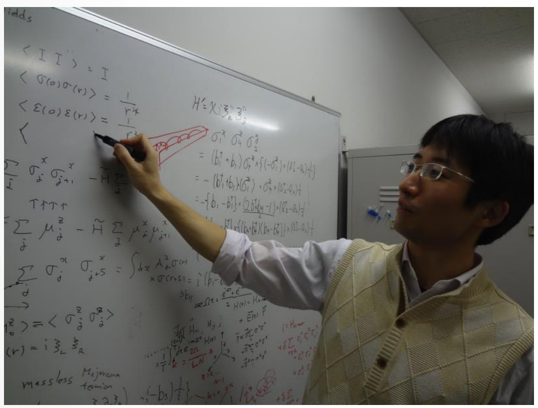
Control of Topology in Quantum Materials by Laser

Takashi Oka
(Max Planck Institute PKS, CPfS)



Masahiro Sato
(Ibaraki)

Shintaro Takayoshi
(U-Geneva)

Leda Bucciantini
(MPI PKS&CPfS)

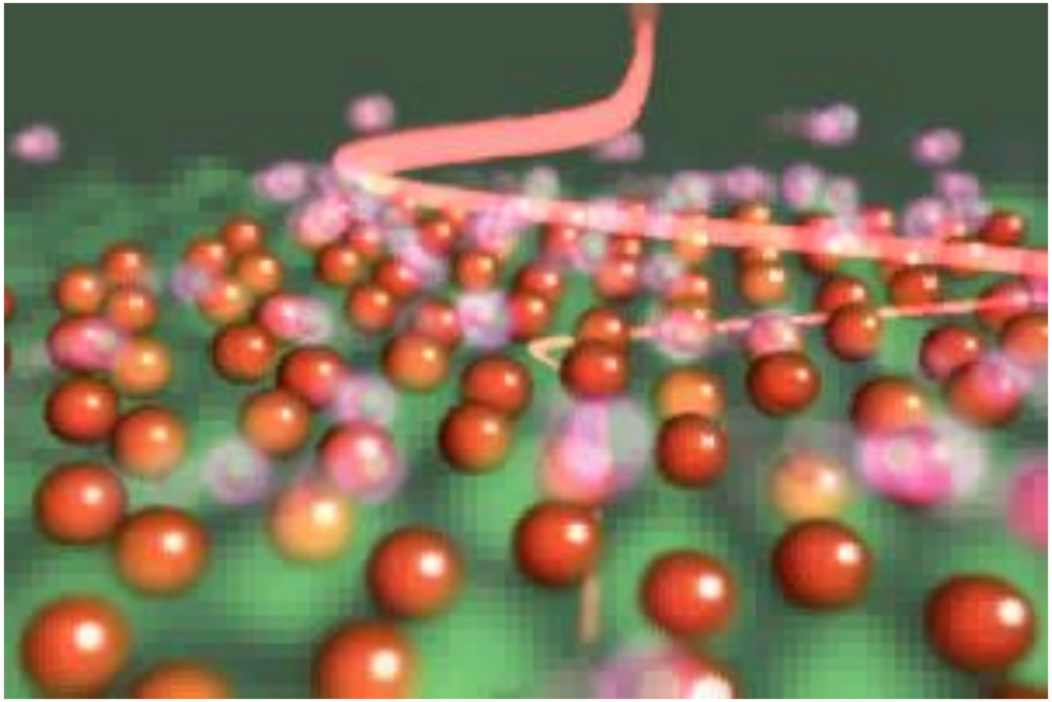
革新的研究開発推進プログラム
ImPACT

graphene, quantum spin, Hall effect, periodic driving

time reversal symmetry

Material × Laser

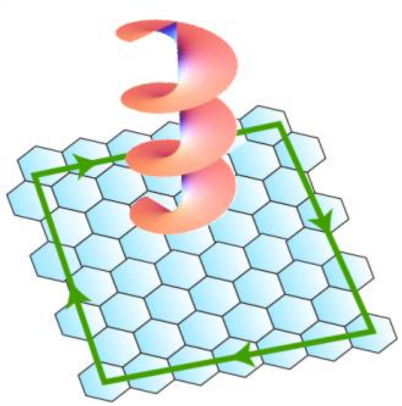
→ new function



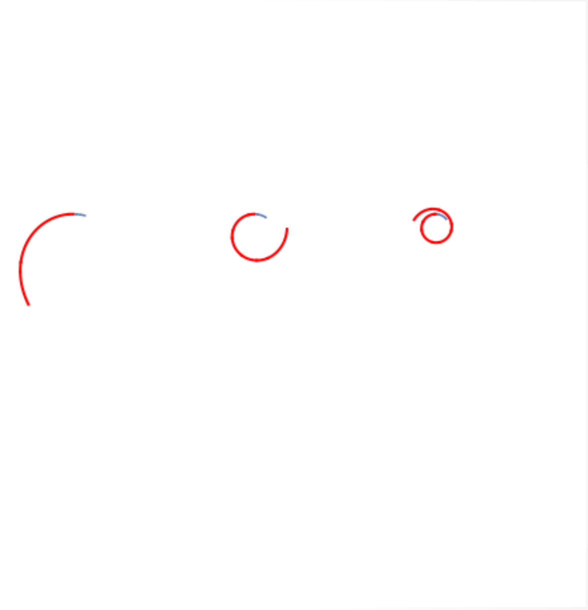
animation by **K. Tanaka (Kyoto)**

Three examples

Floquet topological states



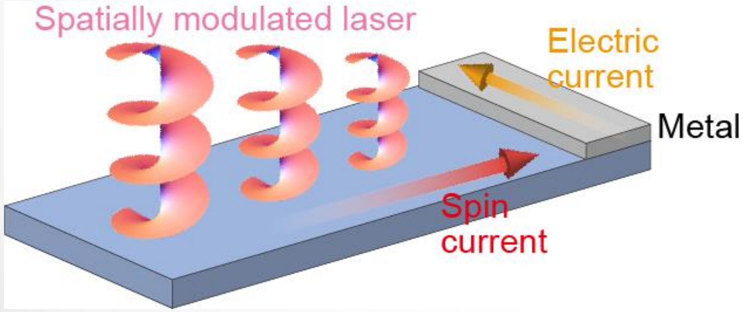
Heterodye Hall effect



Out of plane
Magnetic field

$$B_z = B \cos \Omega t$$

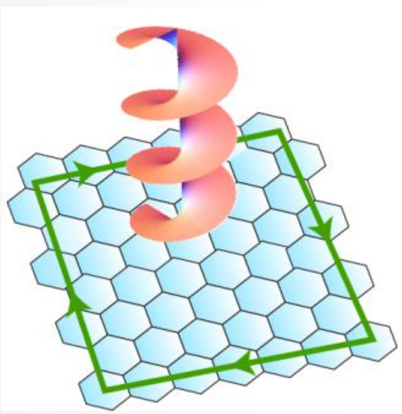
Ultrafast spintronics

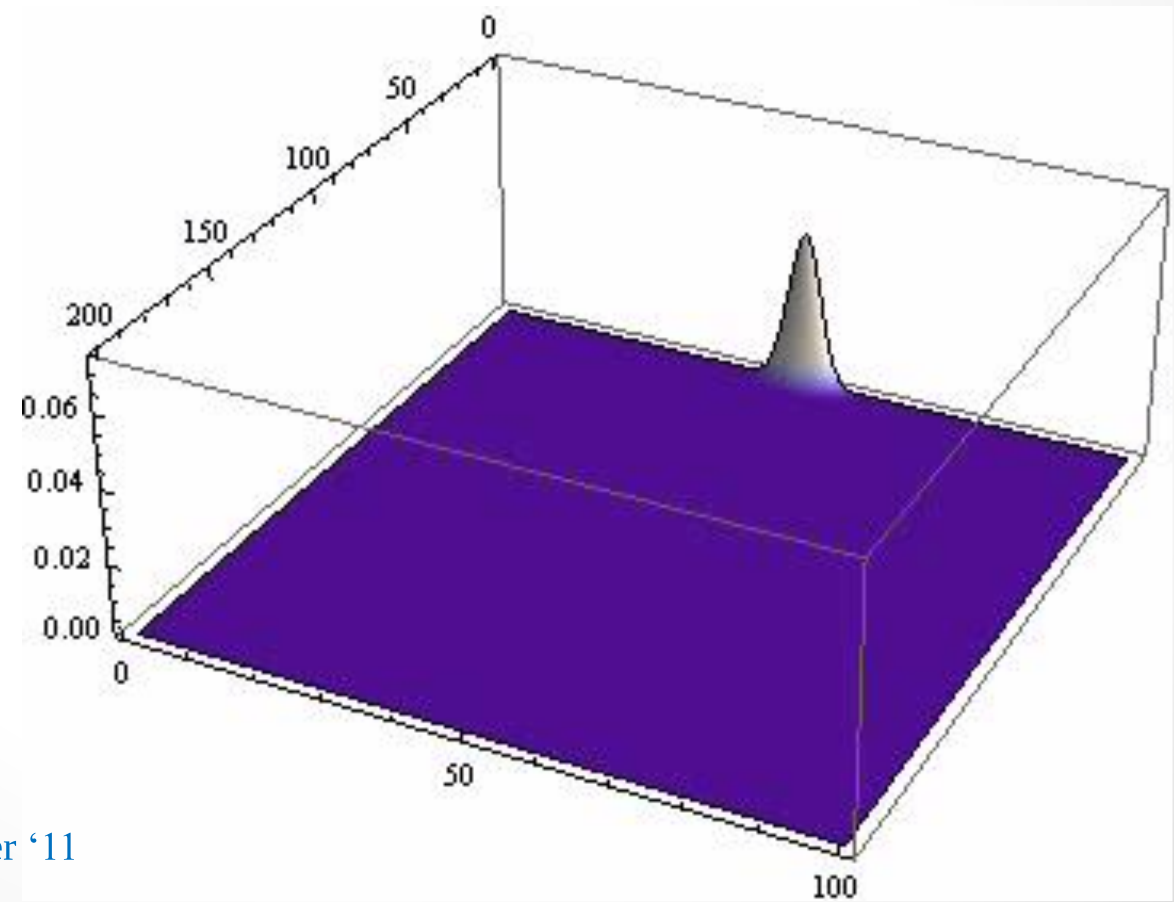


1. Floquet topological insulator

Multiband system (graphene) × Circularly polarized Laser

⇨ Control of topology (Chern number)





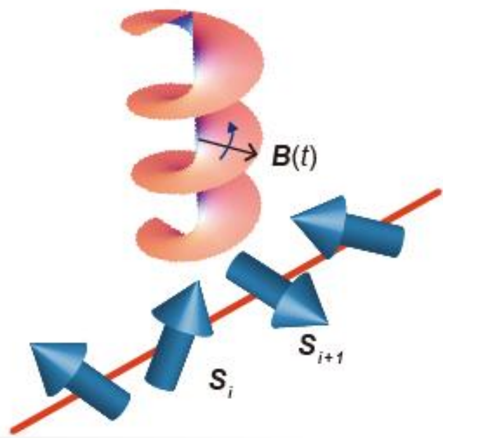
TO, Aoki '09
Lindner, Rafael, Galitsky '11
Kitagawa, TO, Fu, Brataas, Demler '11

2. Ultrafast spintronics

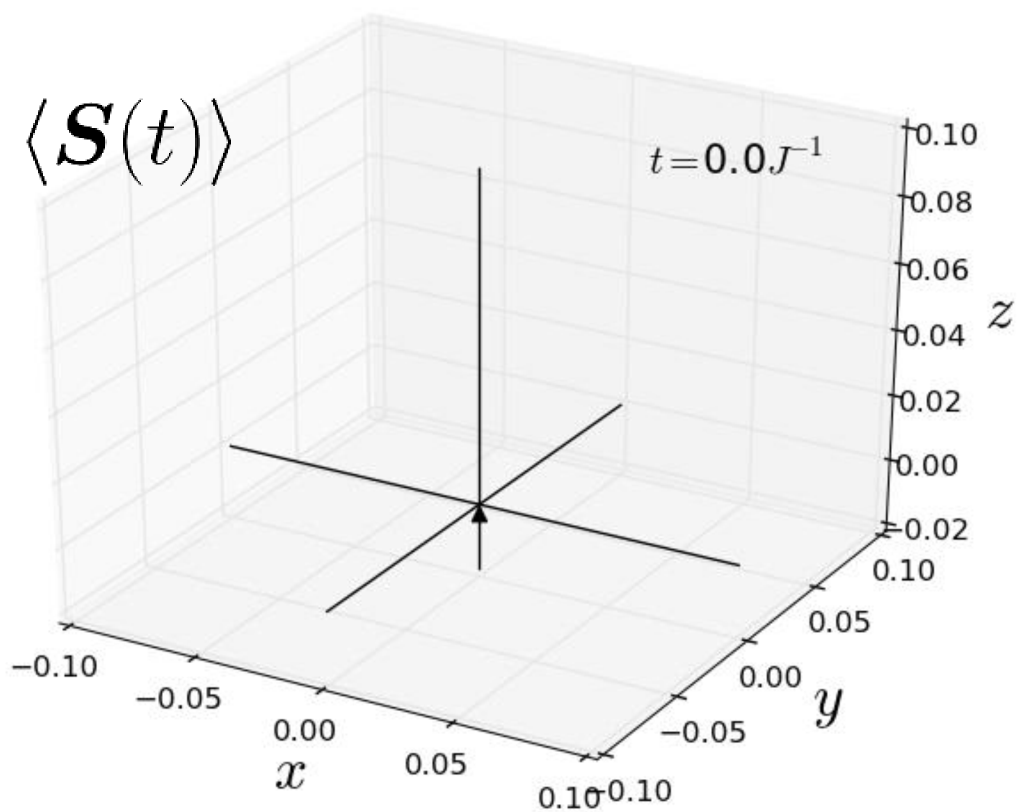
Quantum magnets × **Circularly polarized Laser**

➡ THz magnetization and spin current

$$\boldsymbol{B}(t) = (B\cos\Omega t, B\sin\Omega t)$$



$$\langle \boldsymbol{S}(t) \rangle$$



Takayoshi, Aoki, TO, PRB2014
Takayoshi, Sato, TO, PRB2014
Sato, Takayoshi, TO, '16

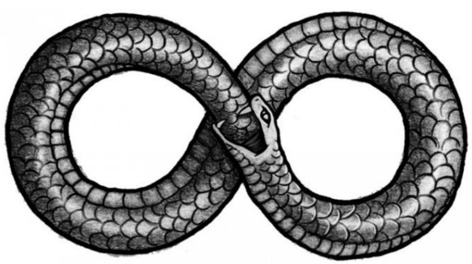
3: Heterodyne Hall Effect: Quantization of a Snake State

Charged particles × **Oscillating magnetic fields**

→ Heterodyning Hall Effect

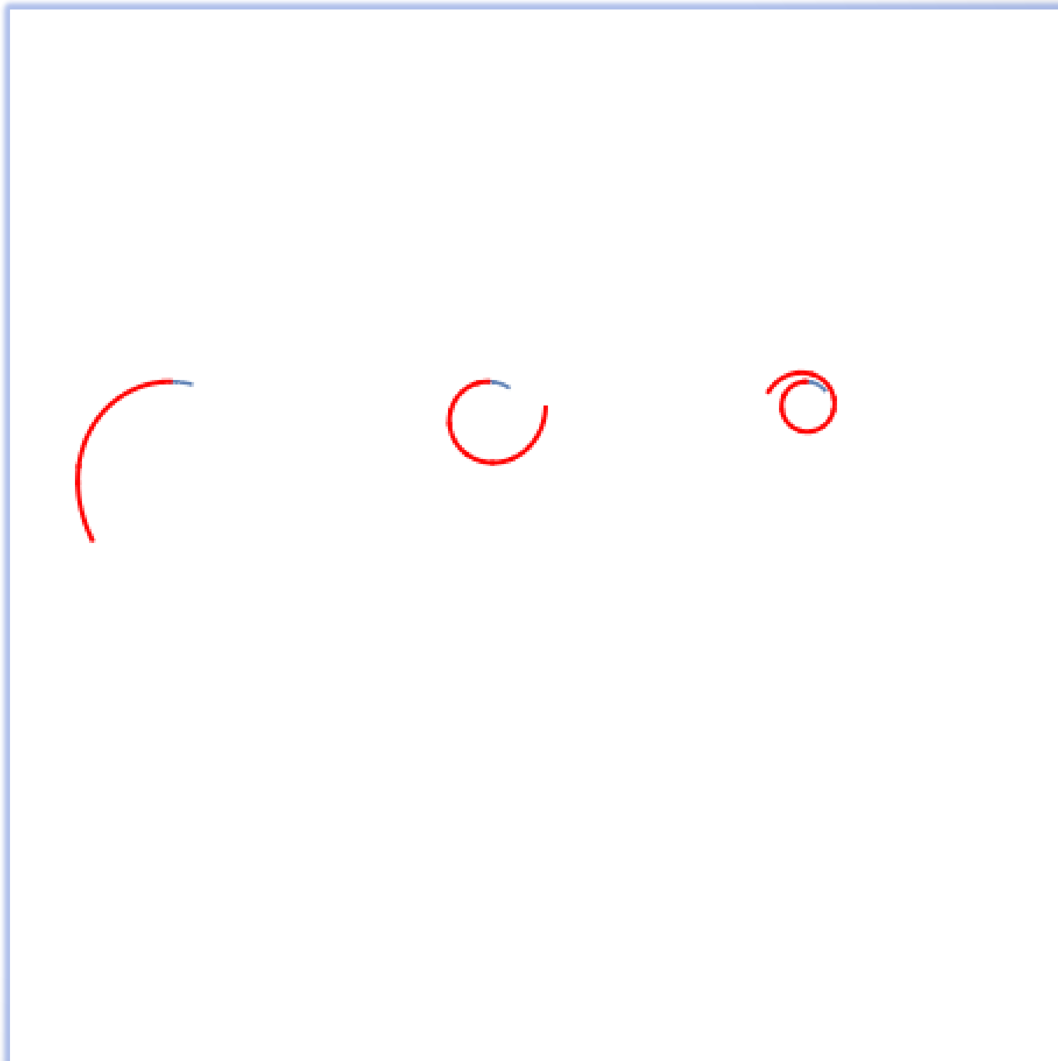
Out of plane
Magnetic field

$$B_z = B\cos\Omega t$$

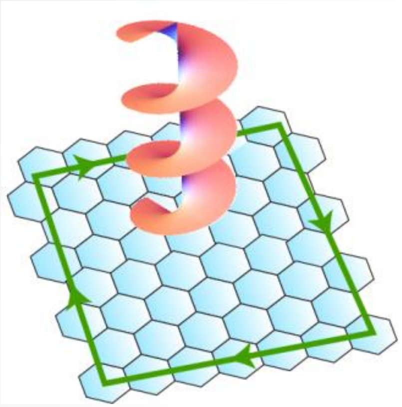


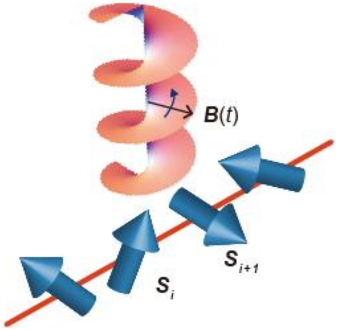
http://jeroenvanhonk.com/

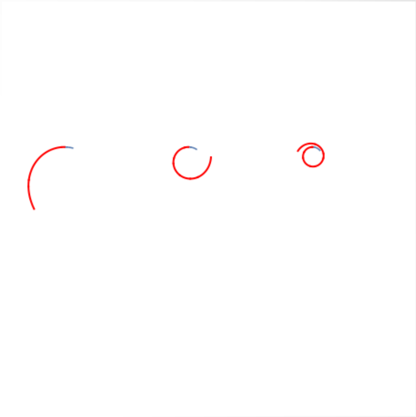
Bucciantini, TO, *to submit*



Light matter coupling







$$H(t) = \sum_{ij} t_{ij} e^{-i\phi_{ij}(t)} c_i^\dagger c_j$$

$$H(t) = H_{\text{spin}} + \boldsymbol{B}(t)\cdot\boldsymbol{S} + \boldsymbol{E}(t)\cdot\boldsymbol{P}$$

$$H(t) = \frac{(\boldsymbol{p} - \boldsymbol{A(t)})^2}{2m}$$

⇨ Periodically driven system

$$H(t) = H(t+T)$$

Floquet theory basics (1/3)

Use Fourier transformation

time periodic system

$$i\partial_t \psi = H(t)\psi \qquad H(t) = H(t+T) \qquad \Omega = 2\pi/T$$

$$\Psi(t) = e^{-i\varepsilon t} \sum_m \phi^m e^{-im\Omega t}$$

Floquet Hamiltonian (static eigenvalue problem)

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn} \phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n$$

ε: Floquet quasi-energy

$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

comes from the $i\partial_t$ term

$$H_m = \mathcal{H}^{m0}$$

~ absorption of *m* “photons”

Floquet theory basics (2/4)

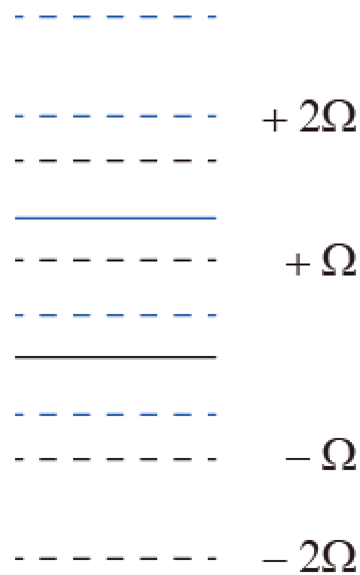
Time-periodic quantum system = Floquet theory (exact) ~ effective theory

$$i\partial_t \psi = H(t)\psi \qquad \mathcal{H}\phi = \varepsilon\phi \qquad H_{\rm eff}$$

$$H(t) = H(t+T)$$

two states + periodic driving

Floquet theory



n-photon dressed state

Floquet side bands

projection to the original Hilbert space

Hilbert sp. size
= original system

Floquet theory basics (3/3)

Floquet-Magnus expansion (stroboscopic dynamics)

$$H_{\mathrm{eff}} = \frac{i}{T} \ln \hat{T} e^{-i \int_0^T H(s) ds}$$

$$H_{\mathrm{eff}} = H_0 + \sum_{m>0} \frac{[H_{-m}, H_m]}{m\Omega} + \frac{1}{3} \sum_{m,n\neq 0} \frac{[H_{-m}, [H_{m-n}, H_n]]}{nm\Omega^2} + \frac{1}{2} \sum_{m,n\neq 0} \frac{[H_m, [H_0, H_{-m}]]}{m^2\Omega^2} + \cdots$$

* This expansion is divergent in many-body states

Floquet topological insulator

TO, Aoki '09
Kitagawa, TO, Fu, Brataas, Demler '11
Jotzu *et al.* (ETH), Nature '14

1st order

$$H_{\rm eff} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

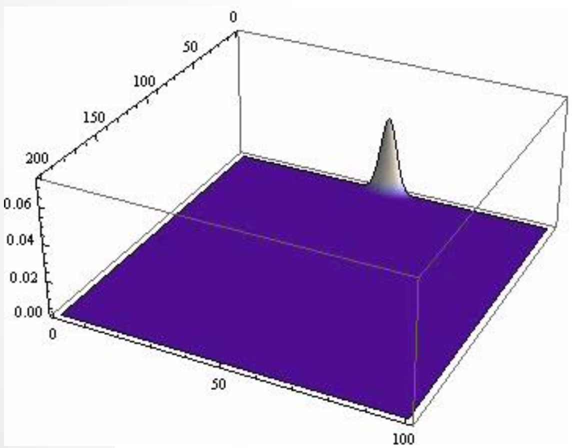
$A = F/\Omega$

applied to honeycomb lattice

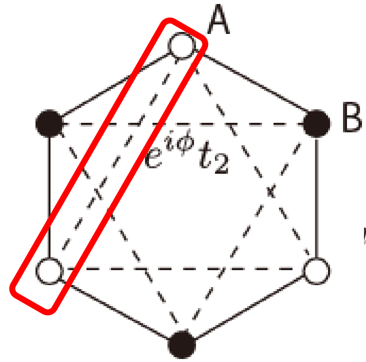
n. hopping + n. hopping = n.n. hopping with phase $\pi/2$

honeycomb + circularly polarized light

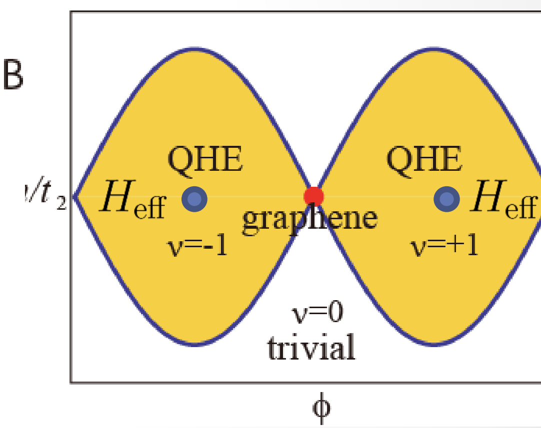
Haldane's Model of QHE without LL (1988)

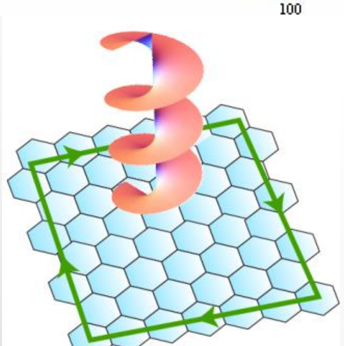


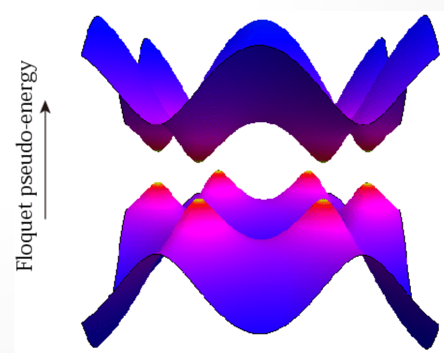
1-st order



local magnetic field ϕ
AB-level offset m

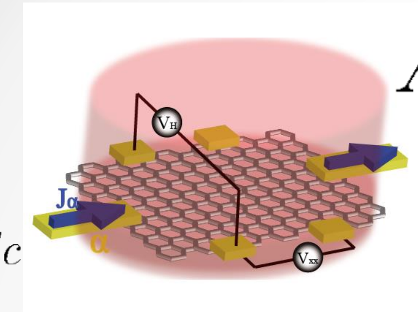






Non-equilibrium Kubo formula for photo-induced transport

TO, Aoki 2009
Torres-Kunold PRB '05
for microwave-assisted zero-resistance states



Large A_{ac} small E_{dc}

$$J^i_{\rm dc} = \sigma_{ij}(\boldsymbol{A}_{\rm ac})E^j_{\rm dc}$$

two time formalism

$$\sigma_{ab}(\boldsymbol{A}_{ac}) = i\int \frac{d\boldsymbol{k}}{(2\pi)^d}\sum_{\alpha,\beta\neq\alpha}\frac{[f_\beta(\boldsymbol{k})-f_\alpha(\boldsymbol{k})]}{\varepsilon_\beta(\boldsymbol{k})-\varepsilon_\alpha(\boldsymbol{k})}\frac{\langle\langle\Phi_\alpha(\boldsymbol{k})|J_b|\Phi_\beta(\boldsymbol{k})\rangle\rangle\langle\langle\Phi_\beta(\boldsymbol{k})|J_a|\Phi_\alpha(\boldsymbol{k})\rangle\rangle}{\varepsilon_\beta(\boldsymbol{k})-\varepsilon_\alpha(\boldsymbol{k})+i\eta}$$

ε_α Floquet's quasi-energy

f_α occupation fraction

inner product = time average

$$\langle\langle\Phi_\alpha|\Phi_\beta\rangle\rangle = \frac{1}{T}\int_0^T\langle\Phi_\alpha(t)|\Phi_\beta(t)\rangle$$

Floquet TKNN formula

TO, Aoki 2009

$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2\int\frac{d\boldsymbol{k}}{(2\pi)^d}\sum_\alpha f_\alpha(\boldsymbol{k})\left[\nabla_{\boldsymbol{k}}\times\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle\Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$$

Photo-induced Berry curvature (Chern density)

