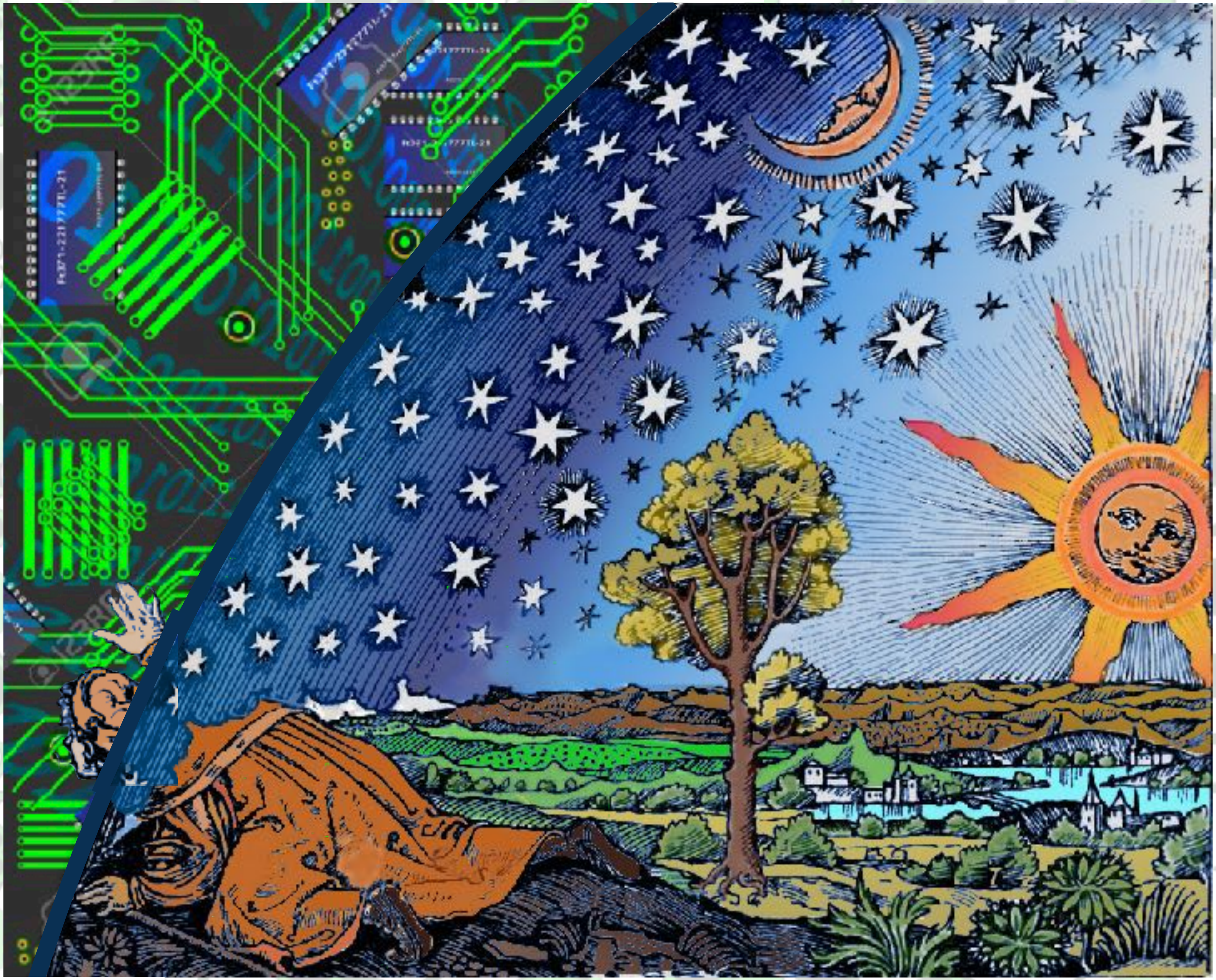
Computing Reality



D. B. Kaplan ~ ICTS Bengaluru ~ 31/1/18

Monthly Notices of the Royal Astronomical Society, Vol. 91, p.456-466, 1931

The Highly Collapsed Configurations of a Stellar Mass.
By S. Chandrasekhar.

(*Communicated by Professor E. A. Milne.*)

§ 1. Professor Milne in his recent paper * on "The Analysis of Stellar Structure" has put forward some essentially new considerations on the possible steady-state configurations of stellar aggregates of varying mass, luminosity, and opacity. One of the main consequences of the analysis is the explanation not only of the existence of white dwarfs—his collapsed configurations—but also of the principal physical characteristics of these configurations. The following is devoted to the development of Milne's theory of these collapsed configurations a stage further.

§ 2. Milne's estimates for the central density and temperature of these collapsed configurations indicate that in some cases we pass beyond the range of validity of the degenerate form of the Fermi-Dirac equation of state ($p = K\rho^{5/3}$). It can be shown that the pressure of an electron gas which is highly degenerate and which has a very highly predominant relativistic-mass variation effect, takes the limiting form †

$$p = \frac{n^{4/3}hc}{8}\left(\frac{3}{\pi}\right)^{1/3} \quad . \quad . \quad . \quad . \quad (1)$$

(c = velocity of light, h = Planck's constant) if the following two conditions are satisfied :—

What is computational physics?

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Not...

Great theorists...

H $\mathcal{L}$

...Computational physicists

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...but a branch of physics that requires great creativity and greatly deepens our understanding of physics.

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- Chirality, extra dimensions and topology

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...Computational physicists

- The study of (next to) nothing
- Chirality, extra dimensions and topology
- The study of something, signs of trouble, and the quantum computer

I. The study of (next to) nothing

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Electromagnetism

photon

photons only couple to charge
and not each other

wave solutions can be superimposed

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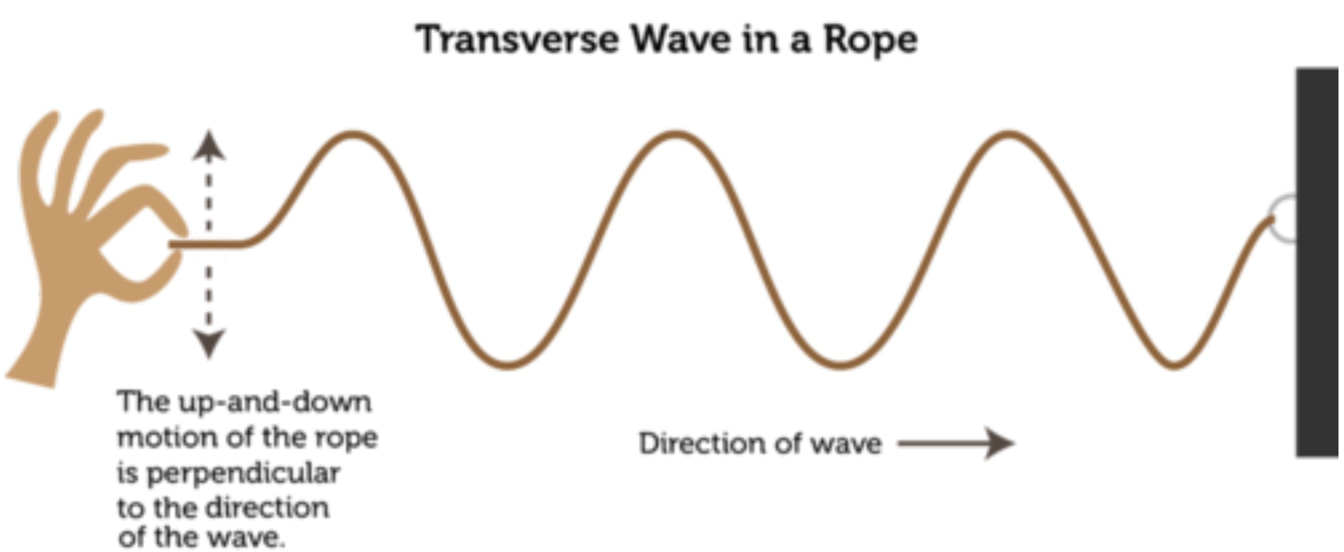
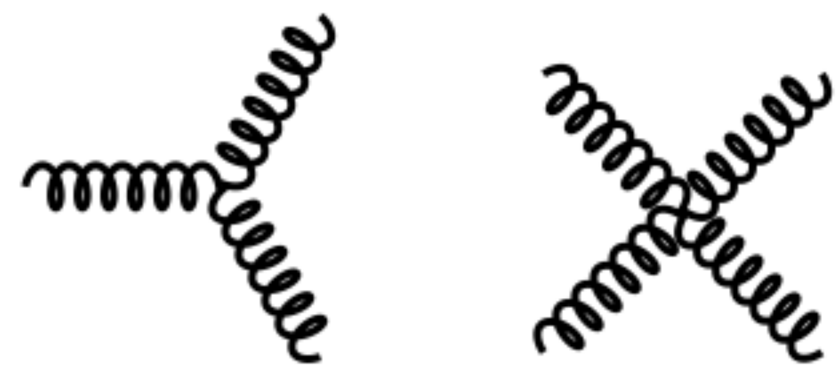


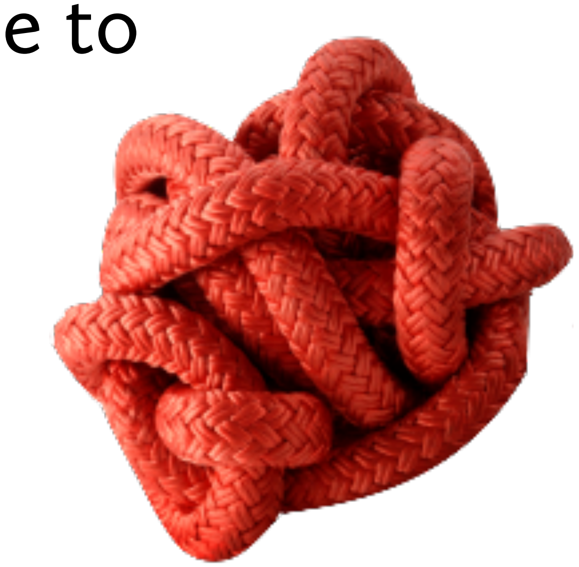
wave solutions can be superimposed

Strong interactions (QCD)

gluon

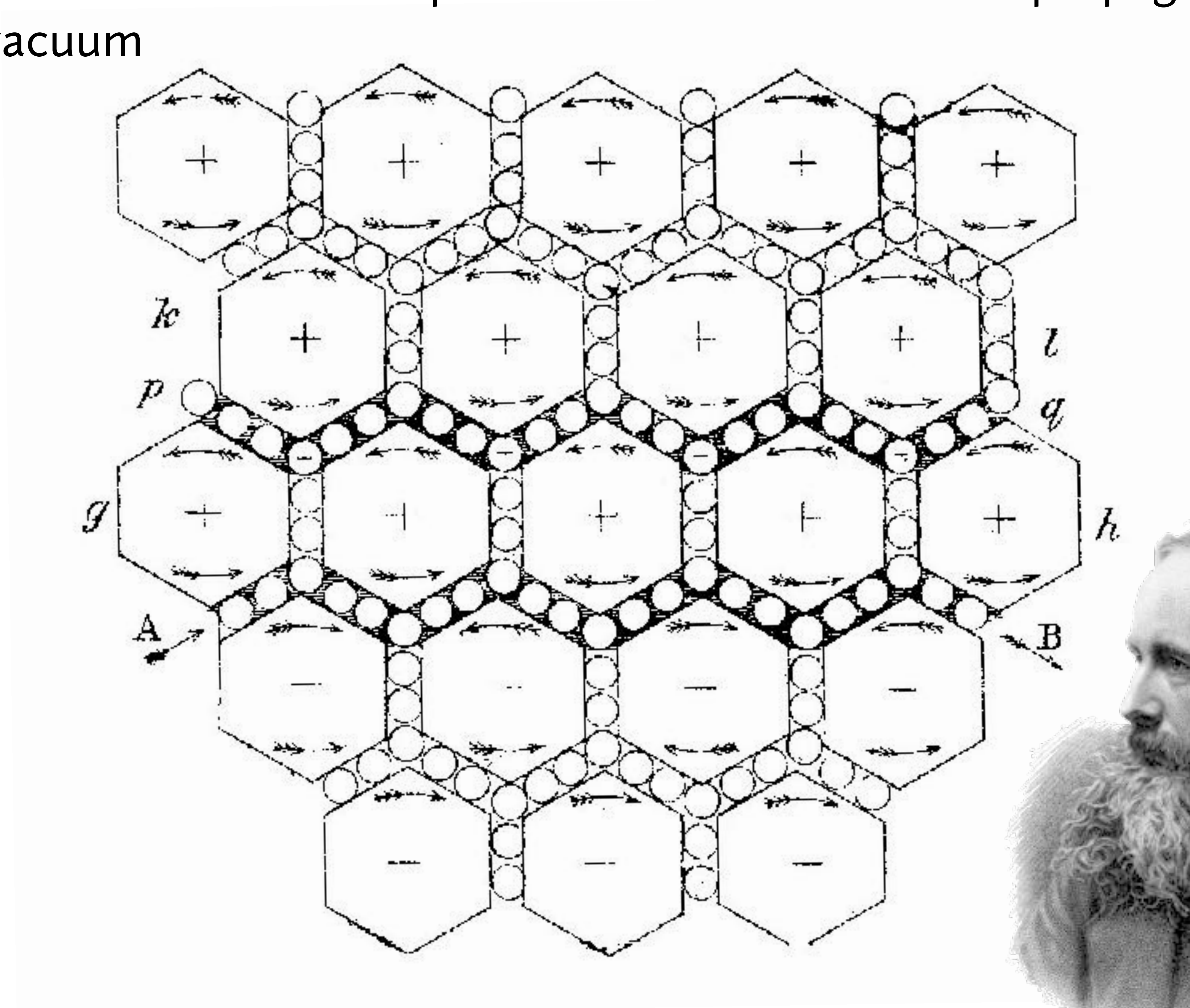


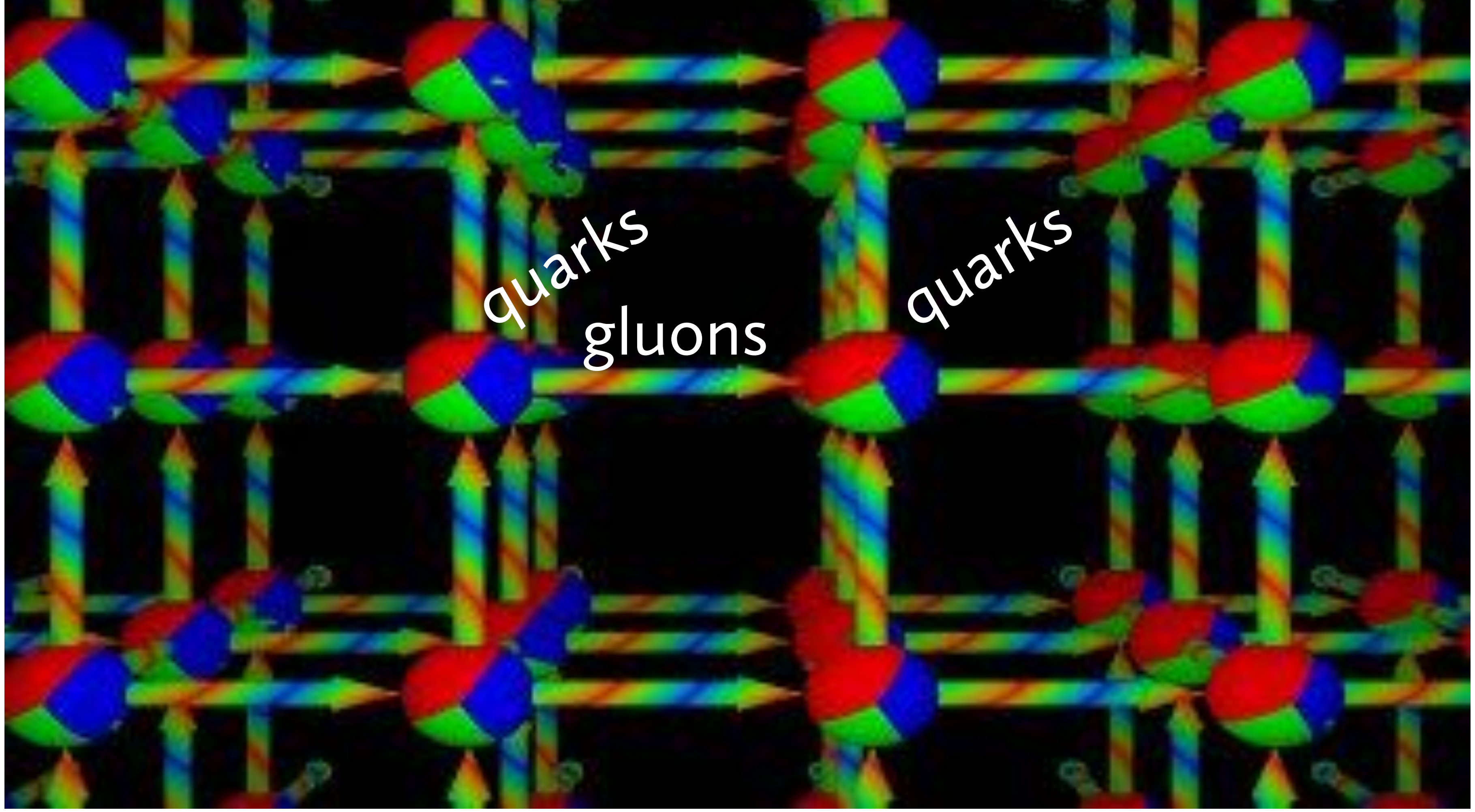
gluons carry the charge they couple to



nonlinear interactions are complicated!

Maxwell's first concept of "luminous aether": the propagation of light in vacuum

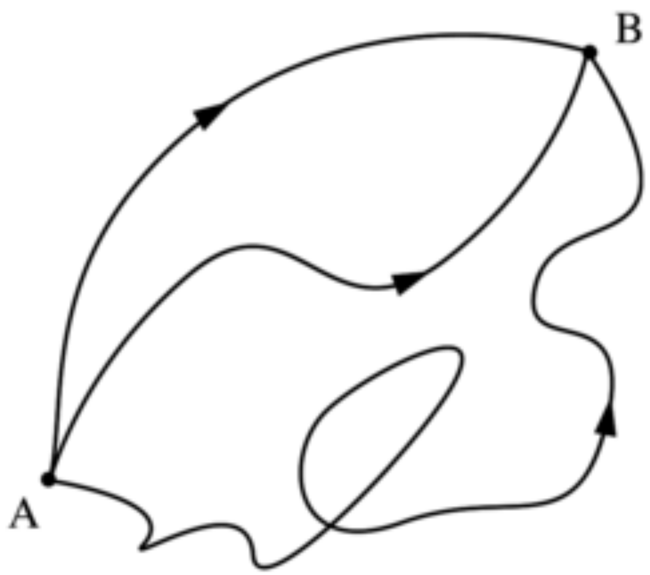




Ken Wilson

To put QCD on a computer, first approximate spacetime as a lattice of points.

The tool for lattice field theory computations:



Feynman path integral

Richard Feynman

Sum over "paths" through field space, weighted by exp[-classical action]

Quantum field theory becomes a task of computing a huge integral

32 x 32 x 32 x 64 site lattice for QCD:
millions of degrees of freedom

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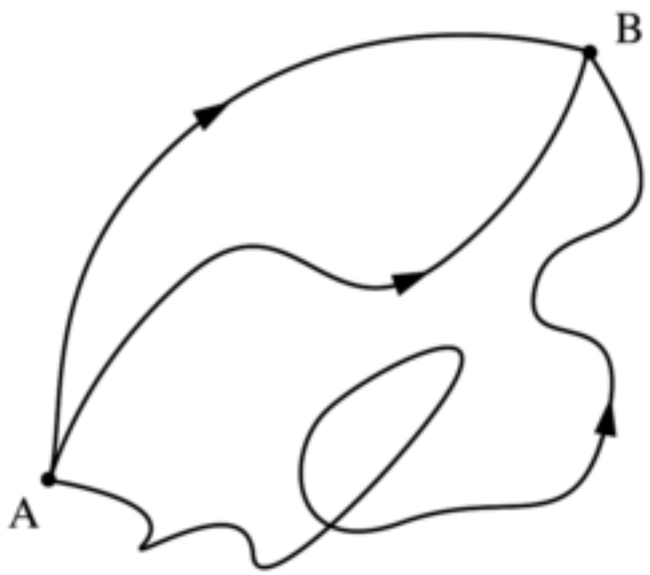

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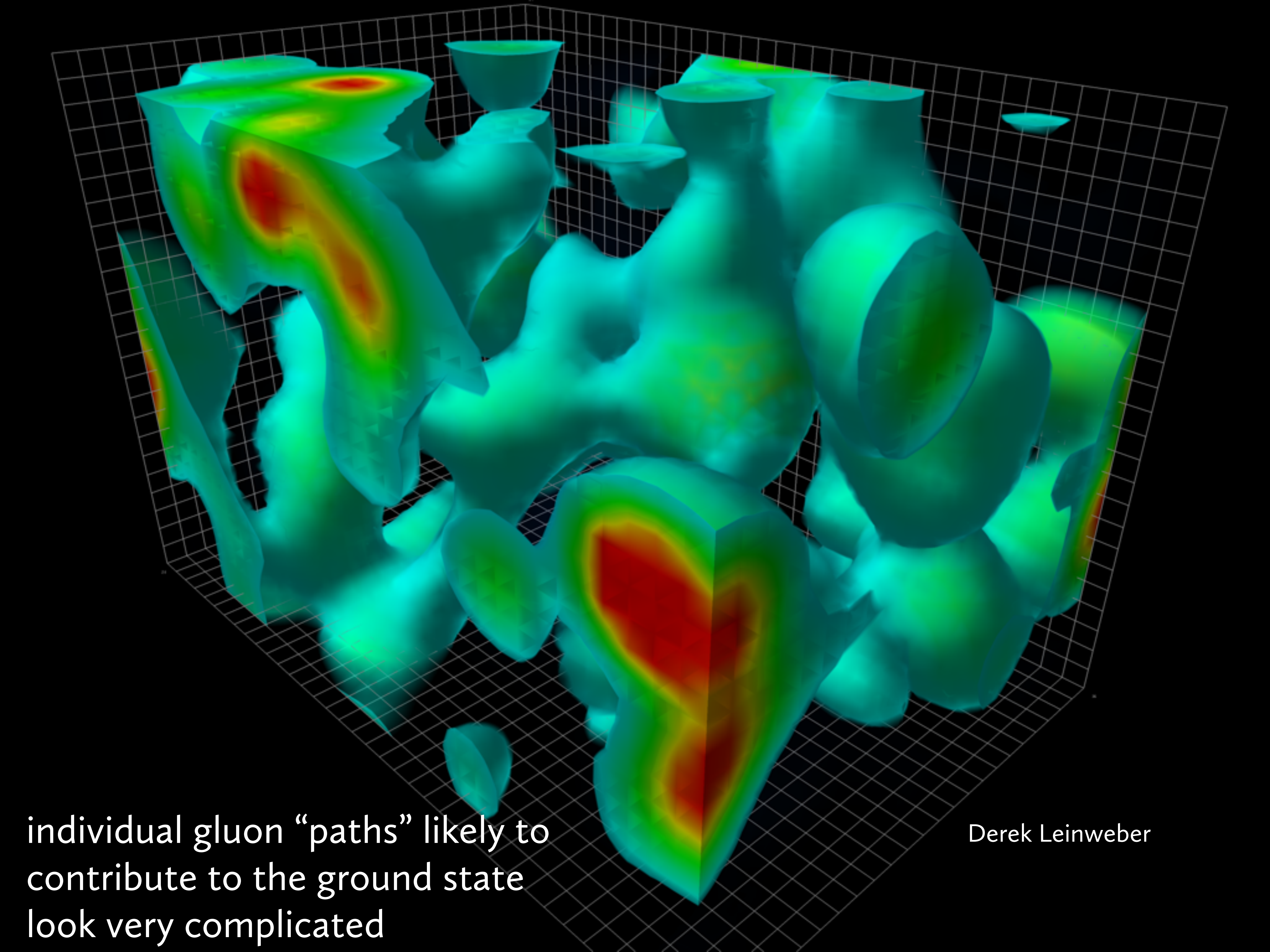
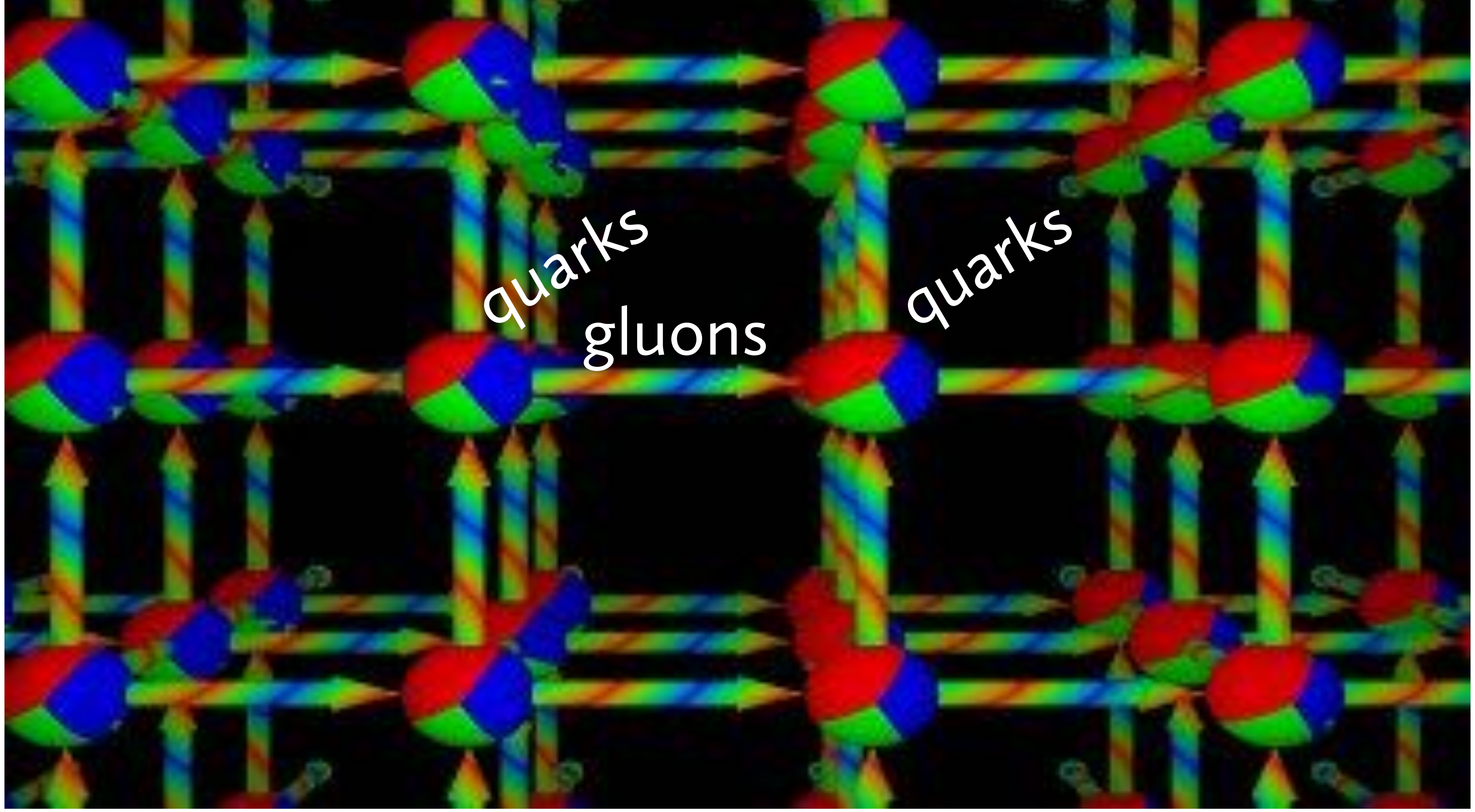

individual gluon “paths” likely to contribute to the ground state look very complicated
Derek Leinweber



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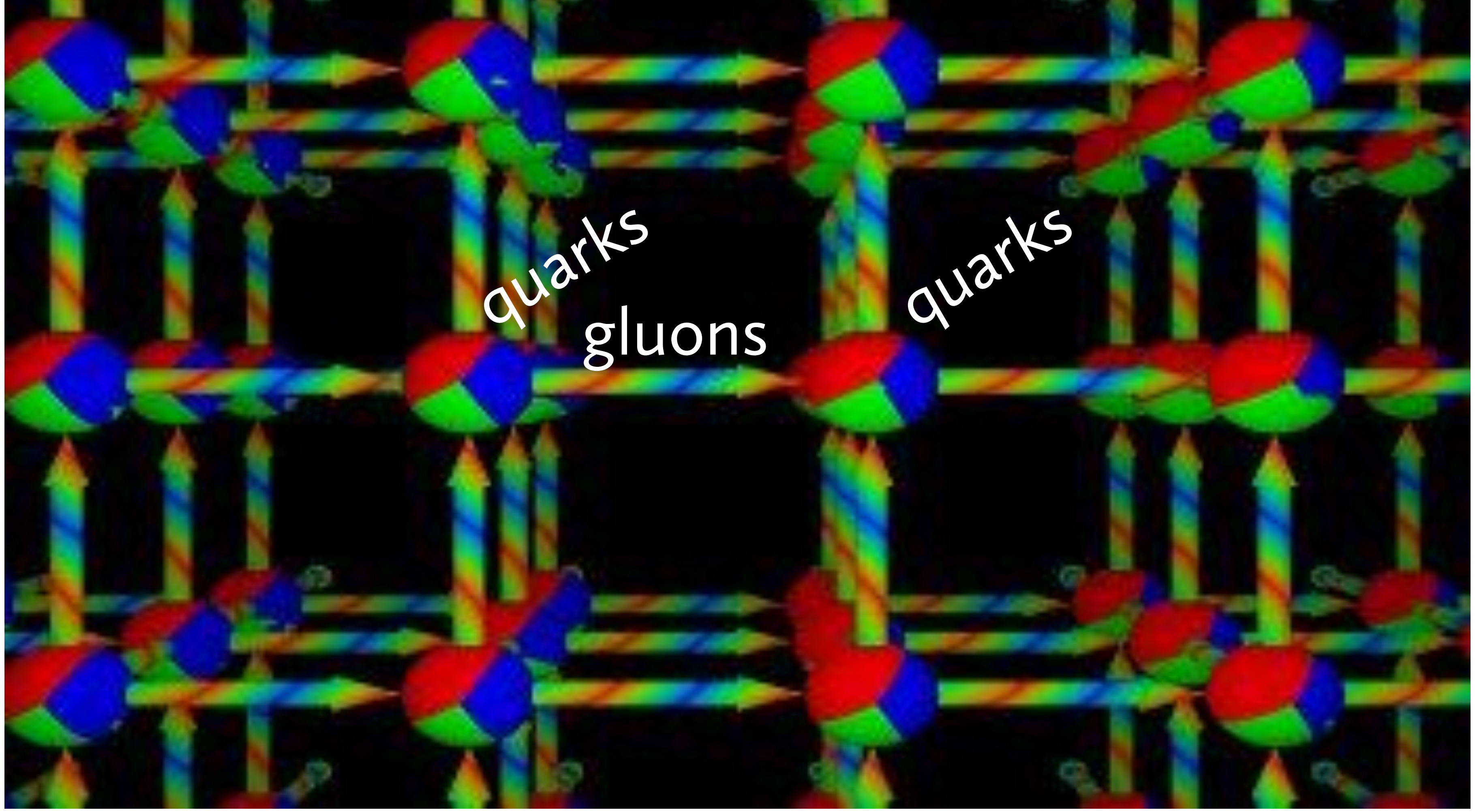

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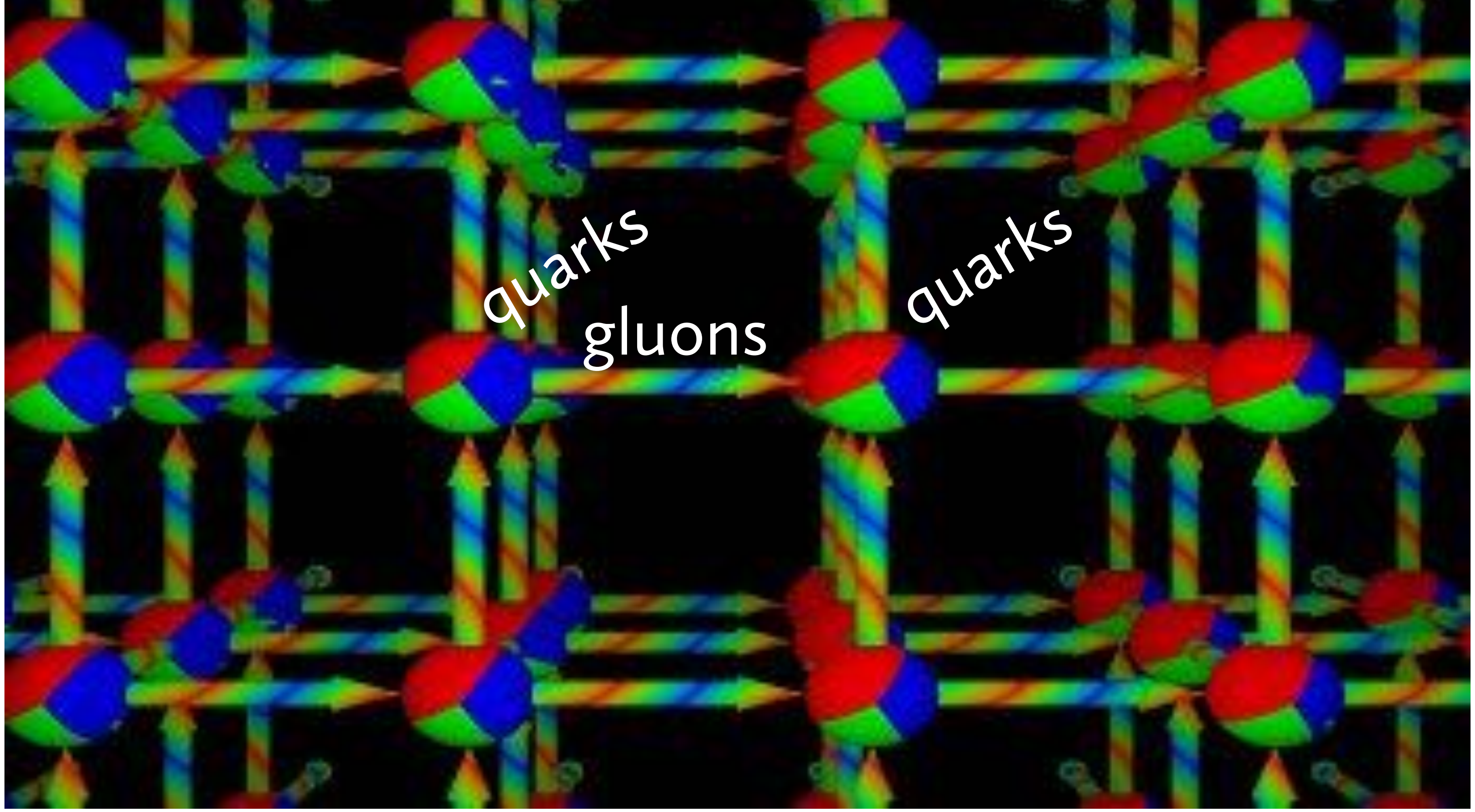
But:

- How can a lattice end up looking like continuous space-time?
- How can a computation that is necessarily finite work for quantum field theory, which is notorious for infinities and requires “renormalization”?
- What is gauge symmetry, and how do we see phenomena such as confinement and chiral symmetries of quarks?



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All of these questions arise even when studying the vacuum… “nothing”

The QCD vacuum looks like “nothing” to us as water to fish.

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Potential for a PR problem

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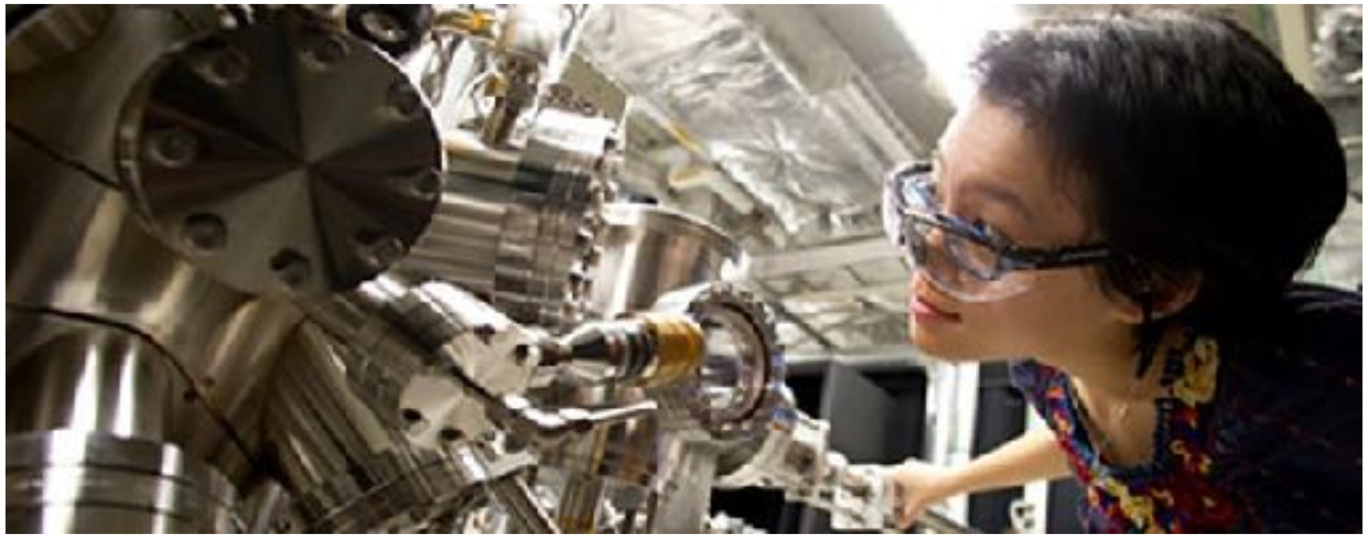
Potential for a PR problem

Wilson took inspiration from condensed matter experimentalists, who can study the ground state of materials without crawling inside them.



