DE}$ AND decreasing Ω_m. (Since $\Omega_m h^2$ must beheld constant one should simultaneously increase h.)

To break this degeneracy one must combine CMB information with independent information about the values of Ω_m and h in order to determine $w_{\rm DE}$.

[See also: Bond, Efstathiou and Tegmark, astro-ph/9702100]



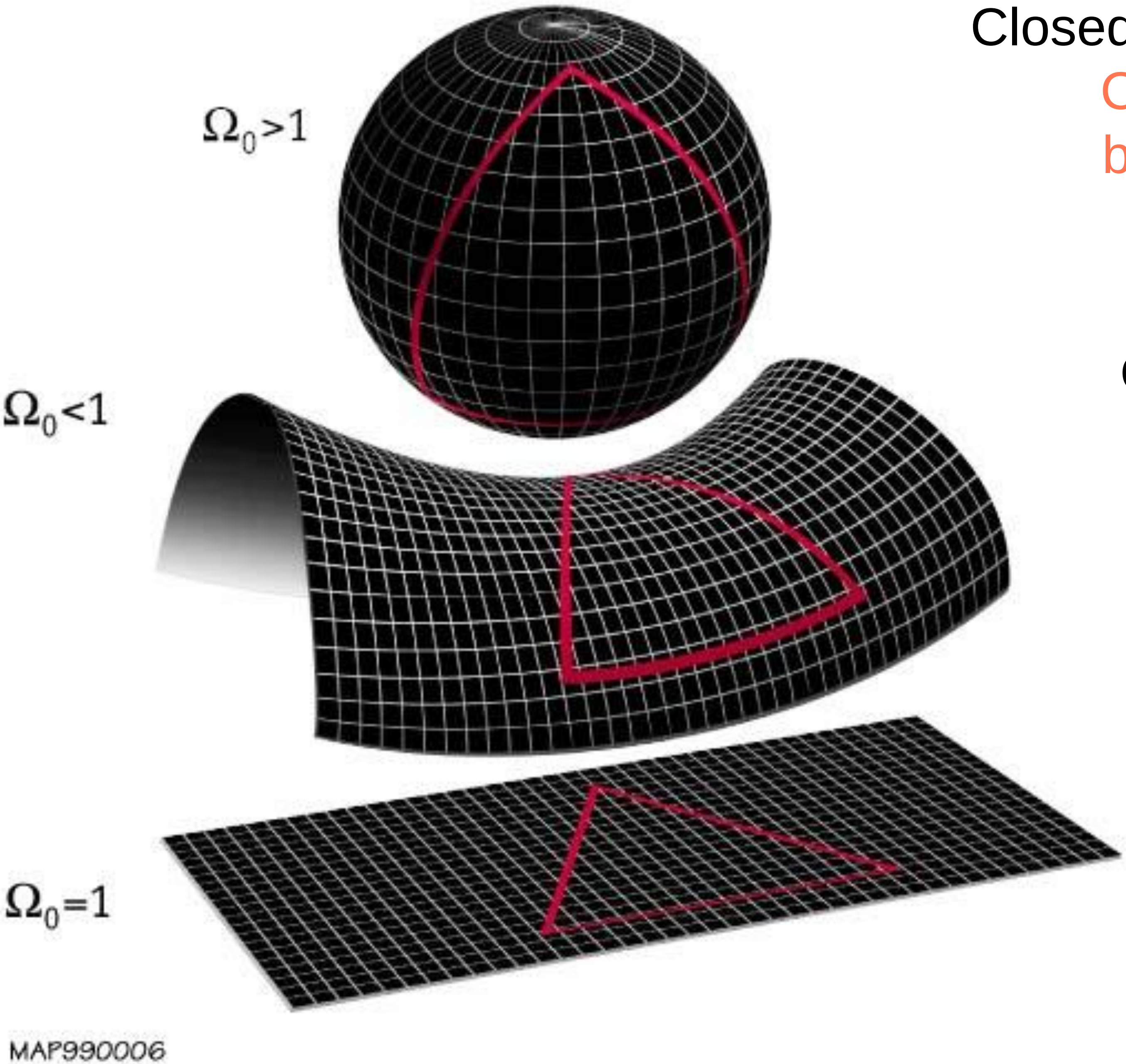
KE > PE

KE = PE

Expands forever

KE < PE

Ends in Big Crunch



Closed finite universe
Collapses to
big crunch
KE < PE

Open universe
(infinite)
KE > PE
Expands forever

Flat universe
(infinite)
KE = PE

Matter is moved from its initial location $(\mathbf{q})$ to its final position $(\mathbf{x})$ by means of the Zeldovich transformation

