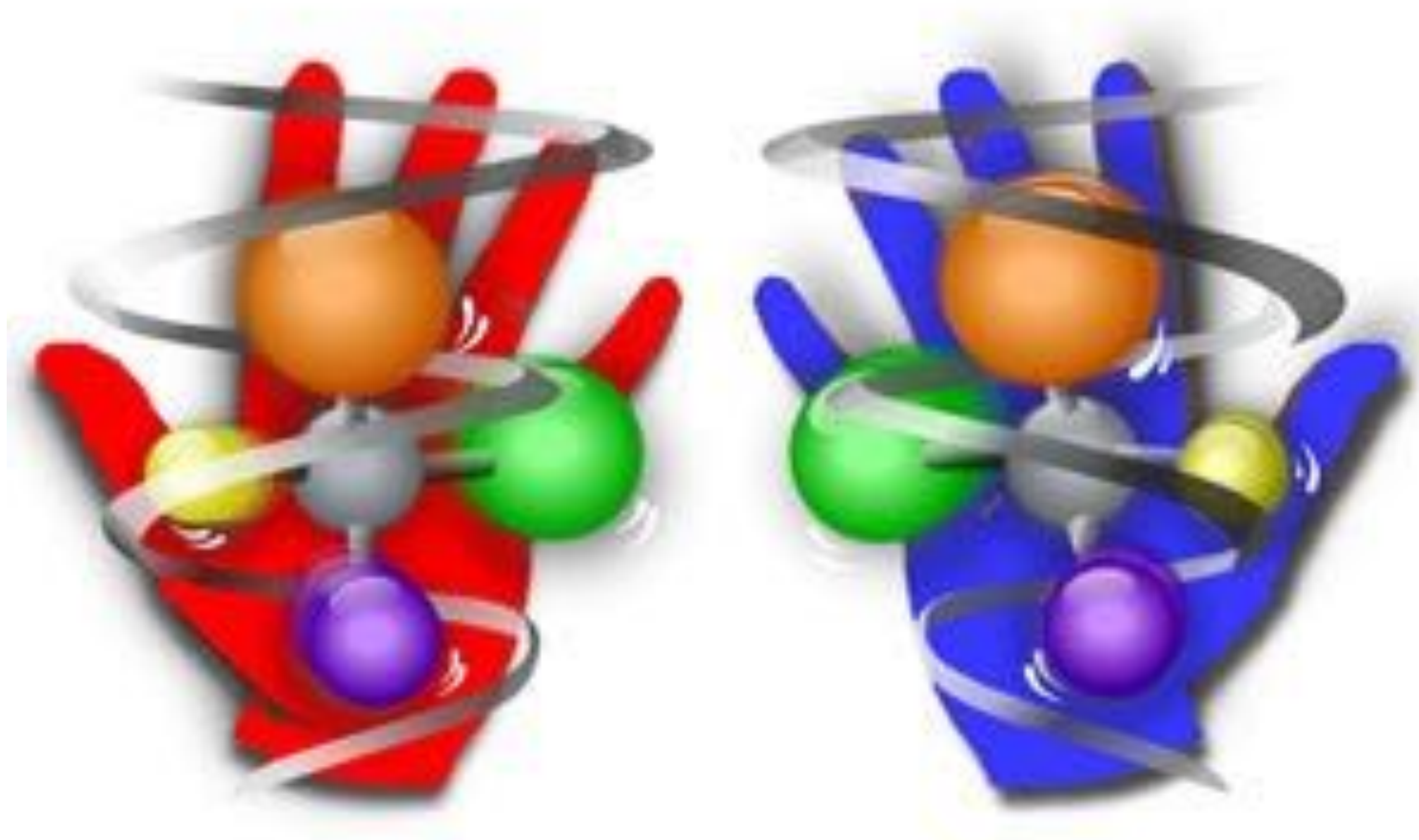
The mystery of chiral gauge theories



What is a chiral gauge theory?

- In 3+1 dimensions, can write all fermions as LH Weyl spinors
- They transform as some representation R of the gauge group
- If R is complex ▸ "chiral gauge theory"
  - Fermion masses break gauge symmetry
  - Constrained by anomalies: $\mathrm{Tr}(\{T_a,T_b\}T_c)_{L-R} = 0$.
    - In d=1+1: $\mathrm{Tr}\,(T_a T_b)_{L-R} = 0$
  - Nielsen-Ninomiya escape involves $\{\gamma_5, D\} \neq 0$... can this be consistent with gauge invariance?
- SM is a chiral gauge theory! No known nonperturbative regulator!
- Unknown: is <u>any</u> anomaly-free χGT possible?

What about gauged chiral symmetries??

A notoriously hard problem.

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- ★ …all likely to suffer from sign problems…

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★ …all likely to suffer from sign problems…

If all a lattice chiral gauge theory can do is reproduce SM perturbation theory at great computational cost, not so interesting!

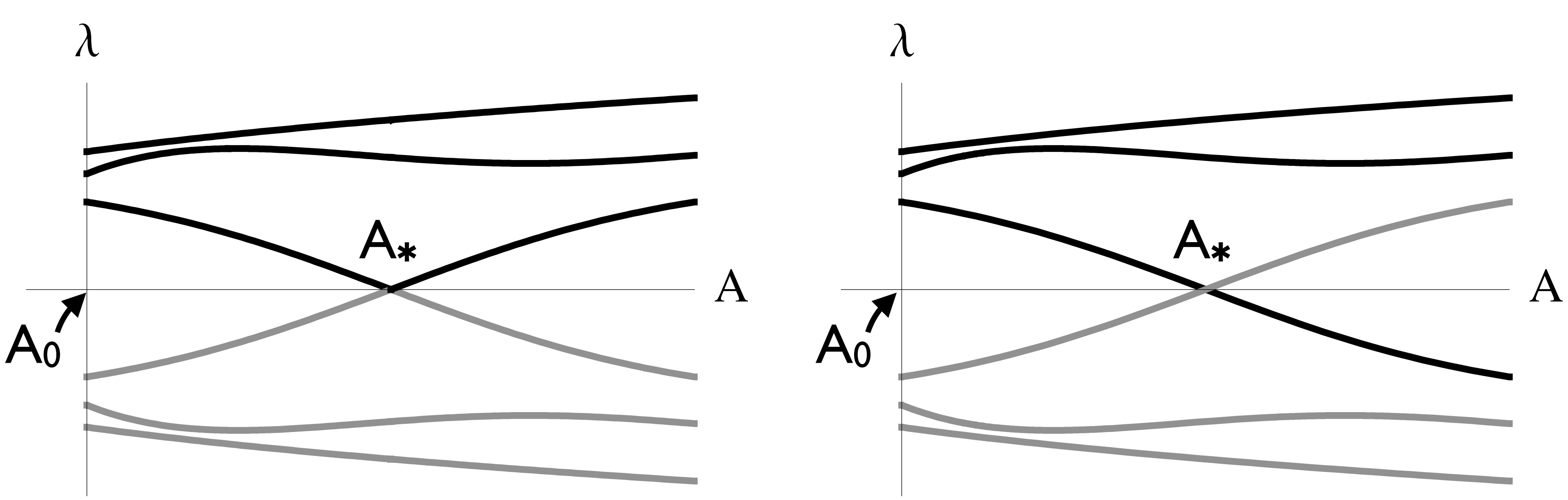
My own motivation: might we have completely missed interesting and bizarre nonperturbative physics in the SM?

A chiral gauge theory is the square root of a Dirac theory:

Δ(R) = Euclidian fermion determinant for XGT with fermions in representation R:

$$\Delta(R)\Delta(R^*) = |\Delta(R)|^2 = \det \not{D}_R \qquad \Longrightarrow \qquad \Delta(R) \sim \sqrt{\det \not{D}_R}$$

Is there a natural way to take the square root? Eg, take product of 1/2 of Dirac eigenvalues?



Will generally lead to a nonlocal and/or non-analytic theory

eigenvalue flow of Dirac operator as function of gauge field

$$\Delta(R) = |\Delta(R)| e^{i\phi[A]}$$

- $|\Delta(R)|$ = positive real square root of the Dirac operator
- What is the phase φ?

Alvarez-Gaume, Della Pietra[2] Phys. Lett. B 166 (1986) 177 :

For an anomaly free theory, $\phi[A] - \phi[A_0] = \pi\eta[H]$

η is the "eta-invariant" of an operator = sum of signs of eigenvalues

$$\eta = \lim_{s \to 0^+} \sum_{\lambda} \frac{\lambda}{|\lambda|} |\lambda|^{-s}$$

Here: operator is $H = i\gamma_5\, \partial/\partial t + \not{D}(A_t)$

Where $A_t(x)$ interpolates from $A_0(x)$ at $t= -\infty$, to $A(x)$ at $t= +\infty$

Anomaly-free theory: $\phi[A] - \phi[A_0] = \pi\eta[H]$

H: $H = i\gamma_5\, \partial/\partial t + \not{D}(A_t)$

η: $\mathrm{Tr}\, \epsilon(H)$

A_t: interpolating field in 5th dimension (t)

Anomalous theory: additional contribution to φ from 5d Chern-Simons operator for A_t

...this all looks sort of familiar from the discussion of the overlap operator!

Hold onto that thought!

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Try to tune toward continuum

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Excellent physicists…nice work…ugly theory

Old: E. Eichten, J. Preskill, Nucl. Phys. B268 (1986) 179

Recent: M. DeMarco, X.-G. Wen, arXiv:1706.04648

Current: D. Schaich, S. Catterall, "Phases of a strongly coupled four fermion theory", arXiv:1710.08137

Very nice idea...but will dynamics cooperate?
How does theory fail when anomalous?
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M. Grabowska, DBK
Phys. Rev. Lett. 116 (2016) 211602
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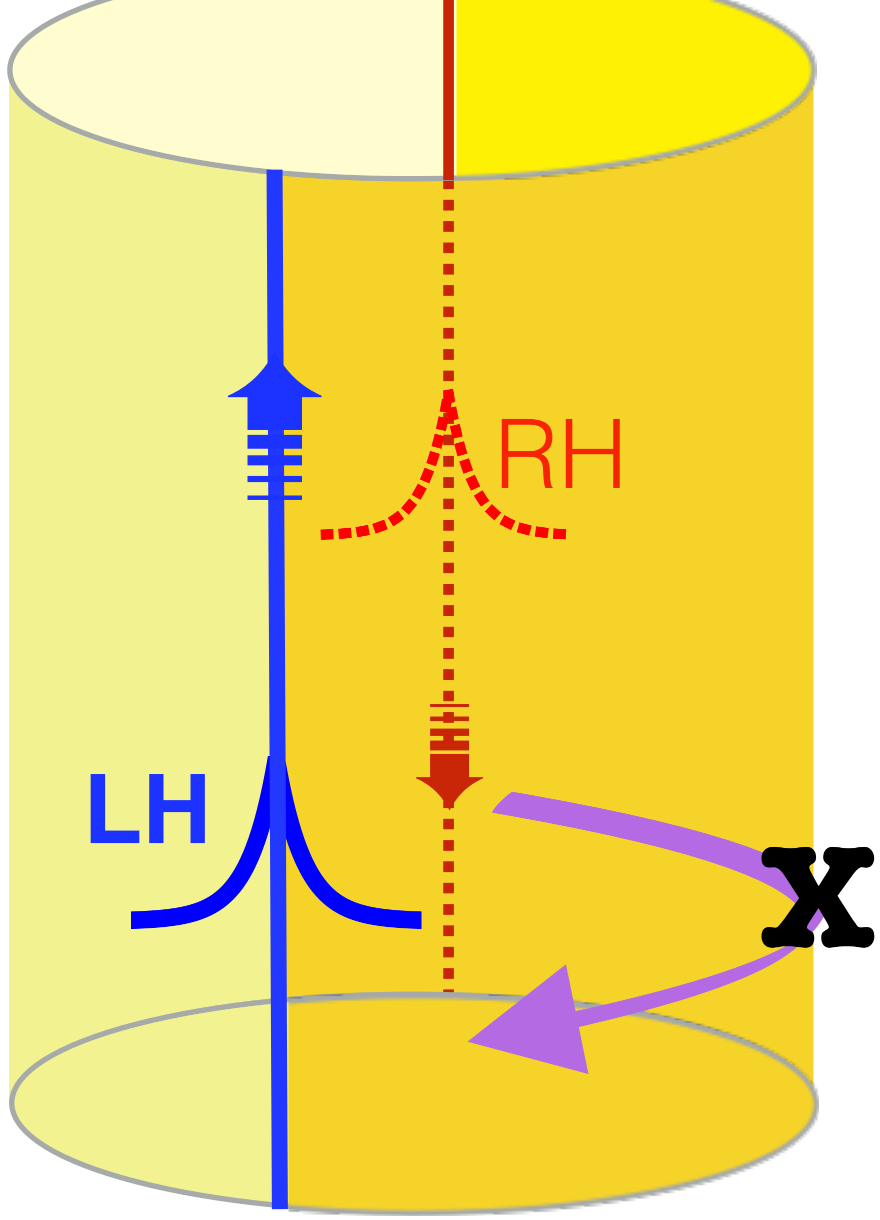
Why domain wall fermions?

