Entanglement generation in periodically driven integrable systems

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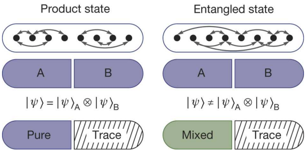
Current Frontiers in Condensed Matter Research, ICTS

Plan of the talk

- Ground states versus highly excited states of local Hamiltonians
- Steady states of driven systems
- Approach to steady state
- Dynamic phase transition
- Floquet Hamiltonian: Explaining the transition
- Entanglement properties of the steady state
- Conclusions and future directions

Entanglement

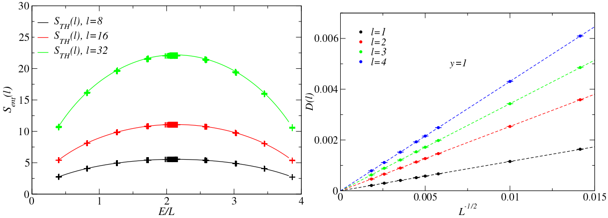


R. Islam et al, Nature (2015)

- Entanglement entropy of ground states of many-body Hamiltonian follow area law (M. Hastings) $\rightarrow S = -\mathrm{Tr}[\rho_l \log(\rho_l)] \sim l^{d-1}$
- May lead to classification of states with *non-local* order (subleading terms in entanglement entropy).
- Universal behaviour at higher-dimensional critical points? [in 1D, central charge of the associated CFT]
- Typical high-energy eigenstates however follow the volume law $S \sim l^d$ (ETH, Deutsch, Srednicki, Rigol+Dunjko+Olshanii, Kim+Ikeda+Huse)

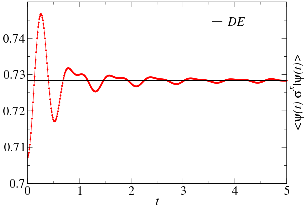
High energy eigenstates of the TFIM

- We use unbiased statistical sampling of eigenstates at a given finite energy density to get local properties of *typical* eigenstates [S. Nandy, A. Sen, A. Das, A. Dhar, arXiv:1605.09225].



- Local properties of the typical eigenstates follow a *Gibbs ensemble* in-spite of the integrability of the model!
- Atypical eigenstates follow appropriate (truncated) Generalized Gibbs ensembles locally.

Driven systems: Steady state



- Does an ensemble description exist for steady states of driven quantum systems? Guiding principles?
- Lots of progress in recent years for quenches [Rigol, Dunjko, Olshanii (2008)] and periodically driven systems [Lazarides, Das, Moessner (2014)]
- Steady state description for generic drives still an open issue.

Class of integrable models

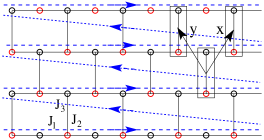
- In 1D, transverse field Ising model (TFIM)

$$H = -\sum_j (h\sigma_j^x + \sigma_j^z \sigma_{j+1}^z)$$

- In 2D, Kitaev model—

$$H_{\mathrm{2D}} \;=\; \sum_{j+l=\mathrm{even}} (J_1 \sigma_{j,l}^x \sigma_{j+1,l}^x + J_2 \sigma_{j-1,l}^y \sigma_{j,l}^y + J_3 \sigma_{j,l}^z \sigma_{j,l+1}^z)$$

(see Chen+Nussinov, 2008)



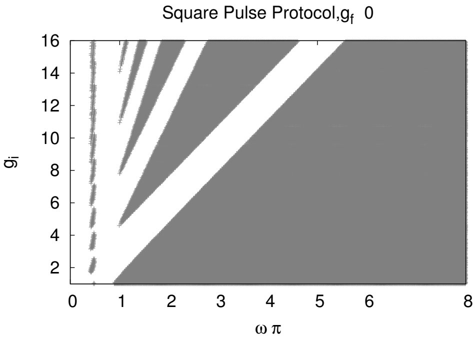
- Jordan-Wigner transformation: [lectures by D. Sen]

$$\sigma_n^x = 1 - 2c_n^\dagger c_n \quad \sigma_n^z = -(c_n + c_n^\dagger) \prod (1 - 2c_m^\dagger c_m),$$

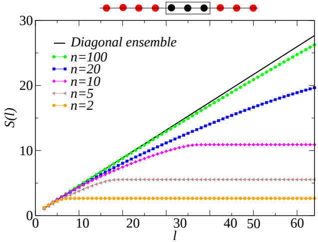
Approach to steady state: Dynamic transition

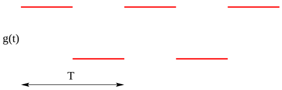
- Hamiltonian connects $|\uparrow\rangle_{\vec{k}} = c^{\dagger}_{\vec{k}} c^{\dagger}_{-\vec{k}} |0\rangle$ with $|\downarrow\rangle_{\vec{k}} = |0\rangle$ where $|0\rangle$ denotes vacuum of c fermions, and $c^{\dagger}_{k}|0\rangle$ with $c^{\dagger}_{-k}|0\rangle$.
- Dynamics through $H_{\vec{k}} = (g(t) - b_{\vec{k}})\tau_3 + \Delta_{\vec{k}}\tau_1$.
  Need to find $(u_{\vec{k}}(t), v_{\vec{k}}(t))^T$.
- Choose $g(t)$ to be a periodic function of time.
- Subject of this talk:Relaxational behaviour of entanglement entropy and local (in space) observables to the final ($n \to \infty$) steady state values, where n is number of drive cycles.