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Where D(t) is the density contrast predicted by linear theory:
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Introducing a new time coordinate: $T = D(t)$ we get

$$\mathbf{r} = \mathbf{q} + \mathbf{v}(\mathbf{q})T$$

The Zeldovich approximation is therefore equivalent to the simple inertial motion of particles !

An essential feature of inertial motion from random initial conditions is that nearby particle trajectories intersect leading to the formation of singularities (caustics), where the density field becomes very large.

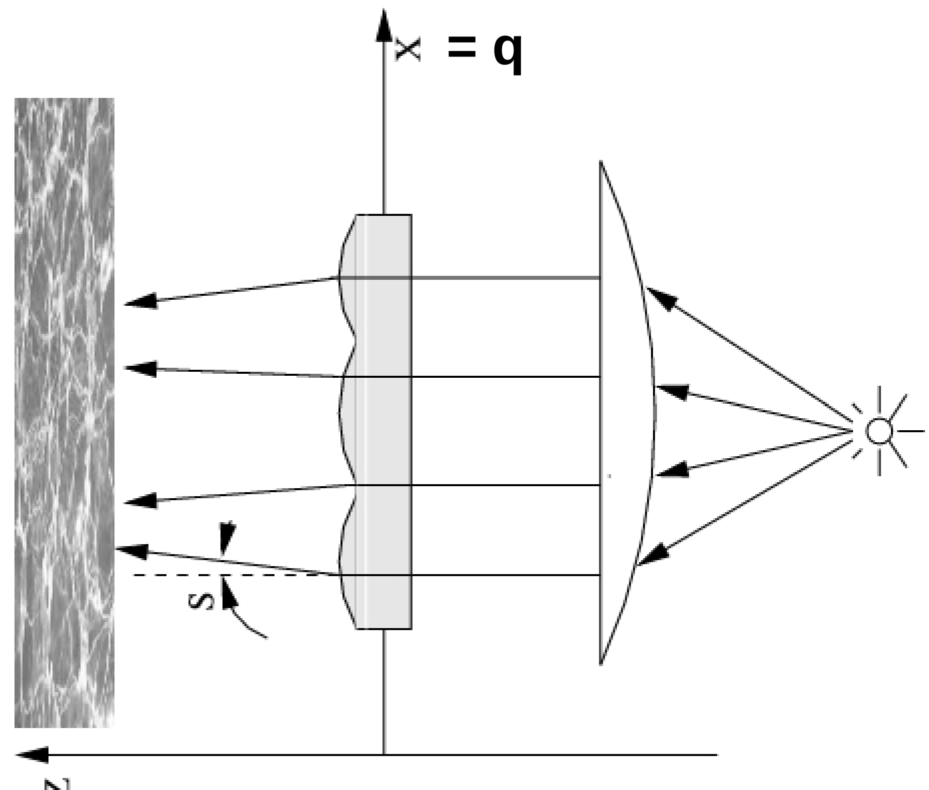
A similar effect is seen in the propagation of light as it passes through a plate of glass or water. After passing through glass/water neighboring light trajectories intersect to form caustics where the intensity of light is exceedingly bright !

The Zeldovich approximation which describes how particles move under the influence of a spatially random gravitational field is very similar to the propagation of light rays in geometrical optics.

A light ray which enters a glass plate of thickness h at $\mathbf{q}$ will emerge at

$$R(z, \mathbf{q}) = \mathbf{q} + \mathbf{s}z$$

where z is the distance to the screen and $s_i = -(n-1)\dfrac{\partial h(\mathbf{q})}{\partial q_i}$ is the angle of deflection; n is the index of refraction.