- ★ Must explain why anomalous gauge symmetries fail to have a continuum limit
- ★ Domain wall fermions "know" about anomalies via bulk Chern-Simons currents
- ★ η-invariant makes extra dimension look natural...with 5d-dependent gauge fields



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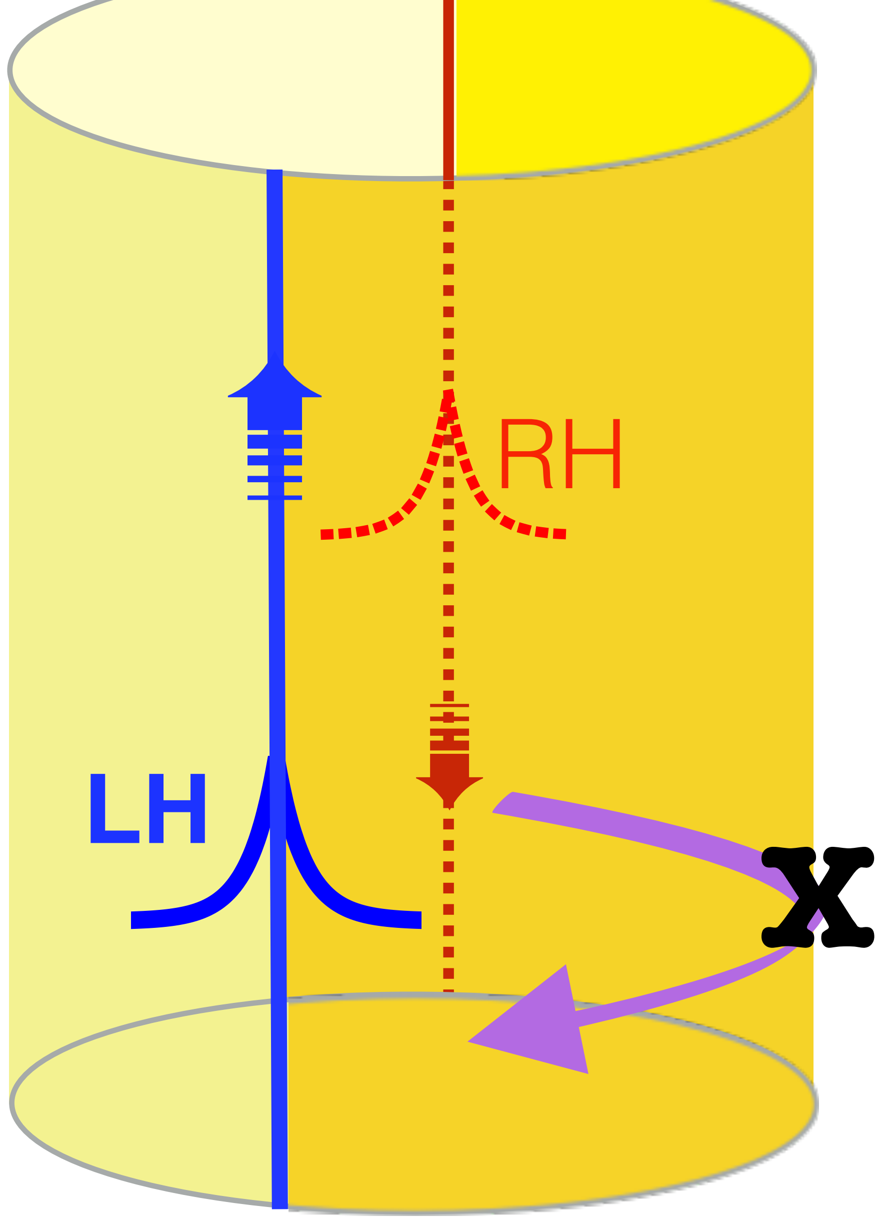

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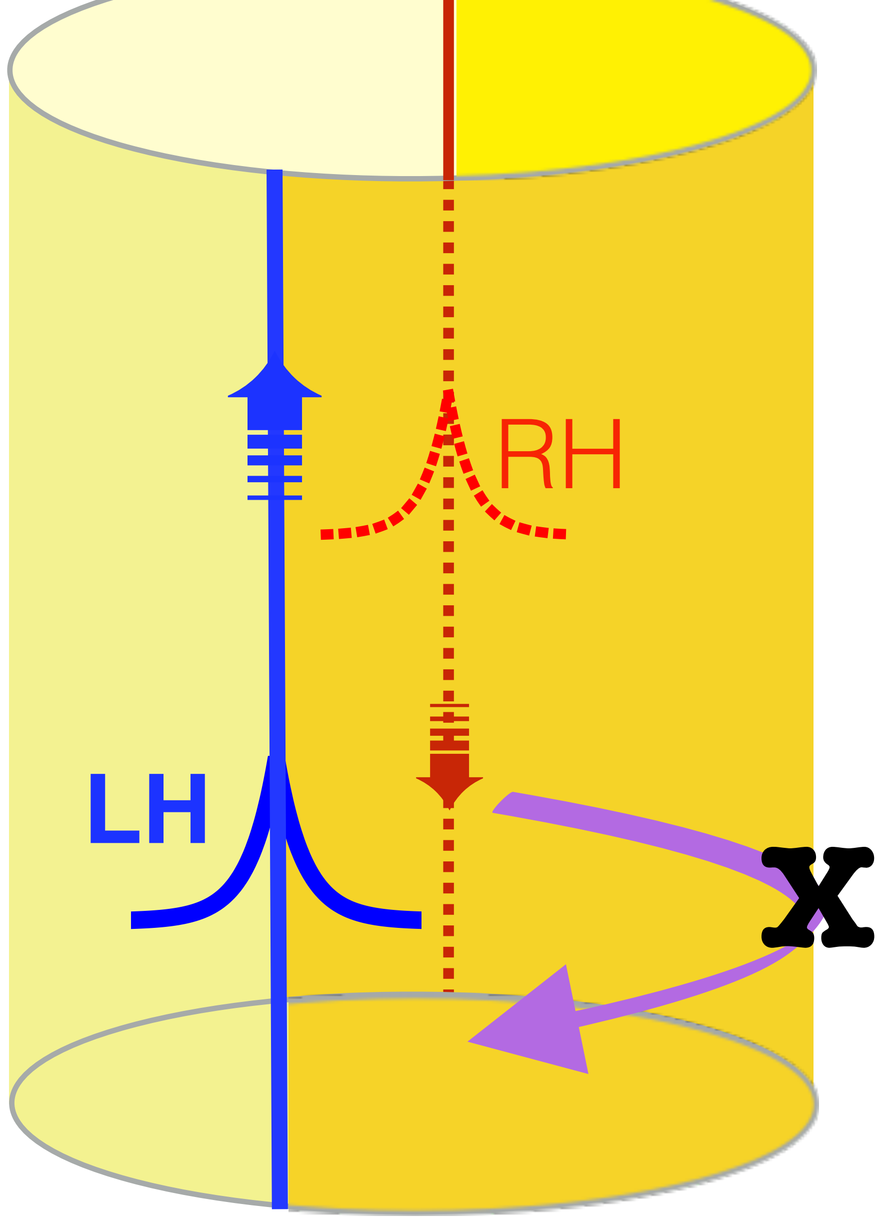

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Aside: In condensed matter systems -
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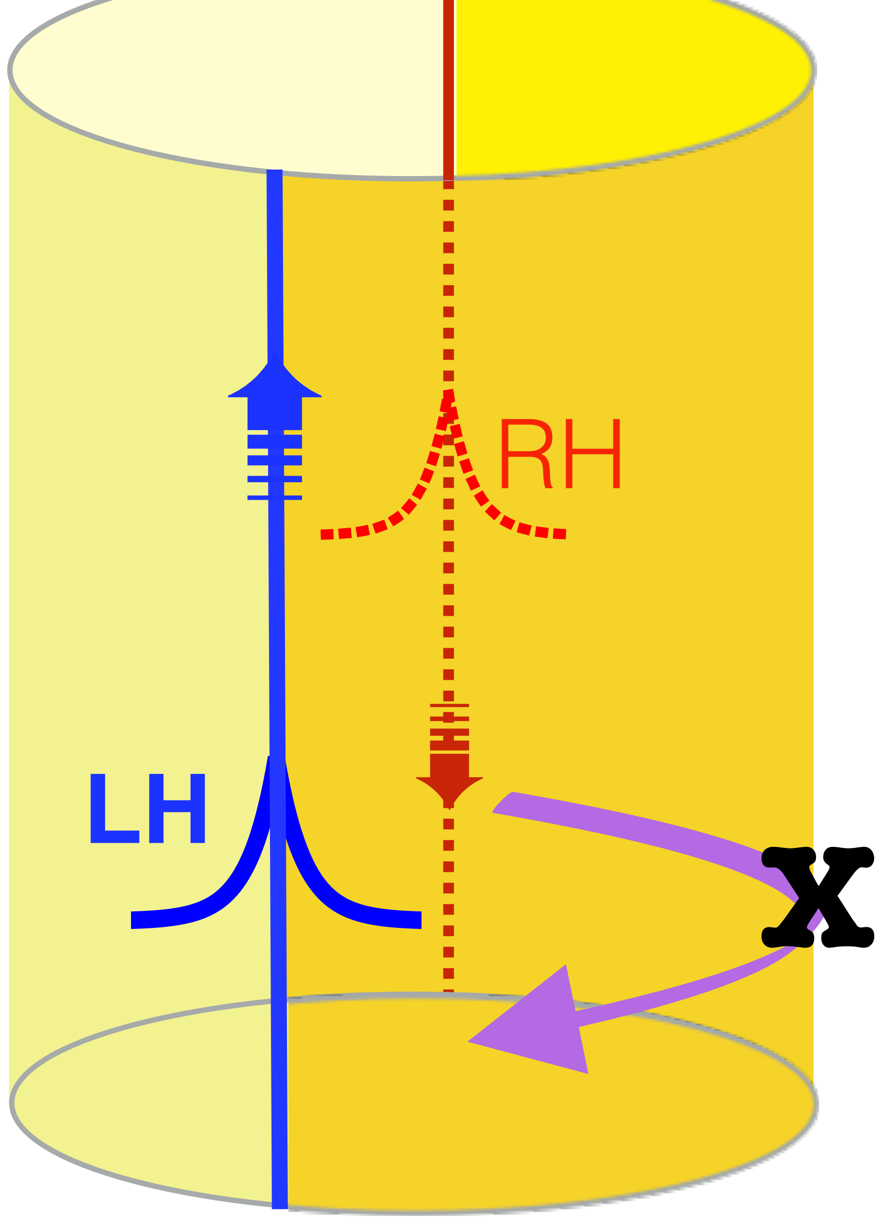