Some results



Entanglement generation after n drive cycles





- Need the knowledge of two $l \times l$ matrices (Peschel et. al.)–

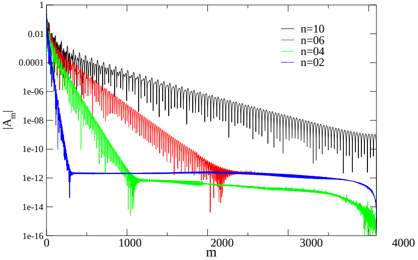
$$
\begin{aligned}
C_{ij} &= \langle c^{\dagger}_{\vec{i}} c_{\vec{j}} \rangle_n = 2 \sum_{\vec{k} \in \mathrm{BZ}/2} |u_{\vec{k}}(t)|^2 \cos(\vec{k} \cdot (\vec{i} - \vec{j}))/L^d \\
F_{ij} &= \langle c^{\dagger}_{\vec{i}} c^{\dagger}_{\vec{j}} \rangle_n = 2 \sum_{\vec{k} \in \mathrm{BZ}/2} u^{*}_{\vec{k}}(t) v_{\vec{k}}(t) \sin(\vec{k} \cdot (\vec{i} - \vec{j}))/L^d
\end{aligned}
$$

- $S(l)$ satisfies area law for small n. This length scale, however, diverges as $n \to \infty$. (some similarities to quench in non-integrable models, Huse et al)

Qualitative criteria for a non-area law

- Hastings' theorem applicable for ground states of local Hamiltonians
- However, $|\psi(t)\rangle$ is not the ground state of $H(t)$
- Construct $\mathcal{H}_t$ for which $|\psi(t)\rangle$ is the ground state.
- Has the form $\mathcal{H}_t = \epsilon_{kt}\tau_3 + \Delta_{kt}\tau^+ + \Delta^*_{kt}\tau^-$
- Demand the correct form for $|\psi(t)\rangle$ in the adiabatic and the sudden quench limit.
- $\epsilon_{kt} = \Delta_k(|u_k(t)|^2 - |v_k(t)|^2)/(2|u_k(t)||v_k(t)|)$
  $\Delta_{kt} = \Delta_k \exp(i(\alpha_{kt} - \beta_{kt}))$ where
  $\alpha_{kt}(\beta_{kt}) = \mathrm{Arg}[u_k(t)(v_k(t))]$
- In real space, $\mathcal{H}_t = \sum_{i,j}(A_{i-j}c_i^\dagger c_j + B_{i-j}c_ic_j + h.c.)$

Behaviour of $\mathcal{H}_t$

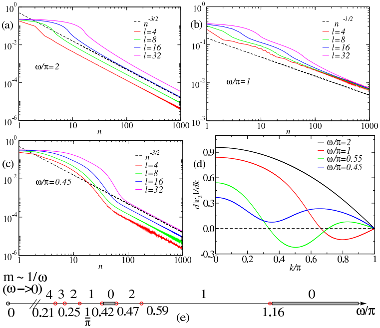


- Magnitude of the hopping elements A_{i-j} show a $\exp(-r/R_t)$ decay that indicates a short-ranged $\mathcal{H}_t$.
- However, R_t increases rapidly with number of periods n.

Floquet Hamiltonian

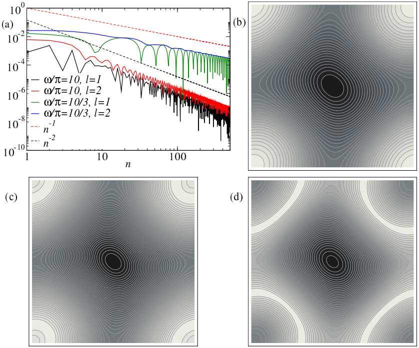
- For stroboscopic measurements at the end of n drive cycles, system described by Floquet Hamiltonian $\rightarrow U_{\vec{k}} = e^{-iH_{\vec{k}F}T}$
- For these integrable systems, $H_{\vec{k}F} = \vec{\sigma} \cdot \vec{\epsilon}_{\vec{k}}$. Thus $U_{\vec{k}} = e^{-i(\vec{\sigma}\cdot\vec{n}_{\vec{k}})\phi_{\vec{k}}}$ with $\vec{n}_{\vec{k}} = \frac{\vec{\epsilon}_{\vec{k}}}{|\vec{\epsilon}_{\vec{k}}|}$ and $\phi_{\vec{k}} = T|\vec{\epsilon}_{\vec{k}}|$
- $$\langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_n = \langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_\infty - \frac{1}{(2\pi)^d} \int_{\vec{k}\in BZ/2} d^d k \cos(\vec{k}\cdot(\vec{i}-\vec{j}))$$
$$\times (1 - \hat{n}^2_{\vec{k}3}) \cos(2n\phi_{\vec{k}})$$
$$\langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_n = \langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_\infty + \frac{1}{(2\pi)^d} \int_{\vec{k}\in BZ/2} d^d k \sin(\vec{k}\cdot(\vec{i}-\vec{j}))$$
$$\times \left[\hat{n}_{\vec{k}3}(\hat{n}_{\vec{k}1} + i\hat{n}_{\vec{k}2}) \cos(2n\phi_{\vec{k}}) + i(\hat{n}_{\vec{k}1} + i\hat{n}_{\vec{k}2}) \sin(2n\phi_{\vec{k}}) \right]$$
- Stationary points of $\phi_{\vec{k}}$ control the late time relaxations.