The 'WHY NOW' ? Conundrum

Cosmic Coincidence: We live during an epoch when the density of dark matter and dark energy are comparable.

- Big Bang nucleosynthesis prevents Λ from being large at $z \sim 10^9$.
- CMB observations prevent Λ from being large at $z \sim 10^3$.
- Λ must be subdominant at $z > 1$ otherwise galaxy formation will be suppressed.

In the past $(a \ll a_0)$ $\Omega_m \to 1$, $\Omega_\Lambda \to 0$.

In the future $(a \gg a_0)$ $\Omega_m \to 0$, $\Omega_\Lambda \to 1$.

$$\Omega_m \sim \Omega_\Lambda \sim O(1) \text{ today !}$$

What makes the present epoch so special ?

Tracker potentials satisfy $V''V/V'^2 \geq 1$ and approach a common evolutionary path from a wide range of initial conditions.

$$V' = \frac{dV}{d\phi}$$

The potential $V(\phi) \propto \phi^{-p},\ p > 0$ is an example of a tracker.

$$\text{During tracking } \frac{\rho_\phi}{\rho_B} \propto t^{\frac{4}{2+p}} \Rightarrow \frac{\rho_\phi}{\rho_B} \text{ grows with time!}$$

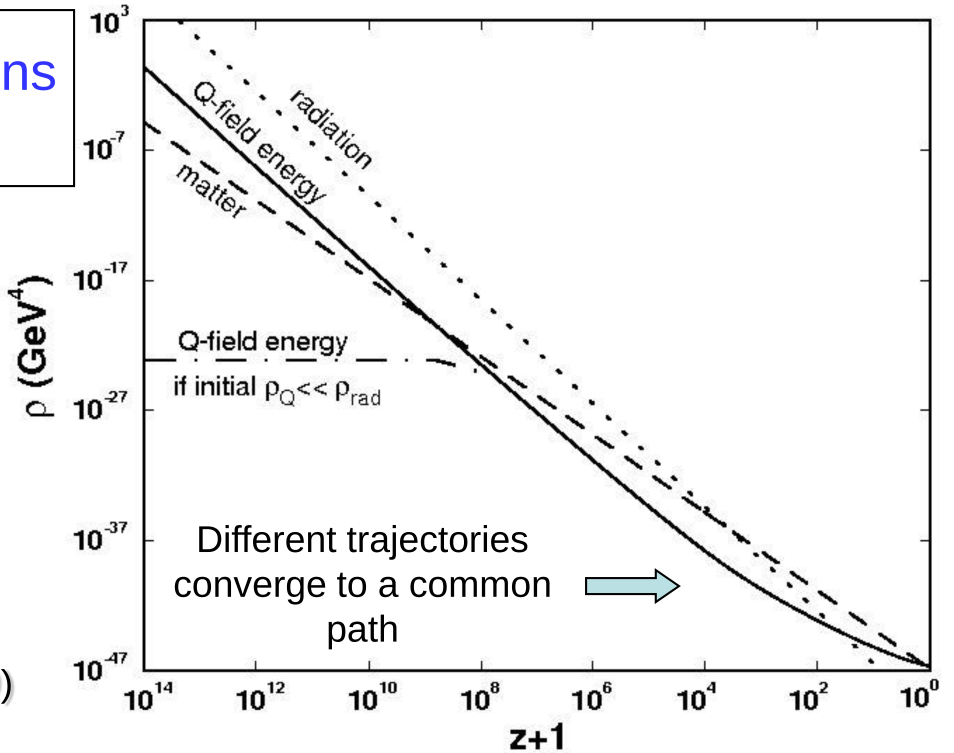
The scalar field density can dominate the matter/radiation density at late times even if it was initially subdominant !