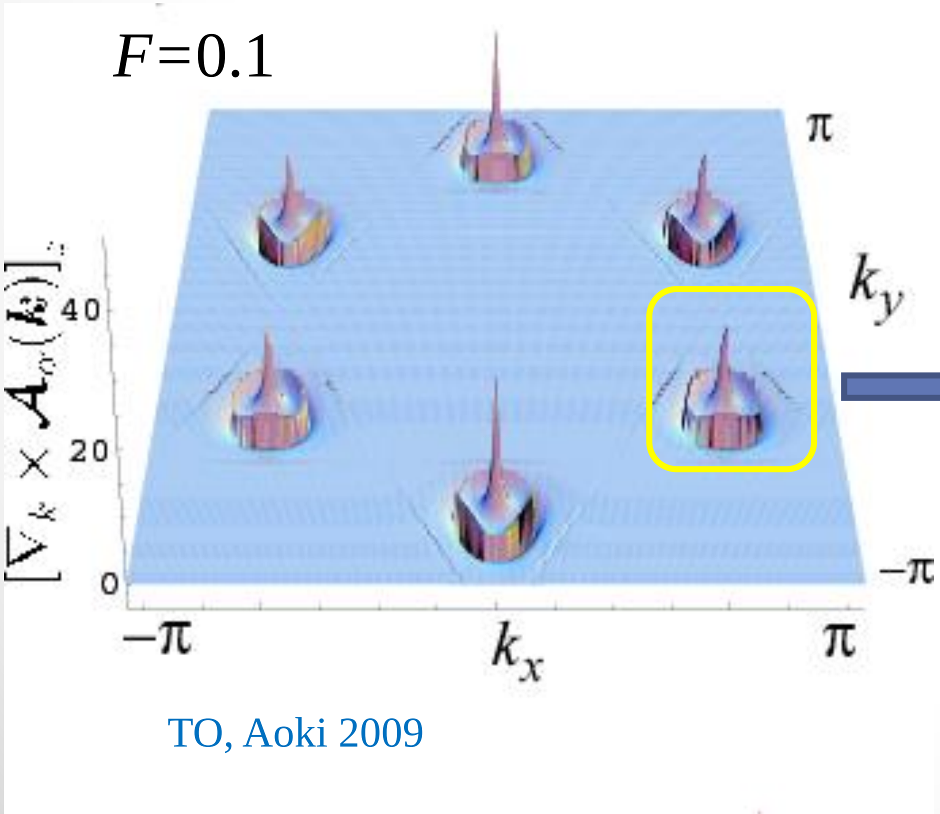
Berry`s curvature

$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_\alpha f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle\Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$$

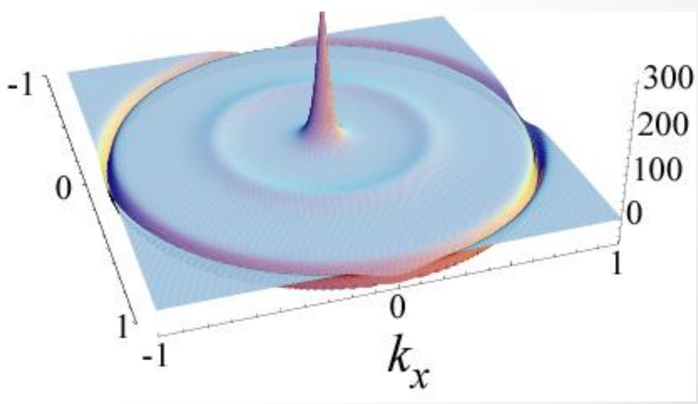
Floquet states

Photo-induced Berry curvature



Peaks at the Dirac cone

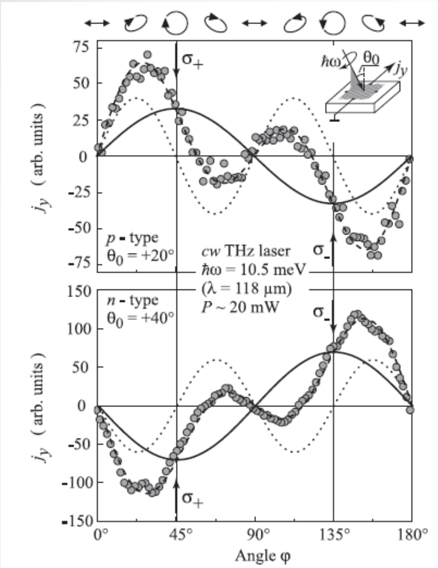
$$\left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z \sim \pm\tau_z \frac{1}{2}\kappa(|\boldsymbol{k}|^2 + \kappa^2)^{-3/2}$$



TO, Aoki 2009

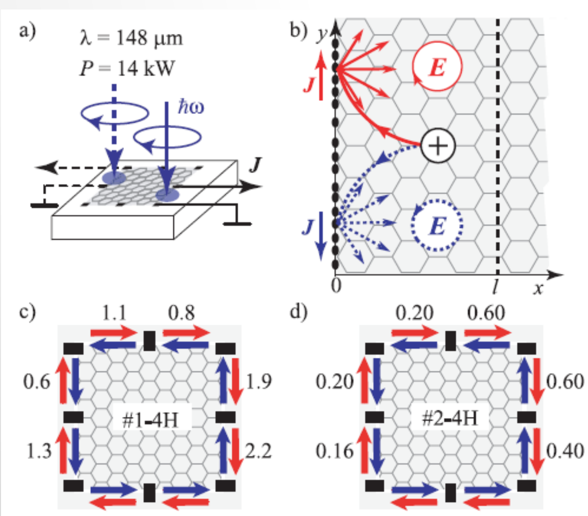
Ganichev's group (graphene)

Laser induced Hall effect



Karch *et al.* PRL '10

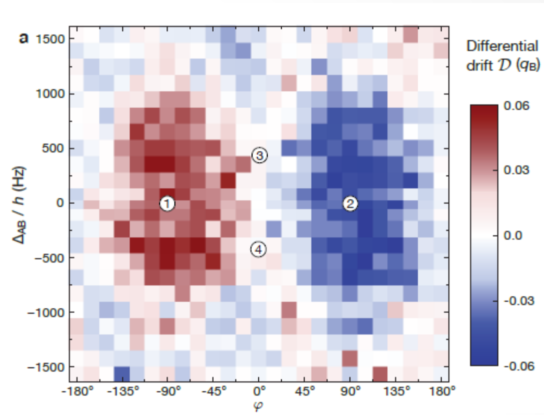
Laser induced chiral edge current



Karch *et al.* PRL '11

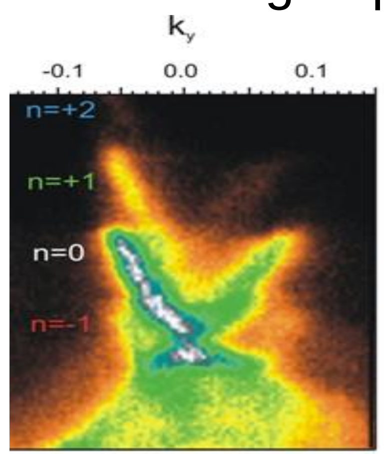
Esslinger's group (optical lattice)

Haldane model

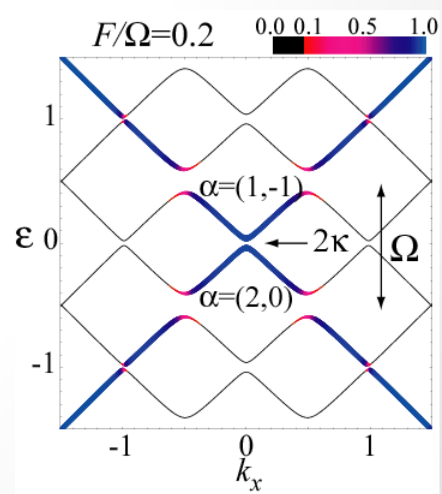


Jotzu *et al.* (ETH), Nature '14

Gedik's group (TI surface state)



Wang *et al.* Science '13



TO, Aoki '09

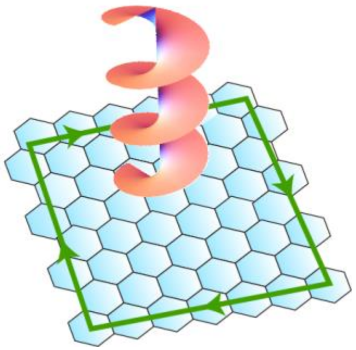
and others...

key idea

Circularly polarized laser

➡ Control of Chirality

Floquet TI



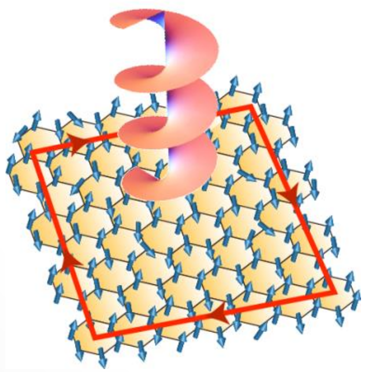
Chirality

Chern number

(axial vector field

in Floquet Weyl semimetal)

magnets



magnetization

vector chirality

scalar chirality

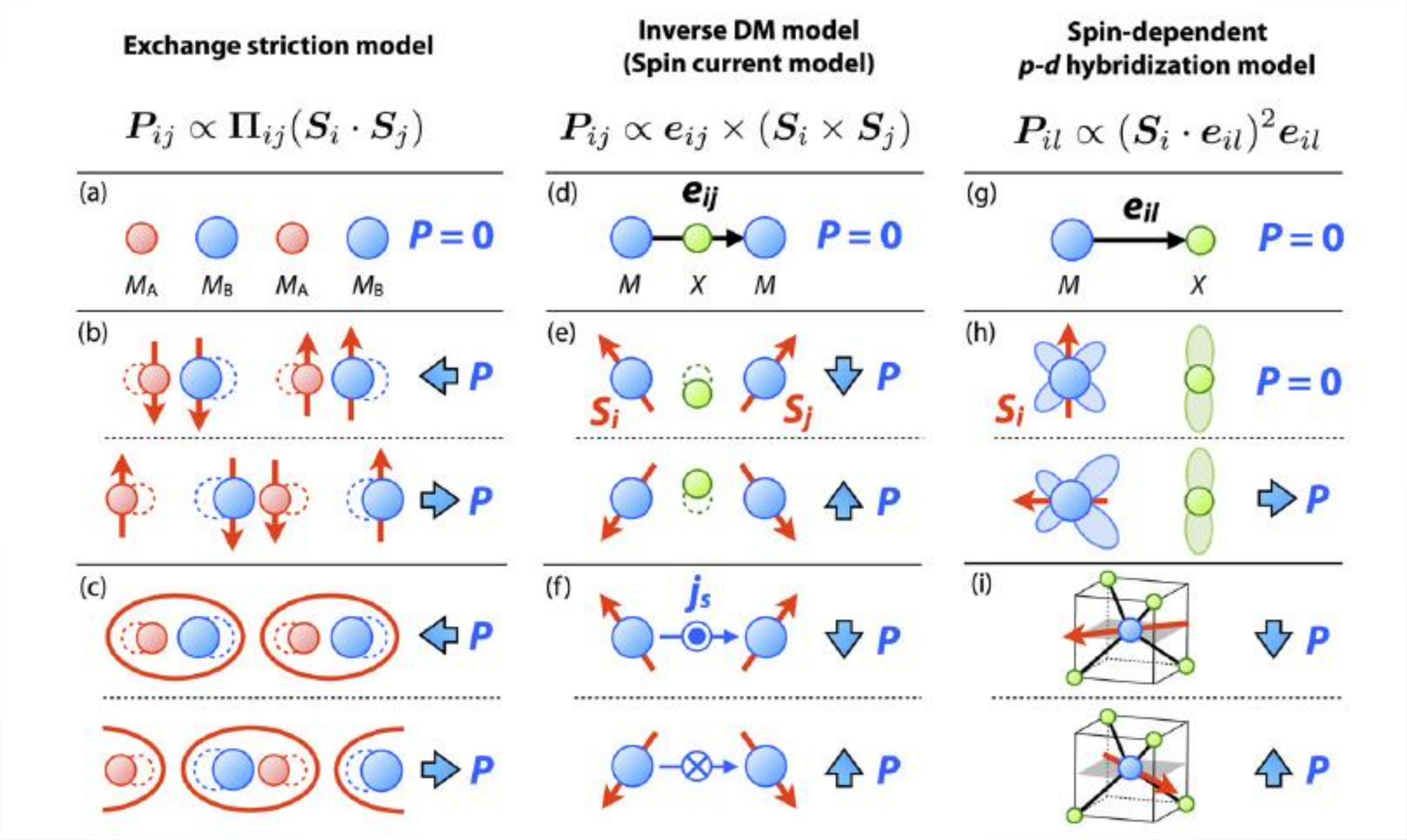
and spin current

➡ **2. Ultrafast spintronics**

2. Ultrafast spintronics in multiferroics

$$H(t) = H_{\rm spin} + \boldsymbol{B}(t)\cdot\boldsymbol{S} + \boldsymbol{E}(t)\cdot\boldsymbol{P}$$

magneto-optical coupling in **multiferroics**



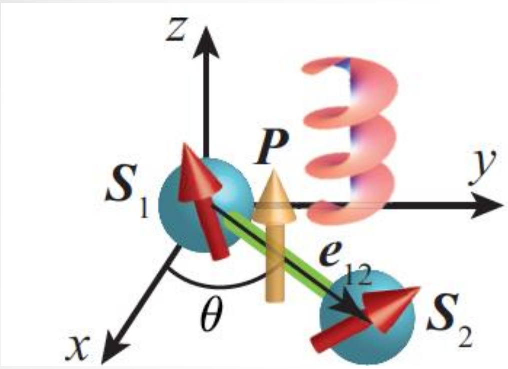
review: Tokura, Seki, Nagaosa, Rep. Prog. Phys. ‘14

Application to multiferroic quantum magnets

Sato, Takayoshi, TO, arXiv'16

$$H(t) = H_{\text{spin}} + \boldsymbol{B}(t)\cdot\boldsymbol{S} + \boldsymbol{E}(t)\cdot\boldsymbol{P}$$

Zeeman **magneto-optical coupling**



circularly polarized laser

$$\boldsymbol{E} = E_0(\cos\Omega t, \sin\Omega t),\ \boldsymbol{B} = B_0(\sin\Omega t, \cos\Omega t)$$

↓ Fourier transform

$$H_1 = E_0(P_x - iP_y)/2 + B_0(S_y - iS_x)/2$$

$$H_{-1} = E_0(P_x + iP_y)/2 + B_0(S_y + iS_x)/2$$

$$P \sim S\cdot S \text{ or } S\times S$$

effective terms $[H_{-1}, H_1]/\Omega$

$$H_\Omega = \frac{E_0E_0}{\Omega}(3\ \text{spin term}) + \frac{E_0B_0}{\Omega}(2\ \text{spin term}) + \frac{B_0B_0}{\Omega}(1\ \text{spin term})$$

$\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)$

scalar chirality

Sato, Sasaki, TO, arXiv'15

kitaev model

$\boldsymbol{a}\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)$

vector chilarity

Sato, Takayoshi, TO, arXiv'16

Zeeman term

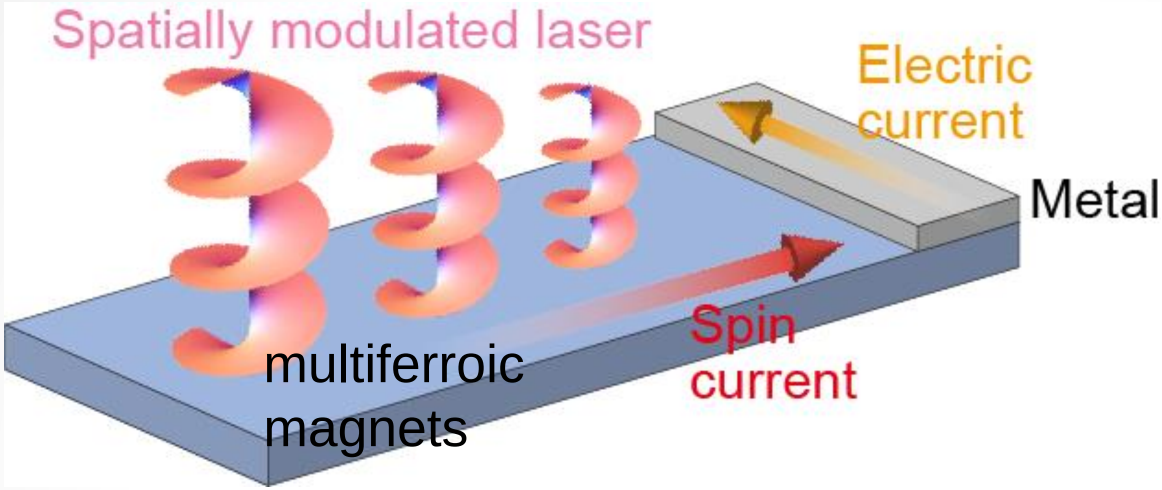
Takayoshi, Aoki, TO, PRB'14

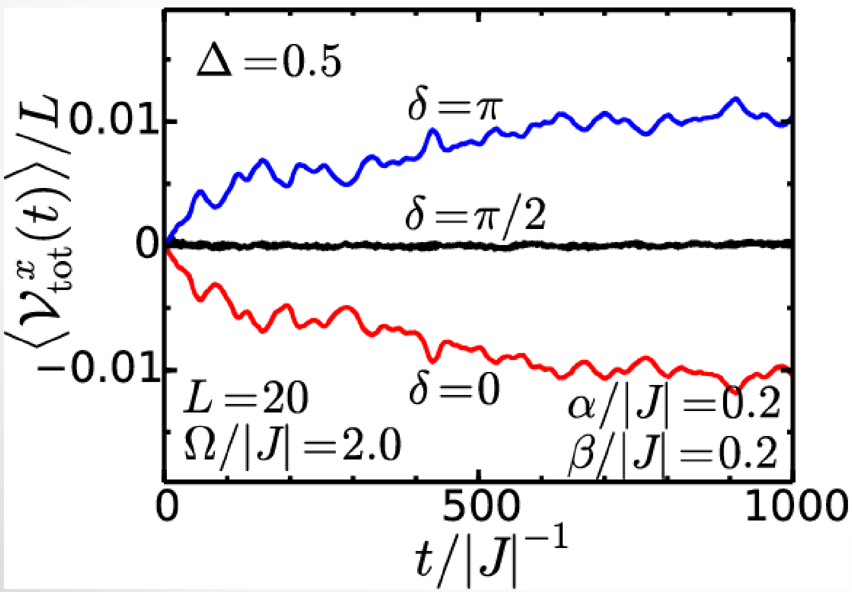
Takayoshi, Sato, TO PRB'14

Ultrafast spintronics (THz spin current generation)

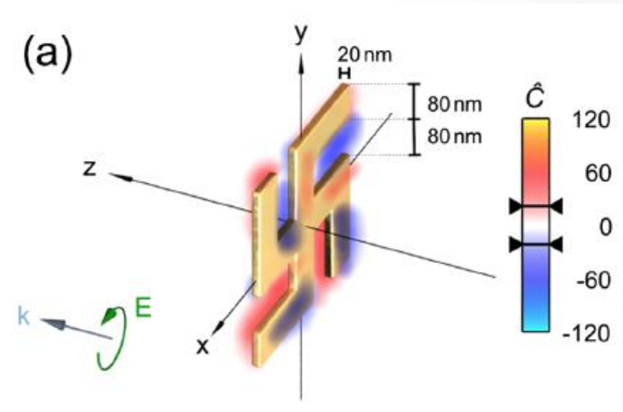
proposed setup

Sato, Takayoshi, TO, arXiv'16





cf) chiral plasmonic nanostructure

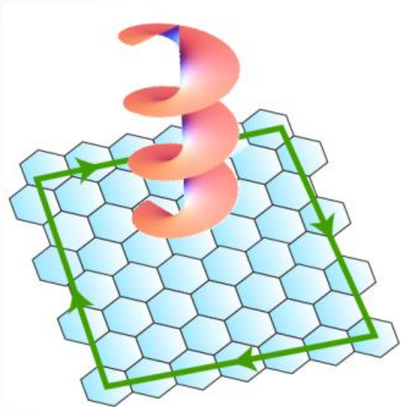


Schaferling, Dregely, Hentschel, Giessen PRX'12

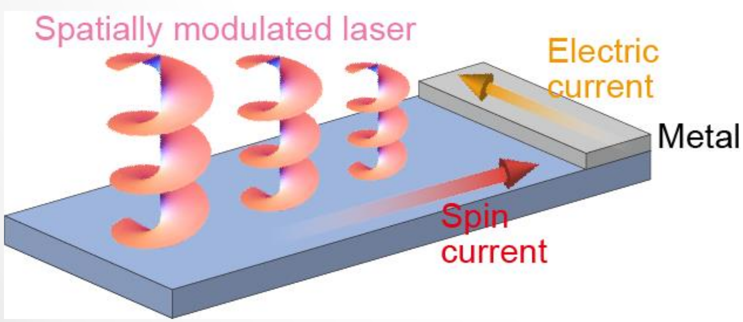
strong modulated circularly polarized field

Three examples

Floquet topological states



Ultrafast spintronics



Heterodye Hall effect

Out of plane Magnetic field

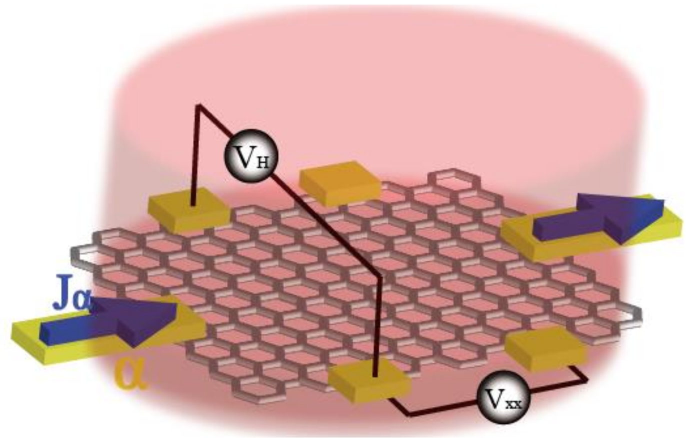
$$B_z = B \cos \Omega t$$

3. Heterodyne Hall Effect: Quantization of a Snake State

TO, Bucciantini, *in prep*

driving field (local oscillator) $A_{\mathrm{ac}} e^{-i\Omega t}$

input signal $E^n e^{-in\Omega t}$



output signal $j(m\Omega) e^{-im\Omega t}$

Heterodyne (=frequency mixing) response

$$j_a(m\Omega) = \sum_b \sum_{n=-\infty}^{\infty} \sigma_{ab}^{m,n} E_b^n$$

c.f) Floquet Chern insulator was characterized by $\sigma_{xy}^{0,0}$

3. Heterodyne Hall Effect: Quantization of a Snake State

TO, Bucciantini, *in prep*

main result:

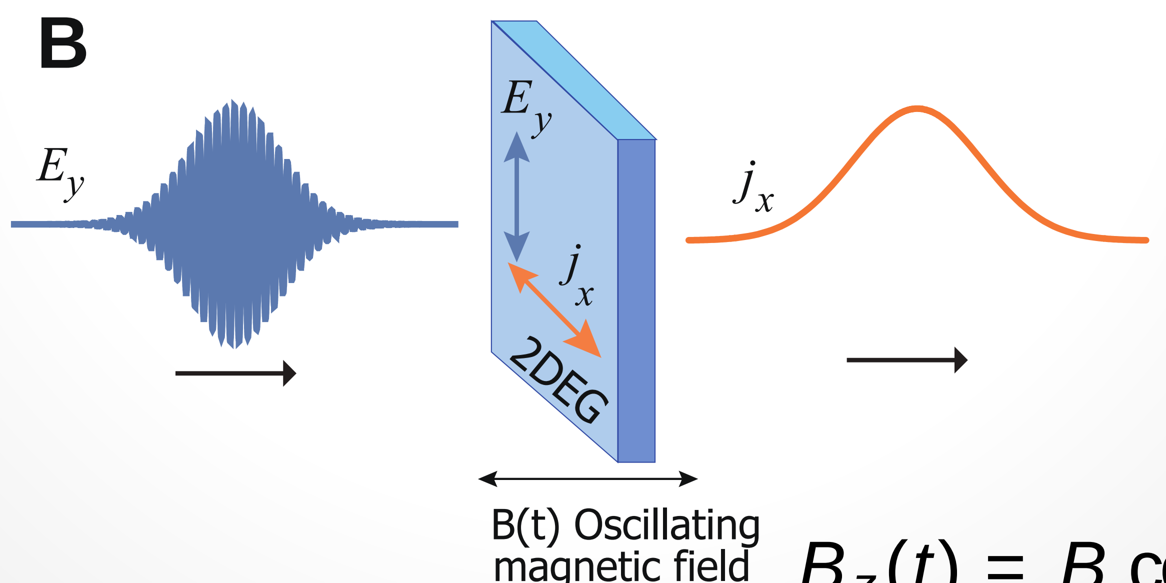
In 2D electron gas, subject to field $A_y = B_0 \cos(\Omega t) x$, Landau-quantization can take place, and the heterodyne Hall coefficient is

$$j_y = \sigma_{yx}^{0,1} E_x^1 \qquad \sigma_{yx}^{0,1} = \frac{e^2}{h} Q\nu$$

ν: filling fraction

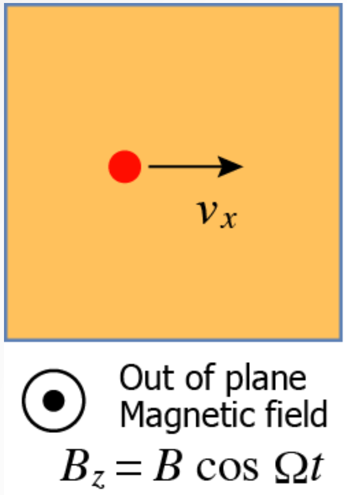
Q: numerical factor

*we don't know if this is topological



$B_z(t) = B \cos \Omega t$

"Snake-states" in oscillating magnetic field



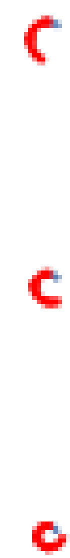
classical EOM

$$m_e \left(\frac{d}{dt} + \eta \right) \boldsymbol{v}(t) = q \left(\boldsymbol{E} + \frac{1}{c} \boldsymbol{v}(t) \times \boldsymbol{B}(t) \right)$$

cyclotron frequency

$\omega_c = qB/m_e c$

free motion with finite initial velocity (τ=0, E=0)



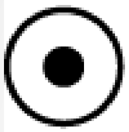
ω_c/Ω=3.0

5.0

6.0

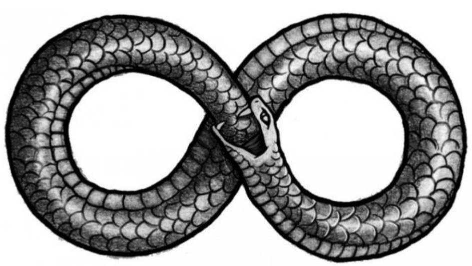
cf) J. E. Muller, PRL '92, "snake state in static nonuniform B"

Closed orbit for special ratios $r = \omega_c/\Omega$

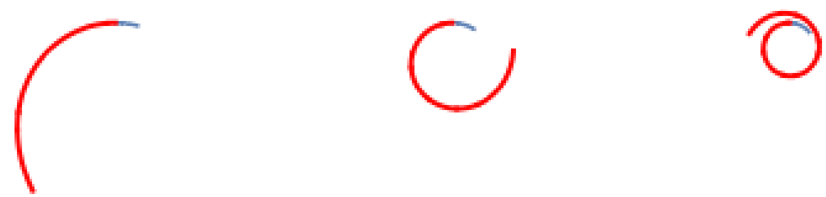


Out of plane
Magnetic field

$$B_z = B \cos \Omega t$$



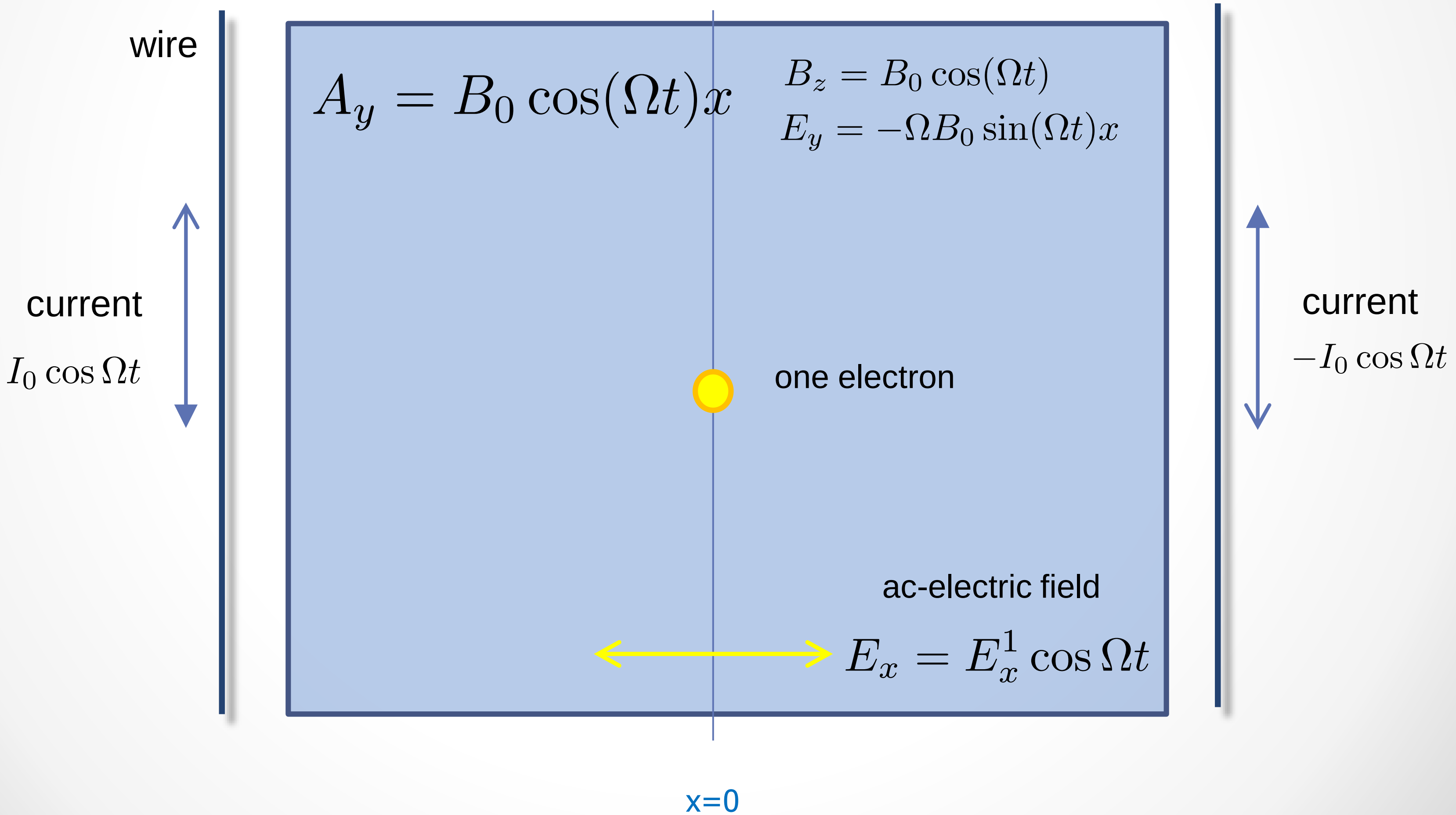
http://jeroenvanhonk.com/



$r=\omega_c/\Omega=2.40$	5.52	8.66
$\alpha=1$	$\alpha=2$	$\alpha=3$

Quantum Heterodyne Hall Effect

$$H = \frac{1}{2m}(\boldsymbol{p} - e/c\boldsymbol{A})^2$$



Husimi transformation

Husimi (Taniuti) PTP '53

$$H(p_x, p_y - qA_y; t) = \frac{1}{2m_e}\left[p_x^2 + \left(p_y - qA_y - \frac{e}{c}B_z(t)x)^2\right)^2\right]$$