Lattice QCD looks like the physics of a (complicated) crystal

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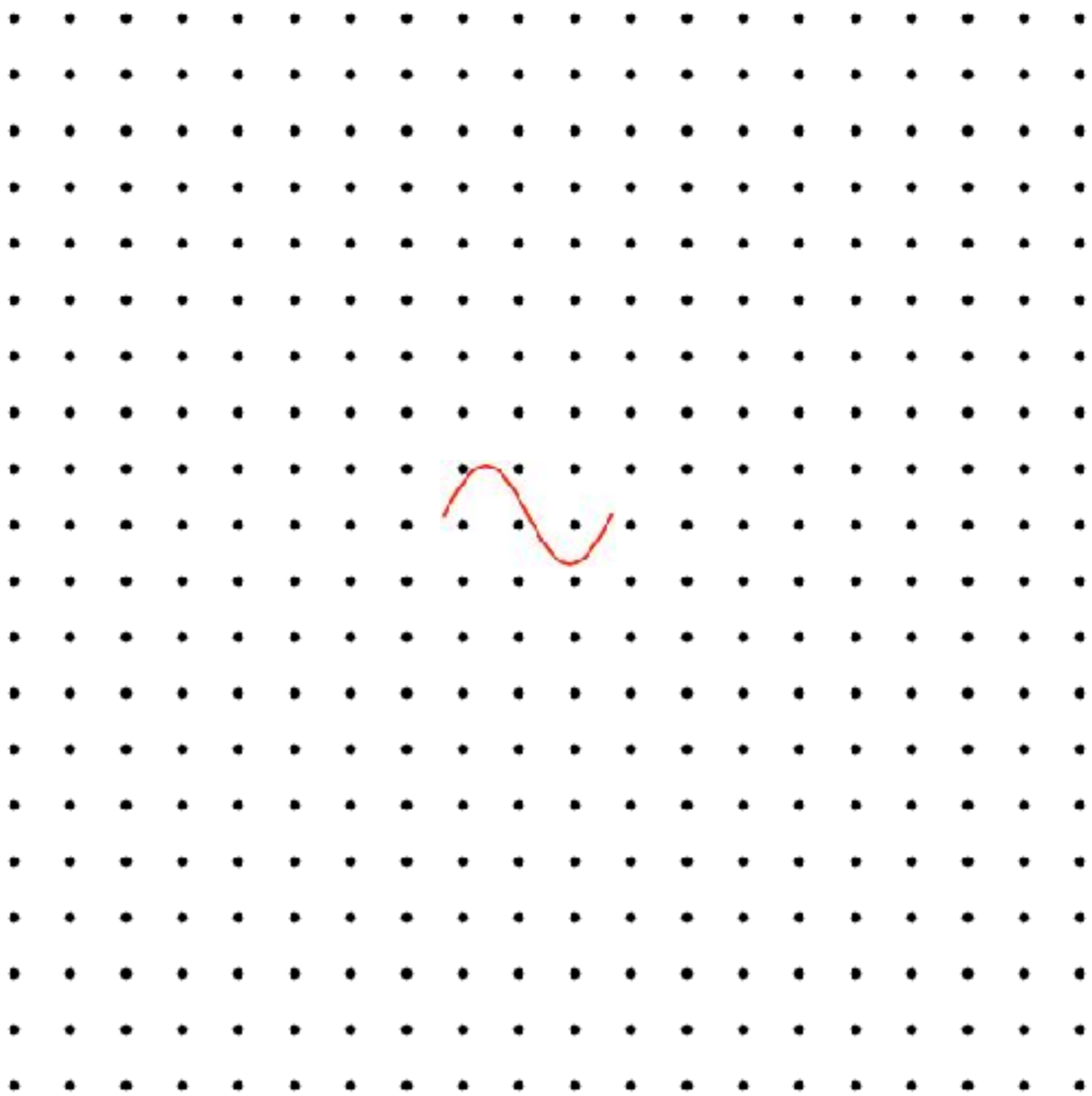
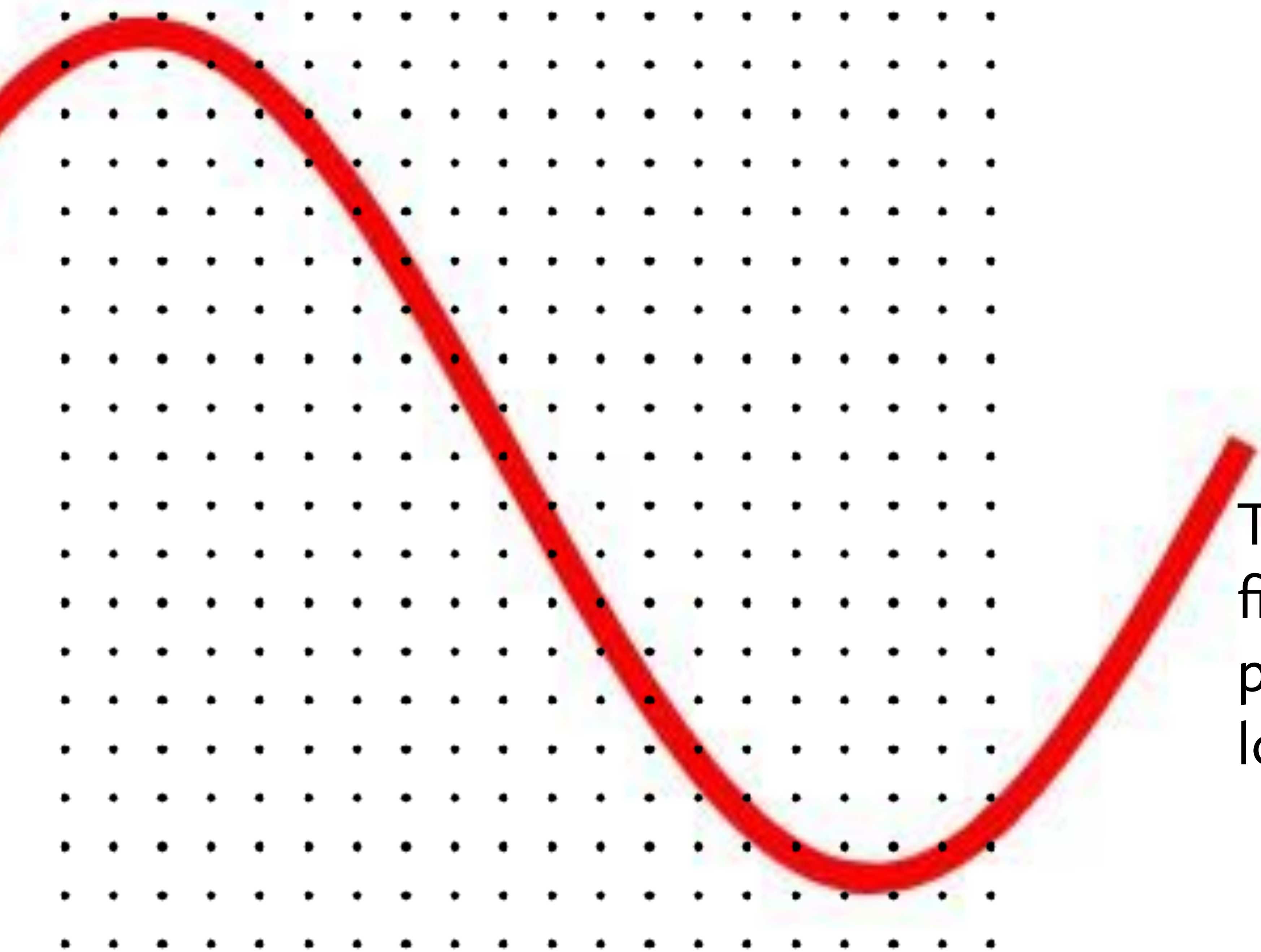
For example, we need to recover rotation and translation invariance



Think of lattice spacing as fixed; require physical particles to have very long wavelengths



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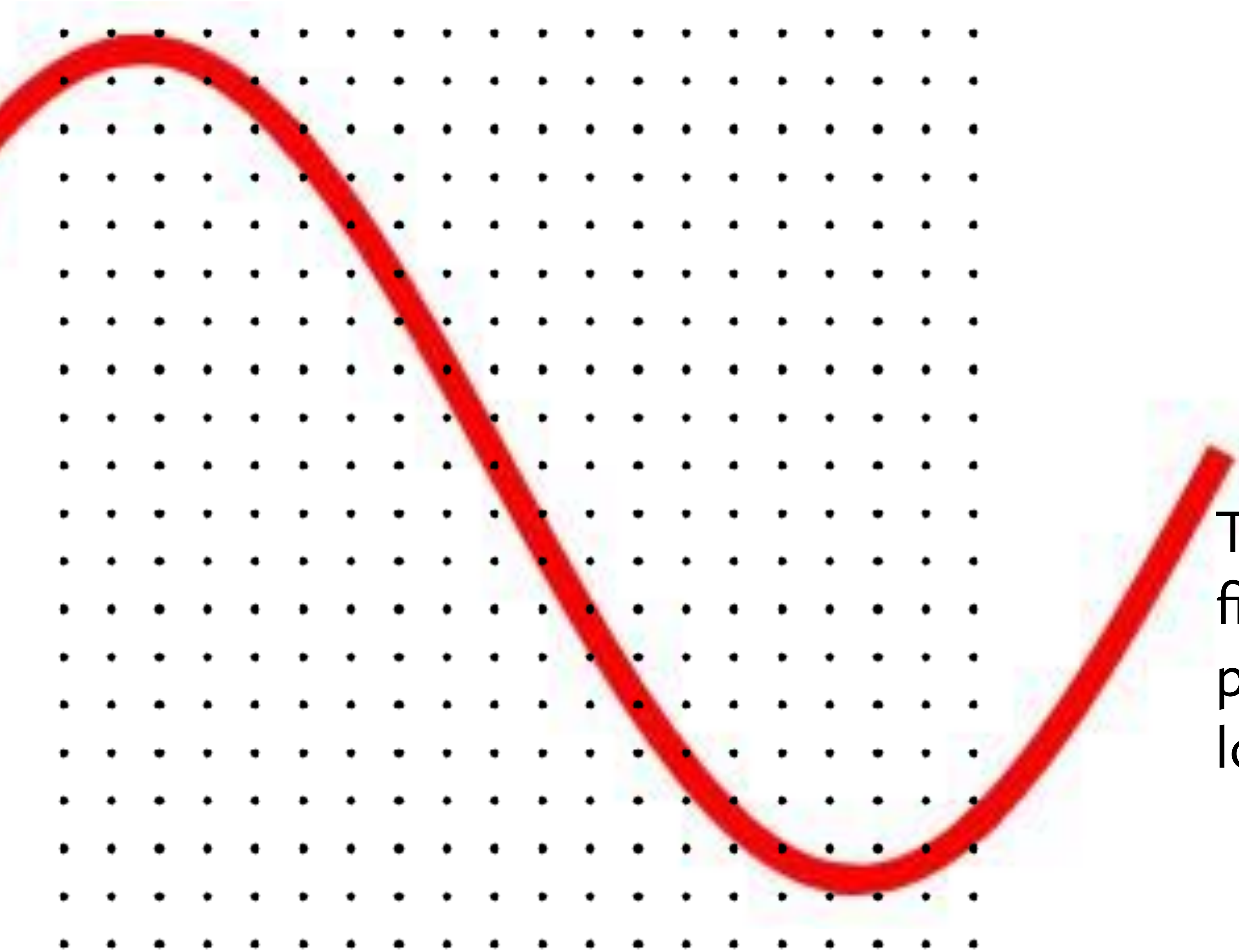

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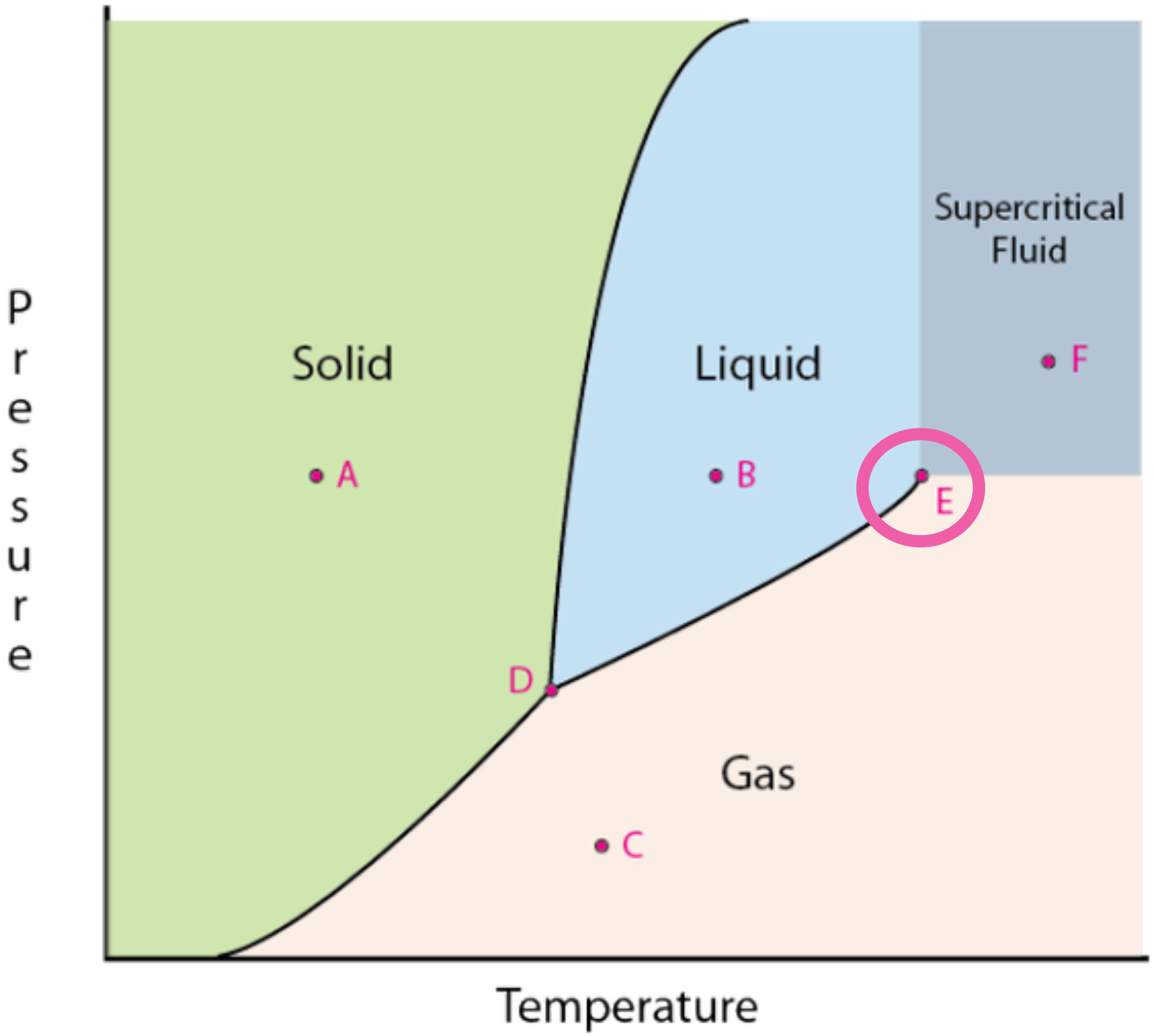
In condensed matter systems, this usually only occurs near 2nd order phase transitions



For example, tuning pressure and temperature in a liquid to point E

In a quantum field theory one tunes the coupling constant(s).

These constants are finite… no manipulation of infinities

Before Wilson:

- renormalization meaning the hiding infinities
- bad theories being ones where you cannot hide them

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- renormalization = tuning lattice parameters toward 2nd order phase transition with correlation lengths >> the length scales we can probe experimentally

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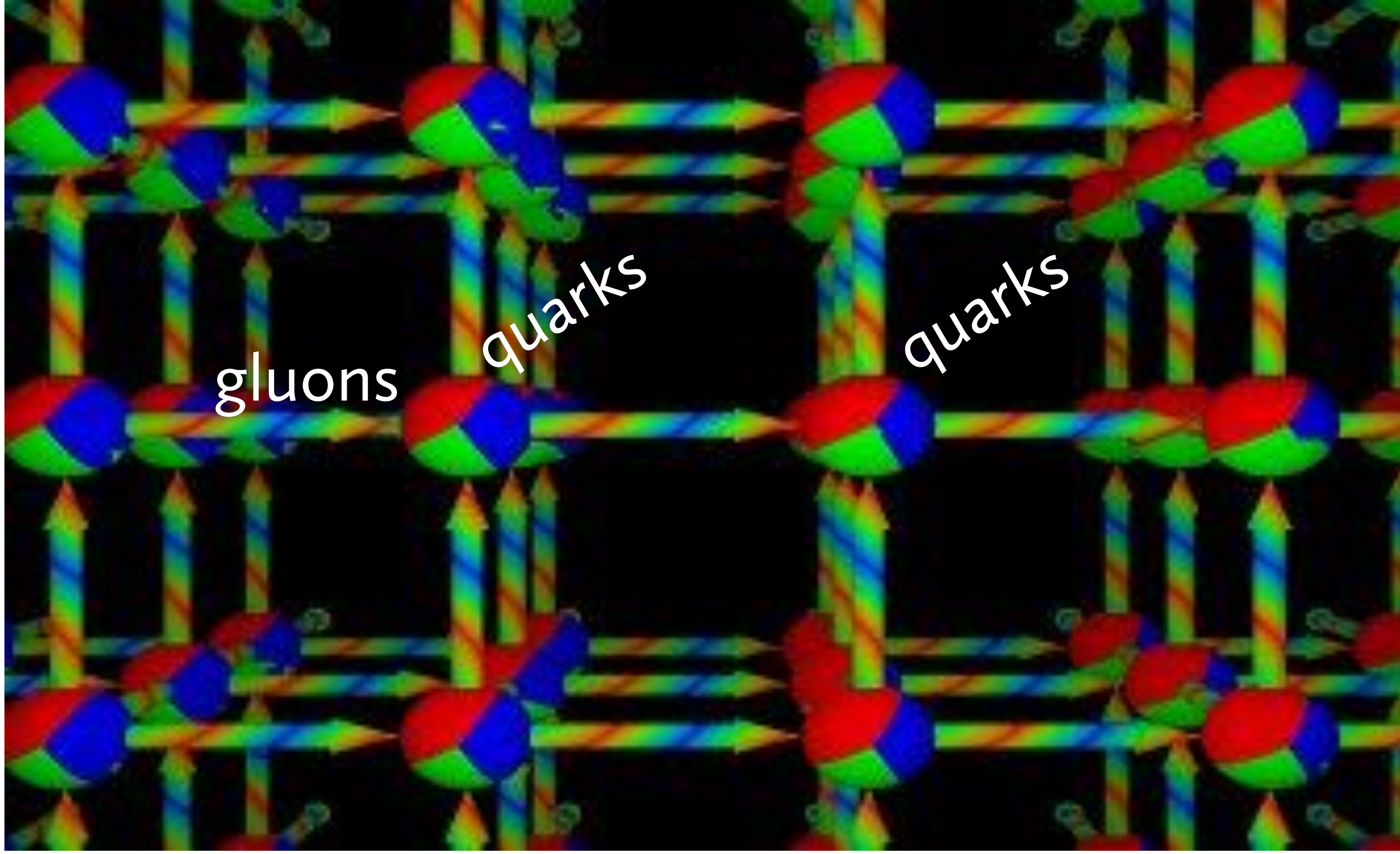
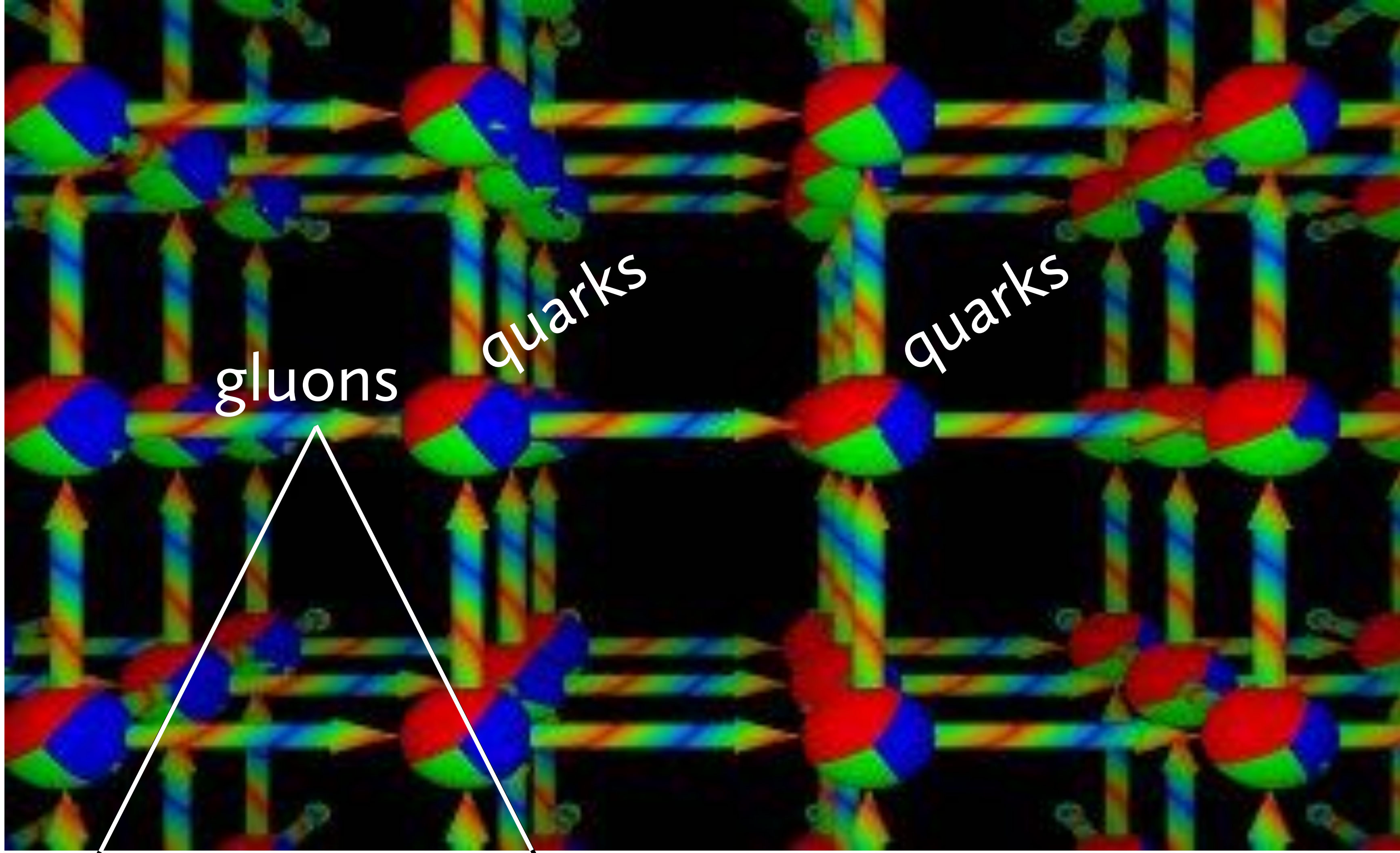
- Higgs in the Standard Model: "good" ‣ "peculiarly fine tuned"

More conceptual insights from Wilson's lattice: gauge symmetry and confinement



Gluons allow quark phases to rotate independently…

More conceptual insights from Wilson's lattice: gauge symmetry and confinement



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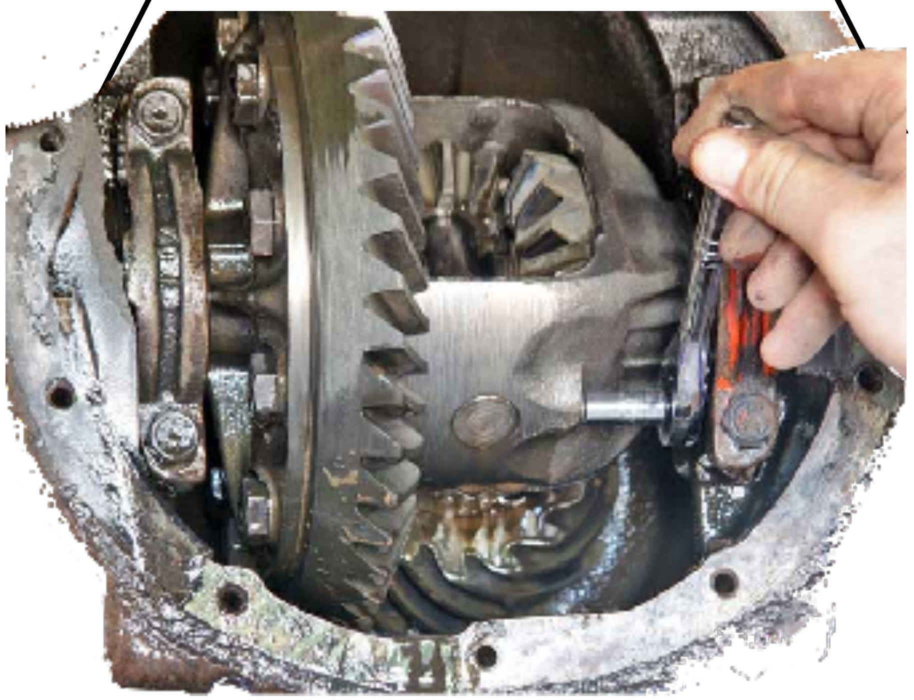

Like the differential in a car

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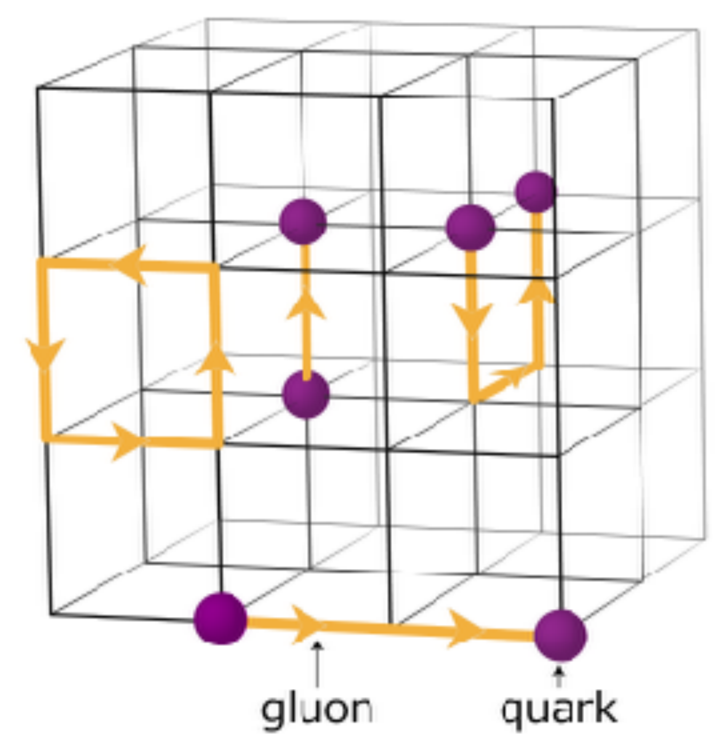

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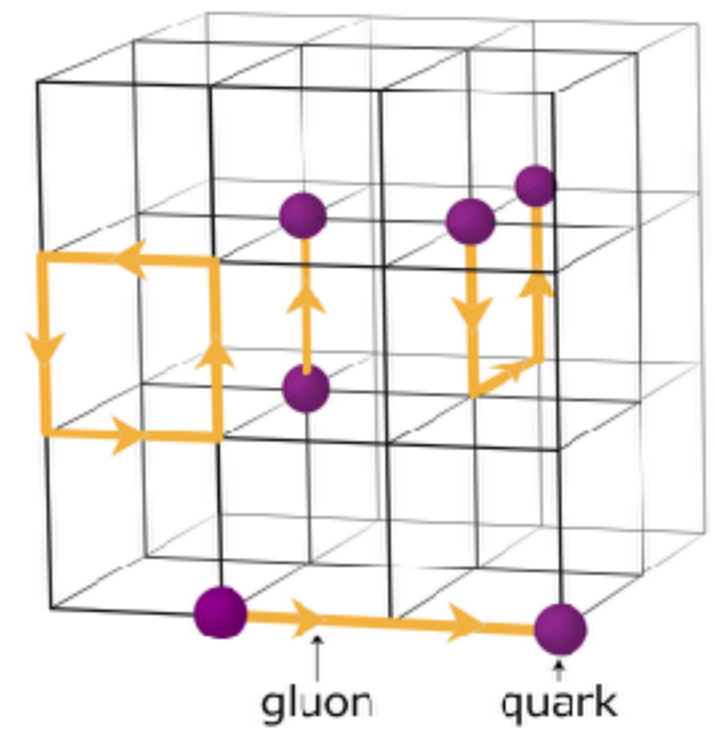

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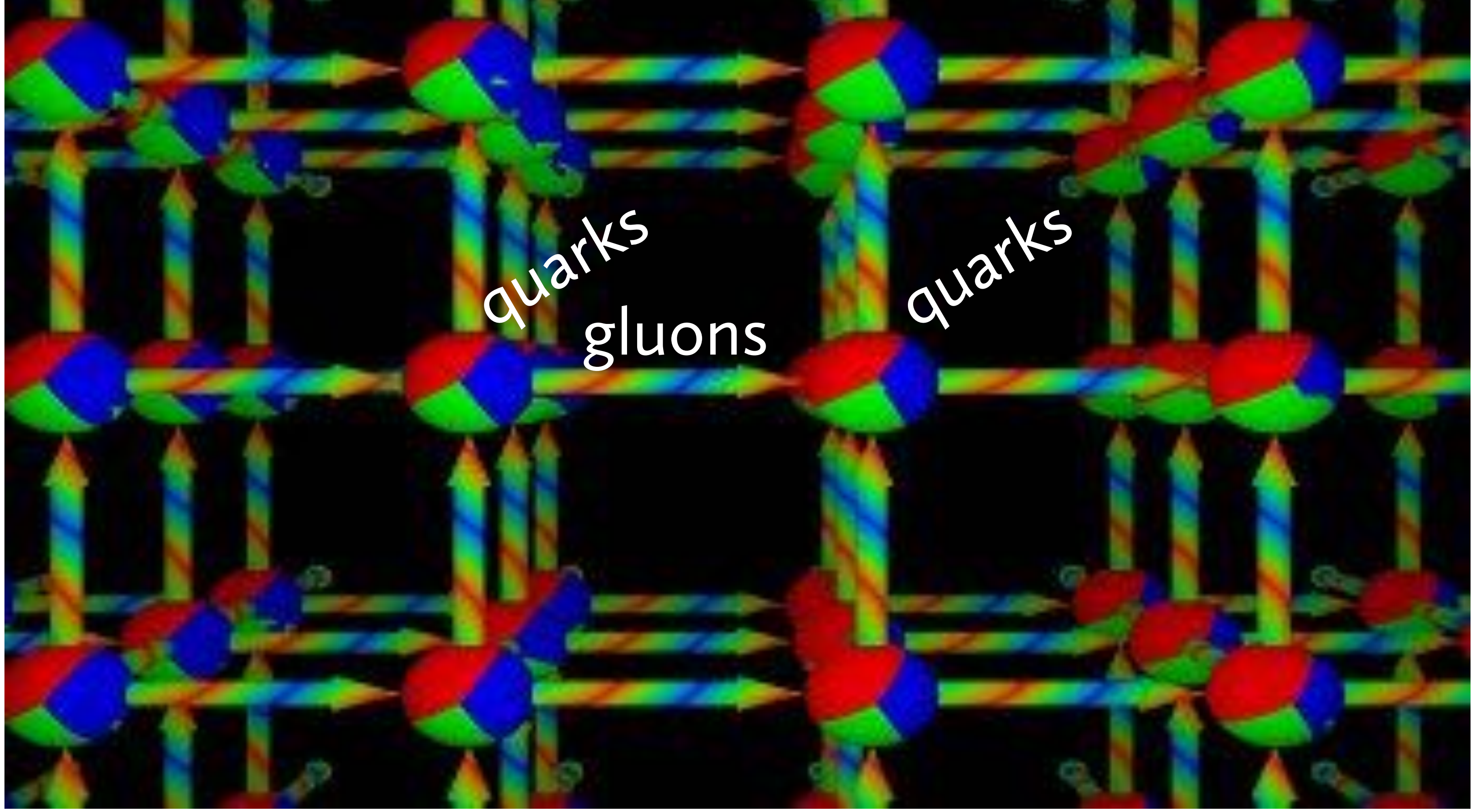
Wilson discovered how to show that QCD confines quarks by studying the property of such loops



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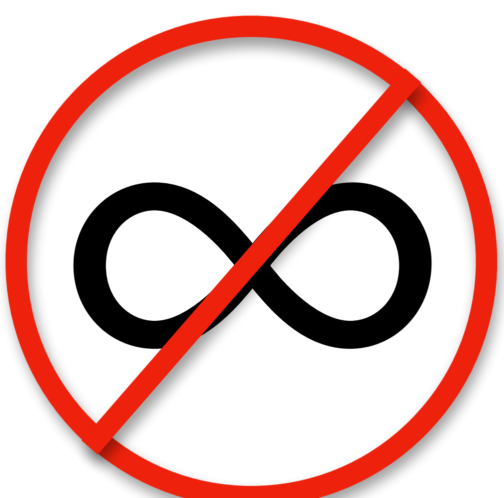
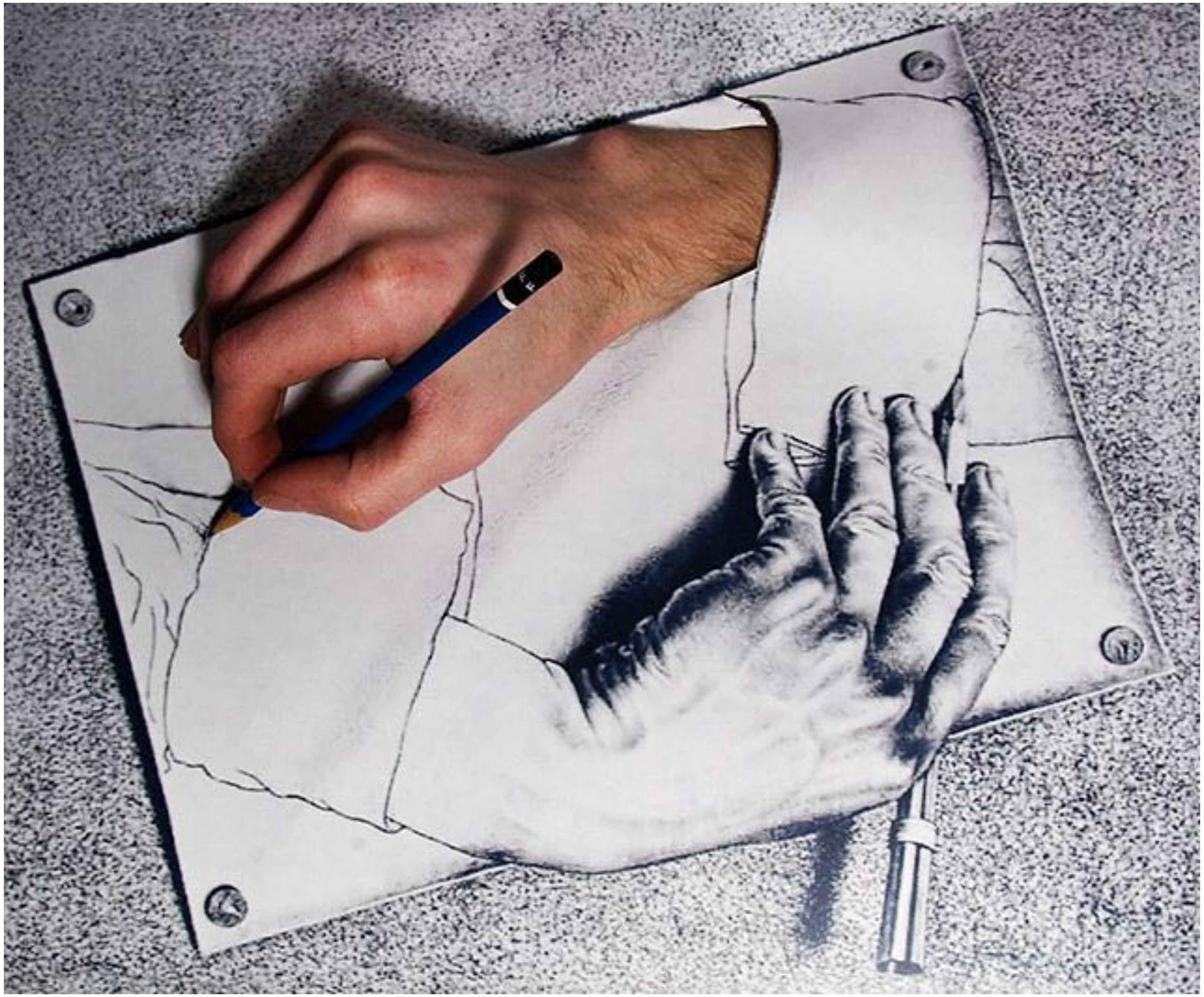
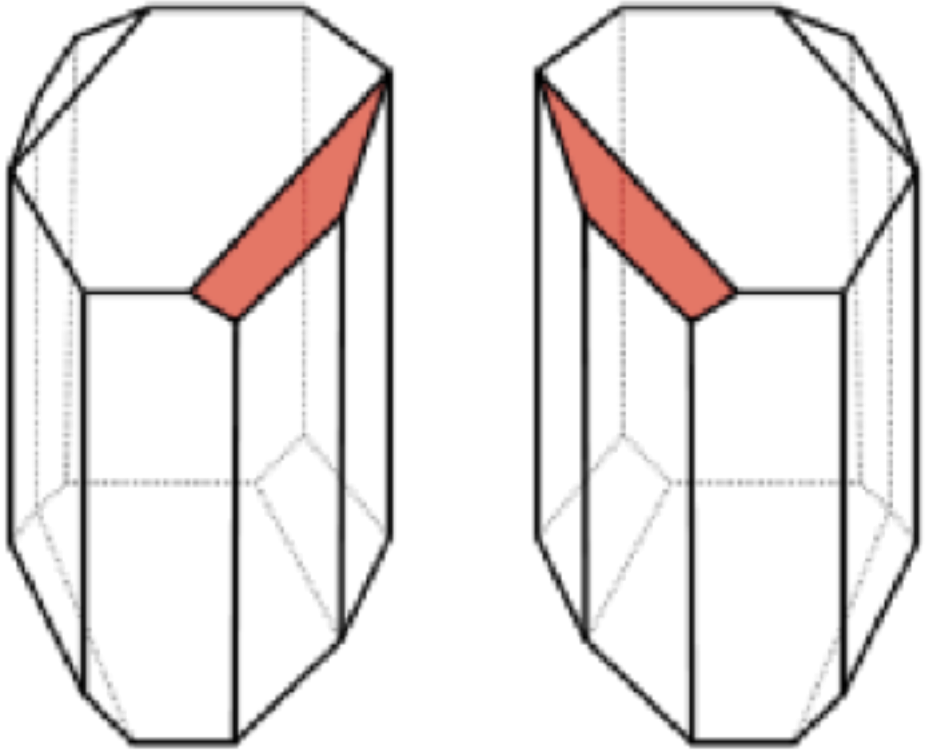


By asking how to put QCD on a computer Wilson transformed how people <u>think</u> about quantum field theory

- Renormalization group
- Effective field theory
- Confinement, etc.

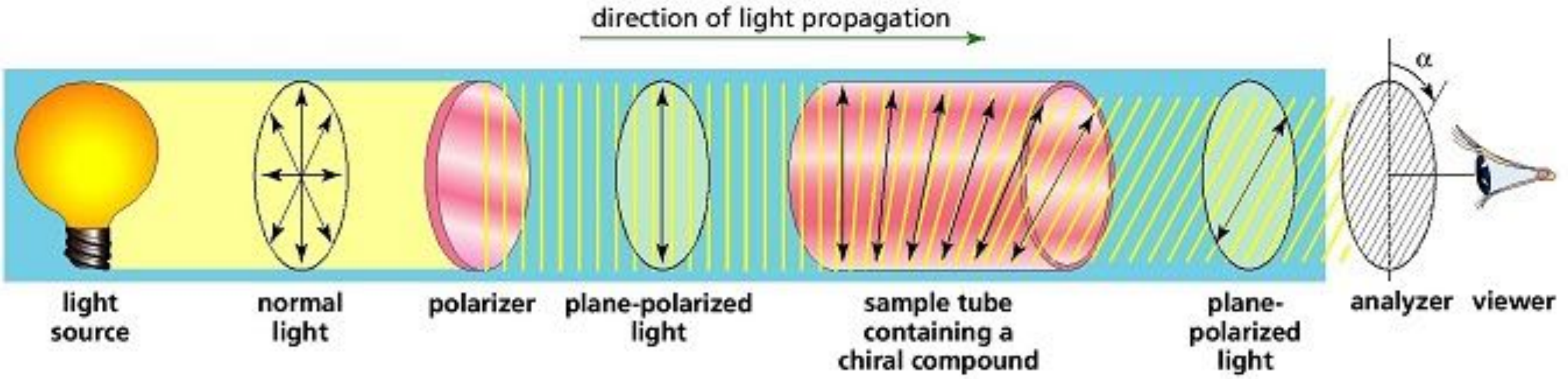
II. Chirality, extra dimensions, and topology





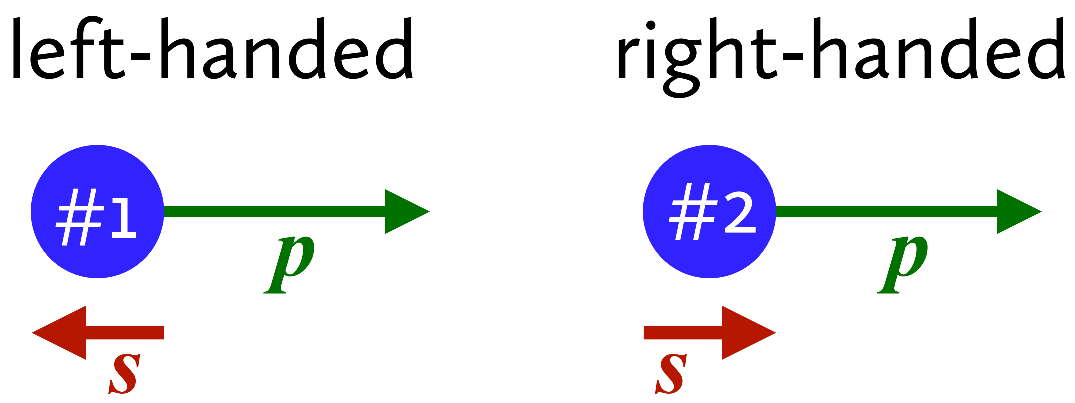
Left- and right-handed tartaric acid crystals

Louis Pasteur



Chirality for fermions?