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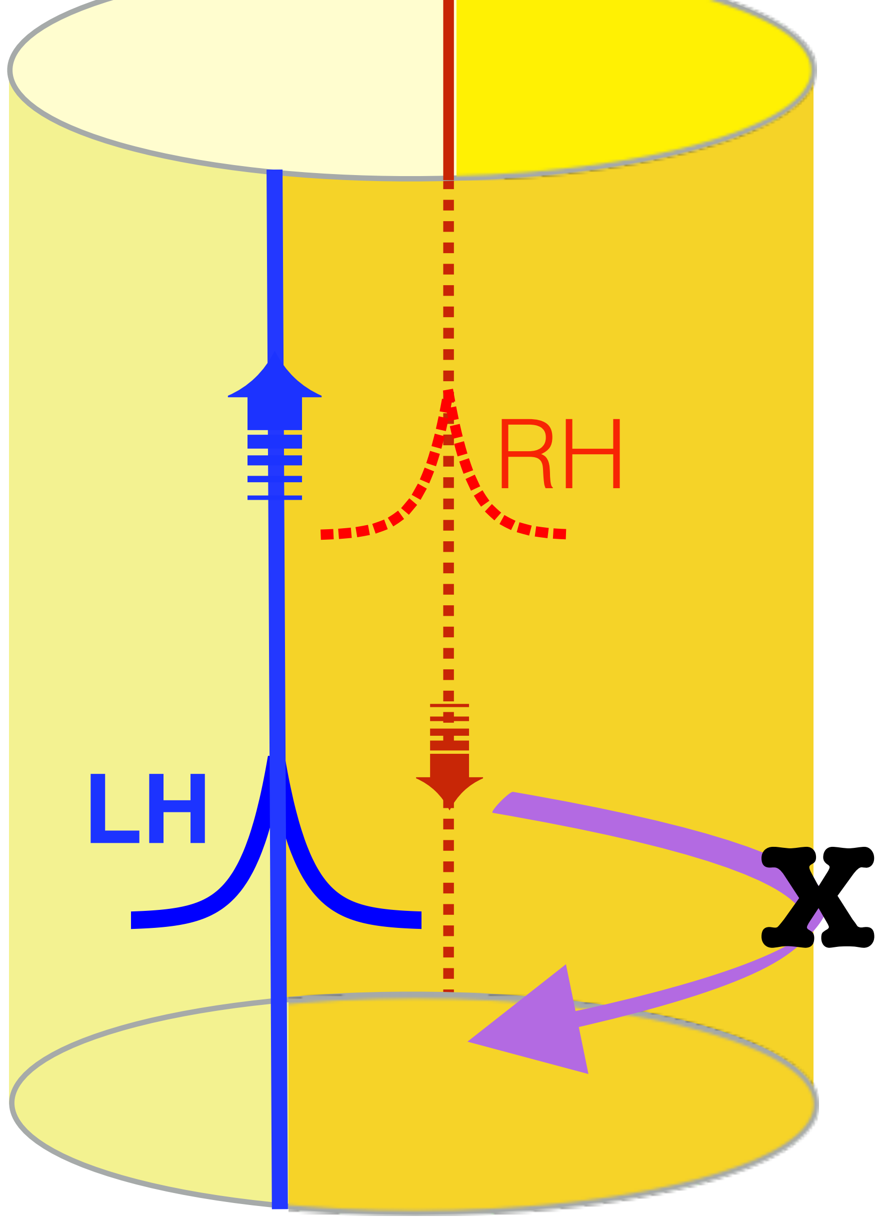

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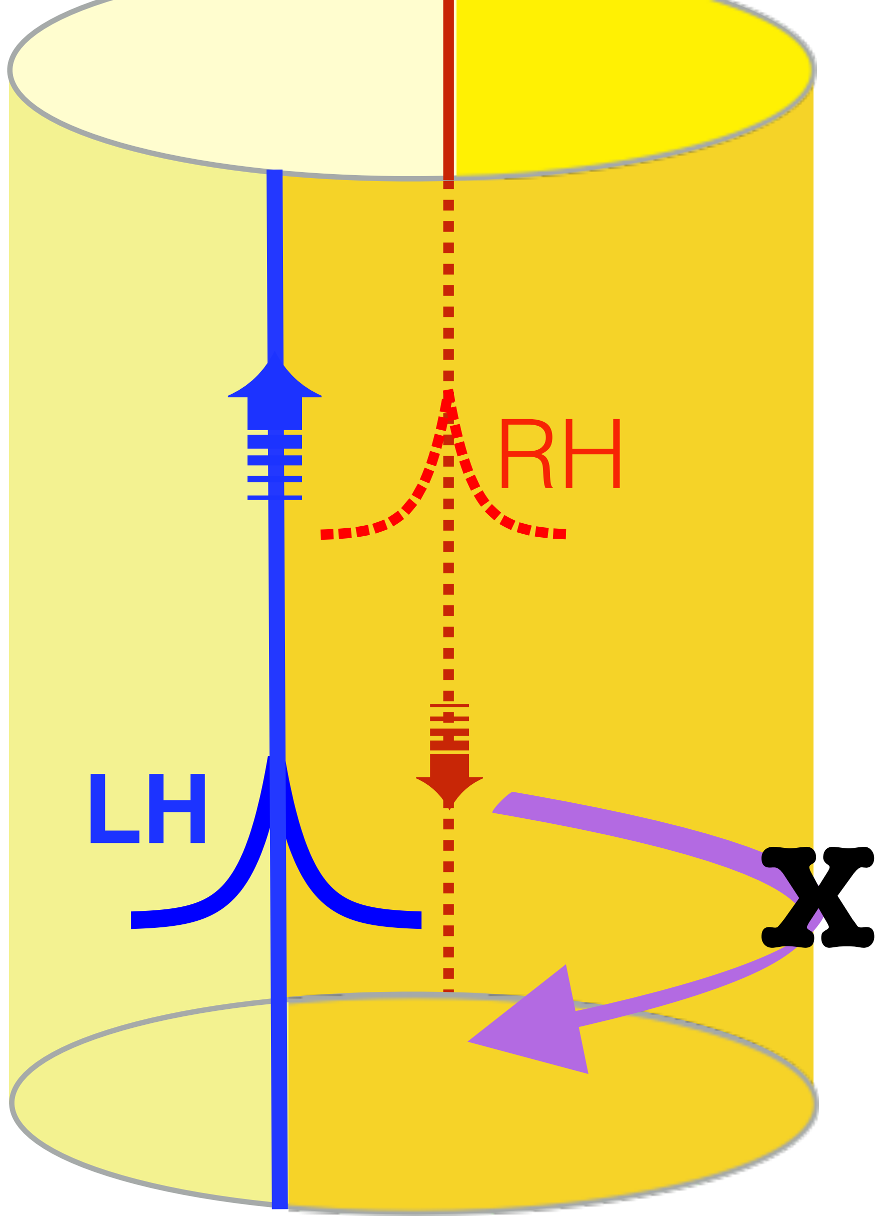

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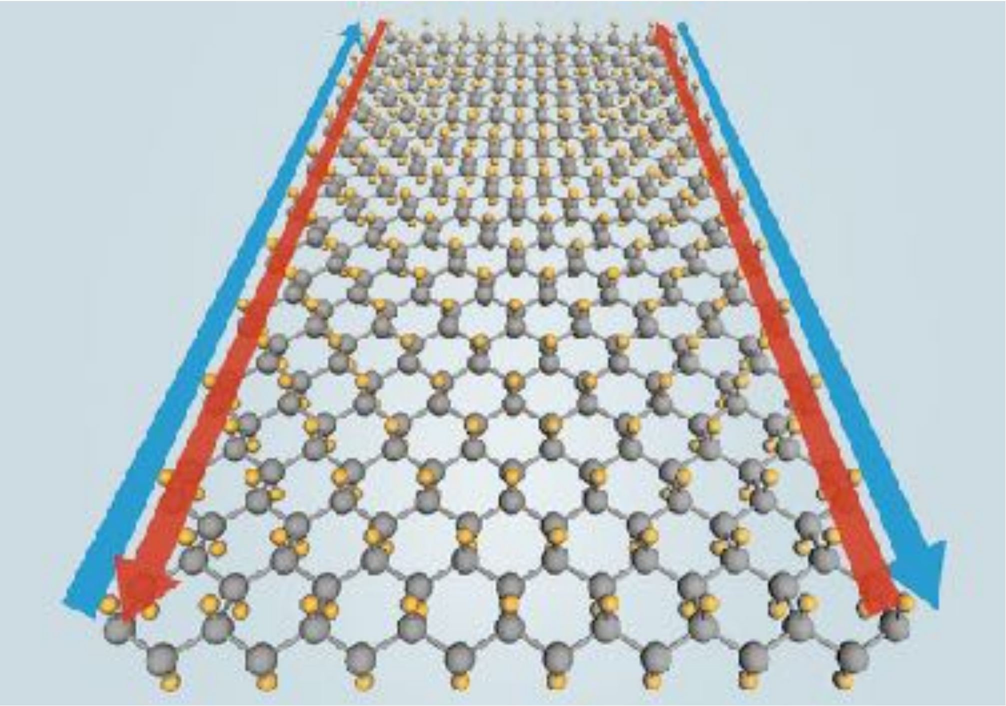
- ★ There <u>is</u> a bulk $U(1)_A$ current ("Spin Hall Current")