= driven Harmonic oscillator with an oscillating potential

(i) Quantum oscillator without driving

$$H(t) = \frac{p_x^2}{2m_e} + \frac{m_e\omega^2(t)}{2}x^2$$

$$\omega(t) = \omega_c \cos \Omega t$$

+

(ii) Classical driven oscillator

$$m_e\ddot{X} + m_e\omega(t)^2 X = S(t)$$

$$S(t) = \omega(t)(\hbar k_y - qA_y)$$

solution

$$\Psi_n(\boldsymbol{x}, t) = e^{-\frac{i}{\hbar}E_n t} e^{ik_y y}\varphi_n(x - X(t), t)\exp\left[\frac{i}{\hbar}\{m_e\dot{X}(t)(x - X(t)) + \int_0^t dt' L(t') - L_0 t\}\right]$$

$$L = \frac{1}{2}m_e\dot{X}^2 - \frac{1}{2}m_e\omega(t)^2 X^2 + XF(t)$$

pseudo-energy

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m^*}$$

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{1}{T}\int_0^T L(t')dt'$$

Wave function

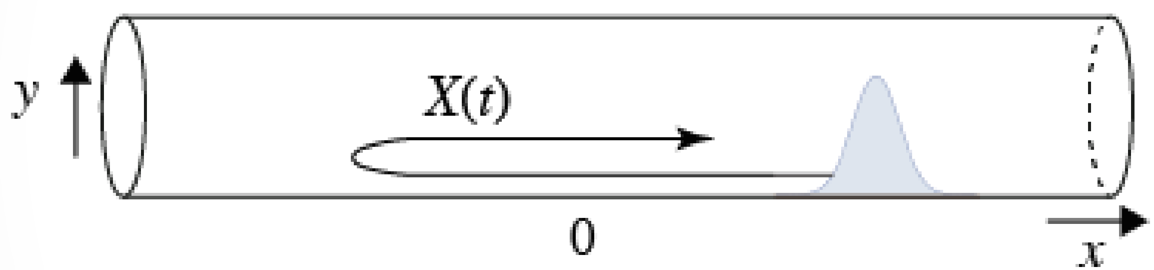
$$\Psi_n(\boldsymbol{x},t) = e^{-\frac{i}{\hbar}E_n t} e^{ik_y y} \boxed{\varphi_n(x - X(t), t)} \exp\left[\frac{i}{\hbar}\{m_e \dot{X}(t)(x - X(t)) + \int_0^t dt' L(t') - L_0 t\}\right]$$

wave packet

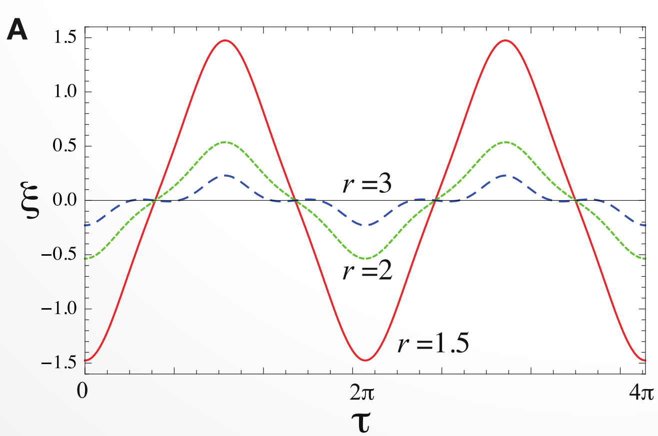
$$\varphi_n(x) \sim e^{-x^2/2l^2} H_n(x/l)$$

wave packet center

$$X(t) = -(l_B r)^2 \xi(\tau) \left(k_y - \frac{qE_x^1}{\hbar\omega_c}\right)$$



motion of the center



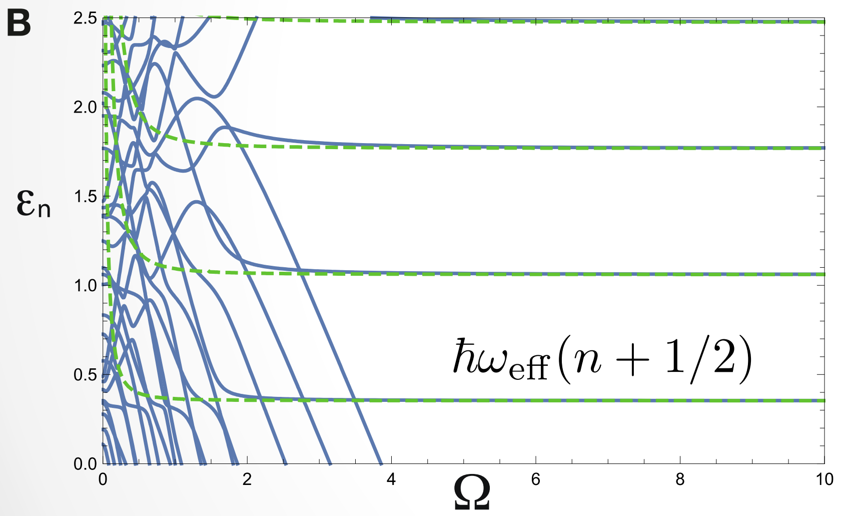
Mathieu eq. with a source

$$\xi''(\tau) + (a - 2q\cos 2\tau)\xi(\tau) = -\cos\tau$$

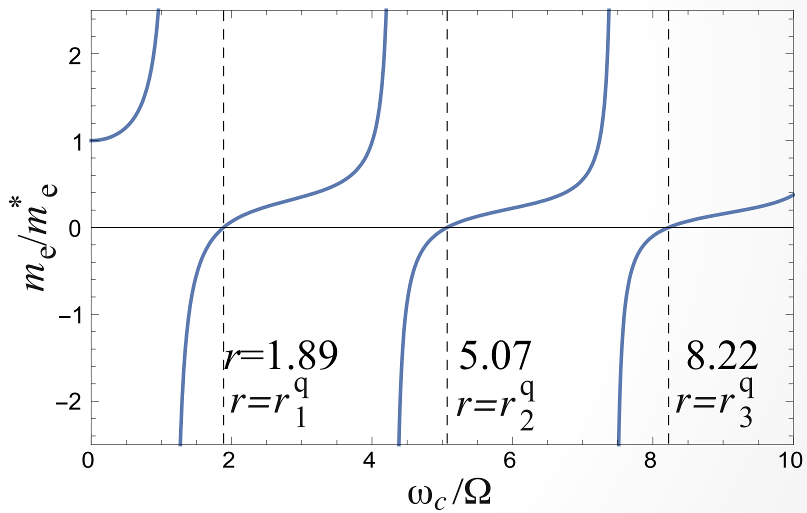
Floquet quasi-energy Spectrum

$$E_n(k_y) = \epsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{\hbar^2}{2m_e}\left(1 - \boxed{\frac{m_e}{m_e^*}}\right)\left(k_y - q\frac{E_x^1}{\hbar\omega_c}\right)^2$$

Floquet energy of the undriven QHO



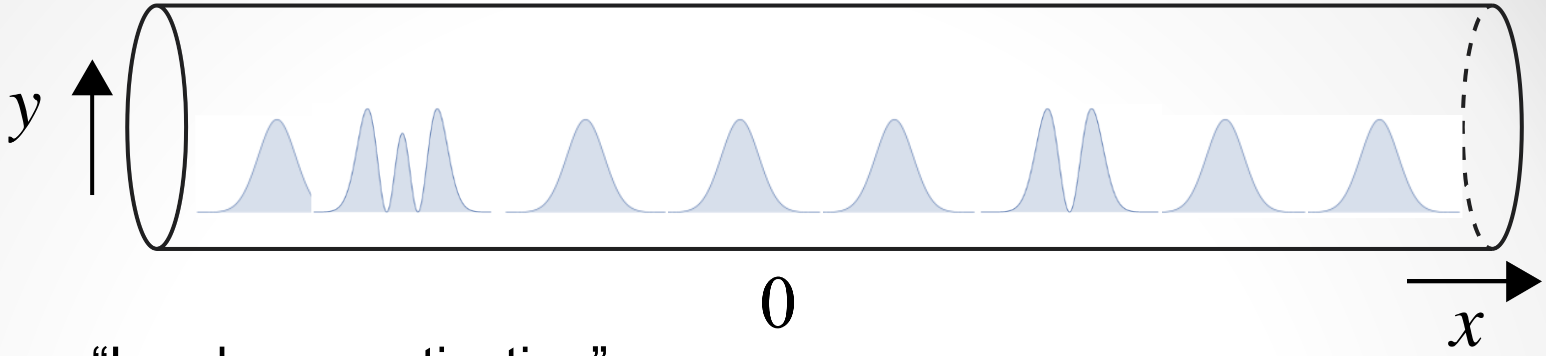
effective mass



Landau quantization when the effective mass diverges

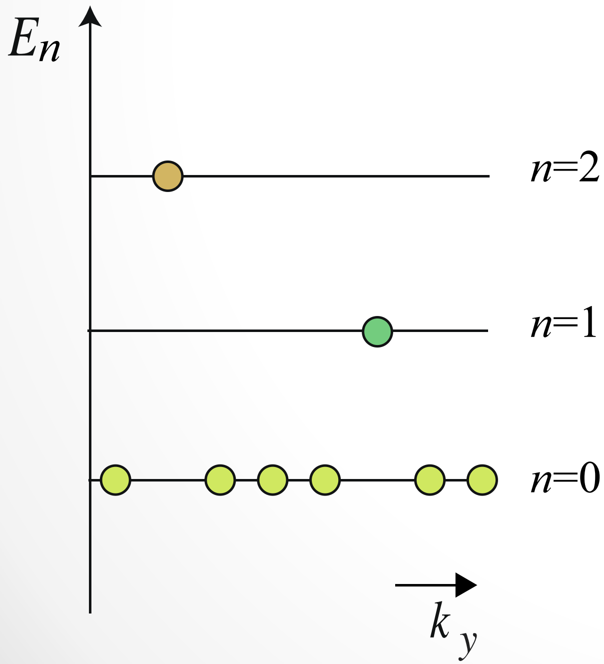
$$m_e^* \to \infty$$

Many-body state and heterodyne Hall effect

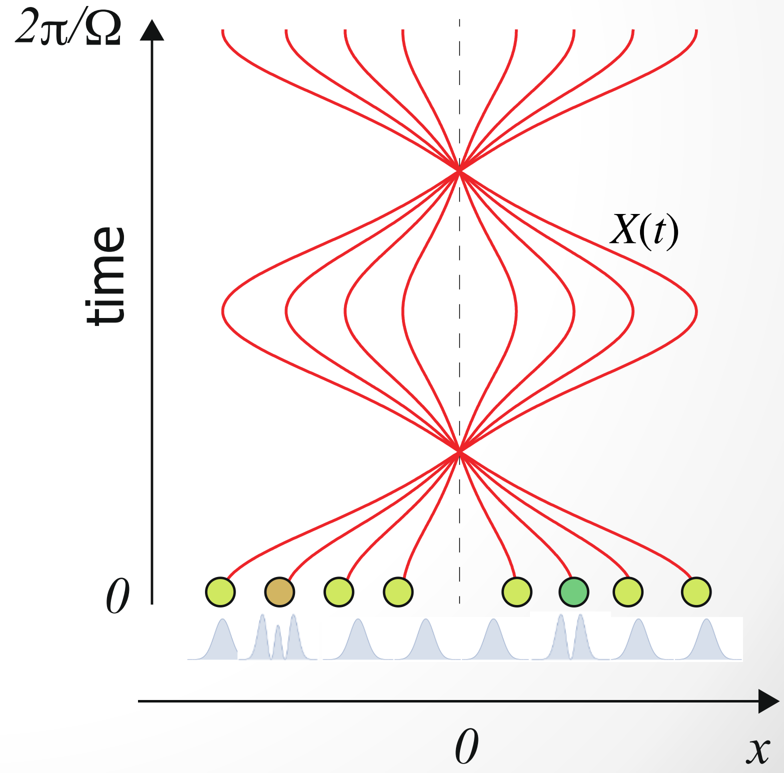


$m_e^* \to \infty$ "Landau quantization"

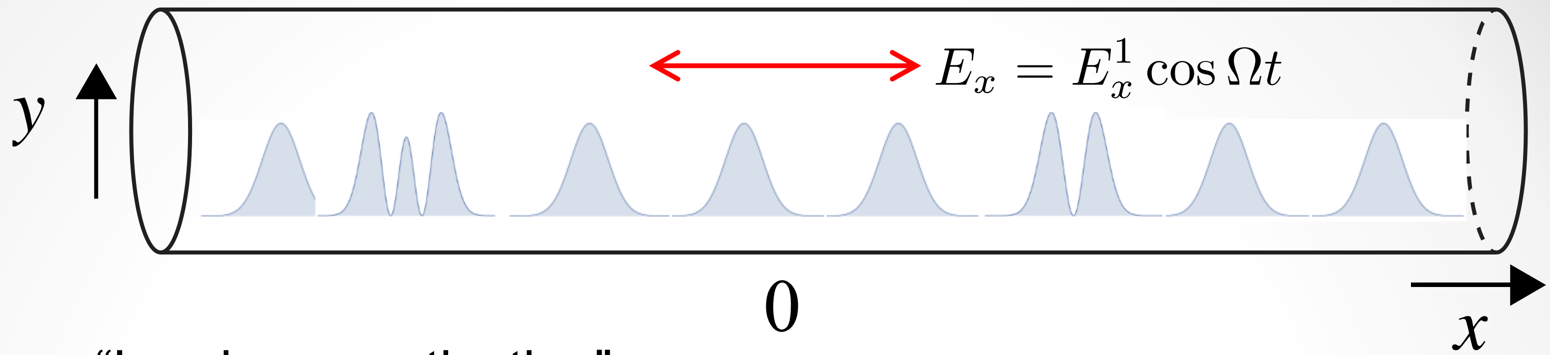
$$E_n(k_y) = \epsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{\hbar^2}{2m_e}\left(1 - \frac{m_e}{m_e^*}\right)\left(k_y - q\frac{E_x^1}{\hbar\omega_c}\right)^2$$



motion of the wave packets

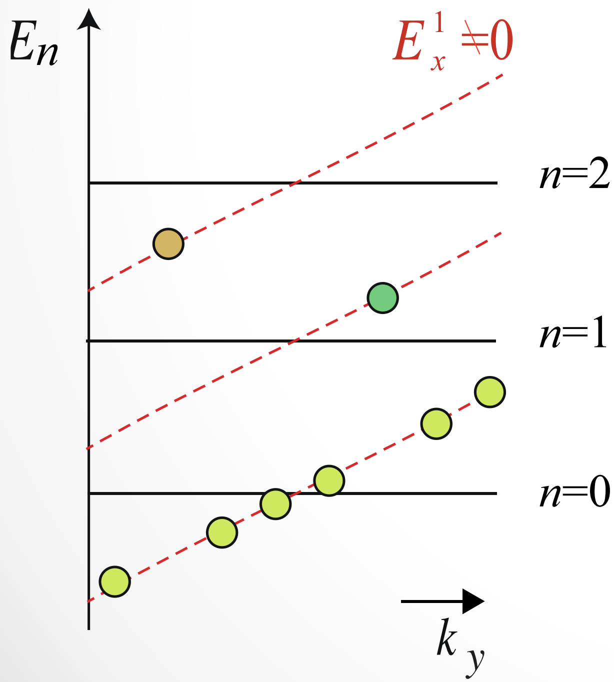


Many-body state and heterodyne Hall effect



$m_e^* \to \infty$ “Landau quantization”

$$E_n(k_y) = \epsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{\hbar^2}{2m_e}\left(1 - \frac{m_e}{m_e^*}\right)\left(k_y - q\frac{E_x^1}{\hbar\omega_c}\right)^2$$



$$J_y(k_y) = q\frac{\partial E_n}{\hbar \partial k_y}$$

Total dc-current

$$j_y = \frac{1}{L_x L_y} \sum_{k_y} f_n(k_y) J_y(k_y)$$

$$= \sigma_{yx}^{0,1} E_x^1$$

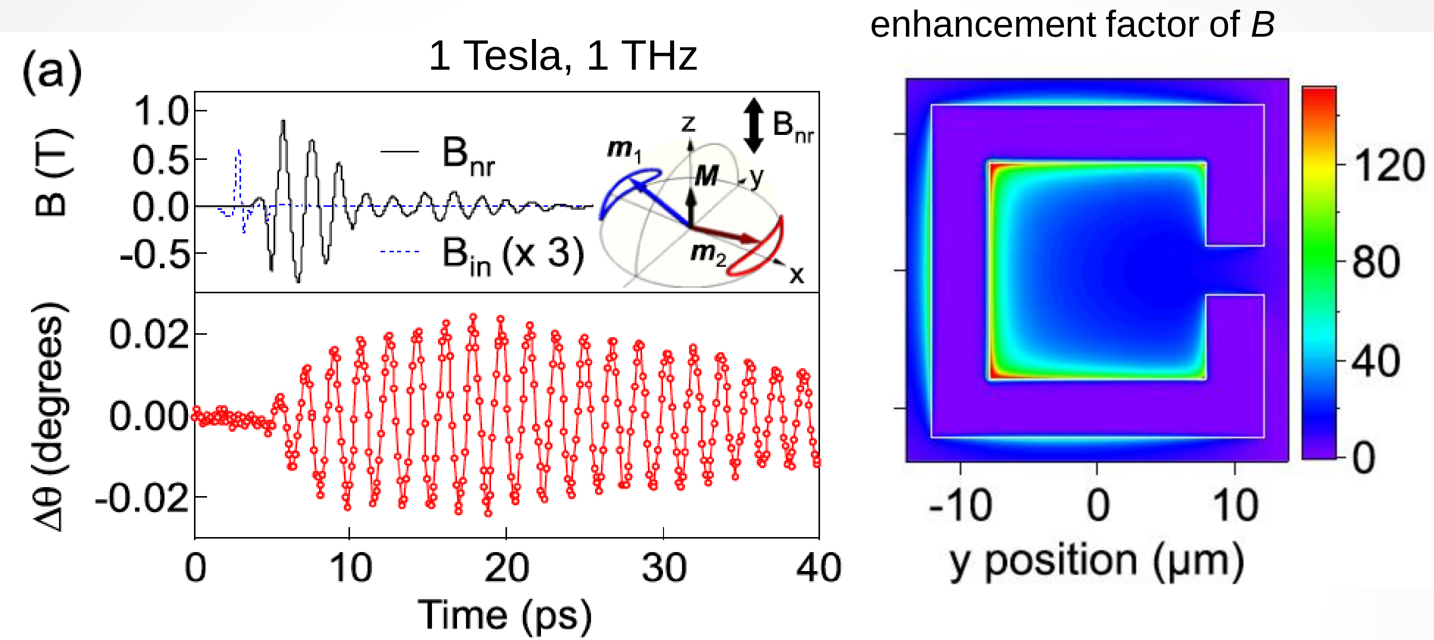
$$\sigma_{yx}^{0,1} = \frac{e^2}{h} Q\nu$$

LL filling

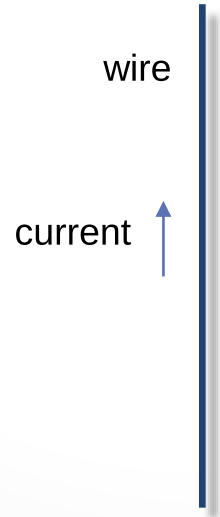
$$\nu = N_e / N_\Phi$$

α	1	2	3	4
r_α^q	1.89	5.07	8.22	
Q_α	0.22	0.15	0.12	

How to realize? THz metamaterial



Mukai, *et al.* (K. Tanaka grp.) New J. Phys.'16

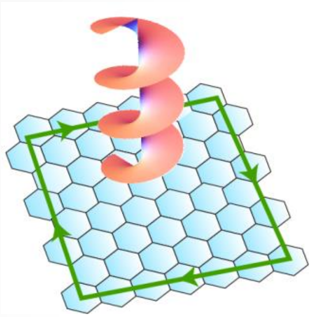


$$A_y(x,t) = \frac{\mu_0}{4\pi}\int j_y(x',t)/|x-x'|dx$$

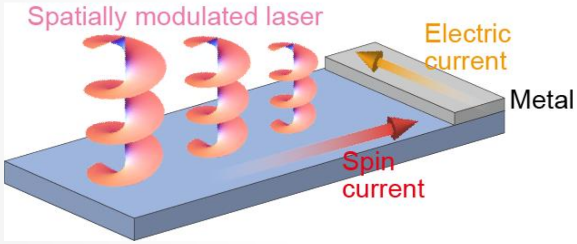
Conclusion

New states can be introduced by laser.

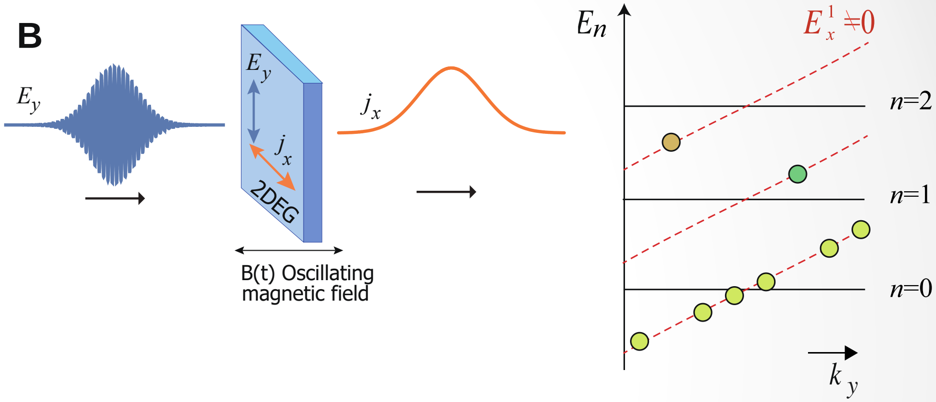
Floquet topological states



Ultrafast spintronics



Heterodye Hall effect



Q1. Is this topological?
TKNN for heterodyne?
Q2. What happens with interaction?
Q3. Dirac materials?

Q1. We found resonant enhancement.
How can we understand it? ESR?

$$N_\Phi = \frac{L_x L_y}{2\pi l_B^2 r^2 \max \xi}$$

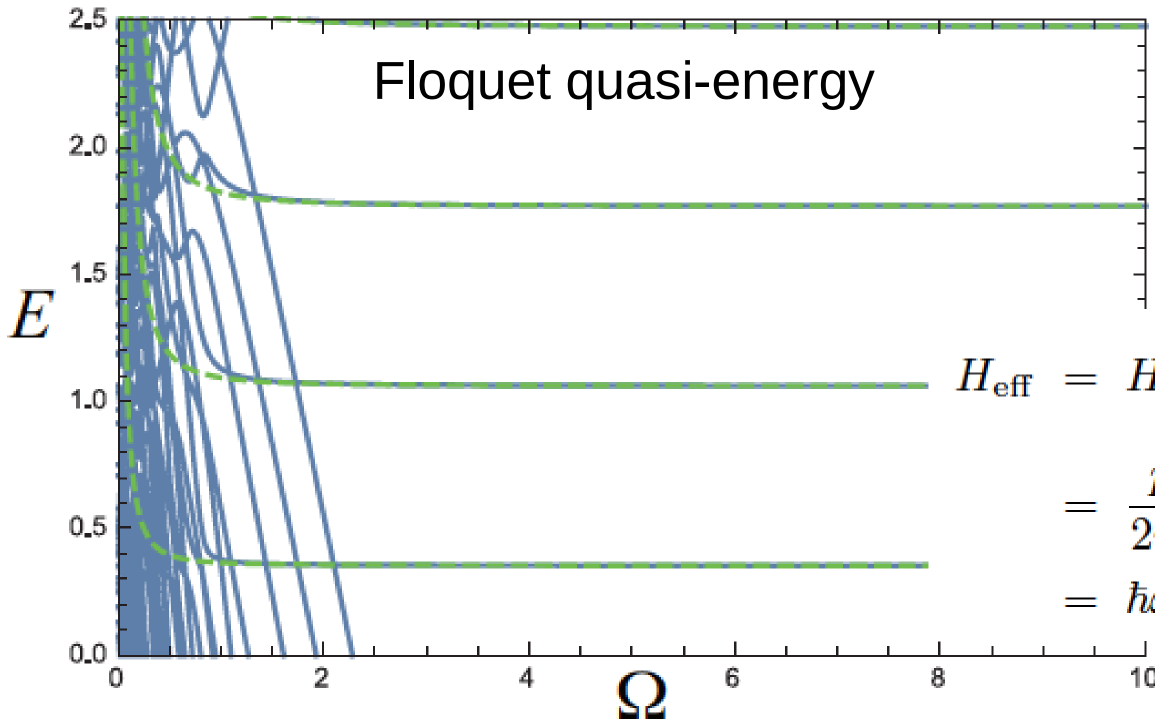
$$Q \ = \ \left(1 - \frac{m_e}{m_e^*}\right) / \left(2r^2 \max \xi\right)$$

(i) Quantum oscillator without driving

$$H(t) = \frac{p_x^2}{2m_e} + \frac{m_e \omega^2(t)}{2} x^2$$

$$\omega(t) = \omega_c \cos \Omega t$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\varphi_i(t) = E_i \varphi_i(t)$$



high-frequency expansion

$$\begin{aligned} H_{\text{eff}} &= H_0 + \frac{[[H_{-2}, H_0], H_2]}{(2\Omega)^2} + \mathcal{O}(\frac{1}{\Omega^4}) \\ &= \frac{p^2}{2m_e} + \frac{m_e \bar{\omega}^2}{2}\left(1 + \frac{1}{8}\left(\frac{\bar{\omega}}{\Omega}\right)^2\right) x^2 \\ &= \hbar \omega_{\text{eff}}(\Omega)(n + 1/2) \end{aligned}$$

Harmonic oscillator with a renormalized potential

(ii) Classical driven oscillator

$$m_e \ddot{X} + m_e \omega(t)^2 X = S(t)$$

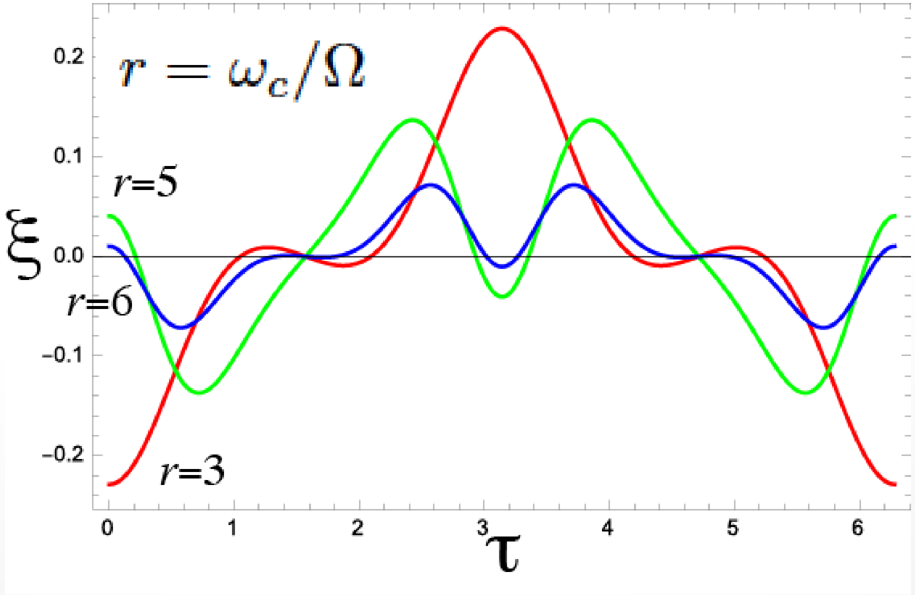
$$S(t) = \omega(t)(\hbar k_y - q A_y)$$

Mathieu's differential equation with a source term

$$\frac{d^2\xi}{d\tau^2} + (a - 2q\cos 2\tau)\xi = -\cos\tau$$

$$X(t) = -\left(\frac{\omega_c p_y}{\Omega^2 m_e}\right)\xi(\Omega t) = -(l_B r)^2 \xi(\Omega t) k_y$$