Helicity: spin dotted into momentum



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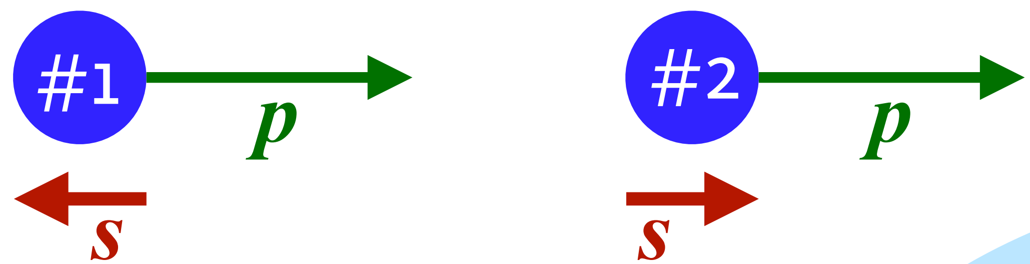
left-handed　　　right-handed



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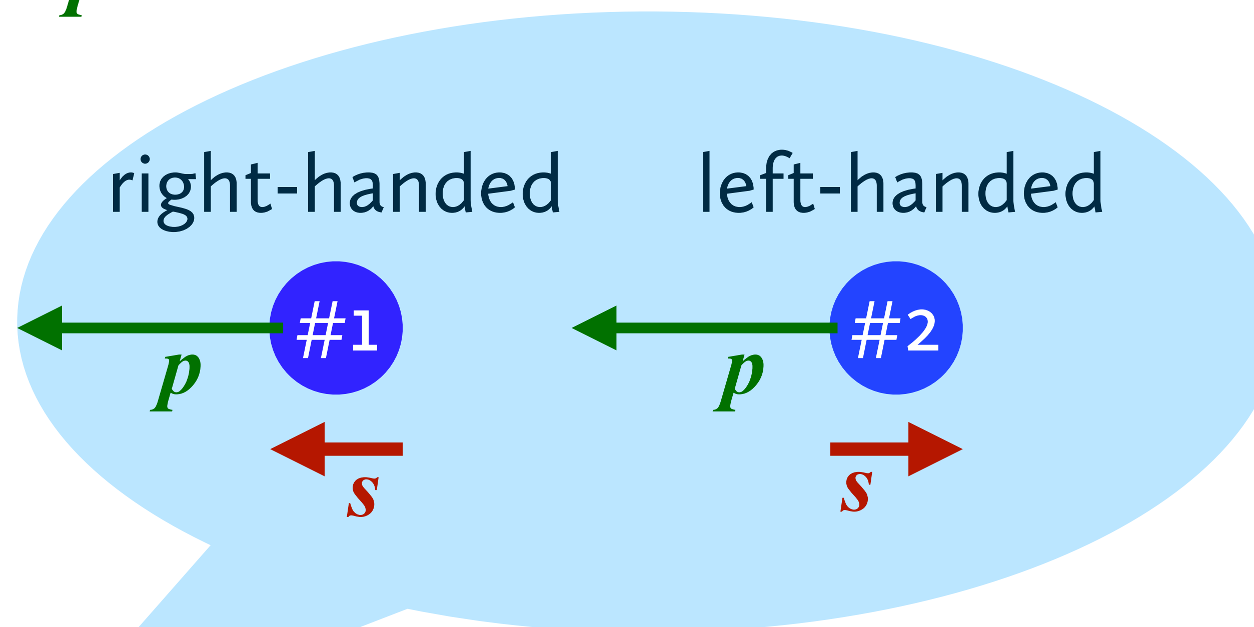

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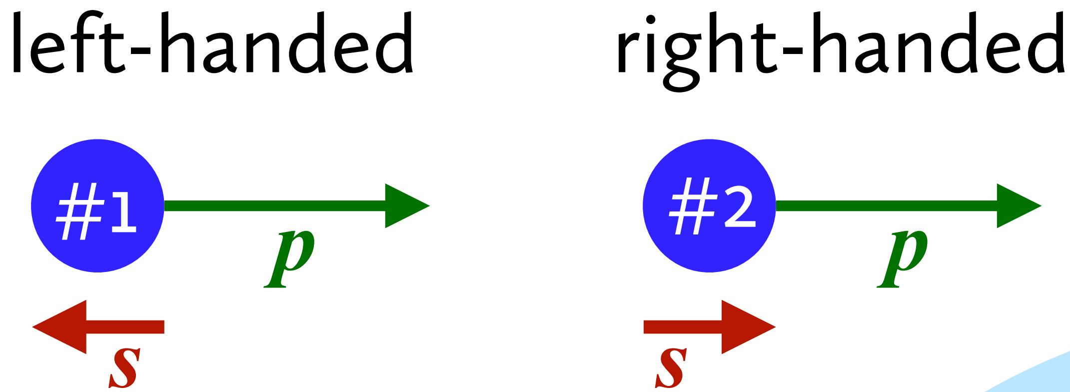

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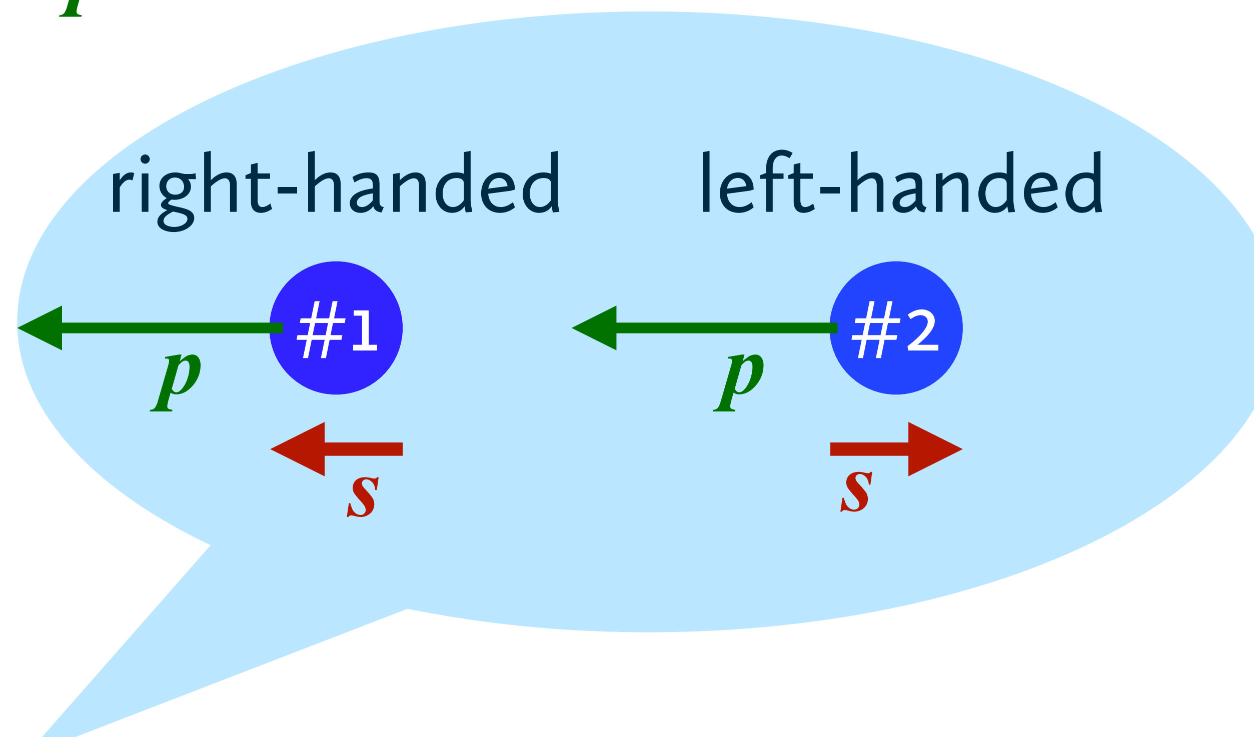
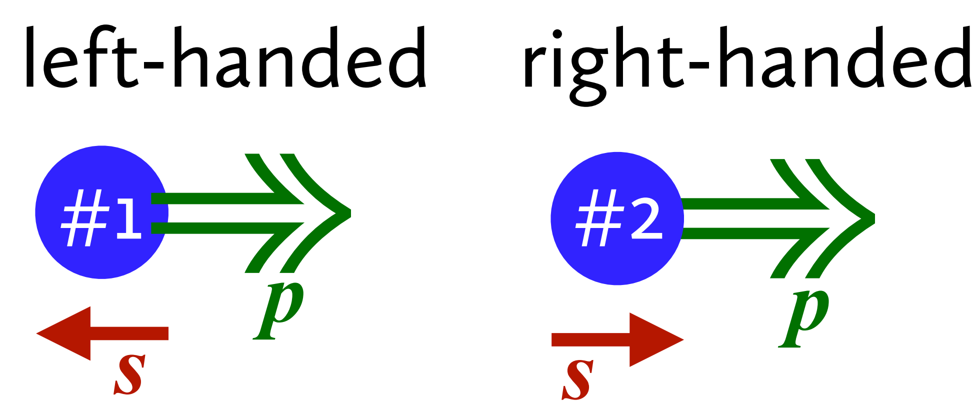


<u>...unless particles are massless moving @ speed of light!</u>



Helicity for <u>massless</u> particles = “chirality” …. apparently a conserved quantity

Conservation law implies symmetry: “chiral symmetry”

Emmy Noether

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Can define an <u>approximate</u> chiral symmetry obeyed by massive particles

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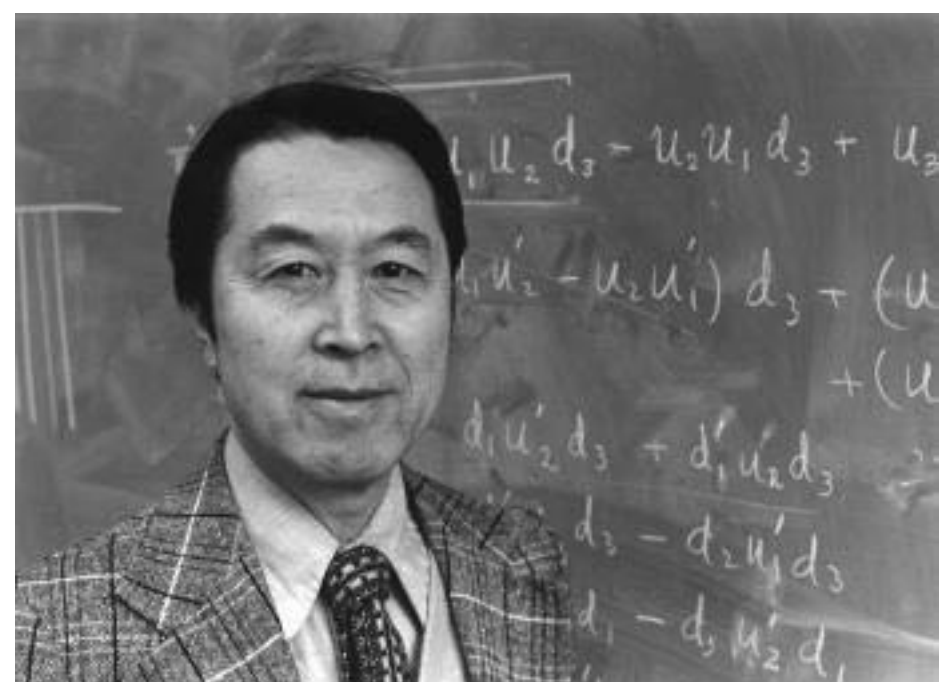
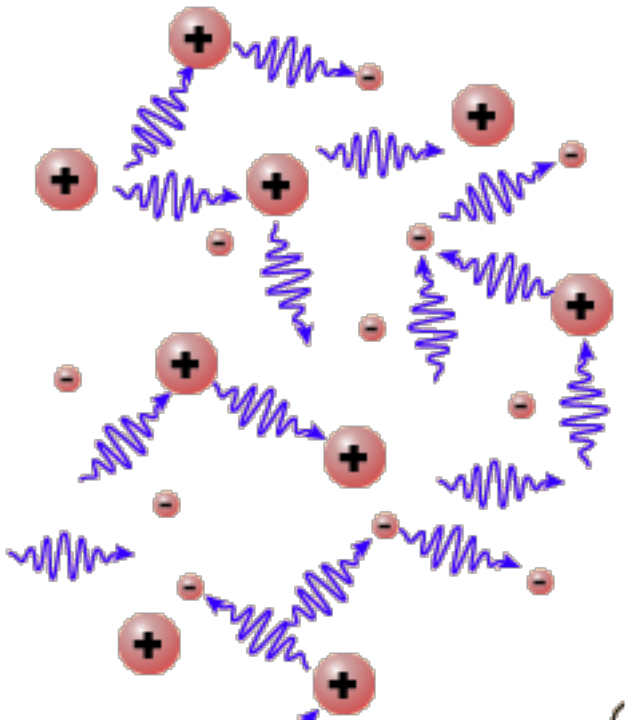
Approximate chiral symmetry is an important feature of QED…
And a very important feature of the QCD!



Yochiro Nambu

Nambu was fascinated by the BCS theory of superconductivity



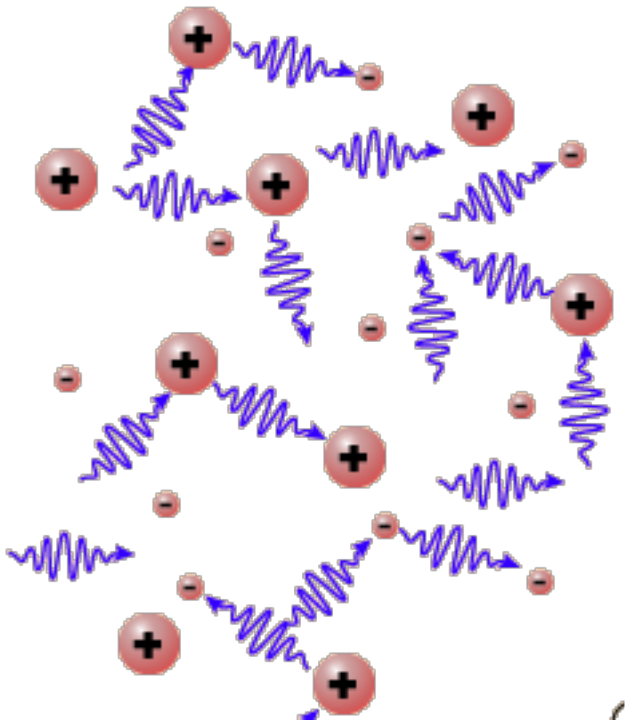
The ground state in a superconductor has complicated correlations between electrons

Nambu was fascinated by the BCS theory of superconductivity

Nambu suggested that the universe has a similarly complicated ground state (vacuum)

the vacuum is full of correlated quark-antiquark pairs

...and this spontaneously breaks approximate chiral symmetry, explaining the lightness of the pion mesons



The ground state in a superconductor has complicated correlations between electrons

To do lattice QCD computations requires knowing how to formulate the lattice action in a way that preserves chiral symmetry....

...but all straightforward approaches (e.g. Wilson's) destroy chiral symmetry, for a deep reason

Problems realizing chiral symmetry in lattice QCD...
The key is strange behavior in the continuum: "the anomaly"!

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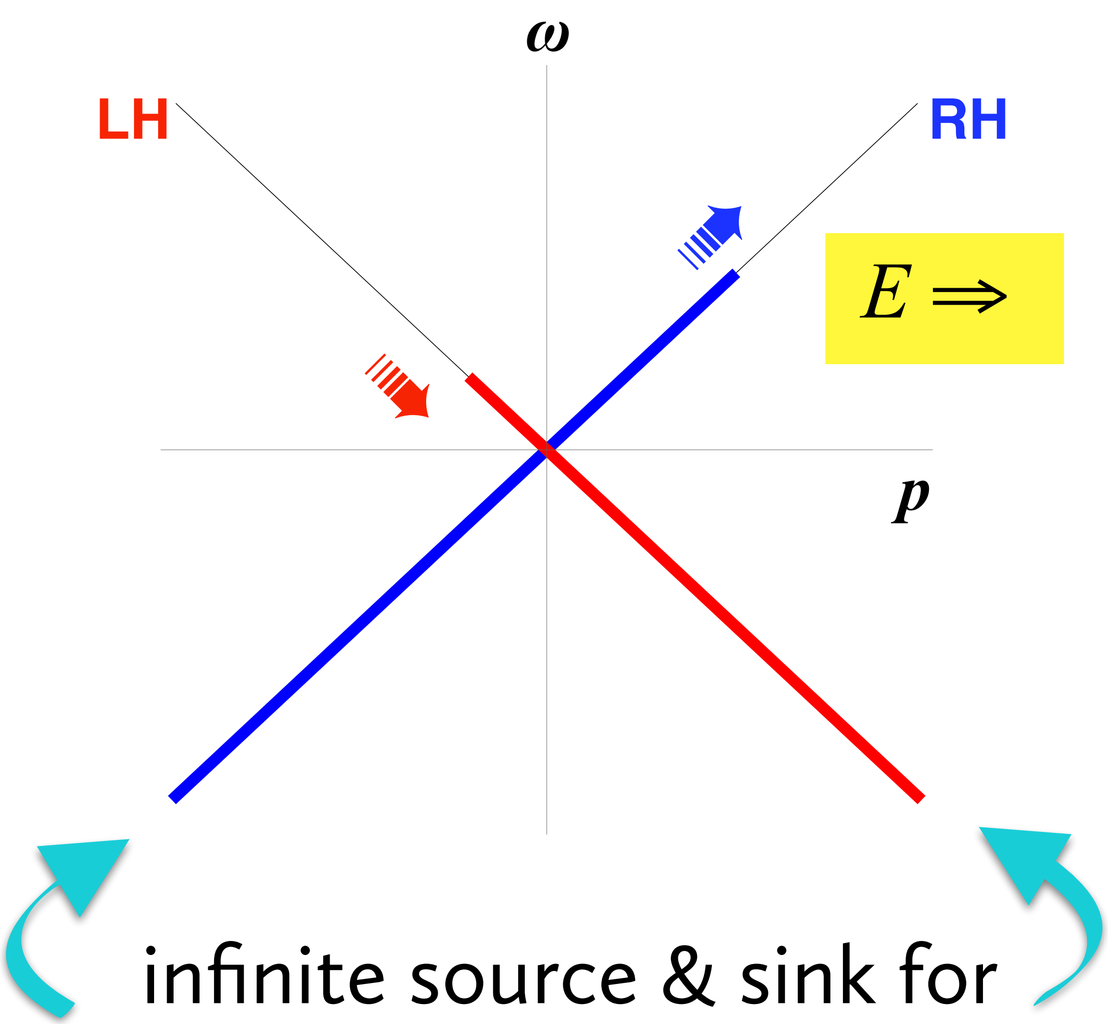
Consider massless Dirac fermions in an electric field E, 1+1 dim



infinite source & sink for fermions

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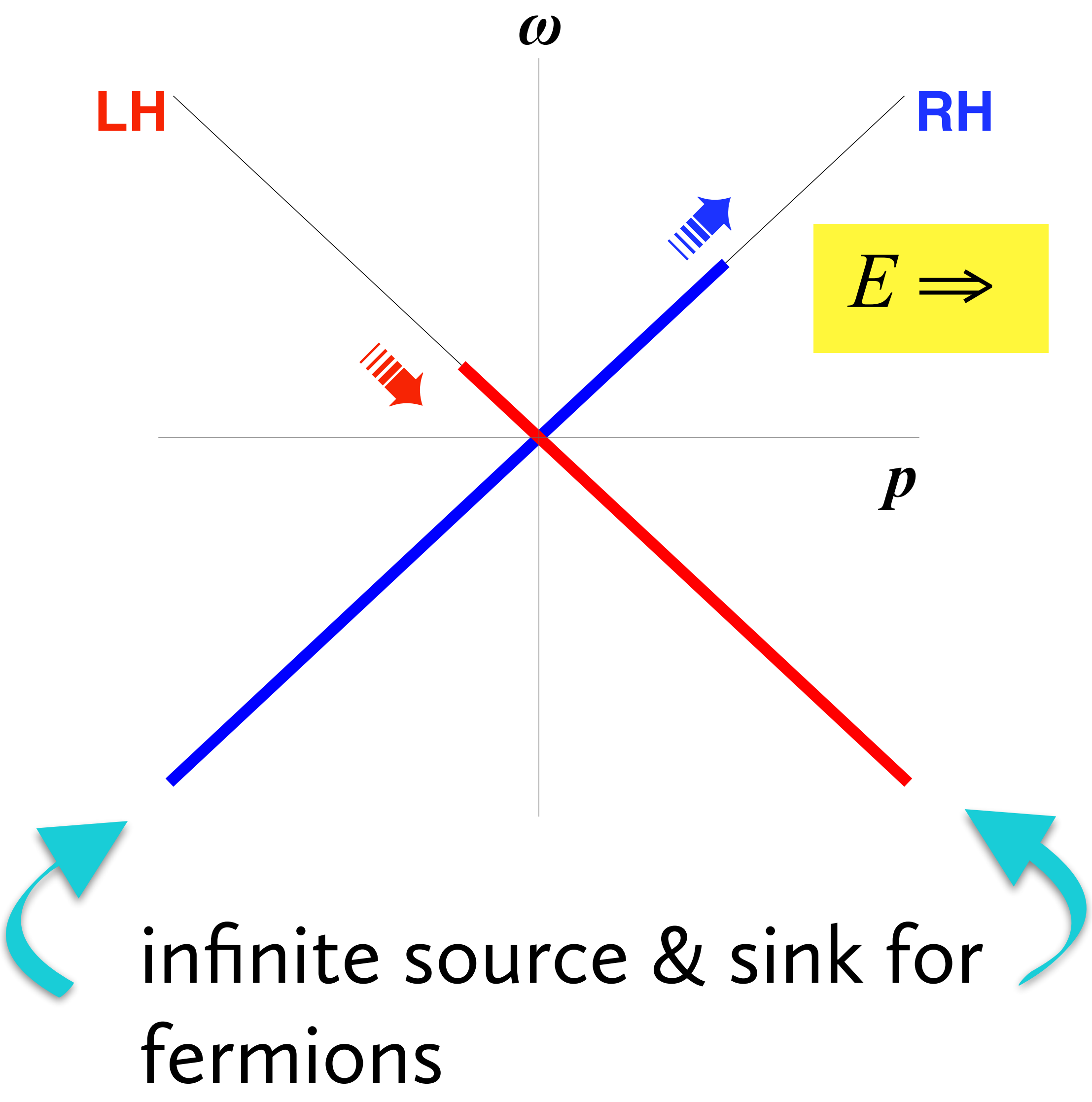

$$dp = qEdt$$

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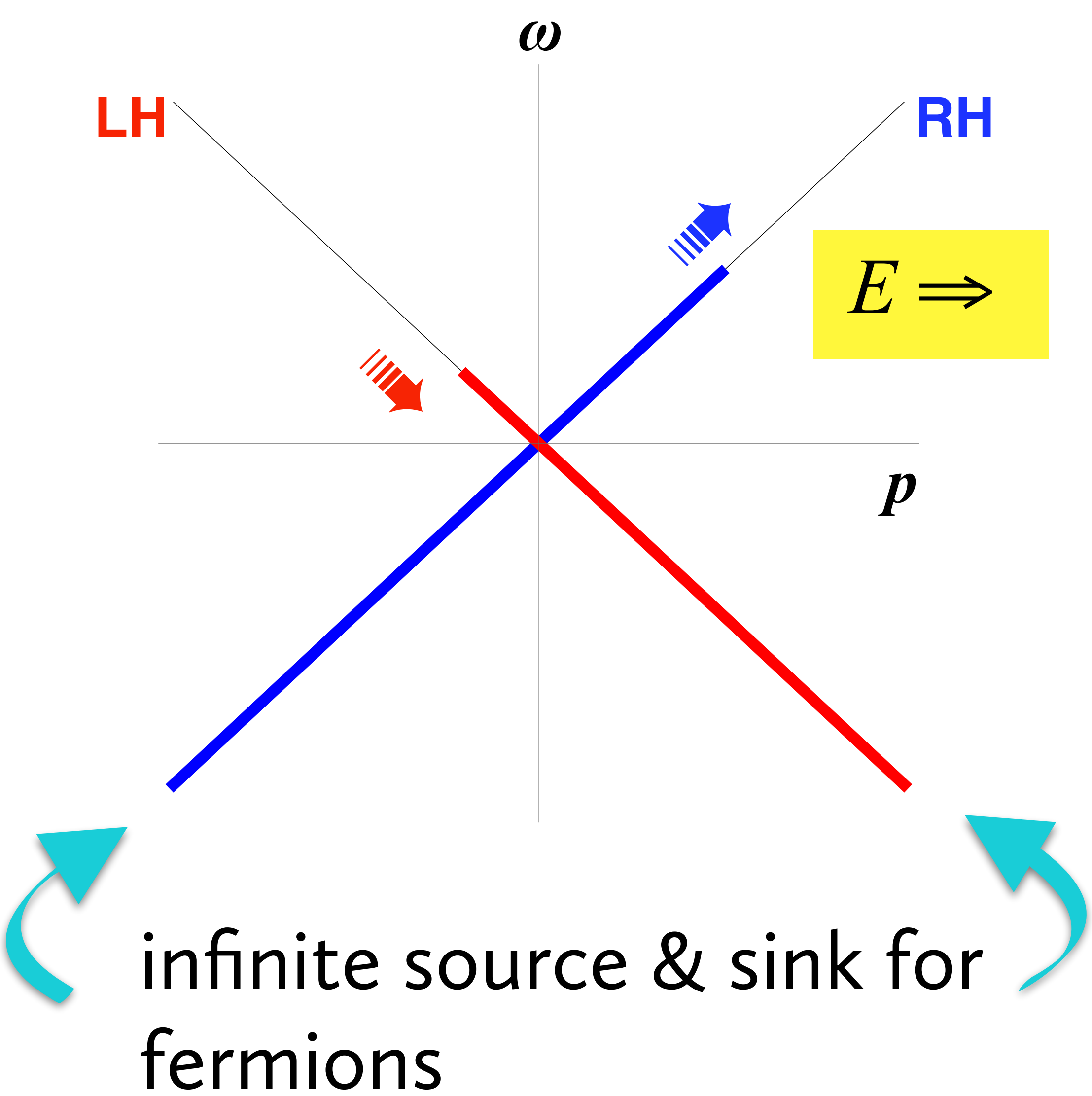

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$$dn_R = +\frac{dp}{2\pi} \qquad dn_L = -\frac{dp}{2\pi}$$

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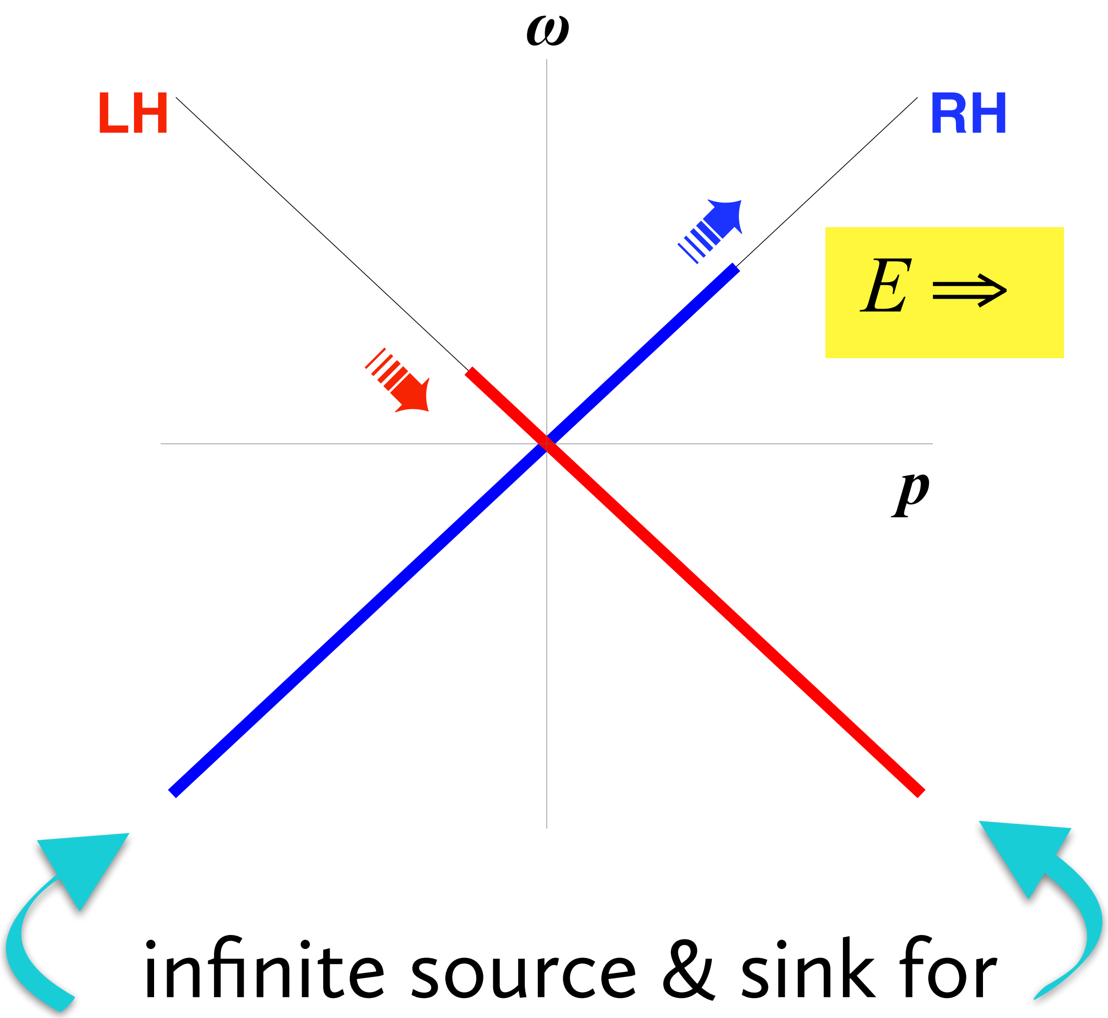

infinite source & sink for fermions

$$dp = qE\,dt$$

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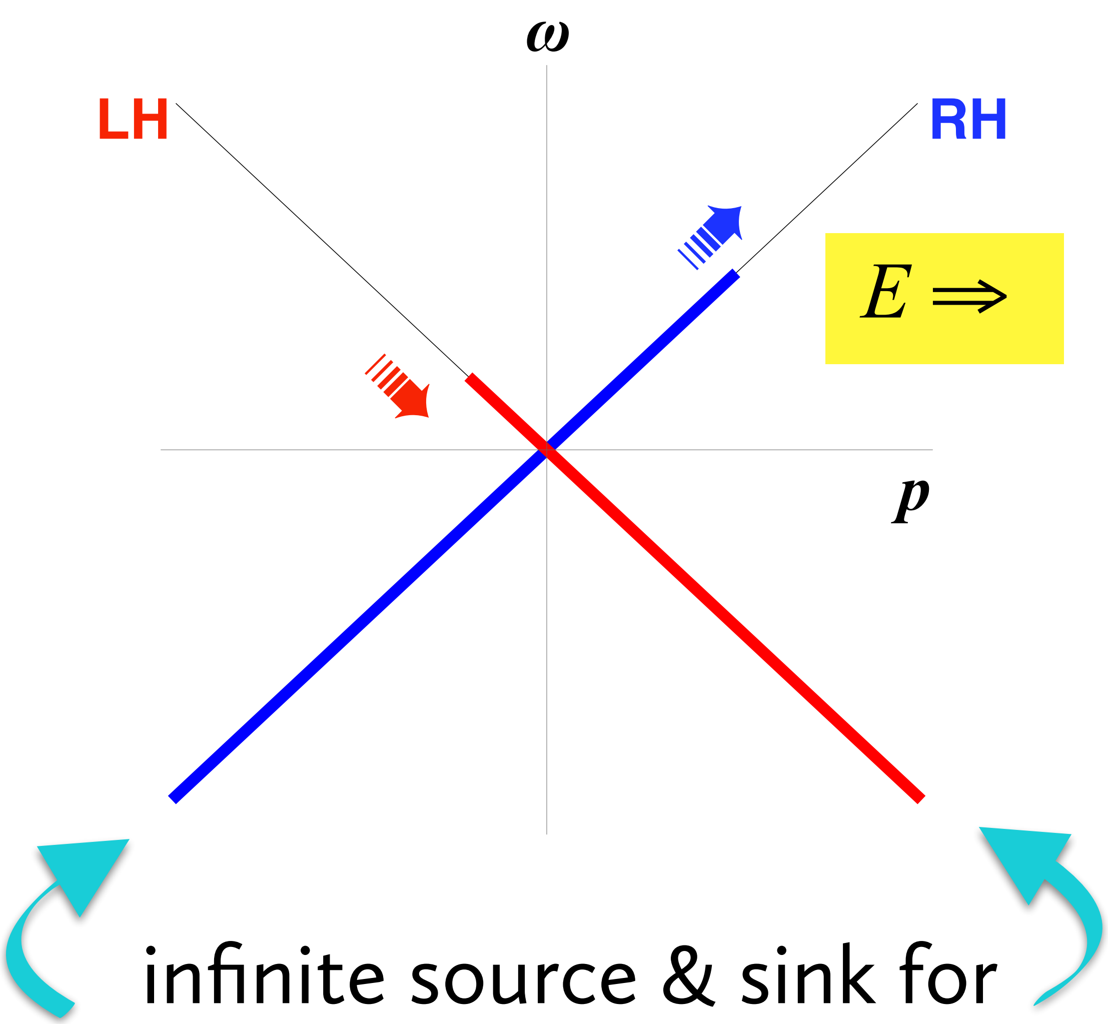
$$\frac{d(n_R - n_L)}{dt} = \frac{qE}{\pi}$$

d=1+1 anomaly

Problems realizing chiral symmetry in lattice QCD...
The key is strange behavior in the continuum: "the anomaly"!

Consider massless Dirac fermions in an electric field E, 1+1 dim



infinite source & sink for fermions

$$dp = qE\,dt$$

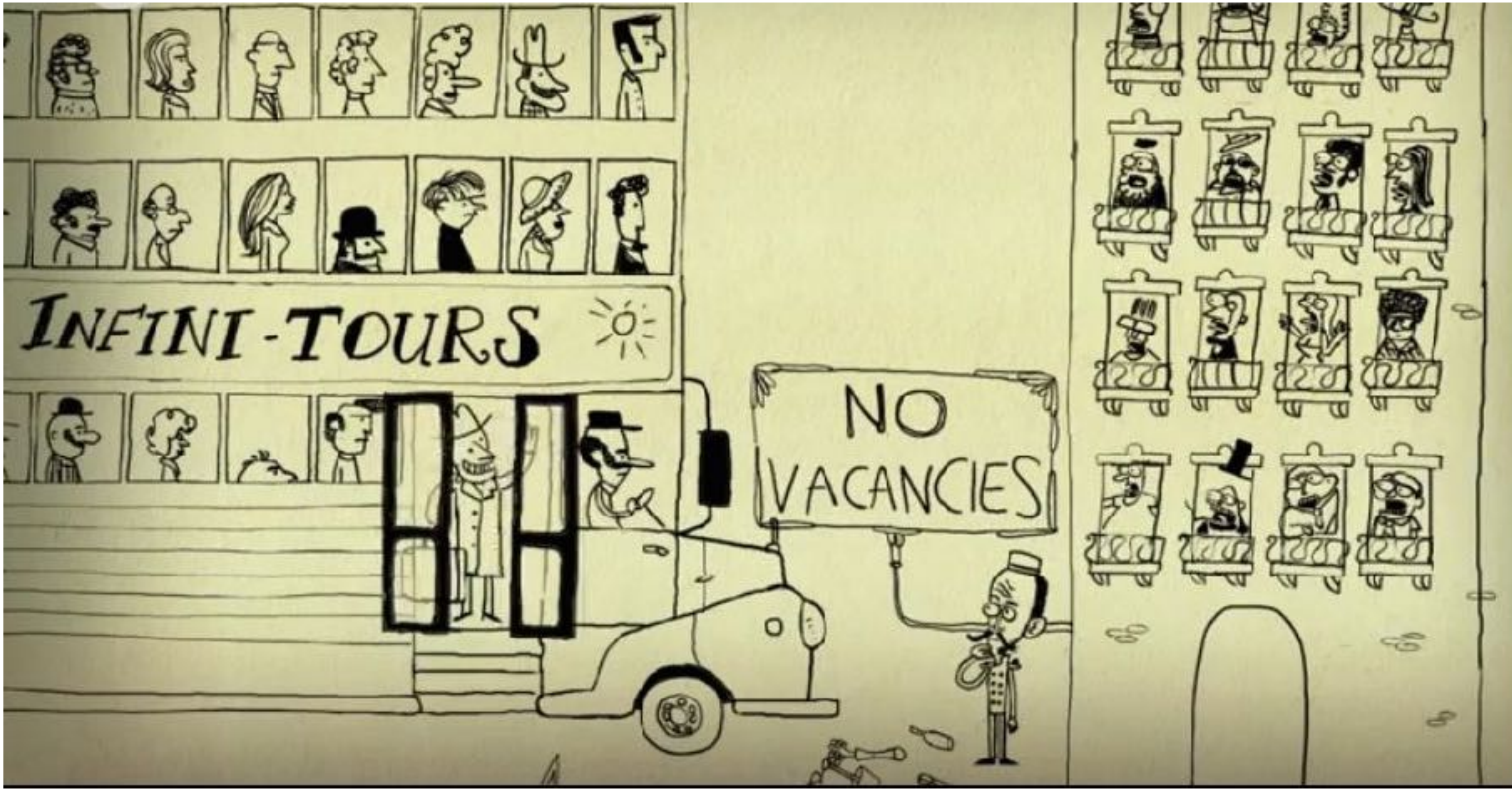
$$dn_R = +\frac{dp}{2\pi} \qquad dn_L = -\frac{dp}{2\pi}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{qE}{\pi}$$

d=1+1 anomaly

In the continuum, chiral symmetries of the classical theory can be violated in the quantum theory due to infinite Dirac sea...not possible on lattice!

In the continuum, the Dirac sea is filled…but is a Hilbert Hotel which always has room for more



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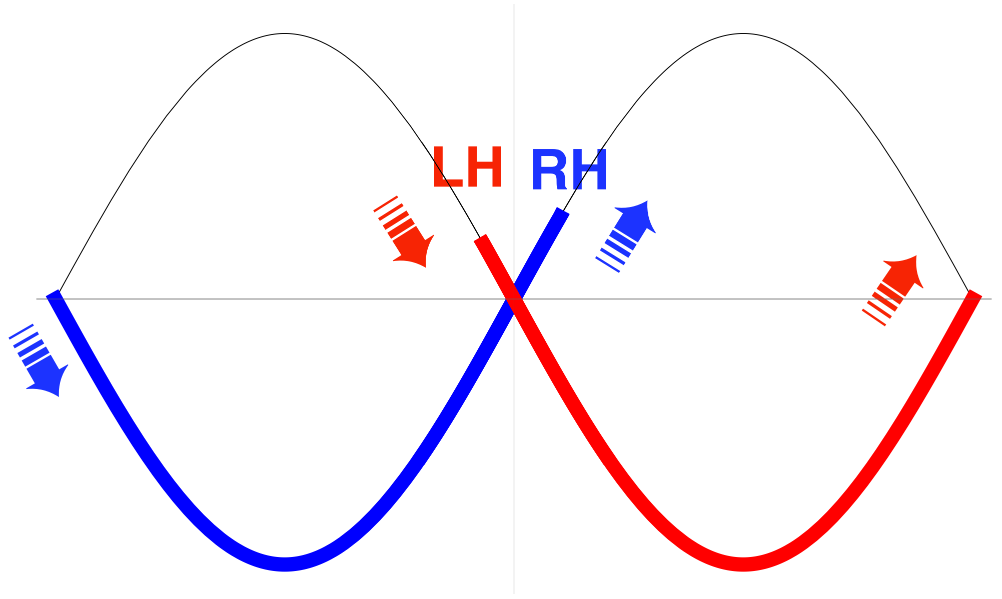
Only finite hotels exist in a computer

No infinite Dirac sea on the lattice:

Can reproduce continuum physics for long wavelength modes…

…but **no** anomalies in a system with a finite number of degrees of freedom



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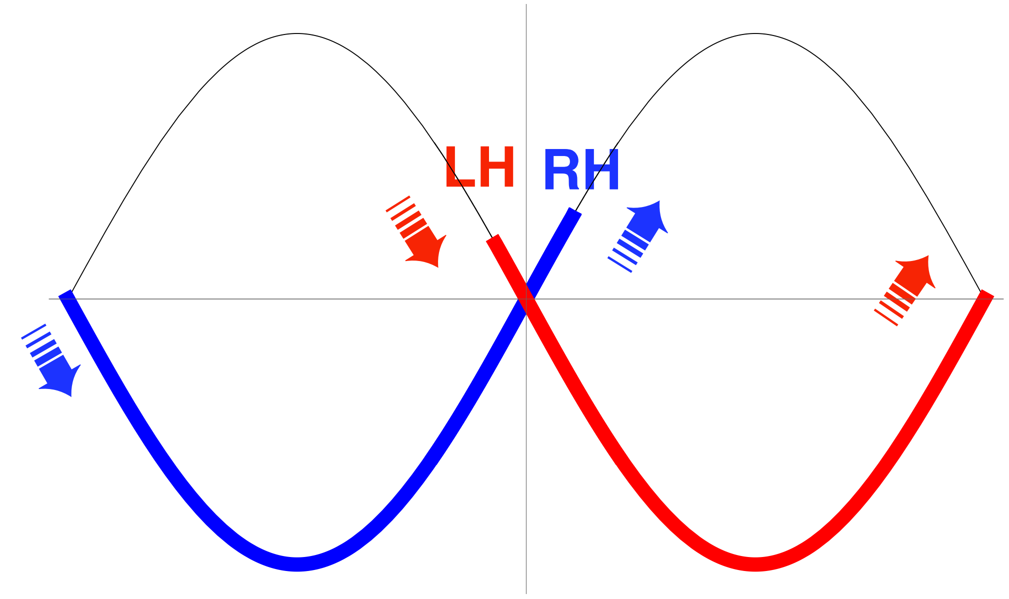

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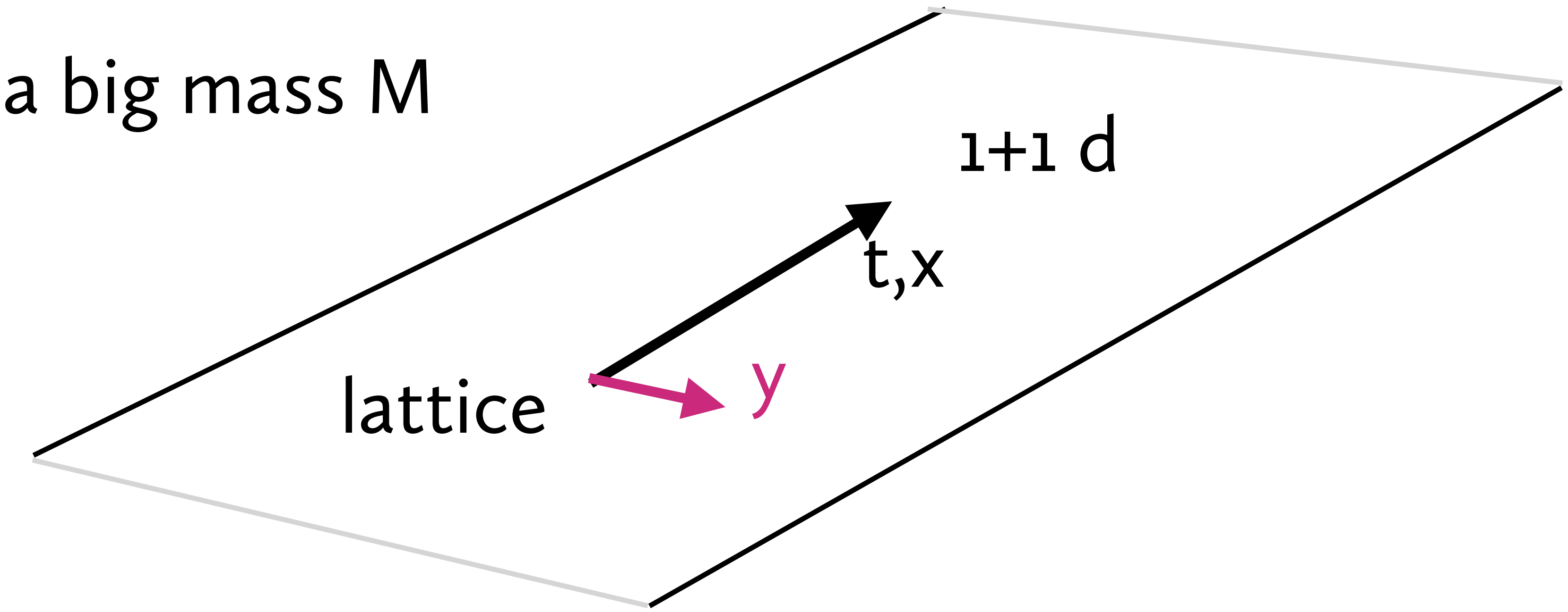
A solution for QCD is to formulate it in 5 dimensions!