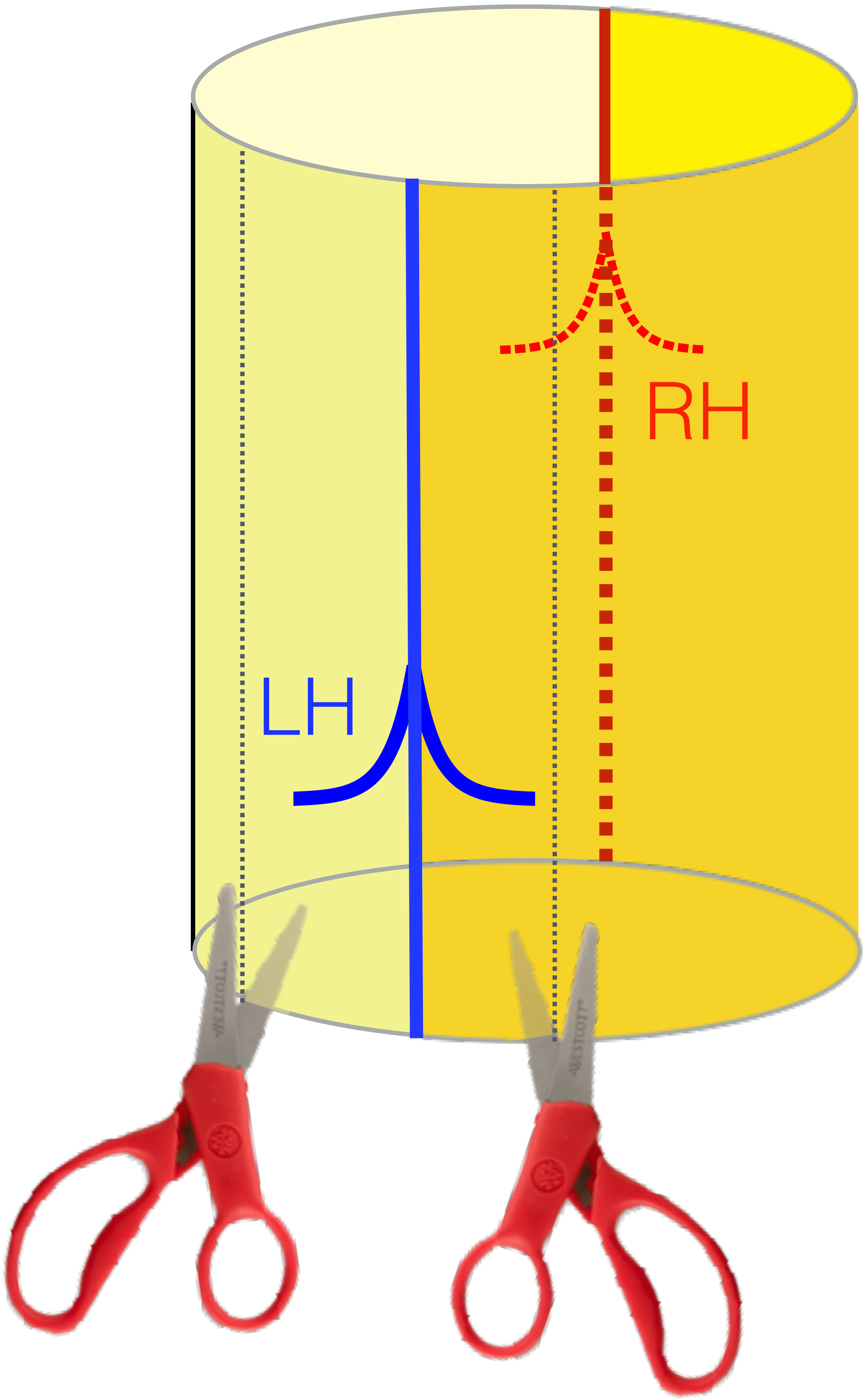
Simplest example(s): have a Dirac fermion live at each surface



Two ways to achieve?

- 2 5d fermions Ψ with same charge q, but opposite sign mass $\pm\Lambda$
- 2 5d fermions Ψ with opposite charges $\pm$q but same sign mass Λ

This is exactly the Kane-Mele model for QSHE



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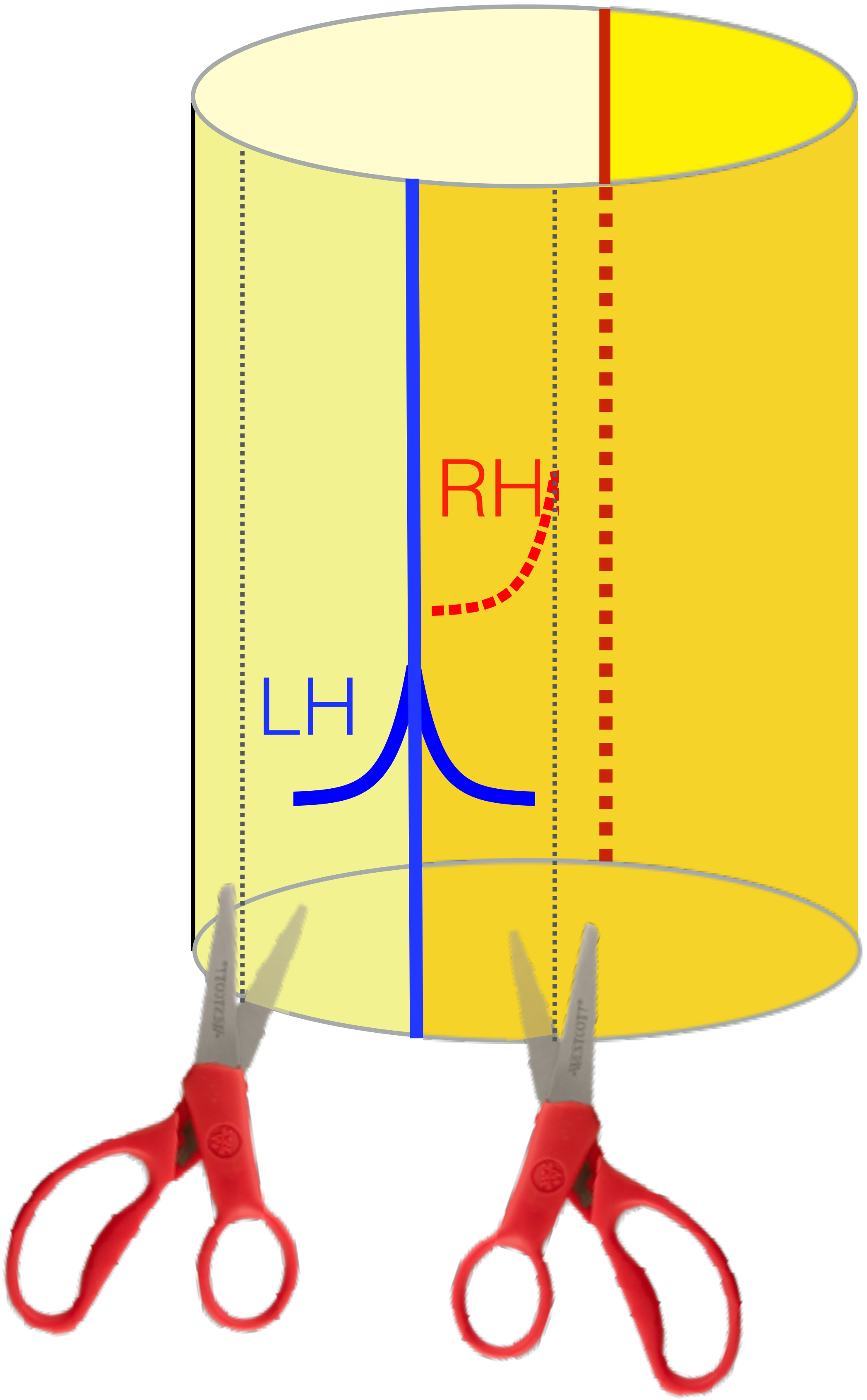

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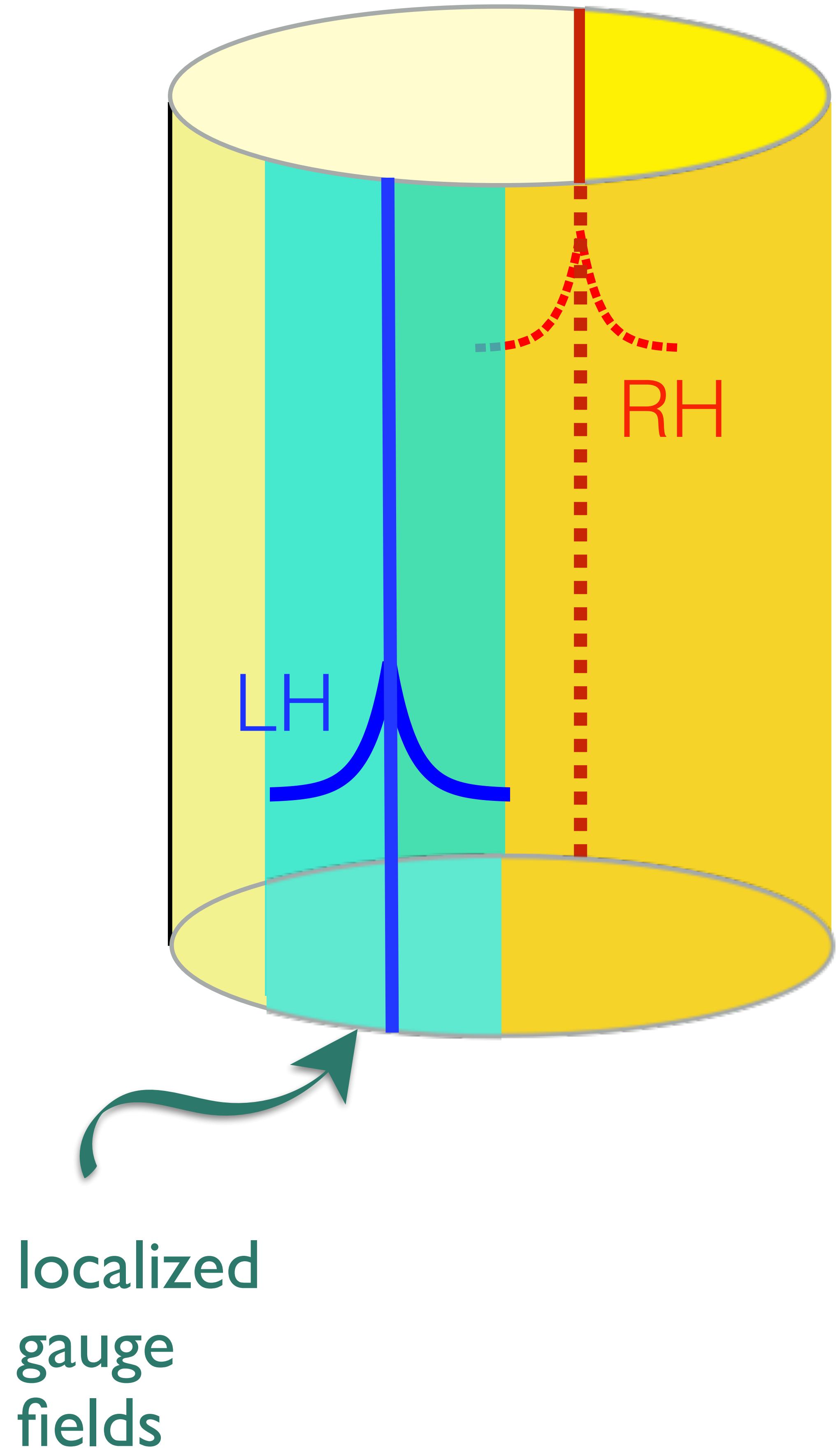