"snake state": trajectory of the wave center

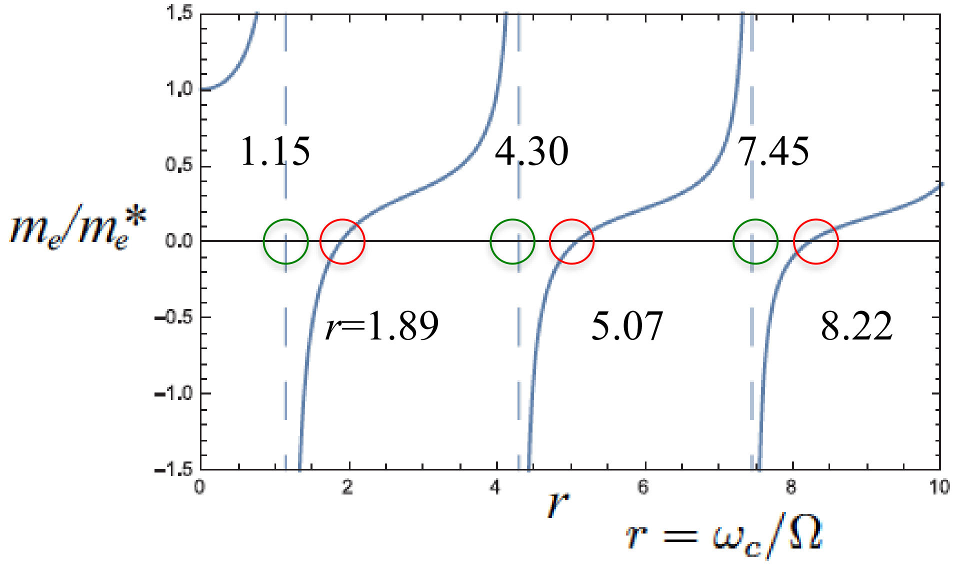


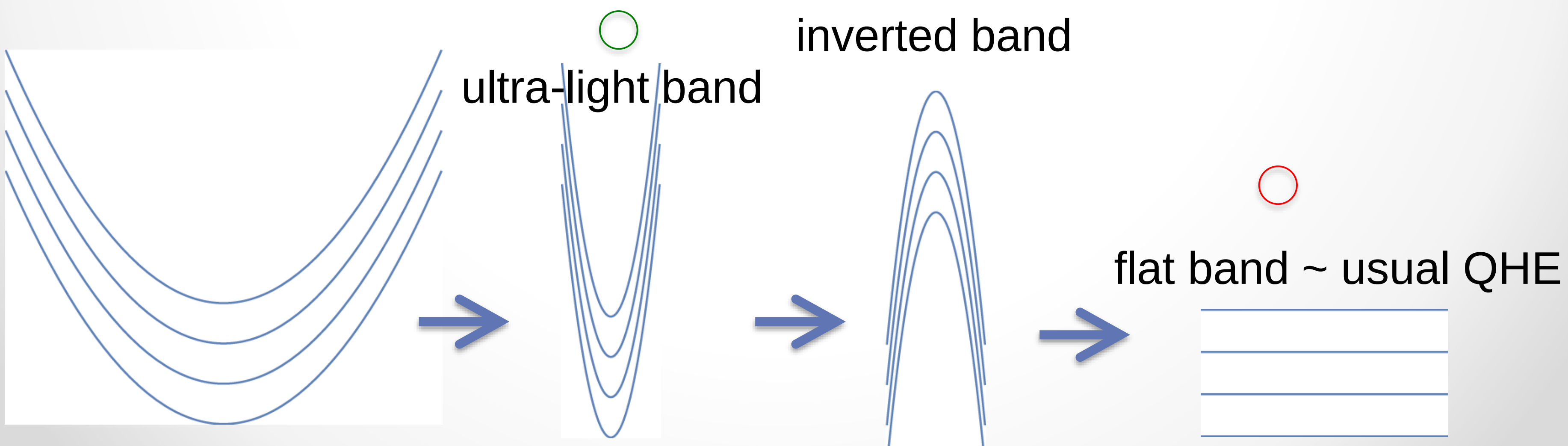
Spectrum

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m^*}$$

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{1}{T}\int_0^T L(t')dt'$$

Effective mass





Effect of electric fields

$$H(p_x, p_y; t) = \frac{1}{2m_e}\left[p_x^2 + \left(p_y - \frac{q}{c}B_z(t)x\right)^2\right] + qE_x(t)x$$

$$= \frac{p_x^2}{2m_e} + \frac{p_y^2}{2m_e} + \frac{m_e(\omega(t))^2}{2}x^2 - \omega(t)p_y x + qE_x(t)x$$

ac-field

$$E_x = E_x^1 \cos\Omega t$$

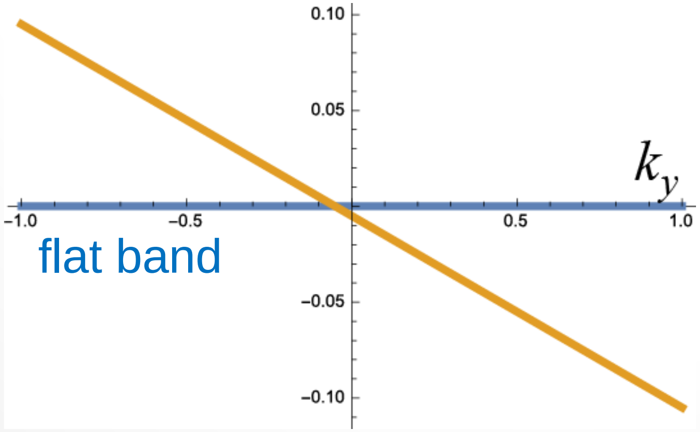
shift of k_y momentum

$$k_y \to k_y + \frac{eE_x^1}{\hbar\omega_c}$$

deformation of spectrum

$$E_n(p_y) = \varepsilon_n + \frac{p_y^2}{2m^*} \to E_n(p_y) = \varepsilon_n + \frac{p_y^2}{2m_e} + \frac{1}{2m_e}\left(\frac{m_e}{m^*} - 1\right)\left(p_y + \frac{eE_x^1}{\omega_c}\right)^2$$

linear dispersion (finite E_x^1)



current (for LL filling N)

$$j_y = \frac{q^2}{h}N\frac{\left(\frac{m_e}{m^*} - 1\right)}{2r^2\max(\xi(\Omega t))}E_x^1$$

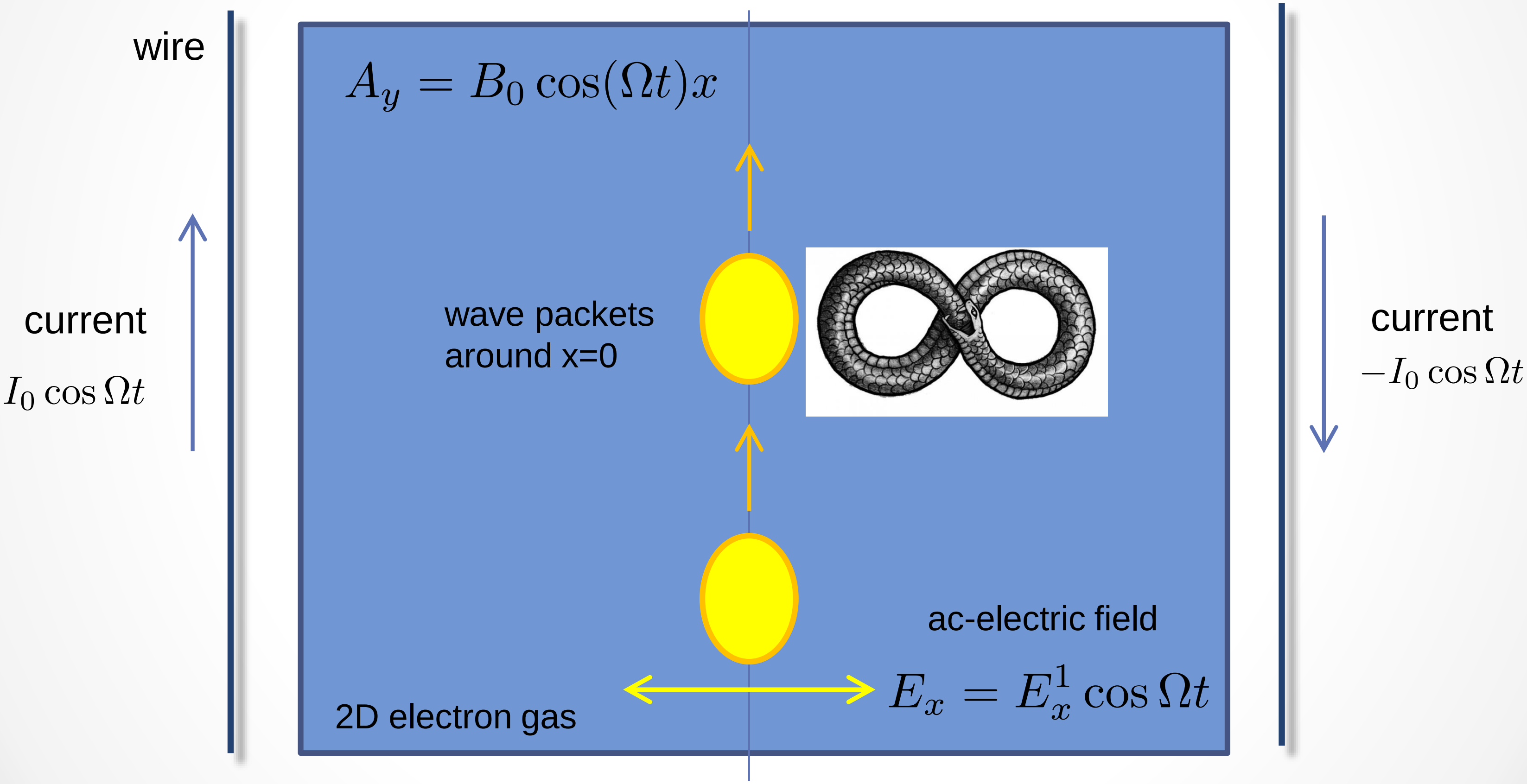
cf) integer QHE

$$j_y = \frac{q^2}{h}NE_x^{\mathrm{dc}}$$

Conclusion and outlook

quantization condition (flat band condition)

$$r = \omega_c/\Omega = 1.89, 5.07, 8.22\ldots \qquad \omega_c = qB/m_e c$$



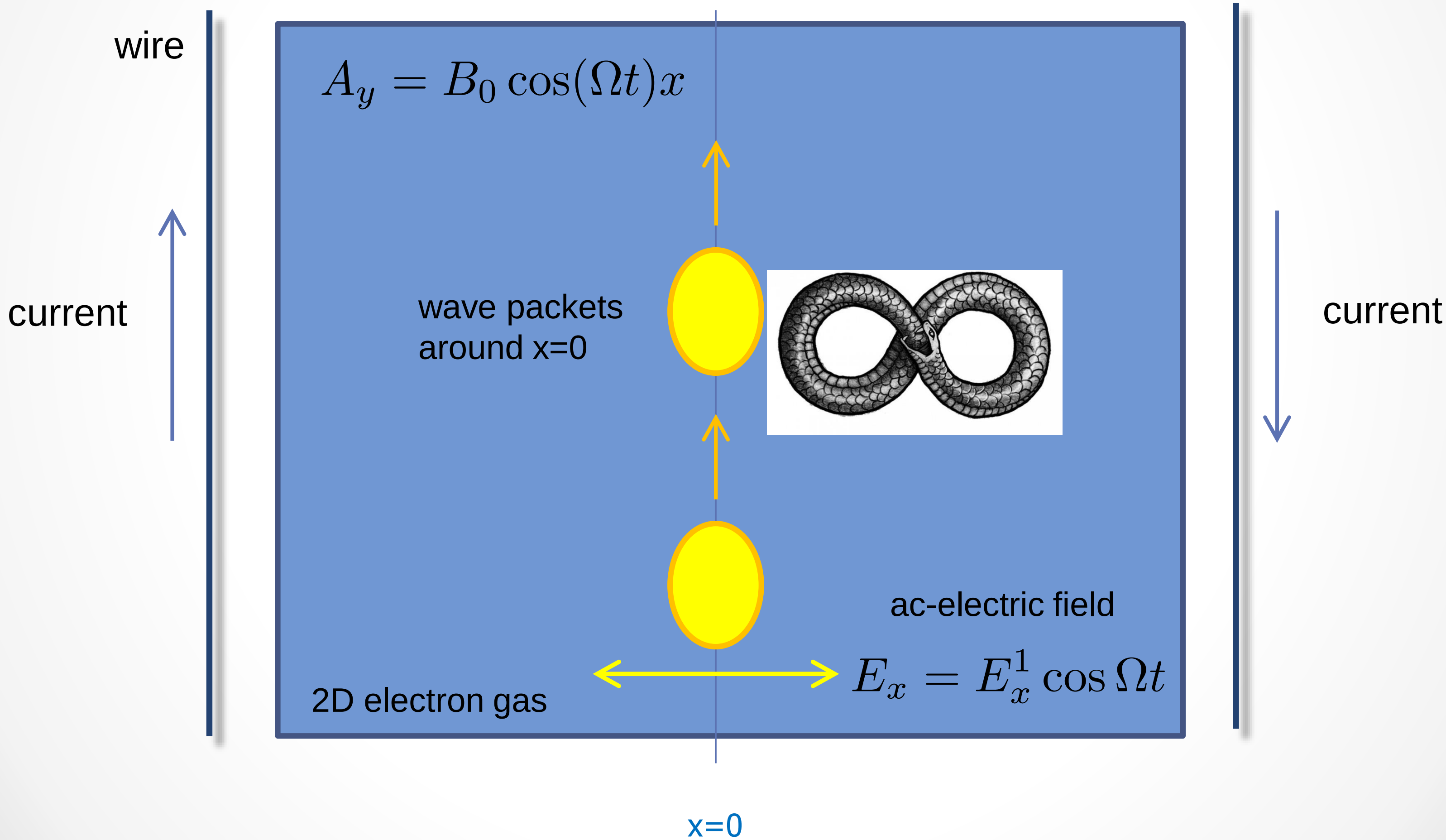
Question: Effect of Coulomb interaction

Wigner crystal? fractional QHE??

Conclusion

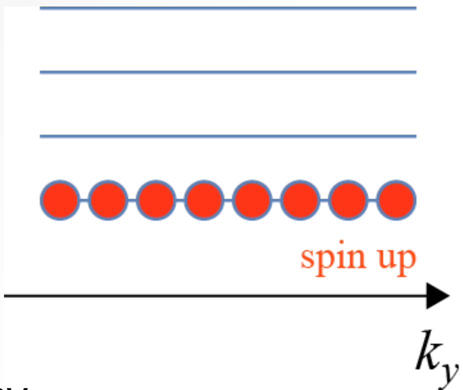
quantization condition (flat band condition)

$$r = \omega_c/\Omega = 1.89, 5.07, 8.22\ldots \quad \omega_c = qB/m_e c$$



heterodyne: ac-field is converted to dc-current (rectification)

Many-body state



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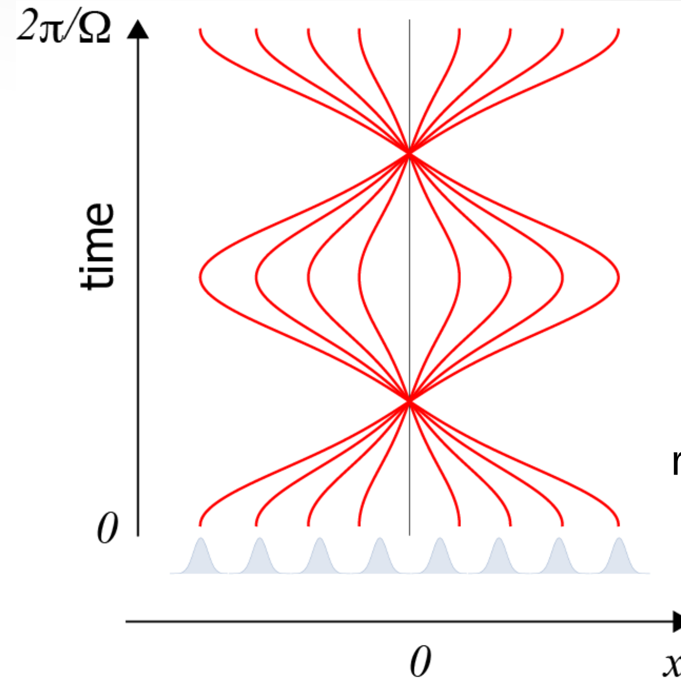
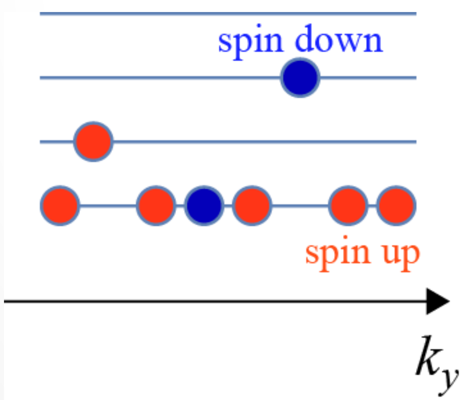


r=1.89

degeneracy

$$N_\phi = \frac{L_x L_y}{2\pi l_B^2 r^2 \max(\xi(\Omega t))}$$



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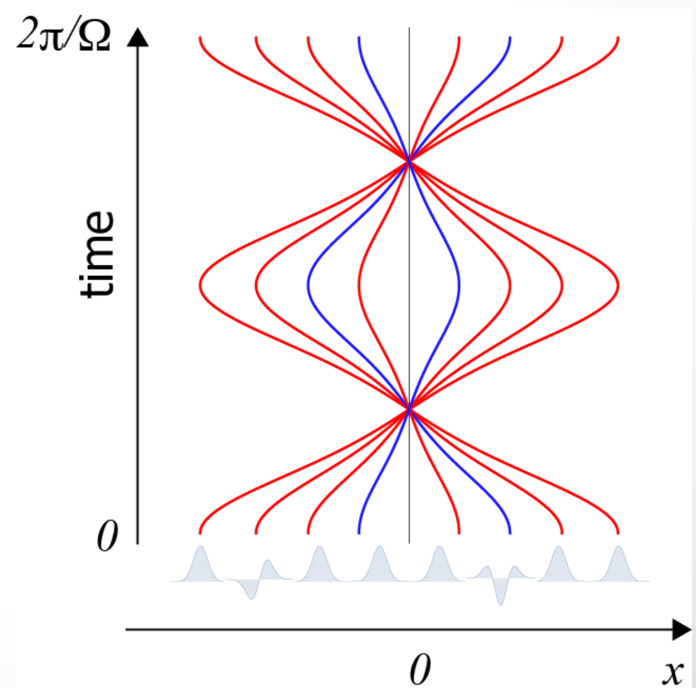
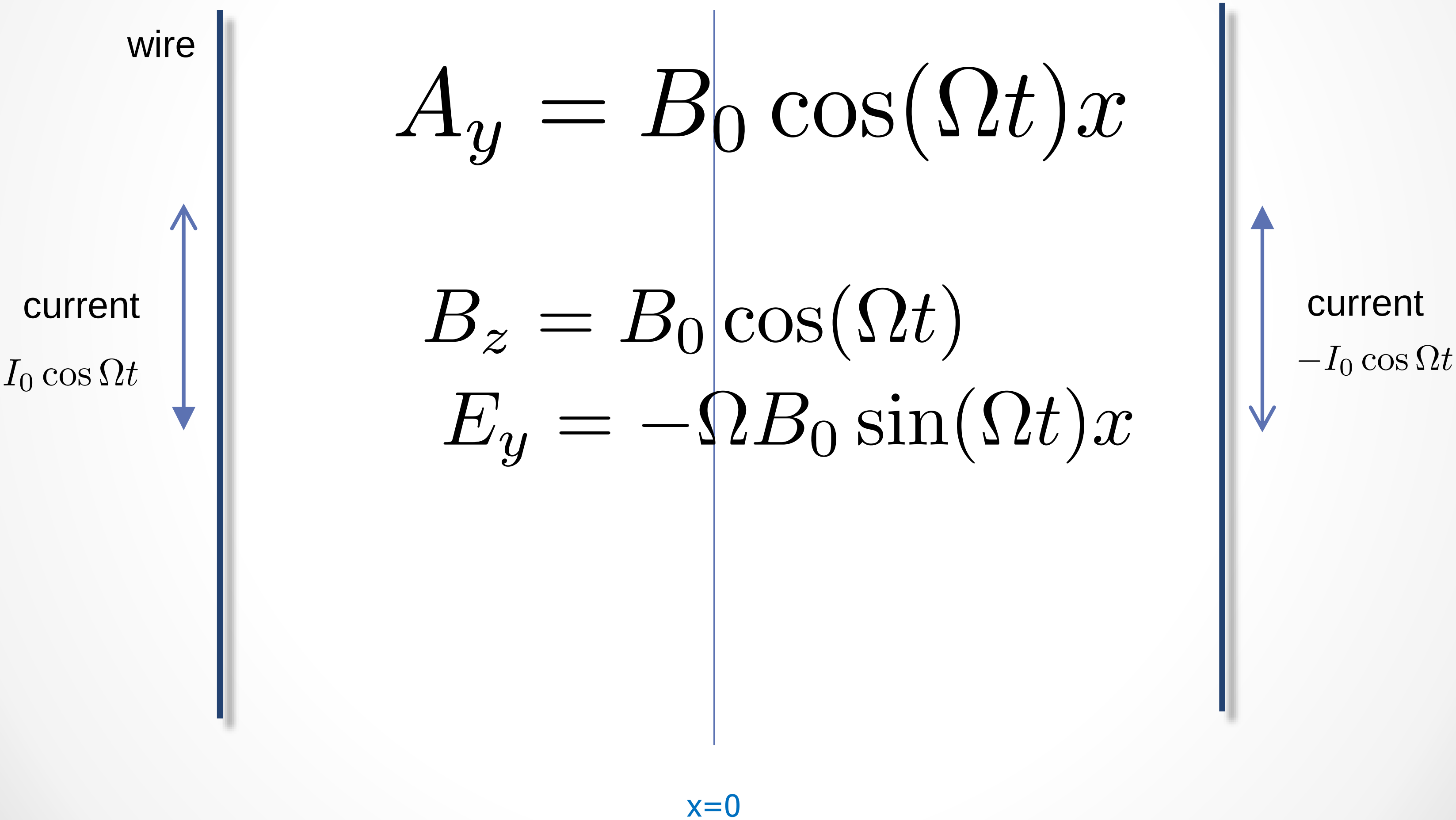


All electrons approaches the origin (x=0)

strongly interacting problem

with Leda Bucciantini (*in progress*)



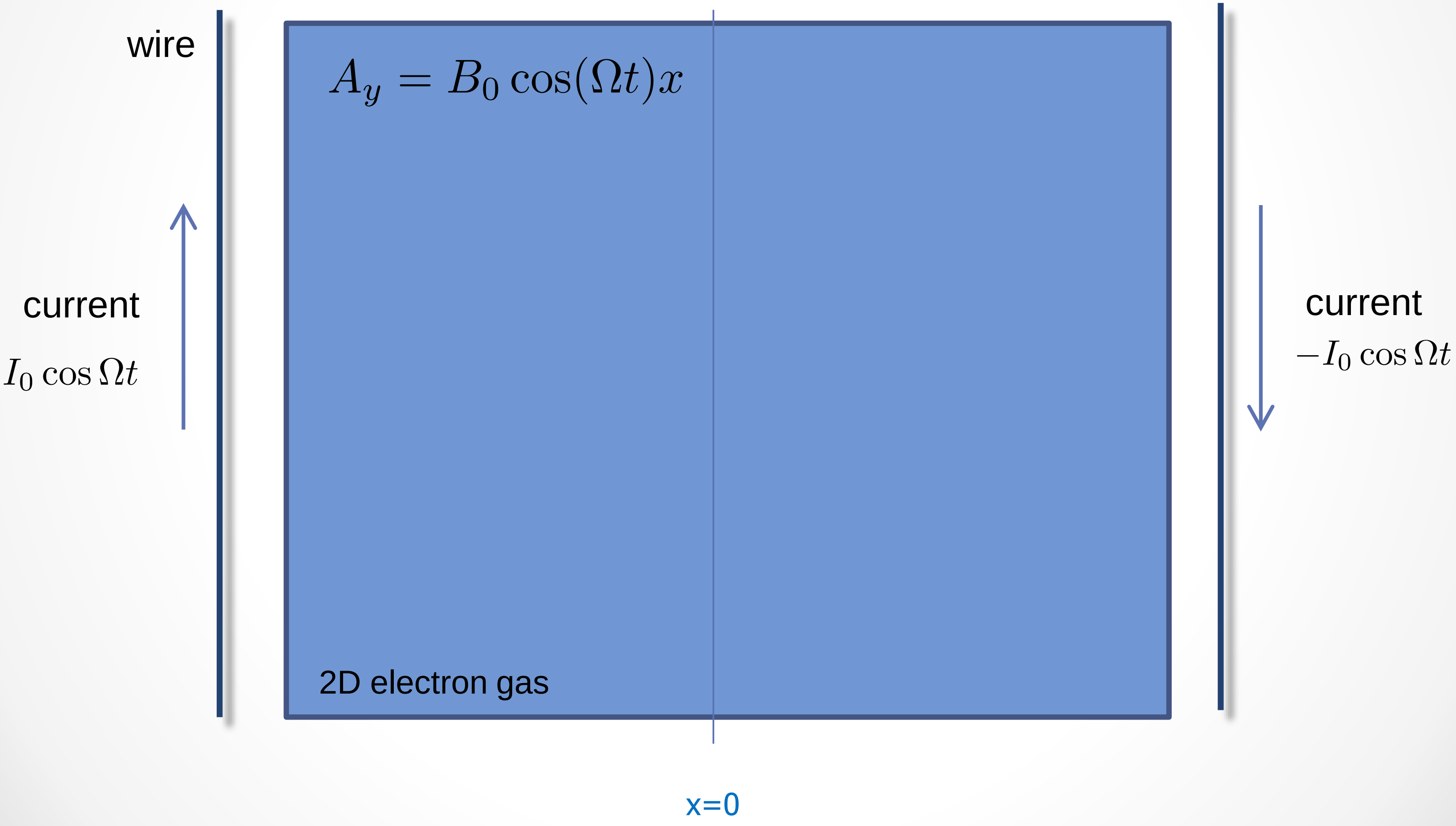
$$A_y = B_0 \cos(\Omega t) x$$

$$B_z = B_0 \cos(\Omega t)$$

$$E_y = -\Omega B_0 \sin(\Omega t) x$$

What will happen?

with Leda Bucciantini (*in progress*)

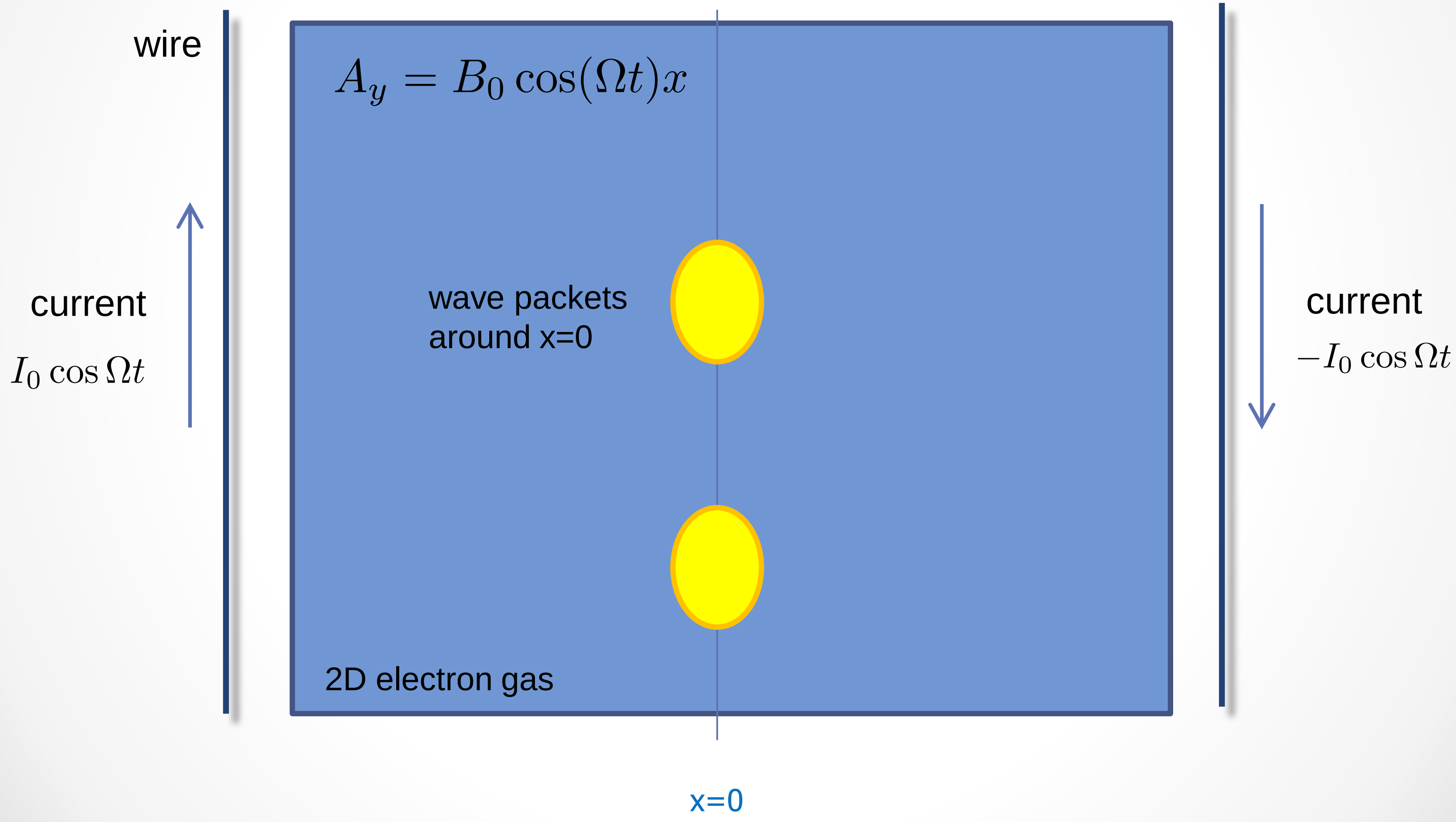


What will happen?