1D Ising model



- $H_{\vec{k}} = (g(t) - b_{\vec{k}})\tau_3 + \Delta_{\vec{k}}\tau_1$. Here $g(t) = h(t), b_k = \cos(k), \Delta_k = \sin(k)$
- Distance measure $\mathcal{D} = \mathrm{Tr}[(\mathcal{C}_\infty(l) - \mathcal{C}_n(l))^\dagger(\mathcal{C}_\infty(l) - \mathcal{C}_n(l))]^{1/2}/(2l)$
- For $\omega \gg 1$, $H_F \sim (1/T)\int_0^T H(t)dt$. Stationary points at $k = 0, \pi$ only.
- New stationary point emerges at $\sim 1.16\pi$ for this protocol.

2D Kitaev Model

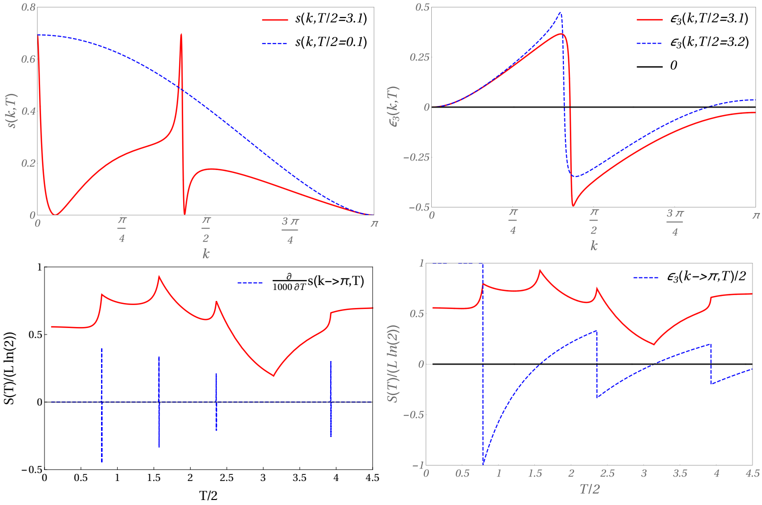


- $H_{\vec{k}} = (g(t) - b_{\vec{k}})\tau_3 + \Delta_{\vec{k}}\tau_1$. Here $g(t) = J_3(t), b_{\vec{k}} = J_1\cos(k_x) + J_2\cos(k_y), \Delta_{\vec{k}} = J_1\sin(k_x) + J_2\sin(k_y)$
- Special symmetry of the Kitaev model $H_{\vec{k}} = h[g_p(k_x) + \alpha_p g_p(k_y); \beta(t)]$ where $g_1 = \cos(k_i)$, $g_2 = \sin(k_i)$, $\beta(t) = J_3(t)/J_1$ and $\alpha_1 = \alpha_2 = J_2/J_1$.
- Implies that if $\partial|\epsilon_{\vec{k}}|/\partial k_x = 0$ then $\partial|\epsilon_{\vec{k}}|/\partial k_y = 0$

Properties of the Diagonal ensemble

- Use
  $\langle\psi(nT)|O_k|\psi(nT)\rangle = p_k\langle 1_k|O_k|1_k\rangle + (1-p_k)\langle 2_k|O_k|2_k\rangle$
  where $p_k = |\langle 1_k|\psi_k(t=0)\rangle|^2$ (thus cross-terms are dropped).
- Total entropy per site of the DE
  $\frac{S_{tot}}{L} = \frac{1}{\pi}\int_0^\pi S(k)dk$
  where $S(k) = -p(k)\log(p(k)) - (1-p(k))\log(1-p(k))$
- $S(k)$ maximized for $p(k) = 1/2$.
- S_{tot}/L shows a rich behaviour as a function of ω.
- Can be connected to special values ω_c where the number of zeroes of $\vec{\epsilon}_{\vec{k}3}$ in $k \in [0,\pi]$ changes by one.

Diagonal ensemble (continued)



Conclusions and future directions

- Two dynamical regimes for relaxation of correlation functions in periodically driven many-body systems
- These regimes separated by a dynamical transition
- Non-monotonic behaviour of the entanglement entropy of the steady state as a function of ω (related to change in number of zeroes of $\epsilon_3(k)$)
- Generalizations to models with disorder, and to other kinds of integrable models??
- Non-integrable (generic) models??