Alleviates the problem of initial conditions for large values of p

The equation of state of the tracker is

$$w_\phi = \frac{p w_B - 2}{p + 2}$$

$$\Rightarrow \ w_\phi \simeq w_B \text{ as } p \longrightarrow \infty$$

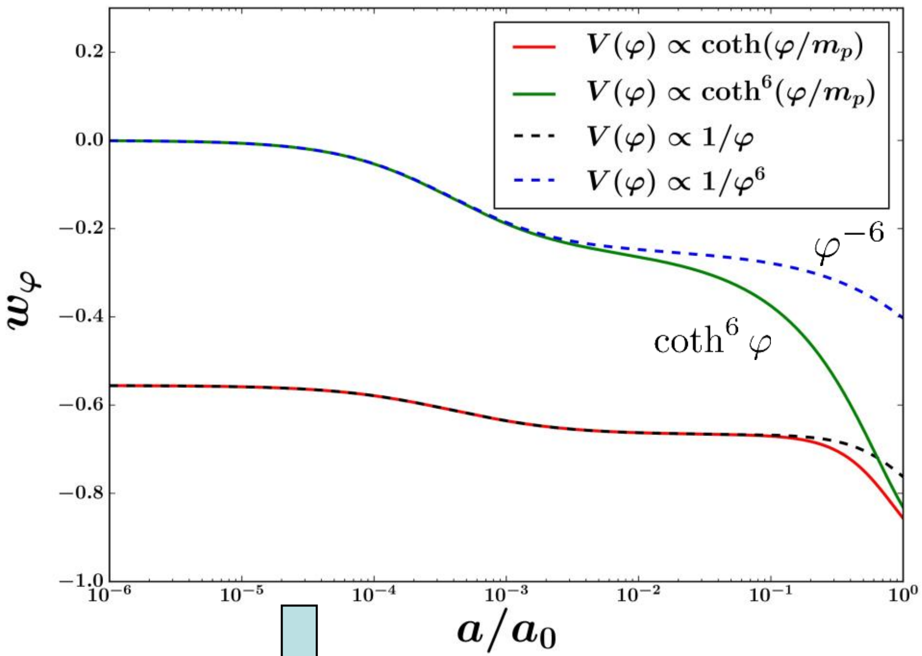


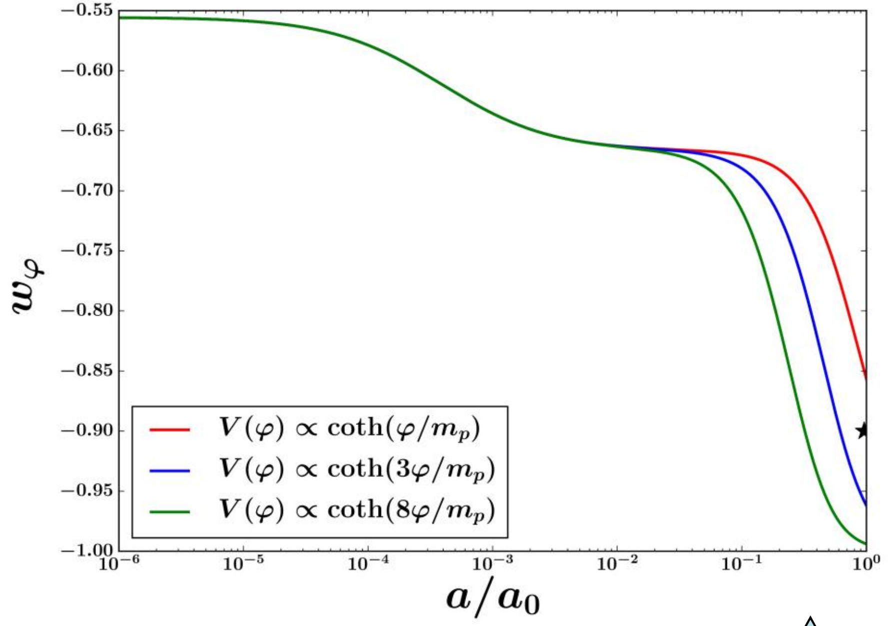
Ratra & Peebles (1988), Zlatev, Wang & Steinhardt (1999)

$$V(\varphi) = V_0 \coth^p(\lambda\varphi)$$

For $\lambda\varphi \ll 1$, $V \propto \varphi^{-p} \rightarrow$ IPL tracker **at early times**.

For $\lambda\varphi \gg 1$, $V \simeq V_0 \Rightarrow \Lambda$CDM asymptote at late times





At late times w_φ in $\coth^p(\lambda\varphi)$ **falls below** the EOS in φ^{-p} .