Domain wall fermions: 2d ▸ 3d, or 4d ▸ 5d

DBK 1992

Dirac fermions with a big mass M



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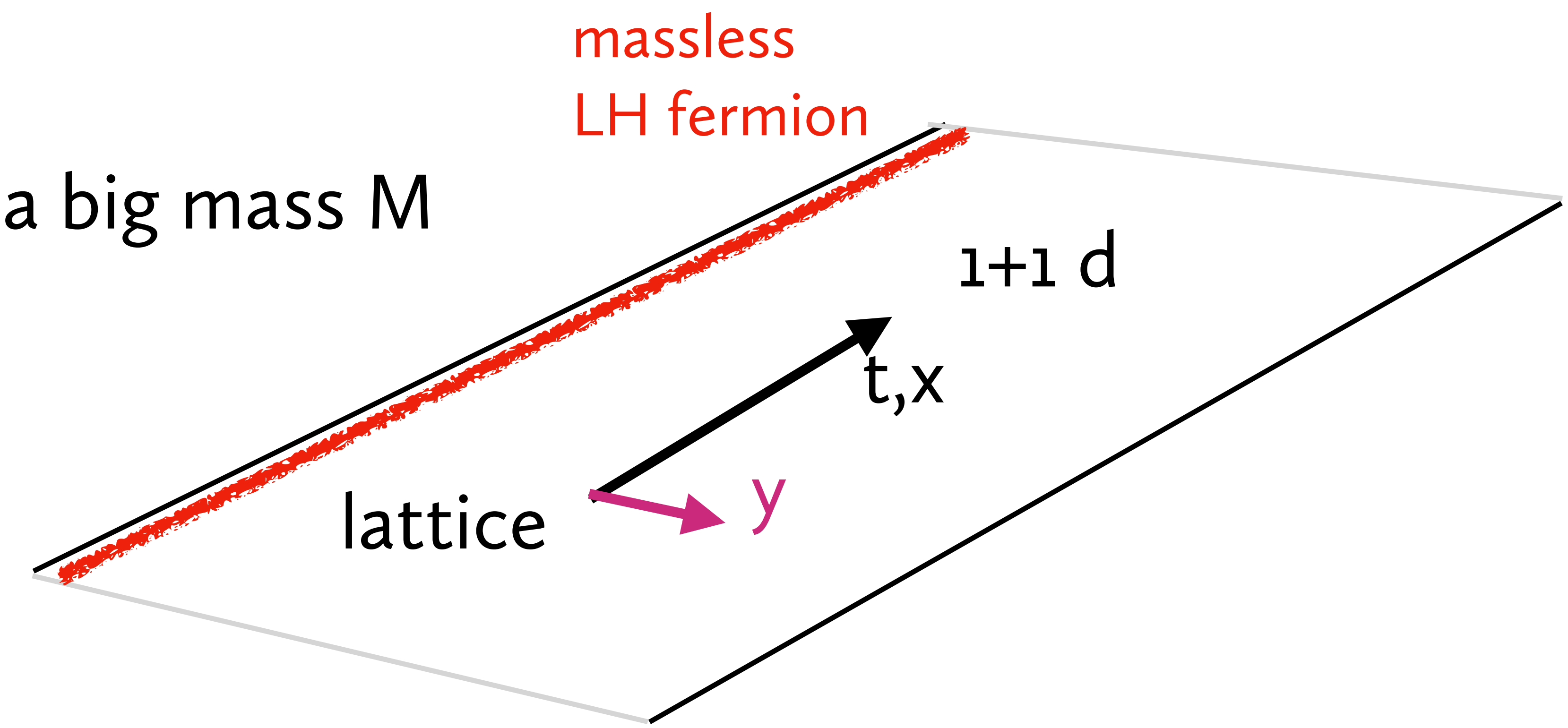

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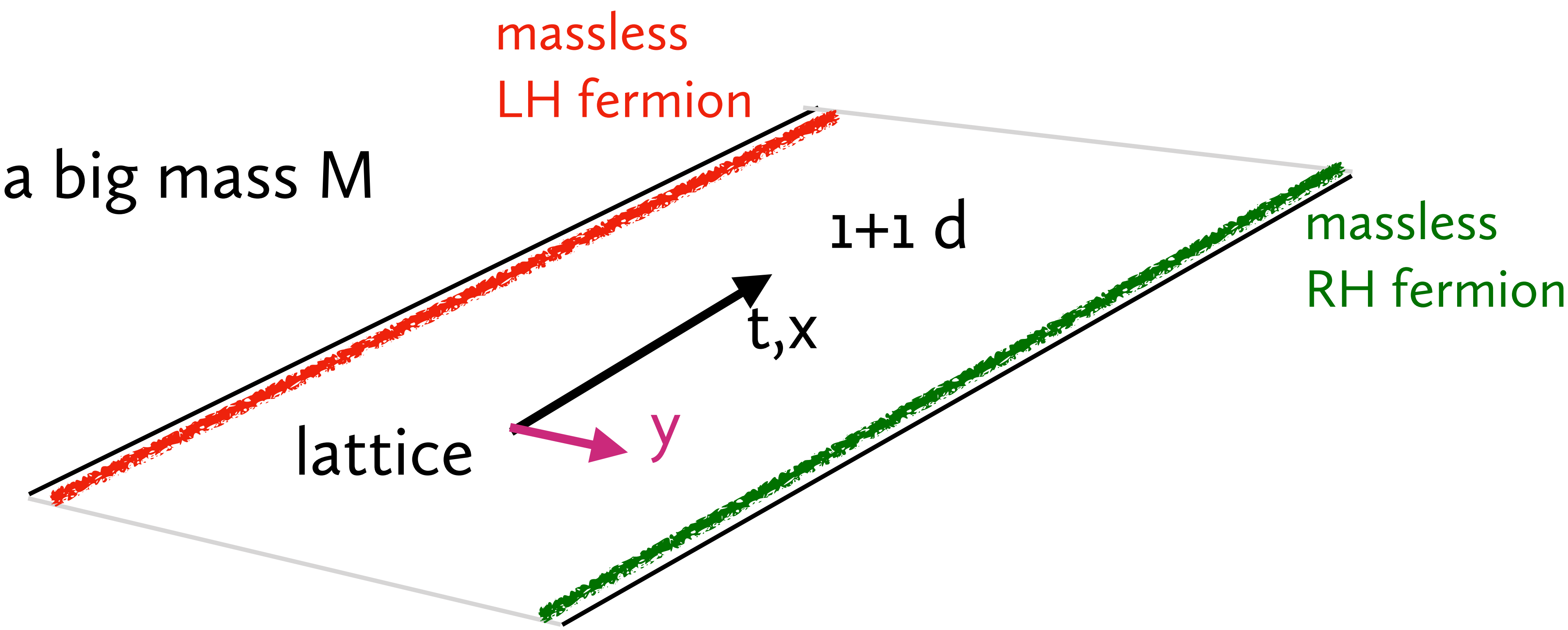

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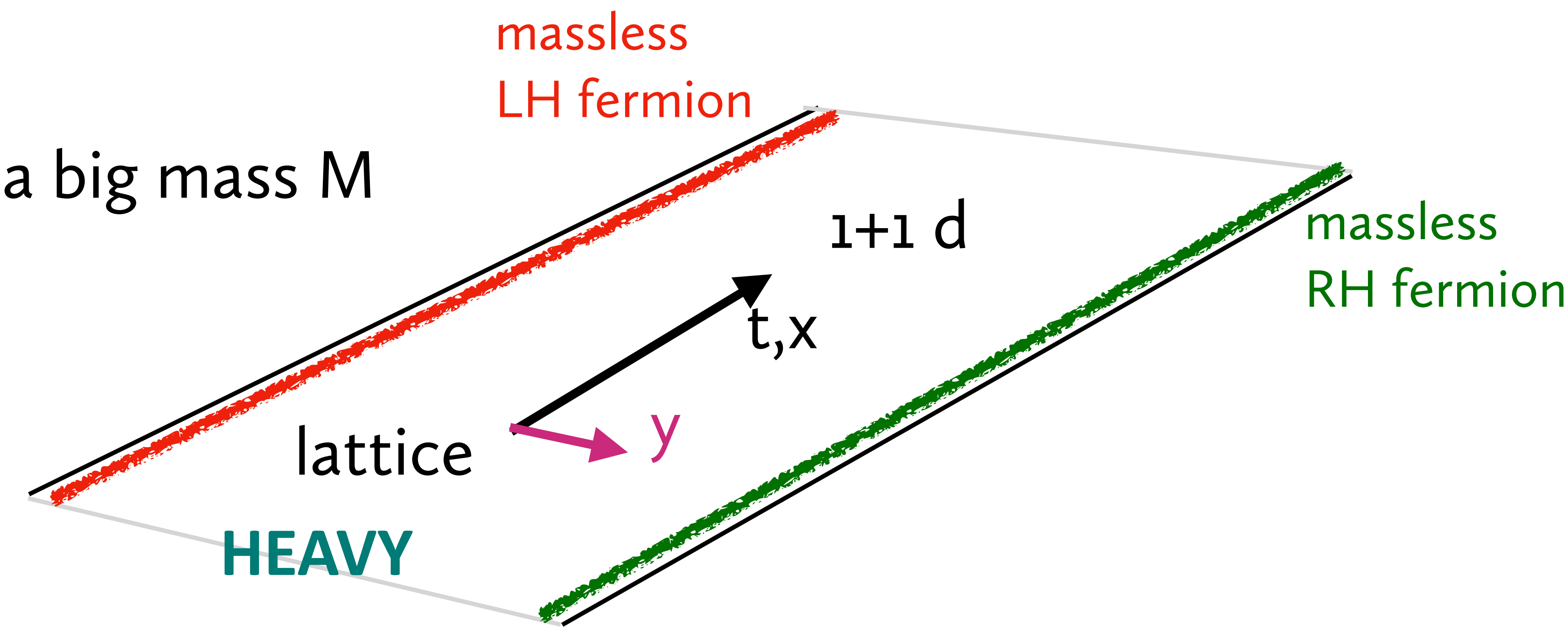

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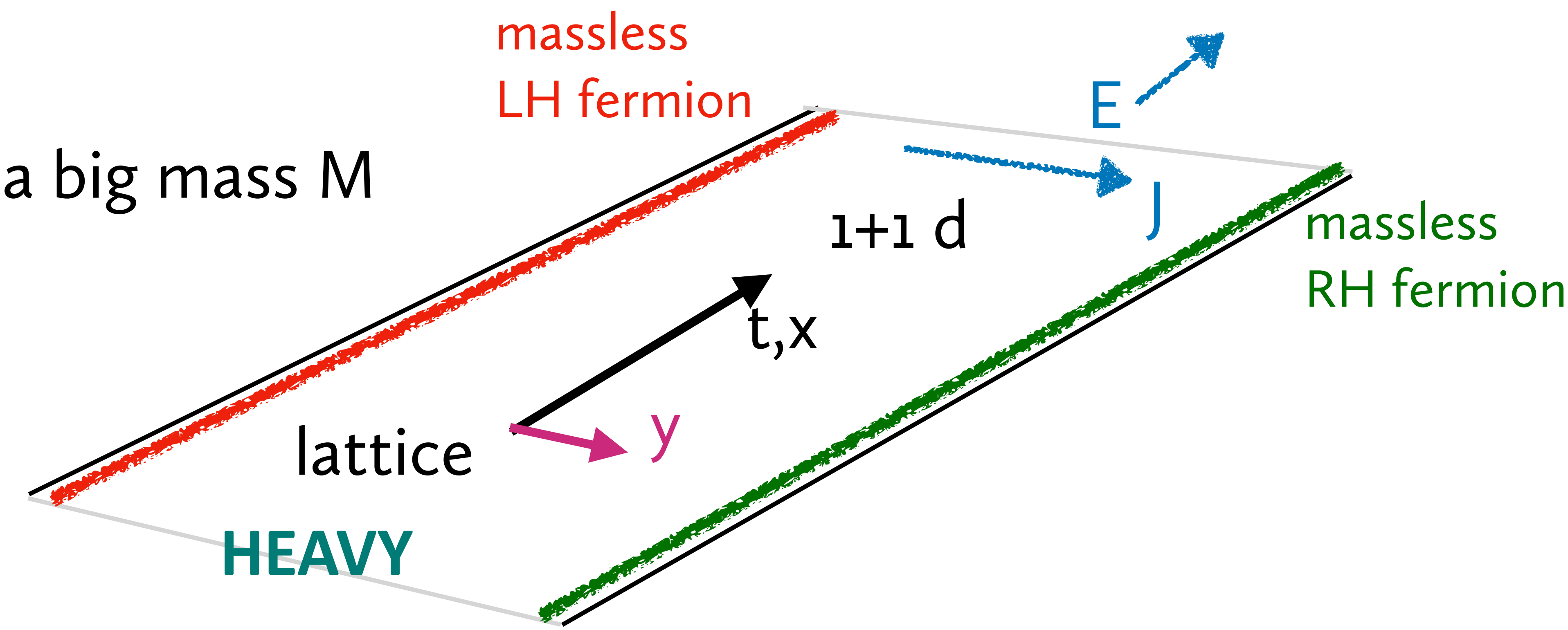

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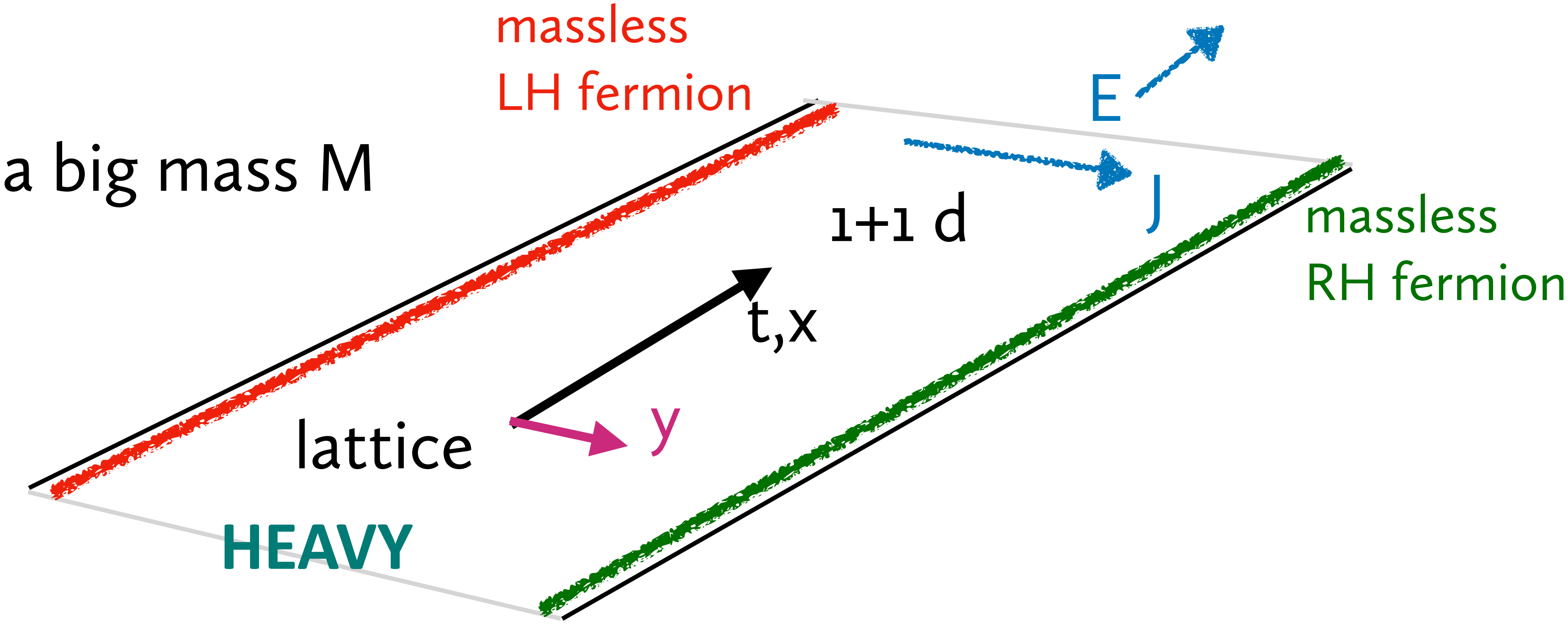

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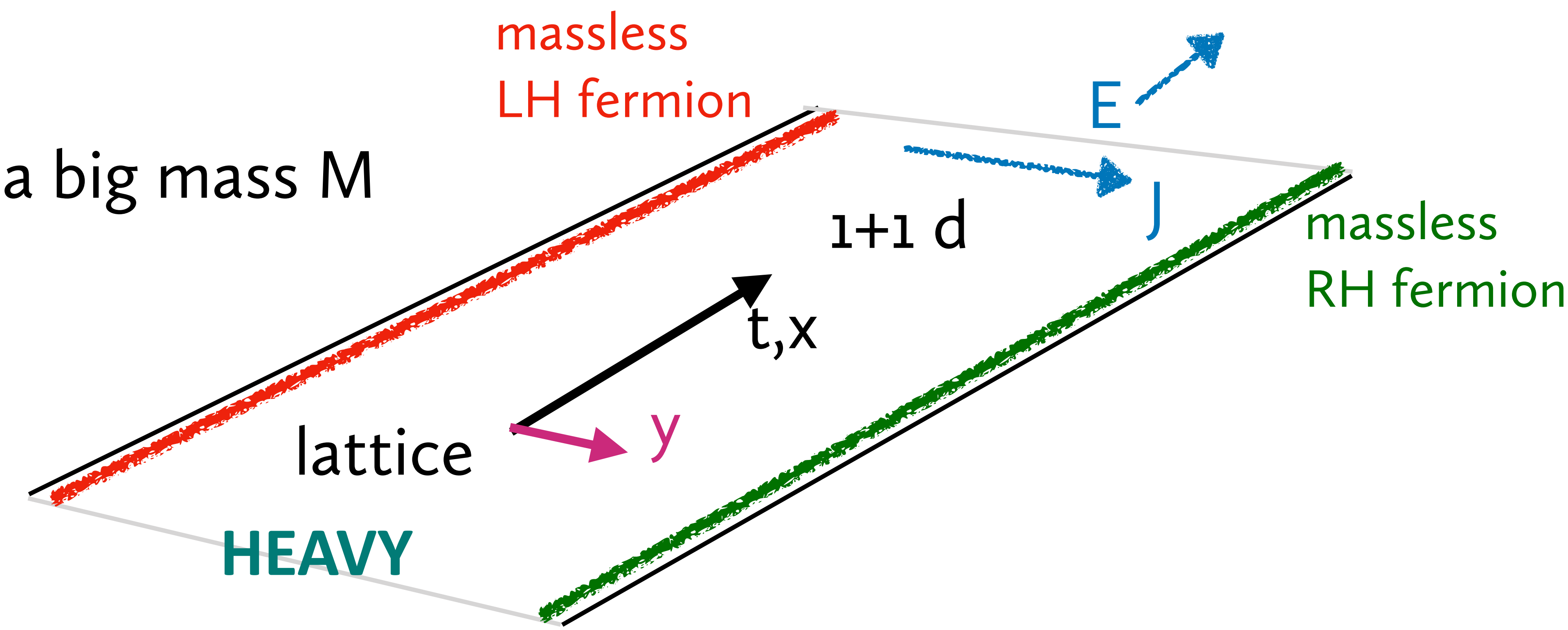

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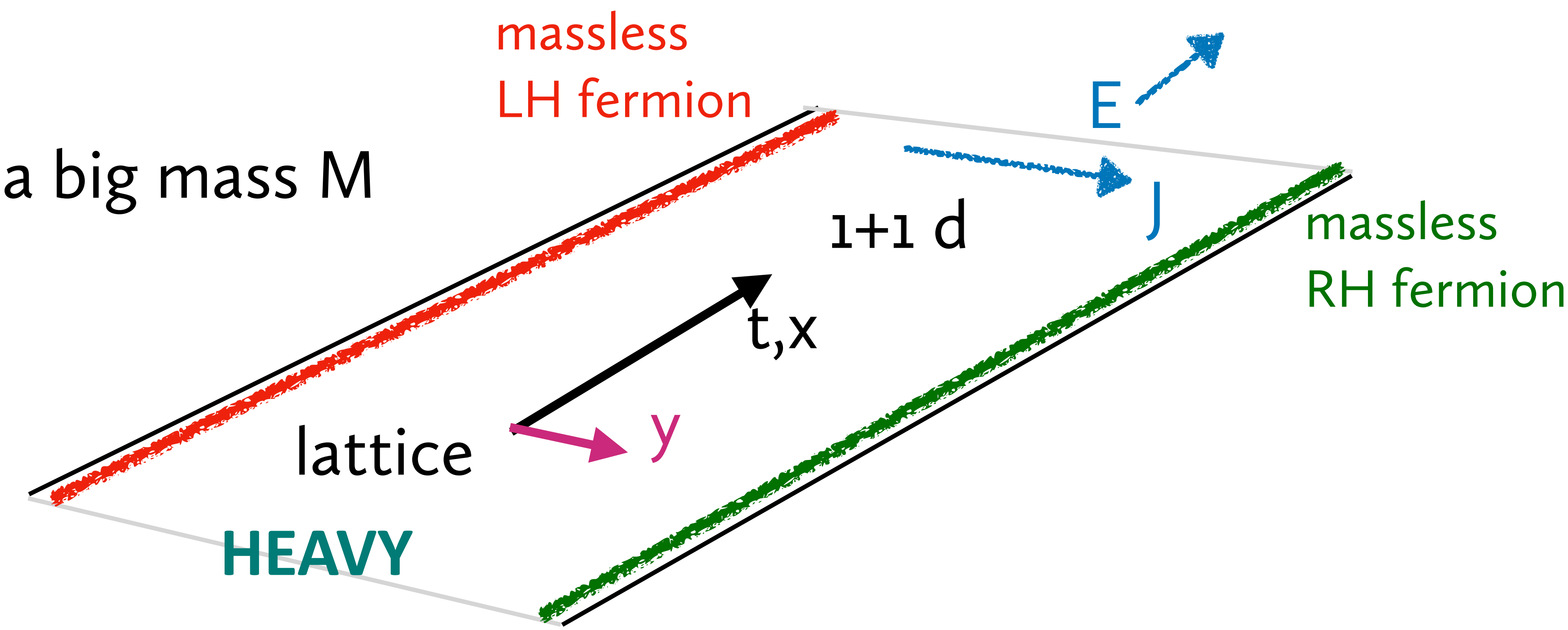

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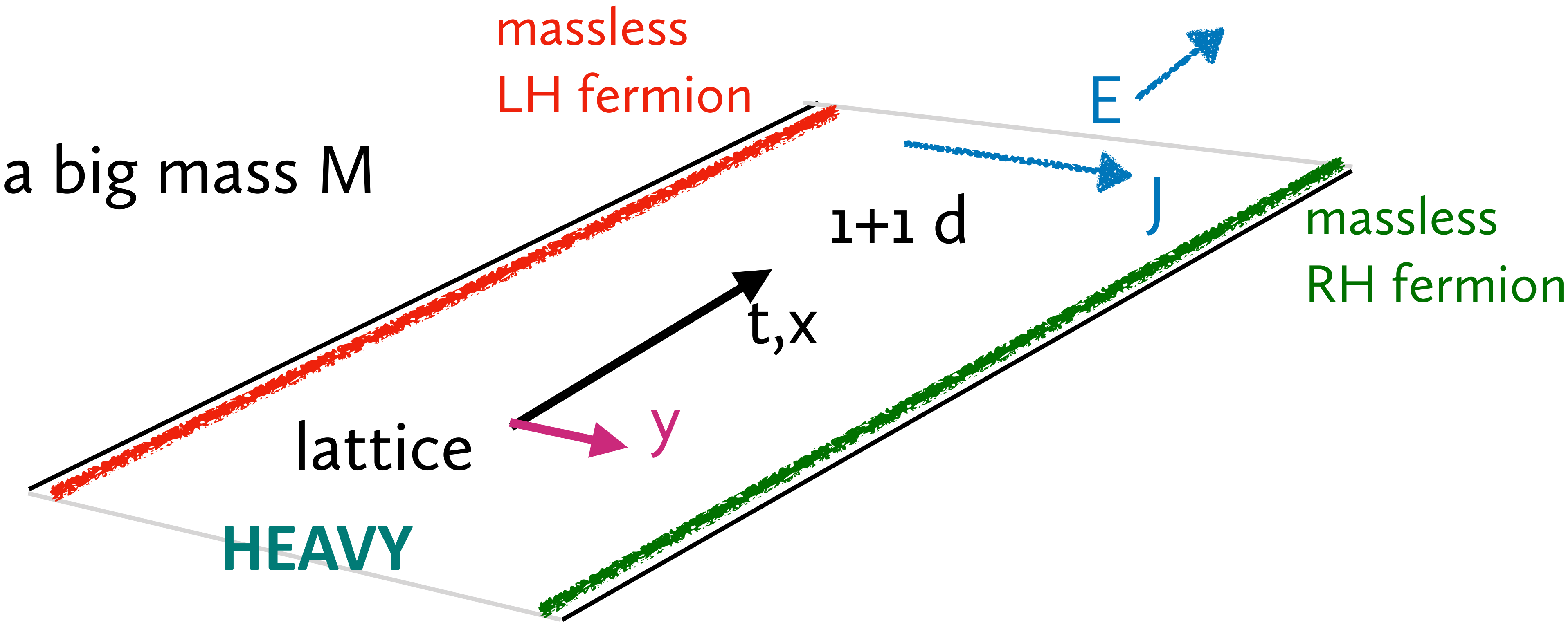

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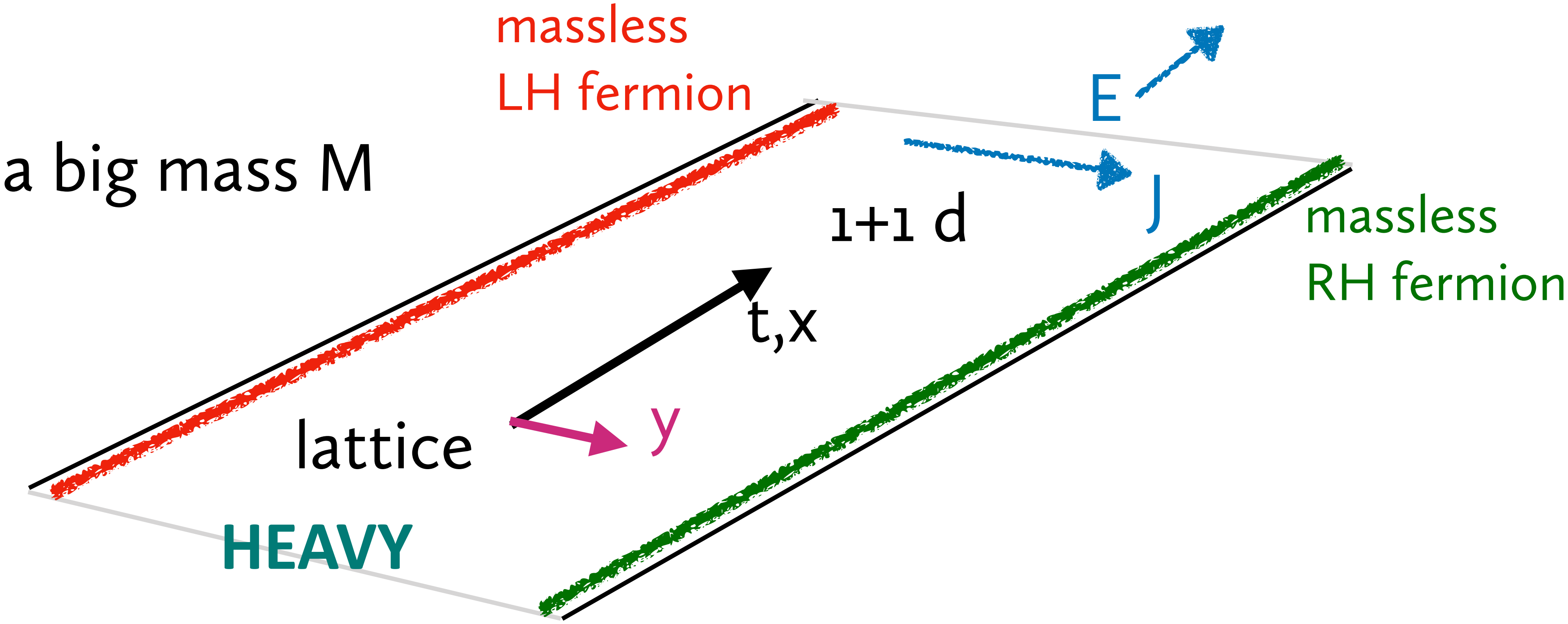

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Topology on a lattice??

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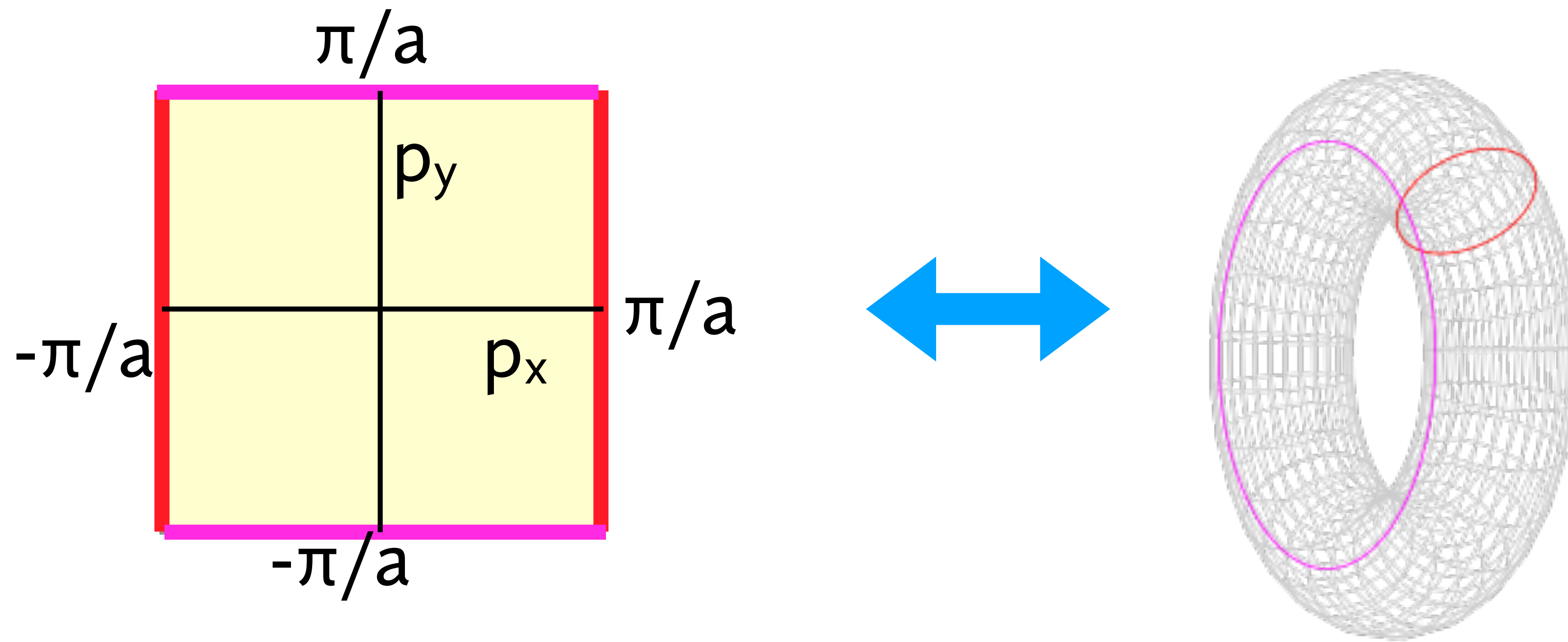

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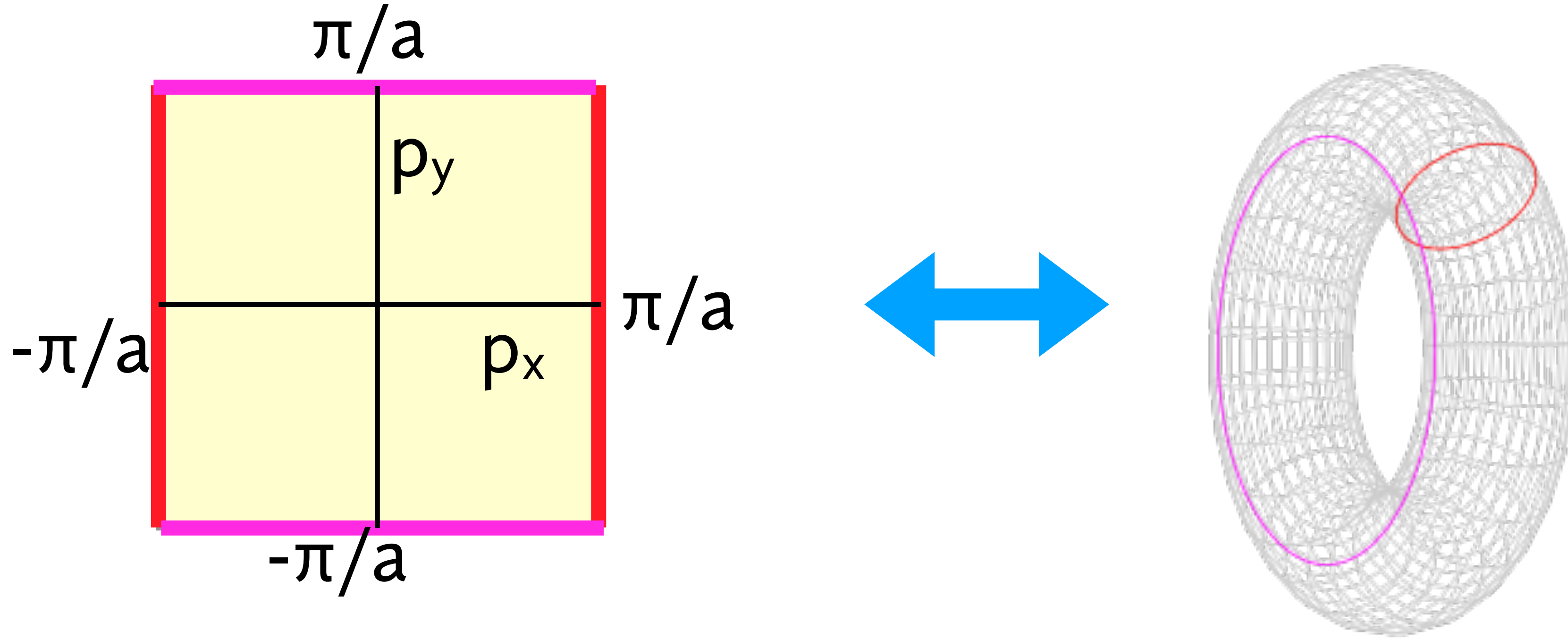

Fermions in the bulk of the lattice have an energy-spin-momentum relation that can be related to mapping the d-momentum space d-torus to a d-sphere.