with Leda Bucciantini (*in progress*)

quantization condition (flat band condition)

$$r = \omega_c/\Omega = 1.89, 5.07, 8.22\ldots \quad \omega_c = qB/m_e c$$



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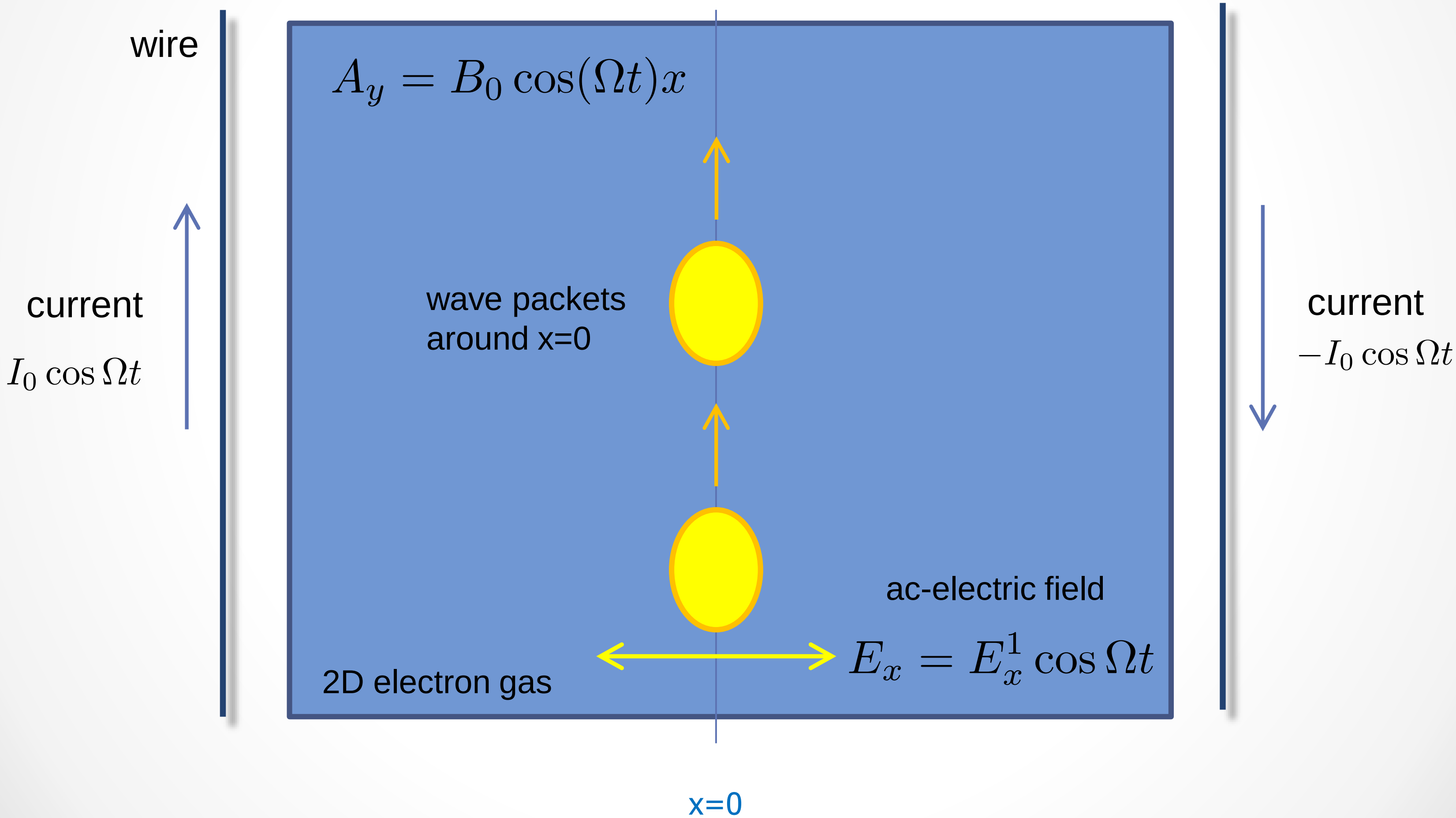

heterodyne: ac-field is converted to dc-current (rectification)

What will happen?

with Leda Bucciantini (*in progress*)

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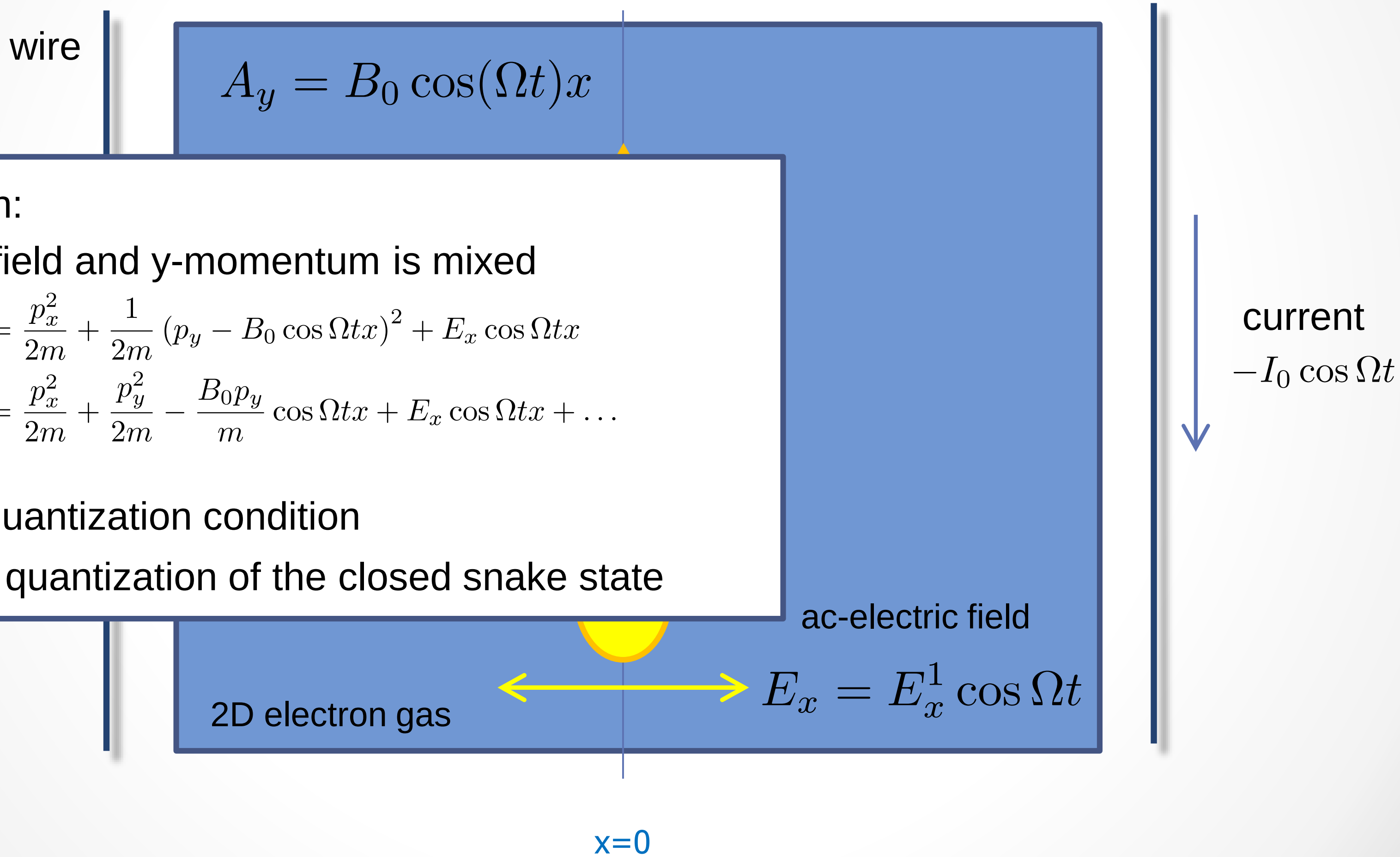


Reason:

1. x-field and y-momentum is mixed

$$H = \frac{p_x^2}{2m} + \frac{1}{2m}\left(p_y - B_0 \cos \Omega t x\right)^2 + E_x \cos \Omega t x$$
$$= \frac{p_x^2}{2m} + \frac{p_y^2}{2m} - \frac{B_0 p_y}{m} \cos \Omega t x + E_x \cos \Omega t x + \ldots$$

2. quantization condition

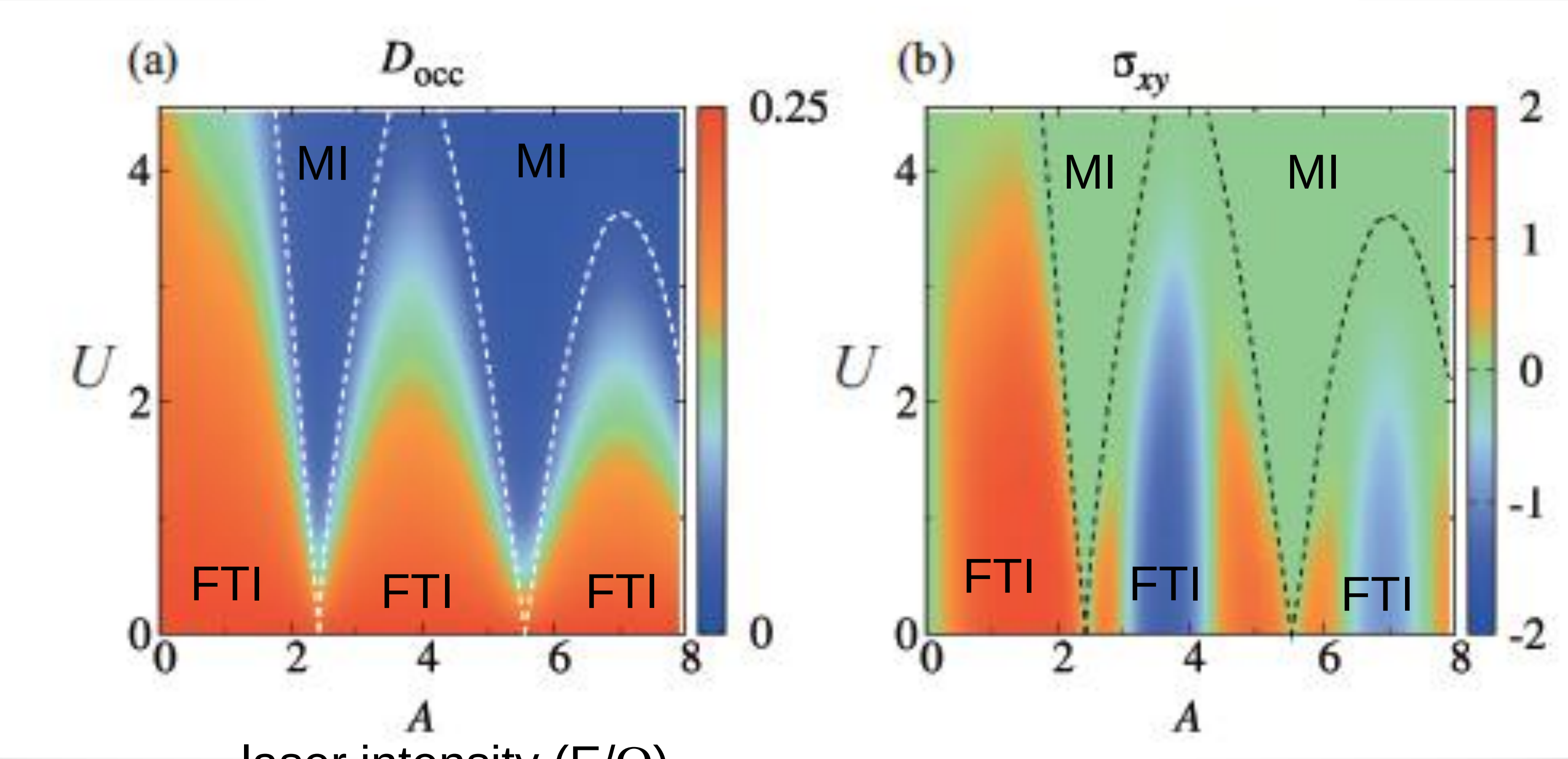
   quantization of the closed snake state

heterodyne: ac-field is converted to dc-current (rectification)

Periodically driven Hubbard model (half-filling, honeycomb +CPL) with local electron bath

Nonequilibrium DMFT Aoki, TO, *et al. RMP* '15
Mikami *et al. PRB* '16

$$\mathcal{H}_{\rm sys}(t) = \sum_{\langle i,j\rangle,\sigma} \left[J_{i,j}(t) c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.}\right] + \sum_i U \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$



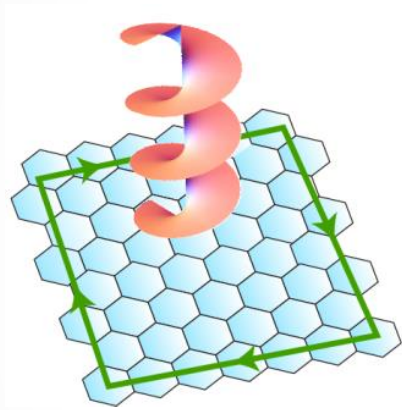
laser intensity (E/Ω)

$$\sigma_{xy}(\nu) = \text{Re}\frac{1}{\nu}\frac{1}{\mathcal{N}S_{\rm cell}}\text{tr}\sum_{\boldsymbol{k}}\int_{-\omega/2}^{\omega/2}\frac{d\nu'}{2\pi} \times\left[\hat{v}^i_{\boldsymbol{k}}\hat{G}^{\rm R}_{\boldsymbol{k}}(\nu'+\nu)\hat{v}^j_{\boldsymbol{k}}\hat{G}^<_{\boldsymbol{k}}(\nu') + \hat{v}^i_{\boldsymbol{k}}\hat{G}^<_{\boldsymbol{k}}(\nu')\hat{v}^j_{\boldsymbol{k}}\hat{G}^{\rm A}_{\boldsymbol{k}}(\nu'-\nu)\right]$$

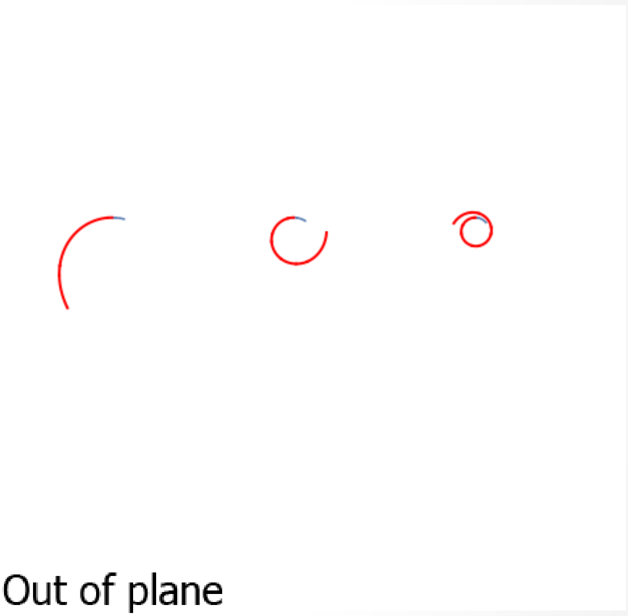
Summary

1. New states of matter in driven nonequilibrium systems
2. Plasmonic enhancement

Floquet topological states



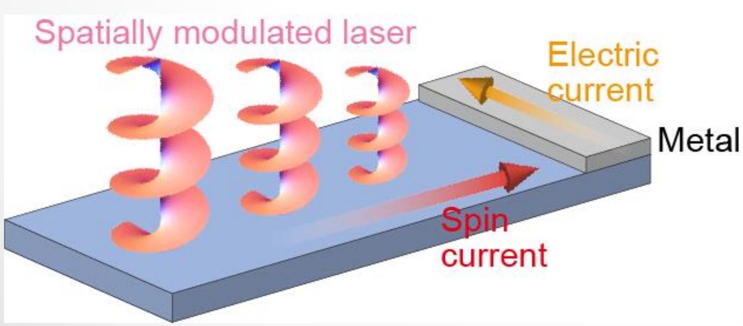
Heterodyning Hall effect



⊙ Out of plane Magnetic field

$$B_z = B \cos \Omega t$$

Ultrafast spintronics



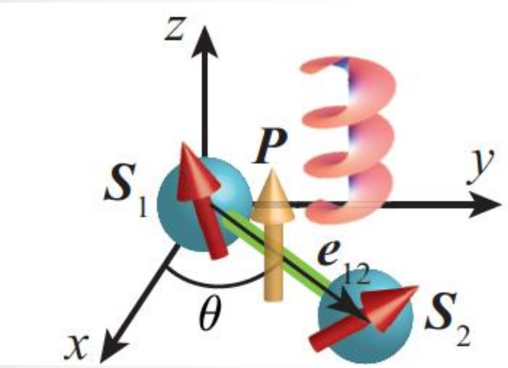
Can we see quantum oscillation when there is a Fermi surface?

Application to multiferroic quantum magnets

Sato, Sasaki, TO, arXiv'15
Sato, Takayoshi, TO, arXiv'16
Kitamura, TO *in prep.*

$$H(t) = H_{\rm spin} + \boldsymbol{B}(t)\cdot\boldsymbol{S} + \boldsymbol{E}(t)\cdot\boldsymbol{P}$$

Zeeman **magneto-optical coupling**



circularly polarized laser

$$\boldsymbol{E} = E_0(\cos\Omega t, \sin\Omega t),\ \boldsymbol{B} = B_0(\sin\Omega t, \cos\Omega t)$$

↓ Fourier transform

$$H_1 = E_0(P_x - iP_y)/2 + B_0(S_y - iS_x)/2$$
$$H_{-1} = E_0(P_x + iP_y)/2 + B_0(S_y + iS_x)/2$$
$$P \sim S\cdot S \text{ or } S\times S$$

effective terms $[H_{-1}, H_1]/\Omega$

$$H_\Omega = \frac{E_0E_0}{\Omega}(3\text{ spin term}) + \frac{E_0B_0}{\Omega}(2\text{ spin term}) + \frac{B_0B_0}{\Omega}(1\text{ spin term})$$

$\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)$ — scalar chirality

$\boldsymbol{a}\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)$ — vector chilarity

Part II Time-dependent gauge field

What happens in an oscillating magnetic field?

$$A_y = B_0 x$$

$$\Downarrow$$

$$A_y = B_0 \cos(\Omega t) x$$

$$B_z = B_0 \cos(\Omega t)$$

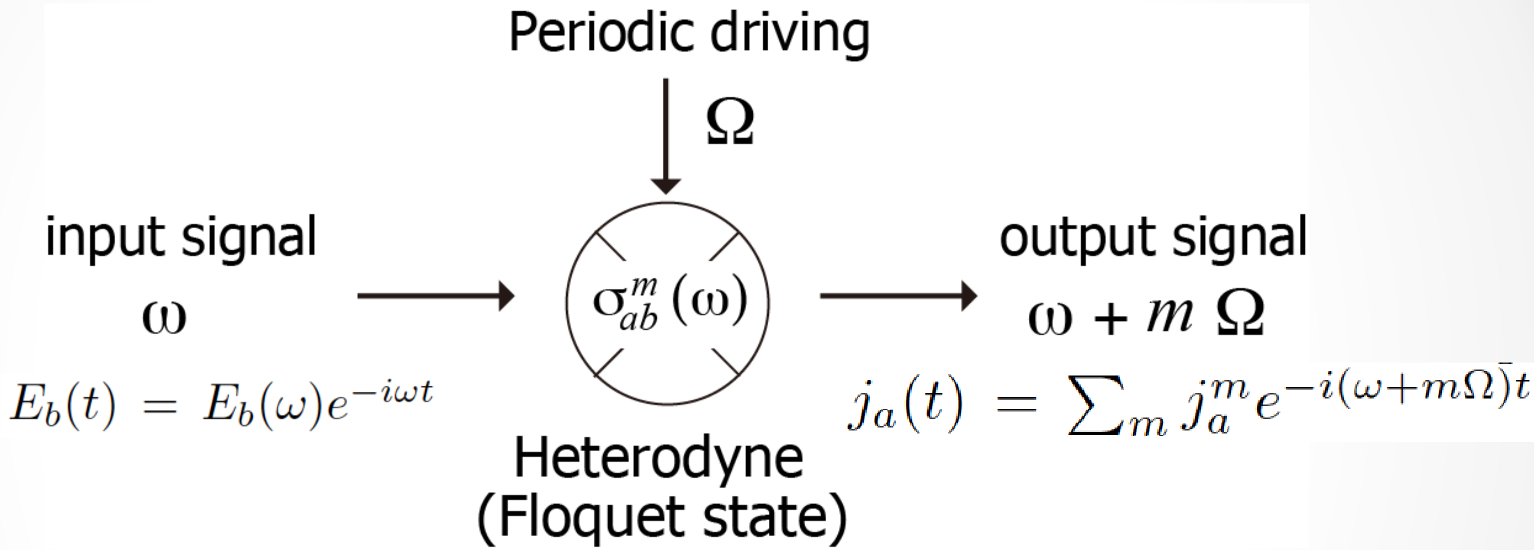
$$E_y = -\Omega B_0 \sin(\Omega t) x$$

"Floquet-Landau quantization"
and "dissipationless heterodyne Hall response"

with Leda Bucciantini (*in progress*)

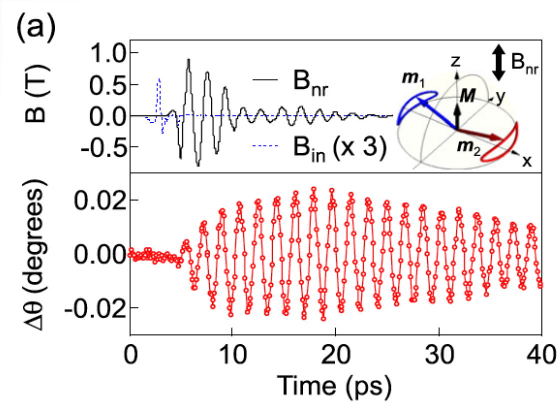
Heterodyne (frequency mixer)

cf) wikipedia

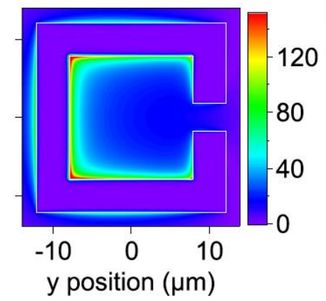


$$j_a^m(\omega + m\Omega) = \sum_a \sigma^m_{ab}(\omega) E_b(\omega)$$

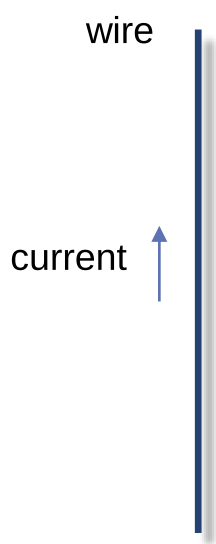
How to realize



enhancement factor of *B*

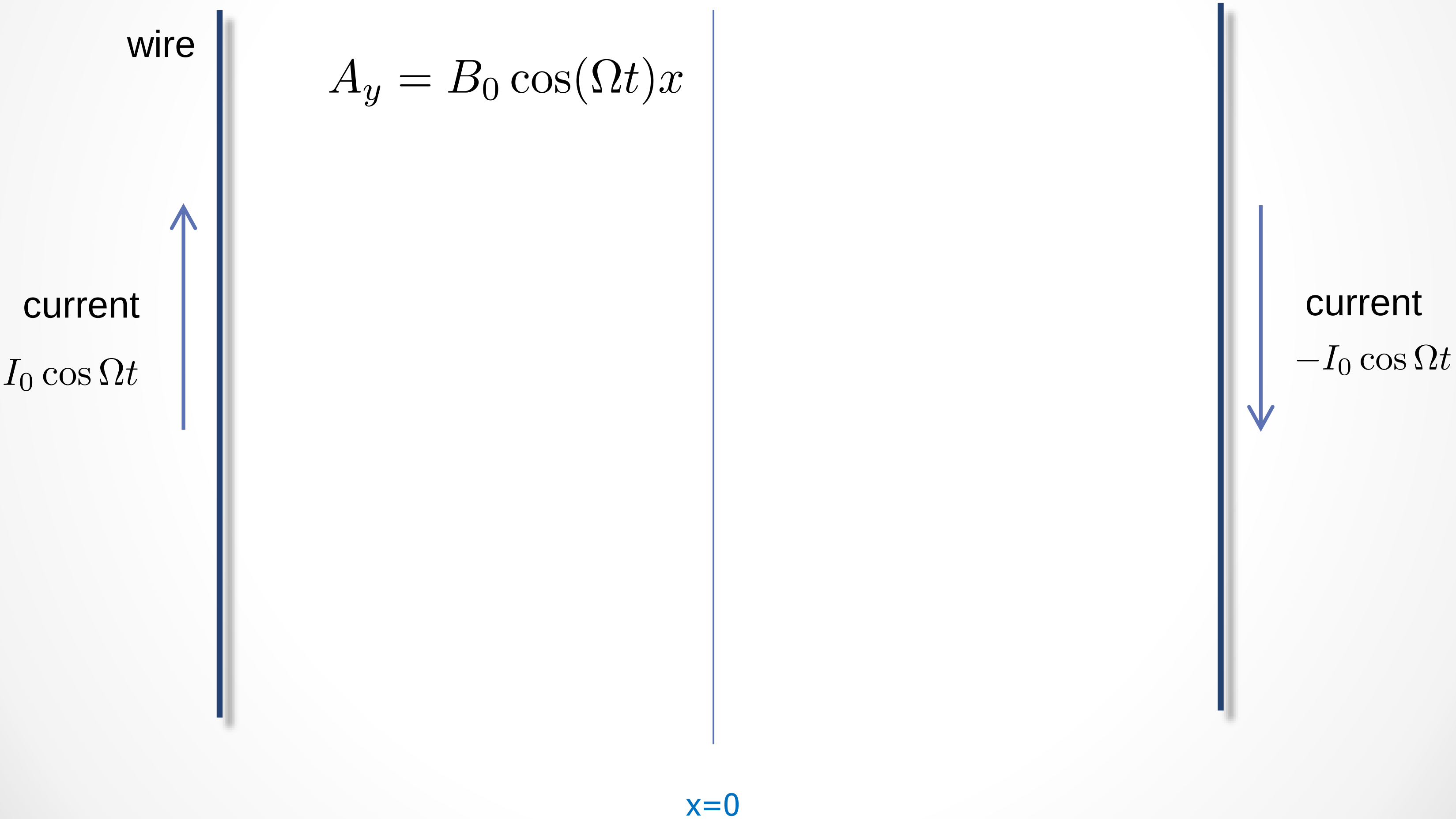


Mukai, *et al.* (Kyoto grp.) New J. Phys.’16

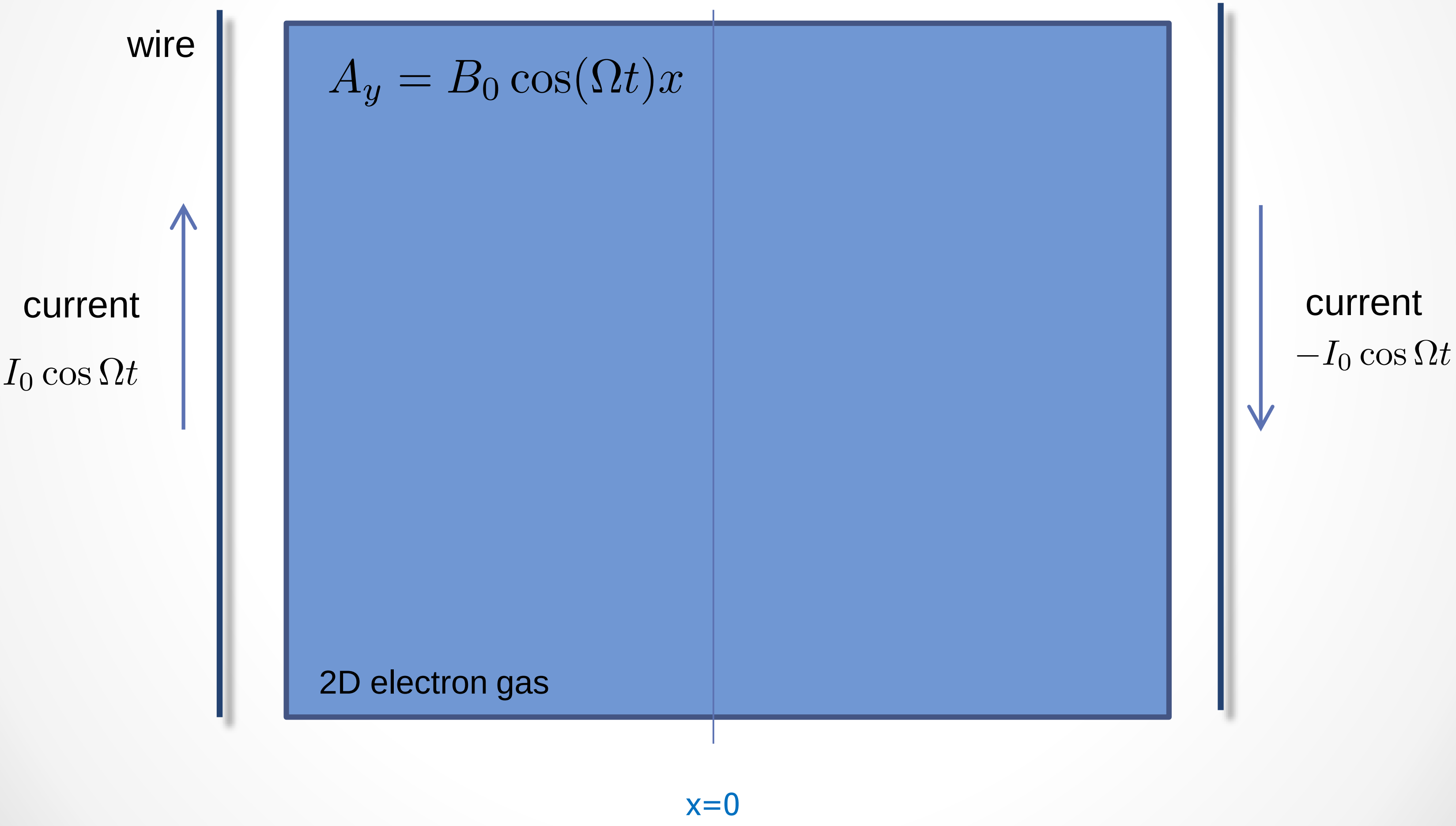


$$A_y(x,t) = \frac{\mu_0}{4\pi}\int j_y(x',t)/|x-x'|dx$$

What will happen?