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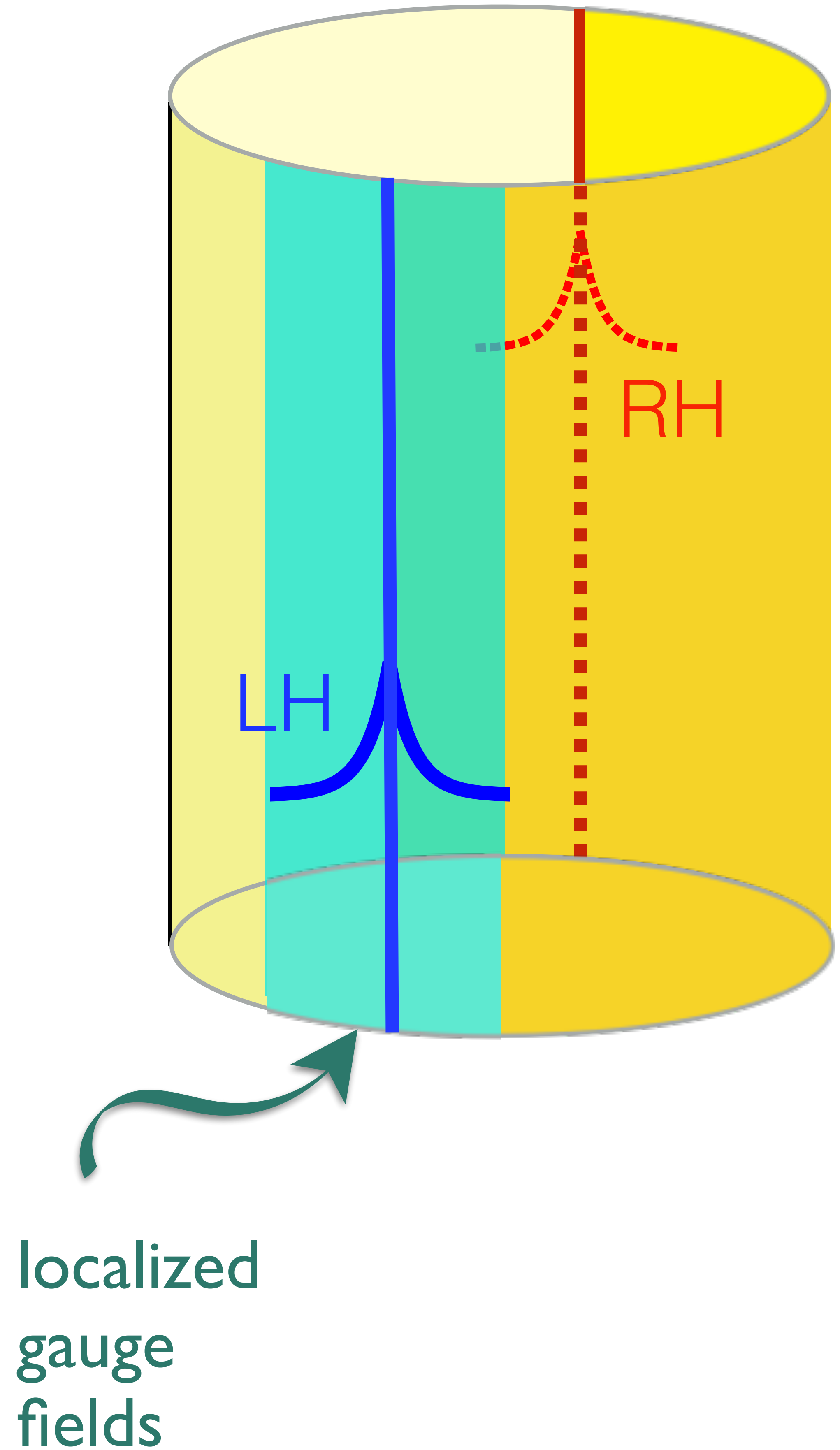

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No: The 5d kinetic term allows fermions to "hop" in the extra dimension; localizing the gauge field would explicitly (or spontaneously) break gauge symmetry.

Proposal: "localize" gauge fields using gradient flow

Dorota Grabowska, D.B.K.

- Phys.Rev.Lett. **116** 211602 (2016) [arXiv:1511.03649]
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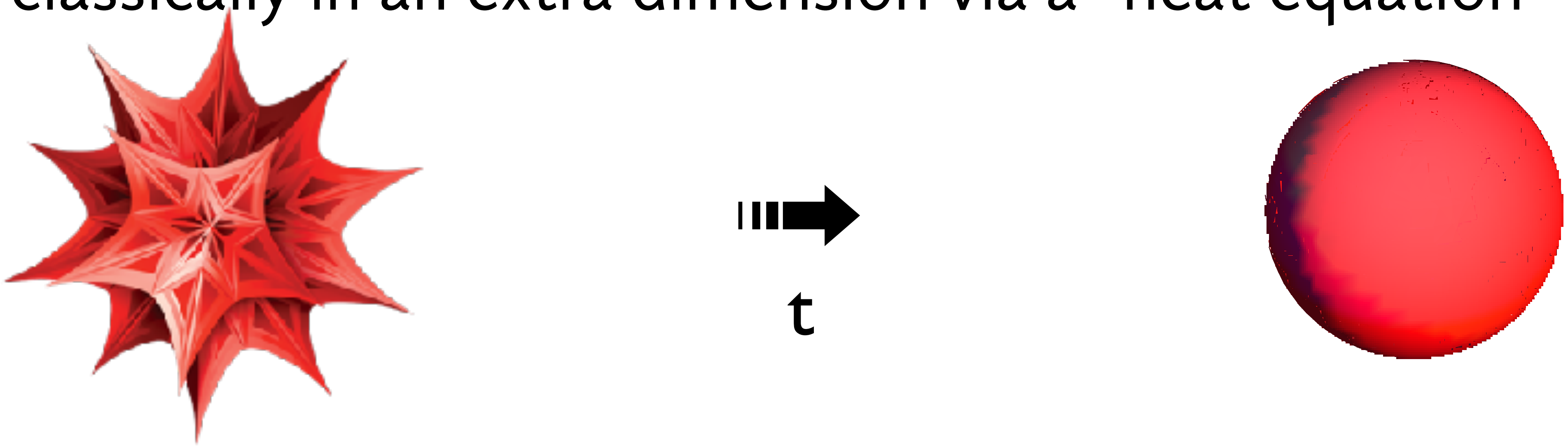

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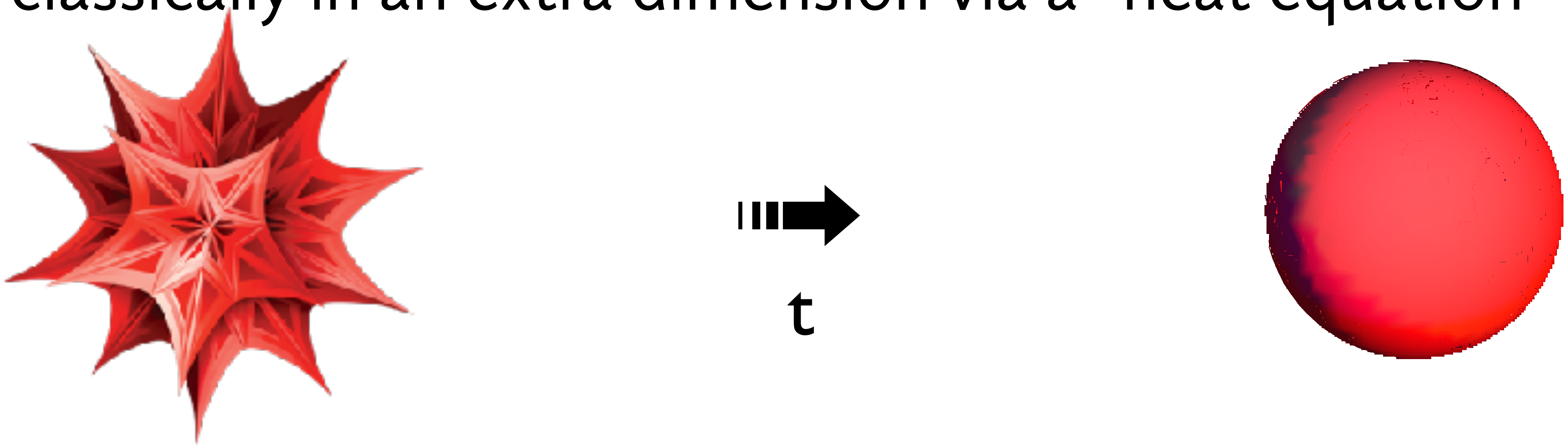

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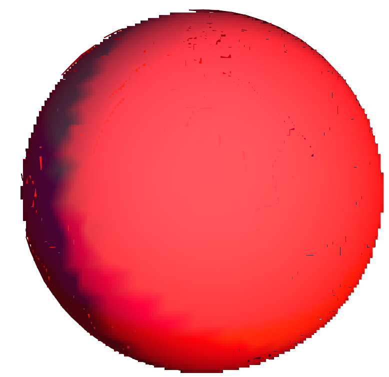

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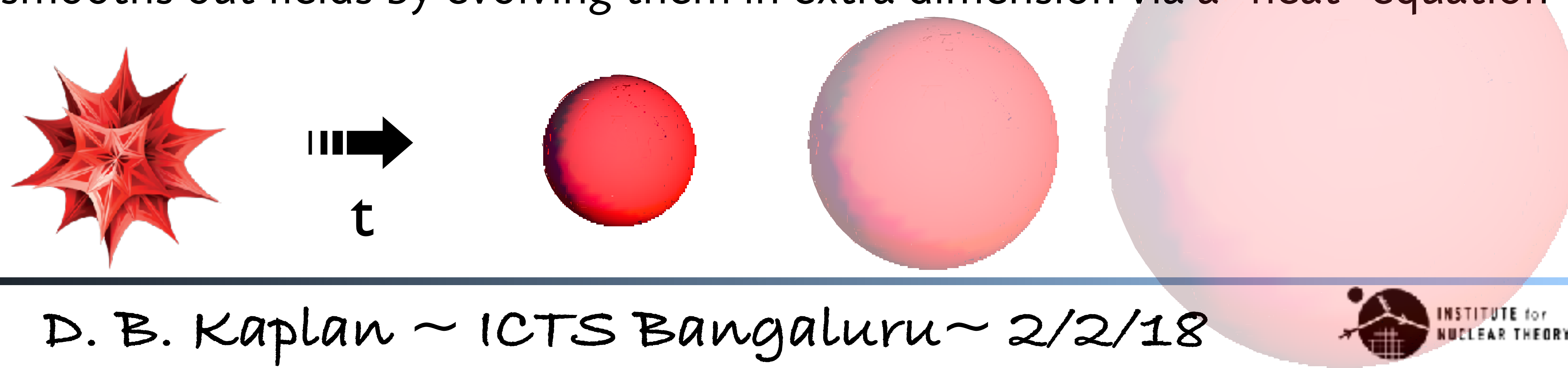

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Example: U(1) gauge theory, 3d bulk, 2d boundaries

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★ *This will allow λ(p) to be localized near t=0 while maintaining gauge invariance*

★ *RH wall has to be at t=∞ for this to make sense in Minkowski spacetime*

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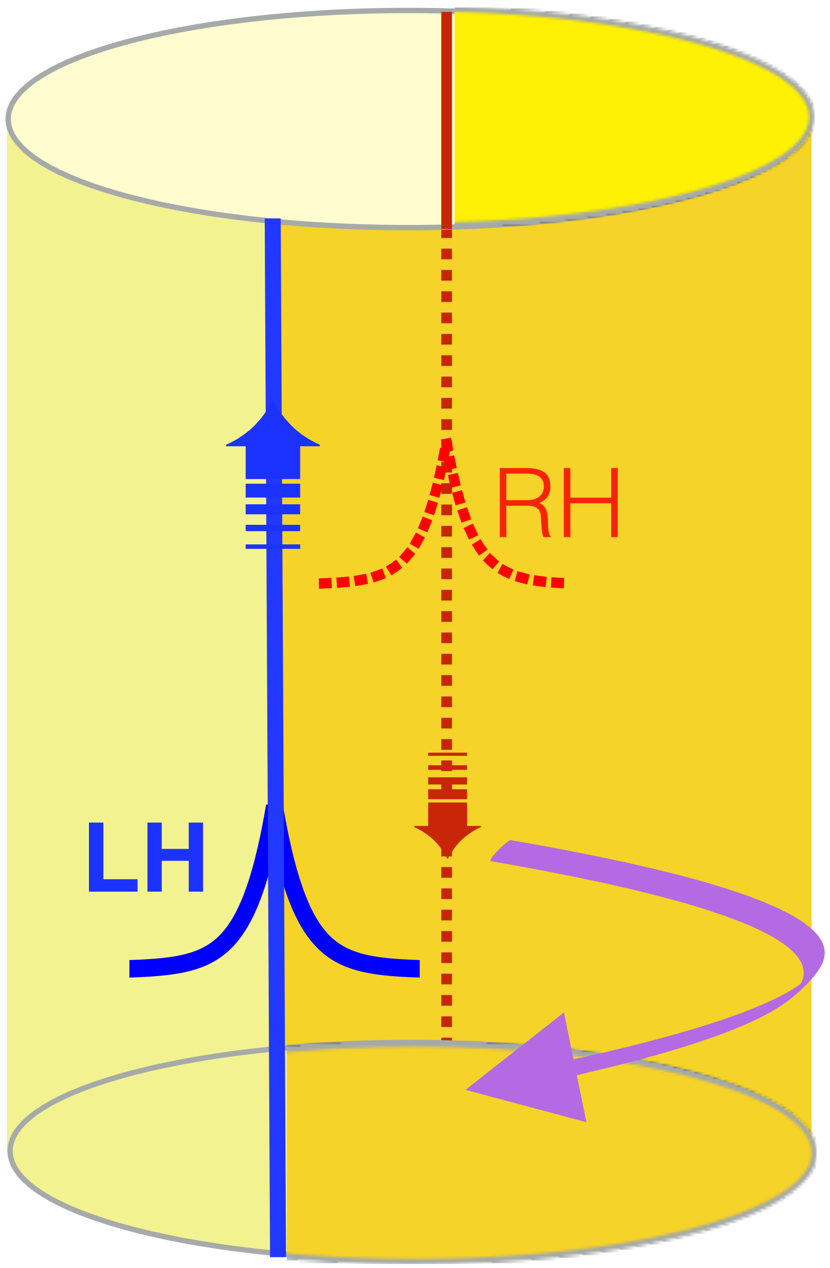
- ★ Can we continue to Minkowski spacetime? **Great question!**

Anomalous representations: Can we show no (local) 4d EFT?