Increasing λ in $\coth(\lambda\varphi)$ makes w_φ drop to even more negative values.

The coth potential can lead to $w_0 \sim -1$ from a **larger** initial basin of attraction than φ^{-p}

The same expansion history, $H(z)$, may result from two very different dark energy models !

Example 1. DE with a constant equation of state $-1 < w < 0$ is described by the potential:

$$V(\phi) = \frac{3H_0^2(1-w)(1-\Omega_{m0})^{1/|w|}}{16\pi G \Omega_{m0}^{\alpha}} \sinh^{-2\alpha}\left(|w|\sqrt{\frac{6\pi G}{1+w}}(\phi - \phi_0 + \phi_1)\right) ,$$

where

$$\alpha = \frac{1+w}{|w|}, \quad \phi_0 = \phi(t_0), \quad \phi_1 = \sqrt{\frac{1+w}{6\pi G}} \frac{1}{|w|} \ln \frac{1+\sqrt{1-\Omega_{m0}}}{\sqrt{\Omega_{m0}}} .$$

• Dark energy with a constant EOS can also arise from a network of cosmic strings $(w = -1/3)$ or domain walls $(w = -2/3)$.

Degeneracy between the two models can be broken by gravitational clustering.

[Sahni & Starobinsky, IJMPD 15 (2006) 2105]

Example 2. The Chaplygin gas which has $p = -A/\rho$ can be described by a minimally coupled scalar field with the potential

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh(2\sqrt{6\pi G}\phi) + \frac{1}{\cosh(2\sqrt{6\pi G}\phi)}\right) ,$$

associated with the canonical Lagrangian density $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.

[Kamenschik, Moschella and Pasquier, PLB, 2001, Gorini et al, PRD 2005]

• However the Chaplygin gas can also be modeled completely differently using a scalar field with the Born-Infeld kinetic term:

$$\mathcal{L} = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} \ .$$

[Bilic et al, PLB 2002, Frolov et al, PLB 2002]

• This illustrates the fact that the equation of state $w(z)$ does not uniquely define an underlying field-theoretic model !

Q-field Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra & Peebles, Wetterich (1988), Ferreira & Joyce (1998)
$m^2\phi^2, \lambda\phi^4$	Frieman et al (1995)
$V_0/\phi^\alpha, \alpha > 0$	Ratra & Peebles (1988)
$V_0 \exp(\lambda\phi^2)/\phi^\alpha, \alpha > 0$	Brax & Martin (1999,2000)
$V_0(\cosh\lambda\phi - 1)^p$,	Sahni & Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$,	Sahni & Starobinsky (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland & Nunes (2000)
$V_0(\exp M_p/\phi - 1)$,	Zlatev, Wang & Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$,	Albrecht & Skordis (2000)

Pseudo-Goldstone boson models – [Frieman et al (1995)]; Supersymmetric gauge theories – [Binetruy (1999), Masiero et al. (1999)]; Supergravity – [Brax & Martin (1999), Albrecht& Skordis (2000), Copeland et al. (2000)]; Extra dimensions – [Bento & Bertolami (1999), Banks et al. (1999), Benakli (1999)]; Vacuum polarization – [Sahni & Habib (1998), Parker & Raval (1999)]; Topological defects – [Spergel & Pen (1997), Bucher & Spergel (1999)]; k-essence – [Armendariz-Picon et al. (2000)]; Scalar-tensor theories – [Perrotta et al. (2000), Boisseau et al (2001) etc]; Non-minimally coupled fields – [Amendola (1999), Chiba (1999), Holden and Wands (2000) etc]; Phantom models [Caldwell (1999)]; Braneworld models – [Deffayet, Dvali & Gabadadze (2001), Sahni & Shtanov (2002)]; Chaplygin gas – [Kamenshchik et al. (2001)]; Cardassian cosmology – [Freese & Lewis (2002)]; Quintessential Inflation – [Vilenkin and Peebles (1999)]; etc.

Some questions for quintessence:

1) Do Q parameters have 'realistic' values ?

a) $\phi/M_{\rm Pl} \to 0$ as $t \to t_0$,

b) $\phi/M_{\rm Pl} \geq 1$ as $t \to t_0$.