This map can be topologically nontrivial

The number of massless fermions at the edge of the topological insulator is determined by the winding number of this map

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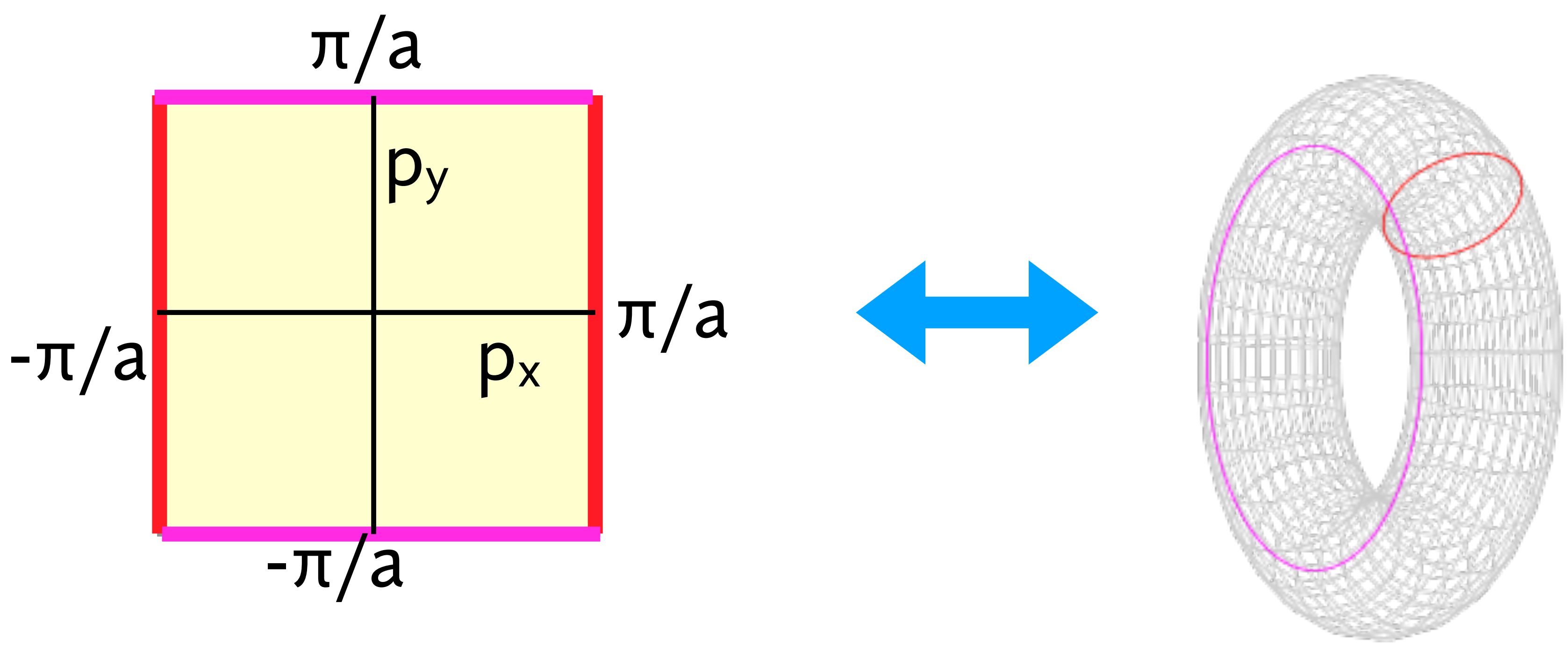

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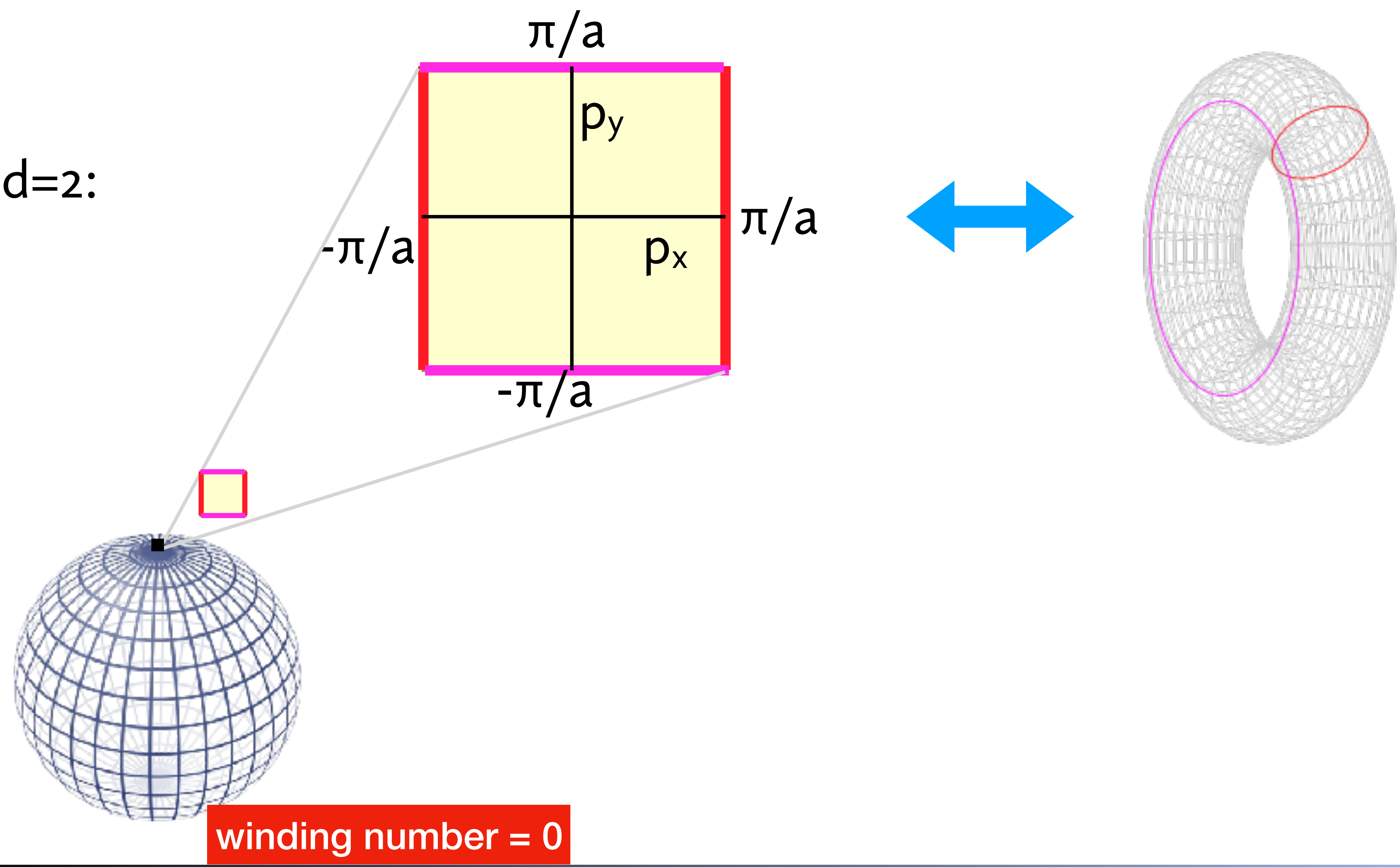

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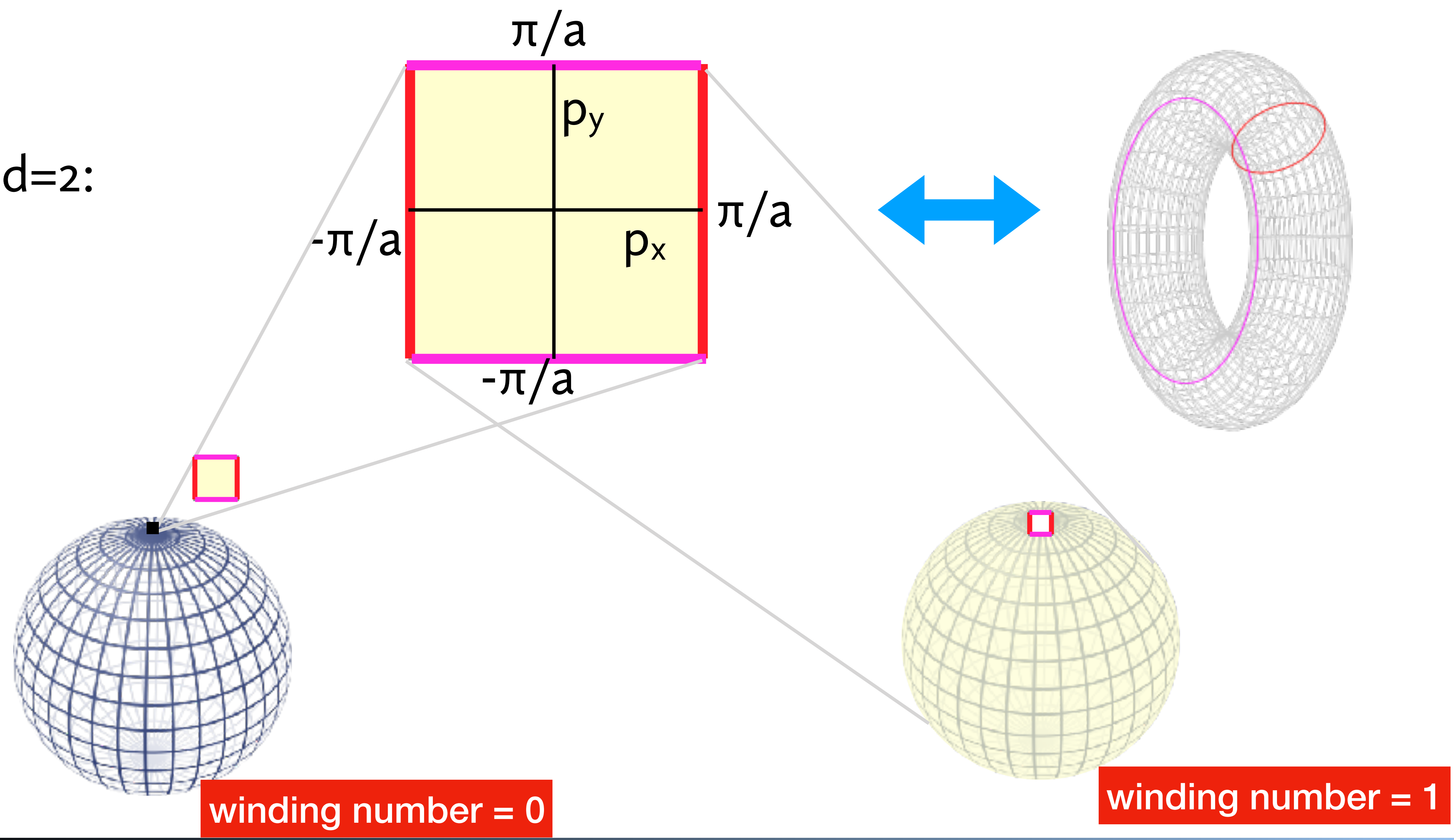


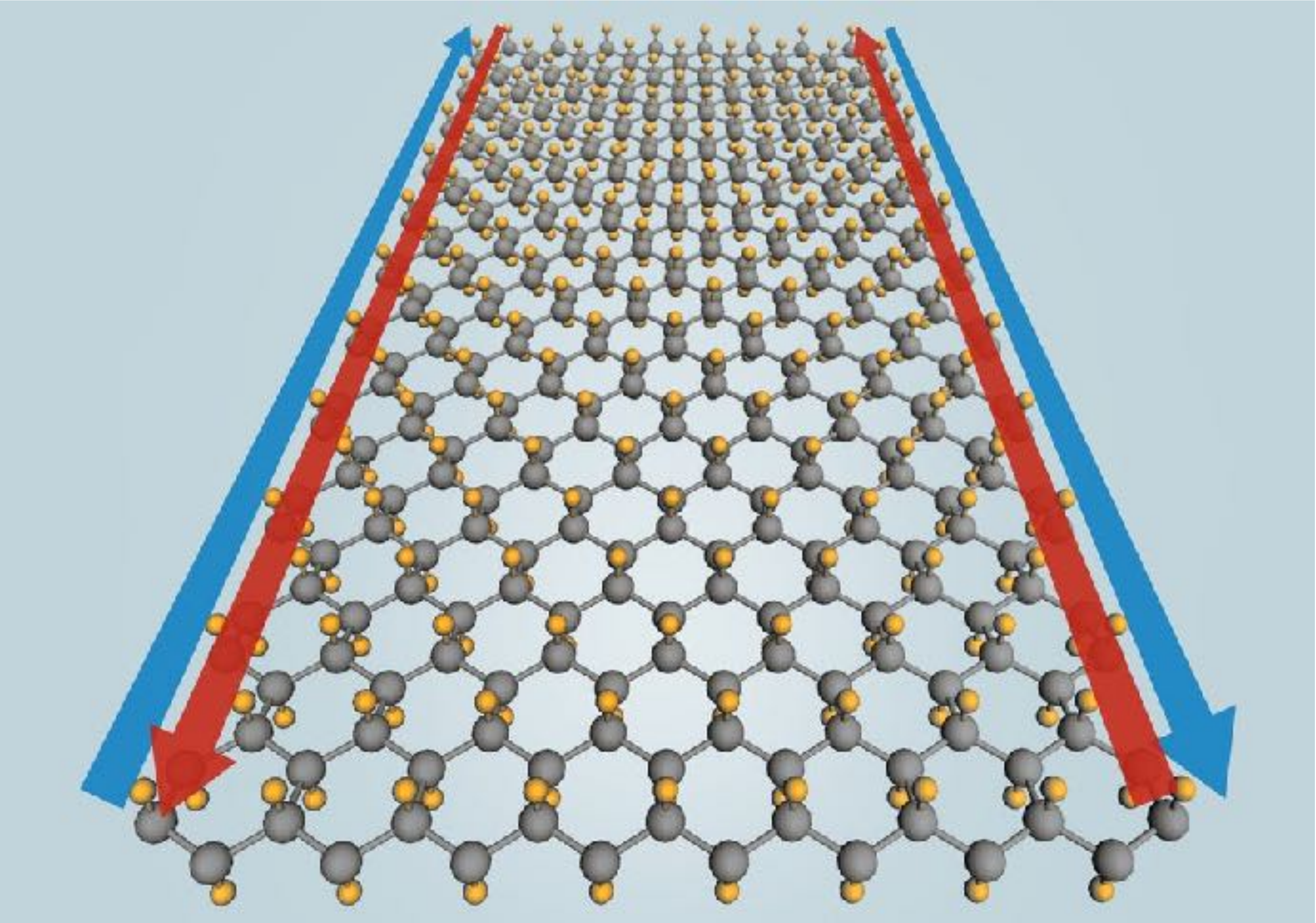
Winding numbers are integers…cannot change continuously

- Guarantees the existence of massless edge states must be insensitive to small continuous changes in parameters, radiative corrections, etc.
- Guarantees quantization of the bulk “Hall current”
- Quantized phenomena can only change when something dramatic happens (eg: vanishing of the mass gap in the bulk of the sample)

Topology explains, for example, how clean results can arise from dirty samples (eg, precision measurement of α in quantum Hall effect)

Domain wall fermions in QFT are examples of the topological insulators discovered in condensed matter physics.

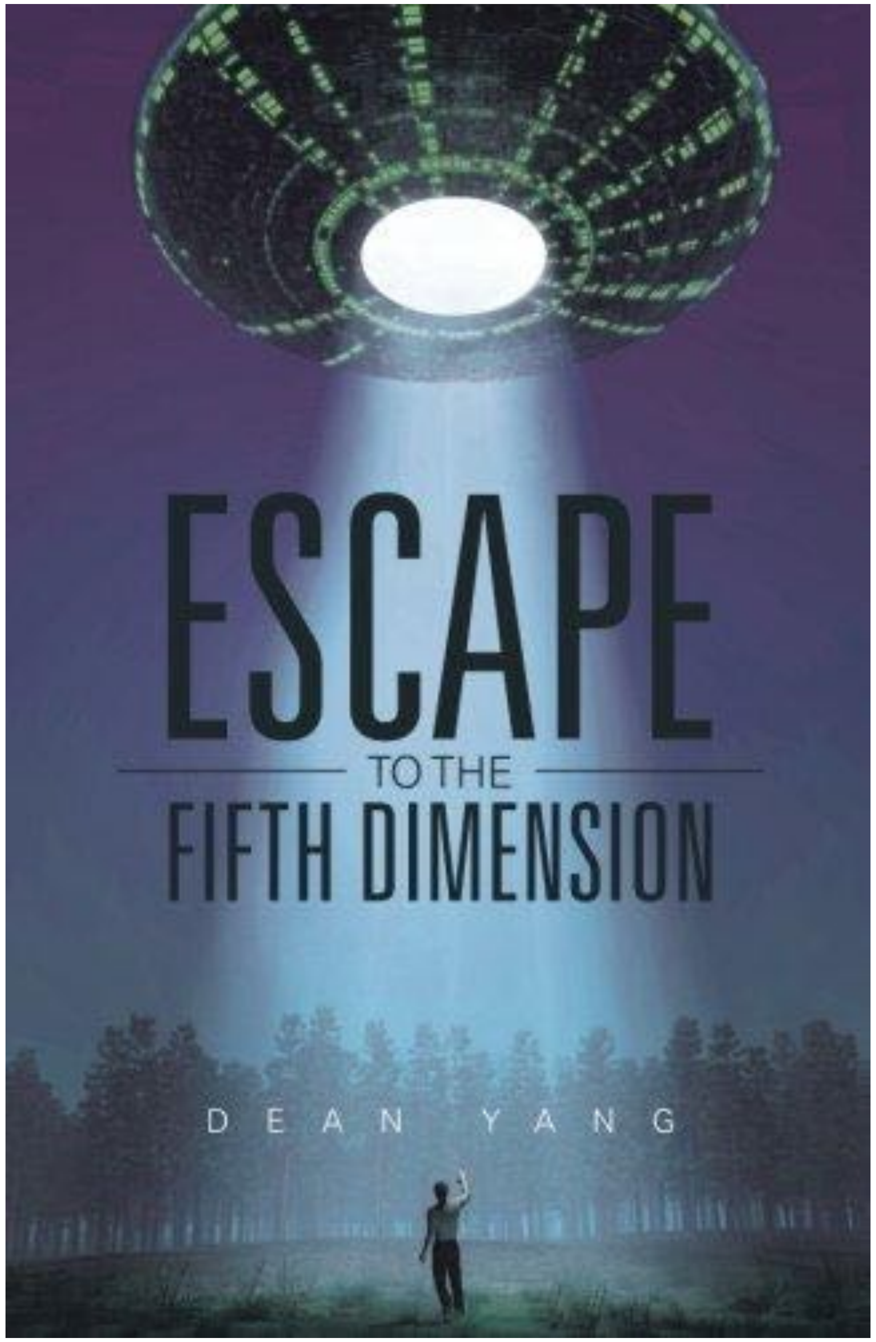
e.g.: Quantum Spin Hall Effect (2004) = lattice field theory model for chiral gauge theories (1993)



stanene

first synthesized at the Indian Institute for Technology (2015)

Many simulations of QCD performed today model our 3+1 dimensional world as the surface of a five-dimensional topological insulator



III. The study of something, signs of trouble and the quantum computer

Fish are more interested in other fish than water

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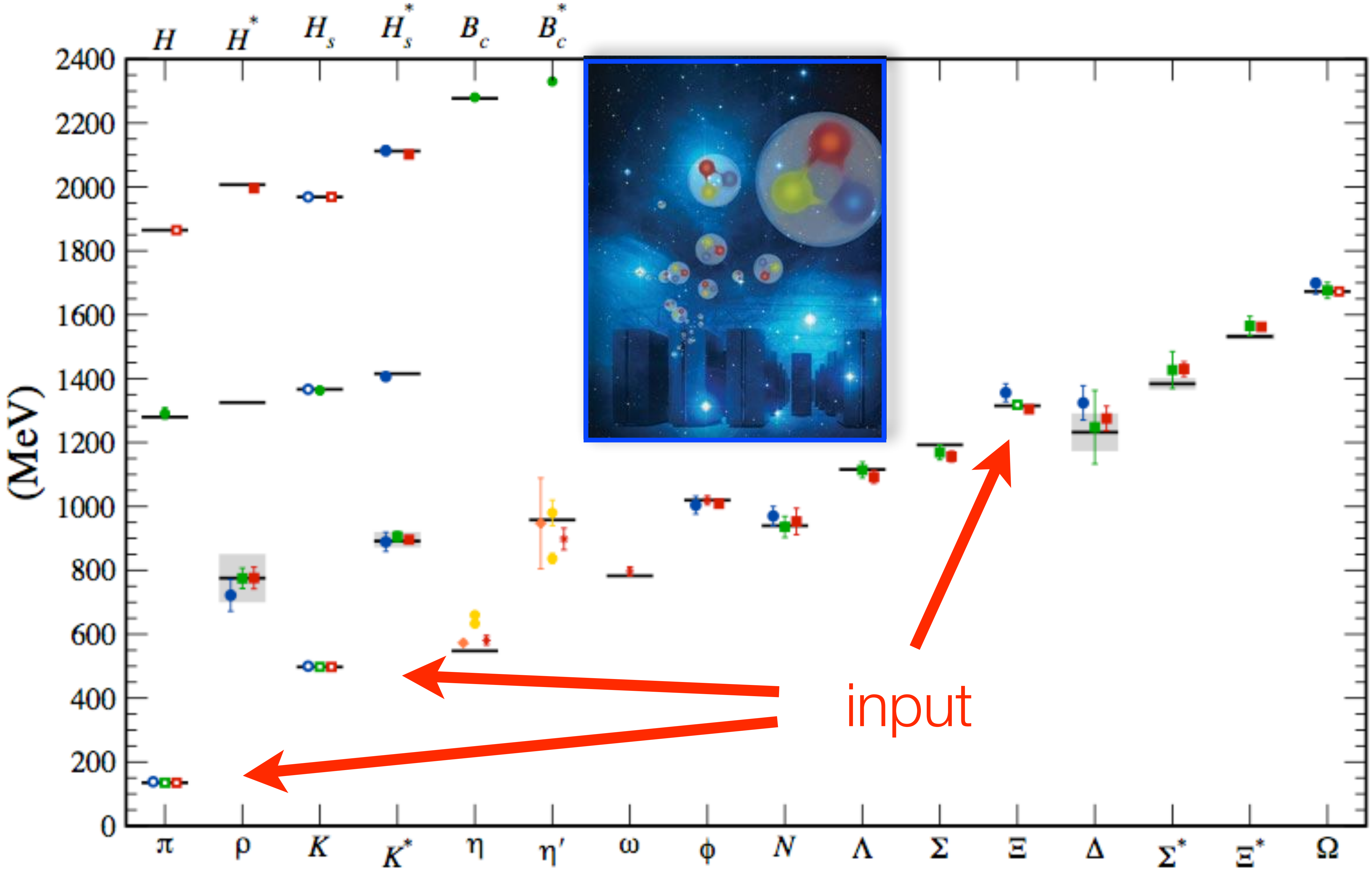
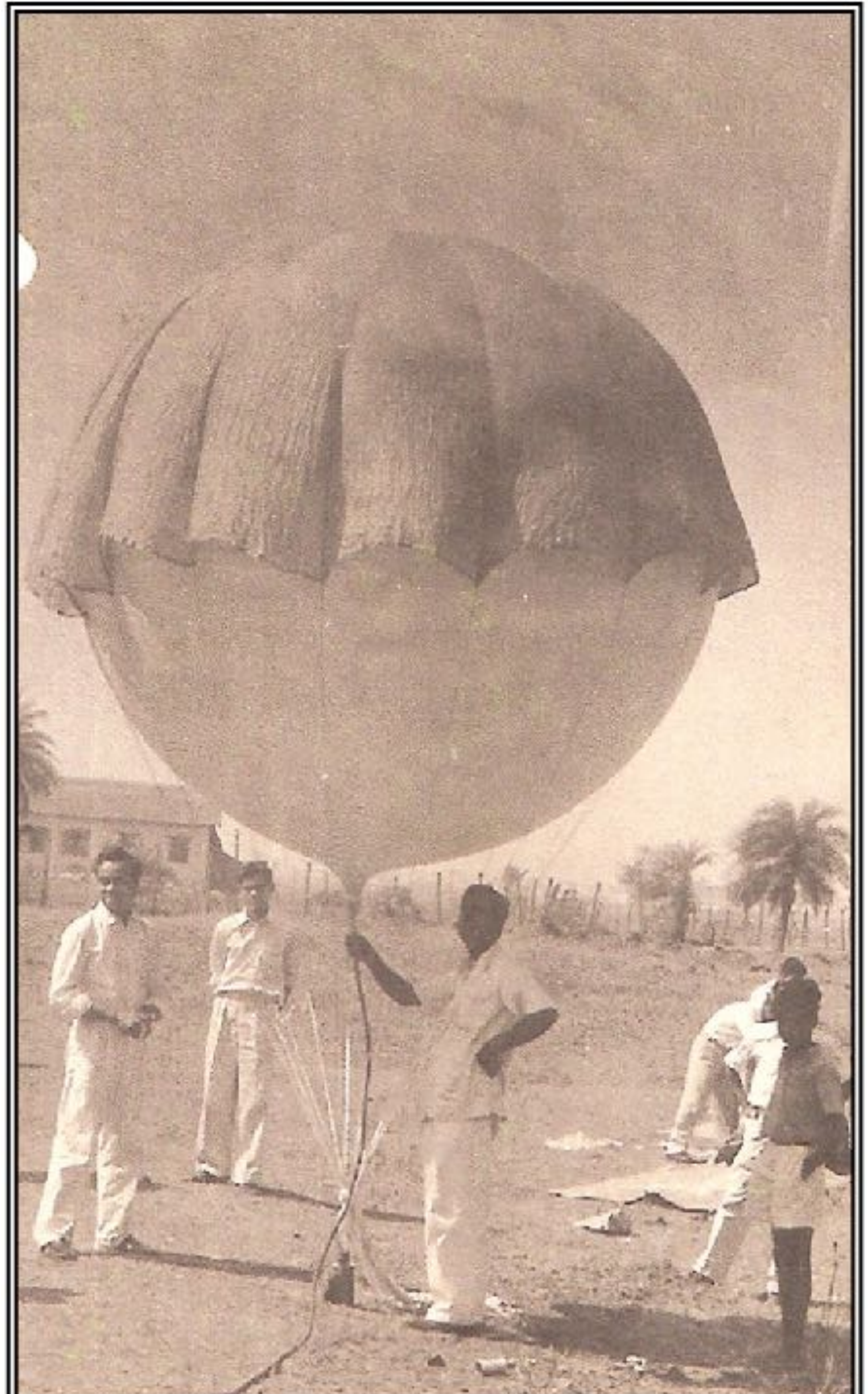
We have these wonderful theories of matter…can we understand macroscopic matter?

E.g., what are the properties of matter in a neutron star? Neutrons? Quark matter? Strange matter?

Lattice QCD ▸ high accuracy hadron spectroscopy:



MILC collaboration / Kronfeld



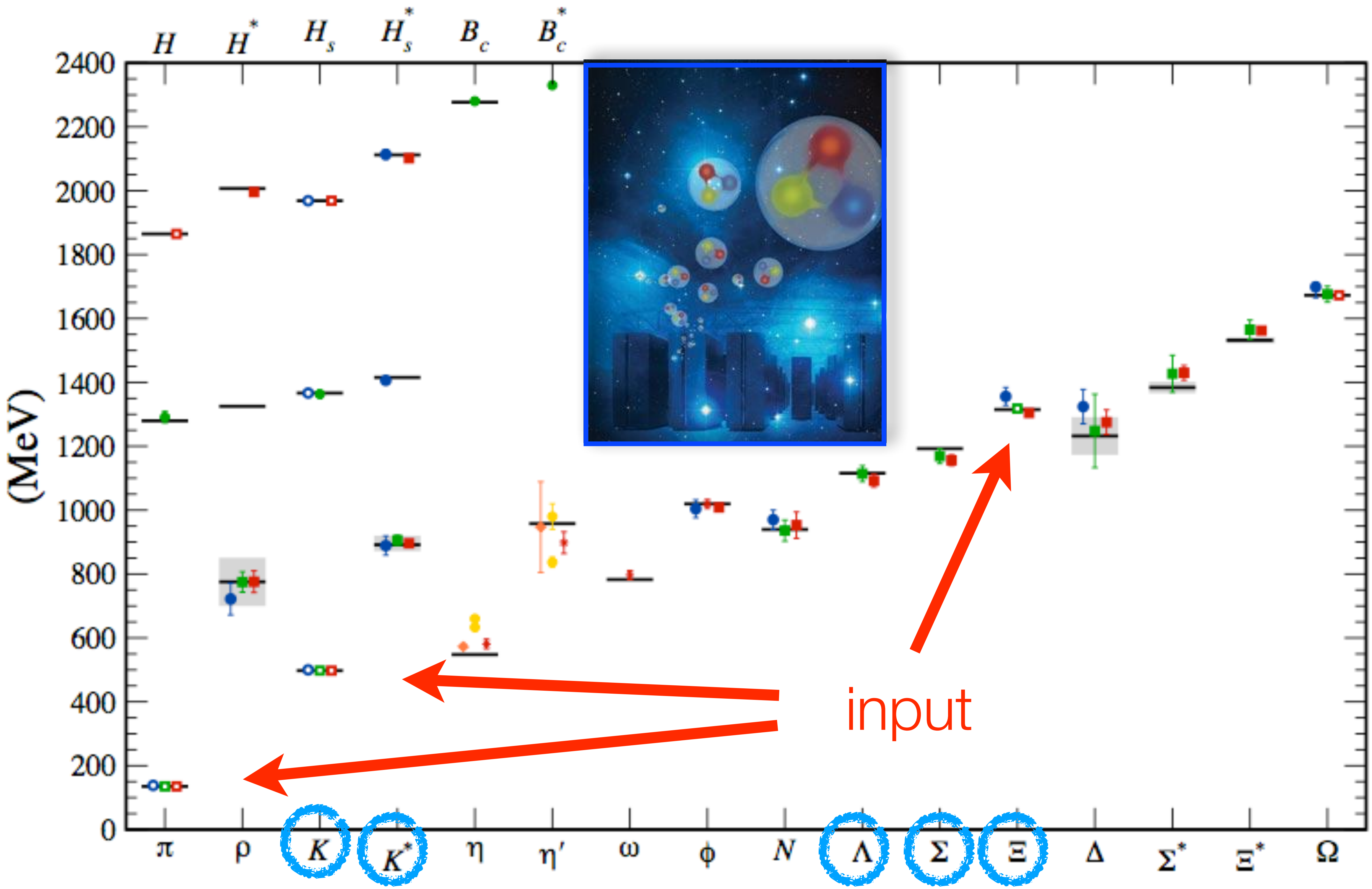
Yash Pal, 2005, about cosmic ray research at TIFR in the late 1950s:

We found that there was another ‘funny particle produced along with it, so it became clear that somehow one ‘funny’ requires another funny’ to be together and once they separate, then they are acting differently. So there is another property – this came to be known as Strangeness – just a few months later Gell Mann used along with the term ‘associated production’.

In a sense I would say that ‘associated production of particles’ was first discovered in this lab. We didn’t call it that but we remarked on it – and of this type, so many are produced together, and that was a very important insight and recognized world over.

Look, the total number of strange particles in the world was 20 and about 8 or 10 were discovered here – at that time! Then the whole world changed. This was the High Energy Physics of that era!

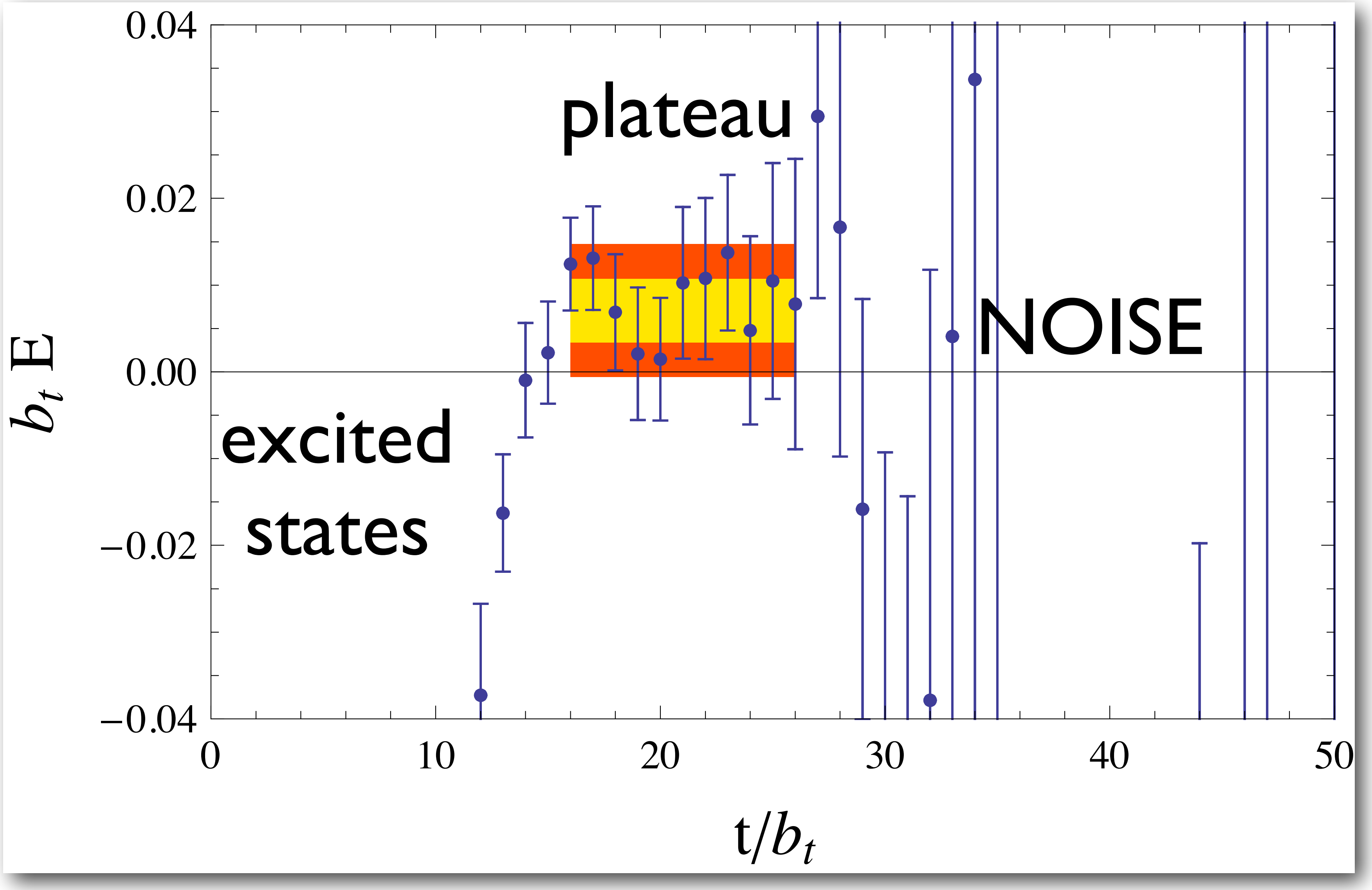
High accuracy hadron spectroscopy:



MILC collaboration / Kronfeld

However, it gets harder to extract a signal from even small nuclei

Triton B.E. S. R. Beane et al. (NPLQCD), Phys. Rev. D 80, 074501 (2009) $(m_\pi = 390\ \mathrm{MeV})$



data should plateau at ground state energy for large τ

Back to Wilson's implementation of the Feynman path integral

32^3 x 64 lattice size: millions of degrees of freedom
Hilbert space size ~ e^{millions}. Lattice QFT: sample it!

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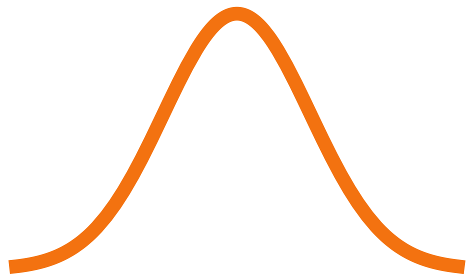
1d wave function



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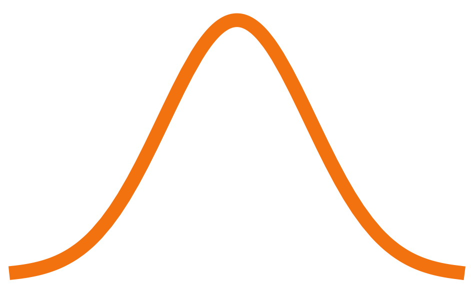

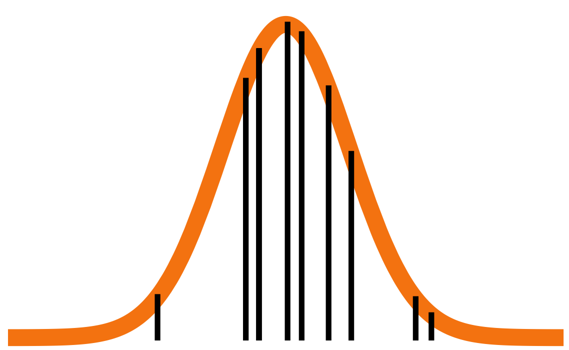

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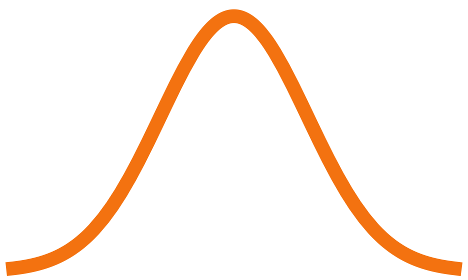
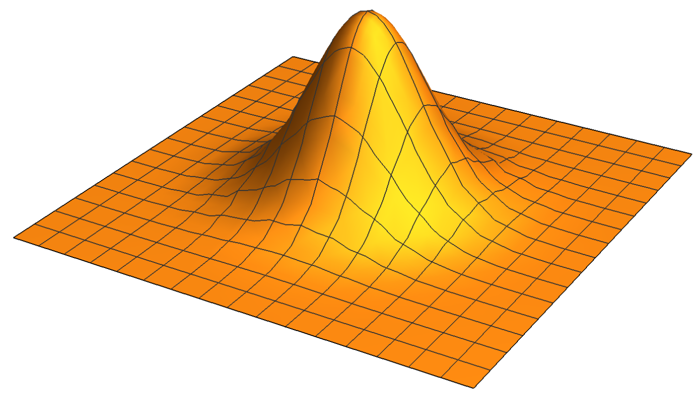
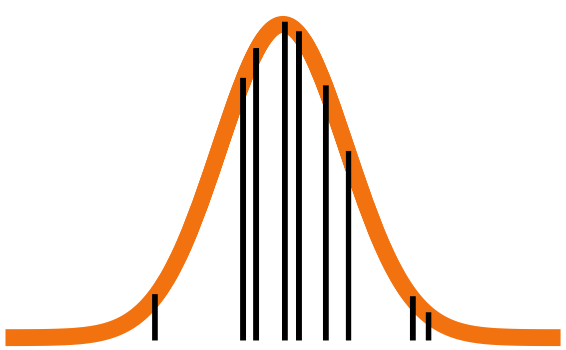


2d wave function





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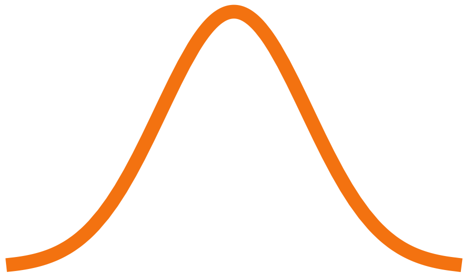

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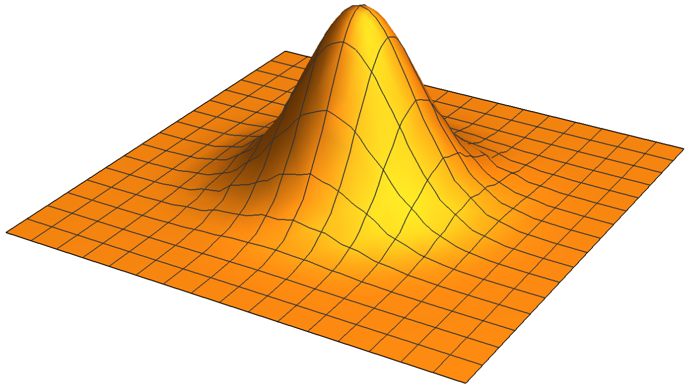

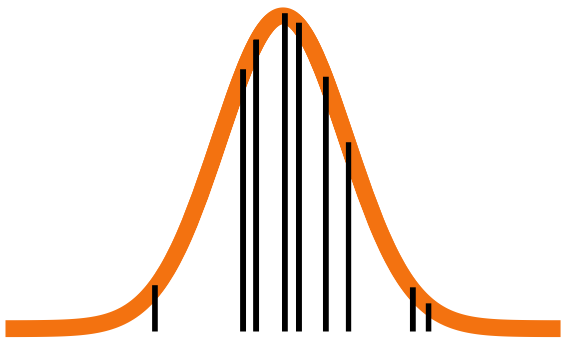

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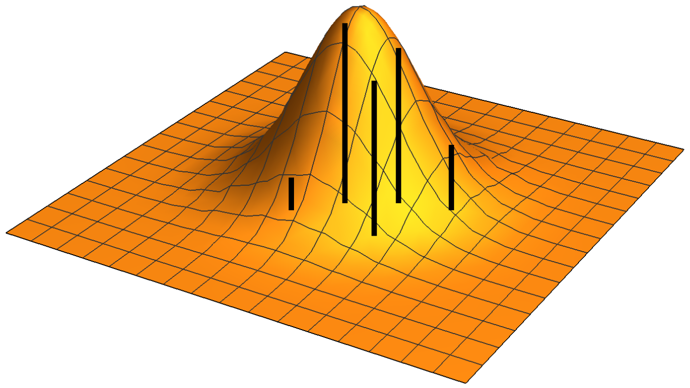

2d wave sampling

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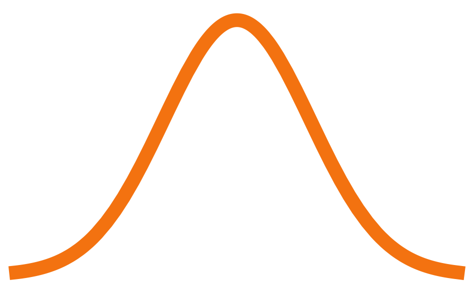
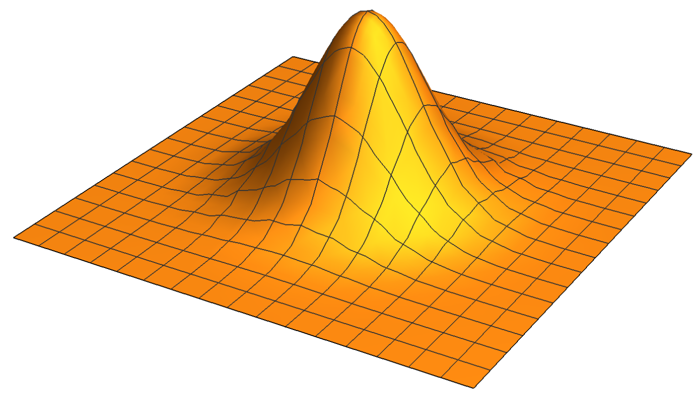
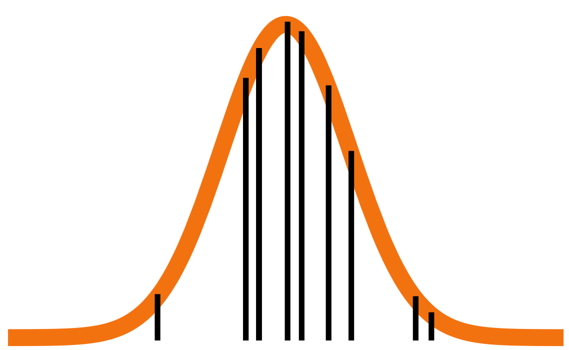
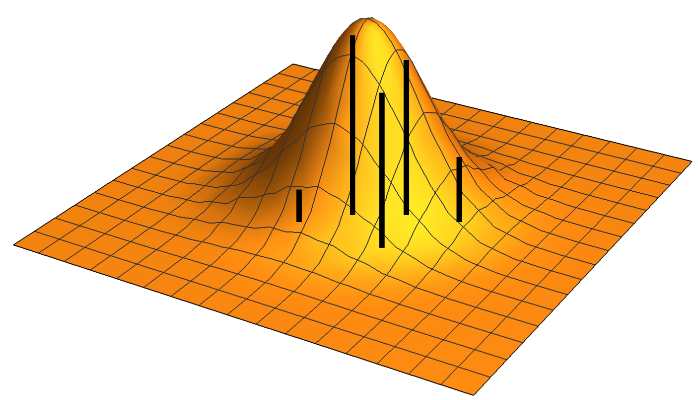
1d wave function | 2d wave function | … e^{millions} d wave function