Yes! integrating out bulk fermions leads to CS operator that depends on the nasty nonlocal $\underline{A}_\mu(x,t)$

t = extra dimension

Do not get a 4d (2d) field theory

The algebraic condition for the coefficient of this bad operator to vanish?

☛ Independent gauge anomaly cancellation of LH, RH fermions.

Example: 3d ▸ 2d U(1) gauge theory without anomaly cancellation:

Integrating out bulk modes generates a nonlocal term:

$$S_3^{\text{bulk}} \propto \int d^2x \int ds[1-\epsilon(s)]\epsilon_{abc}\bar{A}_a\partial_b\bar{A}_c$$

← 3d gauge fields

$$= 2\int d^2x d^2y \left(\frac{\partial_\mu\partial_\alpha}{\Box}A_\alpha(x)\right)\Gamma(x-y)$$

$$\times\left(\frac{\partial_\mu\partial_\beta}{\Box}\epsilon_{\beta\gamma}A_\gamma(y)\right)$$

2d gauge fields

$$\Gamma(r) = [\delta^2(r) - (\mu^2/4\pi)e^{-\mu^2 r^2/4}] \qquad \mu = \sqrt{\Lambda/L_3}$$

Not a local 4d theory

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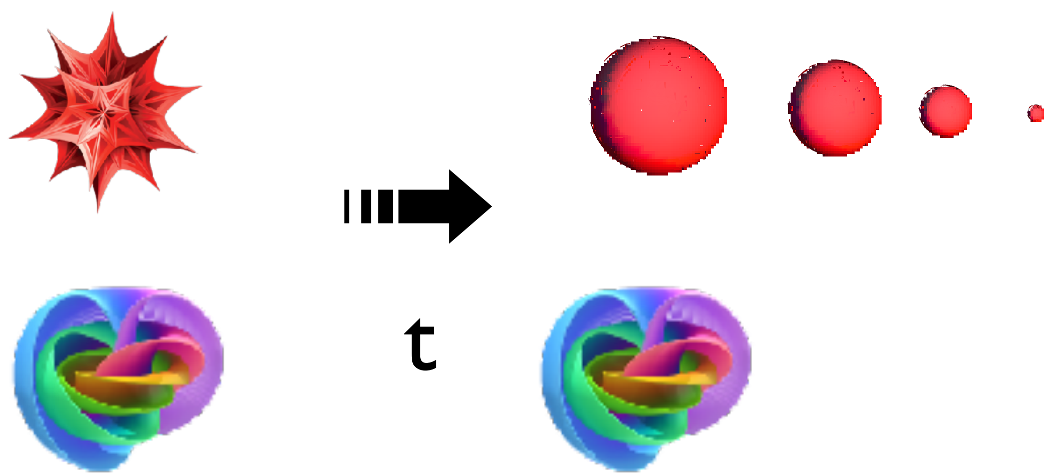

Exact solutions to Euclidian eqs of motion don't flow

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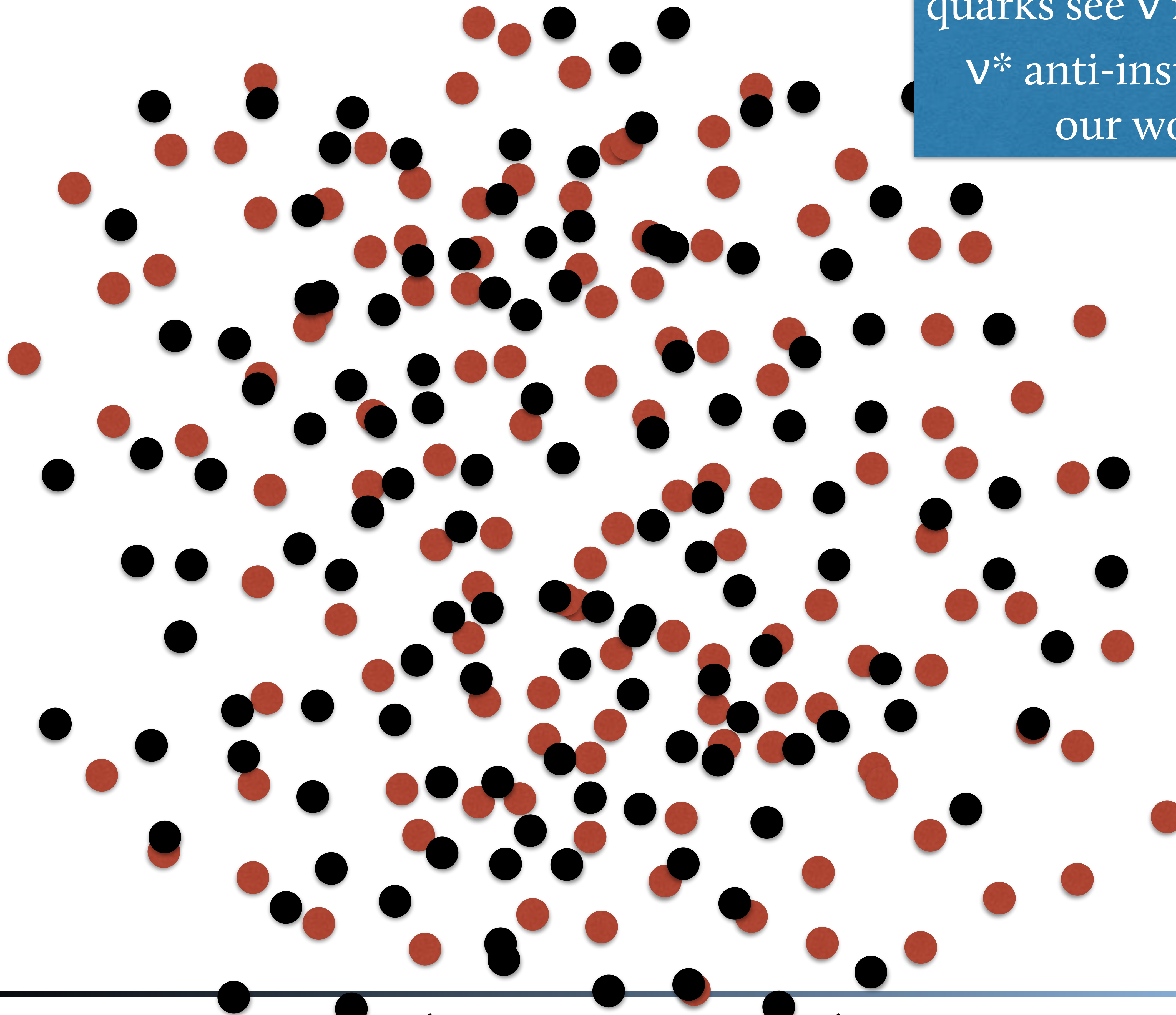
For each winding number ν there are an infinite number of exact solutions with ν instantons arranged in different locations…

Expect these to be attractive fixed points of gradient flow

…but not for ν+n instantons plus n anti-instantons.

Matter-fluff interactions?

quarks see ν instantons + ν* anti-instantons in our world…



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quarks see ν instantons + ν* anti-instantons in our world…

Fluff quarks only see (ν-ν*) instantons in the mirror world

A crude calculation (e.g., guess!):

when matter sees 10001 instantons and 10000 anti-instantons, **the one instanton that fluff sees is not spatially correlated with them**.

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fluff

Action induced by instantons:

$$S = \ln\left[\sum_{n=0}^{\infty}\left(\sum_{\bar{n}=0}^{n} \frac{\mathcal{O}^n \bar{\mathcal{O}}^{\bar{n}}}{n!\bar{n}!}\mathcal{F}^{n-\bar{n}} + \sum_{\bar{n}=n+1}^{\infty} \frac{\mathcal{O}^n \bar{\mathcal{O}}^{\bar{n}}}{n!\bar{n}!}\bar{\mathcal{F}}^{\bar{n}-n}\right)\right]$$

$$= \Lambda^{8-6N_f} \int d^4x\ \det \bar{q}_L q_R(x) \int \frac{d^4y}{V}\ \det \bar{\varphi}_L \varphi_R(y) + h.c. + \ldots$$

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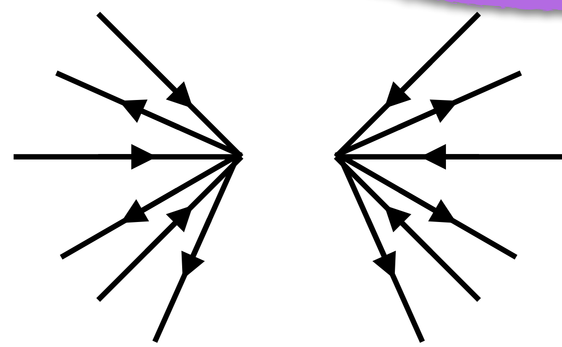
matter  fluff

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no transfer of energy or momentum
(follows from no-spatial-correlations guess)

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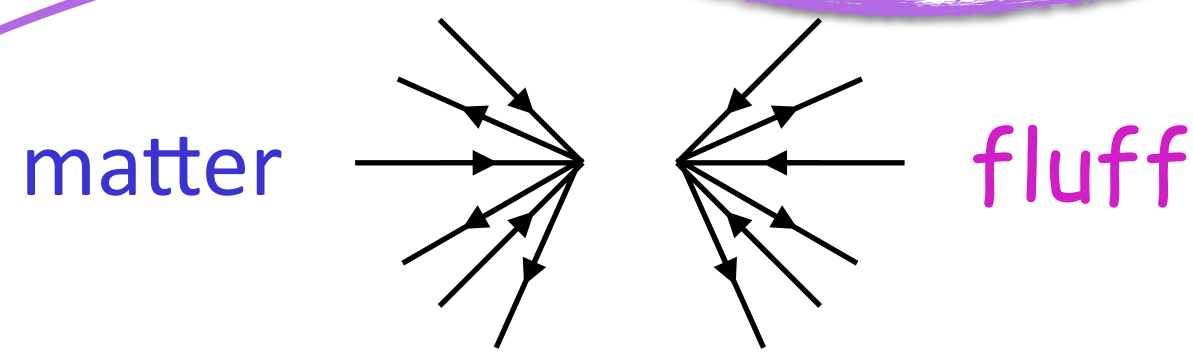

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The fluff operator looks like a conventional coupling constant for ordinary matter…but it is a dynamical quantum variable; reminiscent of Coleman's wormholes.