For $V = \frac{M^{4+\alpha}}{\phi^\alpha}$, $V_0 \simeq 10^{-47}\,\text{GeV}^4$ therefore $M \sim 0.1$ GeV if $\alpha = 2$. Smaller values of M are implied for smaller α.

2) Could quantum corrections to the quintessence potential become important at $\phi/M_{\rm Pl} \geq 1$? [Kolda and Lyth (1999)].

3) Nature of the Q-field [modulii, KK-relic's, bulk-induced, vacuum polarization, k-essence etc.]

4) 'Quintessential inflation': Could the inflaton be quintessence ? [Peebles and Vilenkin (1999)].

The general brane-world action [Sahni & Shtanov 2002]

$$S = M^3 \left[\int_{\rm bulk} (\mathcal{R} - 2\Lambda_{\rm b}) - 2 \int_{\rm brane} K \right] + \int_{\rm brane} \left(m^2 R - 2\sigma \right) + \int_{\rm brane} L\left(h_{ab}, \phi \right) \quad (1)$$

includes several important models as subclasses:

- $\text{General Relativity } (M = 0, \Lambda_{\rm b} = 0)$
- $\text{The DGP brane } (\Lambda_{\rm b} = 0, \sigma = 0)$
- $\text{The Randall} - \text{Sundrum model } (m = 0, \ \text{hence } \ell = 0)$

Eqn. (1) leads to the equations of motion:

$$m^4 \left(H^2 + \frac{\kappa}{a^2} - \frac{\rho + \sigma}{3m^2} \right)^2 = \epsilon M^6 \left(H^2 + \frac{\kappa}{a^2} - \frac{\Lambda_{\rm b}}{6} - \frac{C}{a^4} \right)$$

$$\Rightarrow H^2(a) = \frac{A}{a^3} + B + \frac{2}{\ell^2} \left[1 \pm \sqrt{1 + \ell^2 \left(\frac{A}{a^3} + B - \frac{\Lambda_{\rm b}}{6} - \frac{C}{a^4} \right)} \right], \quad \text{where } \ell = \frac{2m^2}{M^3}$$

- The minus sign above leads to the Phantom brane with w < -1.

Can one distinguish between degenerate dark energy models which, by virtue of having identical expansion histories H(z), will also give identical results for geometrical tests of dark energy based on standard candles and rulers ?

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2} \quad \text{where} \quad d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

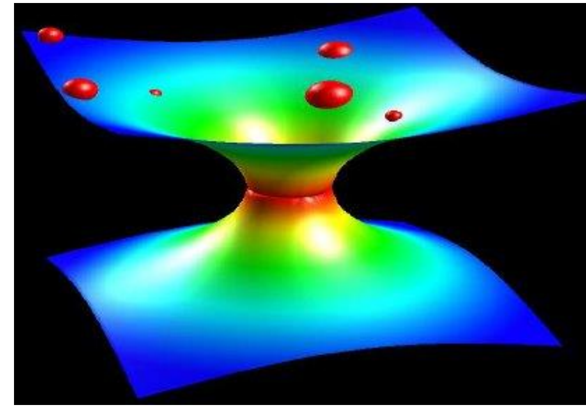
So the light flux from a distant supernova will be identical in models which have the same expansion history – H(z) !

Such quasi-degenerate models can still be distinguished since the assembly of structure in the cosmic web will occur at different rates !