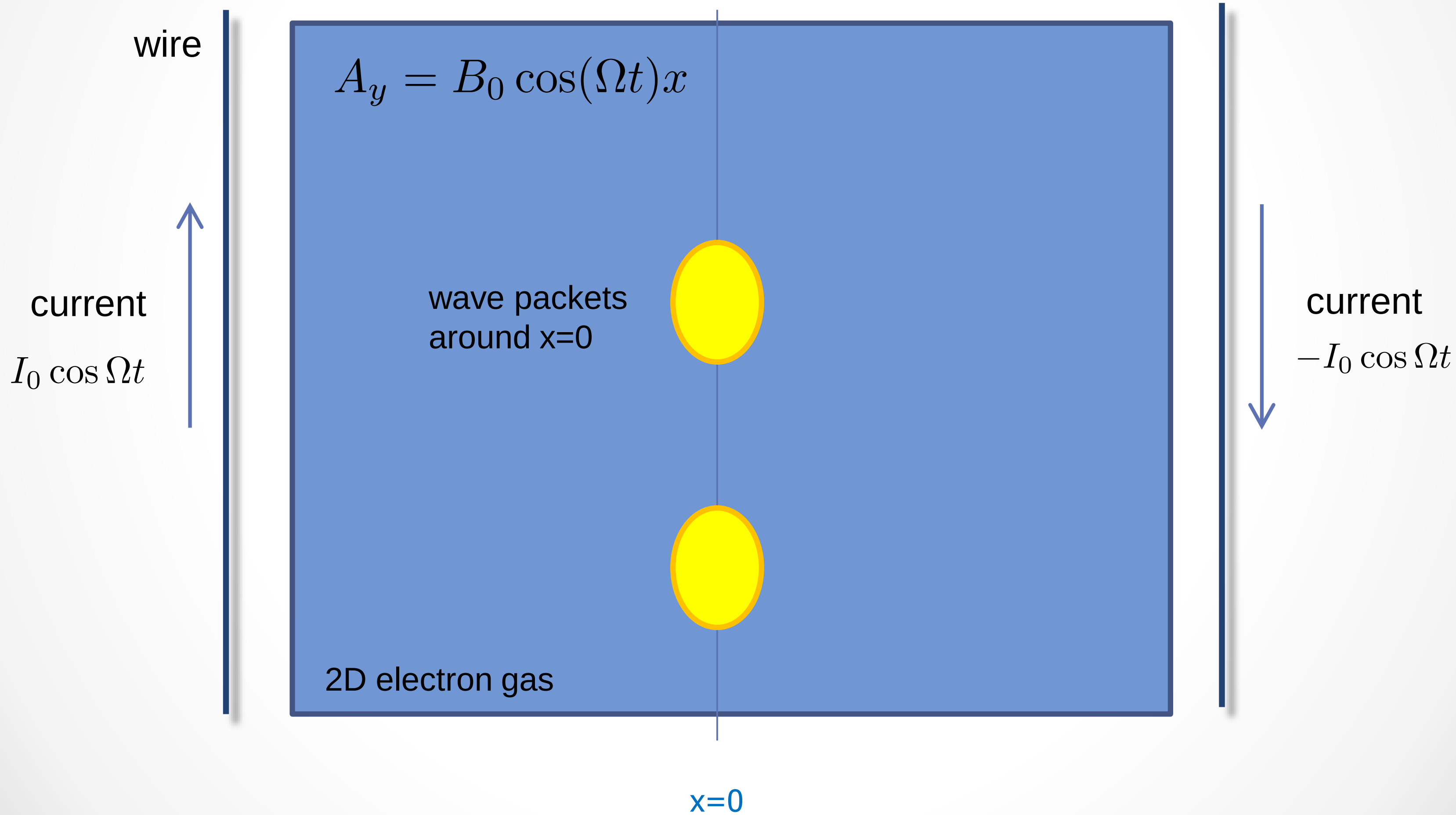
What will happen?



What will happen?

quantization condition (flat band condition)

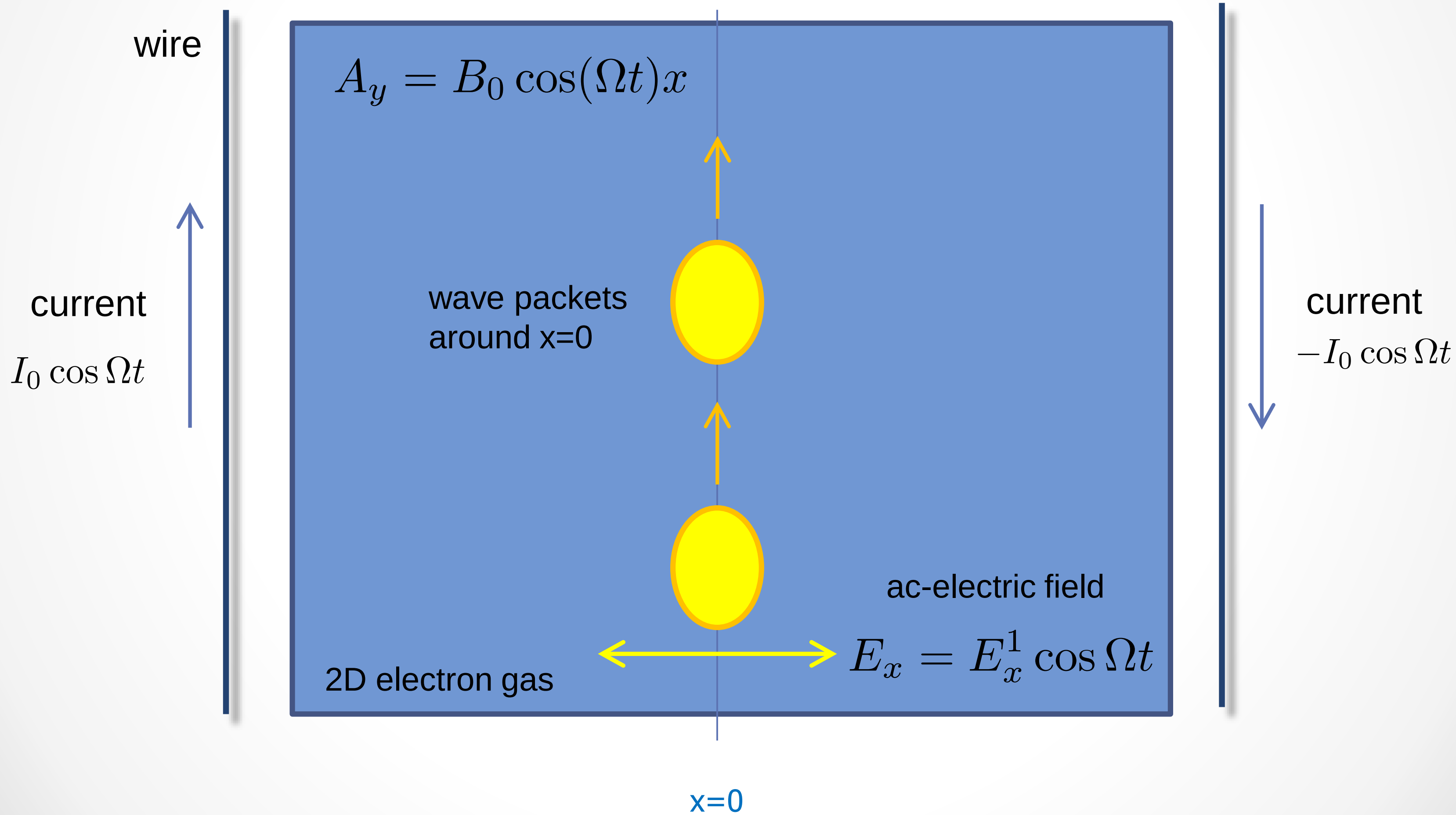
$$r = \omega_c/\Omega = 1.89, 5.07, 8.22\ldots \quad \omega_c = qB/m_e c$$



What will happen?

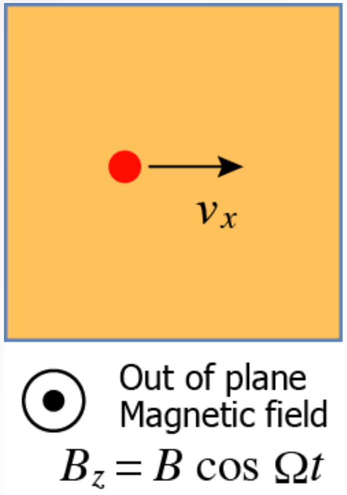
quantization condition (flat band condition)

$$r = \omega_c/\Omega = 1.89, 5.07, 8.22\ldots \quad \omega_c = qB/m_e c$$



heterodyne: ac-field is converted to dc-current (rectification)

”Snake-states” in oscillating magnetic field



TO, Bucciantini *in progress*

classical EOM

$$m_e\left(\frac{d}{dt} + 1/\tau\right)\boldsymbol{v} = q\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)$$

cyclotron frequency

$\omega_c = qB/m_e c$

free motion with finite initial velocity (τ=0, E=0)

ω_c/Ω=3.0

5.0

6.0

cf) J. E. Muller, PRL ‘92, “snake state by static nonuniform B”

motion in static E-field

Out of plane
Magnetic field

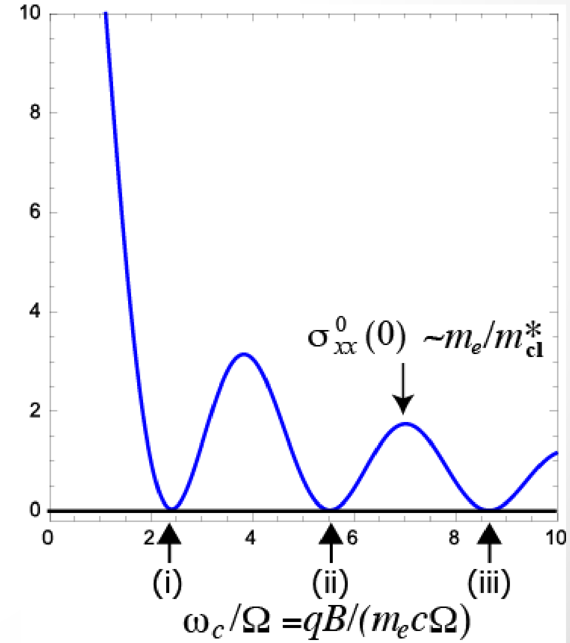
$B_z = B \cos \Omega t$

In plane
Electric field

E_y

$\omega_c/\Omega=3$ 5 6

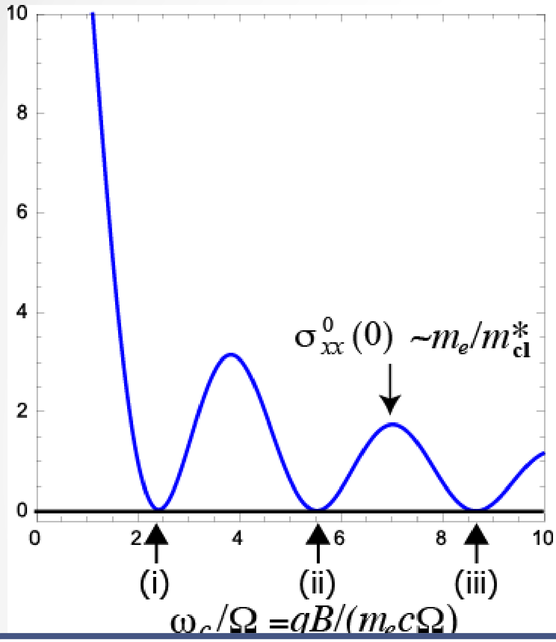
conductivity

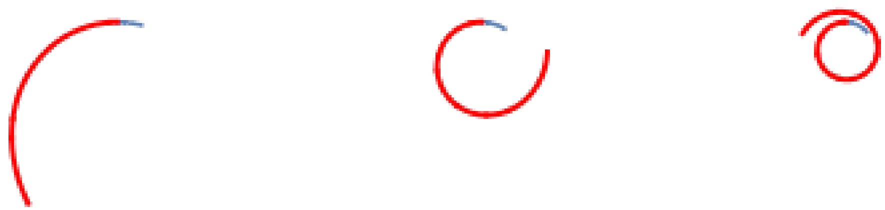


(τ=0.05, E_y=1)

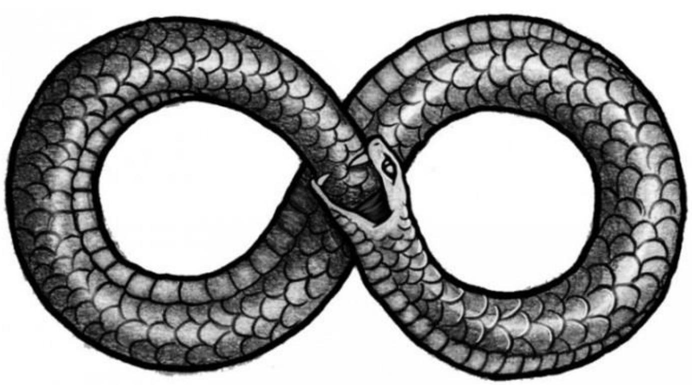
Periodic orbits

(τ=0.0, E_y=0)





Ω=2.41	5.52	8.66
(i)	(ii)	(iii)
α=1	α=2	α=3



http://jeroenvanhonk.com/ozymandias-vs-ouroboros

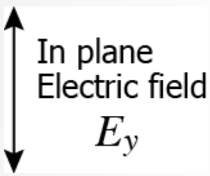
Dissipationless heterodyning Hall current (classical)

ω_c/Ω

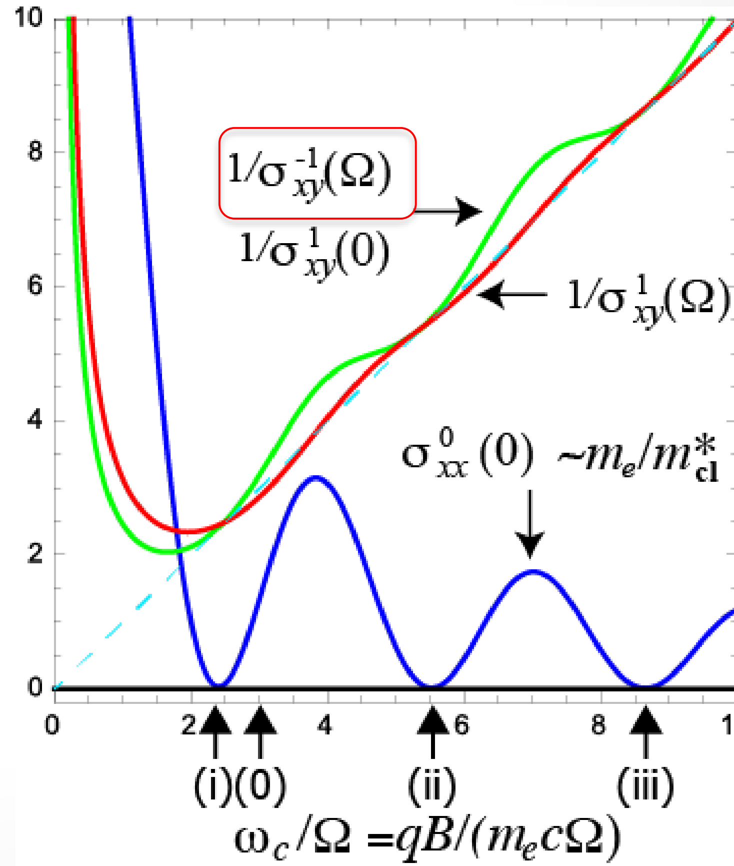
$=3$

5

6

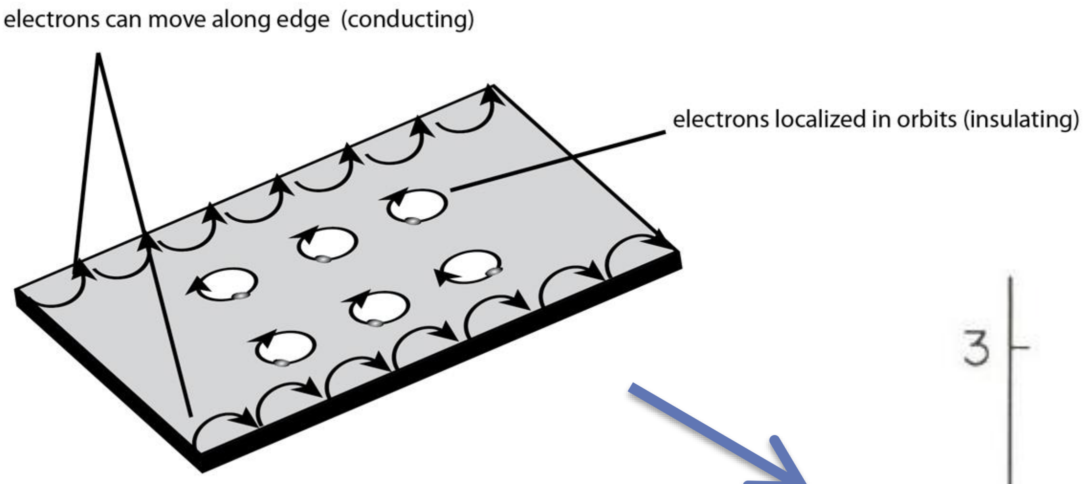


$$E_y^1 \cos \Omega t$$

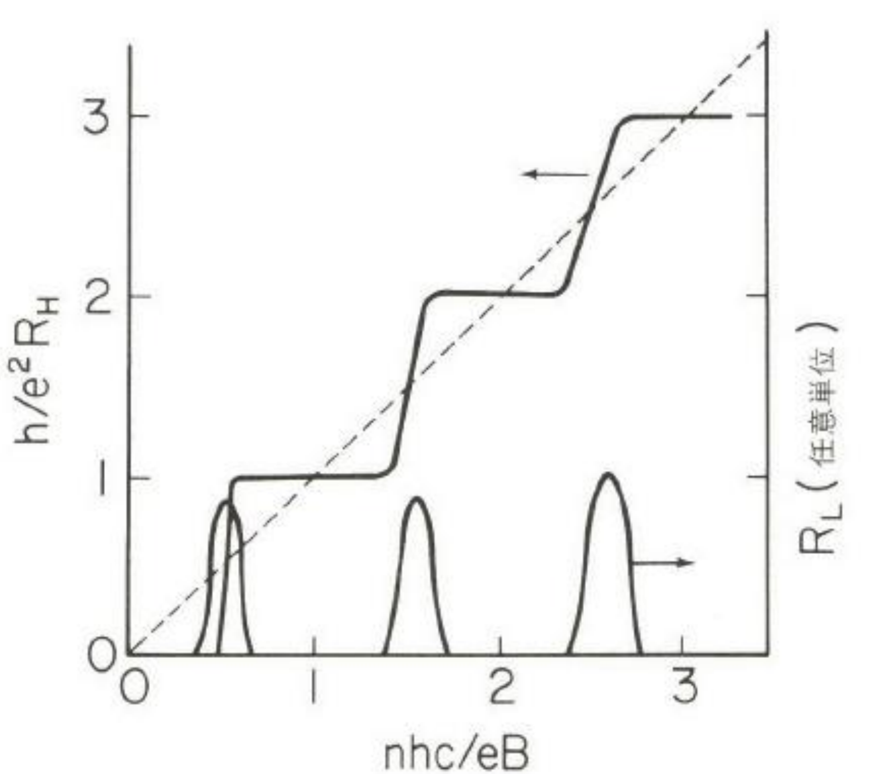


Quantum Hall effect

Classical periodic orbit



1st quantization



2nd quantization

Fractional QHE

Landau levels + disorder

Quantum charged particles in an oscillating magnetic fields

TO, Bucciantini *in progress*

$$H(p_x, p_y - qA_y; t) = \frac{1}{2m_e}\left[p_x^2 + \left(p_y - qA_y - \frac{e}{c}B_z(t)x)^2\right)^2\right]$$

= driven Harmonic oscillator with an oscillating potential

(i) Quantum oscillator without driving

$$H(t) = \frac{p_x^2}{2m_e} + \frac{m_e\omega^2(t)}{2}x^2$$

$$\omega(t) = \omega_c \cos\Omega t$$

+

(ii) Classical driven oscillator

$$m_e\ddot{X} + m_e\omega(t)^2 X = S(t)$$

$$S(t) = \omega(t)(\hbar k_y - qA_y)$$

Husimi (Taniuti) PTP '53

solution

$$\Psi_n(\boldsymbol{x}, t) = e^{-\frac{i}{\hbar}E_n t}e^{ik_y y}\varphi_n(x - X(t), t)\exp\left[\frac{i}{\hbar}\{m_e\dot{X}(t)(x - X(t)) + \int_0^t dt' L(t') - L_0 t\}\right]$$

$$L = \frac{1}{2}m_e\dot{X}^2 - \frac{1}{2}m_e\omega(t)^2X^2 + XF(t)$$

pseudo-energy

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m^*}$$

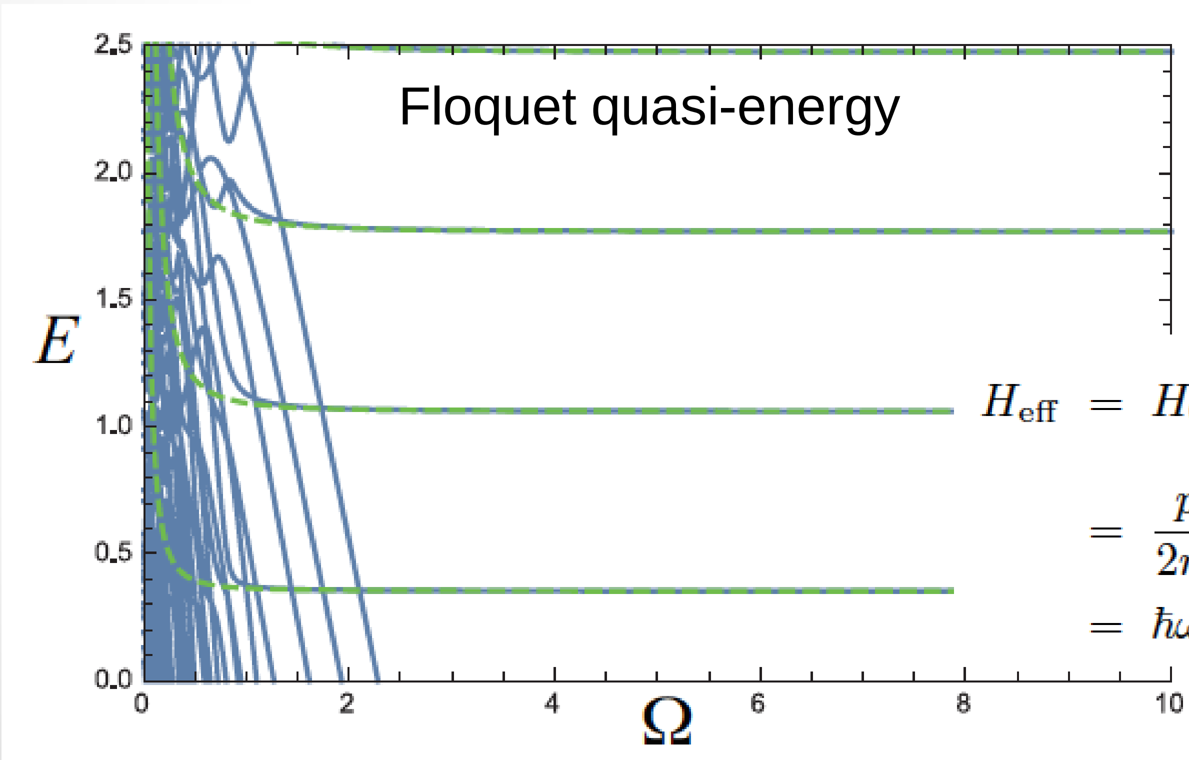
$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{1}{T}\int_0^T L(t')dt'$$

(i) Quantum oscillator without driving

$$H(t) = \frac{p_x^2}{2m_e} + \frac{m_e\omega^2(t)}{2}x^2$$

$$\omega(t) = \omega_c \cos \Omega t$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\varphi_i(t) = E_i\varphi_i(t)$$



high-frequency expansion

$$\begin{aligned} H_{\rm eff} &= H_0 + \frac{[[H_{-2}, H_0], H_2]}{(2\Omega)^2} + \mathcal{O}(\frac{1}{\Omega^4}) \\ &= \frac{p^2}{2m_e} + \frac{m_e\bar{\omega}^2}{2}\left(1 + \frac{1}{8}\left(\frac{\bar{\omega}}{\Omega}\right)^2\right)x^2 \\ &= \hbar\omega_{\rm eff}(\Omega)(n + 1/2) \end{aligned}$$

Harmonic oscillator with a renormalized potential

(ii) Classical driven oscillator

$$m_e \ddot{X} + m_e \omega(t)^2 X = S(t)$$

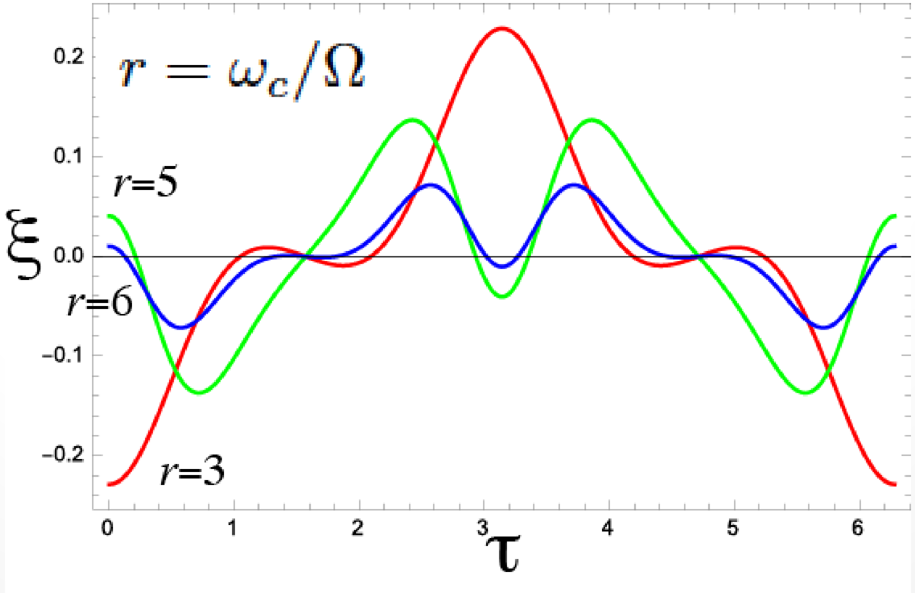
$$S(t) = \omega(t)(\hbar k_y - qA_y)$$

Mathieu's differential equation with a source term

$$\frac{d^2\xi}{d\tau^2} + (a - 2q\cos 2\tau)\xi = -\cos\tau$$

$$X(t) = -\left(\frac{\omega_c p_y}{\Omega^2 m_e}\right)\xi(\Omega t) = -(l_B r)^2 \xi(\Omega t) k_y$$