Does this make sense? Does it even have correct volume scaling?? Can this be simulated for a vector gauge theory?

These strangely interacting mirror fermions get a name: Fluff

Is massless colored fluff a solution to the strong CP problem??

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What happens with colored fluff in SM?

(fluff partners of SM quarks)

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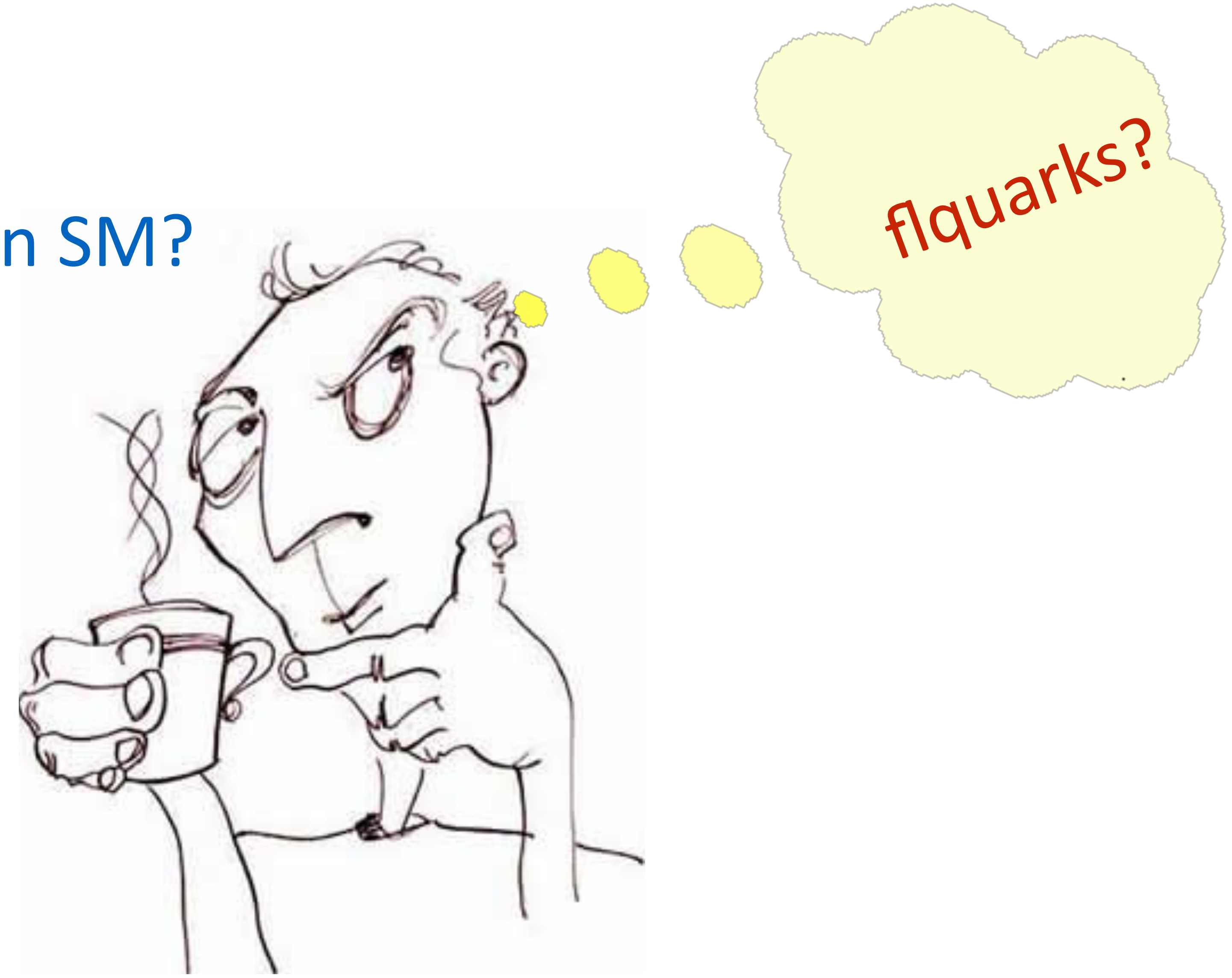

Does it confine? Interact with quarks?

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Toy model for confinement of ordinary quarks:

Consider a <u>random</u> (abelian) magnetic field $B_z = b(x,y)$ and a Wilson loop in x-y plane

$$W_C[b] = P e^{i \oint_C A\cdot d\ell} = e^{i \int_S d^2x\, b(x)} = e^{i \int d^2p\, \tilde{b}(p) g(p)\, d^2p}$$

Fourier transform of function that equals 1 inside loop, 0 outside.

where

$$g(\mathbf{p}) = \int_S d^2\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}} \ .$$

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It has an expectation value:

$\longleftarrow W_C \longrightarrow$

$$\langle W_C \rangle = \mathcal{N} \int D\tilde{b}\; e^{-\frac{1}{2\lambda^2}\int \frac{d^2\mathbf{p}}{(2\pi)^2}|\tilde{b}(\mathbf{p})|^2} e^{i\int \frac{d^2\mathbf{p}}{(2\pi)^2}\tilde{b}(\mathbf{p})g(\mathbf{p})} = e^{-\frac{1}{2}\lambda^2\int \frac{d^2\mathbf{p}}{(2\pi)^2}|g(\mathbf{p})|^2}$$

Path integral for gaussian random b field

$$= e^{-\frac{1}{2}\lambda^2 A}$$

are of loop...signal of confinement

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Same toy model for colored fluff:

At extra dimension coordinate s, random magnetic field has flowed to

$$\tilde{\beta}(\mathbf{p}, s) = \tilde{b}(\mathbf{p}) e^{-\mathbf{p}^2 s/\Lambda}$$

Λ sets speed of gradient flow

s is the extra dimension

form factor due to flow

separation between domain walls

Average over random b, and find

★ Area law (confinement) if quarks are separated by distance $r \gg \sqrt{\frac{L_5}{\Lambda}}$

★ Perimeter law (deconfinement) for smaller r.

★ As L_5 becomes infinite, no confinement

Anomaly-free representations: Can we obtain the effective 4d theory for the fermions, at nonzero lattice spacing & infinite extra dimension? **A chiral overlap operator?**

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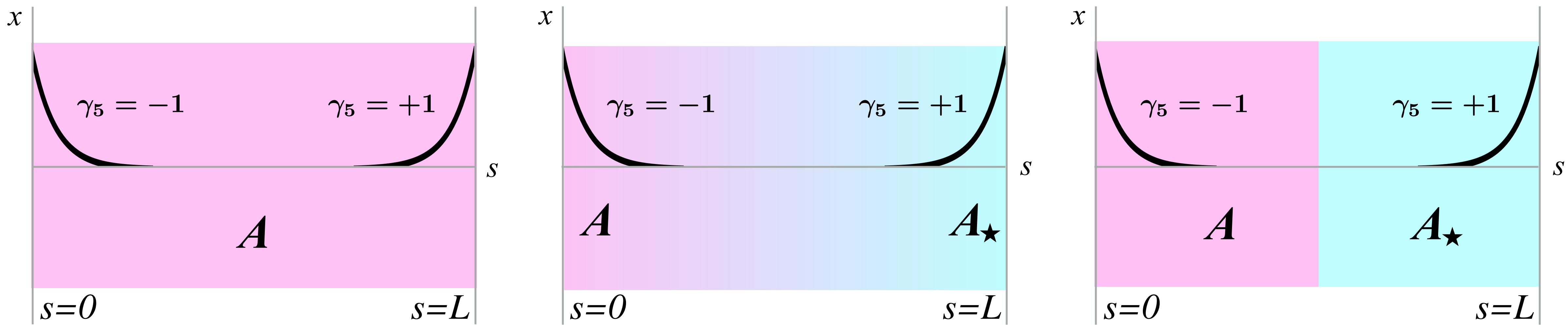

Usual DWF | Gradient flow | Crude approximation

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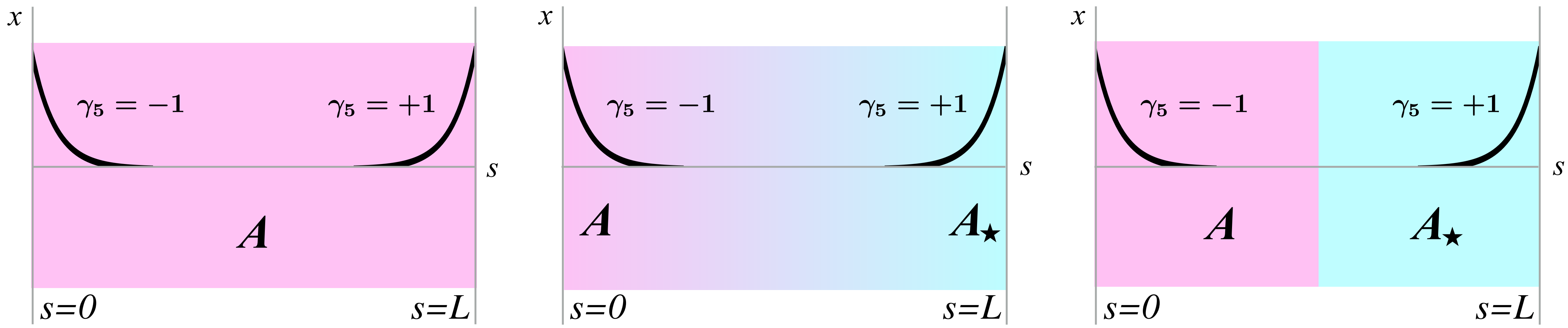

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$$D = \lim_{L_5\to\infty} 1+\gamma_5\frac{1-T^L}{1+T^L} = 1+\gamma_5\epsilon(H)$$

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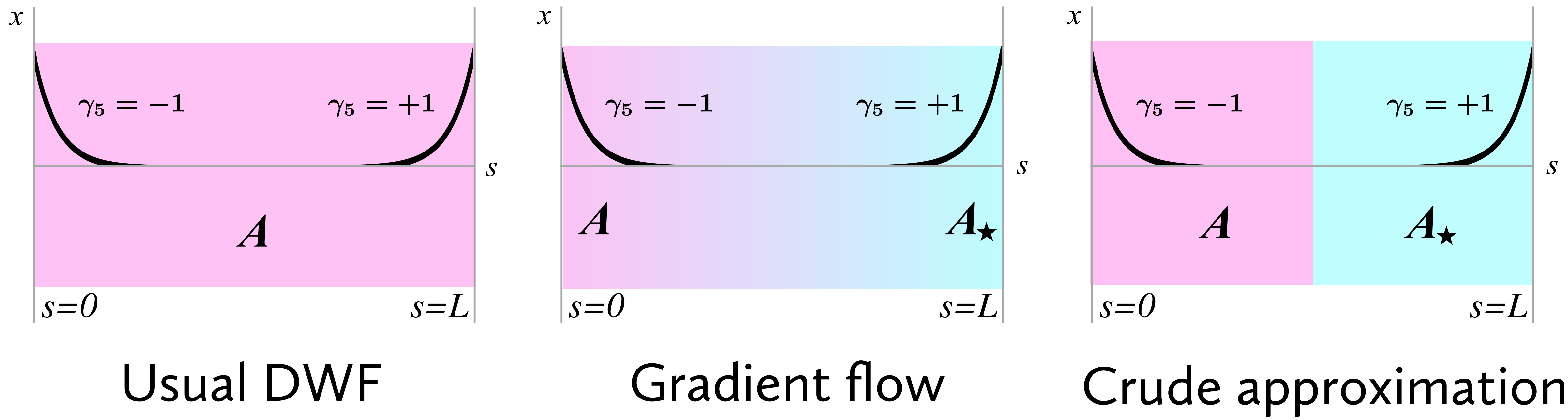