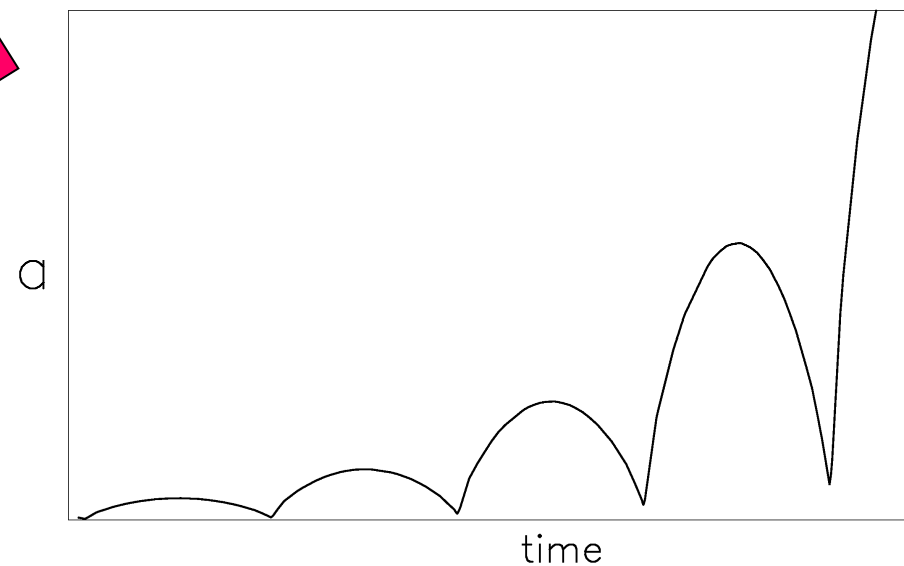
The underlying physical reason is that the perturbation equation governing the growth of inhomogeneities feels the nature of gravity ! So the cosmic web grows at a different rate in modified gravity theories than in General Relativity.

Bouncing Cosmology. $H^2 = \frac{\kappa}{3}\rho\left\{1 - \frac{\rho}{\rho_c}\right\}$ $\qquad \mathbf{v} = H\mathbf{R}$

Universe bounces when $\rho \simeq \rho_c$



Which can lead to **Cyclic Cosmology**

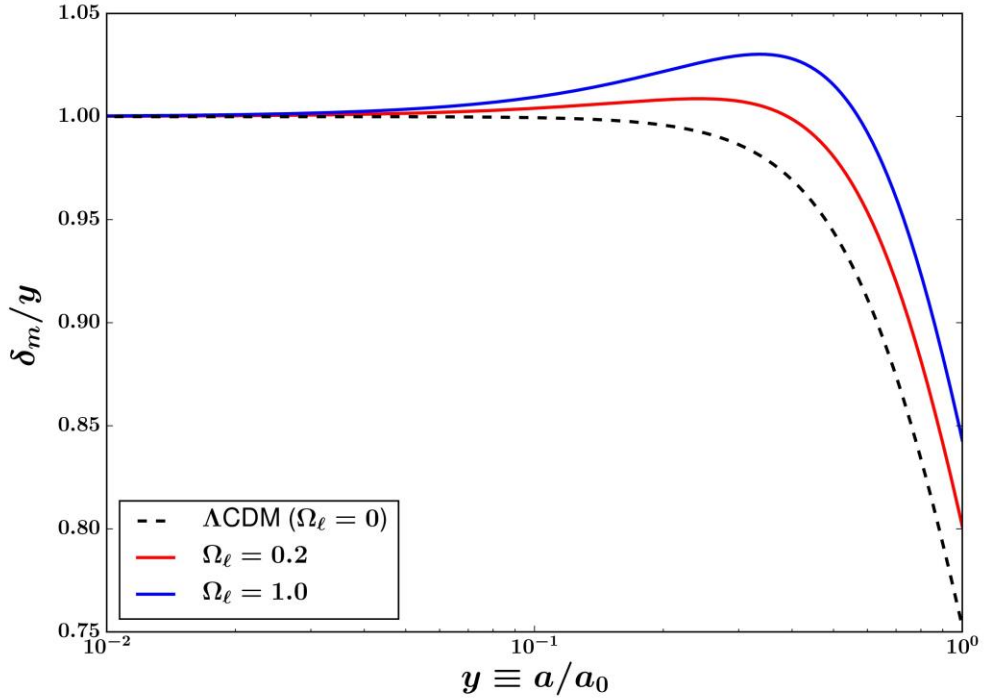


The Universe becomes larger and older with each new cycle, and soon begins to resemble our Own Universe.

[VS, Shtanov and Toporensky, CQG 2015]

Oscillatory Universe

A key test which can distinguish the Phantom Brane from LCDM is gravitational clustering, which proceeds at a **faster rate** on the brane than in LCDM.



[Bag et al. 2018]

Upcoming experiments such as Euclid and SKA will help reveal the nature of Dark Energy.