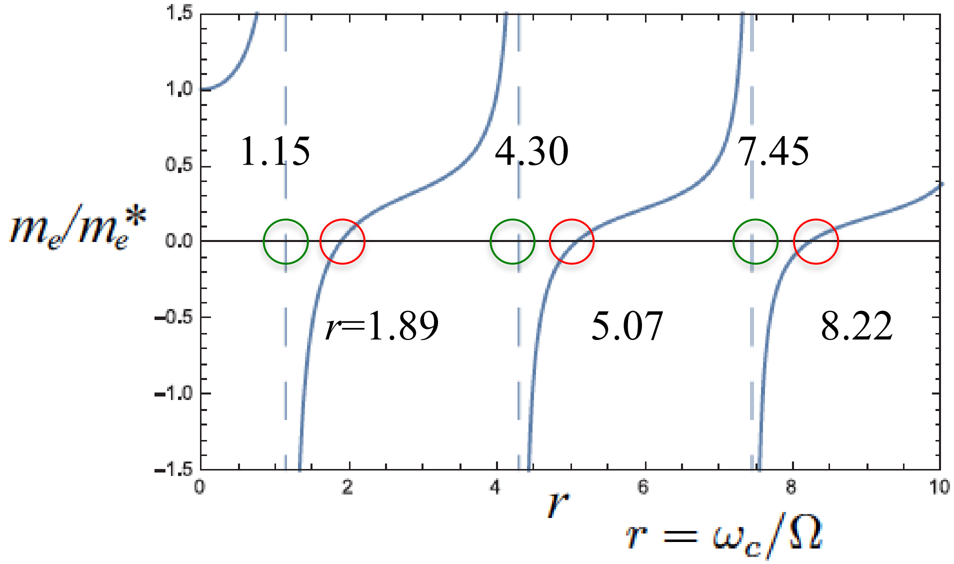
"snake state": trajectory of the wave center

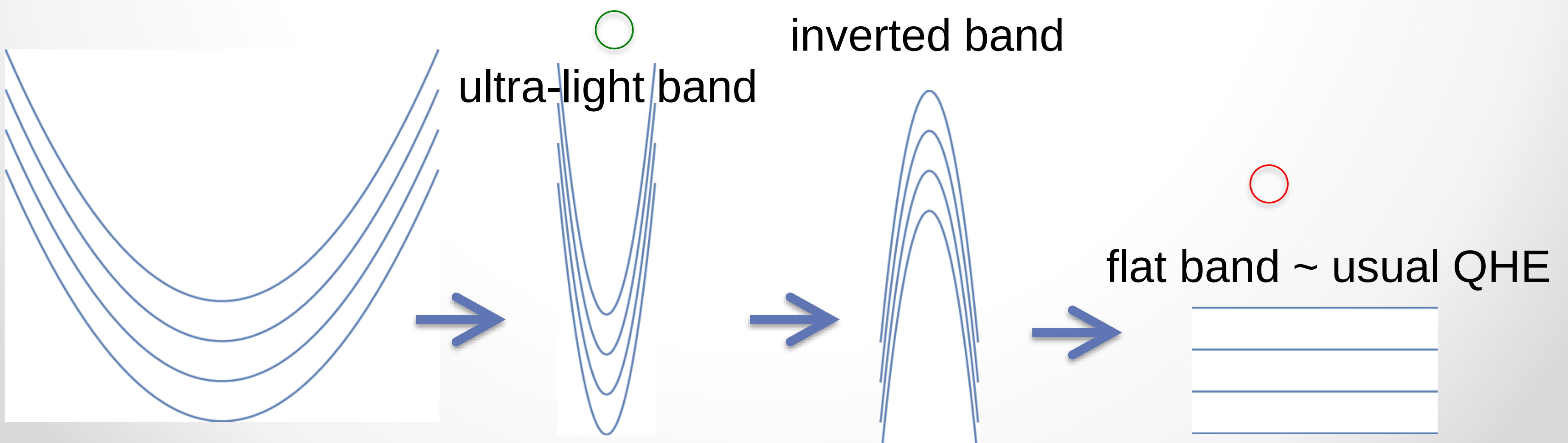


Spectrum

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m^*} \qquad E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{1}{T}\int_0^T L(t')dt'$$

Effective mass





Effect of electric fields

$$H(p_x, p_y; t) = \frac{1}{2m_e}\left[p_x^2 + \left(p_y - \frac{q}{c}B_z(t)x\right)^2\right] + qE_x(t)x$$

$$= \frac{p_x^2}{2m_e} + \frac{p_y^2}{2m_e} + \frac{m_e(\omega(t))^2}{2}x^2 - \omega(t)p_y x + qE_x(t)x$$

ac-field

$$E_x = E_x^1 \cos \Omega t$$

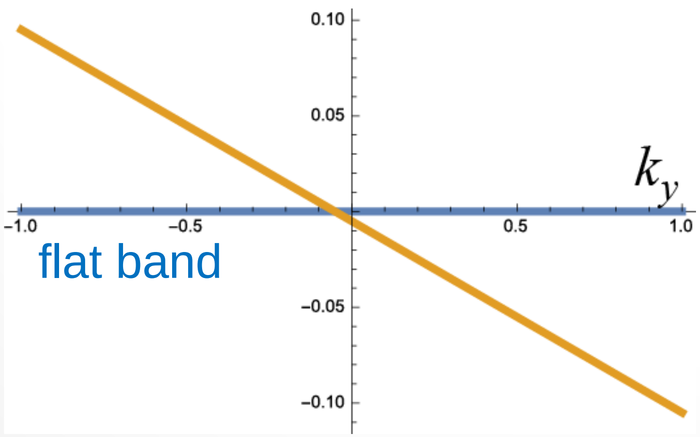
shift of k_y momentum

$$k_y \to k_y + \frac{eE_x^1}{\hbar\omega_c}$$

deformation of spectrum

$$E_n(p_y) = \varepsilon_n + \frac{p_y^2}{2m^*} \to E_n(p_y) = \varepsilon_n + \frac{p_y^2}{2m_e} + \frac{1}{2m_e}\left(\frac{m_e}{m^*} - 1\right)\left(p_y + \frac{eE_x^1}{\omega_c}\right)^2$$

linear dispersion (finite E_x^1)



current (for LL filling N)

$$j_y = \frac{q^2}{h}N\frac{\left(\frac{m_e}{m^*} - 1\right)}{2r^2 \max(\xi(\Omega t))}E_x^1$$

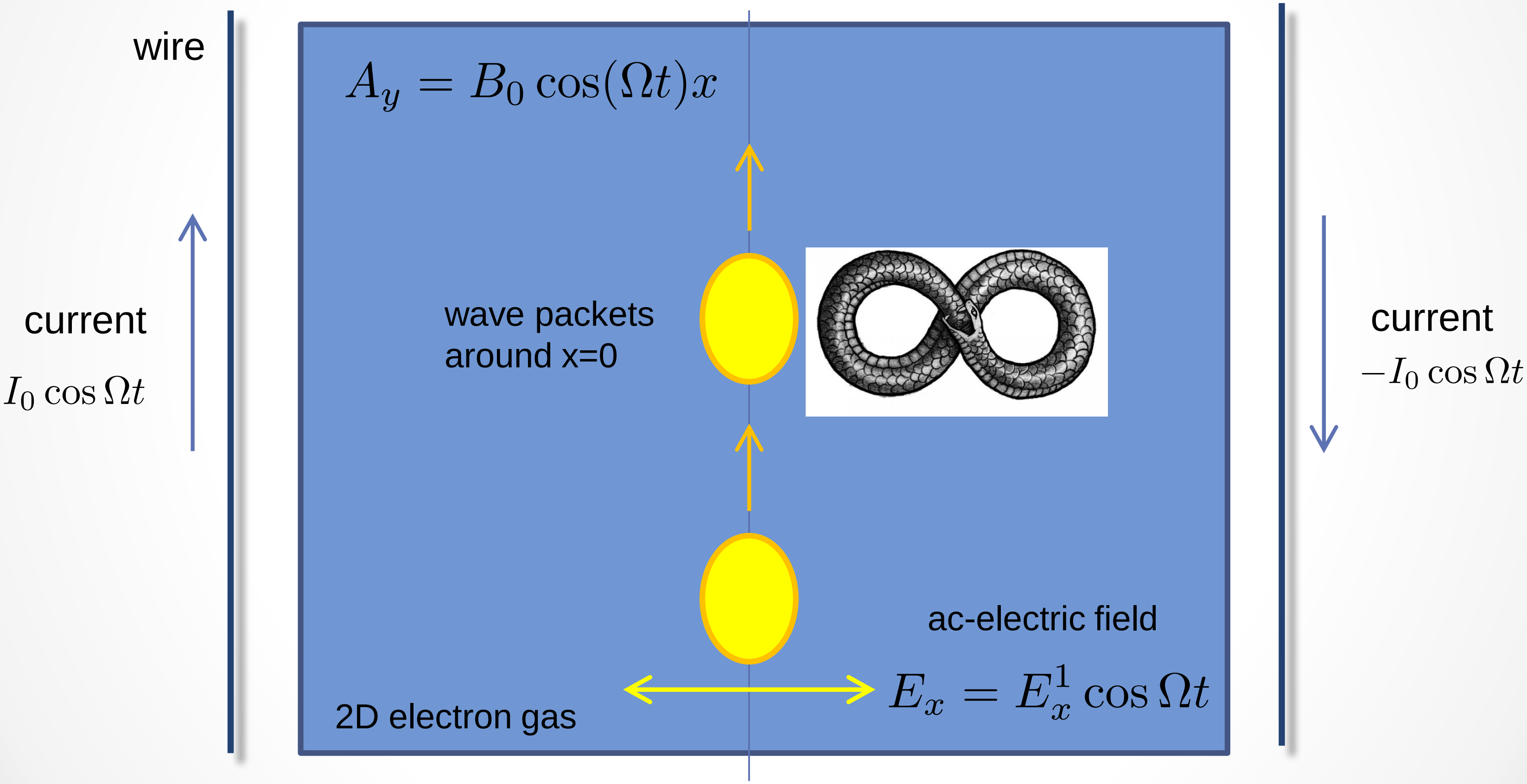
cf) integer QHE

$$j_y = \frac{q^2}{h}NE_x^{\rm dc}$$

Conclusion and outlook

quantization condition (flat band condition)

$$r = \omega_c/\Omega = 1.89, 5.07, 8.22\ldots \qquad \omega_c = qB/m_e c$$



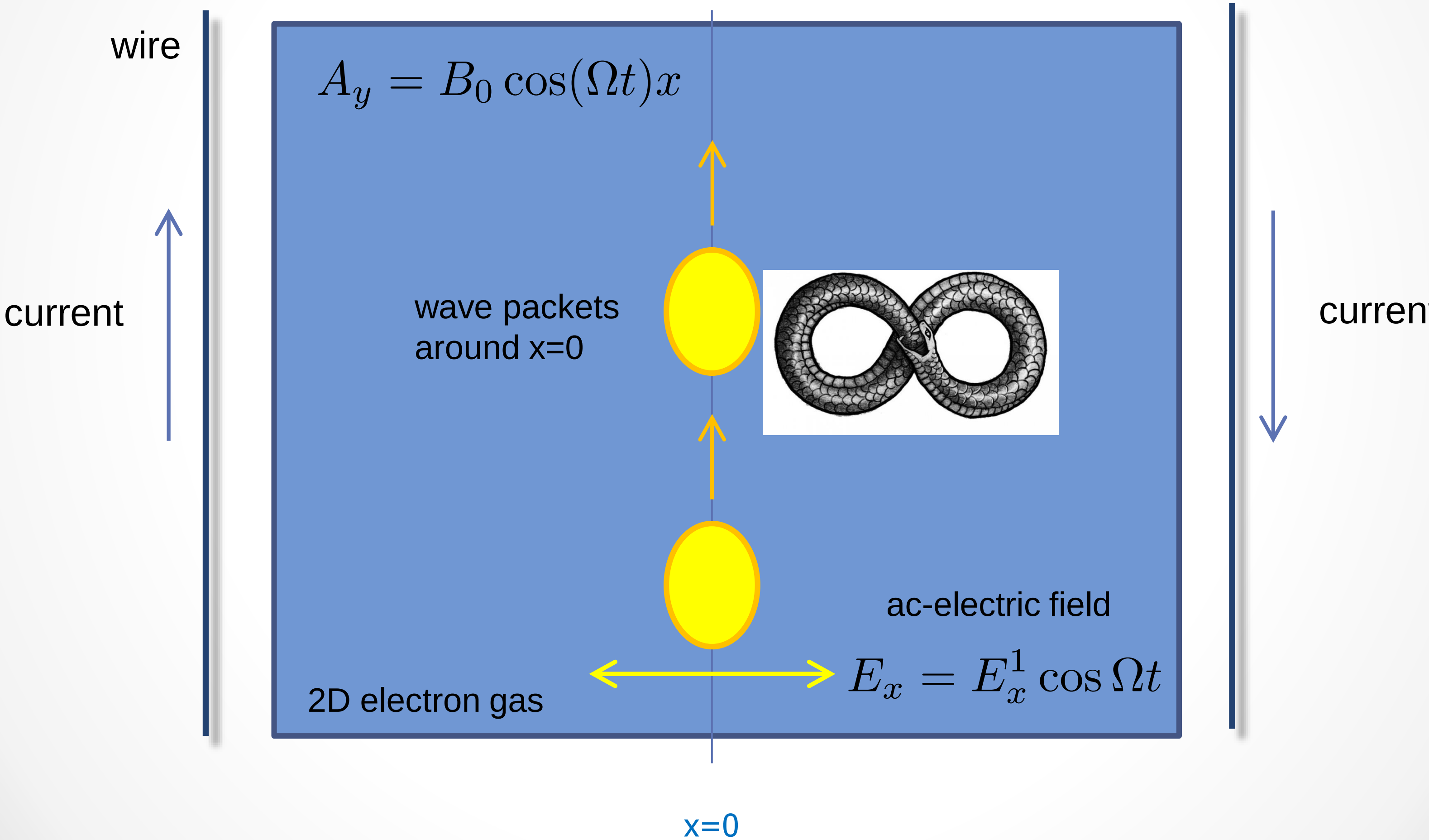
Question: Effect of Coulomb interaction

Wigner crystal? fractional QHE??

Conclusion

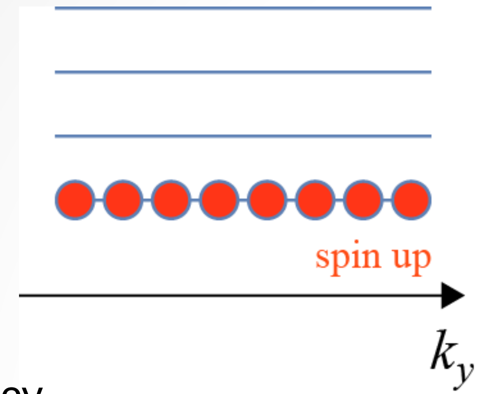
quantization condition (flat band condition)

$$r = \omega_c/\Omega = 1.89, 5.07, 8.22 \ldots \qquad \omega_c = qB/m_e c$$

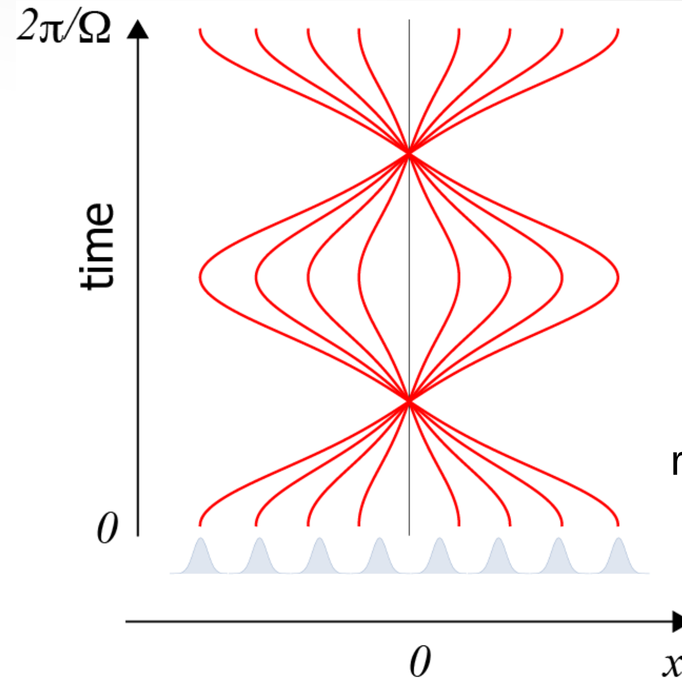


heterodyne: ac-field is converted to dc-current (rectification)

Many-body state



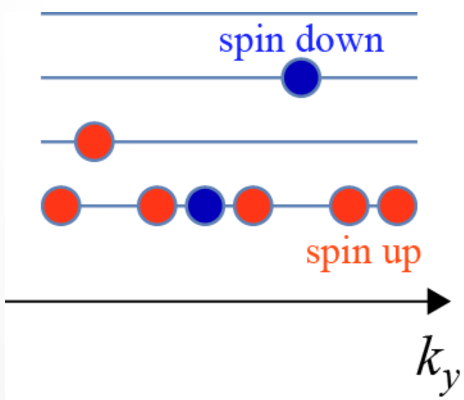
↔



r=1.89

degeneracy

$$N_\phi = \frac{L_x L_y}{2\pi l_B^2 r^2 \max(\xi(\Omega t))}$$



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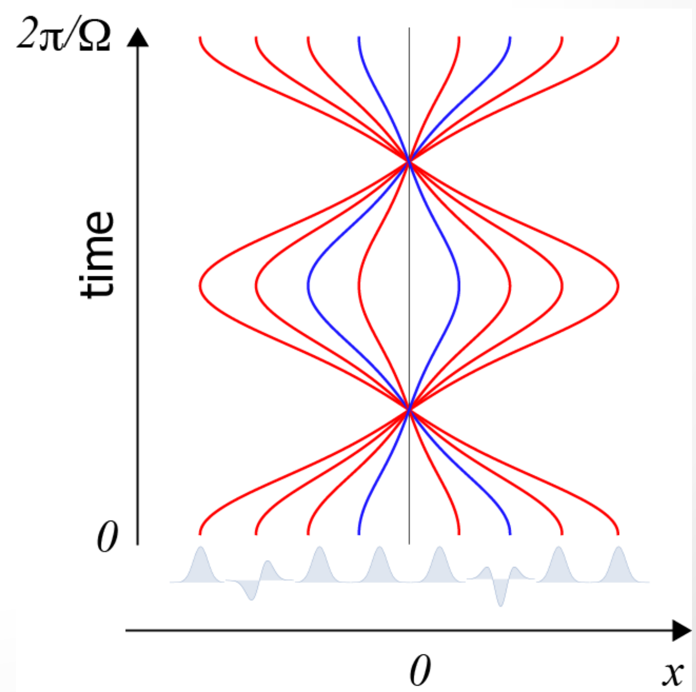


All electrons approaches the origin (x=0)

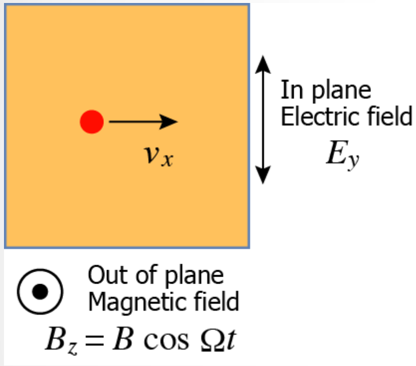
strongly interacting problem

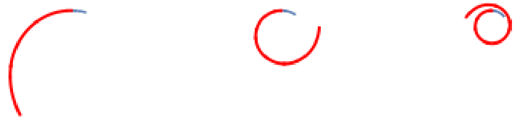
Summary and future directions

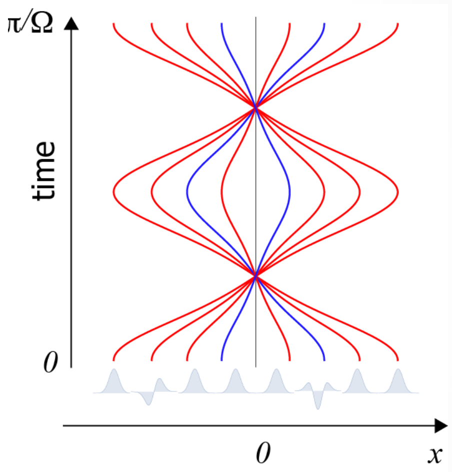
2D particles in oscillating magnetic fields shows interesting behaviors

classical

quantum







- Kubo formula for heterodyne conductivity
- Streda formula
- disorder (robustness)
- interaction (Fractional state?)

Effect of electric fields 1

$$E_y = E_y^1 \sin \Omega t \qquad A_y = \frac{E_y^1}{\Omega} \cos \Omega t$$

Classical driven oscillator

$$m_e \ddot{X} + m_e \omega(t)^2 X = S(t)$$

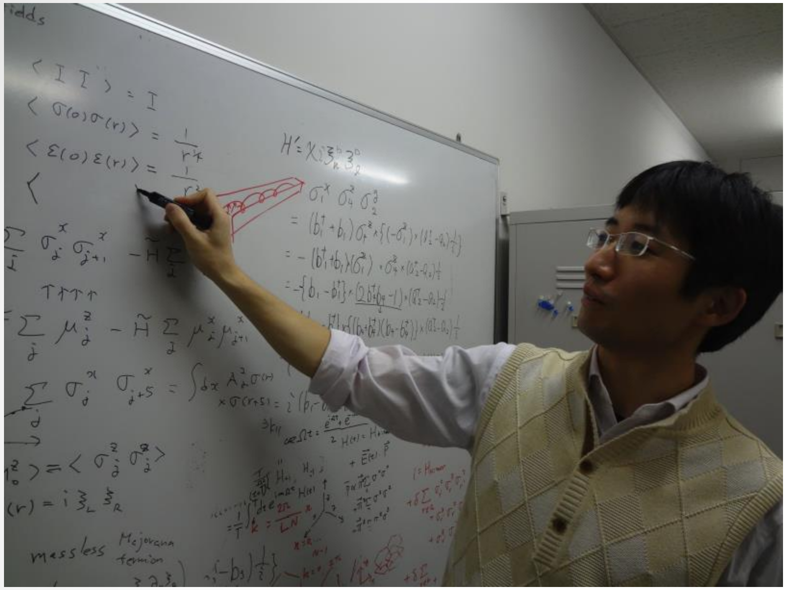
$$S(t) = \omega(t)(\hbar k_y - q A_y)$$

shift of position

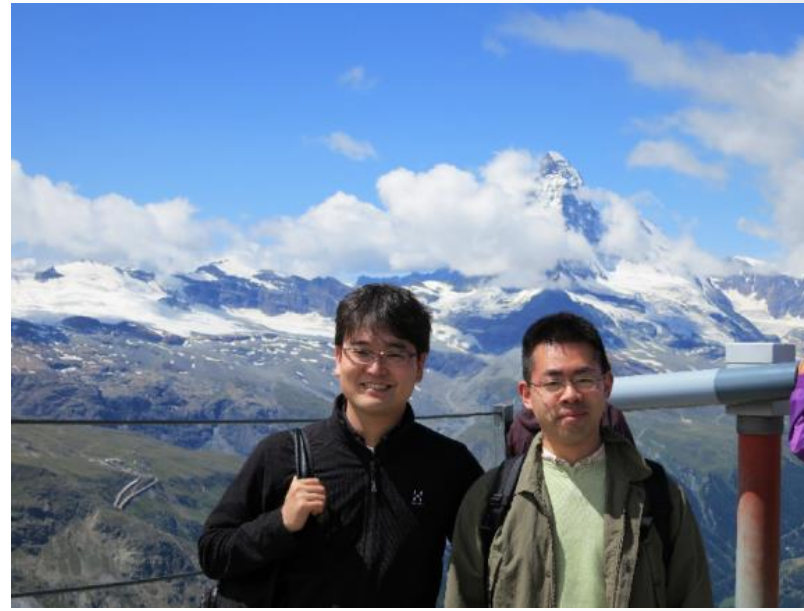
$$X \to X - \frac{c E_y^1}{B \Omega}$$

no effect in the current

Application to quantum magnets



M. Sato (Aoyama Gakuin U.
→Japan Atomic Energy Agency)

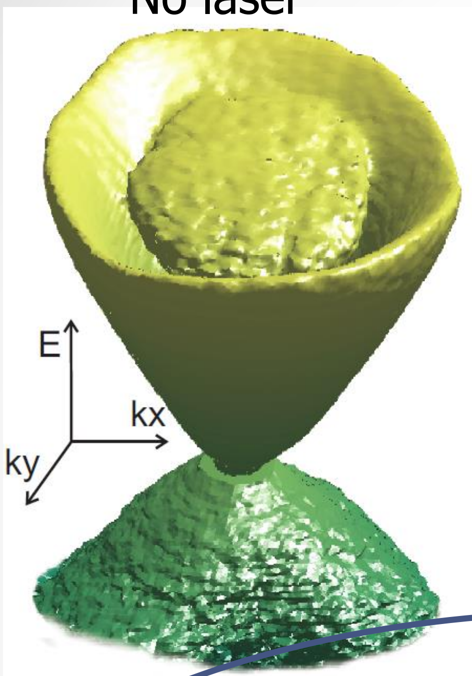


S. Takayoshi
(U-Tokyo→NIMS→U-Tokyo→U-Geneva)

Experiment using time resolved ARPES

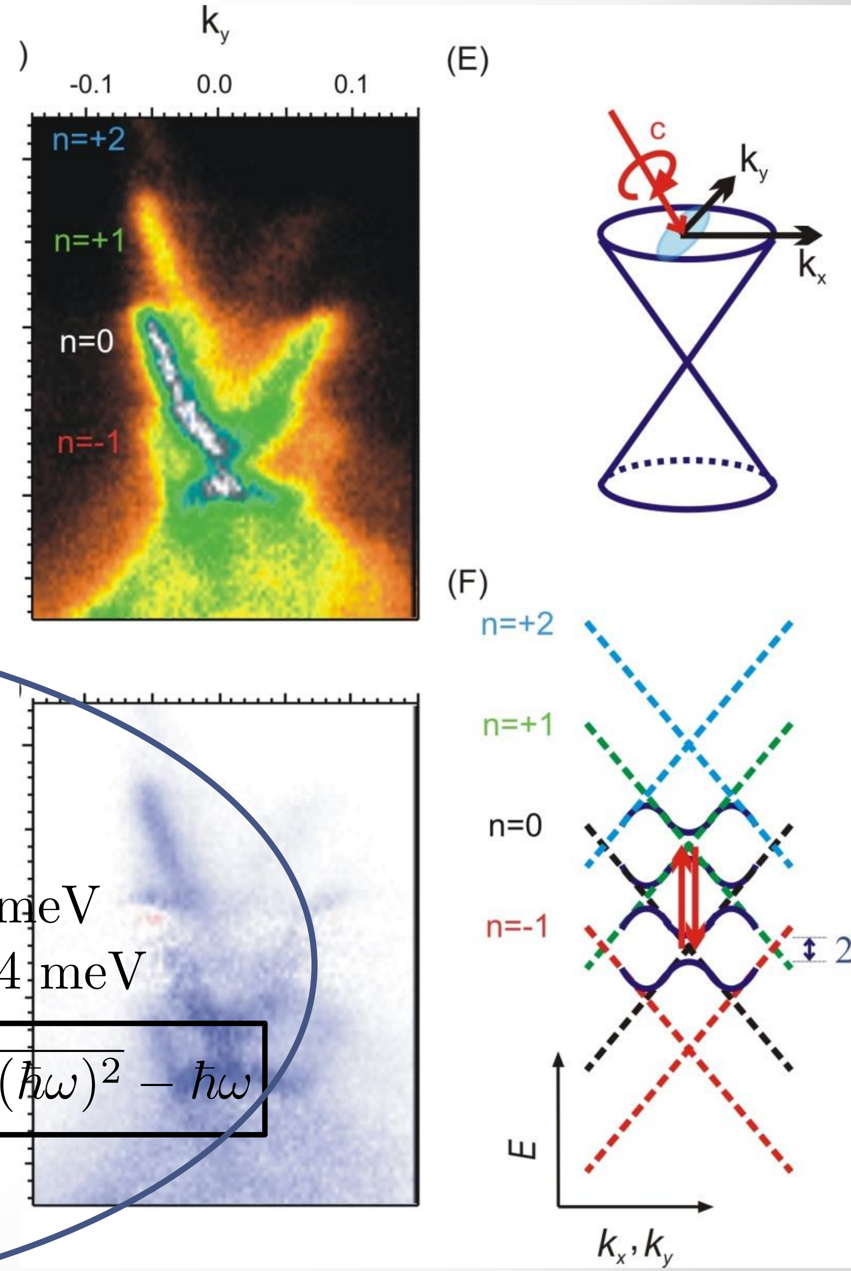
surface Dirac state of a TI

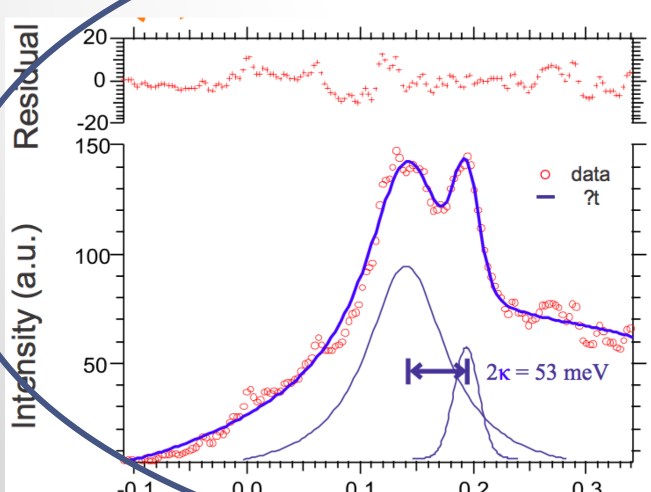
No laser



circularly polarized laser

Gedik (MIT) Science '13





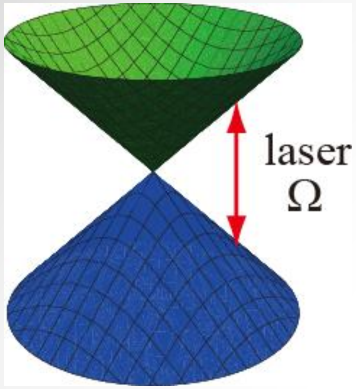
Exp.: $2\kappa = 53$ meV

Theory: $2\kappa = 54$ meV

$$2\kappa = \sqrt{4V^2 + (\hbar\omega)^2} - \hbar\omega$$

TO, Aoki '09

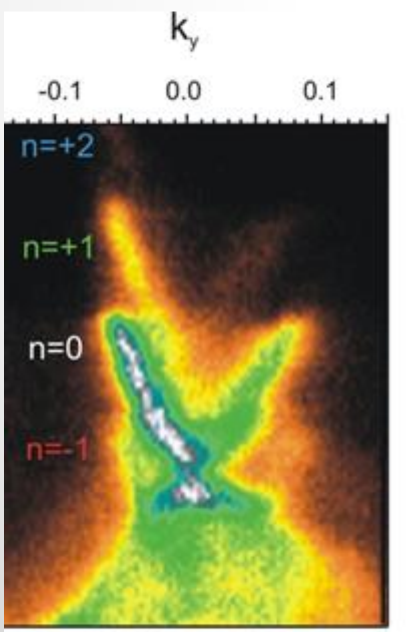
Floquet spectrum: Dirac model + circularly polarized laser



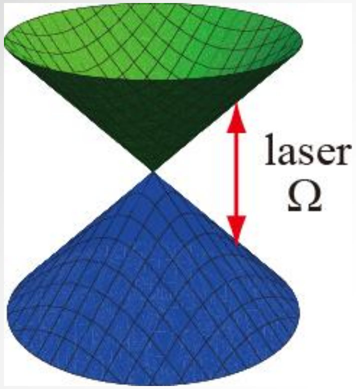
$$H_{\mathrm{Dirac}} = \begin{pmatrix} 0 & k \\ \bar{k} & 0 \end{pmatrix}$$

$k = k_x + ik_y$

??