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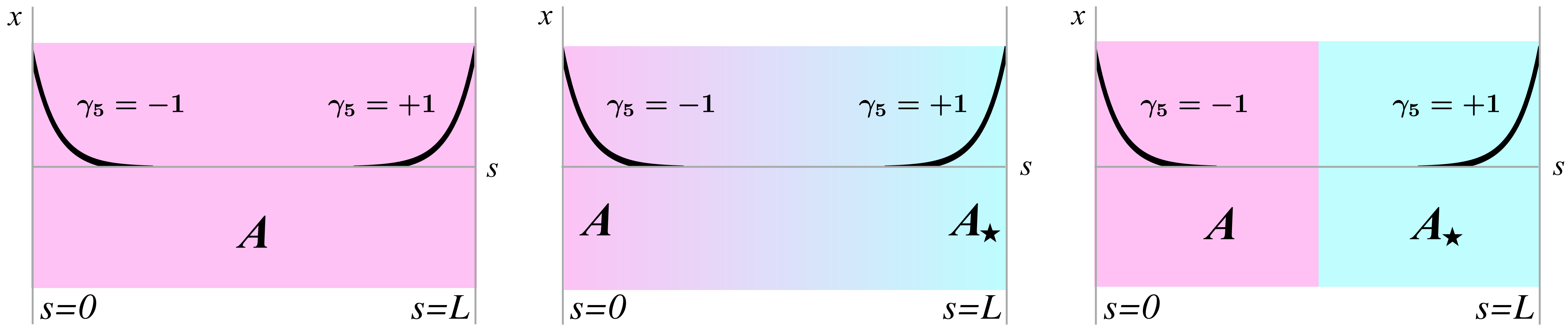

Usual DWF   Gradient flow   Crude approximation

$$D_\chi = \lim_{L\to\infty} 1 + \gamma_5 \frac{1 - T_\star^{L/2} T^{L/2}}{1 + T_\star^{L/2} T^{L/2}} = 1 + \gamma_5 \mathcal{E}$$

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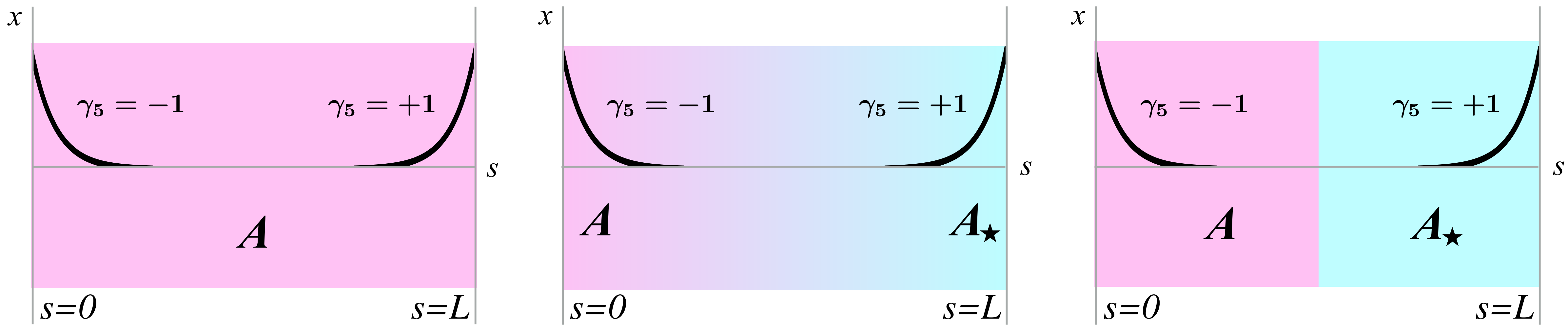

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$$\mathcal{E} = 1 - (1 - \epsilon_\star) \frac{1}{1 + \epsilon\epsilon_\star} (1 - \epsilon)$$

$$\epsilon = \epsilon(H[A])$$
$$\epsilon_\star = \epsilon(H[A_\star])$$

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★ D_χ has the desired long wavelength expansion

$$D_\chi = \begin{pmatrix} 0 & \sigma_\mu D_\mu(A) \\ \bar{\sigma}_\mu D_\mu(A_\star) & 0 \end{pmatrix} + O(a)$$

...LH chiral fermion that sees gauge field A
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★ Ambiguities in index theorem due to abrupt flow from A to $A_\star$

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$$\nu \equiv \frac{1}{2}\,\mathrm{Tr}\epsilon, \qquad \nu_\star \equiv \frac{1}{2}\,\mathrm{Tr}\epsilon_\star$$

Expect:

$$-\mathrm{Tr}\gamma_5 \hat{\mathcal{D}}_\chi = -\mathrm{Tr}\hat{\mathcal{E}}_\chi = -(\nu + \nu_\star)$$

$$\hat{\mathcal{E}}_\chi = \left[1 - (1-\epsilon_\star)\frac{1}{1+\epsilon\epsilon_\star}(1-\epsilon)\right]$$

If ν = ν★ (flow preserves winding number) we get the expected result.

If ν ≠ ν★ (flow violates winding number) find 0/0 because (1+εε★) is not invertible

★ Problem with sudden flow approximation

A ▸ A★ suddenly in middle of bulk, so fermions with local interactions can couple simultaneously to both gauge fields…

then the gauge invariant field (A - A★) can be generated in the effective theory as bulk modes are integrated out — can give rise to strange nonlocal interactions for matter

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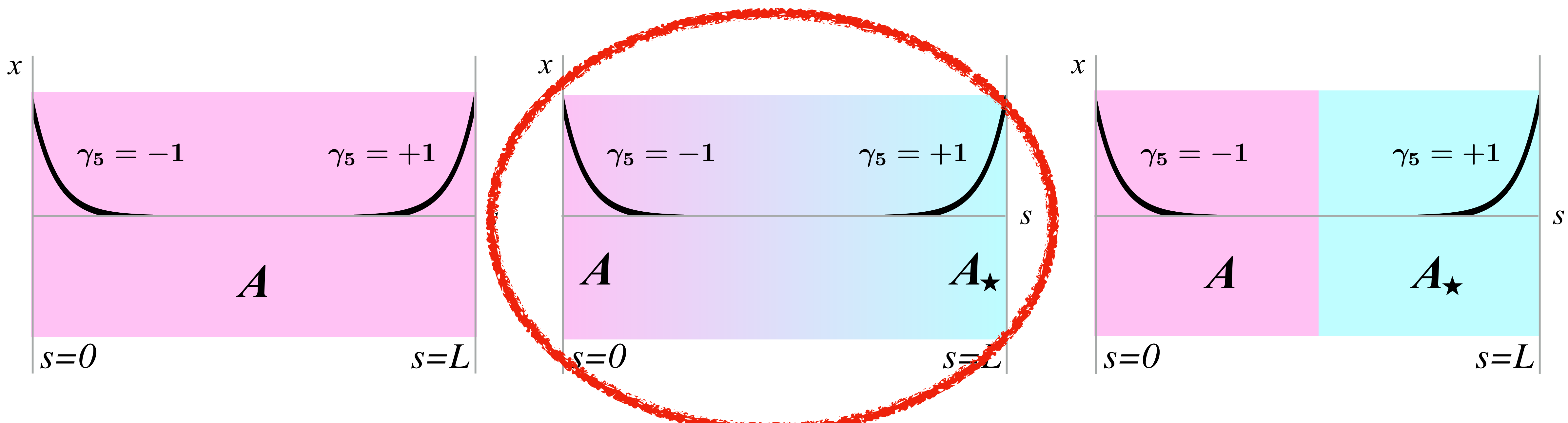
Would like an overlap operator for gradual flow...but do not know a closed form expression.



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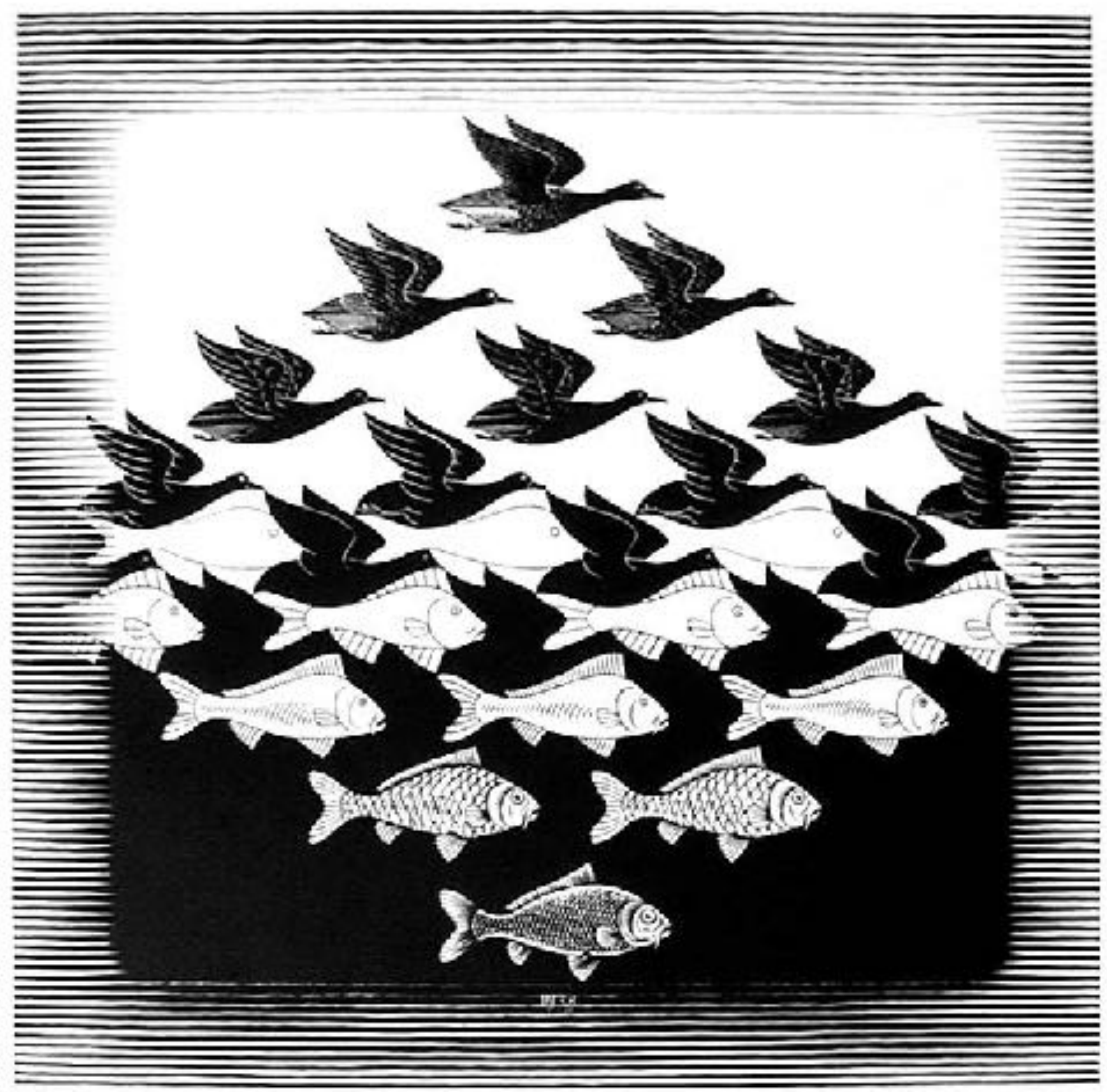
★ ...if that all goes well, what is the phenomenology of fluff?

Aimed for:



Found:

Future goal: see if there exists a way for the overlap operator to more closely reproduce Alvarez-Gaumé et al. paper (lattice realization of the η-invariant)