1d wave sampling | 2d wave sampling

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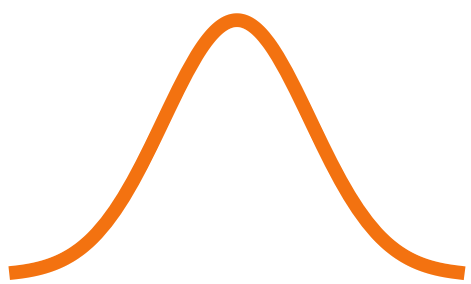

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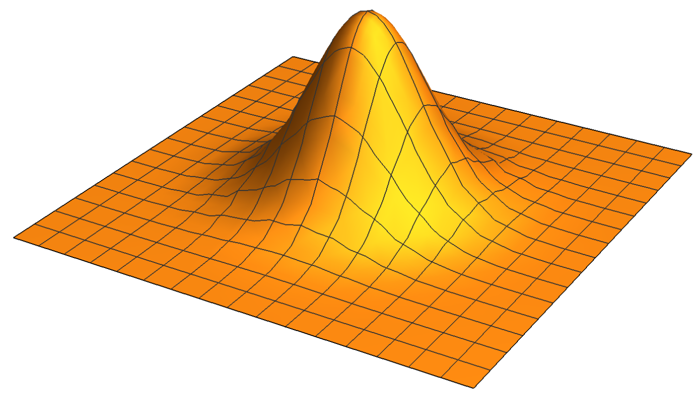
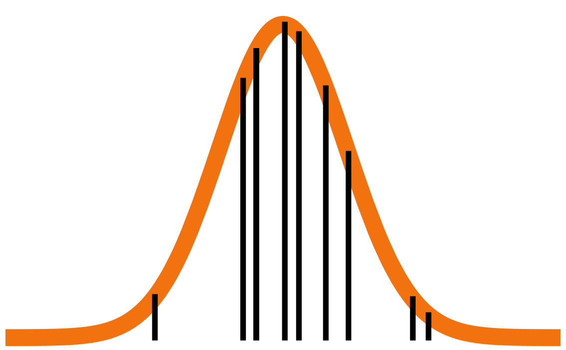


…

e^{millions} d wave function



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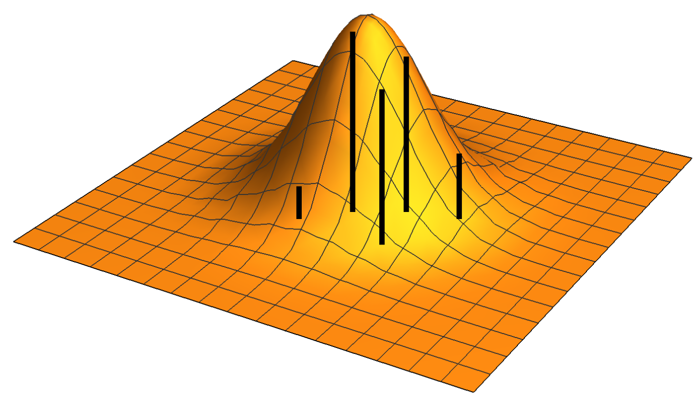
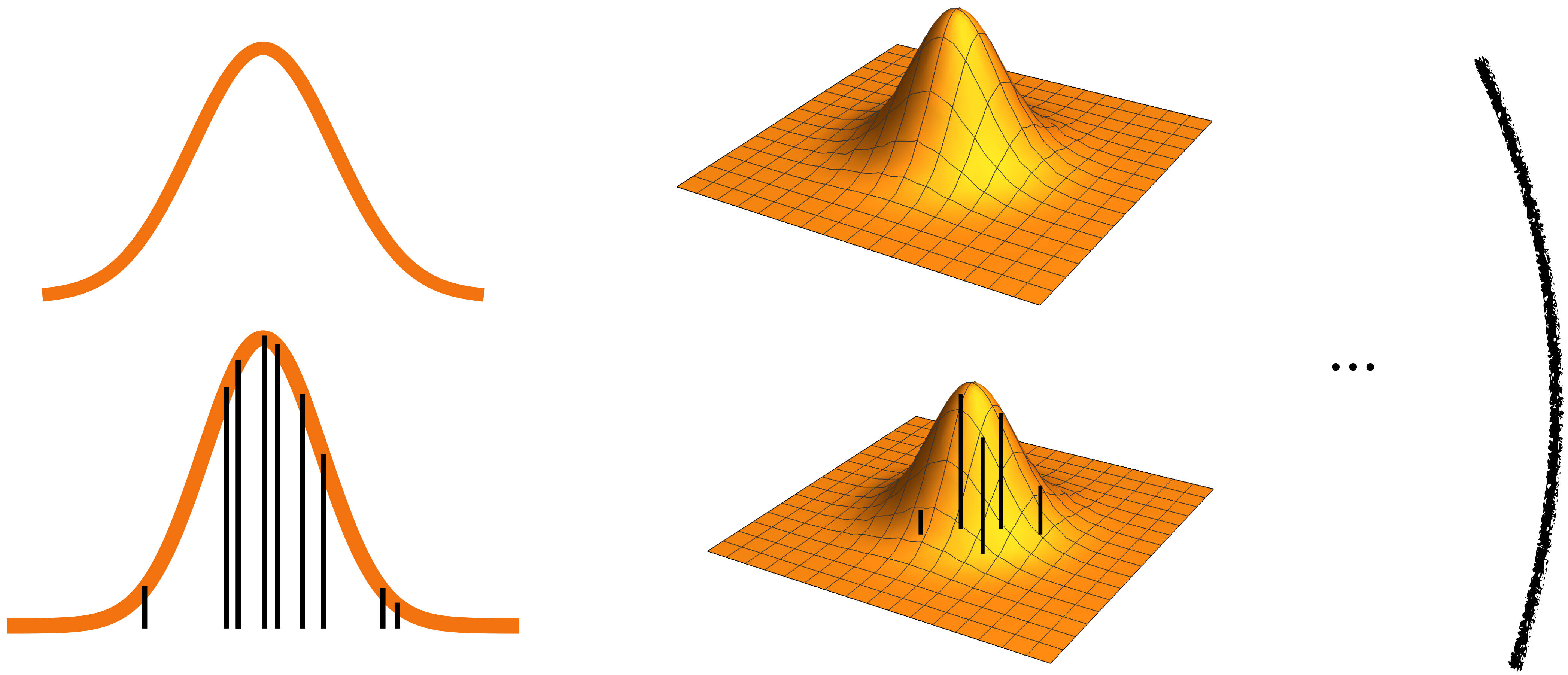
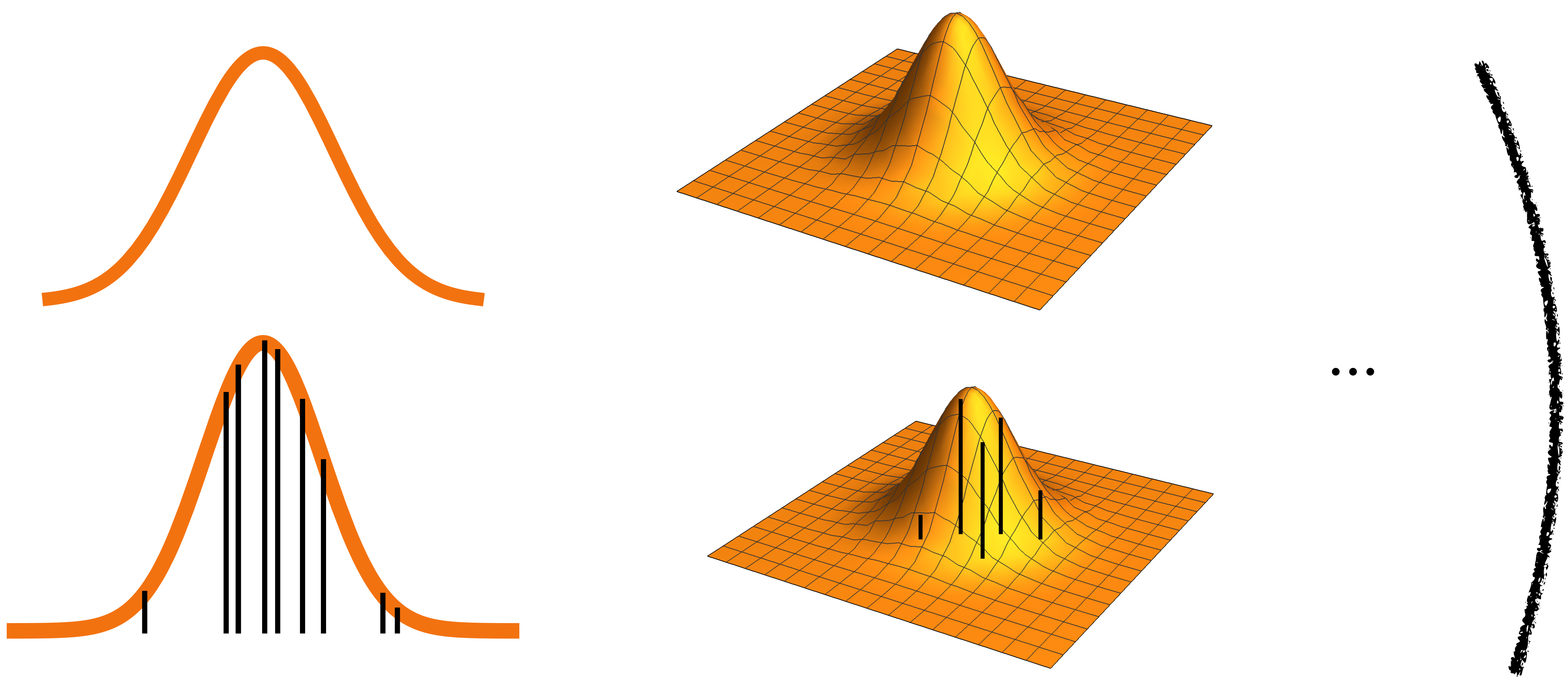


2d wave sampling

<u>Amazingly</u>, the ground state wave function of glue + quark/antiquark pairs is possible to sample effectively



at zero
baryon
number



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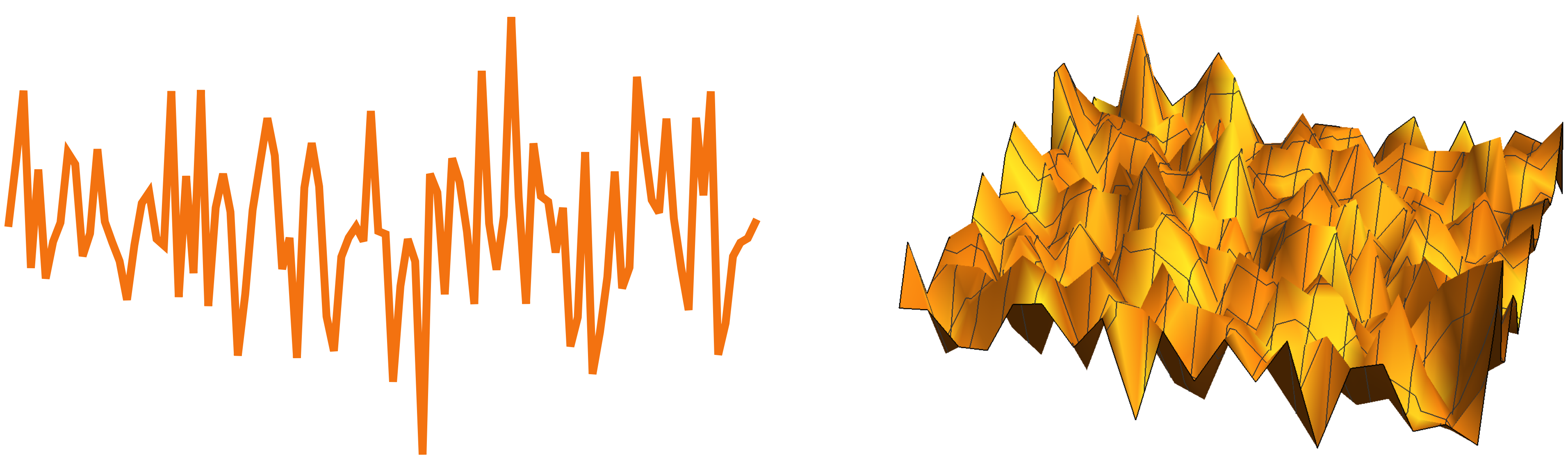
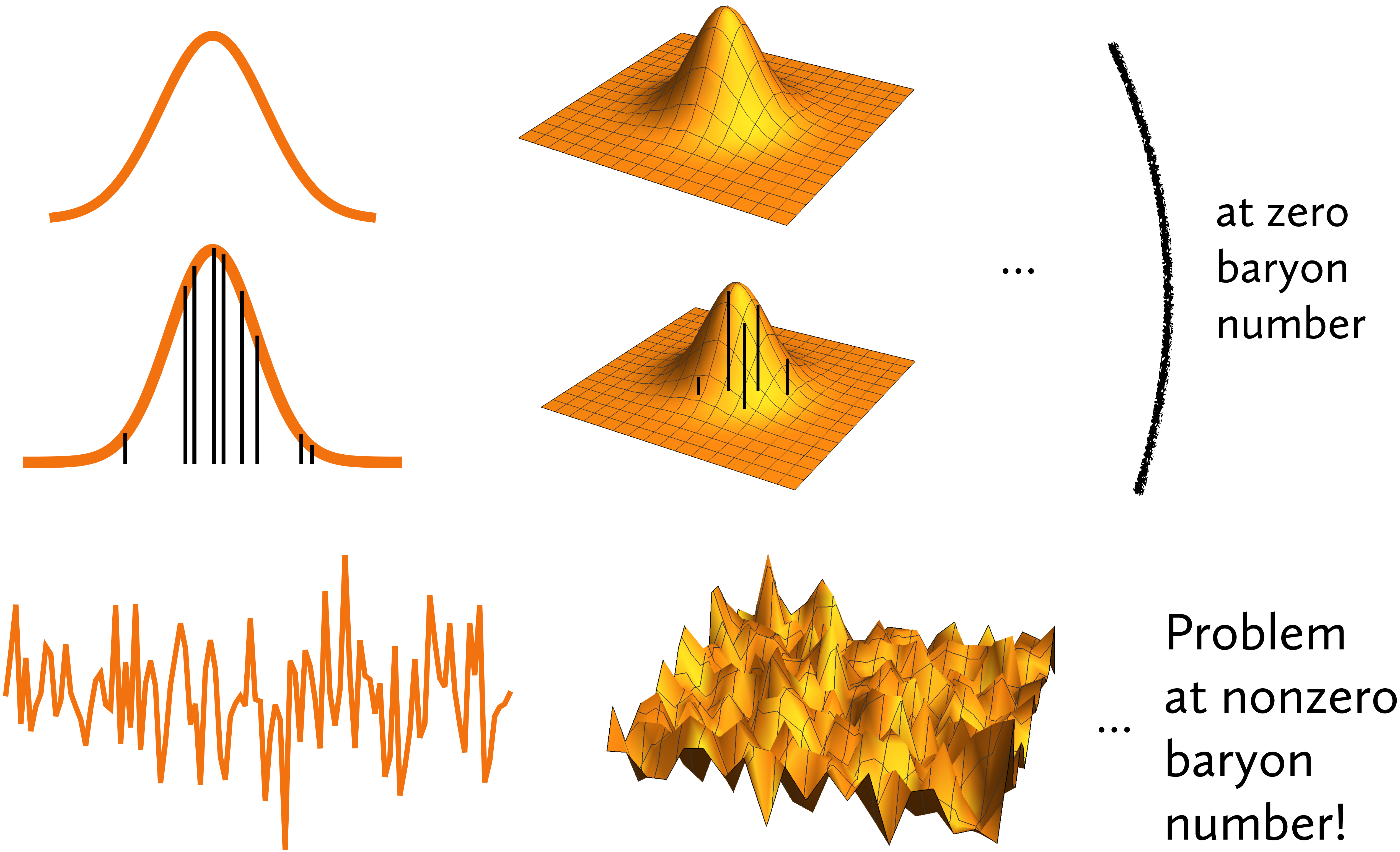


…

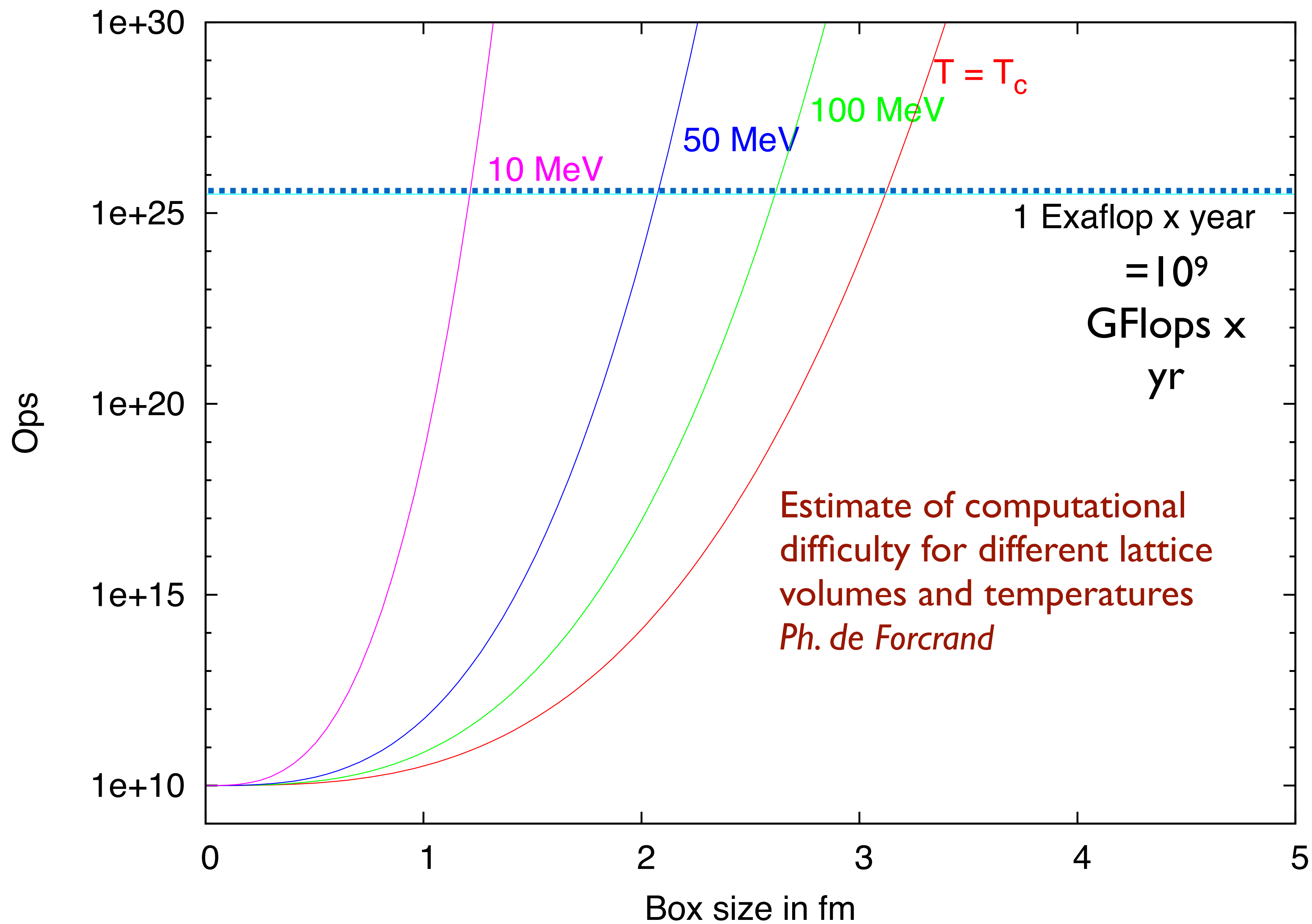
Problem at nonzero baryon number!



One needs an exponentially large number of samples to approximate the wave function

Problem gets worse with more nucleons! How hard is the sign problem?

CPU effort to study matter at nuclear density in a box of given size
Give or take a few powers of 10...



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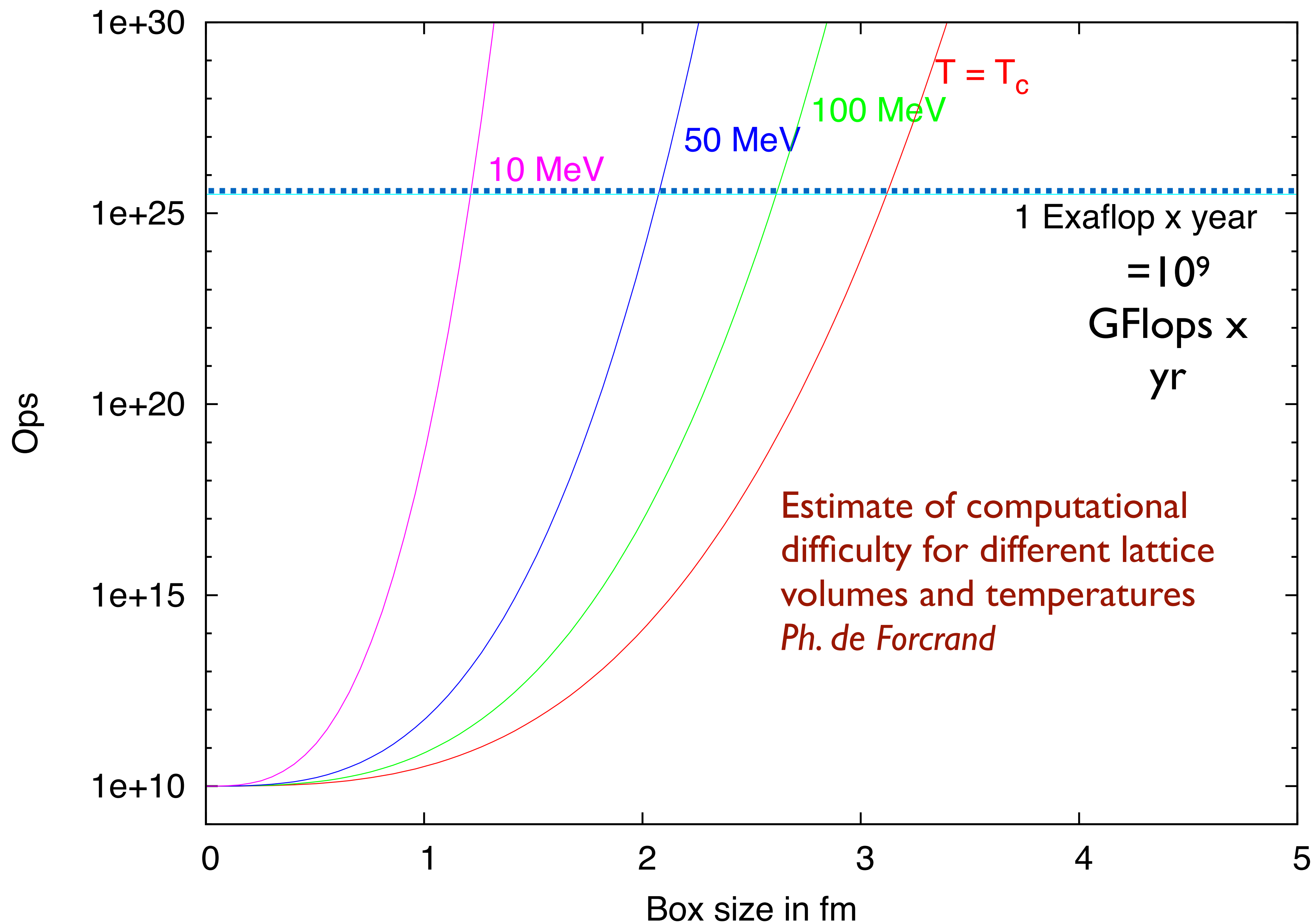
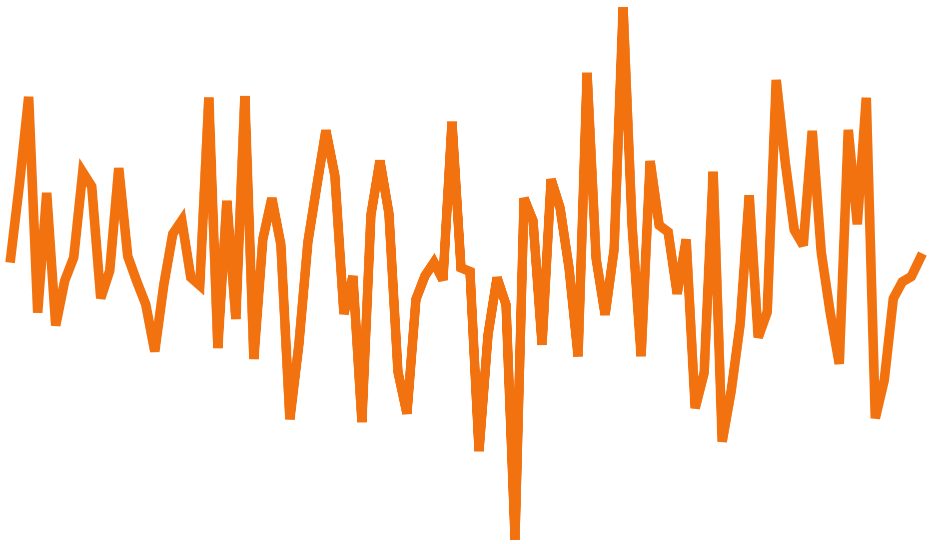
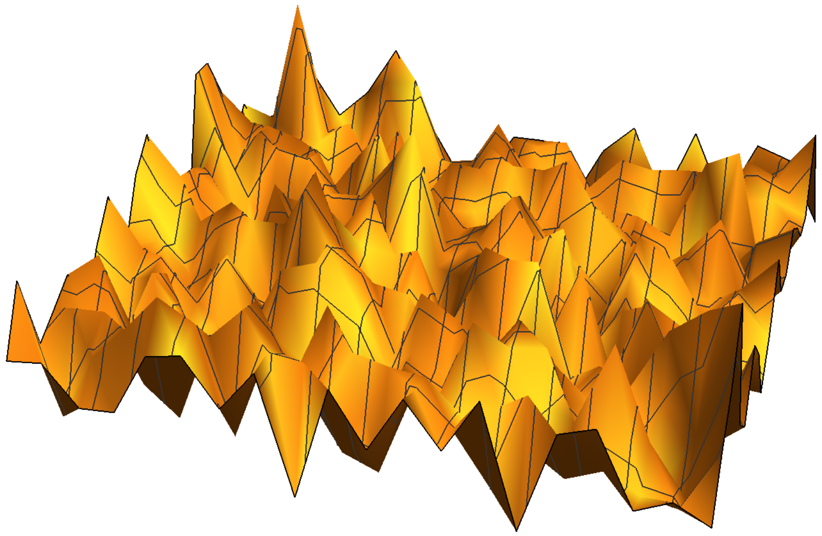


e.g.: T=10 MeV, L=3fm, ρ=nuclear density: > $10^{10\text{-}20}$ exaflop-yrs?!

Apparently at significant baryon number density the wave function explores a large Hilbert space.





... Problem at nonzero baryon number!

Hilbert space is VERY BIG, growing exponentially with the number of particles.

Classical computers are ill-equipped for such problems.

Quantum computers to the rescue?

Richard Feynman
(again)

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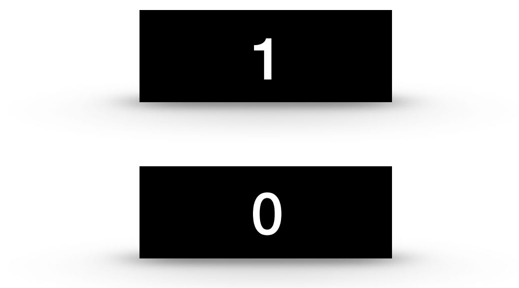
Classical bit



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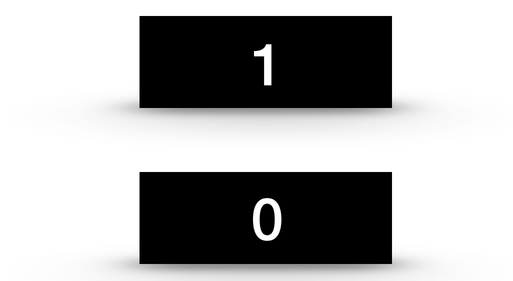
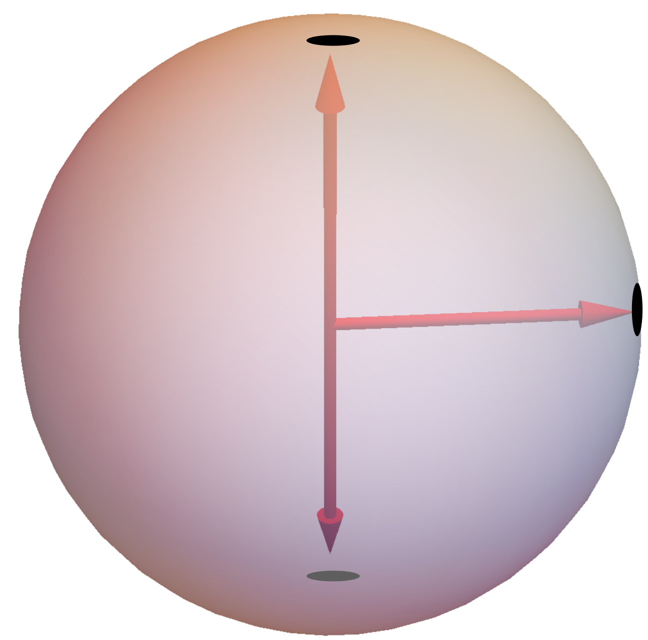


Quantum bit (qubit)

$|1\rangle$



$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$|0\rangle$

state 1:

state 2:

bit flip = 2x2 matrix acting on 2nd bit

can get from any initial state to any final one by a sequence of single-bit flips

2 classical bits

state 1: 00

state 2: 01

bit flip = 2x2 matrix acting on 2nd bit

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2 classical bits

state 1: **00**

state 2: **01**

bit flip = 2x2 matrix acting on 2nd bit

can get from any initial state to any final one by a sequence of single-bit flips

2 qubits

state 1: $|00\rangle$

state 2: $(|00\rangle + |11\rangle)/\sqrt{2}$

requires 4x4 matrix acting on both qubits

cannot get from initial state to final state by a sequence of individual bit flips due to <u>entanglement</u>

2 classical bits

state 1: 00

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2 qubits

state 1: $|00\rangle$

state 2: $(|00\rangle + |11\rangle)/\sqrt{2}$

requires 4x4 matrix acting on both qubits

cannot get from initial state to final state by a sequence of individual bit flips due to entanglement

N classical bits: N 2x2 matrices needed to get from one state to the next

N qubits: 2^Nx 2^N matrix needed to get from one state to the next

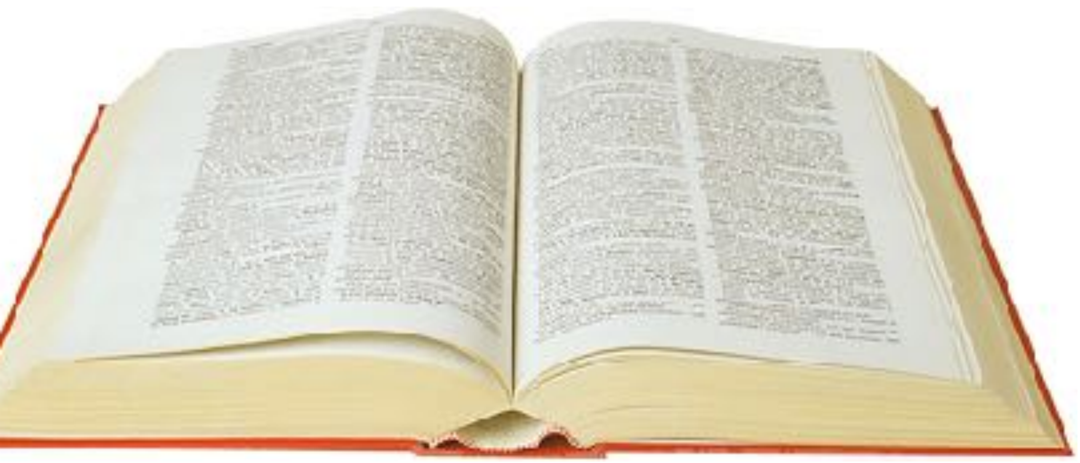
Classical bit stream

11011001000110101011011110...

Classical bit stream

110110010001101011011110...

Classical book



Tear out 1% of the pages and
you lose 1% of the information

Classical bit stream

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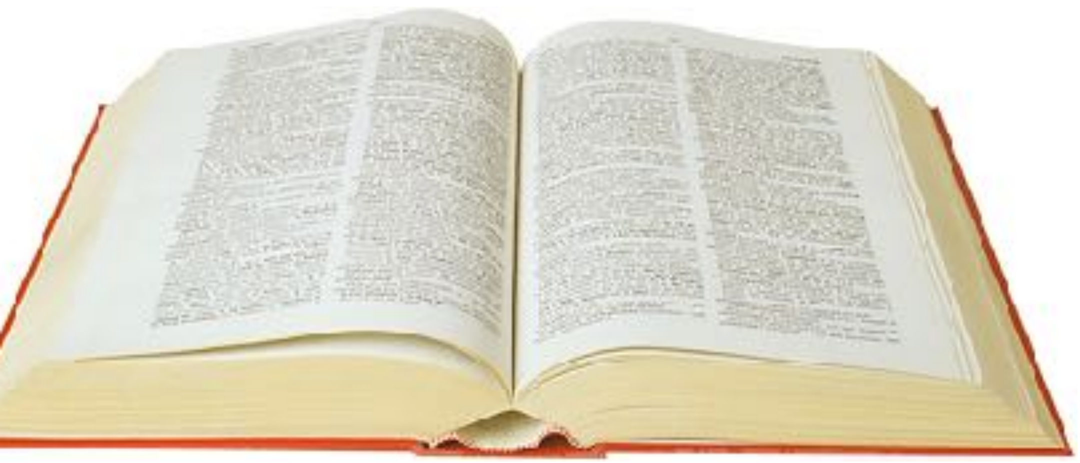
Classical book



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Qbit stream

$$c_1|110010111\ldots\rangle + c_2|001000110\ldots\rangle + c_3|101010001\ldots\rangle + \ldots$$

Classical bit stream

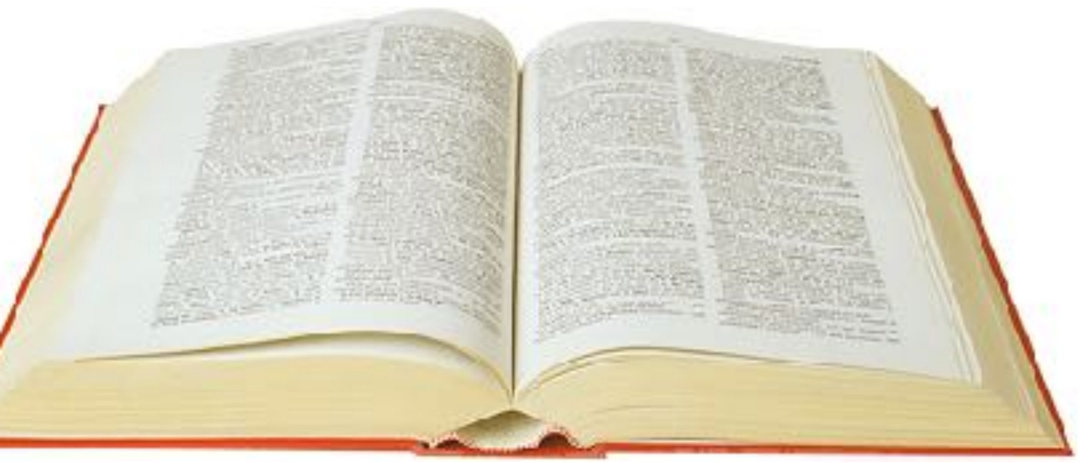
110110010001101011011110...

Qbit stream

$$c_1|110010111\ldots\rangle + c_2|001000110\ldots\rangle + c_3|101010001\ldots\rangle + \ldots$$

Entanglement!

Classical book



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Classical bit stream

11011001000110101101111 0...

Qbit stream

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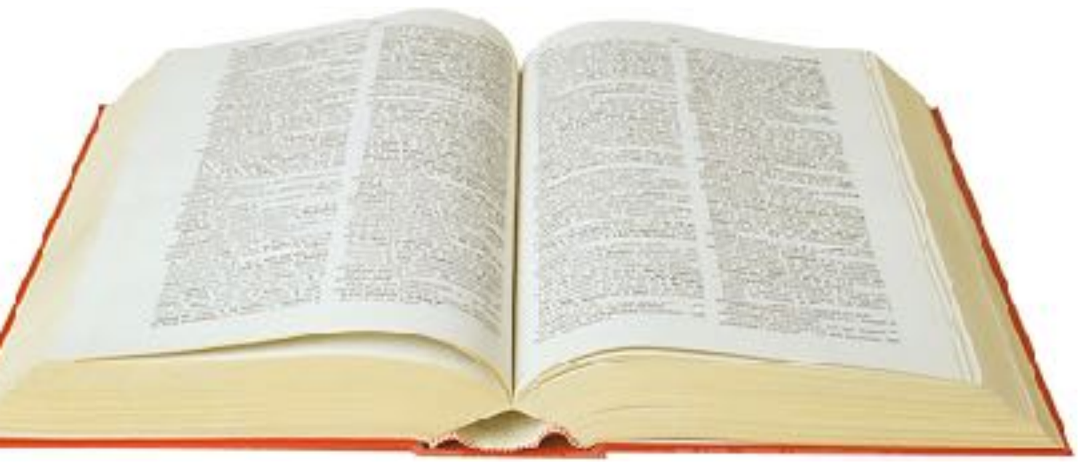
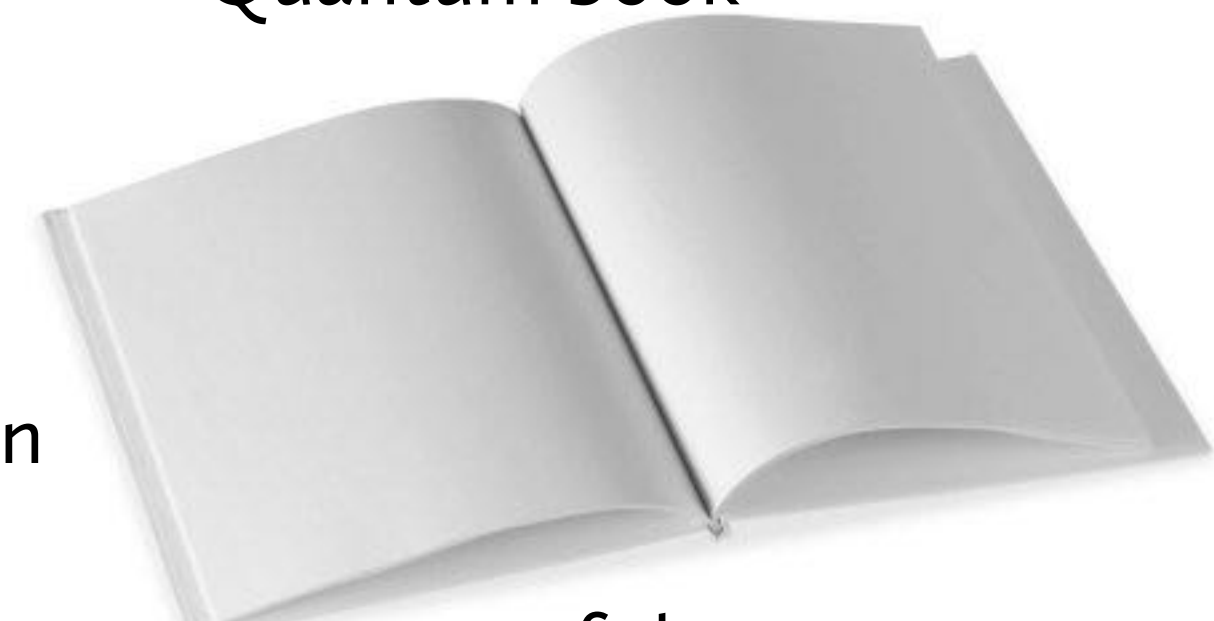


Quantum book



Tear out 1% of the pages and you lose 1% of the information

No information on any one page of the quantum book…tearing out a page is like losing resolution in a photograph…but each page tells you something about what is on the other pages

Classical bit stream

11011001000110101101110...