Floquet spectrum: Dirac model + circularly polarized laser



coupling to AC field

$$\boldsymbol{k} \rightarrow \boldsymbol{k} + \boldsymbol{A}(t)$$

$$k = k_x + ik_y$$
$$\boldsymbol{A}(t) = (F/\Omega \cos \Omega t, F/\Omega \sin \Omega t)$$
$$A = F/\Omega$$

time dependent Schrodinger equation

$$i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k$$

Floquet theory

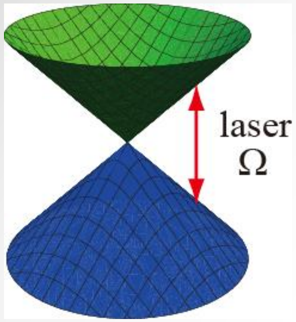
$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

truncated at m=0,+1, -1 for display

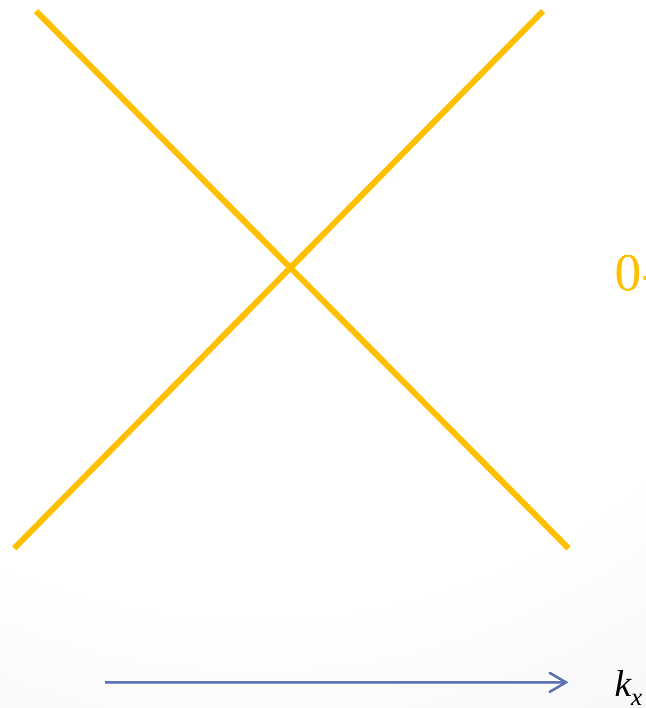
Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009



$$H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

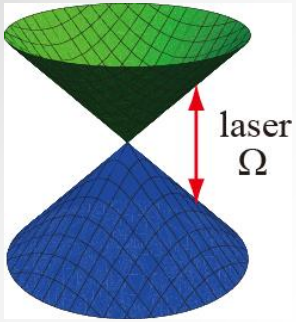
0-photon absorbed state



0-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

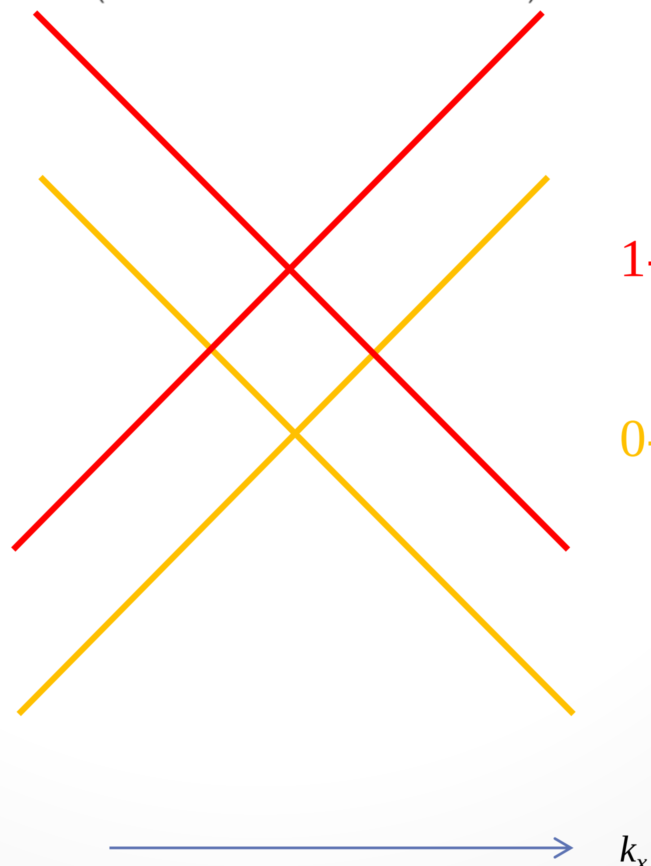
TO, Aoki 2009



$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

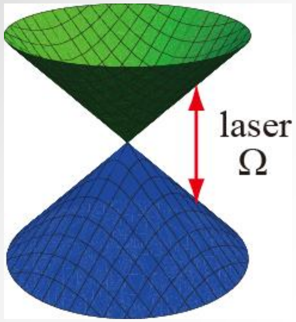


1-photon absorbed state

0-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009

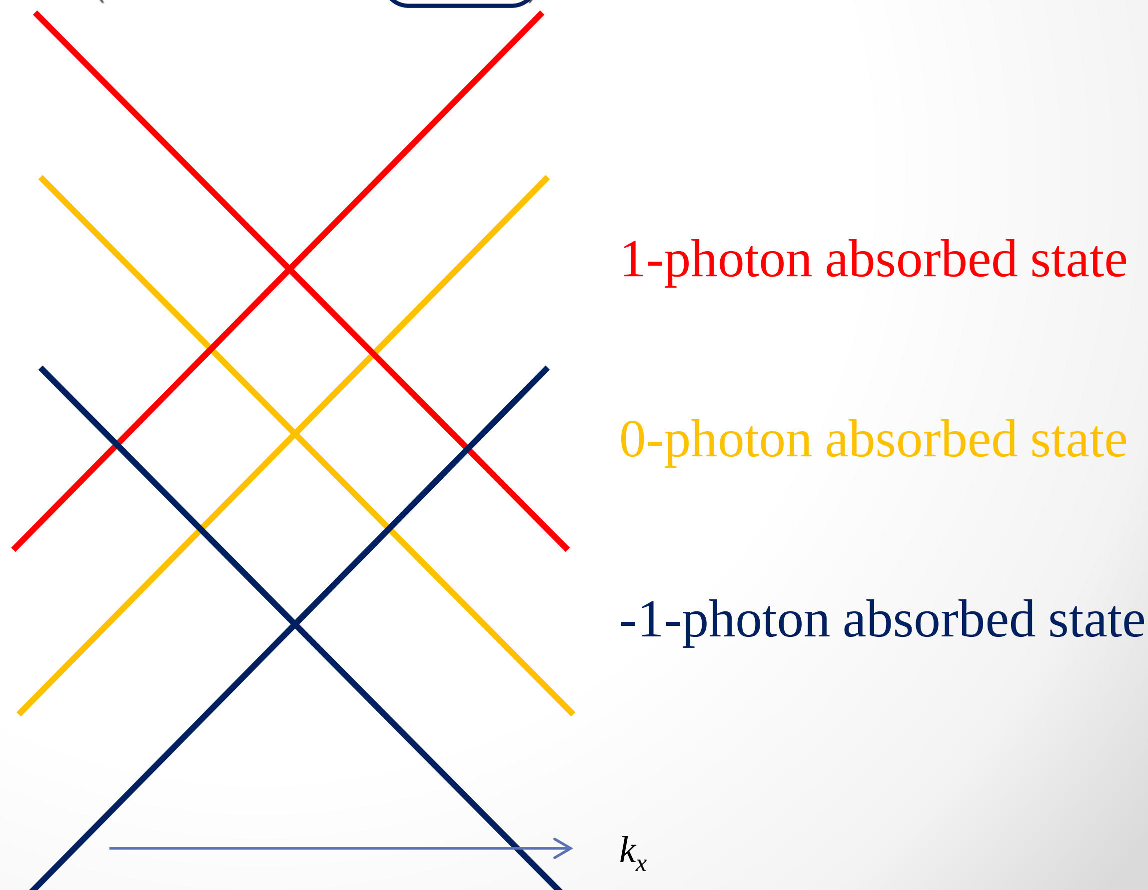


$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

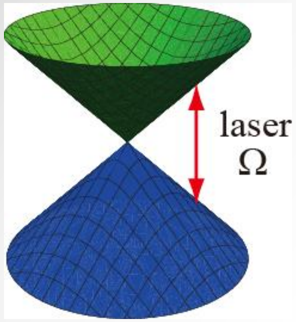
0-photon absorbed state

-1-photon absorbed state



Floquet spectrum: Dirac model + circularly polarized laser

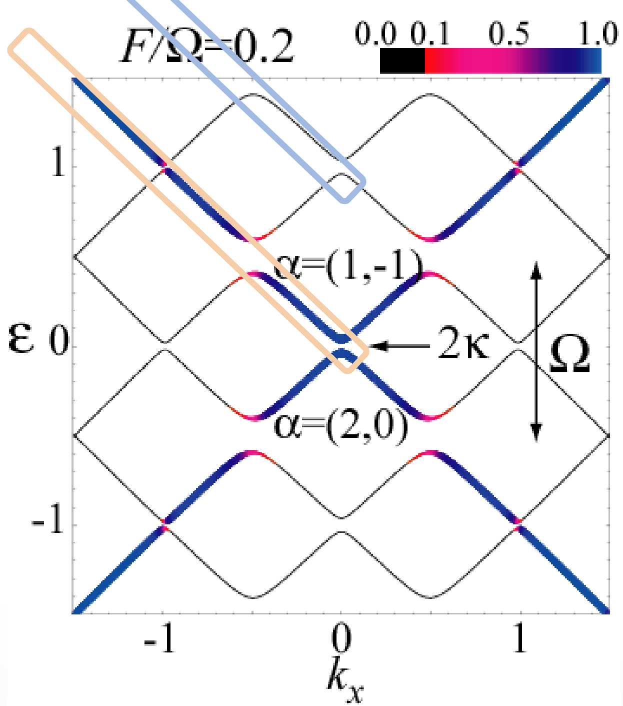
TO, Aoki 2009

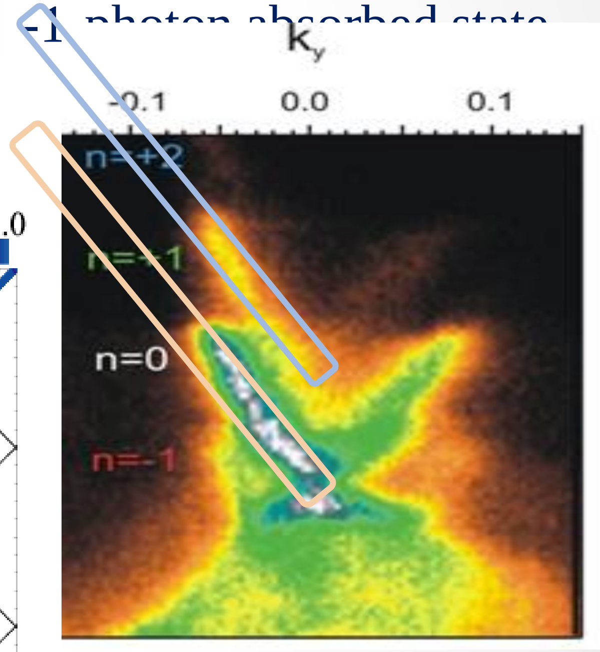


$$H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

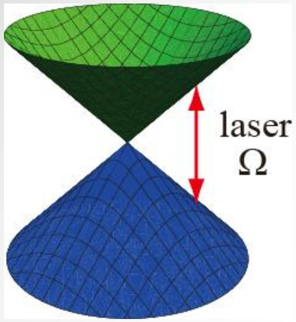




-1-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009

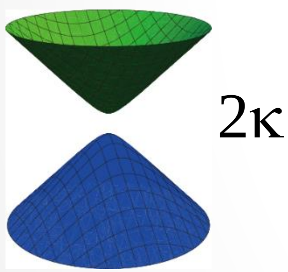


$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

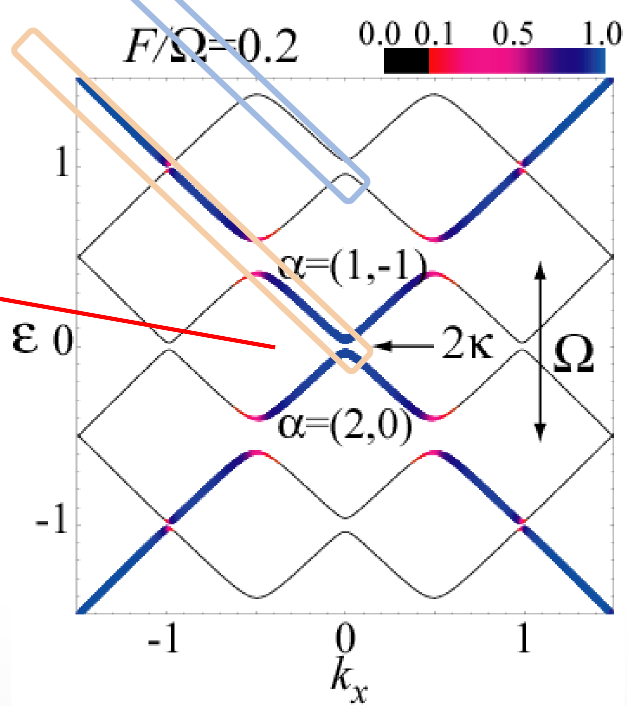
0-photon absorbed state

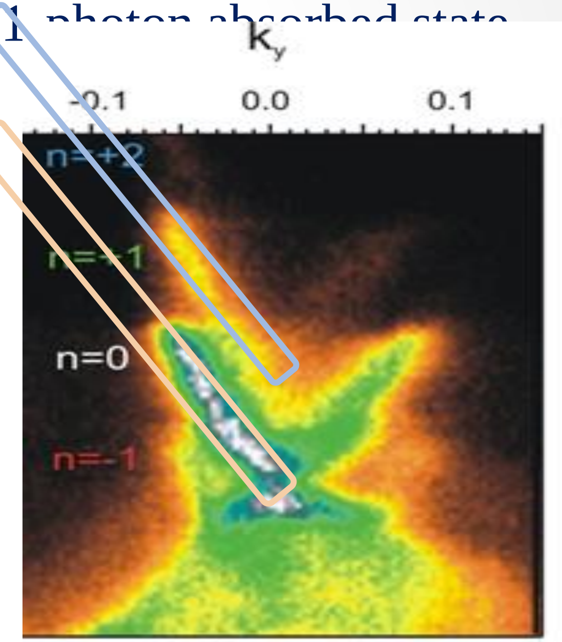
near Dirac point



Dirac gap

$$\kappa = \frac{\sqrt{4A^2+\Omega^2}-\Omega}{2} \sim A^2/\Omega$$





-1-photon absorbed state

Floquet topological insulator

TO, Aoki '09
Kitagawa, TO, Fu, Brataas, Demler '11

2D Dirac electron in circularly polarized laser

$$H(t) = k_x\sigma_x + k_y\sigma_y + Ae^{i\Omega t}\sigma_- + Ae^{-i\Omega t}\sigma_+$$

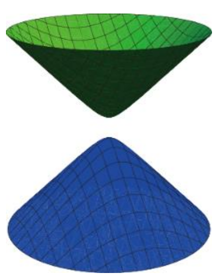
1st order

$$\sim A\sigma_- \qquad \sim A\sigma_+$$

$$H_{\rm eff} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

$$= k_x\sigma_x + k_y\sigma_y + \frac{A^2}{\Omega}\sigma_z + \dots$$

Dirac point

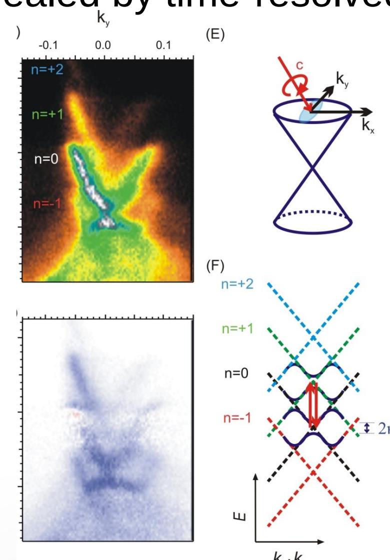


2κ

Dynamical gap

$$\kappa = \frac{\sqrt{4A^2+\Omega^2}-\Omega}{2} \sim A^2/\Omega$$

revealed by time-resolved ARPES

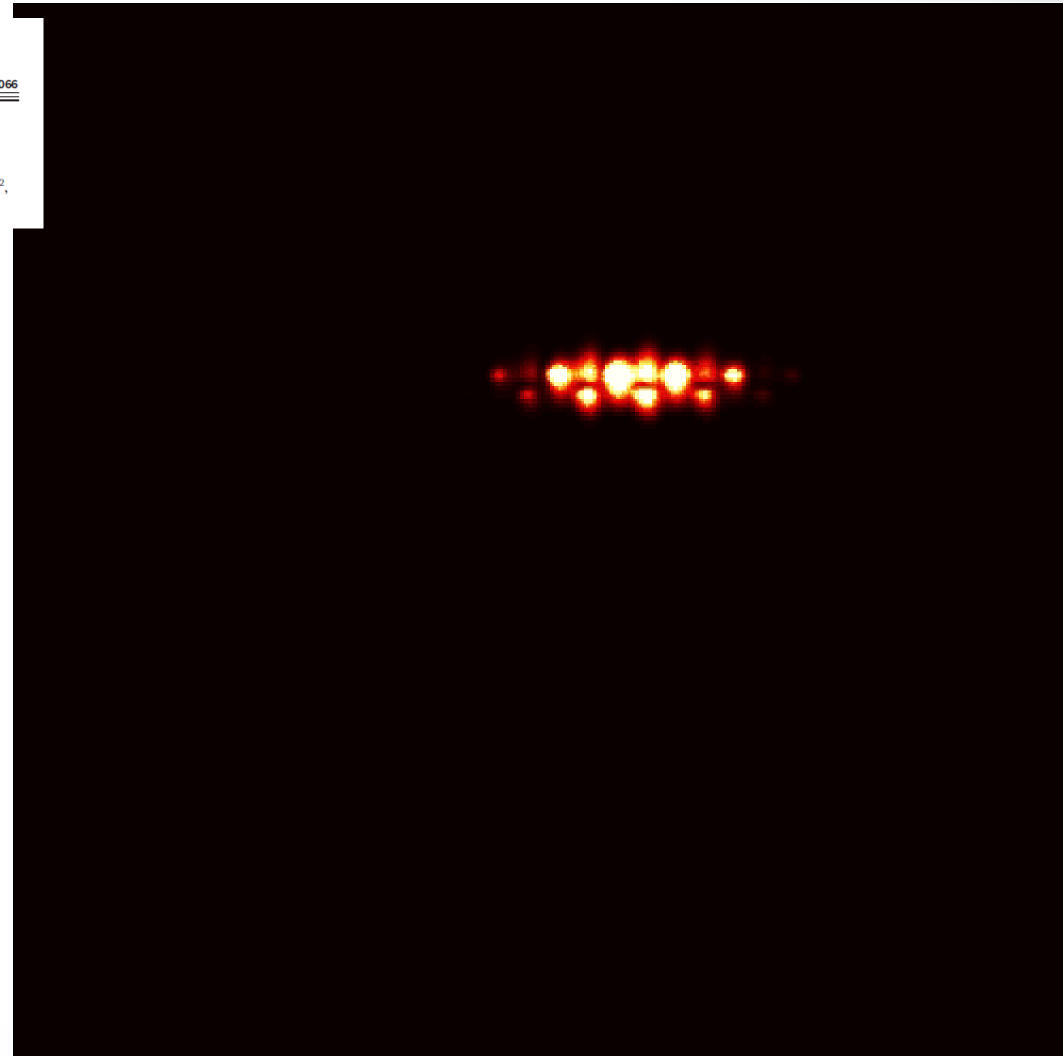


Wang, Gedik et al. (MIT) Science'13

Other experiments

Photonic Floquet topological band

Realization in photonic crystals



LETTER

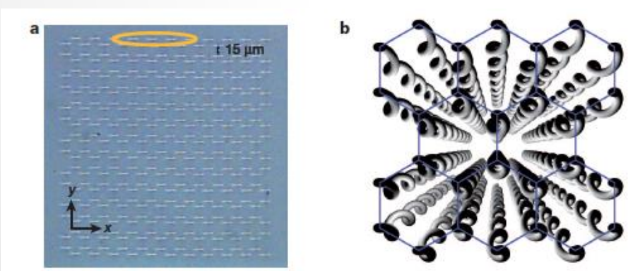
doi:10.1038/nature12066

Photonic Floquet topological insulators

Mikael C. Rechtsman[1]*, Julia M. Zeuner[2]*, Yonatan Plotnik[1]*, Yaakov Lumer[1], Daniel Podolsky[1], Felix Dreisow[2], Stefan Nolte[2], Mordechai Segev[1] & Alexander Szameit[2]

Rechtsman *et al.* Nature '13

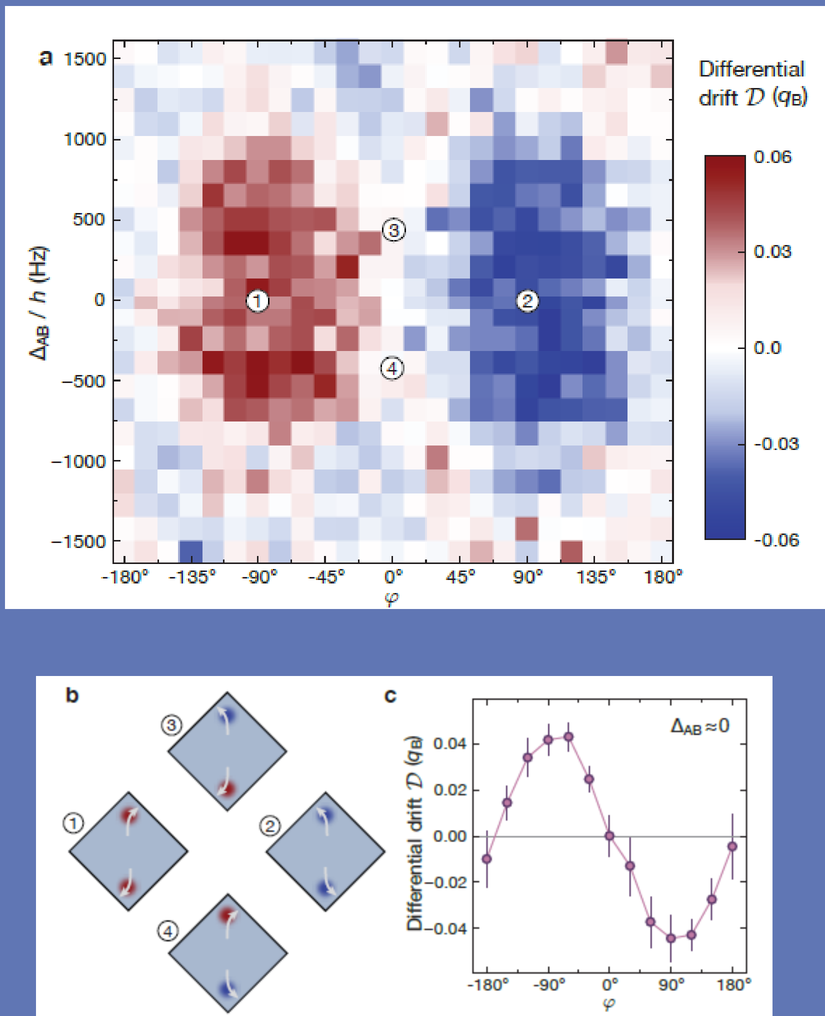
Haifa & Abbe center of photonics



Use z-direction as the "time" axes

doi:10.1038/nature13915

Drift measurement ~ conductivity

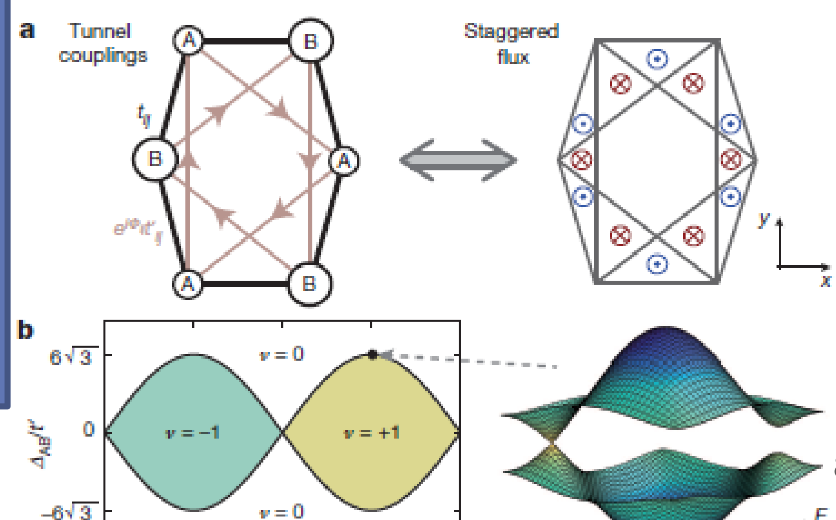


f the topological Haldane
ions

[1], Thomas Uehlinger[1], Daniel Greif[1] & Tilman Esslinger[1]

ETH group, Nature '14

tunnelling[13]. In higher dimensions this allowed the study of phase transitions[14,15], and topologically trivial staggered fluxes were realized[16,17]. Furthermore, uniform flux configurations were observed using rotation and laser-assisted tunnelling[18,19], although for the latter method, heating seemed to prevent the observation of a flux in some experiments[20]. In a honeycomb lattice, a rotating force, as proposed by T. Oka and H. Aoki, can induce the required complex tunnelling[7]. Using arrays of coupled waveguides, a classical version of this proposal was used to study topologically protected edge modes in the inversion-symmetric regime[21]. We



transition between topologically distinct regimes. By identifying the vanishing gap at a single Dirac point, we map out this transition line experimentally and quantitatively compare it to calculations using Floquet theory without free parameters. We verify that our approach

current status

non-interacting examples

Floquet Chern insulator

TO-Aoki PRB'09

Floquet Weyl semimetal (2D to 3D)

R. Wang, *et al*. EPL '14
(Ebihara-TO-Fukushima '15),

No! →

many-body examples

superfluid-Mott transition
Eckardt-Weiss-Holthaus PRL '05

Floquet fractional Chern insulator

Grushin-Gomez-Leon-Neupert PRL '14

dynamical Cooper paring

Knap-Babadi-Rafael-Martin-Demler '15

No! →

closed system

no go theorem

Alessio-Rigol PRX'15

$$\dot{C} = 0$$

thermalization (heats up to T=∞)

Lazarides-Das-Moessner, PRL '15

$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

only valid for short time

Kuwahara-Mori-Saito '15
Abanin

open system/ many-body

Floquet Landauer-Buttiker

(Moskalets-Buttiker '02), Kitagawa-TO-Brataas-Fu-Demler '11

Floquet Dynamical mean field theory

(Tsuji-TO-Aoki '08), Mikami *et al*. '16

Holographic Floquet Weyl semimetal

Hashimoto-Kinoshita-Murata-TO *in prep*

Floquet master eq. (phonon)

Hossein-TO-Mitra '14, '15
Hossein-Mitra '16

Phase diagram of laser induced topological state
(C: Chern number = # of edge channels)

Mikami *et al. PRB* ’16
Kundu, et al. PRL ‘14