Qbit stream

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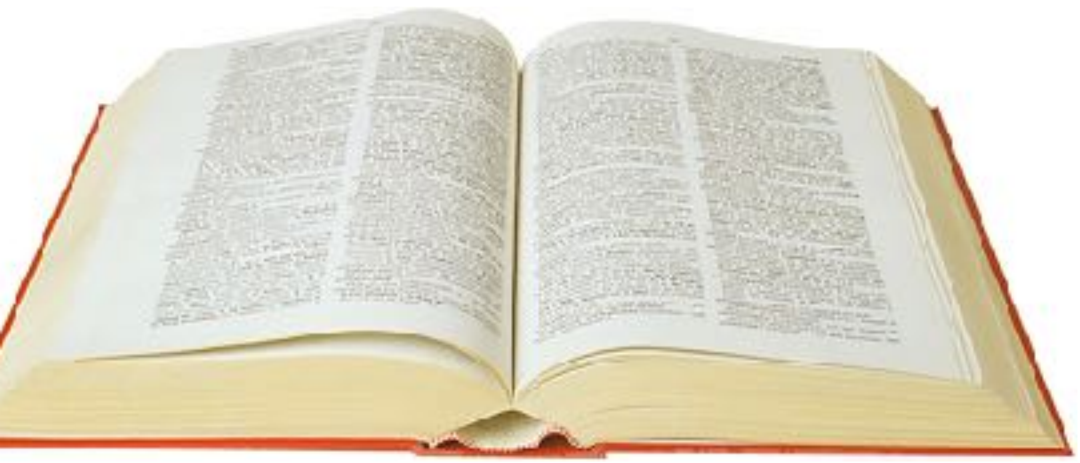
Entanglement!

Classical book



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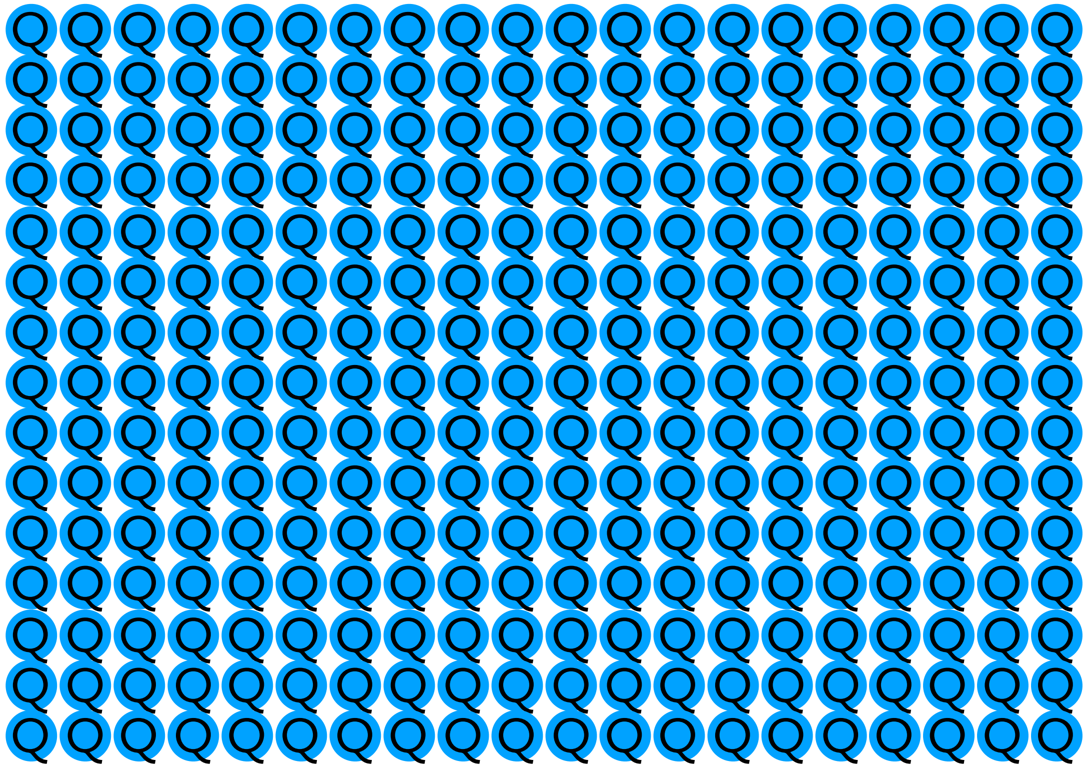
Spukhafte Fernwirkung

…but quantum books contain vastly more information than classical ones



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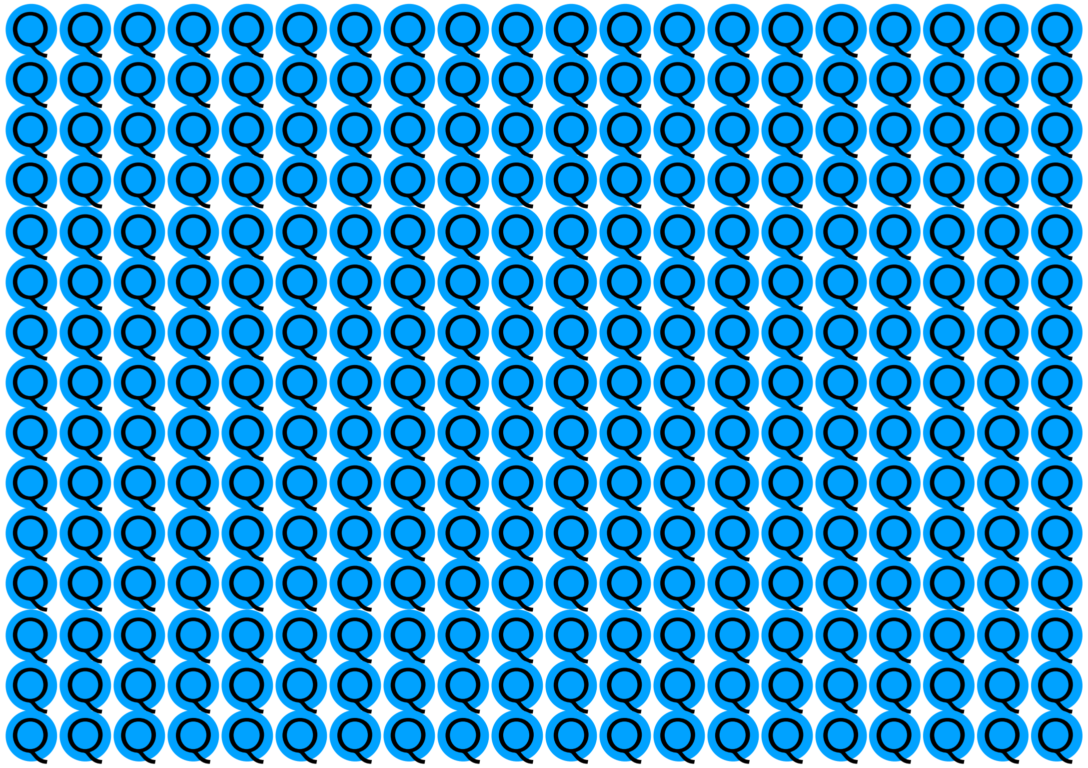

300 qubits

$\sim 2^{300}$

The number of classical bits required to encode the information in 300 qubits is more than the total number of atoms in the Universe!

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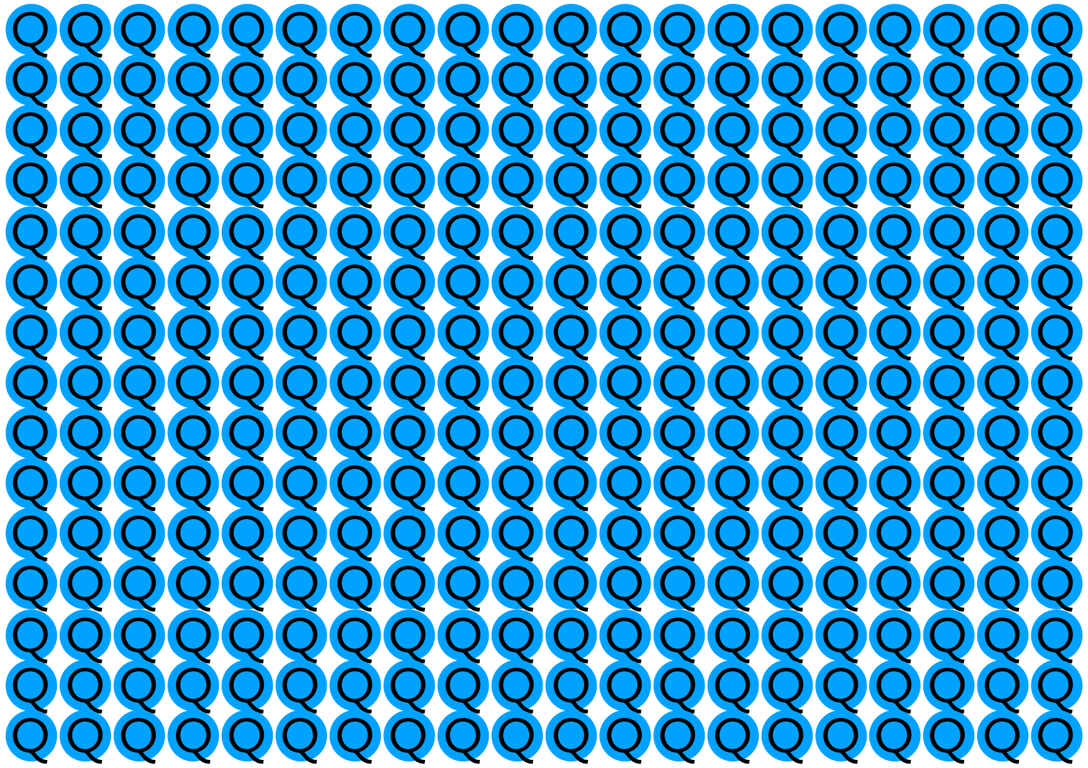

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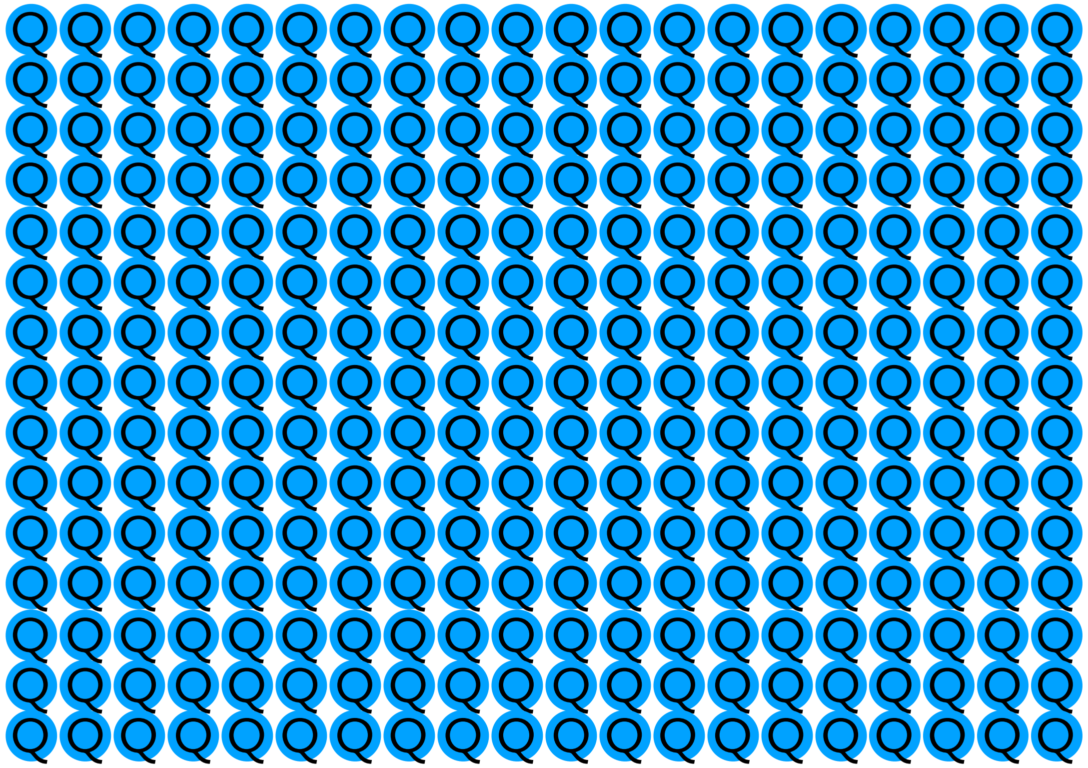

300 qubits > 1 classical universe

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So how do you solve hard scientific problems on a quantum computer?

On a quantum computer:

- initialize qubits in some state
- evolve it with a Hamiltonian one constructs as a sequence of gates
- measure matrix elements in final state

Algorithms on a quantum computer don't look like classical algorithms!

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2 examples:

Quantum Phase Estimation Algorithm

The Quantum Adiabatic Algorithm

Suppose $|\psi>$ is the eigenvector of a unitary operator U ($= e^{-iHt}$), represented by m qubits:

$$U\,|\psi> = e^{2\pi i\theta}\,|\psi>$$

and you want to determine θ to accuracy $1:2^{-n}$

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Quantum Phase Estimation Algorithm:

- Introduce n auxiliary qubits
- Entangle them with 1…n applications of U to $|\psi\rangle$
- Measure the auxiliary qubits…which will with high probability give θ as a binary number

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Quantum Phase Estimation Algorithm:

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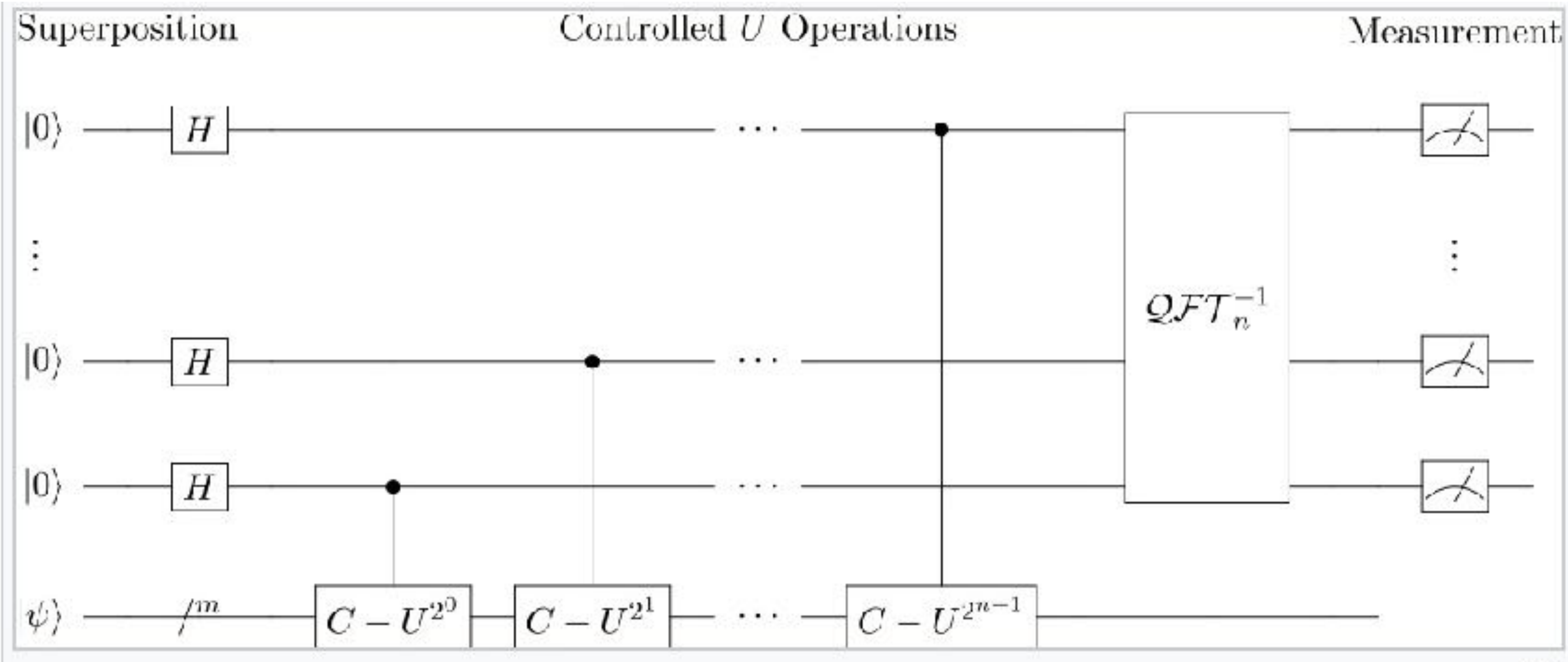
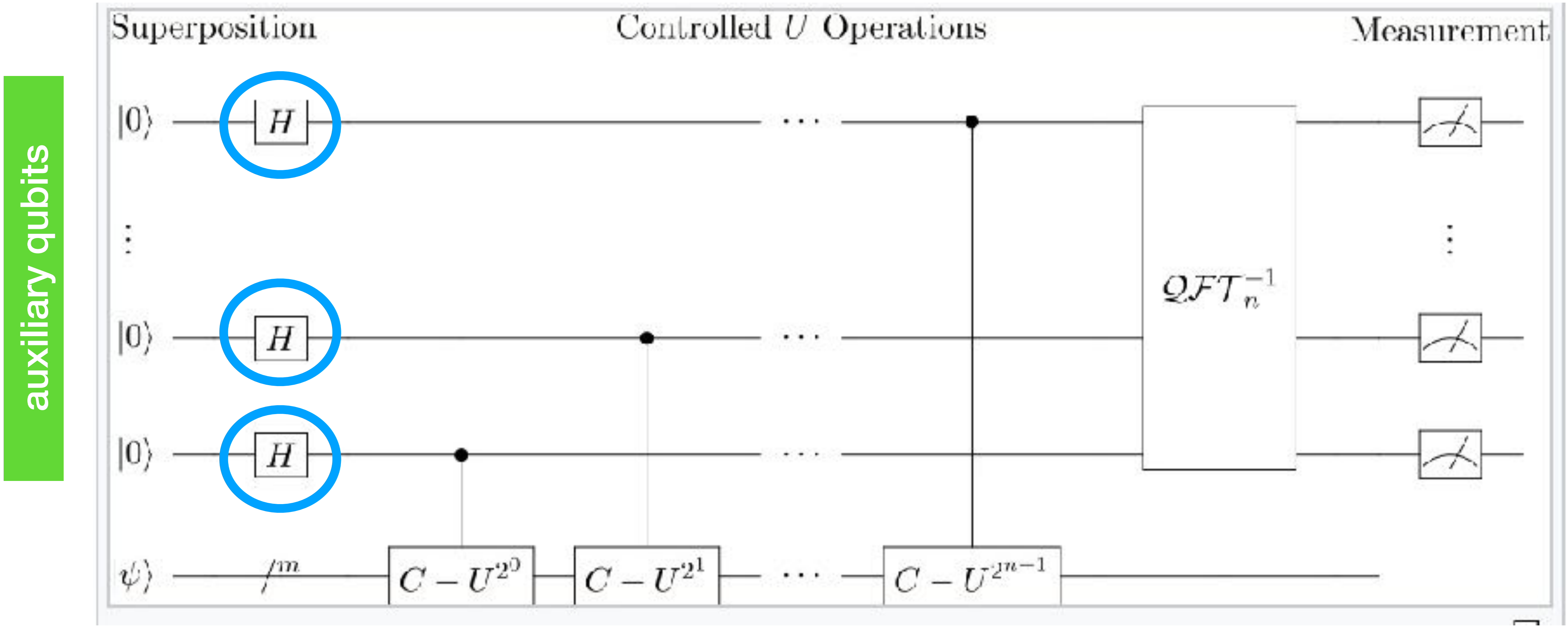
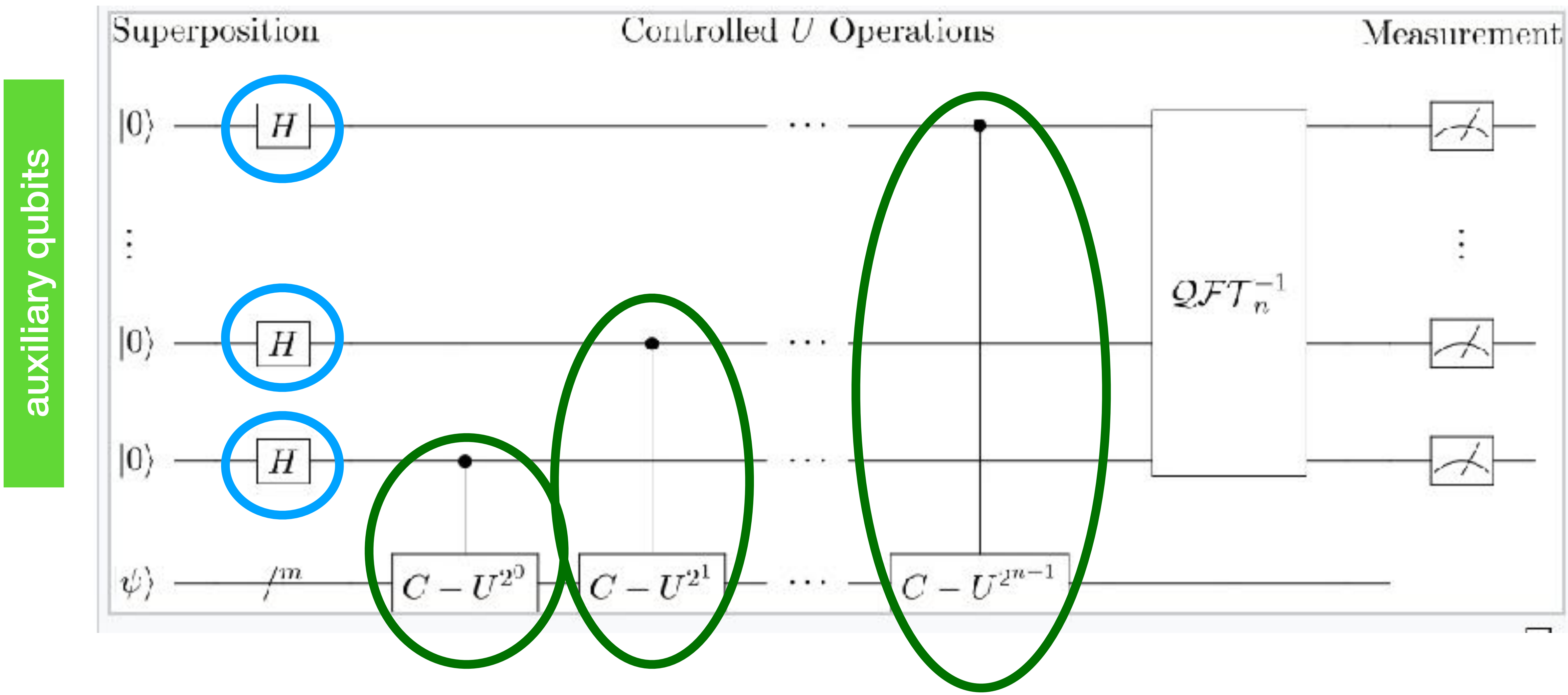
✦ This can be applied to general $|\psi\rangle$ where $U = e^{-i\,H\,dt}$
✦ Output are the eigenvalues of H weighted by $|\langle\psi|n\rangle|^2$

auxiliary qubits





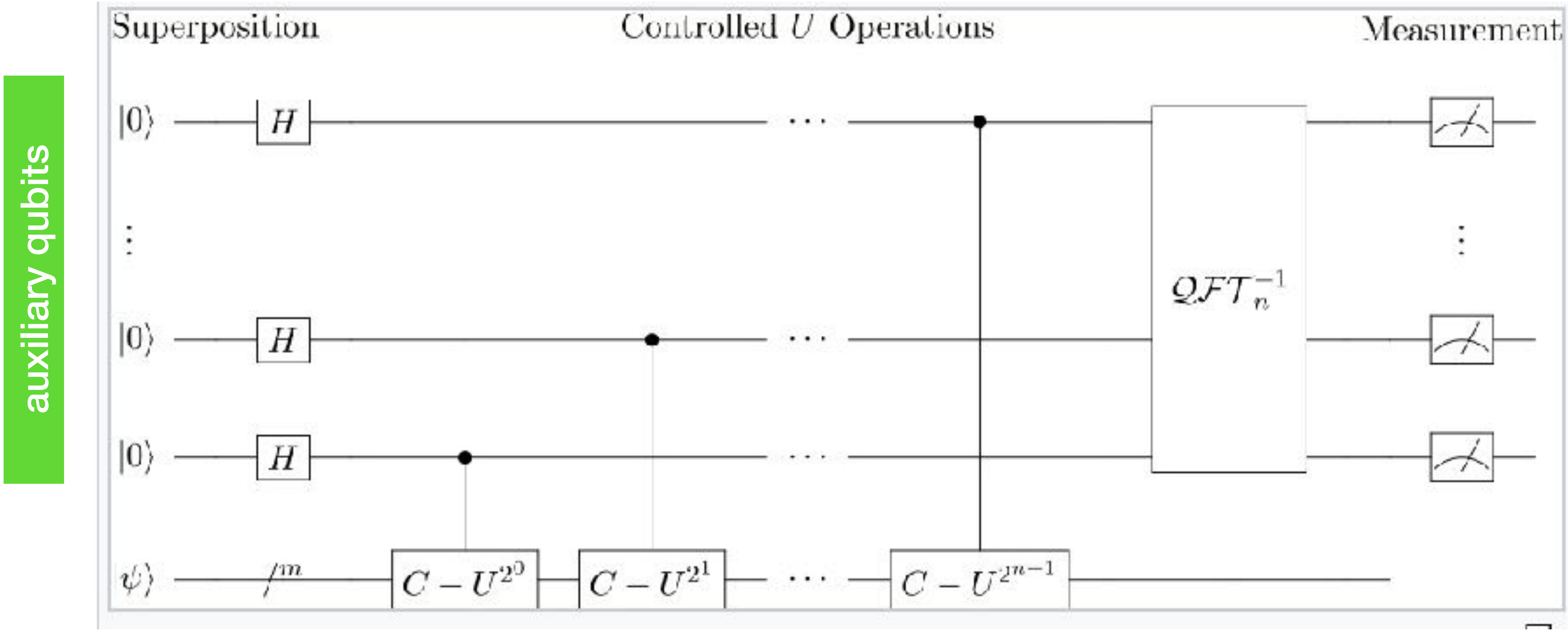
Hadamard gates give you the state: $2^{-n/2}\,(|0\rangle + |1\rangle)^{\otimes n}\,|\psi\rangle$

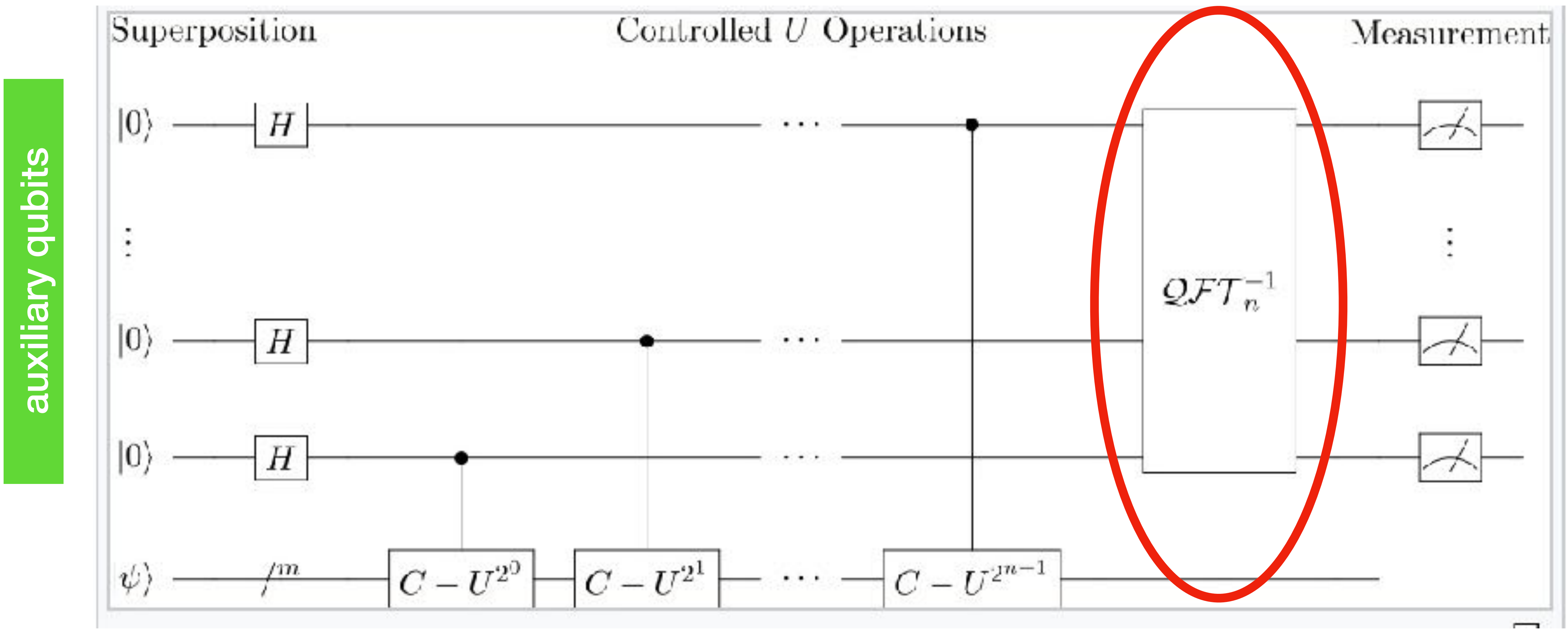


Hadamard gates give you the state: $2^{-n/2}\,(|0\rangle + |1\rangle)^{\otimes n}\,|\psi\rangle$

Controlled phase rotations by U then give you the state

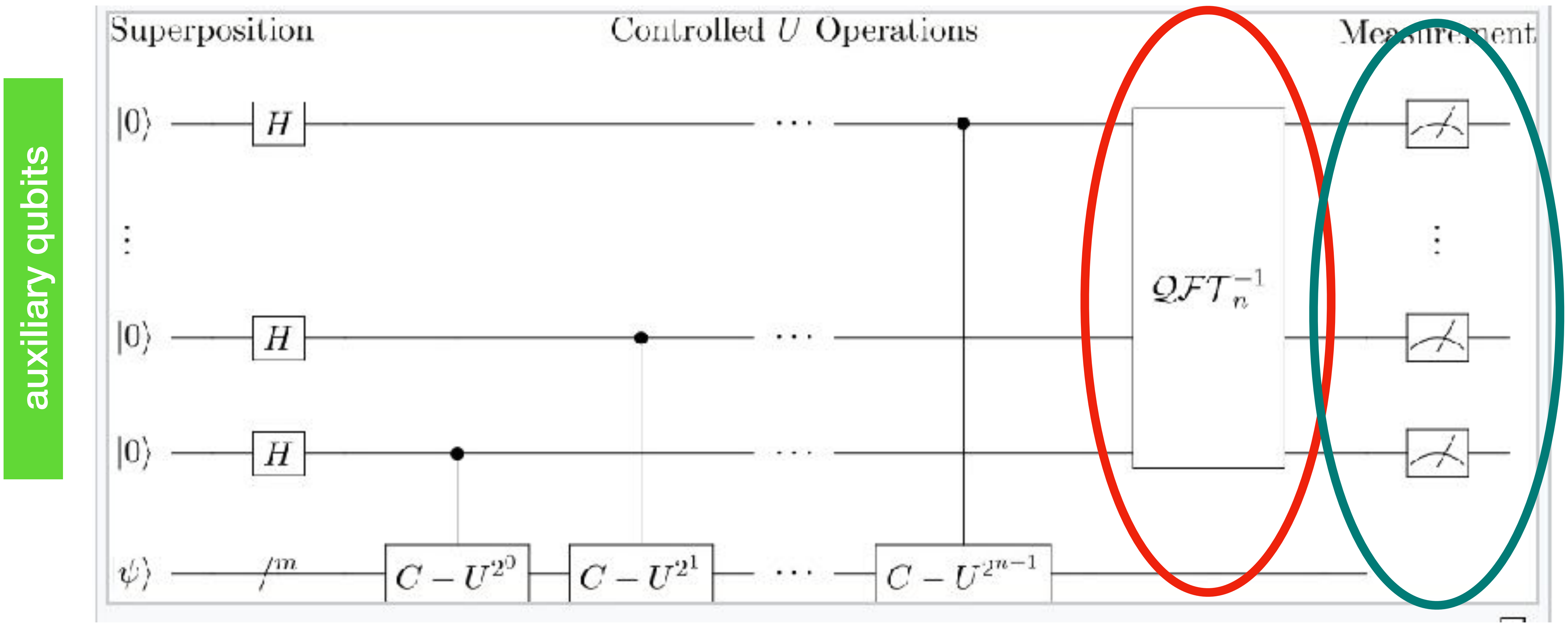
$$\frac{1}{2^{\frac{n}{2}}}\underbrace{\left(|0\rangle + e^{2\pi i 2^{n-1}\theta}|1\rangle\right)}_{1^{st}\ qubit} \otimes \cdots \otimes \underbrace{\left(|0\rangle + e^{2\pi i 2^{1}\theta}|1\rangle\right)}_{n-1^{th}\ qubit} \otimes \underbrace{\left(|0\rangle + e^{2\pi i 2^{0}\theta}|1\rangle\right)}_{n^{th}\ qubit} = \frac{1}{2^{\frac{n}{2}}}\sum_{k=0}^{2^n-1} e^{2\pi i\theta k}|k\rangle.$$


auxiliary qubits
Superposition
Controlled U Operations
Measurement
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|\psi\rangle$
H
H
H
m
$C - U^{2^0}$
$C - U^{2^1}$
$C - U^{2^{n-1}}$
QFT_n^{-1}



Inverse Quantum Fourier Transform

- Classical computer: FT requires $O(n2^n)$ gates
- Quantum computer: FT requires only $O(n^2)$ gates



Inverse Quantum Fourier Transform

- Classical computer: FT requires $O(n2^n)$ gates
- Quantum computer: FT requires only $O(n^2)$ gates

Measure auxiliary qubits:

{1,0,0,1,1...} ▸ $\theta = 1 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + \ldots$

Quantum Phase Estimation requires |<ψ|0>|^2 not be small…might be difficult for a strongly coupled many-body theory.

Another option:

Quantum Adiabatic Algorithm for finding the ground state of a Hamiltonian

$$H(s) = (1-s)H_0 + sH_1 \qquad 0 \le s \le 1$$

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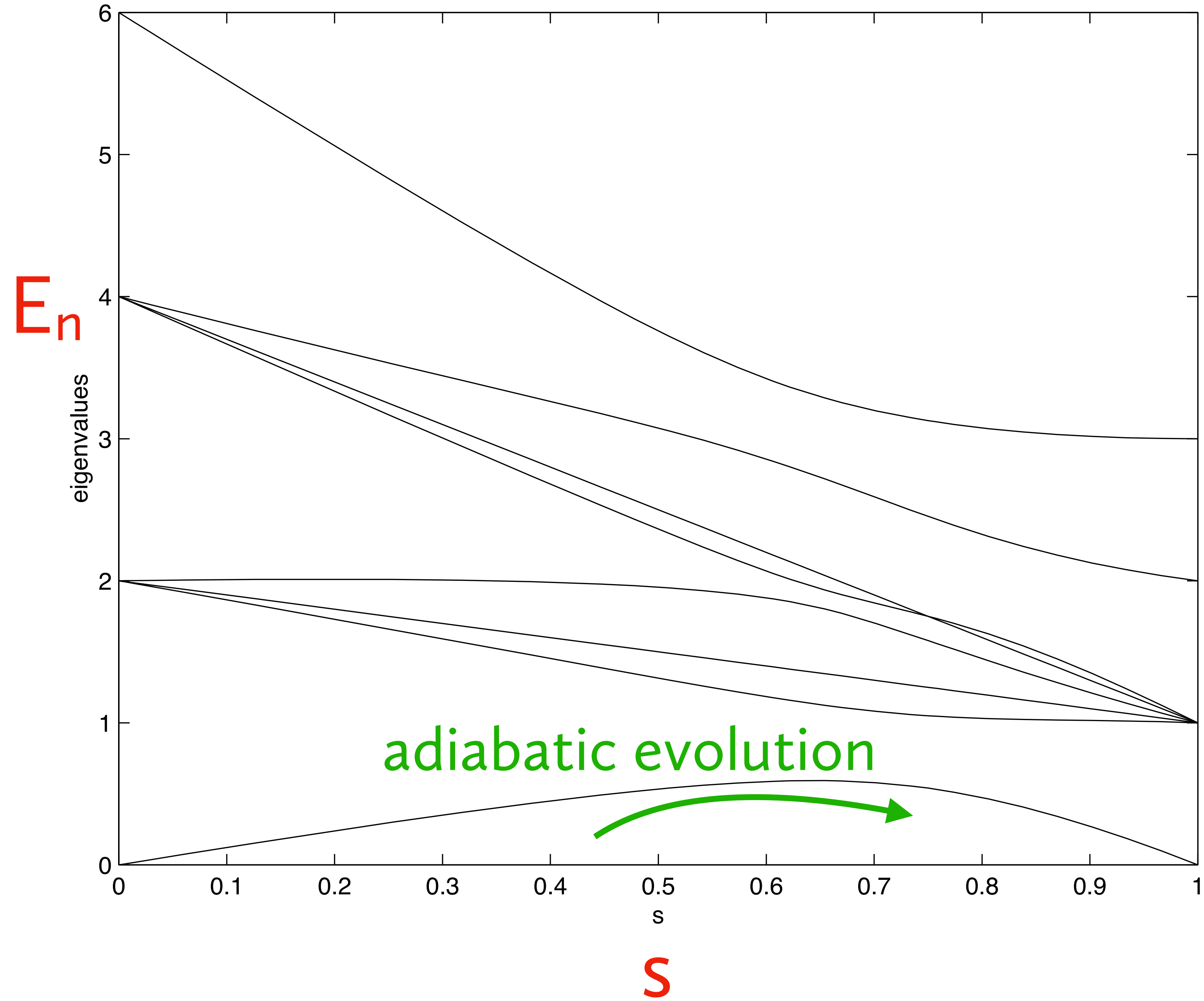
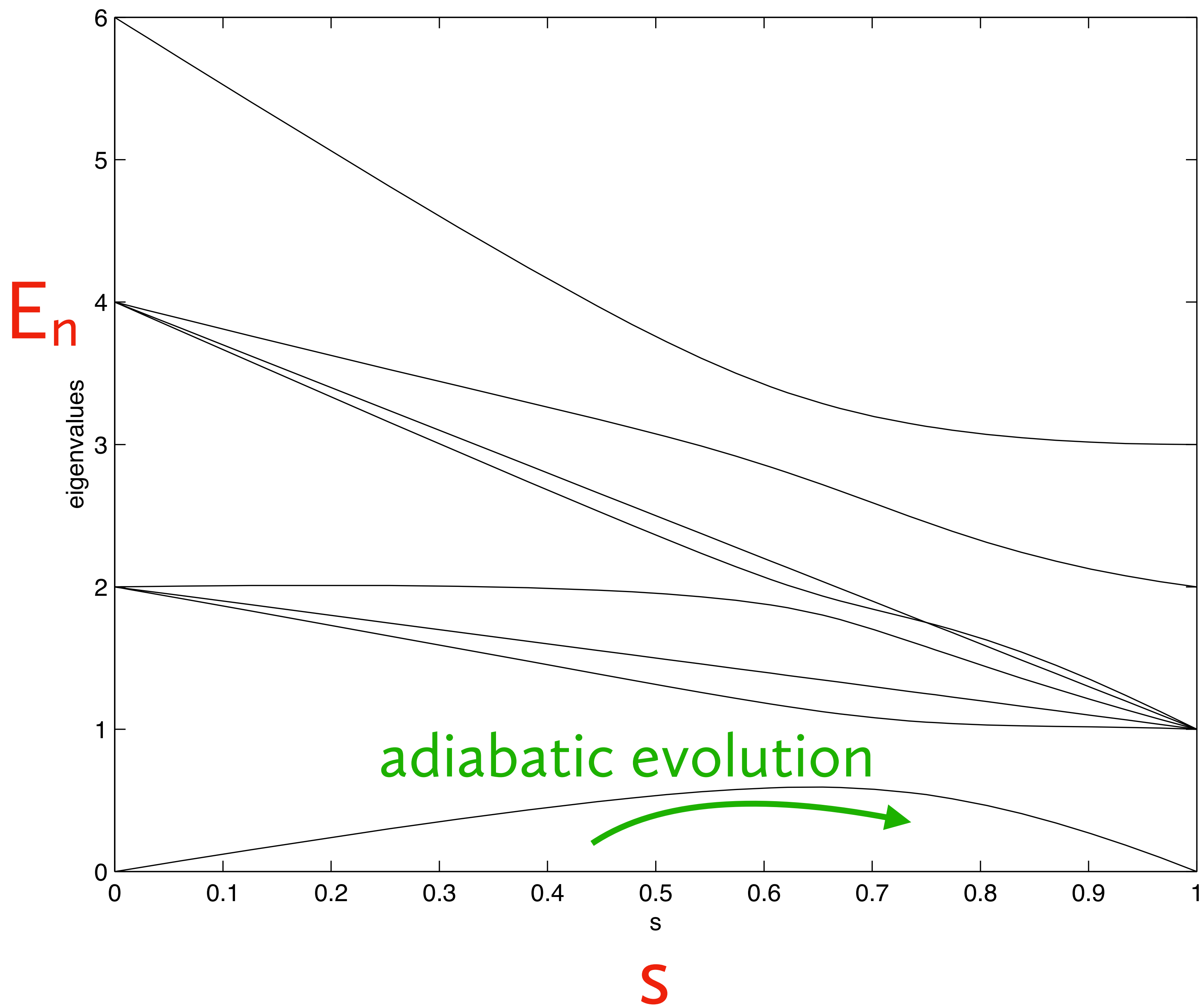
simple Hamiltonian → H_0

interesting Hamiltonian → H_1



Edward Farhi, Jeffrey Goldstone, Sam Gutmann, Michael Sipser arXiv:quant-ph/0001106

- Initialize qubits for known ground state of H_0



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- Initialize qubits for known ground state of H_0
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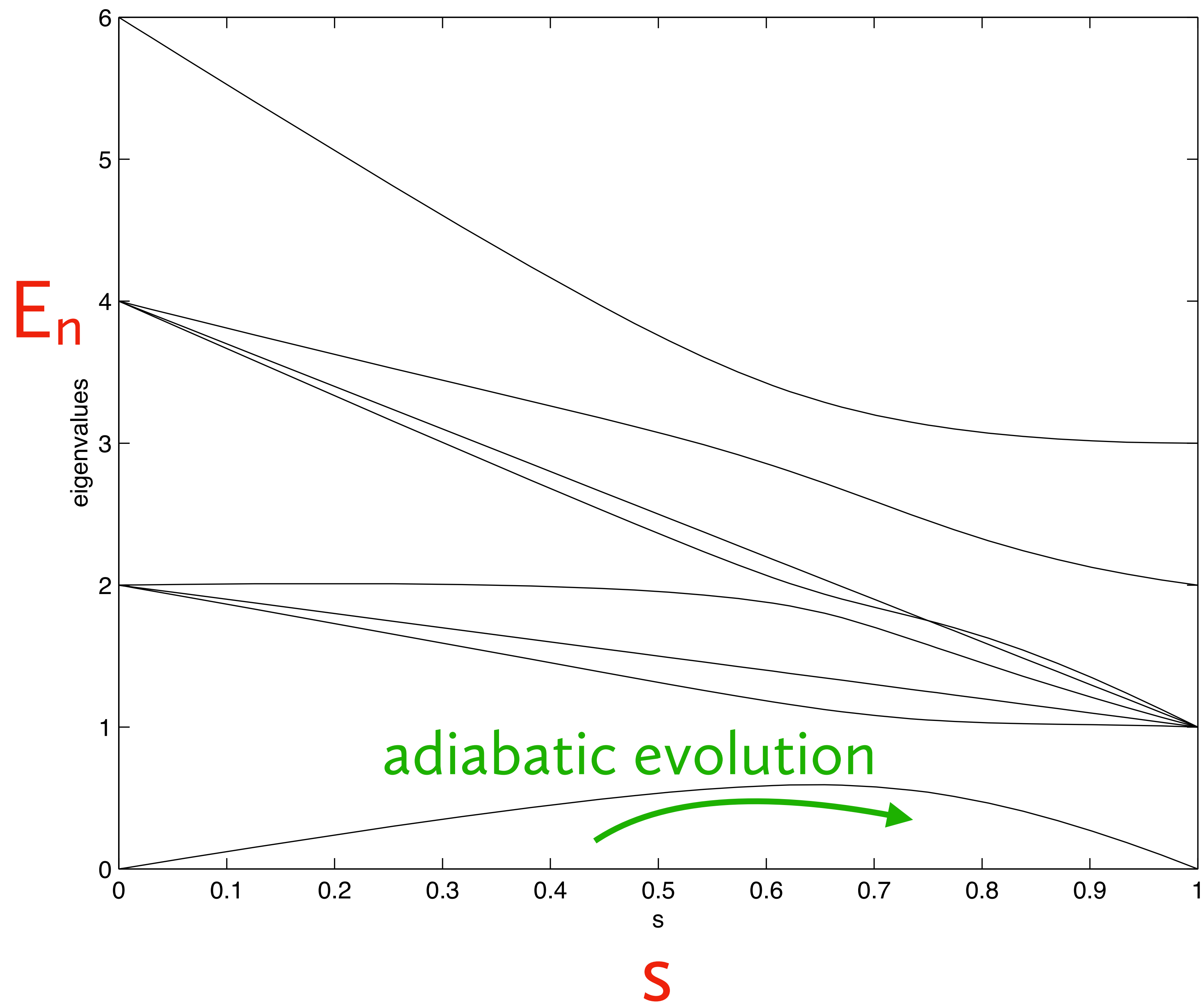

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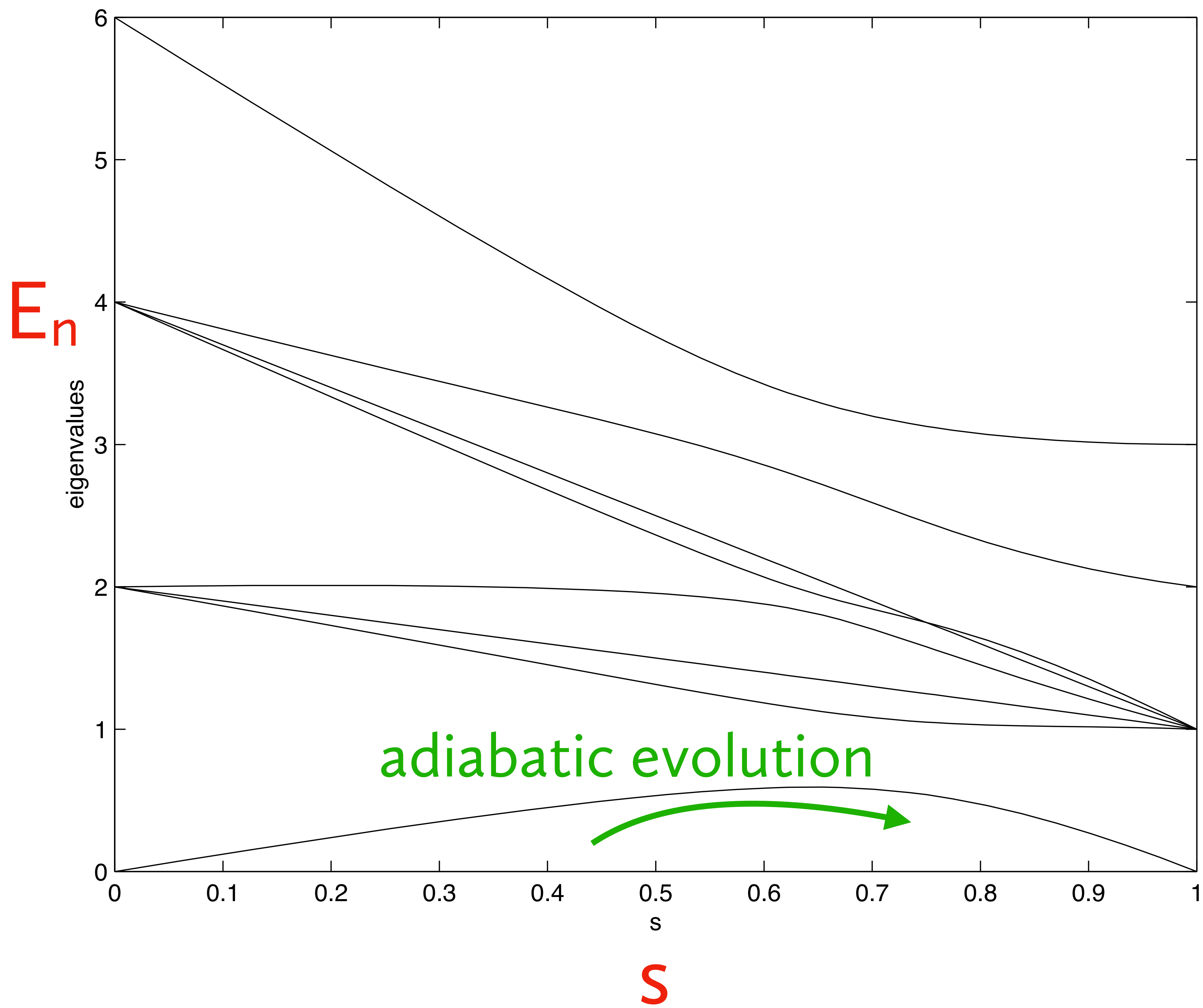

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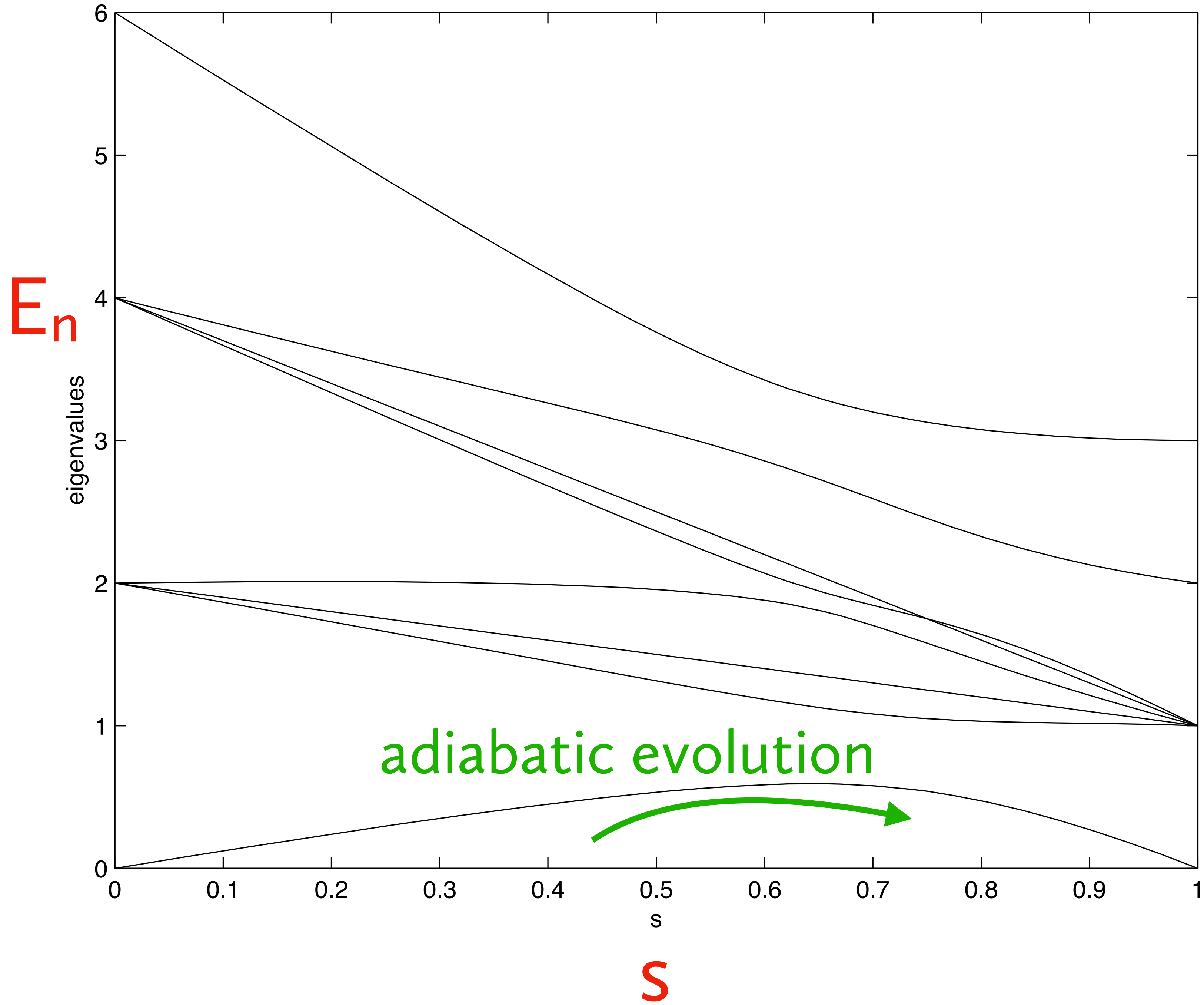
- Measure desired matrix elements



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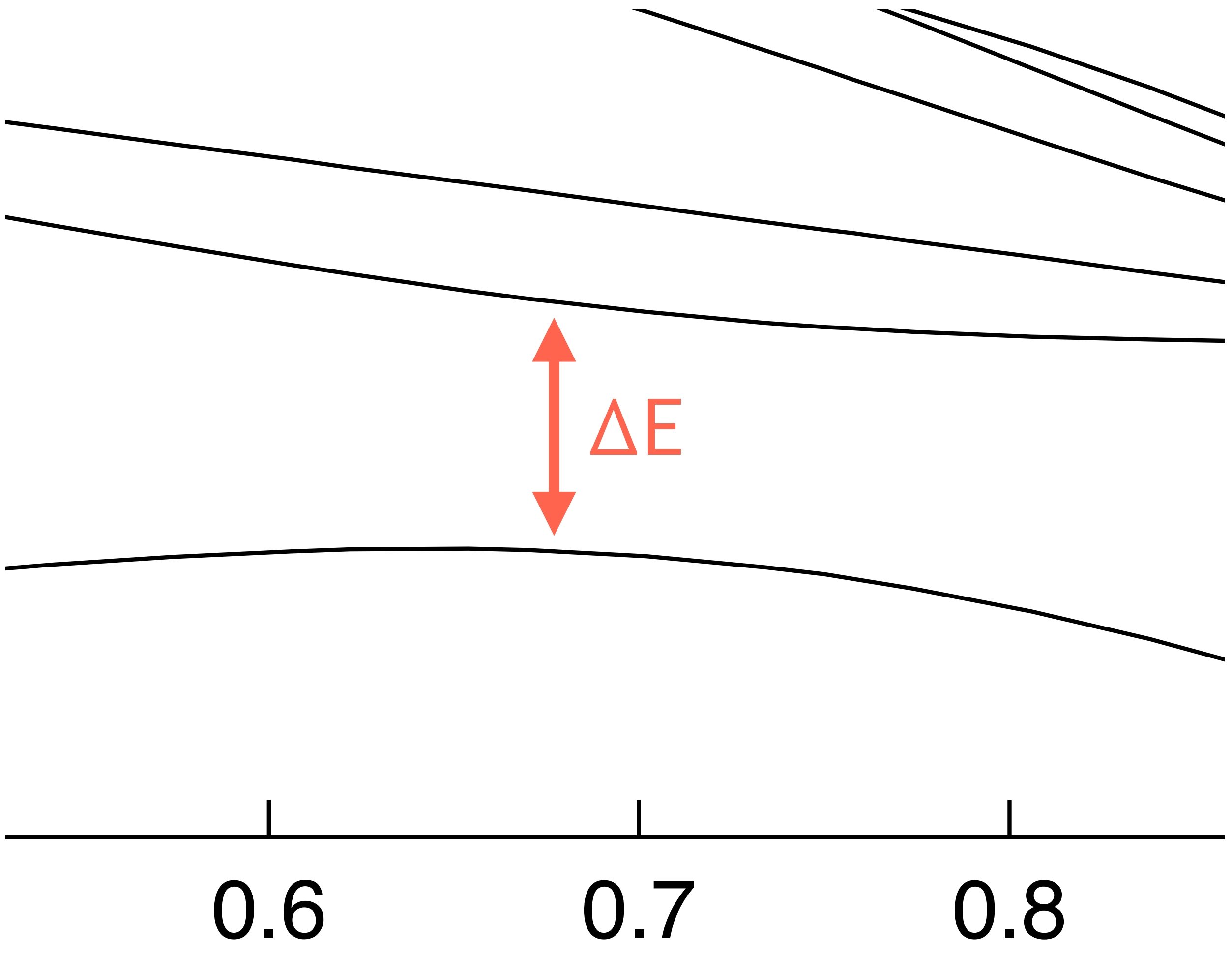
Drawback of the Quantum Adiabatic Algorithm:

Adiabatic theorem requires evolution time scales as

$$t \sim \frac{1}{\Delta E^2}$$



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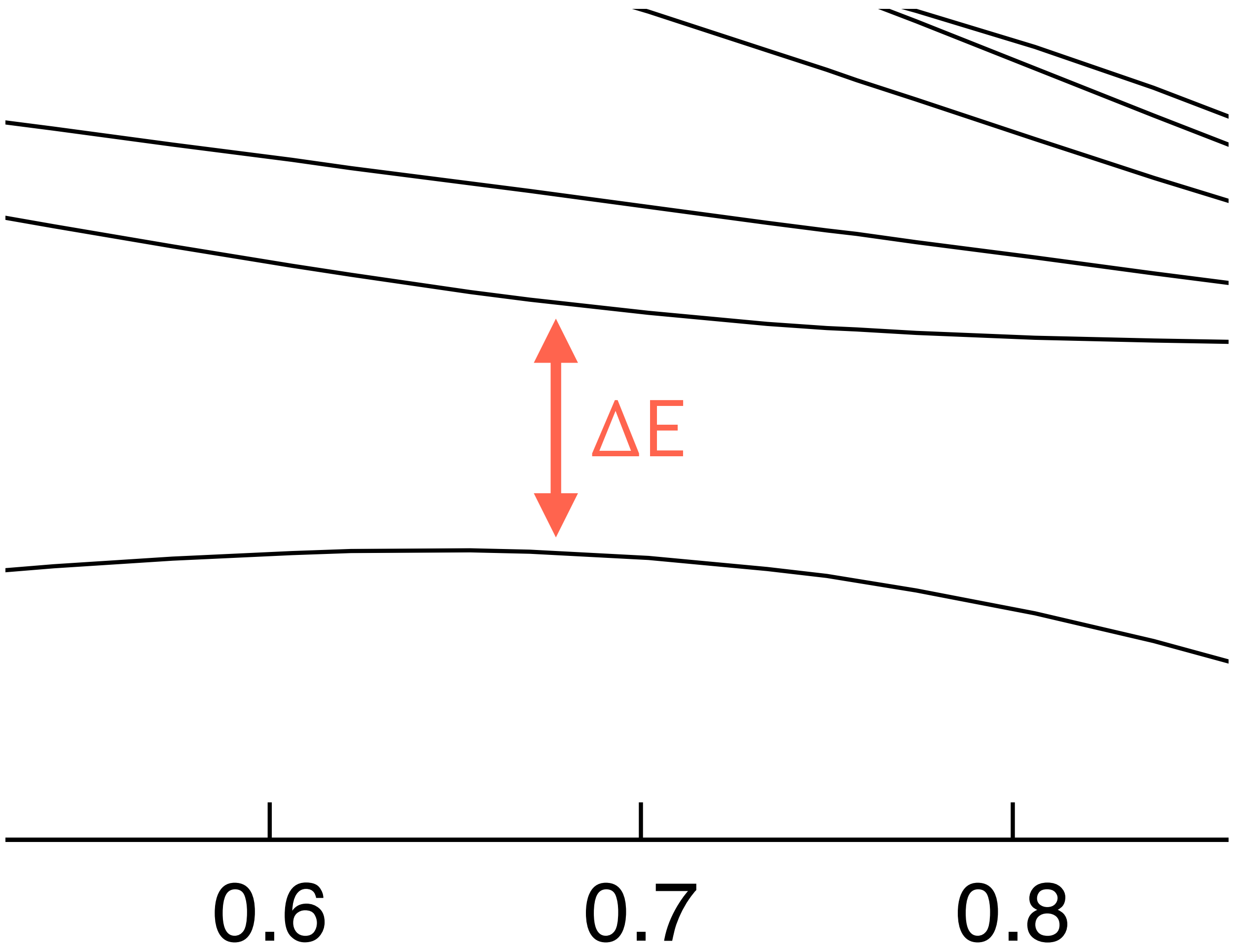


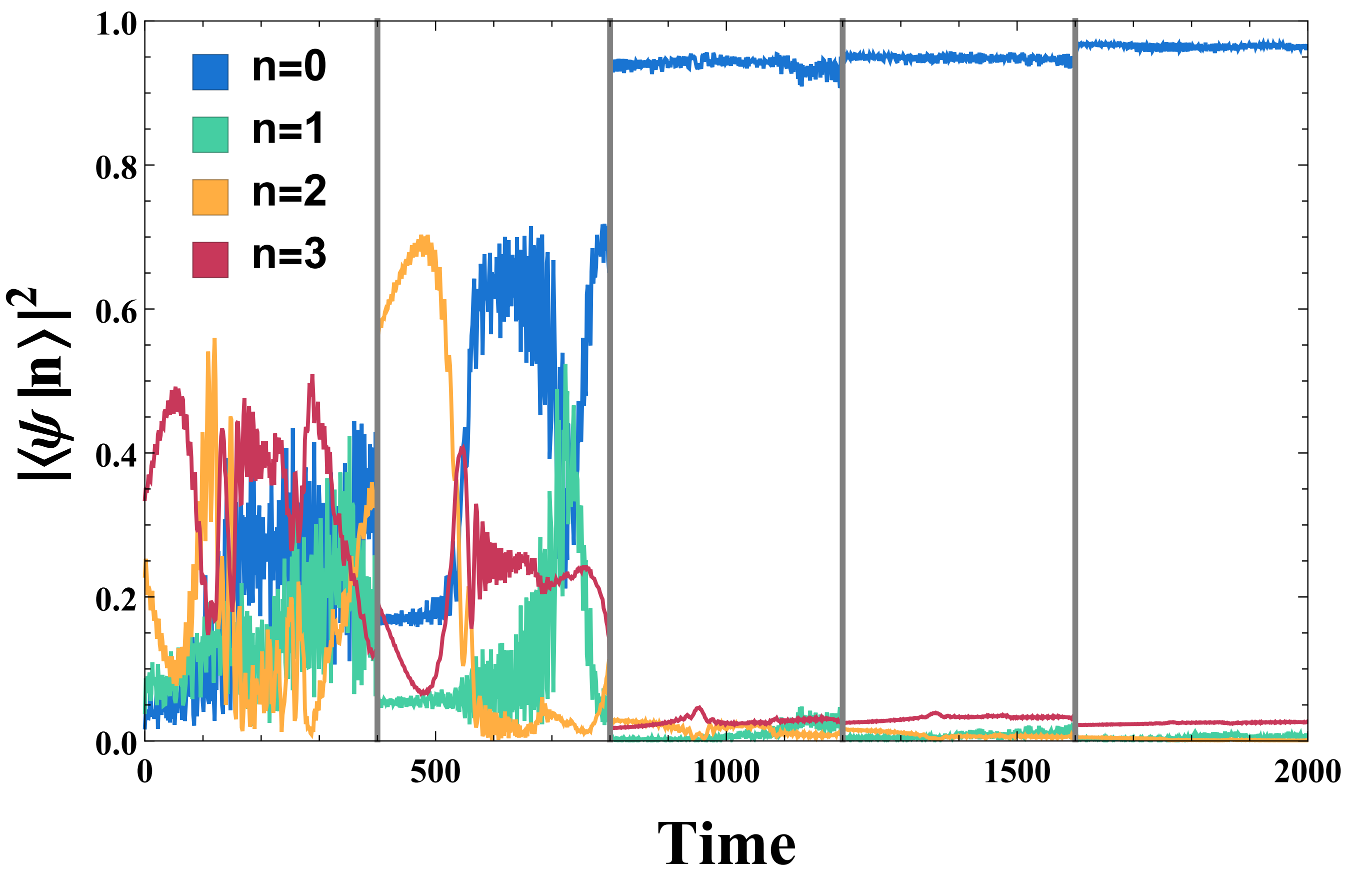
For N-particle systems might expect $\Delta E \sim \exp(-N)$?!

Yet another idea:

“Spectral Combing”

DBK, N Klco, A Roggerro, E-print 1709.08250 (quant-ph)

Multiple coolings simulated for N=3 1d Ising model



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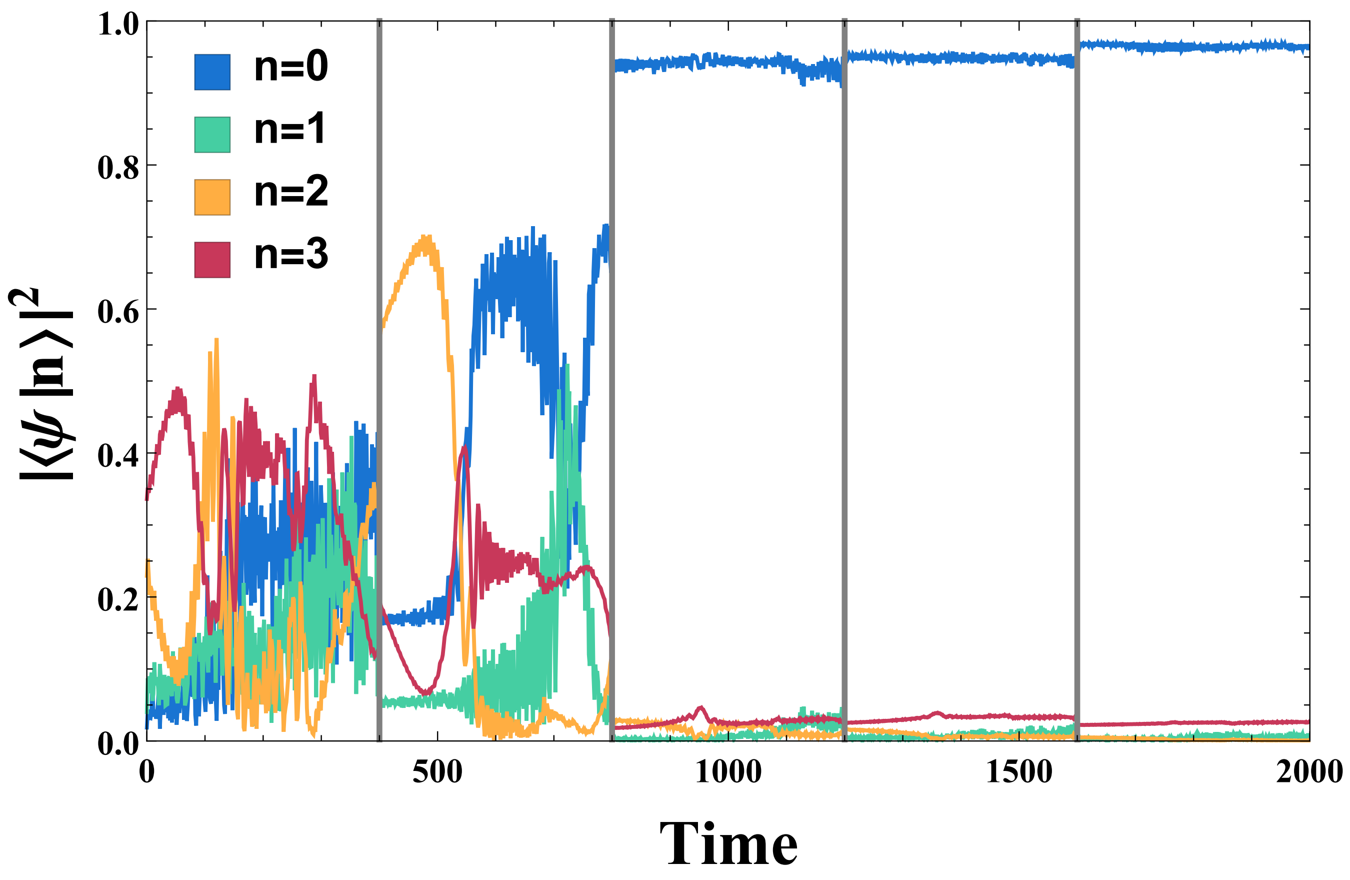

- Optimal approaches for scientific computing are not known, although many ideas.
- Some appear to exhibit exponential speed-up relative to classical algorithms

Will we soon be solving problems that are exponentially hard on a classical computer?

- Nuclear physics & nuclear matter
- Quantum chemistry of complex molecules
- Interesting condensed matter models, such as the Hubbard Model…

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Regardless, quantum computers change how we think about physics:

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- the line between computers and nature gets blurred
- will we some day require quantum gravity computers?

Are We Living in a Computer Simulation? FREE

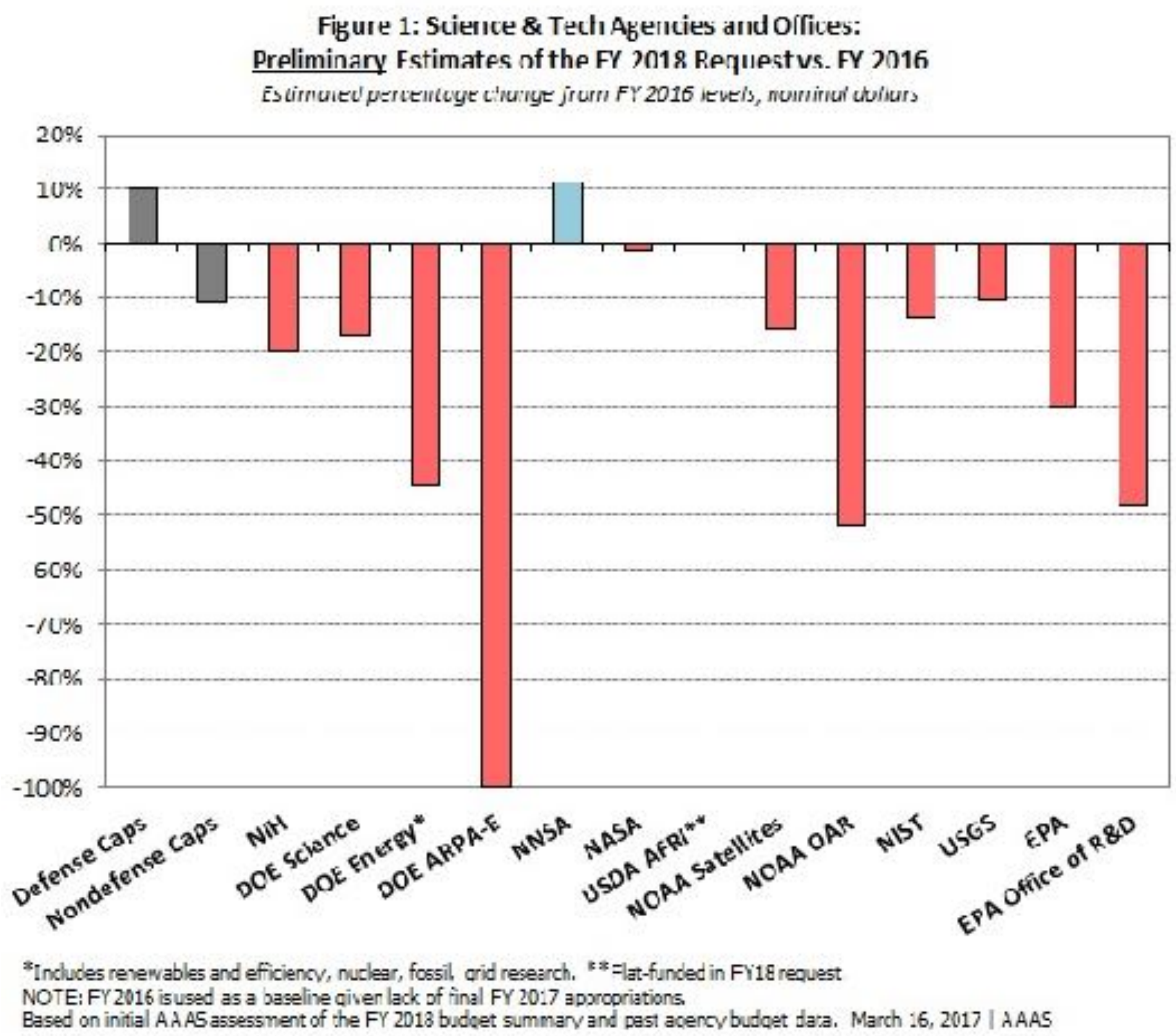
Nick Bostrom

The Philosophical Quarterly, Volume 53, Issue 211, 1 April 2003, Pages 243–255,
https://doi.org/10.1111/1467-9213.00309
Published: 28 April 2003

Abstract

I argue that at least one of the following propositions is true: (1) the human species is very likely to become extinct before reaching a 'posthuman' stage; (2) any posthuman civilization is extremely unlikely to run a significant number of simulations of its evolutionary history (or variations thereof); (3) we are almost certainly living in a computer simulation. It follows that the belief that there is a significant chance that we shall one day become posthumans who run ancestor-simulations is false, unless we are currently living in a simulation. I discuss some consequences of this result.

What if the Being simulating us is having its budget cut?



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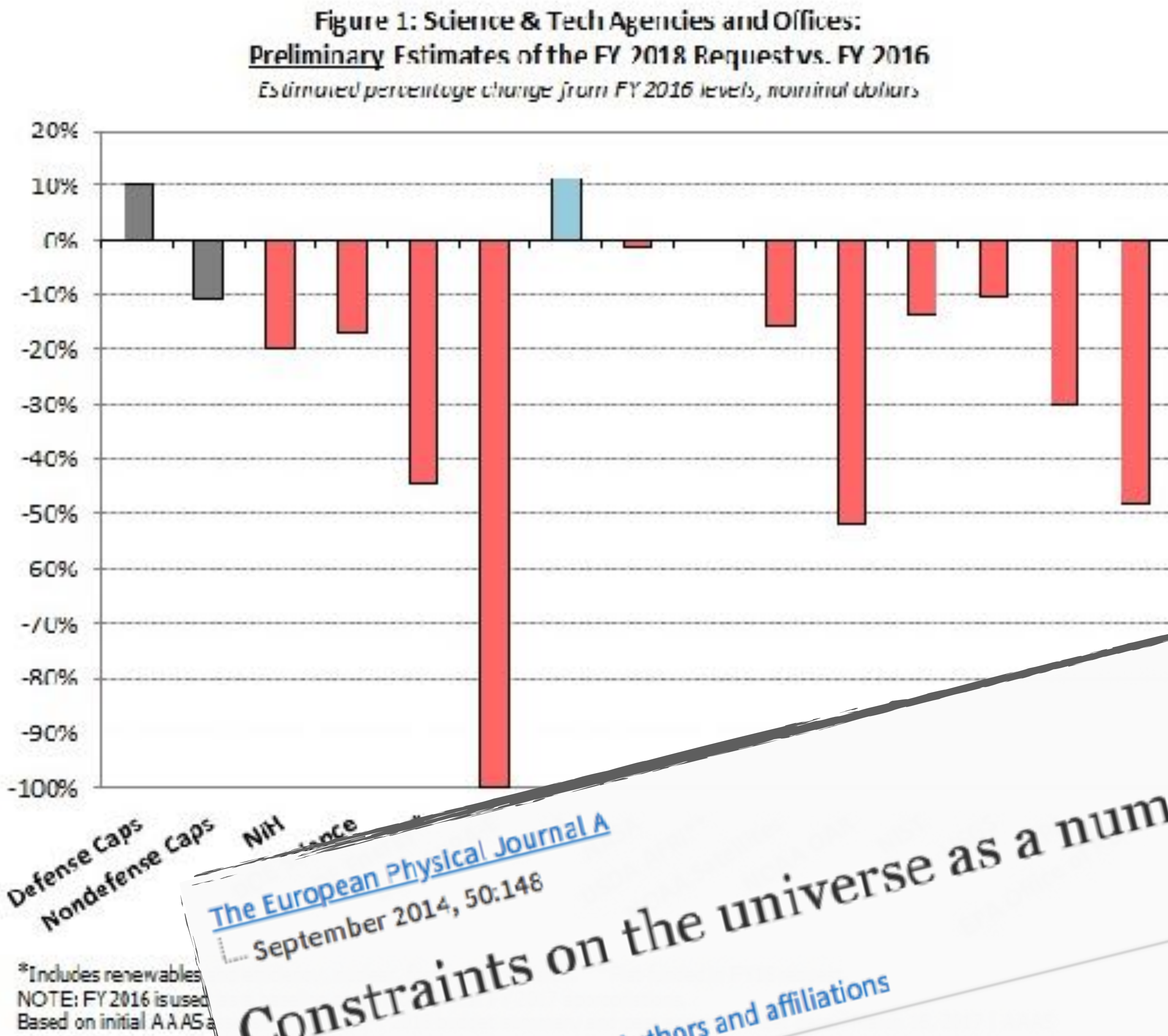

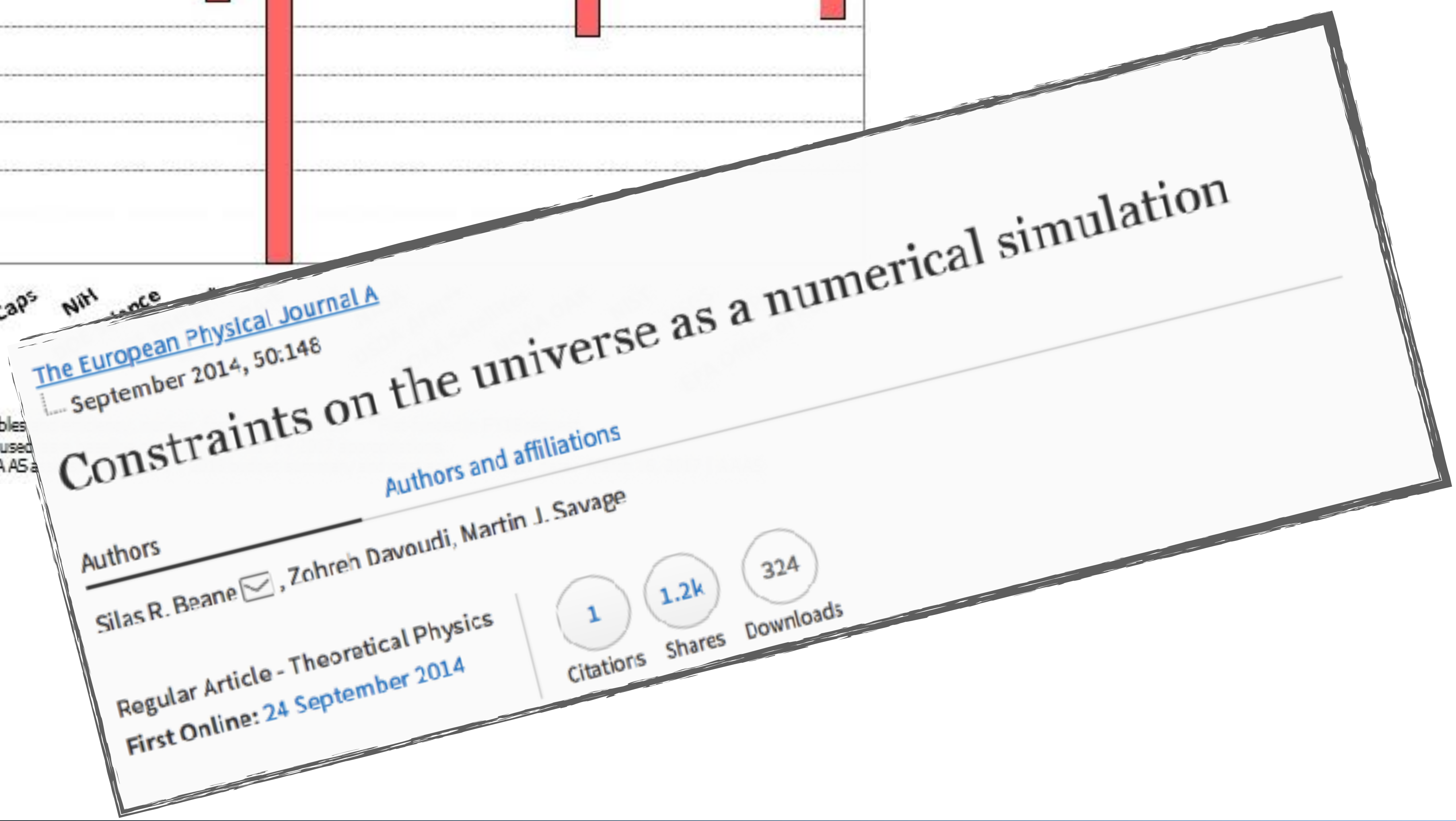
The European Physical Journal A
September 2014, 50:148

Constraints on the universe as a numerical simulation

Authors and affiliations

Authors

Silas R. Beane, Zohreh Davoudi, Martin J. Savage

Regular Article - Theoretical Physics
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Summary

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To understand a theory well enough to compute with it can lead to far ranging theoretical advances and new insights

This is a branch of science that is vibrant and creative... and with the advent of quantum computing we can hope for new revolutions in understanding to come.